RAIRO-Oper. Res. 55 (2021) S1551–S1583
https://doi.org/10.1051/ro/2020040

RAIRO Operations Research
www.rairo-ro.org

IMPACT OF AN INPUT-OUTPUT SPECIFICATION ON EFFICIENCY SCORES IN DATA ENVELOPMENT ANALYSIS: A BANKING CASE STUDY

Martin Boďa[2,*] and Zuzana Piklová[1]

Abstract. The paper stresses the importance of making an appropriate specification of inputs and outputs in technical efficiency measurement and provides empirical evidence that this initial step of an efficiency measurement project should not be underestimated. Oriented on a case study of Slovak commercial banks for the period from 2005 to 2016, the paper explores to what extent different input-output specifications affect the comparability or congruence of technical efficiency scores in a banking application produced by four different data envelopment models differing in the efficiency measure and orientation. Building on the long-standing controversy in the banking literature about the most appropriate description of banking production, the paper compares technical efficiency scores for 9 input-output specifications of the intermediation approach, 9 specifications of the production-like approaches and 3 network integrated specifications. All these specifications were empirically applied earlier in the literature. The efficiency scores produced by different input-output specifications and models are confronted by six measures of association or dependence, and their levels are explained in a regression framework. The results attest that the choice of the input-output set is a critical judgemental input to efficiency measurement since there is vast diversity in efficiency scores of input-output sets coming from different approaches but also for input-output sets associated with the same approach. In addition, intermediation input-output specifications tend to produce higher efficiency scores than production-like specifications.

Mathematics Subject Classification. 90C08, 90B30, 90B90.

Received March 27, 2019. Accepted April 18, 2020.

1. Introduction

No matter whether accomplished by means of data envelopment analysis (DEA) or by a different methodology, every task of correct efficiency measurement requires that the right model of production is identified and that the aptest input-output specification is made. In some cases this may be a challenge, because there are uncertainties about the status of one or more production variables, or because various researches, if being in general agreement, might come up with different specifications of input and output variables. Examples when the former case happens is with the role of research income in the process of university research (see *e.g.* [2], pp. 4100, 4101), the role of Ph.D. students in the education process of universities (*e.g.* [26], p. 2), the role of mains length in the process of water distribution (see *e.g.* [31], p. 198) or the role of deposits in bank production (see *e.g.* [9],

Keywords. Data envelopment analysis, input-output set, efficiency score, banking, dependence, repeated measures analysis.

[1] Matej Bel University in Banská Bystrica, Faculty of Economics, Tajovského 10, 975 90 Banská Bystrica, Slovakia.

[2] Jan Evangelista Purkyně University in Ústí nad Labem, Faculty of Natural Sciences, České mládeže 8, 400 96 Ústí nad Labem, Czech Republic.

[*]Corresponding author: martin.boda@outlook.com

Article published by EDP Sciences

p. 198). As to the latter case, in many areas where efficiency measurement based on DEA is applied, there are a variety of choices concerning the input-output set, possibly made with an effort to tailor the efficiency analysis to particular conditions or sprouting from the analyst's personal experience, an understanding of the production process studied or issues such as data accessibility. Examples are specifications of input-output sets varying in detailedness and size in efficiency assessments of primary and secondary schooling (*e.g.* [18], Tab. 1), universities (*e.g.* [77], Tab. 2), police forces (*e.g.* [42], pp. 34–36), water distribution services (*e.g.* [31], Tab. 2), rail freight transport services (*e.g.* [66], Tab. 2), health-care providers (*e.g.* [25]), agricultural sectors of different countries (*e.g.* [59], Tab. 1) or banks (*e.g.* [4], Tab. 2; [33], Tab. 2). In no respect is the intent of this paper to criticise any particular specification of inputs and outputs or question its credibility; yet, the fact remains that every particular specification may have a bearing on the results of the analysis and that different input-output specifications may lead to different efficiency scores and findings. The potential variety of results is further amplified by other modelling choices (associated chiefly with the efficiency measure and orientation of the DEA model). Surprisingly, it appears that it is not standard to simultaneously investigate various specifications and evaluate their impact on the results as a robustness check, at least not in an exhaustive way.

Having these considerations in mind and specialized to commercial banks, the aim of the paper is to explore and demonstrate to what extent different specifications of the input-output set affect the results of efficiency measurement in a banking case study focused upon commercial banks. The pursuit of this question is of notable import for there is an intense debate in the literature about the true nature and content of the production process of commercial banks (*e.g.* [1], pp. 4, 5). The different interpretations are an apple of discord and give rise to a wealth of diverse input-output specifications causing incomparability of results between studies. The long-standing main concern in the banking literature is about the dual role of deposits. The traditional approaches posit deposits either on the input side (the intermediation approach) or the output side (the production and other associated approaches), whereas a newer approach is to treat the production process in commercial banks as a two-stage process in which deposits act as a link between the initial production stage and the terminal intermediation stage. Making a reference to the input-output specifications encountered in the DEA literature, the paper compares nine intermediation and nine production-like ("black-box") input-output specifications and confronts them with three integrated ("two-stage network") input-output specifications. For simplicity, all these specifications abstain from a possible presence of undesirable outputs of banking production (*viz.* non-performing loans). In the comparison, a total of four DEA models are employed at variable returns to scale (VRS) with each input-output specification to control for basic configurations in DEA efficiency measurement concerning the definition of the efficiency measure and orientation. The four DEA models considered arise in a matrix format as they encompass both radial (Debreu-Farrell) and non-radial (Pareto-Koopmans slacks-based) efficiency measures contra two orientations (*i.e.* input, and output). Using annual data for Slovak commercial banks pooled for the period from 2005 to 2016 and with restriction to technical efficiency measurement, the conducted analysis sheds some light on practical issues embodied in the following question: With what gravity may the choice of an input-output specification affect the results of an efficiency assessment? In the context of the present analytical set-up this question can be further specialized: What effect does preference for a particular approach have upon comparability of the results? The question is next protracted to the issue of practical usefulness of the results: What is the lesson for input-output selection in real applications?

There, of course, are several choices to be made in a project of efficiency measurement that are aptly summarized and highlighted *e.g.* by Dyson *et al.* [34], or by Emrouznejad and de Witte [35], and only one of them is given scrutiny in this paper with a focus on banking. The paper strives to provide an insight into the impact of crucial judgmental choices required of the analyst to be made at the very beginning of a DEA-based performance assessment of commercial banks, but it is not concerned with choosing the most appropriate or descriptive input-output set. Toward this end, there are a good many valid approaches of both a statistical and non-statistical nature that have been developed to aid the process of variable selection in DEA. Notwithstanding, the paper compares and examines the effect of different plausible input-output specifications adopted in applications of DEA centred on commercial banks, and seeks to formulate useful guidance for the analyst in practice.

The findings are not surprising as they evince clearly that the choice of an input-output set is a pivotal step in banking efficiency measurement. Although anticipated and consistent with conventional wisdom, these findings are clearly needed since they provide evidence necessary to contradict the prevalent practice of DEA researchers to take inputs and outputs for granted without inspecting the validity of their specification, which is raised as an issue for banking applications *e.g.* by Ouenniche *et al.* ([70], p. 154). The findings are not abated in relevance by centring upon a particular case study and a particular data set and highlight that the choice of inputs and outputs should not be underestimated chiefly in situations where there are uncertainties about the role of one or more production variables.

The remainder of the paper is organized into four more sections. With a focus upon efficiency assessment of commercial banks, Section 2 highlights motivation and explains the difficulties with choosing an input-output set for commercial banks, and then elaborates a short literature review. Section 3 gives technical details on the four DEA models employed in the comparison. After Section 4 describes the data set, it reports the results of the comparison and enquires into the patterns with which efficiency measurement results vary under different analytical choices. Finally, Section 5 summarizes the findings and draws conclusions alongside practical implications.

2. Motivation and literature review

The correct identification of the input-output set is highlighted in textbooks or introductory texts as the backbone or even "the most important stage" of a DEA assessment as is acknowledged *e.g.* by Thanassoulis ([86], p. 89). That being said, Cook *et al.* [26] point out that some researchers tend to skip the stage of acquainting themselves thoroughly with the nature and singularities of the production process, and are inclined to adopt an arbitrary choice of inputs or outputs, although model (or rather input-output) misspecification is an issue known also in DEA (*e.g.* [41, 65, 82]). This impression of arbitrary input-output choices also arises whilst studying input-output sets that are frequently applied in DEA assessment focused upon commercial banks (*e.g.* [4], Tab. 2; [33], Tab. 2; [67], Tab. 1). Nonetheless, this diversity in input-output sets applied to commercial banks emanates in part from the fact that for banking there exist four main theoretical concepts that rationalize banking behaviour and explain the very essence of banking production by noticing different features of banking operations. Ahn and Le ([1], pp. 9–16) provide an excellent description and summary of merits and shortcomings for the intermediation approach, the production approach, the user-cost approach and the value added approach that are applied in modelling bank behaviour. Refraining from the philosophical construals of these approaches, an important distinction arises in respected of the role attributed to deposits which are then (possibly broken down into a few subcategories) interpreted as an input or output of banking production. Whenever deposits appear on the input side, commercial banks are viewed as intermediation links between surplus-fund units (depositors) and deficit-fund units (debtors), and deposits are an input in production of loans (the intermediation approach and in most cases also the user-cost approach). Conversely, placing deposits on the output side entails that commercial banks are viewed predominantly as production facilities of banking services whose function is to provide deposits and loans to customers as their outputs. Albeit the classification of these approaches may be, and in theory actually is, more delicate, the treatment of deposits as an input is associated in the paper only with the "intermediation approach" and the opposite case when the deposits are taken for an output may be designated as the "production or quasi-production approach" (yet, in headings of tables and graphs for convenience referred to only as the "production approach"). The prefix "quasi" warns that specification of deposits as an output need not done truly and conceptually under the production approach. The selective overview of input-output sets employed in banking efficiency research reported in Appendix A gives a testimony on the inclination of researchers to apply different perspectives on banking production. It must be admitted that the presented specifications avoid undesirable outputs of banking production (typically non-performing loans). The reason being, since there are various methods of handling undesirable outputs, their consideration in an input-output set would inject inevitably another (here peripheral) factor into the comparison. In order to simplify the comparison, undesirable banking outputs are overlooked in the comparison (as is done in many studies).

Nine of the input-output specifications answer to the intermediation approach (adopted by 11 DEA studies) and nine comply with the production or quasi-production approach (adopted by 9 DEA studies). Needless to say, the factors at play behind these selections are not only the analyst's discretion, judgement and understanding of banking enterprise, but there are issues such as data unavailability, data quality and an effort to include all relevant variables and omit those redundant. In the context of banking efficiency assessment, the effect of adopting an approach or choosing an input-output set within an approach upon efficiency scores and results has not been consistently explored. Several studies entertained differing specifications of inputs and outputs usually as a robustness check without carrying out a more systematic sensitivity analysis. For example, Kenjegalieva *et al.* [58] as well as Boďa and Zimková [16] performed their analysis using three different specifications, one specification per a different approach. Two different specifications were simultaneously considered, inter alia, by Avkiran [3], Jemric and Vujcic [53], Tortosa-Ausina [91], Sathye [79], Drake *et al.* [32], Kouki and Al-Nasser [60] and Boďa and Zimková [17]. Several few examples in the literature can be readily produced. All these cited studies, the papers organized in Appendix A as well as the overwhelming majority of banking studies in general favour input-output specifications that model banking business as a one-stage process in the style of a black-box. Nevertheless, it is now recognized that it is more appropriate, although not completely flawless, to interpret banking operation in a two-stage framework (see [1], pp. 24, 25), in which commercial banks produce deposits first (Stage 1 answering to the production function) and only then transforms them into loans or other earning assets (Stage 2 answering to the intermediation function). The overview in Appendix B lists 3 two-stage network input-output specifications that stipulate that provision of banking services foregoes financial intermediation. Other examples where a very similar approach has found application are Fukuyama and Weber [39], Fukuyama and Matousek [38] and Wang *et al.* [95], but these studies consider bad loans as an undesirable output of banking production. There are also cases when a completely different internal structure is postulated, *e.g.* as in Gulati and Kumar [45], or where the network framework is extended toward a dynamic structure [40]. Because the network approach houses both partial "black-box" treatments of deposits in an aggregative network fashion, it is labelled here as the "integrated network approach".

The paper does not explore contexts in which it is necessary to make a decision regarding the most descriptive specification of specification of inputs and outputs, to which end there are a number of approaches available operating in some objectivized fashion. These approaches were devised either to tackle the restrictive requirement that the input-output set must be known *a priori* and identified on the basis of discretion, judgment and expertise or to find a tenable trade-off between the number of units being assessed and the number of production variables so that the computed efficiency scores preserve discriminatory power. The former strand of approaches is concerned with model (or, more precisely, input-output) specification, whilst the interest of the latter strand is variable reduction. To mention a few of the former strand, there are stepwise approaches based on the principle of forward selection or backward elimination of variables (*e.g.* [72, 94]), on the study of correlations between variables [29, 52], on principal component analysis [93], or on regression analysis [78]. Another possibility is to search exhaustively and or selectively through the space of all possible input-output specification and decide on the basis of Akaike's information criterion accounting for possible stochasticity in data [62]. The issue of finding a balance between the number of units and the number of variables is addressed *e.g.* by Toloo *et al.* [87] who propose mixed linear integer programming models to select production variables.

It turns out that investigations of a character similar to the present study are extremely scarce. As a matter of fact, aside from the studies cited earlier that espoused several concurrent input-output specifications and compared the results of two or three variants, any other research that would deal with the effect of choosing from amongst a multitude of sound input-output specifications in a comprehensive way is unbeknownst to the authors.

3. Technicalities of the DEA models applied in the comparison

All the models for efficiency measurement employed in the paper and presented below are considered in envelopment form. There are several reasons for this choice. First, the presentation is then in a unified format and demonstrates clearly direct and interpretable links amongst radial DEA models, on the one hand, and between one-stage and two-stage network DEA models, on the other. Second, technical efficiency measurement undertaken in envelopment space tallies with the frontier concepts and apparatus of production economics. Third, although in the case of two-stage models there is some disagreement between DEA models formulated in multiplier space and DEA models constructed in envelopment space [24], envelopment DEA models are more adaptable and versatile across different configurations of returns to scale. That being said, only the case of VRS is given an interest here.

The first three parts of this section first present "black-box" one-stage and two-stage network DEA whose efficiency scores are juxtaposed and subjected to analysis in the paper, and then short comments are appended on handling non-positive data present in the data set to be used. The last part of this section makes a brief observation on making raw efficiency scores resulting from different DEA models comparable.

3.1. "Black-box" one-stage DEA models

It is supposed that there are N production units that transmute P identified inputs into R identified outputs. The production activity of any production unit o, $o \in \{1, \ldots, N\}$, is described by input vector $\mathbf{x}_o = (x_{o1}, \ldots, x_{oP})'$ and output vector $\mathbf{y}_o = (y_{o1}, \ldots, y_{oR})'$. All these production vectors are required positive. The disagreement between an actually observed production activity and its efficient projection assuming a position on the estimated efficient frontier gives rise to slacks that are presented by their respective vectors. The vectors of input and output slacks for production unit o are written as $\mathbf{s}_o^{\mathbf{x}} = (s_{o1}^{\mathbf{x}}, \ldots, s_{oP}^{\mathbf{x}})'$ and $\mathbf{s}_o^{\mathbf{y}} = (s_{o1}^{\mathbf{y}}, \ldots, s_{oR}^{\mathbf{y}})'$, and, by the philosophy of their construction, are non-negative. In addition, a vector of zeros of an appropriate length is, in what follows, denoted by $\mathbf{0}$ and a similar vector of ones by $\mathbf{1}$.

The models are exposited in envelopment space spanned by observed production activities using intensity vectors $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_N)'$ that come to satisfy both $\lambda_1, \ldots, \lambda_N \geq 0$ and the sum restriction $\sum_i \lambda_i = 1$. The compliance with the indicated sum restriction is in tune with the adoption of VRS. For constant returns to scale (CRS), $\sum_i \lambda_i$ would be left free and the sum restriction would be dropped. Some other useful details on the one-stage DEA models described here can be found in Ray ([74], pp. 56–59, 82–88, 124–126) and in Tone [88].

3.1.1. Radial models

The input-oriented radial measure of efficiency θ_o is obtained for unit o by solving the linear program

$$\min \ \theta_o \tag{3.1}$$

subject to

$$\sum_{i=1}^{i=N} \lambda_i \mathbf{x}_i \leq \theta_o \mathbf{x}_o, \qquad \sum_{i=1}^{i=N} \lambda_i \mathbf{y}_i \geq \mathbf{y}_o, \tag{3.1a}$$

$$\boldsymbol{\lambda} \geq \mathbf{0}, \qquad \mathbf{1}'\lambda = 1; \tag{3.1b}$$

and the linear model that yields the output-oriented analogue η_o for unit o then reads

$$\max \ \eta_o \tag{3.2}$$

subject to

$$\sum_{i=1}^{i=N} \lambda_i \mathbf{x}_i \leq \mathbf{x}_o, \qquad \sum_{i=1}^{i=N} \lambda_i \mathbf{y}_i \geq \eta_o \mathbf{y}_o, \tag{3.2a}$$

$$\boldsymbol{\lambda} \geq \mathbf{0}, \qquad \mathbf{1}'\lambda = 1. \tag{3.2b}$$

Constraints (a) in both programs (3.1) and (3.2) represent the traditional envelopment conditions for the input and output variables. A property essential to practical efficiency measurement is the fact that $\theta_o, \eta_o^{-1} \in (0, 1]$, where the value of unity is attained only at full technical efficiency.

3.1.2. Slacks-based models

The non-radial measurement of efficiency is accomplished here *via* the slacks-based measure (SBM) which was considered in DEA literature on several occasions but exhaustively accommodated in a linear programming framework first by Tone [88]. As such, the measure is defined as a solution to a linear program if an input or output orientation is favoured. The input-oriented of the SBM π_o is obtained optimizing the objective function

$$\min\ \pi_o = 1 - \frac{1}{P} \sum_{p=1}^{p=P} s^{\mathbf{x}}_{op}/x_{op} \tag{3.3}$$

subject to the set of conditions

$$\mathbf{x}_o = \sum_{i=1}^{i=N} \lambda_i \mathbf{x}_i + \mathbf{s}^{\mathbf{x}}_o, \qquad \mathbf{y}_o = \sum_{i=1}^{i=N} \lambda_i \mathbf{y}_i - \mathbf{s}^{\mathbf{y}}_o, \tag{3.3a}$$

$$\mathbf{s}^{\mathbf{x}}_o \geq \mathbf{0}, \qquad \mathbf{s}^{\mathbf{y}}_o \geq \mathbf{0}, \qquad \boldsymbol{\lambda} \geq \mathbf{0}, \qquad \mathbf{1}'\lambda = 1, \tag{3.3b}$$

whereas the output oriented variant ω_o follows from the optimization

$$\max\ \omega_o = 1 + \frac{1}{R} \sum_{r=1}^{r=R} s^{\mathbf{y}}_{or}/y_{or} \tag{3.4}$$

applied with respect to the same set of conditions (3.3a) and (3.3b). These oriented slacks-based measures are restricted in values, *i.e.* $\pi_o, \omega_o^{-1} \in (0, 1]$, where the value of unity points to full technical efficiency.

3.2. Two-stage network DEA models

In recognition of subtleties and delicacies of production, it is assumed in a two-stage network that the production process can naturally be broken down into two serial connected stages. As before, there are N production units, but their production is divided into Stages 1 and 2. Stage 1 transmutes P_1 identified inputs into R_1 identified outputs that are unlinked to Stage 2. Stage 2 transforms P_2 identified inputs into R_2 identified outputs that are unlinked to Stage 1. Both stages are connected through L_{12} intermediate products (links) that represent extra outputs of Stage 1 (different from the R_1 unlinked outputs) and extra inputs to Stage 2 (distinct from the P_2 unlinked inputs). The network situation in question is portrayed by the schema in Figure 1. The simplification to merely two stages is without any apparent loss of generality, because the approach can be generalized freely to multiple connected stages.

The input and output vectors of Stage k (whereas $k \in \{1, 2\}$) of each production unit o, $o \in \{1, \ldots, N\}$, are written as $\mathbf{x}^k_o = (x^k_{o1}, \ldots, x^k_{oP_k})'$ and $\mathbf{y}^k_o = (y^k_{o1}, \ldots, y^k_{oR_k})'$. In this notation the right-hand superscript distinguishes between Stages 1 and 2. The linking products from Stage 1 to Stage 2 are for production unit o organized into the vector $\mathbf{z}^{12}_o = (z^{12}_{o1}, \ldots, z^{12}_{oL_{12}})'$. All production variables are assumed positive, and the relative importance given to Stages 1 and 2 is represented by positive weights w^1 and w^2 such that $w^1 + w^2 = 1$. Also slacks vectors are differentiated as to whether they arise in Stage 1 or 2. For production unit o, vectors $\mathbf{s}^{\mathbf{x}\cdot k}_o = (s^{\mathbf{x}\cdot k}_{o1}, \ldots, s^{\mathbf{x}\cdot k}_{oP_k})'$ and $\mathbf{s}^{\mathbf{y}\cdot k}_o = (s^{\mathbf{y}\cdot k}_{o1}, \ldots, s^{\mathbf{y}\cdot k}_{oR_k})'$ describe input and output slacks of Stage k ($k \in \{1, 2\}$). All of these slacks vectors are non-negative.

The attainable production activities are identified in envelopment space by means of intensity vectors $\boldsymbol{\lambda}^k = (\lambda^k_1, \ldots, \lambda^k_N)'$ different for each Stage k ($k \in \{1, 2\}$). In addition to the non-negativity condition and the sum constraint required separately for each stage under VRS, the network models considered here are implemented with fixed links. Under this treatment, the linking activities are kept unchanged (as if being non-discretionary) that is represented by a set of conditions imposed on the links in the form

$$\mathbf{z}^{12}_o = \sum_{i=1}^{i=N} \lambda^k_i \mathbf{z}^{12}_i, \qquad (k \in \{1, 2\}), \tag{3.5}$$

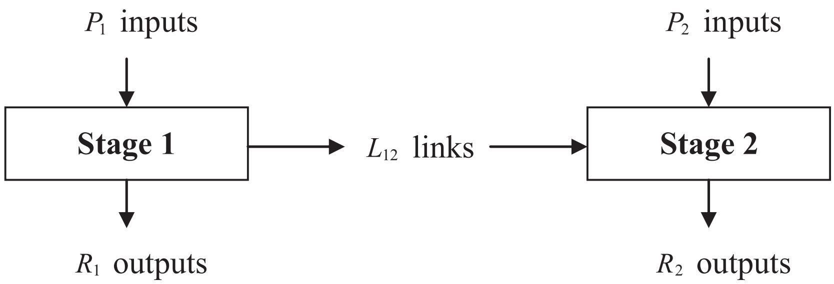

FIGURE 1. Simple two-stage network structure considered.

which warrants that the linking products are matched between Stages 1 and 2 and are not affected through the optimization procedure of efficiency measurement.

The slacks-based network DEA models in envelopment space were developed by Tone and Tsutsui [89,90] and derived from the concept of production possibility sets. Some concerns about this approach were raised by Chen *et al.* [24] who demonstrated that multiplier and envelopment DEA models are footed on different approaches to efficiency measurement and are different concepts. Discrepancies arise especially with stage-specific efficiencies. The oriented network models below come from generalizations of radial efficiency measurement in envelopment space for the given two-stage network structure.

3.2.1. Radial models

The overall input-oriented radial measure of efficiency θ_o^{12} for unit o is defined as a weighted average of stage-specific input-oriented radial measures, θ_o^1 and θ_o^2, as yielded by the linear program

$$\min\ \theta_o^{12} = w^1\theta_o^1 + w^2\theta_o^2 \tag{3.6}$$

subject to

$$\sum_{i=1}^{i=N} \lambda_i^k \mathbf{x}_i^k \leq \theta_o^k \mathbf{x}_o^k, \qquad \sum_{i=1}^{i=N} \lambda_i^k \mathbf{y}_i^k \geq \mathbf{y}_o^k \qquad (k \in \{1,2\}), \tag{3.6a}$$

$$\sum_{i=1}^{i=N} \lambda_i^k \mathbf{z}_i^{12} = \mathbf{z}_o^{12} \qquad (k \in \{1,2\}), \tag{3.6b}$$

$$\boldsymbol{\lambda} \geq \mathbf{0}, \qquad \mathbf{1}'\lambda = 1 \qquad (k \in \{1,2\}). \tag{3.6c}$$

In the same vein, the overall output-oriented radial measure of efficiency η_o^{12} for unit o is defined as a weighted average of stage-specific output-oriented radial measures, η_o^1 and η_o^2, and is the result of the linear program

$$\max\ \eta_o^{12} = w^1\eta_o^1 + w^2\eta_o^2 \tag{3.7}$$

subject to

$$\sum_{i=1}^{i=N} \lambda_i^k \mathbf{x}_i^k \leq \mathbf{x}_o^k, \qquad \sum_{i=1}^{i=N} \lambda_i^k \mathbf{y}_i^k \geq \eta_o^k \mathbf{y}_o^k \qquad (k \in \{1,2\}), \tag{3.7a}$$

$$\sum_{i=1}^{i=N} \lambda_i^k \mathbf{z}_i^{12} = \mathbf{z}_o^{12} \qquad (k \in \{1,2\}), \tag{3.7b}$$

$$\boldsymbol{\lambda} \geq \mathbf{0}, \qquad \mathbf{1}'\lambda = 1 \qquad (k \in \{1,2\}). \tag{3.7c}$$

Understandably, the difference between programs (3.6) and (3.7) rests in both their objective functions and constraints (a). In either program, the optimizations that are undertaken simultaneously for Stages 1 and 2 and that are regulated by constraints (a) and (c) would be separate but for the required match in the links implemented by constraint (b). The domain of stage-specific efficiencies $\theta_o^1, \theta_o^2 \in (0, 1]$ implies that also $\theta_o^{12} = w^1\theta_o^1 + w^2\theta_o^2 \in (0, 1]$. Similarly, $\eta_o^1, \eta_o^2 \in [1, \infty)$ signifies that $(\eta_o^{12})^{-1} = (w^1\eta_o^1 + w^2\eta_o^2)^{-1} \in (0, 1]$.

3.2.2. Slacks-based models

In preparation for the formulation of two-stage slacks-based DEA models in line with Tone and Tsutsui [89, 90], introduce partial input and output efficiencies π_o^k and ω_o^k associated with any Stage k ($k \in \{1, 2\}$) by expressions

$$\pi_o^k = 1 - \frac{1}{P_k}\sum_{p=1}^{p=P_k} s_{op}^{\mathbf{x}\cdot k}/x_{op}^k \qquad \omega_o^k = 1 + \frac{1}{R_k}\sum_{r=1}^{r=R_k} s_{or}^{\mathbf{y}\cdot k}/y_{or}^k, \qquad (k \in \{1, 2\}). \tag{3.8}$$

Then, the input-oriented SBM efficiency π_o for production unit o with fixed links is the result of program

$$\min \ \pi_o = w^1\pi_o^1 + w^2\pi_o^2 \tag{3.9}$$

subject to

$$\mathbf{x}_o^k = \sum_{i=1}^{i=N} \lambda_i^k \mathbf{x}_i^k - \mathbf{s}_o^{\mathbf{x}\cdot k}, \qquad \mathbf{y}_o^k = \sum_{i=1}^{i=N} \lambda_i^k \mathbf{y}_i^k + \mathbf{s}_o^{\mathbf{y}\cdot k} \qquad (k \in \{1, 2\}), \tag{3.9a}$$

$$\sum_{i=1}^{i=N} \lambda_i^k \mathbf{z}_i^{12} = \mathbf{z}_o^{12} \qquad (k \in \{1, 2\}), \tag{3.9b}$$

$$\mathbf{s}_o^{\mathbf{x}\cdot k} \geq \mathbf{0}, \qquad \mathbf{s}_o^{\mathbf{y}\cdot k} \geq \mathbf{0}, \qquad \boldsymbol{\lambda}^k \geq \mathbf{0}, \qquad \mathbf{1}'\boldsymbol{\lambda}^k = 1 \qquad (k \in \{1, 2\}). \tag{3.9c}$$

In order to obtain the output-oriented SBM efficiency ω_o under this set-up, it is only necessary to replace the objective function in (3.9) by

$$\max \ \omega_o = w^1\omega_o^1 + w^2\omega_o^2, \tag{3.10}$$

whilst keeping the conditions (a)–(c) intact. By the definition of partial stage-specific efficiencies and their overall network aggregates, it becomes obvious that $\pi_o, \omega_o^{-1} \in (0, 1]$. It is but convenient that both oriented variants of network SBM efficiencies are ipso facto constructed from linear programs.

3.3. Handling non-positive data or absent variables

The previous exposition requires that all production data are positive and this desideratum may be relaxed for radial DEA models (no matter whether of a one-stage or two-stage structure) if there are occasional zeros. Nonetheless, this solution is not possible for slacks-based efficiency measurement. If there are occasional zeros or negative data (for some reason) whose occasionality means that they plague the entire vector of inputs, outputs or links of a production unit, then these remedies may be systemized:

– Negative values in radial DEA models may be replaced by zeros for the unit being assessed.
– Non-positive values in one-stage SBM models may be processed for the unit being assessed by one of the approaches advocated by Tone ([88], pp. 506, 507). For both negative and zero values for the unit being assessed in the input or output vector may be overlooked and the corresponding slacks may be neglected. This means an adjustment in the definition of the SBM, from which the respective slacks term simply disappears. However, in contrast to the recommendation of Tone [88], here the denominators $\frac{1}{P}$ and $\frac{1}{R}$ in the SBM are not updated and the respective slack is simply put to zero.
– Non-positive values in two-stage network SBM models may be dealt with by means of the approach described by Tone and Tsutsui ([89], pp. 23, 24) in their discussion paper.

Each of these approaches only modifies the production data of the unit being assessed and only at the time of being assessed. In estimating the production possibility set, production data remain as they are observed and are untouched.

Another complication emerges in the situation when in a network structure the output part of Stage 1 and/or the input part of Stage 2 are not present. Stages 1 and 2 are interconnected by intermediate products, but as such either Stage 1 does not produce outputs that leave the network or Stage 2 does not consume inputs that enter the network. It is not impossible that both such singularities happen at a time. In such a case, the entire side of production of the stage in question is modelled by an artificial fixed input or output consisting of zeros (in the radial two-stage network case) or ones (in the SBM network case). This addendum does not affect the efficiency measurement in any harmful way and makes the framework described above workable in more general settings. In radial network models, a fixed zero output added to Stage 2 and/or a fixed zero input added to Stage 1 have no effect upon radial contractions or expansions; whereas in SBM network models, a fixed unit output appended to Stage 2 and/or a fixed unit input appended to Stage 2 under VRS mean that the associated slacks are zero and their influence in efficiency measurement is fully eliminated.

3.4. Assuring higher comparability of efficiency scores yielded by different DEA models

Comparability of efficiency scores produced by different DEA models is not warranted automatically, and some basic rules for comparing efficiency scores should be adhered to. Since efficiency scores in two-stage network models aggregate merely efficiency scores ascertained in partial two stages in an average-like manner, they inherit limitations to comparability from "black-box" one-stage models and the conventions ensuring their comparability pass onto them directly from these "black-box" one-stage models. In consequence, it suffices to comment on the issue of comparability of efficiency scores in "black-box" one-stage models.

The distinction between efficiency scores in oriented radial and slacks-based DEA models is that whereas the former measure the average proportional input contractions or output expansion to achieve technical efficiency, the latter measure the average non-proportional input contractions and output expansion. In spite of higher informational content of slacks-based efficiency scores, this logic of construction permits straightforward comparisons of θ with π and of η^{-1} with ω^{-1}. These rules are respected in the comparative analysis to come.

4. Data and results

The analysis began with application of the four DEA models detailed in the earlier sections to annual data on Slovak banking institutions respecting the $9 + 9 = 18$ "black-box" one-stage and 3 two-stage network input-output specifications enumerated in Appendices A and B. In accord with Appendices A and B, these specifications are further labelled as I1–I9, P1–P9 and N1–N3, respectively. This scheme of application led to $6 \times (18+3) = 126$ sets of technical efficiency scores that were further investigated with an emphasis laid upon the approach and a particular input-output specification. Hence, the analysis was oriented on assessment of differences or consonance of technical efficiency scores yielded by various approaches and input-output specifications for a given choice of the DEA model (*i.e.* its orientation and efficiency measure).

The analysis was conducted entirely in program R [73] with codes compiled *ad hoc* with the aid of packages `lpSolve` [11], `Benchmarking` [13], `Rcsdp` [19], `kdecopula` [68], `glmulti` [21], and `lme4` [7]. Program R was also used in preparing statistical reports and graphical presentations.

The data came from separate IFRS financial statements of individual Slovak banking institutions (both commercial banks per se and branch offices of foreign banks) compiled by TREND Analyses of News and Media Holding, a. s. (the former TREND Holding, s. r. o.). The data sample covered the period of 12 years from 2005 to 2016; yet, the nominal number of commercial banks across these years varied between 15 (in 2013) and 24 (in both 2007 and 2008). In total, the sample contained nominally 244 bank-year observations. The effective number of observations was affected by missingness or negativity of some data points, in consequence of which each input-output specification listed in Appendices A and B induced a differing set of banks that actually figured in the calculations. Another reason for fluctuations in bank numbers is that some small banks or branch offices of foreign banks entered and left the Slovak banking market. The samples for the input-output specification considered and listed in Appendices A and B were fairly representative of the Slovak banking sector as they encompassed more than 50% of the operations of the entire sector. Bank-year data for different years were pooled into one data set under the assumption of time-invariant production technology in Slovak banking. The premise of a single production frontier is barely objectionable given the fact that since the accession of Slovakia to the European Union the Slovak banking sector has been free of structural reforms and seen equally resilient and stable. The year-end balance sheet items stated in thousand € were properly deflated by the implicit price deflator published by Eurostat to assure their comparability.

A basic descriptive summary of the production variables appearing in the adopted input-output specifications is provided in Appendix C and a similar report for the calculated efficiency scores is displayed in Appendix D. As is declared in C, with almost each variable there were occasional zeros or absent data points, which curtailed the effective size of the data set for a particular input-output specification (which became more amplified when the variables were assembled for that given specification). OEA (other earning assets) is the variable with a conspicuously high frequency of absent data points or with zero values, which is a consequence of the fact that especially branch offices of foreign banks provide retail and corporate services only and to not report investments in securities. Three variables (EQ, NcFTI and TOOI) come to be reported for some bank-years with negative values, which is the consequence of their construction since they originate as subtractions of other variables. The number of calculated efficiency scores for each input-output specification is part of the report in Appendix D and ranged from 103 (for N2) to 239 (for I8). Naturally, the highest numbers of efficiency scores are available for the input-output specifications which encompass OEA as a production variable. Several observations can be compiled after having inspected the patterns in the efficiency scores summarized in Appendix D. Owing to pronounced sternness of radial DEA models, generally higher efficiency scores are naturally measured with radial models than with non-radial models, but orientation of a DEA model does not affect technical efficiency scores substantially. Input-output specifications of the production approach produce more heterogeneous and varied technical efficiency scores than input-output specifications of the intermediation approach. The more intense variability of efficiency scores under the production approach can barely be given a satisfactory explanation. Yet, after a thorough study of the composition and richness of input-output specifications (such as the number of production variables, the proportion of inputs relative to outputs) it seems that the key factor causing more heterogeneous efficiency scores is, NoE (number of employees); either alone or in combination with TFA (total fixed assets). The former is a proxy for labour force, whereas the latter for physical capital; and they both are traditional factors of production. Input-output specifications in which NoE or NoE with TFA are not present tend to have smaller standard deviations of efficiency scores than those in which they are represented. NoE appears with 5 production and 3 intermediation specifications, whereas both NoE and TFA are part of 4 production and 3 intermediation specifications. Apparently, they inject heterogeneity particularly into input-oriented models (RADI and SBMI).

Pearson or Spearman correlation represents the most common device to measuring association or agreement of various approaches in DEA efficiency studies as is testified by Berg *et al.* [8], Hunter and Timme [50], Berger and Humphrey [9], Resti [75], Mostafa [67] and Boďa and Zimková [16]. Though correlation may well capture the strength of linear agreement (Pearson correlation) or rank association (Spearman correlation), it is not a general dependence measure as it fails to describe well the dependence structure between random variables. This is a fact recognized in a comparison of different DEA efficiency scores by Tortosa-Ausina [91] who assessed the dependence structure between pairs of efficiency scores by inspecting a kernel estimate of their bivariate density. Nonetheless, her inspection was merely optical and was conducted by surveying kernel density and contour plots. A similar approach was applied later by Kenjegalieva *et al.* [58], but this study only compared in one graph kernel density estimates of the marginal distributions for different efficiency scores. In neither case, any comprehensive measure of dependence was provided. These observations underlie the motivation for using several measures of association or dependence. In addition to the Spearman correlation coefficients that are reported summarized in Table 1, also more general dependence measures are employed and their summary is provided in Appendix E. The reported measures arising from mutual comparisons of efficiency scores are reported for each DEA model separately and are intended to emphasize especially differences between the three approaches studied.

Irrespective of the DEA model chosen for benchmark, Table 1 testifies that when efficiency scores yielded by diverse input-output specifications are juxtaposed, mostly different impressions are drawn. This is indicated not only by the range of Spearman correlation coefficients in individual comparisons, but also by their mean values corresponding to moderate positive correlation. Apparently, the choice of production variables thus impacts upon the measured level of technical efficiency of commercial banks. In some cases efficiency scores of different input-output specifications are almost irreconcilable (minimum Spearman correlations close to 0) or they convey almost identical information on the ranking of banks (maximum Spearman correlations close to 1). The choice of input-output specification is found of importance also in the case of a single approach (the upper left part of Tab. 1), although the integrated network specifications led to more congruent technical efficiency scores than the intermediation and production specifications did. The higher integrity of the network input-output specifications is perhaps circumstantial as only 3 (fairly similar) network specifications were studied in comparison to 9 intermediation and production specifications. Notwithstanding, the difference between the intermediation and production approach emanating from their disparate economic philosophies is discernible in the means and standard deviations of pairwise Spearman correlation coefficients. The smallest values of these correlation coefficients suggest as though sometimes these two approaches were unrelated. Furthermore, a smaller degree of congruence in technical efficiency scores is observed between any of the "black-box" one-stage approaches and the network two-stage approach, but here the results are more varied according to the DEA model. In some cases it transpires that also the choice of the DEA model may be a factor to which extent technical efficiency scores retain congruence when input-output specifications of different approaches are considered. An apt example presented in Table 1 is the SBMO model when an input-output specification of the production approach and an input-output specification of the network approach may return efficiency scores that are negatively correlated (if weakly).

A similar picture of the congruence of efficiency scores arising from different input-output specifications is given by a quintet of other dependence measures whose detailed overview is given in Appendix E. The information goes in the style of Table 1 and is structured for combinations of approaches and for DEA models, but is also reinforced by identified pairs of input-output sets that yield minimum and maximum values of dependence measures. The five dependence measures considered are employed intensively in modelling dependence *via* copulas and their list covers Gini's rank association coefficient (Gini's γ), the medial correlation coefficient (Blomqvist's β), van der Waerden's coefficient, mutual information and Linfoot's correlation coefficient. In calculating these dependence measures, the bivariate density of a pair of efficiency scores was first estimated using a Gaussian product kernel density estimator with the mirror-reflection method of Gijbels and Mielniczuk [44]. The procedure made use of the fact that efficiency scores normalized to range [0,1] can be manipulated as pseudo-random observations. The bandwidth was selected automatically by minimizing the mean integrated square error using the Frank copula as the reference family (see [68]). These dependence measures were calculated from the estimated copula densities (all measures) using quasi Monte Carlo methods (all measures except the medial correlation coefficient). Gini's rank association coefficient, the medial correlation coefficient, van der Waerden's coefficient and Linfoot's correlation coefficient take values between -1 and $+1$ for inverse and positive association, respectively. Mutual information yields values from 0 (when there is no mutual information) to $+\infty$. The definition and details on these measures of dependence frequent in modelling with copulas are available with Nelsen ([69], p. 182), Joe ([55], pp. 57, 58), Genest and Verret [43], Joe [54] and Linfoot [64]. The overall picture offered by the five dependence measures in Appendix E is in tune with the general observations inferred from Table 1. Naturally, the particular values attained by these dependence measures are chiefly affected by the allowable range of values they can take. Save mutual information and up to some exceptions, these dependence measures tally in mean and standard deviation and roughly also in the smallest and largest attained values. It seems that when efficiency scores arising from different input-output specifications of the same approach are matched against themselves, it is usually very difficult to identify the combinations in which they are most dissimilar or congruent (the dependence measures are comparatively lowest or largest, respectively). Insofar as

TABLE 1. Descriptive statistics of Spearman correlation coefficients between the calculated efficiency scores.

DEA model	RADI	RADO	SBMI	SBMO	RADI	RADO	SBMI	SBMO
	Intermediation approach specifications against intermediation approach specifications*				Production approach specifications against production approach specifications*			
Minimum	0.4187	0.4204	0.3606	0.3538	0.2136	0.2208	0.1947	0.1854
Maximum	0.9439	0.9248	0.9524	0.9566	0.9496	0.9643	0.9419	0.9392
Mean	0.6770	0.6750	0.6703	0.6416	0.5913	0.6125	0.5452	0.5627
Standard deviation	0.1730	0.1579	0.1893	0.1745	0.2067	0.1953	0.2267	0.2196
	Integrated network approach specifications against integrated network approach specifications§				Intermediation approach specifications against production approach specifications†			
Minimum	0.7205	0.5062	0.7580	0.3125	0.0842	0.1658	0.0504	0.0894
Maximum	0.8166	0.7714	0.8269	0.7757	0.7364	0.7611	0.7775	0.7059
Mean	0.7678	0.6014	0.7970	0.5031	0.4571	0.4860	0.4510	0.4282
Standard deviation	0.0480	0.1476	0.0353	0.2422	0.1326	0.1342	0.1470	0.1544
	Intermediation approach specifications against integrated network approach specifications‡				Production approach specifications against integrated network approach specifications‡			
Minimum	0.1179	0.3655	0.0985	0.1309	0.2523	0.2124	0.2491	−0.0772
Maximum	0.7324	0.8833	0.7733	0.8228	0.8448	0.5921	0.8392	0.6232
Mean	0.4775	0.6314	0.4986	0.5630	0.6287	0.4082	0.5828	0.2837
Standard deviation	0.1484	0.1598	0.1546	0.1761	0.1718	0.1256	0.1739	0.2078

Notes. The Spearman correlation coefficients reported in this table are yielded by their traditional sample estimators. (*)Since for both the intermediation and production approach as many as 9 input-output sets were considered, these summaries of pairwise Spearman correlation coefficients pertain to all possible $9 \times 8/2 = 36$ pairs of input-output sets for the intermediation or production approach. (§)Since only three input-output sets were considered for the integrated network approach, these summaries are drawn only from all possible $3 \times 2/2 = 3$ pairwise Spearman correlation coefficients. (†)Since 9 input-output sets were considered for both the intermediation and production approach, these summaries are computed from pairwise Spearman correlation coefficients applied to all possible $9 \times 9 = 81$ combinations arising by matching the input-output sets of both approaches. (‡)Since 9 input-output sets were considered for both the intermediation and production approach and three input-output sets for the integrated network approach, these summaries describe pairwise Spearman correlation coefficients applied to all possible $9 \times 3 = 27$ combinations of input-output sets.

only three integrated network input-output specifications are accounted for, this difficulty does not emerge in that case. On the contrary, the most disparate specifications are N3/N2 and the most similar specifications N2/N1. A comparatively higher level of orderliness in this sense is ascertained for paired specifications of different approaches. When intermediation specifications are confronted against production specifications, the least similar efficiency scores are universally observed for I9/P8, whereas the most similar efficiency scores alternate for I7/P9 (especially for RADI, SBMI, RADO) and I6/P5 (especially for SBMO). When comparisons are done for intermediation and integrated network specifications, the smallest (or none) degree of congruence is found for I1/N1 (for RADI, SBMI), and I9/N2 (for RADO, SBMO). The most similar specifications are then I6/N2 (for the RADI, SBMI), and I1/N1 (for RADO, SMBO). Eventually, when juxtaposing production and integrated network specifications, the most similar specifications are detected to be P8/N3 (for RADI, SBMI) and P7/N2 (for RADO, SBMO), whereas the most congruent specifications are P5/N1 (for RADI, SBMI), and P1/N1 (for RADO, SBMO). Hence, it thus transpires that alongside the particular choice of production variables, both the efficiency measure and orientation of the model are factors of achieving altogether similar or different efficiency scores.

A graphical visualization of the information contained in Appendix E is further provided in Appendix F in the form of 3D scatter graphs. For the four DEA models considered, these 3D scatter graphs display the estimated joint densities of efficiency scores between the least similar and most similar input-output specifications of two distinct approaches. The pairs of the least congruent and most similar pairs of efficiency scores in question were identified by Spearman correlation coefficients that are additionally reported in the labels of individual scatter graphs. To which situation a scatter graph pertains is identified in its label either by "Max" or "Min", and the value in brackets is the estimated Spearman correlation coefficient. The shapes of the estimated joint densities shed light on the form of association between the extreme pairs of efficiency scores that emerge by matching input-output specifications of different approaches. Of course, the most dissimilar input-output specifications are typified by flat joint densities of paired efficiency scores provided that the associated Spearman correlation coefficient is roughly to about 0.10. These flat joint densities with occasional petit twists resemble a bivariate uniform distribution, in which case efficiency scores yielded by different approaches are not in any relationship whatsoever. For higher values the flat pattern tends to be disrupted and indicative of a slight regularity. On the other hand, for the most similar input-output specifications the estimated joint densities are scarcely elliptical or bell-shaped, and they merely demonstrate that traditional correlation measures may not be descriptive enough for these are suited to measure association between elliptically distributed variables. This is a justification of using more general dependence measures such as those reflected in Appendix E. Moreover, the estimated joint densities of the most similar input-output specifications show a relatively frequent presence of (clustered) observations for which efficiency scores are in disagreement. It is sort of disconcerting that the bivariate densities of efficiency scores yielded by the most similar input-output specifications where Spearman correlation coefficients are roughly above 0.60 are awash with irregularities and marked with frequent bends, hills and peaks. The visualized tendency to irregular patterns implies that there is little warranty even for the most congruent input-output specifications that if a bank is found performing favourably or poorly under one approach a similar finding is established for another approach. All in all, different approaches may lead (and they frequently do lead) to different efficiency scores and may see the same bank in a different light.

From an economist's point of view, it is of relevance to inquire whether treatment of deposits or breakdown of the banking production process into subsequent stages affects the results of efficiency measurement to a considerable extent. The present study is limited to nine intermediation and production input-output specifications and to three integrated network specifications and is undertaken with the use of a particular data set. In this context, the measures of dependence and association reported in Table 1 and partially also the estimated density plots in Appendix E evince that moving from one approach to another may lead to a different assessment and intimate other conclusions. The association between efficiency scores of the same approach is unsurprisingly generally higher than between distinct approaches, which is in particular true when the intermediation approach is confronted with the production approach or with the integrated network approach (see *e.g.* Tab. 1). In contrast, production specifications are in greater harmony with integrated network specifications. This emphasizes the importance of deposits in modelling banking production. That being said, a more conscientious examination also points out the role of other variables than deposits themselves. For instance, the greatest similarity between the production and intermediation approach was found for specifications I6/P5 and I7/P9. The former differ only in deposits swapped between the two sides of the production process, whereas for the latter alongside the swapped role of deposits there is one more variable considered for P9. The universally poorest match between efficiency scores was detected for I9/P8, where I9 is restricted only to balance sheet items plus employee numbers as production variables in contrast to P8 which considers a number of income statement items. It thus also matters whether production variables are compiled from the balance sheet or also from the income statement.

As pointed out by a reviewer, another issue linked with comparability of different efficiency assessments is the congruence in their discriminatory power. Different input-output specifications lead traditionally to different designations of technically efficient units acting as benchmarks to inefficient units. In step with the definitions adopted here for efficiency measures, such units are identified by efficiency scores of one. The discrimination power of a certain input-output specification in the context of the present study may be depicted by the relative frequency of bank-years (here representing units under assessment) identified as fully efficient. It being understood, this percentage itself says nothing about the quality of the discrimination and is useful merely in a comparison to find out which input-output specification is more lenient and which is more stringent. Hence, in addition to the direct comparisons of efficiency scores undertaken so far, the analysis is extended to compare and contrast the percentages of technically efficient bank-years identified by intermediation, production and integrated network input-output specifications as well as the percentages of bank-years pinpointed as technically efficient simultaneously by two disparate approaches. Appendix G presents descriptive summaries of the percentages of technically efficient bank-years under each approach and simultaneously for two distinct approaches. Notwithstanding that slight distortions are at play due to the fact that the calculated percentages and their statistics are related to the number of valid efficiency scores, there are clear patterns that present themselves regardless of the DEA model and its orientation. Hence, in terms of discrimination, the following ordering is applicable: the intermediation approach (the highest frequencies of efficient bank-years) $\succ$ the production approach $\succ$ the integrated network approach (the smallest frequencies of efficient bank-years). This goes in conformity with the findings of this study that for Slovak commercial banks that efficiency scores under the intermediation approach tend to be higher than those under the production approach. Perceptibly smaller factions of efficient bank-years under network input-output specifications signal that it is demanding to show performance both in intermediation and production operations, which is an observation that holds especially for the input orientation. The ordering made above is also applicable when it comes down to the evaluation of comparability between the approaches. Intermediation input-output specifications are most alike, whereas integrated network specifications are most dissimilar in terms of bank-years unanimously identified as technically efficient. When comparing input-output specifications between different approaches, the integrated network approach stands out again as it agrees with the other two approaches in many fewer efficient bank-years. Finally, intermediation and production input-output specifications are jointly less discriminatory than integrated specifications themselves as is found especially for input-oriented DEA models (for RADI and SBMI the mean percentages of simultaneously efficient bank-years with combined intermediation and production specifications are roughly twofold in comparison to the mean percentages with integrated network specifications). This last observation merely entails the fact that a simultaneous application of two black-box approaches cannot supplant reasonably and fully a properly defined two-stage network model.

The patterns of efficiency levels are further examined in a regression framework by regressing calculated efficiency scores upon factors that decide their size and variation. Albeit the task is fairly similar to two-stage evaluation of contextual variables common in DEA (*e.g.* [56], and references therein), it is conceptually different. The reason being, these factors now are not external outside the model, but are generated within as they spring out of the technical configuration of the DEA model (*i.e.* the chosen efficiency measure and its orientation) and by a particular input-output selection. Thus, instead of censored or ordinary least squares regression, the linear mixed effect model is utilized, also in response to repeated measures character of data. As a matter of fact, this investigation tallies with the recommendation of Ouenniche *et al.* ([70], p. 154) to perform a second-stage analysis of this sort using a *linear* [sic] regression framework to ascertain the validity of inputs and outputs employed. Nonetheless, regressions are now deployed to examine how the chosen input and outputs affect efficiency scores, and the analysis is extended by a search for an appropriate regression specification in order to determine the most influential determinants of efficiency scores. This search is data-driven and is implemented as a standard subset selection problem of identifying a parsimonious set of the most relevant regressors (*e.g.* [28, 47, 48]). The regression framework reposing on the linear mixed effect model is here adopted with the full understanding that there are two opposing methodological views regarding how a second stage analysis should be handled. Whilst Banker and Natarajan [5] advocate the use of ordinary least squares for a second stage, Simar and Wilson [80, 81] develop a coherent statistical model built on plausible assumptions of the data generating mechanism that requires truncated regression instead. Nonetheless, the simulation study by Banker *et al.* [6] casts doubt about the usefulness of the latter approach as its reliable performance requires that the data-generating process is specified *a priori* correctly, which is not applicable in the present context when the right input-output specification is unbeknownst and there are no contextual variables under consideration.

The primary ambition is to furnish the analyst with insights how a specific choice of the input-output set may shape the results of an efficiency measurement. Since the integrated network input-output specifications under consideration are few, the performed regression analysis formulated in a most general fashion is limited only to "black-box" input-output specifications. The list of the factors accounted for as explanatory variables comprise: (i) bank size (derived from the volume of credit activity and measured as log(TL)), (ii) number of inputs and outputs, (iii) approach adopted, (iv) efficiency measure specified, (v) orientation chosen, (vi) individual bank effects, (vii) inclusion of individual production variables. Inasmuch as the factors (ii) and (vii) supply clearly overlapping information, the regression analysis is performed in two variants and the detailed results are organized in Appendix H. The purpose of the factor (i) is to attenuate possible disparities between smaller and larger banks that may translate into efficiency scores and to secure in this manner a control for heteroskedasticity. One variant of regression runs seeks to explain the calculated efficiency scores against the factors (i)–(vi), whereas the other variant regresses the scores explicitly against (i), (vi) and (vii) and implicitly considers also (iii), (iv) and (v). Individual regressions are fitted separately to bank-year data for distinct DEA models (RADI, RADO, SBMI, SMBO) and for all them pooled. Separate regressions are based on a total of 3044 observations of the efficiency scores and regressors as opposed to pooled regressions make use of 12 176 observations altogether. The consequence of utilizing a bank-year data set is that repeated measurements of efficiency are available on the same bank because each bank is characterised by several scores calculated for distinct years. Efficiency scores of a single bank are naturally correlated rather than independent, which is the reason why the regressions are formulated as linear mixed effect models (see *e.g.* [30]) and estimated by restricted maximum likelihood (REML). Bank-specific effects represented by (vi) are interpreted as random effects, whilst all other regressors are treated as fixed effects. The Gaussianity assumption for both random effects and errors is sufficiently descriptive as is proven by traditional regression diagnostics implemented with the fitted regressions.

The regression output in Appendix H presents estimated fixed and random effects with significance flags, but is also equipped with measures of traditional goodness of fit applied in regression. The first table in Appendix H gives a concise report on the intensity and manner with which size ("log(TL)"), richness of an input-output set ("ins", "outs"), the approach to interpreting the role of deposits ("is_IA") and other DEA configuration (seen in differences between the models and in "is_sbm", "is_out" in the pooled regression) present themselves upon efficiency scores. Neither qualitative nor quantitative differences emerge between the fitted regression models. Two well-known facts transpire among the results: First, richer input-output sets (higher numbers of deployed production variables) generally yield favourable (higher) efficiency scores (for the estimated production possibility sets are tighter and the distances of inefficient units from the frontier are inevitably measured shorter). Second, SBM efficiency scores are more stringent (lower) than those measured in a radial framework. Aside from these reiterated findings, there are also some other regularities: (1) bank size is conducive to efficient operations as larger banks display on average higher efficiency scores, (2) intermediation specifications have efficiency scores on average roughly 0.10 higher than production and quasi-production specifications, (3) output-oriented efficiency scores are less stringent (*i.e.* greater) than input-oriented counterparts.

The remaining tables in Appendix H provide an assessment of the intensity and manner with which size ("log(TL)") and inclusion of individual production variables ("i_TC" to "O_TD") impact upon efficiency scores. Three fitted regression are reported for each DEA configuration and pooled data set: a regression with full entry of regressors (regardless of their optimality and explanatory potential) and regressions with regressors selected by optimizing the Bayesian information criterion (BIC) and the small-sample corrected Akaike information criterion (AICc) (for details see *e.g.* [20]). A more compact form of the results is Table 2 that shows the regressors ordered by their influence on efficiency scores. The ordering is meaningful since all the regressors listed in Table 2 are represented through dummy variables. The ranks are based on mean absolute values of the estimated regression coefficients and supplemented with information on significance.

TABLE 2. Regressors ordered by their influence upon the calculated efficiency scores.

Regressor	o_II	o_OEA	i_TOOE	i_NIE	log(TL)	o_TD	i_TOE
Mean absolute regression coefficient	0.215^{*}	0.183^{***}	0.148	$0.099^{\bullet}$	0.081^{***}	0.079^{**}	$0.067^{\bullet}$
Rank	1	2	3	4	5	6	7
Regressor	i_TP	i_TC	i_PE	i_NoE	i_IE	i_TFA	o_NonII
Mean absolute regression coefficient	$0.067^{\bullet}$	0.059	0.049	0.044	0.040	0.034	0.033
Rank	8	9	10	11	12	13	14

Legend. In consistence with the labels introduced and employed in Appendices A and C, production variables are identified by these abbreviations: II – interest income, OEA – other earning assets, TOOE – total other operating expense, NIE – non-interest expense, TL – total loans, TD – total deposits, TOE – total operating expense, TP – total (loss) provisions, TC – total costs, PE – personnel expense, NoE – employee number, IE – interest expense, TFA – total fixed assets, NonII – non-interest income. As in Appendix G, now the additional prefixes "i_" and "o_" point out that a production variable at which they stand is placed on the input or output side, respectively. This is coded by means of a dummy variable that attains a value of 1 if the production variable in question appears as an input (for "i_") or output (for "o_"), and a zero value elsewhere. The symbols in the superscripts are associated with the significance levels of estimated regression coefficients: three asterisks "***" mean that a variable is found significant at 0.001 for all cases, two asterisks "**" indicate ever-present significance at 0.01, one asterisk "*" indicates ever-present significance at 0.05, whereas the dot symbol "$^{\bullet}$" denotes ever-present significance at 0.10.
Notes. The influence of individual regressors is assessed by means of the average value of estimated regression coefficients reported in the tabular output of Appendix H. In this, regression coefficients are considered in their absolute value and then averaged for all regressions and DEA models (not excluding pooled regressions). Exclusion of the pooled regressions has no qualitative effect since it does not alter the ranking.

As with the previous regressions, the size factor "log(TL)" proves itself valid in catching heterogeneity of efficiency scores and its magnitude did not change substantially. Also adoption of the approach represented here by "o_TD" is of a similar net effect upon efficiency scores. The estimated regression coefficients for "o_TD" are all negative, which entails that placing deposits on the input side (within an intermediation specification) induces higher efficiency scores in general. Nonetheless, both these factors operate with a nearly mediocre effect (ranks 5 and 6). Unexpectedly, a marginal influence is attributable to factors that form traditional input factors of production, *i.e.* labour (here "i_NoE") and capital (here "i_TFA"). Both variables naturally embody information on the size of operations, and it is odd that their role in explaining efficiency is subtle (ranks 11 and 13). In contrast, of the greatest comparative influence upon efficiency scores are interest income (here through "o_II") and other earning assets (here through "o_OEA"), both placed on the output side and with an increasing average effect upon efficiency. Whereas the latter output is typical across diverse input-output specifications, the former output appears only once in I8. The effect of including these two factors into an input-output set is just about fivefold in comparison to the effect of including employee number and total fixed assets.

Although limited to this particular case study, the novel message of this regression analysis of efficiency scores for banking applications is that intermediation input-output specifications may put units being assessed into a more favourable light than production or quasi-production specifications. Of course, qualification of deposits as an input or output of banking production is a sovereign economic question, but a practical consequence of embracing the intermediation approach (placing deposits on the input side) is that resultant efficiency scores can be expected on average greater. Although employee number and total fixed assets are barely omitted from input-output specification, being inevitable pre-conditions for economic estimation of the production possibility set, their actual effect upon efficiency scores is found inconsequential.

5. CONCLUSIONS AND IMPLICATIONS

The examination undertaken in this study was motivated by the obvious fact that the choice of input and output variables is an indispensable and introductory part of efficiency assessment, though frequently underrated even in situations where one or more variables may be designated either as input or output. One such situation emerges in efficiency assessments in banking where there are different philosophical construals of deposits that tempt researchers into considering imaginative input-output sets for commercial banks. Apparently, a limitation of this study is that its scope is restricted to a particular question of efficiency measurement in banking and that it is built as a case study of Slovak commercial banks for the period of 12 years between 2005 and 2016. Not with standing, the results are sufficiently persuasive to demonstrate the point that the choice of an input-output set should not be underestimated, which is a legacy pertinent not only to banking, but also outside this field.

The study analyzed the impact of choosing a particular input-output specification upon technical efficiency scores of Slovak commercial banks whereas considering the claims of the leading approaches to banking enterprise about the role and position of deposits. The analysis was comparative and covered intermediation approach specifications (with deposits on the input side), production and quasi-production approach specifications (with deposits on the output side) and integrated network approach specifications (with deposits acting as a link between the production and intermediation stage of banking operations). In order to marginalize a possible effect of the DEA model adopted for efficiency measurement, a total of four DEA models were employed in estimating the technical efficiency scores for every specification. The informational similarity or congruence of efficiency scores yielded by different input-output specifications was measured with the aid of Spearman correlation coefficients and five other general dependence measures and visualized by estimated bivariate densities. Finally, a more insightful inquiry in a regression spirit was undertaken in order to ascertain how the calculated efficiency scores vary with particular input-output selections and other analytical choices.

The results are clearly suggestive that the view on technical efficiency is affected by the chosen production variables and that this effect varies in magnitude. In the particular context being examined, the distinction between the intermediation and production-like approaches was manifested in technical efficiency scores that detected the differing economic characteristics of these two approaches. Ipso facto, this ascertainment is both natural and barely unexpected. For instance, Hunter and Timme [50] compared three input-output specifications in a stochastic frontier design and found that efficiency estimates were substantially sensitive as to the input-output position of deposits. A similar limited comparison was already considered in DEA. Drake *et al.* [32] confronted two input-output specifications, whilst Boďa and Zimková [16] juxtaposed technical efficiency scores produced by three distinct specifications with similar conclusions. Interestingly, albeit working in the methodology of stochastic frontier analysis, Hunter and Timme ([50], p. 183) called for more empirical research into the compatibility of different input-output sets, and Iršová and Havránek ([51], p. 247) emphasized the need to compare simultaneously different input-output specifications. On the contrary, using the methodology of stochastic frontier analysis, Berger *et al.* [10] reported consistency between the intermediation and production approaches when applied to bank branches. Despite this slight contradiction, the evidence gathered by the papers cited afore and established by the present study indicates that not only does it matter whether deposits are specified as an input or output, but also what particular specification of other production variables is entertained. If the intention is not to particularize one function of commercial banks (*i.e.* banks as financial intermediaries or money creators), it is more suitable to blend both aspects of banking enterprise in a two-stage network manner to combine both microeconomic and macroeconomic outlooks on commercial banks. It appears that such a network approach produces efficiency scores at large comparable to the production or intermediation "black-box" approaches, although a higher degree of congruence with the network approach was generally observed for the production approach. Alas, this statement is considerably limited by a small number of network input-output specifications examined. In addition, a considerable amount of heterogeneity was not also rare for technical efficiency scores arising from different input-output specifications under the same approach. The paper abstained from commenting about the appropriateness of some input-output specifications, but a discussion may be held over the choice of inputs and outputs that are encountered in efficiency studies. It is just another point that highlights that the issue of selecting appropriate inputs and outputs should not be neglected, which is further underlined by the observation that production variables drawn from balance sheets tend to shed different light on the assessment than those compiled form income statements. Comparatively a lesser factor upon the results of technical efficiency assessment is the choice of the DEA model (the underlying efficiency measure and its orientation). The impact of both radiality/non-radiality and orientation of the model on the final ranking seems marginal in comparison to the impact of the chosen inputs and outputs.

These observations are also confirmed by the regression analysis of the calculated efficiency scores whose results reveal some additional regularities. Firstly, intermediation input-output specifications (with deposits posited on the input side) tend to have generally higher efficiency scores than production and quasi-production specifications. Secondly, although employee number and total fixed assets are traditional input factors of banking enterprise, their contribution to efficiency scores is (on average) slim and negligible in comparison to other variables, but they introduce more heterogeneity into efficiency scores. At any rate, the established regularities follow only from the present case study of Slovak commercial banks and whether they appertain to other production situations necessitates further case studies. Still, the ascertainment suggests that a project of efficiency measurement should be preferably based on a sparser input-output set in order to avoid (perhaps unnecessary) inflation of efficiency scores and that espousal of the intermediation approach may lead to less critical (higher) efficiency scores than those that would follow with a specification with deposits construed as an output.

Given the conceptual and philosophical otherness between approaches applicable to banking production, it is questionable to what degree the recommendation of Cooper *et al.* ([27], p. 166) to use different models whenever there are doubts about the features of production possibilities is useful. That being said, it is only rational to accommodate several input-output specifications at a time compliant with the analyst's *a priori* beliefs and to compare the results generated by different choices. Possible variations in the results then not only act as a robustness check, but especially give an impetus for further analysis whether there is a discrepancy and what triggers it.

An undeniable outcome of the present study interesting from a regulator's or a macroeconomist's point of view is the finding that Slovak commercial banks fare better in their intermediation activities than in (volume) production of banking services, which is a consequence of the fact that efficiency scores under the intermediation approach tend to be greater than those of the production approach. This pattern cannot be interpreted as the superiority of the intermediation approach in explaining the operations of Slovak commercial banks for both approaches as such are valid (banks are both intermediaries and producers of bank services); yet, what is not certain and is highly controversial is the choice of inputs and outputs that would describe banking operations under either approach. The ascertained pattern is also not attributable to a "more favourable" number of production variables under the intermediation approach. As follows from Appendix A, the nine intermediation specifications considered here exploit an average of 5.33 production variables (between 4 and 7) unlike the nine production specifications that make use of 5.44 production variables on average (between 4 and 8). Since the chief distinction between these two approaches is the treatment of deposits, it matters whether deposits are specified as input or output. Simply speaking, Slovak commercial banks are more successful in using deposits than in producing them, which preordains a good financial position for the Slovak banking sector. It has been felt for some time that the Slovak banking sector is saturated by deposits, for which reason the retail business strategies of many Slovak commercial banks have been reoriented on transforming deposits into different kinds of investments such as mutual fund shares [15, 17]. The deposit-minimizing strategy foisted by the intermediation approach can be therefore seen functioning, effective and compliant with the ultimate task of promoting economic development and growth (*e.g.* [61]).

Conflict of interest

The authors declare that they have no conflict of interest.

APPENDIXES (*see next page*)

Appendix A.

Table A.1. Overview of the "black-box" input-output sets applied in the comparison.

Code	Study	DEA specification	Balance sheet items							Income statement items										Others
			TL	OEA	TP	TFA	TD	TDSF	OPA	TC	PE	IE	NcFTI	NIE	NonII	TOOI	TOE	TOOE	II	NoE
Intermediation approach specifications																				
I1	Casu and Girardone [23]	RADI VRS	O	O			I			I										
	Casu and Molyneux [22]	RADI VRS	O	O			I			I										
I2	Drake *et al.* [32] [model A]	SBMI VRS	O	O	I		I						O			O	I			
I3	Kamarudin *et al.* [57]	RADI VRS	O	O		I		I			I									
I4	Triki *et al.* [92]	RADI VRS	O	O		I		I		I					O					
	Kouiki and Al-Nasser [60] [model B]	RADI CRS	O	O		I		I		I					O					
I5	Kouiki and Al-Nasser [60] [model A]	RADI CRS	O	O		I		I		I										
I6	Sufian *et al.* [85]	RADI VRS	O	O		I	I								O					I
I7	Rezitis [76]	RADO CRS	O	O		I	I													I
I8	Felix Ayadi *et al.* [36]	RADI CRS	O				I				I	I			O				O	
I9	Boďa and Zimková [17] [model B]	SBMN CRS/VRS	O			I	I													I
Production and quasi-production approach specifications																				
P1	Bhattacharyya *et al.* [12]	RADO VRS	O	O			O					I					I			
P2	Boďa and Zimková [17] [model A]	SBMN CRS/VRS	O			I	O													I
P3	Pastor *et al.* [71]	RADI CRS	O				O		O		I			I						
P4	Gunay and Tektas [46]	RADO CRS	O	O			O				I	I					I§			
P5	Subramanyam and Reddy [83]	RADI CRS/VRS	O	O		I	O								O					I
P6	Resti [75]†	RADI CRS/VRS	O			I	O								O					I
P7	Berg *et al.* [8] [model B]	RADI CRS	O				O										I			I*
P8	Drake *et al.* [32] [model B]	SBMI VRS	O	O	I		O						O	I		O		I		
P9	Sufian [84]	RADI VRS	O	O		I	O					I								I

Notes. In DEA specifications, "RADI" and "RADO" denote radial models with an input or output orientation, whereas "SBMI" and "SBMN" represent input-oriented and non-oriented SBM models. The abbreviations "CRS" and "VRS" represent constant and variable returns to scale, respectively. The dual designation "CRS/VRS" means that both return to scale specifications are considered in turn. Input and output items are further listed using letters "I" and "O", respectively. Production variables are identified by these abbreviations: TL – total loans, OEA – other earning assets (*i.e.* securities or investments), TP – total (loss) provisions, TFA – total fixed assets, TD – total deposits, TDSF – total deposits [TD] plus short-term funds, OPA – other productive assets (*i.e.* deposits with banks plus other earning assets [OEA]), TC – total costs, PE – personnel expense, IE – interest expense, NcFTI – net commission, fee and trading income, NIE – non-interest expense, NonII – non-interest income, TOOI – total other operating income, TOE – total operating expense, TOOE – total other operating expense (*i.e.* total operating expense [TOE] less personnel expense [PE]), II – interest income, NoE – employee number. All these items are measured in appropriate monetary amounts (year-end or average balances for balance sheet items and year-end amounts for income statement items) except the number of employees [NoE], which is measured by full-time equivalents (year-end or average numbers).

(§)In fact, the study by Gunay and Tektas [46] considers administrative expense for total operating expense. (†)This study applies this input-output configuration in cost efficiency considerations. (*)Actually, Berg *et al.* [8] use man hours instead of employee numbers.

Appendix B.

Table B.1. Overview of the two-stage network input-output sets applied in the comparison.

Code	Study	DEA specification	Balance sheet items					Income statement items				Others
			TL	OEA	TFA	TD	EQ	PE	IE	NcFTI	TOOE	II	NoE
N1	Fukuyama and Matousek [37]	RADI VRS	O (S2)	O (S2)	I (S1)	L (S1/S2)							I (S1)
	Holod and Lewis [49]	RADN VRS	O (S2)	O (S2)	I (S1)	L (S1/S2)							I (S1)
N2	Boďa [14]	SBMN CRS/VRS	O (S2)	O (S2)	I (S1)	L (S1/S2)	I (S2)						I (S1)
N3	Lim and Randawa [63]	RADI CRS/VRS	O (S2)			L (S1/S2)	I (S1)	I (S1)	I (S1)	O (S1)	I (S2)	O (S2)	

Notes. In DEA specifications, "RADI" and "RADN" denote input-oriented or non-oriented radial models, whereas represents a non-oriented SBM model. The abbreviations "CRS" and "VRS" represent constant and variable returns to scale, respectively. The dual designation "CRS/VRS" means that both return to scale specifications are considered in turn. Input, linking and output items are further listed using letters "I", "L", "O", respectively, and "S1", "S1/S2", "S2" in the brackets indicated to which stage they pertain. Production variables are identified by these abbreviations: TL – total loans, OEA – other earning assets (*i.e.* securities or investments), TFA – total fixed assets, TD – total deposits, EQ – shareholders' equity, PE – personnel expense, IE – interest expense, NcFTI – net commission, fee and trading income, TOOE – total other operating expense (*i.e.* total operating expense less personnel expense [PE]), II – interest income, NoE – employee number. All these items are measured in appropriate monetary amounts (year-end or average balances for balance sheet items and year-end amounts for income statement items) except the number of employees [NoE], which is measured by full-time equivalents (year-end or average numbers).

Appendix C.

Table C.1. Descriptive statistics of the production variables.

	Balance sheet items									Others
	TL	OEA	TP	TFA	TD	TDSF	OPA	EQ		NoE
Minimum	112	0	0	104	4 614	0	910	−3585		10
Maximum	9 434 668	3 544 628	447 331	249 308	10 407 245	10 407 245	4 526 563	1 405 042		4832
Mean	1 514 179	693 839	64 426	37 351	1 733 125	1 765 136	366 713	281 185		911
Median	639 957	178 421	25 345	12 983	713 603	693 405	156 388	120 994		390
Standard deviation	2 009 683	1 004 000	90 458	57 079	2 333 907	2 376 051	594 437	324 797		1297
# of NAs	1	99	7	0	0	9	2	20		13
	Income statement items									
	TC	PE	IE	NcFTI	NIE	NonII	TOOI	TOE	TOOE	II
Minimum	611	102	4	−83 822	9	0	−32 001	611	509	2
Maximum	253 779	122 235	261 244	176 522	60 533	139 717	158 033	253 779	157 651	580 543
Mean	51 386	22 951	36 456	19 657	6130	26 699	7497	51 787	28 679	108 631
Median	29 988	11 668	19 242	4475	1561	8304	985	30360	15 651	51 977
Standard deviation	67 720	30 973	47 522	36 780	11 666	40 516	16 842	67 889	37 277	141 193
# of NAs	2	2	1	13	3	3	24	4	4	1

Notes. Production variables are identified by these abbreviations: TL – total loans, OEA – other earning assets (*i.e.* securities or investments), TP – total (loss) provisions, TFA – total fixed assets, TD – total deposits, TDSF – total deposits [TD] plus short-term funds, OPA – other productive assets (*i.e.* deposits with banks plus other earning assets [OEA]), EQ – shareholders' equity, TC – total costs, PE – personnel expense, IE – interest expense, NcFTI – net commission, fee and trading income, NIE – non-interest expense, NonII – non-interest income, TOOI – total other operating income, TOE – total operating expense, TOOE – total other operating expense (*i.e.* total operating expense [TOE] less personnel expense [PE]), II – interest income, NoE – employee number. All these items are measured in euros (year-end amounts for balance sheet and income statement items) except the number of employees [NoE], which is measured by full-time equivalents (year-end or average numbers).

Appendix D.

Table D.1. Descriptive statistics of the calculated efficiency scores.

DEA model	RADI	RADO	SBMI	SBMO	RADI	RADO	SBMI	SBMO
	Intermediation approach specifications				*Production approach specifications*			
	I1 (# of valid efficiency scores: 142)				*P1* (# of valid efficiency scores: 140)			
Minimum	0.2406	0.0856	0.1947	0.1577	0.2346	0.0953	0.1901	0.1321
Maximum	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Mean	0.7639	0.7531	0.7256	0.6908	0.7330	0.7501	0.6546	0.6547
Median	0.8030	0.8178	0.7586	0.7239	0.7527	0.7688	0.6270	0.6597
Standard deviation	0.1955	0.2265	0.2044	0.2442	0.2134	0.2173	0.2392	0.2529
	I2 (# of valid efficiency scores: 136)				*P2* (# of valid efficiency scores: 229)			
Minimum	0.2406	0.1178	0.1864	0.0146	0.0305	0.0085	0.0230	0.0003
Maximum	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Mean	0.8434	0.8542	0.7857	0.6582	0.4357	0.5346	0.4025	0.4768
Median	0.9258	0.9391	0.8426	0.7163	0.3864	0.5090	0.3454	0.4323
Standard deviation	0.1888	0.1805	0.2196	0.3277	0.3237	0.2795	0.3090	0.2944
	I3 (# of valid efficiency scores: 141)				*P3* (# of valid efficiency scores: 238)			
Minimum	0.2786	0.1084	0.1584	0.1870	0.0605	0.0175	0.0516	0.0014
Maximum	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Mean	0.7735	0.7884	0.6923	0.7424	0.5761	0.6382	0.5077	0.3725
Median	0.8446	0.8765	0.7289	0.8178	0.5658	0.6645	0.4575	0.2456
Standard deviation	0.2121	0.2198	0.2497	0.2421	0.2898	0.2623	0.2963	0.3250
	I4 (# of valid efficiency scores: 141)				*P4* (# of valid efficiency scores: 140)			
Minimum	0.2600	0.1616	0.1407	0.1919	0.2346	0.1790	0.1832	0.1582
Maximum	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Mean	0.8305	0.8336	0.7383	0.7377	0.7653	0.7804	0.7116	0.6898
Median	0.8743	0.8937	0.7505	0.7561	0.7977	0.8074	0.7263	0.7272
Standard deviation	0.1761	0.1812	0.2294	0.2474	0.2119	0.2072	0.2282	0.2495
	I5 (# of valid efficiency scores: 141)				*P5* (# of valid efficiency scores: 134)			
Minimum	0.2406	0.0856	0.1401	0.1577	0.0787	0.0297	0.0509	0.0176
Maximum	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Mean	0.7907	0.7853	0.6936	0.7366	0.6437	0.7072	0.6144	0.6122
Median	0.8405	0.8698	0.7129	0.8299	0.6880	0.7514	0.6499	0.6490
Standard deviation	0.1962	0.2251	0.2401	0.2492	0.3259	0.2780	0.3287	0.3191
	I6 (# of valid efficiency scores: 134)				*P6* (# of valid efficiency scores: 227)			
Minimum	0.1370	0.1534	0.0969	0.1969	0.0322	0.0085	0.0320	0.0005
Maximum	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Mean	0.8003	0.8249	0.7210	0.7341	0.4963	0.6011	0.4549	0.4944
Median	0.8940	0.9223	0.7895	0.8074	0.4533	0.5763	0.4178	0.4180
Standard deviation	0.2205	0.2037	0.2625	0.2593	0.3338	0.2823	0.3170	0.3086
	I7 (# of valid efficiency scores: 135)				*P7* (# of valid efficiency scores: 226)			
Minimum	0.1370	0.0790	0.0969	0.1464	0.1628	0.0136	0.1013	0.0004
Maximum	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Mean	0.7733	0.7887	0.6915	0.7465	0.5301	0.5830	0.4742	0.4933
Median	0.8506	0.8955	0.7219	0.8420	0.4862	0.5442	0.4167	0.4634
Standard deviation	0.2263	0.2332	0.2607	0.2535	0.2347	0.2233	0.2551	0.2523

Table D.1. Continued.

DEA model	RADI	RADO	SBMI	SBMO	RADI	RADO	SBMI	SBMO
	I8 (# of valid efficiency scores: 239)				*P8* (# of valid efficiency scores: 136)			
Minimum	0.3086	0.2671	0.2465	0.0605	0.3801	0.4048	0.2402	0.0211
Maximum	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Mean	0.8326	0.8401	0.7875	0.7255	0.8632	0.8720	0.8096	0.7251
Median	0.8701	0.8868	0.7957	0.7599	0.9972	0.9981	0.9851	0.9699
Standard deviation	0.1614	0.1628	0.1817	0.2470	0.1997	0.1862	0.2435	0.3248
	I9 (# of valid efficiency scores: 230)				*P9* (# of valid efficiency scores: 134)			
Minimum	0.0346	0.0696	0.0206	0.0696	0.1327	0.2086	0.0991	0.1825
Maximum	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Mean	0.5051	0.5939	0.4053	0.5939	0.7099	0.7541	0.6373	0.6631
Median	0.4719	0.6096	0.3455	0.6096	0.7950	0.8201	0.6527	0.6970
Standard deviation	0.2890	0.2667	0.2798	0.2667	0.2632	0.2327	0.2892	0.2767
	Integrated network approach specifications							
	N1 (# of valid efficiency scores: 120)				*N2* (# of valid efficiency scores: 103)			
Minimum	0.5080	0.0846	0.5049	0.1559	0.0731	0.5685	0.0340	0.3205
Maximum	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Mean	0.7448	0.7757	0.7291	0.6988	0.5616	0.8865	0.5696	0.8065
Median	0.7235	0.8337	0.7083	0.7012	0.5706	0.9220	0.5866	0.8488
Standard deviation	0.1652	0.2209	0.1627	0.2192	0.2385	0.1138	0.2522	0.1878
	N3 (# of valid efficiency scores: 171)							
Minimum	0.1851	0.0530	0.1537	0.0470				
Maximum	1.0000	1.0000	1.0000	1.0000				
Mean	0.6081	0.3846	0.5705	0.3568				
Median	0.6131	0.2962	0.5627	0.2741				
Standard deviation	0.2208	0.2613	0.2148	0.2656				

Appendix E.

Table E.1. Descriptive statistics of dependence measures between the calculated efficiency scores.

Dependence measure	Gini's rank association coefficient	Medial correlation coefficient	Van der Waerden's coefficient	Mutual information	Linfoot's correlation coefficient	Gini's rank association coefficient	Medial correlation coefficient	Van der Waerden's coefficient	Mutual information	Linfoot's correlation coefficient
Intermediation approach specifications against intermediation approach specifications										
	RADI					SBMI				
Minimum	0.0983	0.0793	0.1147	0.0239	0.2162	0.0687	0.0399	0.1199	0.0167	0.1810
	I9/I5	I9/I7	I9/I2	I9/I5	I9/I5	I9/I6	I9/I6	I9/I5	I9/I1	I9/I1
Maximum	0.8232	0.8801	0.8339	1.2318	0.9565	0.8472	0.9109	0.8501	1.2415	0.9573
	I5/I4	I5/I1	I5/I4	I7/I6	I7/I6	I5/I1	I5/I3	I6/I4	I7/I6	I7/I6
Mean	0.5030	0.4932	0.5370	0.5650	0.6648	0.5022	0.4679	0.5506	0.5841	0.6821
Standard deviation	0.3081	0.3036	0.3225	0.4508	0.3051	0.2882	0.2773	0.3099	0.4575	0.2892
	RADO					SBMO				
Minimum	0.1018	0.0783	0.1132	0.0261	0.2254	0.0456	0.0785	0.0488	0.0300	0.2415
	I9/I2	I9/I7	I9/I2	I9/I2	I9/I2	I8/I3	I9/I3	I8/I3	I9/I1	I9/I1
Maximum	0.8246	0.8778	0.8378	1.3126	0.9631	0.8541	0.8721	0.8711	1.4253	0.9707
	I5/I4	I5/I3	I5/I1	I5/I3	I5/I3	I5/I3	I5/I3	I5/I3	I5/I3	I5/I3
Mean	0.4995	0.4892	0.5352	0.5703	0.6794	0.4775	0.4487	0.5358	0.5535	0.6890
Standard deviation	0.3067	0.3076	0.3168	0.4475	0.2865	0.2850	0.2708	0.3096	0.4381	0.2625
Production approach specifications against production approach specifications										
	RADI					SBMI				
Minimum	0.0404	0.0338	0.0458	0.0016	0.0573	0.0084	−0.0430	0.0143	0.0022	0.0662
	P8/P2	P5/P2	P8/P2	P8/P2	P8/P2	P8/P3	P5/P2	P8/P2	P8/P2	P8/P2
Maximum	0.8596	0.8918	0.9210	1.3576	0.9663	0.8354	0.8998	0.9048	1.2896	0.9613
	P6/P2	P4/P1	P6/P2	P6/P2	P6/P2	P6/P2	P4/P1	P6/P2	P6/P2	P6/P2
Mean	0.3385	0.3144	0.3960	0.3611	0.5632	0.2822	0.2633	0.3317	0.3198	0.5222
Standard deviation	0.2776	0.2821	0.3059	0.3770	0.2911	0.2824	0.2834	0.2980	0.3631	0.3020
	RADO					SBMO				
Minimum	0.0371	0.0384	0.0413	0.0015	0.0542	0.0271	0.0324	0.0299	0.0007	0.0383
	P8/P2	P5/P2	P8/P2	P8/P2	P8/P2	P8/P2	P8/P2	P8/P2	P8/P2	P8/P2
Maximum	0.8283	0.8914	0.8840	1.3085	0.9628	0.8513	0.8685	0.8888	1.3566	0.9663
	P6/P5	P4/P1	P6/P5	P4/P1	P4/P1	P6/P2	P6/P2	P6/P2	P4/P1	P4/P1
Mean	0.3395	0.3114	0.3997	0.3642	0.5816	0.3190	0.2949	0.3658	0.3527	0.5452
Standard deviation	0.2756	0.2848	0.2997	0.3555	0.2772	0.2628	0.2528	0.2838	0.3976	0.2930
Integrated network approach specifications against integrated network approach specifications										
	RADI					SBMI				
Minimum	0.1582	0.1599	0.1912	0.2988	0.6707	0.1583	0.1392	0.2056	0.2511	0.6283
	N3/N2	N3/N2	N3/N2	N3/N1	N3/N1	N3/N2	N3/N2	N3/N2	N3/N1	N3/N1

TABLE E.1. Continued.

Dependence measure	Gini's rank association coefficient	Medial correlation coefficient	Van der Waerden's coefficient	Mutual information	Linfoot's correlation coefficient	Gini's rank association coefficient	Medial correlation coefficient	Van der Waerden's coefficient	Mutual information	Linfoot's correlation coefficient
Maximum	0.7086	0.6501	0.7909	0.7860	0.8902	0.7690	0.7785	0.8168	0.9065	0.9148
	N2/N1	N2/N1	N2/N1	N2/N1	N2/N1	N2/N1	N2/N1	N2/N1	N2/N1	N2/N1
Mean	0.3579	0.3271	0.4142	0.4627	0.7451	0.3738	0.3654	0.4229	0.4806	0.7339
Standard deviation	0.3047	0.2798	0.3281	0.2800	0.1257	0.3428	0.3584	0.3417	0.3692	0.1574
			RADO					SBMO		
Minimum	0.1022	0.1054	0.1247	0.0359	0.2632	0.0562	0.0660	0.0359	0.0233	0.2134
	N3/N2	N3/N2	N3/N2	N3/N2	N3/N2	N3/N2	N3/N2	N3/N2	N3/N2	N3/N2
Maximum	0.7528	0.8345	0.7818	0.9111	0.9156	0.7513	0.7386	0.7927	0.8753	0.9090
	N2/N1	N2/N1	N2/N1	N2/N1	N2/N1	N2/N1	N2/N1	N2/N1	N2/N1	N2/N1
Mean	0.3536	0.3858	0.3748	0.3374	0.5096	0.3341	0.3336	0.3474	0.3111	0.4605
Standard deviation	0.3496	0.3926	0.3555	0.4970	0.3543	0.3678	0.3568	0.3957	0.4887	0.3891
Intermediation approach specifications against production approach specifications										
			RADI					SBMI		
Minimum	0.0044	0.0076	0.0081	0.0011	0.0478	0.0042	0.0077	0.0076	0.0007	0.0383
	I9/P8	I9/P8	I9/P8	I9/P8	I9/P8	I9/P8	I9/P8	I9/P8	I9/P8	I9/P8
Maximum	0.7054	0.6320	0.7753	0.7657	0.8853	0.6992	0.6454	0.7645	0.7496	0.8813
	I7/P9	I7/P9	I7/P9	I7/P9	I7/P9	I7/P9	I6/P5	I7/P9	I7/P9	I7/P9
Mean	0.3096	0.2757	0.3664	0.2546	0.4805	0.3069	0.2731	0.3636	0.2554	0.4777
Standard deviation	0.2190	0.1796	0.2627	0.2713	0.2965	0.2244	0.1878	0.2656	0.2706	0.3026
			RADO					SBMO		
Minimum	0.0190	0.0235	0.0230	0.0013	0.0508	0.0156	0.0190	0.0199	0.0011	0.0468
	I9/P8	I9/P8	I9/P8	I9/P8	I9/P8	I9/P8	I9/P8	I9/P8	I9/P8	I9/P8
Maximum	0.7175	0.6805	0.7811	0.7886	0.8908	0.7209	0.6713	0.7675	0.8054	0.8946
	I7/P9	I6/P5	I7/P9	I7/P9	I7/P9	I6/P5	I6/P5	I7/P9	I6/P5	I6/P5
Mean	0.3165	0.2826	0.3709	0.2716	0.4967	0.3040	0.2716	0.3549	0.2621	0.4631
Standard deviation	0.2336	0.1965	0.2719	0.2864	0.2935	0.2389	0.2024	0.2767	0.2895	0.3222
Intermediation approach specifications against integrated network approach specifications										
			RADI					SBMI		
Minimum	0.0654	0.0600	0.0824	0.0040	0.0896	0.0562	0.0506	0.0717	0.0031	0.0788
	I1/N1	I1/N1	I1/N1	I1/N1	I1/N1	I1/N1	I1/N1	I1/N1	I1/N1	I1/N1
Maximum	0.6170	0.5204	0.7286	0.6712	0.8595	0.6914	0.6169	0.7780	0.7471	0.8807
	I6/N2	I6/N2	I7/N2	I6/N2	I6/N2	I7/N2	I6/N2	I7/N2	I6/N2	I6/N2

TABLE E.1. Continued.

Dependence measure	Gini's rank association coefficient	Medial correlation coefficient	Van der Waerden's coefficient	Mutual information	Linfoot's correlation coefficient	Gini's rank association coefficient	Medial correlation coefficient	Van der Waerden's coefficient	Mutual information	Linfoot's correlation coefficient
Mean	0.2889	0.2561	0.3488	0.2183	0.4624	0.2889	0.2742	0.3642	0.2393	0.4811
Standard deviation	0.1880	0.1451	0.2366	0.2460	0.2732	0.1880	0.1663	0.2456	0.2670	0.2754
			RADO					SBMO		
Minimum	0.0843	0.0888	0.0911	0.0179	0.1874	0.0778	0.0771	0.0888	0.0050	0.1001
	I9/N2	I9/N2	I9/N2	I9/N2	I9/N2	I9/N2	I9/N2	I9/N2	I9/N2	I9/N2
Maximum	0.8226	0.8321	0.8387	1.1544	0.9490	0.7694	0.7238	0.8135	0.9284	0.9186
	I1/N1	I1/N1	I1/N1	I1/N1	I1/N1	I1/N1	I1/N1	I1/N1	I1/N1	I1/N1
Mean	0.4889	0.4556	0.5388	0.4788	0.6432	0.4358	0.3893	0.4934	0.4068	0.5880
Standard deviation	0.2614	0.2263	0.2743	0.4046	0.2803	0.2538	0.2077	0.2849	0.3626	0.3136
	Production approach specifications against integrated network approach specifications									
			RADI					SBMI		
Minimum	0.0552	0.0592	0.0621	0.0030	0.0769	0.0565	0.0594	0.0647	0.0029	0.0761
	P8/N3	P8/N3	P8/N3	P8/N3	P8/N3	P8/N3	P8/N3	P8/N3	P8/N3	P8/N3
Maximum	0.7278	0.7005	0.7945	0.8226	0.8983	0.7241	0.6991	0.7798	0.7894	0.8909
	P5/N2	P5/N1	P5/N2	P5/N2	P5/N2	P5/N1	P5/N1	P5/N1	P5/N1	P5/N1
Mean	0.3327	0.3134	0.3821	0.3154	0.5919	0.3044	0.2784	0.3588	0.2707	0.5471
Standard deviation	0.2360	0.2110	0.2562	0.2532	0.2421	0.2238	0.2042	0.2429	0.2412	0.2479
			RADO					SBMO		
Minimum	0.0353	0.0411	0.0374	0.0021	0.0645	−0.0184	−0.0187	−0.0198	0.0006	0.0349
	P7/N2	P7/N2	P7/N2	P7/N2	P7/N2	P7/N2	P7/N2	P7/N2	P2/N2	P2/N2
Maximum	0.5882	0.4791	0.6951	0.6237	0.8443	0.4982	0.4088	0.6294	0.4968	0.7936
	P9/N2	P1/N1	P9/N2	P9/N2	P9/N2	P1/N1	P1/N1	P1/N1	P1/N1	P1/N1
Mean	0.2629	0.2407	0.3162	0.1755	0.3976	0.1773	0.1612	0.2230	0.1083	0.3028
Standard deviation	0.1864	0.1533	0.2303	0.2229	0.2855	0.1485	0.1267	0.1962	0.1594	0.2665

Notes. The dependence between different pairs of efficiency scores is assessed by means of five dependence measures. In addition to the Gini's rank association coefficient (Gini's γ), they are the medial correlation coefficient (Blomqvist's β), van der Waerden's coefficient, mutual information and Linfoot's correlation coefficient. Like the Spearman correlation coefficient, also Gini's rank association coefficient, the medial correlation coefficient, van der Waerden's coefficient and Linfoot's correlation coefficient range all from -1 to $+1$ for negative and positive association, respectively. Mutual information is here non-normalized and yields values from 0 (when there is no mutual information) to $+\infty$. Higher values indicate higher association and indicate that more information is shared between the efficiency scores in question. The definition and details on these measures of dependence frequent in modelling with copulas can be learnt *e.g.* by consulting Nelsen ([69], p. 182), Joe ([55], pp. 57, 58), Genest and Verret [43], Joe [54] and Linfoot [64]. Furthermore, minimum and maximum values of dependence measures are reported together with combinations of input-output sets at which they arise (and these combinations are stated underneath the minimums and maximums to which they appertain). All these dependence measures were derived from the estimated copula density and with the exception of the medial correlation coefficient they were computed by means of quasi Monte Carlo methods as implemented by Nagler and Wen [68].

Appendix F.

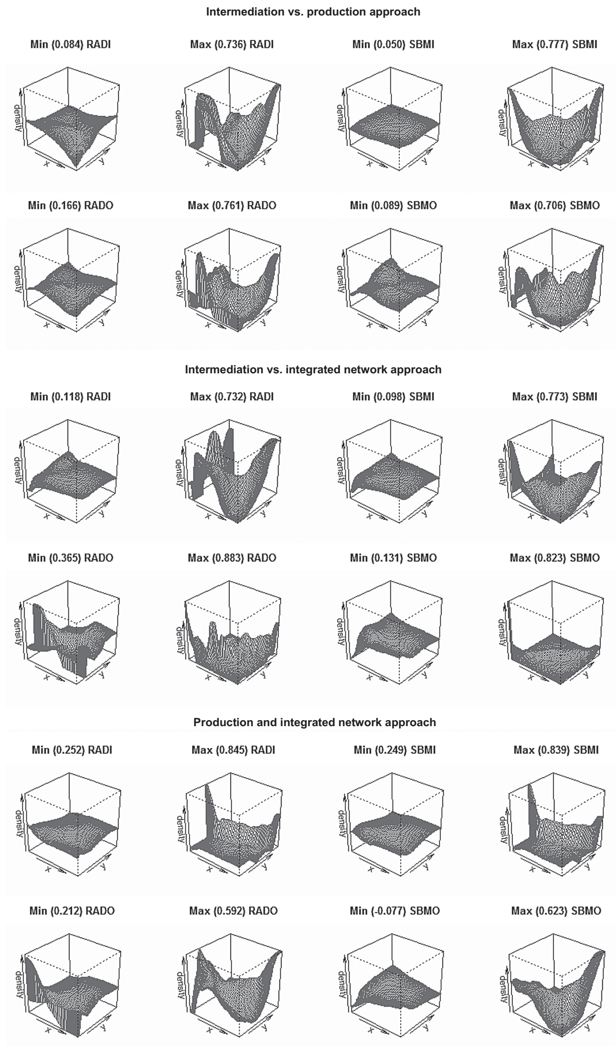

Figure F.1. Kernel estimates of bivariate densities of efficiency scores between the least and most similar specifications of disparate approaches.

In the labels of these plots, "Min" and "Max" identify the least and most similar specifications of the intermediation and integrated network approach, respectively. The Spearman correlation coefficient was applied as a measure of similarity and is displayed in parentheses in each label. For each of the four DEA models considered, the efficiency scores for the intermediation approach specification were matched against the scores of the integrated network approach specification. The smallest and largest Spearman correlations identified the least and most similar pair of efficiency scores. The densities of these pairs were estimated by a Gaussian product kernel density estimator using the reflection method with bandwidth selected on a default basis by minimizing the mean integrated square error using the Frank copula as the reference family (see [44, 68]).

Appendix G.

Table G.1. Descriptive statistics on the percentages of cases identified as technically efficient.

DEA model	RADI	RADO	SBMI	SBMO	RADI	RADO	SBMI	SBMO
	Intermediation approach specifications				Production approach specifications			
Minimum	8.26%	8.26%	8.26%	8.26%	5.75%	5.75%	5.75%	5.75%
Maximum	34.56%	34.56%	33.82%	33.82%	49.26%	49.26%	49.26%	49.26%
Mean	23.69%	23.69%	23.61%	23.61%	19.68%	19.68%	19.68%	19.68%
Standard deviation	8.45%	8.45%	8.33%	8.33%	13.20%	13.20%	13.20%	13.20%
	Integrated network approach specifications							
Minimum	2.40%	1.55%	2.40%	2.40%				
Maximum	11.11%	25.00%	11.11%	25.00%				
Mean	5.31%	11.81%	6.66%	12.10%				
Standard deviation	5.02%	12.00%	4.36%	11.63%				

Notes. The reported summaries relate to the relative frequency of bank-years identified as technically efficient and are derived from valid efficiency scores as reported in the overview of Appendix D. They are compiled from efficiency scores produced by 9 input-output intermediation specifications, 9 input-output production and quasi-production specifications, and – rather trivially – 3 input-output network specifications.

Table G.2. Descriptive statistics on the percentages of cases identified as technically efficient simultaneously by two different input-output specifications.

DEA model	RADI	RADO	SBMI	SBMO	RADI	RADO	SBMI	SBMO
	Intermediation approach specifications against intermediation approach specifications*				Production approach specifications against production approach specifications*			
Minimum	5.75%	5.75%	5.75%	5.75%	3.56%	3.56%	3.56%	3.56%
Maximum	24.81%	24.81%	24.81%	24.81%	21.64%	21.64%	21.64%	21.64%
Mean	15.04%	15.04%	15.04%	15.04%	9.41%	9.41%	9.41%	9.41%
Standard deviation	5.24%	5.24%	5.24%	5.24%	4.82%	4.82%	4.82%	4.82%
	Integrated network approach specifications against integrated network approach specifications§				Intermediation approach specifications against production approach specifications†			
Minimum	0.85%	0.88%	1.71%	0.85%	3.13%	3.13%	3.13%	3.13%
Maximum	2.56%	9.68%	6.45%	9.68%	27.21%	27.21%	26.47%	26.47%
Mean	1.95%	3.82%	3.58%	4.08%	10.61%	10.61%	10.60%	10.60%
Standard deviation	0.95%	5.08%	2.53%	4.87%	5.31%	5.31%	5.28%	5.28%
	Intermediation approach specifications against integrated network approach specifications‡				Production approach specifications against integrated network approach specifications‡			
Minimum	0.82%	1.04%	1.93%	1.93%	0.83%	1.10%	1.54%	1.54%
Maximum	9.70%	18.10%	9.70%	18.10%	9.70%	17.24%	9.70%	17.24%
Mean	3.44%	8.20%	4.50%	8.46%	3.23%	5.60%	3.91%	5.80%
Standard deviation	2.46%	6.19%	2.11%	5.89%	2.50%	4.70%	2.36%	4.53%

Notes. The reported summaries relate to the relative frequency of bank-years identified as technically efficient concurrently under two different input-output specifications and are derived from valid efficiency scores as reported in the overview of Appendix D. (*)Since for both the intermediation and production approach as many as 9 input-output sets were considered, these summaries pertain to all possible $9 \times 8/2 = 36$ pairs of input-output sets for the intermediation or production approach. (§)Since only three input-output sets were considered for the integrated network approach, these summaries are drawn only from all possible $3 \times 2/2 = 3$ combinations. (†)Since 9 input-output sets were considered for both the intermediation and production approach, these summaries are computed from $9 \times 9 = 81$ combinations arising by matching the input-output sets of both approaches. (‡)Since 9 input-output sets were considered for both the intermediation and production approach and three input-output sets for the integrated network approach, these summaries are computed from all possible $9 \times 3 = 27$ combinations of input-output sets.

Appendix H.

Table H.1. Regressions relating efficiency scores calculated under the "black-box" approaches to the number of production variables and DEA configuration.

DEA model	RADI	RADO	SBMI	SBMO	All (four) pooled
Estimated fixed effects					
Intercept§	-1.027^{***}	-0.910^{***}	-1.031^{***}	-0.957^{***}	-0.999^{***}
log(TL)†	0.090^{***}	0.091^{***}	0.088^{***}	0.089^{***}	0.093^{***}
ins	0.078^{***}	0.070^{***}	0.052^{***}	0.112^{***}	0.078^{***}
outs	0.096^{***}	0.078^{***}	0.106^{***}	0.031^{***}	0.078^{***}
is_IA	0.148^{***}	0.106^{***}	0.149^{***}	0.096^{***}	0.125^{***}
is_sbm					-0.081^{***}
is_out					0.023^{***}
Estimated random effects (effects pertaining to single banks)+					
BRE	−0.078	−0.100	−0.094	−0.092	−0.084
CAC	0.131	0.095	0.112	0.173	0.132
CAG	0.108	0.008	0.089	0.000	0.059
CITI	0.073	0.078	0.118	0.096	0.094
CSOB	−0.070	−0.075	−0.073	−0.064	−0.078
CSOBss	−0.094	−0.098	−0.087	−0.126	−0.097
ING	0.081	0.111	0.082	0.158	0.110
JT	−0.010	0.088	−0.014	0.056	0.030
KOBA	0.120	0.093	0.132	0.138	0.122
OBER	0.076	−0.019	0.055	0.022	0.040
OTP	−0.272	−0.214	−0.302	−0.274	−0.271
POBA	0.017	0.027	0.029	−0.023	0.009
PRIMA	−0.126	−0.098	−0.164	−0.115	−0.131
PRIVAT	0.104	0.105	0.128	0.095	0.113
PSS	0.097	0.082	0.065	0.067	0.074
RBS	0.325	0.243	0.350	0.271	0.310
SBER	−0.260	−0.204	−0.276	−0.275	−0.260
SLSP	−0.027	−0.026	−0.021	0.008	−0.027
SZRB	−0.009	0.031	0.002	0.002	0.008
TATRA	−0.044	−0.063	−0.025	−0.029	−0.050
UNICB	−0.031	−0.041	−0.029	−0.041	−0.043
VUB	−0.020	−0.045	−0.039	−0.016	−0.040
WUST	−0.128	−0.098	−0.143	−0.127	−0.122
HSBC	0.264	0.263	0.358	0.229	0.306
HVB	0.092	0.092	0.081	0.082	0.093
ISTRO	−0.319	−0.237	−0.335	−0.214	−0.296
Quality of the fitted models					
adjusted R^2	0.504	0.520	0.479	0.429	0.715
BIC	−889	−1669	−512	−168	−3241
AICc	−931	−1711	−554	−210	−3308
logLik	472	862	284	112	1663

Notes. The labels "BIC" and "AICc" introduce information criteria BIC and AICc computed under the Gaussian assumption adopted for errors, and the label "logLik" indicates traditional Gaussian log-likelihood. The labels "ins" and "outs" in the fixed effects section indicate the number of inputs and outputs, respectively, and other labels "is_IA", "is_sbm", "is_out" represent dummy variables coding by a value of 1 whether an efficiency score in question is derived from an intermediation approach specification ("is_IA"), answers to an SBM model ("is_SBM"), or is obtained under an output orientation ("is_out"). Each of these dummy variables attains a value of 0 in the opposite case. The labels at estimated random effects BRE to ISTRO denote different banks in the sample. The asterisk symbols in the superscripts of estimated fixed effects are traditional indicators for coefficient significance broadly used in statistics: three asterisks "***" show significance at 0.001.

(§)For RADI to SBMO, estimated fixed-effect intercepts must be adjusted additively by random effects estimated for individual banks in order to identify bank-specific intercepts. (†)The regressor log(TL) is indented to capture heterogeneity induced potentially by size effect.

TABLE H.2. Regressions explaining efficiency scores calculated under the "black-box" approaches by inclusion or exclusion of individual production variables.

DEA model	RADI			RADO			SBMI			SBMO			**All (four) pooled**		
Regression	Full	BIC	AICc	Full	BIC	AICc	Full	BIC	AICc	Full	BIC	AICc	Full	BIC	AICc
Estimated fixed effects$^{\#}$															
Intercept§	-0.586^{***}	-0.516^{***}	-0.546^{***}	-0.559^{***}	-0.500^{***}	-0.516^{***}	-0.621^{***}	-0.553^{***}	-0.655^{***}	-0.498^{***}	-0.403^{***}	-0.474^{***}	-0.596^{***}	-0.539^{***}	-0.596^{***}
log(TL)†	0.080^{***}	0.080^{***}	0.080^{***}	0.084^{***}	0.084^{***}	0.084^{***}	0.079^{***}	0.079^{***}	0.079^{***}	0.079^{***}	0.079^{***}	0.079^{***}	0.083^{***}	0.083^{***}	0.083^{***}
i_TC	0.083^{**}		0.032^{*}	0.049^{*}			0.078^{**}		0.089^{***}	0.025^{ns}			0.059^{***}		0.059^{***}
i_TOE	0.111^{***}	0.073^{***}	0.110^{***}	0.074^{***}	0.036^{***}	0.056^{**}	$0.044^{\bullet}$		0.066^{***}	0.058^{*}		0.058^{*}	0.072^{***}	0.046^{***}	0.072^{***}
i_TP^{+}	$0.052^{\bullet}$			0.077^{**}	0.044^{**}	0.055^{***}	0.095^{**}	0.053^{**}	0.110^{***}	-0.065^{*}	-0.073^{***}	-0.090^{***}	0.040^{**}		0.040^{**}
i_PE	0.049^{*}			0.041^{*}			0.066^{**}		0.061^{**}	0.035^{ns}			0.048^{***}		0.048^{***}
i_TFA	0.029^{ns}		0.033^{ns}	$0.034^{\bullet}$		$0.028^{\bullet}$	-0.028^{ns}	-0.052^{***}		$0.044^{\bullet}$		0.050^{*}	$0.020^{\bullet}$		$0.020^{\bullet}$
i_NoE	0.049^{*}			$0.039^{\bullet}$			0.054^{*}		0.056^{*}	0.029^{ns}			0.043^{***}		0.043^{***}
i_IE	$0.036^{\bullet}$			0.049^{**}		0.034^{*}	0.001^{ns}	-0.058^{***}		0.049^{*}	0.083^{***}	0.040^{*}	0.034^{**}		0.034^{**}
i_NIE	0.162^{***}	0.147^{***}	0.159^{***}	0.127^{***}	0.093^{***}	0.125^{***}	$0.063^{\bullet}$		0.102^{***}	-0.075^{*}	-0.119^{***}	-0.067^{*}	0.069^{***}	0.049^{***}	0.069^{***}
i_TOOE	0.065^{*}		0.057^{*}	0.041^{ns}			0.044^{ns}			0.318^{***}	0.319^{***}	0.307^{***}	0.117^{***}	0.096^{***}	0.117^{***}
o_OEA	0.211^{***}	0.234^{***}	0.222^{***}	0.152^{***}	0.172^{***}	0.159^{***}	0.223^{***}	0.252^{***}	0.229^{***}	0.131^{***}	0.103^{***}	0.133^{***}	0.179^{***}	0.192^{***}	0.179^{***}
o_NonII	0.036^{**}	0.032^{**}	0.028^{**}	0.047^{***}	0.039^{***}	0.044^{***}	0.039^{**}		0.033^{**}	0.005^{ns}			0.032^{***}	0.026^{***}	0.032^{***}
o_II	0.250^{***}	0.269^{***}	0.303^{***}	0.156^{***}	0.196^{***}	0.172^{***}	0.264^{***}	0.359^{***}	0.310^{***}	0.106^{*}		0.132^{***}	0.194^{***}	0.223^{***}	0.194^{***}
o_TD$^{\Delta}$	-0.096^{***}	-0.087^{***}	-0.085^{***}	-0.077^{***}	-0.055^{***}	-0.071^{***}	-0.039^{**}		-0.033^{**}	-0.118^{***}	-0.133^{***}	-0.115^{***}	-0.082^{***}	-0.072^{***}	-0.082^{***}
Estimated random effects (effects pertaining to single banks)															
BRE	−0.103	−0.103	−0.103	−0.116	−0.116	−0.116	−0.118	−0.118	−0.118	−0.119	−0.119	−0.119	−0.112	−0.112	−0.112
CAC	0.126	0.126	0.126	0.092	0.092	0.092	0.108	0.108	0.108	0.168	0.167	0.168	0.126	0.126	0.126
CAG	0.084	0.084	0.084	−0.008	−0.008	−0.008	0.067	0.067	0.067	−0.026	−0.026	−0.026	0.034	0.034	0.034
CITI	0.073	0.073	0.073	0.079	0.079	0.078	0.119	0.119	0.118	0.098	0.098	0.098	0.094	0.094	0.094
CSOB	−0.064	−0.063	−0.064	−0.070	−0.070	−0.070	−0.067	−0.066	−0.067	−0.054	−0.055	−0.054	−0.069	−0.069	−0.069
CSOBss	−0.113	−0.113	−0.113	−0.111	−0.111	−0.111	−0.105	−0.105	−0.105	−0.148	−0.148	−0.148	−0.118	−0.118	−0.118
ING	0.085	0.086	0.085	0.114	0.114	0.114	0.085	0.085	0.085	0.158	0.158	0.158	0.112	0.113	0.112
JT	−0.009	−0.010	−0.009	0.088	0.088	0.088	−0.013	−0.013	−0.013	0.057	0.057	0.057	0.030	0.030	0.030
KOBA	0.114	0.114	0.114	0.089	0.089	0.089	0.127	0.127	0.127	0.130	0.130	0.130	0.116	0.116	0.116
OBER	0.045	0.045	0.045	−0.039	−0.039	−0.039	0.026	0.026	0.026	−0.011	−0.012	−0.011	0.009	0.009	0.009
OTP	−0.272	−0.272	−0.272	−0.213	−0.213	−0.213	−0.302	−0.302	−0.302	−0.276	−0.276	−0.276	−0.270	−0.270	−0.270
POBA	0.016	0.017	0.016	0.027	0.027	0.027	0.028	0.029	0.028	−0.020	−0.020	−0.020	0.011	0.011	0.011
PRIMA	−0.112	−0.111	−0.112	−0.088	−0.088	−0.088	−0.151	−0.152	−0.151	−0.103	−0.103	−0.103	−0.118	−0.118	−0.118
PRIVAT	0.086	0.087	0.086	0.094	0.094	0.094	0.112	0.112	0.112	0.078	0.077	0.078	0.095	0.095	0.095
PSS	0.104	0.104	0.104	0.086	0.086	0.086	0.071	0.071	0.071	0.071	0.071	0.071	0.080	0.080	0.080
RBS	0.294	0.295	0.294	0.224	0.223	0.224	0.322	0.322	0.322	0.240	0.240	0.240	0.279	0.279	0.279
SBER	−0.257	−0.257	−0.257	−0.202	−0.202	−0.202	−0.273	−0.274	−0.273	−0.275	−0.275	−0.275	−0.257	−0.257	−0.257
SLSP	−0.008	−0.008	−0.008	−0.014	−0.014	−0.014	−0.004	−0.004	−0.004	0.025	0.025	0.025	−0.007	−0.007	−0.007
SZRB	−0.019	−0.019	−0.019	0.024	0.024	0.024	−0.008	−0.008	−0.008	−0.011	−0.011	−0.011	−0.003	−0.003	−0.003

Table H.2. Continued.

DEA model	RADI			RADO			SBMI			SBMO			All (four) pooled		
Regression	Full	BIC	AICc	Full	BIC	AICc	Full	BIC	AICc	Full	BIC	AICc	Full	BIC	AICc
Estimated random effects (effects pertaining to single banks)															
TATRA	−0.026	−0.026	−0.026	−0.052	−0.052	−0.052	−0.009	−0.009	−0.009	−0.013	−0.013	−0.013	−0.032	−0.032	−0.032
UNICB	−0.021	−0.022	−0.021	−0.035	−0.035	−0.035	−0.019	−0.020	−0.020	−0.033	−0.033	−0.033	−0.032	−0.032	−0.032
VUB	−0.003	−0.003	−0.003	−0.034	−0.034	−0.034	−0.024	−0.024	−0.024	0.001	0.001	0.001	−0.022	−0.022	−0.022
WUST	−0.142	−0.142	−0.142	−0.107	−0.107	−0.107	−0.156	−0.156	−0.156	−0.144	−0.144	−0.144	−0.137	−0.137	−0.137
HSBC	0.279	0.277	0.279	0.273	0.272	0.272	0.372	0.374	0.371	0.263	0.265	0.263	0.322	0.322	0.322
HVB	0.130	0.131	0.130	0.116	0.116	0.116	0.117	0.116	0.117	0.122	0.122	0.122	0.131	0.131	0.131
ISTRO	−0.287	−0.287	−0.287	−0.216	−0.215	−0.215	−0.304	−0.305	−0.305	−0.178	−0.179	−0.178	−0.262	−0.262	−0.262
Quality of the fitted models															
adjusted R^2	0.555	0.553	0.554	0.545	0.543	0.544	0.527	0.523	0.527	0.478	0.476	0.478	0.606	0.685	0.606
BIC	−1074	−1155	−1124	−1686	−1756	−1736	−664	−750	−701	−296	−380	−351	−3481	−3554	−3481
AICc	−1176	−1215	−1202	−1788	−1822	−1814	−766	−804	−786	−399	−440	−430	−3606	−3635	−3606
logLik	605	618	614	911	922	920	400	411	407	216	230	228	1820	1829	1820

Notes. The descriptor "Full" reports a full model with all regressors entered into the regression equation, whereas the labels "BIC" and "AICc" indicate either models with regressors selected by optimizing the information criteria BIC and AICc or their respective values under the assumption of Gaussianity of errors. The label "logLik" represents traditional Gaussian log-likelihood. The report makes use of the labels for production variables introduced and employed in the tables of Appendices A and C, but now the prefixes "i_" and "o_" point out that a production variable at which they stand is placed on the input or output side, respectively. This is coded by means of a dummy variable that attains a value of 1 if the production variable in question appears as an input (for "i_") or output (for "o_"), and a zero value elsewhere. The labels announcing estimated random effects BRE to ISTRO answer to different banks in the sample. The symbols in the superscripts of estimated fixed effects are traditional labels for coefficient significance common in statistics: three asterisks "***" show significance at 0.001, two asterisks "**" indicate significance at 0.01, one asterisk "*" announces significance at 0.05, whereas the dot symbol "$^{\bullet}$" signals significance at 0.10. Finally, the superscript "^{ns}" informs of a p-value greater than 0.10.

$^{(\#)}$TL are present on the output side in each specifications considered so they are dropped from the list of regressors. OPA appears only in the input-output specification P3 on the output side and is perfectly correlated with NIE and TOOE on the input side, and thus "o_OPA" is skipped. Finally, TD on the output side (production or quasi-production specifications) is collinear with TD and TDSTF on the input side (intermediation specifications). To avoid the collinearity trap, only "o_TD" is listed among the regressors and "i_TD" and "i_TDSTF" are skipped. $^{(\S)}$Estimated fixed-effect intercepts must be adjusted additively by random effects estimated for individual banks in order to identify bank-specific intercepts. $^{(\dagger)}$The regressor log(TL) is indented to capture heterogeneity induced potentially by size effect. $^{(+)}$TP appears on the input side whenever NcFTI and TOOI appear on the output side, which occurs for the input-output specifications I2 and P8. To avoid the collinearity trap, "o_NcFTI" and "o_TOOI" are not considered among the regressors and "i_TP" represents them all. $^{(\Delta)}$This is actually a dummy variable that distinguishes whether a production or quasi-production specification is adopted ("o_TD" = 1) or an intermediation specification is applied ("o_TD" = 0).

Acknowledgements. This paper arose in partial fulfilment of project VEGA #1/0608/19.

References

[1] H. Ahn and M.H. Le, An insight into the specification of the input-output set for DEA-based bank efficiency measurement. *Manage. Rev. Q.* **64** (2014) 3–37.

[2] A. Amirteimoori, A. Emrouznejad and L. Khoshandam, Classifying flexible measures in data envelopment analysis: a slacks-based measure. *Measurement* **46** (2013) 4100–4107.

[3] N.K. Avkiran, The evidence of efficiency gains: the role of mergers and the benefits to the public. *J. Banking Finance* **23** (1999) 991–1013.

[4] B. Banerjee, Banking sector efficiency in new EU member states: a survey of cross-country evidence. *Eastern Eur. Econ.* **50** (2012) 81–115.

[5] R.D. Banker and R. Natarajan, Evaluating contextual variables affecting productivity using data envelopment analysis. *Oper. Res.* **56** (2008) 48–58.

[6] R.D. Banker, R. Natarajan and D. Zhang, Two-stage estimation of the impact of contextual variables in stochastic frontier production function models using data envelopment analysis: second stage OLS versus bootstrap approaches. *Eur. J. Oper. Res.* **278** (2019) 368–384.

[7] D. Bates, M. Maechler, B. Bolker, S. Walker, R.H. Bojesen Christensen, H. Singmann, B. Dai, F. Scheipl, G. Grothendieck, P. Green, J. Fox, lme4: Linear mixed-effects models using "Eigen" and S4. R package version 1.1-19. Available from: https://CRAN.R-project.org/package=lme4 (2018).

[8] S.A. Berg, F.R. Forsund and E.S. Jansen, Technical efficiency of Norwegian banks: the non-parametric approach to efficiency measurement. *J. Prod. Anal.* **2** (1991) 127–142.

[9] A.N. Berger and D.B. Humphrey, Efficiency of financial institutions: international survey and directions for future research. *Eur. J. Oper. Res.* **98** (1997) 175–212.

[10] A.N. Berger, J.H. Leusner and J.J. Mingo, The efficiency of bank branches. *J. Monetary Econ.* **40** (1997) 141–162.

[11] M. Berkelaar, *et al.* lpSolve: Interface to "Lp_solve" v. 5.5 to solve linear/integer programs. R package version 5.6.13. Available from: https://CRAN.R-project.org/package=lpSolve (2015).

[12] A. Bhattacharyya, C.A.K. Lovell and P. Sahay, The impact of liberalization on the productive efficiency of Indian commercial banks. *Eur. J. Oper. Res.* **98** (1997) 332–345.

[13] P. Bogetoft and L. Otto, Benchmarking: Benchmark and frontier analysis using DEA and SFA. R package version 0.26. Available from: https://CRAN.R-project.org/package=Benchmarking (2015).

[14] M. Boďa, Market power and efficiency as the source of performance in banking: a case study of the Slovak banking sector. Int. Rev. Appl. Econ. **32** (2017) 589–619.

[15] M. Boďa, Zmena produktivity vo viacročných obdobiach: Hicksov-Moorsteenov index, jeho dekompozícia a banková aplikácia [Productivity change in multi-year periods: the Hicks-Moorsteen index, its decomposition and bank application]. *Politická ekonomie* **67** (2019) 157–180.

[16] M. Boďa and E. Zimková, Efficiency in the Slovak banking industry: A comparison of three approaches. *Prague Econ. Pap.* **24** (2015) 434–451.

[17] M. Boďa and E. Zimková, Malmquist index analysis of the recent development of the Slovak banking sector from two different angles. *Econ. Change Restructuring* **50** (2017) 95–131.

[18] S. Bradley, G. Johnes and J. Millington, The effect of competition on the efficiency of secondary schools in England. *Eur. J. Oper. Res.* **135** (2001) 545–568.

[19] H.C. Bravo, Rcsdp: R interface to the CSDP semidefinite programming library. R package version 0.1.55. Available from: https://CRAN.R-project.org/package=Rcsdp (2016).

[20] K.P. Burnham and D.R. Anderson, Model Selection and Multimodel Inference: A Practical Information-Theoretic Approach, 2nd edition. Springer, New York (2002).

[21] V. Calcagno, glmulti: Model selection and multimodel inference made easy. R package version 1.0.7. Available from: https://CRAN.R-project.org/package=glmulti (2013).

[22] B. Casu and P. Molyneux, A comparative study of efficiency in European banking. *Appl. Econ.* **35** (2003) 1865–1876.

[23] B. Casu and C. Girardone, Integration and efficiency convergence in EU banking markets. *Omega* **38** (2010) 260–267.

[24] Y. Chen, W.D. Cook, C. Kao and J. Zhu, Network DEA pitfalls: divisional efficiency and frontier projection. In: Data envelopment analysis: a handbook on the modelling of internal structures and networks, edited by W.D. Cook and J. Zhu. Springer, New York (2014) 31–54.

[25] J.A. Chilingerian and D. Sherman, Health-care applications: from hospitals to physicians, from productive efficiency to quality frontiers. In: Handbook on data envelopment analysis, 2nd edition, edited by W.W. Cooper, L.M. Seiford and J. Zhu. Springer, New York (2011), 445–493.

[26] W.D. Cook, K. Tone and J. Zhu, Data envelopment analysis: prior to choosing a model. *Omega* **44** (2014) 1–4.

[27] W.W. Cooper, L.M. Seiford and K. Tone, Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software, 2nd edition. Springer, New York (2007).

[28] D.R. Cox and E.J. Snell, The choice of variables in observational studies. *J. R. Stat. S. Ser. C (Appl. Stat.)* **23** (1974) 51–59.

[29] C. Daraio and L. Simar, Advanced robust and nonparametric methods in efficiency analysis. In: Methodology and Applications. Springer, New York (2007).

[30] C.S. Davis, Statistical Methods for the Analysis of Repeated Measurements. Springer, New York (2002).

[31] K. De Witte and R.C. Marques, Designing performance incentives, an international benchmark study in the water sector. *Cent. Eur. J. Oper. Res.* **18** (2010) 189–220.

[32] L. Drake, M.J.B. Hall and R. Simper, Bank modelling methodologies: a comparative non-parametric analysis of efficiency in the Japanese banking sector. *J. Int. Financial Markets Inst. Money* **19** (2009) 1–15.

[33] M. Duygun Fethi and F. Pasiouras, Assessing bank efficiency and performance with operational research and artificial intelligence techniques: a survey. *Eur. J. Oper. Res.* **204** (2010) 189–198.

[34] R.G. Dyson, R. Allen, A.S. Camanho, V.V. Podinovski and E.A. Shale, Pitfalls and protocols in DEA. *Eur. J. Oper. Res.* **132** (2001) 245–259.

[35] A. Emrouznejad and K. de Witte, COOPER-framework: a unified process for non-parametric projects. *Eur. J. Oper. Res.* **207** (2010) 1573–1586.

[36] O. Felix Ayadi, A.O. Adebayo and E. Omolehinwa, Bank performance measurement in a developing economy: an application of data envelopment analysis. *Manage. Finance* **24** (1998) 5–16.

[37] H. Fukuyama and R. Matousek, Efficiency of Turkish banking: two-stage network system. Variable returns to scale model. *J. Int. Financial Markets Inst. Money* **21** (2011) 75–91.

[38] H. Fukuyama and R. Matousek, Modelling bank performance: a network DEA approach. *Eur. J. Oper. Res.* **259** (2017) 721–732.

[39] H. Fukuyama and W.L. Weber, A slacks-based inefficiency measure for a two-stage system with bad outputs. *Omega* **38** (2010) 398–409.

[40] H. Fukuyama and W.L. Weber, Measuring Japanese bank performance: a dynamic network DEA approach. *J. Product. Anal.* **44** (2015) 249–264.

[41] D.U.A. Galagedera and P. Silvapulle, Experimental evidence on robustness of data envelopment analysis. *J. Oper. Res. Soc.* **54** (2003) 654–660.

[42] I.M. García-Sánchez, L. Rodríguez-Domínguez and J. Parra-Domínguez, Yearly evolution of police efficiency in Spain and explanatory factors. Cent. *Eur. J. Oper. Res.* **21** (2013) 31–62.

[43] C. Genest and F. Verret, Locally most powerful rank tests of independence for copula models. *J. Nonparametric Stat.* **17** (2005) 521–539.

[44] I. Gijbels and J. Mielniczuk, Estimating the density of a copula function. *Commun. Stat. – Theory Methods* **19** (1990) 445–464.

[45] R. Gulati and S. Kumar, Analysing banks' intermediation and operating efficiencies using the two-stage network DEA model: the case of India. *Int. J. Product. Perform. Manage.* **66** (2017) 500–516.

[46] E.N. Gunay and A. Tektas, Efficiency analysis of the Turkish banking sector in precrisis and crisis period: a DEA approach. *Contemp. Econ. Policy* **24** (2006) 418–431.

[47] G. Heinze and D. Dunkler, Five myths about variable selection. *Transplant Int.* **30** (2017) 6–10.

[48] G. Heinze, C. Wallish and D. Dunkler, Variable selection – a review and recommendations for the practicing statistician. *Biometrical J.* **60** (2018) 431–449.

[49] D. Holod and H.F. Lewis, Resolving the deposit dilemma: a new DEA bank efficiency model. *J. Banking Finance* **35** (2011) 2801–2810.

[50] W.C. Hunter and S.G. Timme, Core deposits and physical capital: a reexamination of bank scale economies and efficiency with quasi-fixed inputs. *J. Money Credit Banking* **27** (1995) 165–185.

[51] Z. Iršová and T. Havránek, Bank efficiency in transitional countries: sensitivity to stochastic frontier design. *Transition Stud. Rev.* **18** (2011) 230–270.

[52] L. Jenkins and M. Anderson, A multivariate statistical approach to reducing the number of variables in data envelopment analysis. *Eur. J. Oper. Res.* **147** (2003) 51–61.

[53] I. Jemric and B. Vujcic, Efficiency of banks in Croatia: a DEA approach. *Comp. Econ. Stud.* **44** (2002) 169–193.

[54] H. Joe, Relative entropy measures of multivariate dependence. *J. Am. Stat. Assoc.* **84** (1989) 157–164.

[55] H. Joe, Dependence Modelling with Copulas. CRC Press/Taylor & Francis, Boca Raton (2015).

[56] A.L. Johnson and T. Kuosmanen, One-stage and two-stage DEA estimation of the effects of contextual variables. *Eur. J. Oper. Res.* **220** (2012) 559–570.

[57] F. Kamarudin, F. Sufian, F.W. Loong and N.A.M. Anwar, Assessing the domestic and foreign Islamic banks efficiency: insights from selected Southeast Asian countries. *Future Bus. J.* **3** (2017) 33–46.

[58] K. Kenjegalieva, R. Simper, T. Weyman-Jones and V. Zelenyuk, Comparative analysis of banking production frameworks in Eastern European financial markets. *Eur. J. Oper. Res.* **198** (2009) 326–340.

[59] K. Kočišová, Application of the DEA on the measurement of efficiency in the EU countries. *Agric. Econ. – Czech* **61** (2015) 51–62.

[60] I. Kouiki and A. Al-Nasser, The implication of banking competition: evidence from African countries. *Res. Int. Bus. Finance* **39** (2017) 878–895.

[61] R. Levine, Financial development and economic growth: views and agenda. *J. Econ. Literature* **35** (1997) 688–726.

[62] Y. Li, X. Shi, M. Yang and L. Liang, Variable selection in data envelopment analysis via Akaike's information criteria. *Ann. Oper. Res.* **253** 453–476.

[63] G.H. Lim and D.S. Randawa, Competition, liberalization and efficiency: evidence from a two-stage banking model on banks in Hong Kong and Singapore. *Manage. Finance* **31** (2005) 52–77.

[64] E.H. Linfoot, An informational measure of correlation. *Info. Control* **1** (1957) 85–89.

[65] Y. Luo, G. Bi and L. Liang, Input/output indicator selection for DEA efficiency evaluation: an empirical study of Chinese commercial banks. *Expert Syst. App.* **39** (2012) 1118–1123.

[66] D. Marchetti and P. Wanke, Brazil's rail freight transport: efficiency analysis using two-stage DEA and cluster-driven public policies. *Soc.-Econ. Plan. Sci.* **59** (2017) 26–42.

[67] M.M. Mostafa, Modeling the efficiency of top Arab banks: a DEA-neural network approach. *Expert Syst. App.* **36** (2009) 309–320.

[68] T. Nagler and K. Wen, kdecopula: Kernel smoothing for bivariate copula densities. R package, version 0.9.1. Available from: https://CRAN.R-project.org/package=kdecopula (2017).

[69] R.B. Nelsen, An Introduction to Copulas, 2nd edition. Springer, New York (2006).

[70] J. Ouenniche, S. Carrales, K. Tone and H. Fukuyama, An account of DEA-based contributions in the banking sector. In: Advances in DEA Theory and Applications: With Extensions to Forecasting Models. Wiley, New York (2017) 141–171.

[71] J.M. Pastor, F. Pérez and J. Quesada, Efficiency analysis in banking firms: an international comparison. *Eur. J. Oper. Res.* **98** (1997) 395–407.

[72] J.T. Pastor, J.L. Ruiz and I. Sirvent, A statistical test for nested radial DEA models. *Oper. Res.* **50** (2002) 728–735.

[73] R Core Team, R: A language and environment for statistical computing. Vienna: R Foundation for Statistical Computing. Available from: http://www.r-project.org (2017).

[74] S.C. Ray, Data Envelopment Analysis: Theory and Techniques for Economics and Operations Research. Cambridge University Press, Cambridge (2004).

[75] A. Resti, Evaluating the cost-efficiency of the Italian banking system: What can be learned from the joint application of parametric and non-parametric techniques. *J. Banking Finance* **21** (1997) 221–250.

[76] A.N. Rezitis, Productivity growth in the Greek banking industry: a non-parametric approach. *J. Appl. Econ.* **9** (2006) 119–138.

[77] T. Rosenmayer, Using data envelopment analysis: a case of universities. *Rev. Econ. Perspect.* **14** (2014) 34–54.

[78] J. Ruggiero, Impact assessment of input omission on DEA. *Int. J. Inf. Technol. Decision Making* **84** (2005) 359–368.

[79] M. Sathye, Efficiency of banks in a developing economy: the case of India. *Eur. J. Oper. Res.* **148** (2003) 662–671.

[80] L. Simar and P.W. Wilson, Estimation and inference in two-stage, semi-parametric models of productive efficiency. *J. Econometrics* **136** (2007) 31–64.

[81] L. Simar and P.W. Wilson, Two-stage DEA: caveat emptor. *J. Econometrics* **36** (2011) 205–218.

[82] P. Smith, Model misspecification in data envelopment analysis. *Ann. Oper. Res.* **73** (1997) 233–252.

[83] T. Subramanyam and C.S. Reddy, Measuring the risk efficiency in Indian commercial banking – a DEA approach. *J. Econ. Bus.* **11** (2008) 76–105.

[84] F. Sufian, Determinants of bank efficiency during unstable macroeconomic environment: empirical evidence from Malaysia. *Res. Int. Bus. Finance* **23** (2009) 54–77.

[85] F. Sufian, F. Kamarudin and A. Nassir, Determinants of efficiency in the Malaysian banking sector: does bank origins matter? *Intell. Econ.* **10** (2016) 38–54.

[86] E. Thanassoulis, Introduction to the Theory and Application of Data Envelopment Analysis. Kluwer Academic Publishers, Norwell (2001).

[87] M. Toloo, M. Barat and A. Masoumzadeh, Selective measures in data envelopment analysis. *Ann. Oper. Res.* **226** (2015) 623–642.

[88] K. Tone, A slacks-based measure of efficiency in data envelopment analysis. *Eur. J. Oper. Res.* **130** (2001) 498–509.

[89] K. Tone and M. Tsutsui, Network DEA: a slacks-based measure approach. GRIPS Policy Information Center Discussion Paper: 07-08. Accessed 13 October 2017. Available from: http://www.grips.ac.jp/r-center/wp-content/uploads/07-08.pdf (2007).

[90] K. Tone and M. Tsutsui, Network DEA: a slacks-based measure approach. *Eur. J. Oper. Res.* **197** (2009) 243–252.

[91] E. Tortosa-Ausina, Bank cost efficiency and output specification. *J. Prod. Anal.* **18** (2002) 199–222.

[92] T. Triki, I. Kouki, M.B. Dhaou and P. Calice, Bank regulation and efficiency: What works for Africa? *Res. Int. Bus. Finance* **39** (2017) 183–205.

[93] T. Ueda and Y. Hoshiai, Application of principal component analysis for parsimonious summarization of DEA inputs and/or outputs. *J. Oper. Res. Soc. Jpn.* **40** (1997) 466–478.

[94] J.M. Wagner and D.G. Shimshak, Stepwise selection of variables in data envelopment analysis: procedures and managerial perspectives. *Eur. J. Oper. Res.* **180** (2007) 57–67.

[95] K. Wang, W. Huang, J. Wu and Y.-N. Liu, Efficiency measures of the Chinese commercial banking system using an additive two-stage DEA. *Omega* **44** (2014) 5–20.