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A TRANSSHIPMENT MODEL FOR LOGISTICS MANAGEMENT AT INDIAN OIL CORPORATION

Badri Toppur[1,*] and Atanu Sanyal[2]

Abstract. Three refineries of the Indian Oil Corporation procure crude oil from suppliers, at production sites in the Persian Gulf, West Africa, North Africa, West Asia, and India. The crude oil is shipped to two ports in the state of Gujarat, India, by large tankers and medium sized vessels. From these two ports, the crude oil is pumped to the refineries *via* pipelines. The refineries have known capacity, that are different for the two types of crude oil. In this paper, the scaled-up problem has been modelled, as a transshipment network. Next, the concrete instance of the problem, has been solved using an LP solver. This was followed by post-optimality analysis of the solution. The flow values on all arcs, and optimal product mix, validate actual decisions. Precise shipping requirements obtained from the solution, are shared in advance with marine transporters to improve supply chain coordination.

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1. Introduction

The first oil well was drilled in 1859, by Colonel Edwin Drake, in Titusville, Pennsylvania. Soon Rockefeller commercialised Drake's example in the state of Ohio. Other prospectors followed suit, in various regions of the US. In the years that have passed since, the demand for petroleum products, initially for Kerosene as lamp oil, and then for Gasoline, Diesel, and Aviation fuel, has increased to a point that the demand is high and inelastic. The demand in Europe, still exceeds the supply by a considerable percentage. Some countries that are self-sufficient, still trade in oil. Daniel Yergin, the energy reporter of international repute, has vividly described the history of the energy sector, in his books [24]. The major petroleum companies such as British Petroleum, Shell, ConocoPhillips, Total, ExxonMobil, Texaco, and Chevron, have ruled the market through most of the twentieth century, drilling wells in the North Sea, Arabia, Asia, and other continents. During, and after the second world war, new suppliers have emerged from the Middle-East, especially in Bahrain, Saudi Arabia, Kuwait, UAE, Iran, Iraq, and several other countries where the geologic structure has been studied and discovered to be rich with deposits of the resource. Exploration has been done in many parts of the world, both onshore, and offshore, and there are now producers in Argentina, Azerbaijan, Brazil, Canada, China, Gabon, Ecuador, India, Libya, Malaysia, Mexico, Nigeria, Russia, and Venezuela, to name a few nations. Consequently, a global supply chain

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[1] Rajalakshmi School of Business, Chennai 600124, India.
[2] Panipat Naptha Cracker, Panipat 132140, Haryana, India.
*Corresponding author: badri.toppur@rsb.edu.in

has emerged, of enormous proportions and complexity, with trade between distant parts of the world. The routes have been studied and optimised with great feats of engineering such as the Panama canal and later, the Suez canal. The Panama canal allows cargo traffic from Alaska to the South-eastern states of USA, to bypass sailing around Cape Horn and saves capital cost and voyage time. Similarly, the Suez Canal, allows vessels a sea route to European destinations, such as Rotterdam, or Marseilles-Fos from the ports in the Arabian peninsula. This passage does not require circumnavigating the African cape, and results in similar savings. The ports of Singapore, Malaysia and Indonesia are a popular transit point for traffic to and from the Far East. Pipelines such as the Sumed in Egypt, the Alaskan pipeline up to Valdez, and Trans-Alpine Pipeline in Europe, are a cheap alternative to other modes of transportation. On the other hand, pipeline transportation technology is quite complex. Where the infrastructure, and technology has been put in place, crude oil and product pipelines, have shortened transportation time and reduced the variable costs.

Given the increased multi-player competition that has emerged, replacing the monopolistic situation that was present earlier, companies are always looking to make their own operations more efficient, and maintain the margins. For non-integrated Oil & Gas companies, the supplier selection, and order quantity decisions need to be dynamic, flexible, but rational, responding to changing policies of the regulators such as the Organisation of Petroleum Exporting Companies (OPEC), who affect the oil prices and indirectly, the transportation rates. It is for dealing with this changing scenario, that we have attempted to model the system. In this case study, we look at a transshipment model that has developed around the procurement, and transportation strategies for a petroleum company in India, and suggest an annual economic policy, that is best. Due to this single period focus, the model can be referred to as static, to differentiate it from dynamic models, that schedule activities over a planning horizon of many periods. Using the model developed in this paper, other petroleum companies can gauge, at a tactical level, for their own system configuration, the effect of changing margins, supplies, pipeline and refining capacities, on the objective function, product mix and performance measures of their pipelines and refineries. The model solution also suggests the optimal shipping pattern, useful information for the marine transporters to make their schedules. For instance, if certain vessel types are not available in sufficient number, alternative carriers can be compared for cost and benefit.

The paper is organized in the following manner after this introductory section: the literature survey is in the second section. The data and problem definition shared by the industry practitioners are in the third section. The general transshipment model that has been formulated, is described in the fourth section. Results and conclusions for the concrete case study, are in the final sections.

2. Literature review

Over the last fifty years, the world has witnessed the nationalization of the aforementioned multinational corporations (MNC). This has disrupted to a considerable extent the equitable concessions and agreements existing between the MNCs and the nations in earlier decades. These political issues of expropriation are discussed by Mathew Simmons, David G. Victor, David R. Hults, and Mark Thurber, in their books and is not dwelt upon in this paper, which is more concerned with the tactical and operational aspects, and less with the corporate strategy [20, 22]. Ross and Sloan, in their book, focus on corporate strategy in the petroleum sector, leading to a discussion of leadership models [15]. The policy changes, and the new trade agreements benefit the governments and citizens of these emerging nations and inspire economic development through taxes. High and inelastic demand in many parts of the world, especially Europe, has led to keen market competition. Countries such as Iran and Venezuela have also faced embargoes and sanctions, which is a reason, for consuming refineries to consider a change in supplier. After a disruption, refineries and marketing companies have had to search among more choices, for suitable suppliers. Petroleum expert, Randall has emphasized information integrity, in his book, on enterprise risk management (ERM) [12]. Barbosa-Póvoa in her survey paper, provides a comprehensive analysis of process supply chains [1]. Klepikov and Klepikov, discuss the demand and production scenario, in the context of 12 countries of North Europe that are most significant in the sector [9]. According to their findings, Norway, UK and Denmark are the largest producers of crude oil, as of 2015. When it comes to refinery capacity and throughput, Germany, UK and Netherlands dominate the other countries. The major consumers, are in Germany, Belgium and Poland.

Crude oil price benchmarks such as the Brent, West Texas Intermediate (WTI), and Dubai, are at about 60% of their 2013 value of $100/barrel. This fall in price has largely been ascribed to the advent of alternative fuels such as shale gas in the USA, and the refining of heavy crudes. The emergence of hybrid and completely electric cars that have become commercialised successfully are also driving suppliers to reduce the price. However, all countries are not yet mass producing electric cars, and the conventional automobile will remain extremely popular for decades to come. Therefore prices will be competitive for those markets.

Integer Programming models for various types of problems, such as knapsack problem, the set covering problem, the set partitioning problem, and the fixed charge problems, are available in the textbook by Salkin and Mathur [17]. These integer programming models are most relevant to the petroleum sector, because a large number of models turn out to be Mixed Integer Linear Programs (MILP) or Mixed Integer Non-Linear Programs (MINLP). The earliest applications of mathematical programming (MP) in the petroleum industry are documented by Bodington and Baker [2]. They state in their paper, that the first publication in the area, is from 1955, by Esso Standard Oil Company, giving the solution of refinery problems using linear programming; another pioneering paper was by Charnes, Cooper, and Mellon. Various developments in mathematical programming applications are listed by decade; the emphasis on software development, and model management tools from the nineteen seventies, through the eighties is narrated, in this informative article. Their forecast for the nineties, was that enterprise-wide systems would evolve that use MP. Secondly, non-linear programming would be used more frequently, and thirdly, that direct optimization of simulations would eventually replace LP formulations. In his 1995 paper, Ronen, has described the downstream supply chain from the two main types of manufacturing plants in the oil industry, namely, refineries and lube plants [14]. The light products are those used as fuels, typically kerosene, gasoline or petrol, diesel oil, and aviation fuel. The heavier products from the refinery are raw material for lube plants, where they are converted to lube oils, greases, and waxes. His more recent work is a handbook on transportation with M. Christiansen. In a recent paper, Cerda *et al.* have contributed optimization approaches to blending problems [4].

Many academicians and practitioners have looked specifically, at the maritime inventory routing problem (MIRP) that is the focus of this paper. It concerns the transportation of the crude oil, from the production sites, to the refinery terminals. Sherali *et al.* have presented fleet management models and algorithms in 1999 [18]. The paper by Song and Furman, in 2013 builds upon the methods by Ronen and Christiansen [21]. They set their discussion in a general inventory routing problem for any bulk product, and then focus on maritime inventory routing. They use a time-space network framework for modelling the MIR at an operational scope, and a branch-and-cut algorithm for solving the formulation. Papageorgiou *et al.* have put together, in 2014, a library of maritime inventory routing problem instances, along with their core model, and benchmark results [11]. Siddiqui and Verma, took a bi-objective approach that also considered risk premium in addition to transportation cost in their optimisation model [19]. The scheduling and freight forwarding of tankers and other vessels, is surveyed by Vilhelmsen *et al.* [23]. The work by Souto dos Santos Diz *et al.* in 2017 is specific to the case of improving operations at Brazilian refineries in the NNE region of that country [6]. Oliveira *et al.* give a framework for crude oil scheduling in an integrated terminal-refinery system [10]. A recent contribution to the literature in this area is by de Assis *et al.* who present an MINLP formulation for integrated operation management [5]. Their model of the supply chain is an extremely sophisticated mixed integer non-linear program. LP relaxations of the MINLP, are solved using the theory of McCormick envelopes. Rocha *et al.* have extensively modelled the operations along the entire supply chain at Petrobras, the Brazilian national oil company [13]. Their model is complex, and is hard to solve for industrial size problems. This forces them to use a heuristic to find a feasible solution, and then local search to improve the solution.

Pipeline freight costs in India, are 75% of the cost of the railway freight which was relied upon in an earlier era. The crude oil transportation pipelines from offshore production sites, to the refinery, or from the port-side to the refinery is another stage of petroleum logistics that has been studied scientifically. Cafaro *et al.* have studied the pipeline section of crude oil logistics in their paper [3]. The feasibility of expanding pipeline capacity in Western India, has been reported upon, by Sahith *et al.* [16]. Dyntar and Skvor, present their results, for a set of three refineries, sixteen warehouses, and twenty one pieces of pipeline, in the Czech Republic and Slovakia, using a pipe transportation simulator to do sensitivity analysis. Their effort has been useful to verify the product movement plan, and also the repair plan [7]. The input and output interfaces of their system are Excel based, and the central simulation software is *Witness* from Lanner Group.

The model developed in this paper, looks at both the MIR problem and the pipeline transportation, in the Indian Oil & Gas sector context, and will be useful for other petroleum companies that are using management science techniques for decision support. One of the features of the model that is a value addition, is the handling of flexible refining capacity for two types of crude oil.

3. Problem definition

The petroleum company uses two ports of call on the west coast of India. Those are Vadinar in Jamnagar district, and Mundra, in Kutch District, of Gujarat state. The two terminals are separated by the Gulf of Kutch. The company also operates three refineries in the west of India out of the nine that are operative in all of India. Taken together with two refineries operated by their subsidiaries, Chennai Petroleum Corporation Limited (CPCL), gives them a total capacity of 80.70 million metric tonnes per annum (MMTPA). We are modelling the supply chain in this western region of the country with historic company data, but current data will not make the model obsolete. Second, there is no reason, why the model cannot be extended and scaled-up, to all the ports, and the eleven refineries, nation-wide. In the next section, a general model is presented that can include all the ports and refineries. In discussions of crude oil, one of the many distinctions that are made, is that of the sulphur content. Crudes with high sulphur content (above 0.5%) are referred to as sour and crudes with low sulphur content are referred to as sweet. The sweet crude can be processed into high quality gasoline, and other products such as lubricants, greases and waxes. The sour crude is also used to make a similar range of petroleum products, but requires much more refining. There is also a distinction between heavy or light crude oil based upon the viscosity or specific gravity measurements, but we have not factored in those numbers into the constraints, due to their unavailability. The refineries have different capacities for processing the sour or sweet crude, which is one of the motivations for developing a new mathematical model for making the transportation decisions. One refinery is at Koyali, in the state of Gujarat with a refining capacity of 15 MMTPA, and with a maximum sour crude oil processing capability of 50%. Mathura, in the state of Uttar Pradesh is the location of the second refinery with a refining capacity of 9 MMTPA, and with maximum sour crude oil processing capability of 60%. The third refinery is at Panipat, in the state of Haryana with a refining capacity of 15 MMTPA, with a maximum sour crude oil processing capability of 90%. Refineries can process sweet crude oil in place of sour crude oil. Gross Refinery Margin (GRM) from sweet crude processing is \$8/barrel (bbl) whereas gross refinery margin from sour crude processing is \$6.5/barrel. Each type of crude oil is converted into a range of refined products, heavy and light, so in fact, we have two classes of refined products. About 95% of the crude oil is distilled into products, leaving 5% residue in the form of coke.

One comment about the standard international units is required, since those referring to supply, transportation capacity, and refining capacity are in tonnes and those referring to crude oil prices and refinery margins, are in barrels. The standard international units used are million metric tonnes per annum (MMTPA) and thousand metric tonnes (TMT). Since one metric tonne equals 7.3 barrels of oil, the profit margins, for example, can be converted to \$58 400/TMT for premium gasoline and \$47 450/TMT for ordinary gasoline. After obtaining the product mix in terms of tonnes, we convert it back to barrel terms.

The crude is purchased from three distant production sites of the world, and there is also indigenous crude oil supply from Bombay High, an offshore site in India. The current price of Dubai crude oil is \$61/bbl, the price of Brent crude oil from the North Sea region, is \$63/bbl and that of West Texas Intermediate is \$59. The price of other crude oils is fixed relative to these crude oil price benchmarks. Since we are working with given gross refinery margins in this model, the crude oil prices are not used as parameters at present.

3.1. Vessel capacities and freight charges

Transportation of crude to India is through three types of ships: very large crude carriers (VLCC), Suez Max vessels and Long Range (LRII) vessels. Regional options such as Aframax vessels, and global options such as the Ultra Large Crude Carriers (ULCC) are not used. These vessel categories are defined by their Dead Weight Tonnage (DWT) or carrying capacity. Large carriers must go around the Cape of Good Hope, South Africa, as they cannot pass fully laden, through the Suez Canal, due to the draught, although they may go safely through when the vessel is empty. Large vessels from the US have to navigate around Cape Horn, of South America. Suez Max vessels derive their name from their ability to comply with the restrictions of the shallow Suez Canal. The freight rates are set by WorldScale in London, and the charterer negotiates rates with the shipper with reference to these rates. Transporters incur Sailing cost, Demurrage cost or waiting time cost, and port cost. We refer to the total of these costs as the voyage cost. Some details about the three classes of ships used are given below:

(1) VLCCs have a capacity of 270 Thousand Metric Tonnes (TMT) or approximately, 2.0 million barrels (bbl). They are tankers that are more than 160 000 DWT. When in ballast they can transit the Suez canal, but when laden with cargo, they cannot.
(2) A Suez Max vessel has half the capacity of a VLCC, *i.e.* 135 TMT. They are in the range of 100 000–160 000 DWT. They can transit the Suez canal fully laden.
(3) The LR-II vessel has a capacity of 60 TMT or 0.45 million bbl. They are used for regional movement and their size is less than 80 000 DWT. More than two-thirds of the vessels used in the world fleet are of this size.

Crude from Persian Gulf is transported by VLCC or Suez Max, preferably VLCC at the freight charge of $2.0 million per voyage in VLCC or $1.5 million for Suez Max. The voyage time is 7 days with an inventory carrying cost per day at the rate of 8%. Crude from West Africa is transported exclusively by VLCC. The voyage time is 20 days to travel to Vadinar or Mundra, and the freight cost is $3.5 million per voyage, with inventory carrying cost per day, at the rate of 8%. Crude from North Africa, and West Asia is transported by Suez Max tankers *via* Suez Canal, with a voyage time of 10 days, freight cost of $2.0 million per voyage, and inventory carrying cost per day at the rate of 8%. Crude from Bombay High is transported in LR-II vessels with a voyage time of 2 days, freight charge of $0.4 million per voyage and inventory carrying cost per day at the rate of 8%. As mentioned above, we have made the assumption that the inventory carrying cost is factored into the voyage cost. The inventory management protocols and procedures, at the storage tanks at the ports and at the charge tanks of the refinery, are extremely complicated, and we have not included them in our study.

3.2. Crude oil quality and availability

The main resource for a variety of petroleum products, such as gasoline and lubes is crude oil. The raw material is graded according to certain attributes that affect the quality of the final product. The attributes of the crude oil, for example viscosity, and presence of sulphur and other impurities depend on the geology of the production site. Crude oil that is highly viscous such as that found in Canadian tar sands, are considered heavy crudes, and cause problems in pipeline transportation, because they do not flow easily. The technology to dilute heavy crude oil for enabling flow, or to extract shale gas which is even heavier, is an ongoing project for chemical engineers. For our research we have looked only at the Sulphur content, the absence of which is an indicator of a superior product. The availability at five production sites are listed below. We have converted the availability to tanker loads equivalent, to give the reader an idea of how many trips are required to transport the available supply from the production terminals, to the refinery terminals. In reality, one may use a combination of vessel types, that are of different carrying capacity.

(1) Crude oil from the Persian Gulf region, is sour, and the supply is approximately 15 MMTPA. This yields an upper bound of approximately 56 tanker loads equivalent. Although, all three refineries can process sour crude oil, the Panipat refinery has 90% capacity for sour crude. This could imply, that the crude oil from the Gulf region, is best directed to this refinery. On the other hand, the pipeline and rail freight costs to the other two refineries may be competitive. Making such difficult choices is another justification for why a mathematical model is required.

(2) West African crude oil is sweet, and the supply is approximately 17 MMTPA. This corresponds to an upper bound of 63 tanker loads equivalent. If Suez Max vessels are used this corresponds to approximately 126 vessel loads equivalent. Again, only those many loads need to be purchased, that contribute positively, to the objective function.
(3) North African and West Asian crude oil is sweet, and the supply is approximately 2 MMTPA. This corresponds to an upper bound of 15 Suez Max tanker loads equivalent.
(4) Indigenous crude oil from Bombay High, in India is sweet and the supply is approximately 2 MMTPA. This corresponds to approximately 33 LRII vessel loads equivalent. The ministry has a regulation, that only 750 TMT can be procured per month. Since our model is for an entire year, this regulation, does not effect the formulation at present.
(5) The United States of America, that has, some years back, lifted a ban on the export of petroleum, is the latest supplier of sweet crude oil to the company. One MMTPA is shipped by VLCC taking a month of voyage time, around Cape Horn[3].

3.3. Pipeline capacity and type

There are two pipelines in the service of the company. The freight costs for both pipelines, and railway transportation are given in Table 1. One is the Salaya–Mathura (SMPL) pipeline which is 2660 km long and of 30 MMTPA capacity. It can transport both sour crude oil and sweet crude oil in parcel size of 270 TMT for sour crude oil and between 60 and 135 TMT for sweet crude oil. Pumping rate of one VLCC crude oil is 3.5 days at the rate of 80 TMT/day or 5 84 000 barrels per day (BPD). At Vadinar, Indian Oil Corporation has a vast crude oil tank-farm of 18 tanks with a total capacity of 1.07 MMT. Indian Oil Corporation, also has crude oil storage tank farm at Viramgam in Gujarat with a total capacity of 0.331 MMT. Another storage tank farm at Chaksu in Rajasthan has six tanks with a total capacity of 0.219 MMT. Salaya–Mathura Pipeline branches off at Viramgam through a 148 km pipeline to Koyali Refinery in Vadodara. Further, the pipeline branches off at Chaksu to Mathura, and Panipat refineries. The Koyali refinery, is relatively close to the terminals, compared to the other two which are in the Northern states. The Mundra–Panipat (MPPL) pipeline, is 1194 km long, and supplies Panipat only. It is dedicated to carrying sour crude oil. MPPL originally had a capacity of 6 MMTPA that was augmented some years later to 9 MMTPA. The pipeline consists of a 74 km long pipeline from Mundra to Kandla which was hooked up to the existing system of Kandla–Panipat section of erstwhile Kandla–Bhatinda pipeline at Churwa, near Gandhidham. The pipeline utilizes Adani Port's Single Point Mooring (SPM) offshore crude oil terminal facilities and associated offshore and onshore pipelines. The crude oil tank farm consists of 12 crude oil storage tanks with total capacity of 0.499 MMT at Mundra [8]. For readers not familiar with the geography of the region, the pipeline stations can be easily traced on the detailed map in Figure 1 that is provided courtesy of the company.

Table 1. Pipeline and railroad freight costs.

Pipeline	Station	Rail freight Rs. /TMT	Pipeline freight Rs. /TMT	Rail ($)	Pipeline ($)
Vadinar	Koyali	759 600	569 700	10 851	8139
	Mathura	1 938 600	1 453 950	27 694	20 771
	Panipat	2 181 200	1 635 900	31 160	23 370
Mundra	Panipat	1 901 000	1 425 750	27 157	20 368
	Mathura	2 143 600	–	30 623	–
	Koyali	3 322 600	–	47 466	–

[3]The voyage cost information is not available at present, and so the supplier is not included in the model.

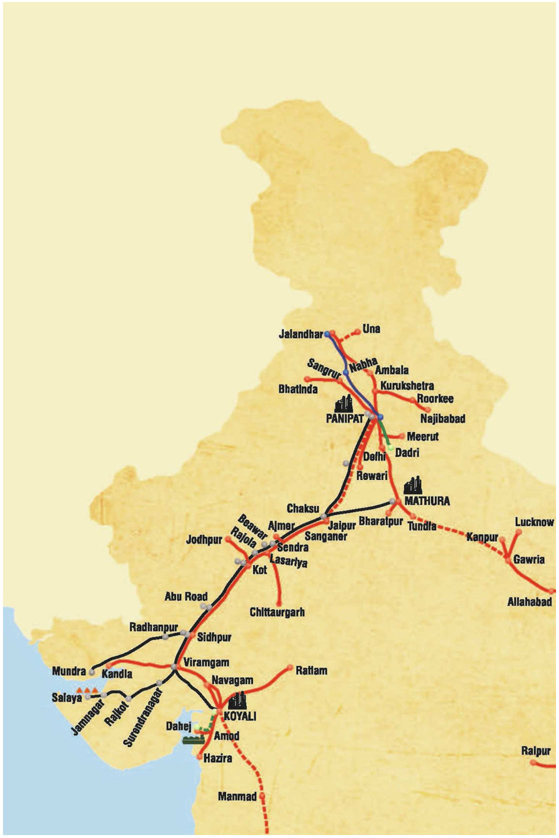

FIGURE 1. Refineries and pipelines in Western India, Source: IOCL.

4. General transshipment model and concrete instance

We first present the general transshipment model of arbitrary size, and then solve a concrete instance of it, with data provided by the company. In the general model, there are m production sites, or supply nodes, n transshipment nodes, and p refineries or demand nodes. The supply at each of the m supply nodes is given to be a_i, for $i = 1, \ldots, m$. The refining capacity at each of the p demand nodes is given to be R_k, for $k = 1, \ldots, p$. The sour crude refining capacity at each of the refineries is r_k, for $k = 1, \ldots, p$. The n transshipment nodes have no supply or demand associated with them by definition, but are included in the route to reduce total transportation cost, either due to distance considerations in a network, or due to benefits as a transportation hub at the interface of two modes of transport such as sea and land. The total supply across the m supply nodes, $\sum_{i=1}^{m} a_i$ is assumed to be equal to the total demand $\sum_{k=1}^{p} b_k$, across the p demand nodes. These and the other symbols denoting coefficients and variables, used in the algebraic formulation are given in Table 2. Following this paragraph, is the objective function, and set of constraints to which it is subject to. The number of barrels can be treated as continuous variables, since they are in millions of barrels, and fractional solutions can be interpreted reasonably. The total shipping costs are predictable, if the entire supply is being purchased, since the number of suppliers are small, and have been chosen strategically. However, if a large number of suppliers and crude oil options, are in the picture, as is the case nowadays, it will be impossible, to purchase all the supply, and the total shipping cost will also be variable factor. The vendor selection, and order size therefore must be decided by linear programming. It may also be the case, that due to the lower profit margin of the product from

TABLE 2. Variables used in general transshipment model.

Serial no.	Name	Definition
1	m	Number of production sites, $i = 1, \ldots, m$
2	n	Number of ports, $j = 1, \ldots, n$
3	p	Number of refineries, $k = 1, \ldots, p$
4	a_i	Supply in thousand tons, at production site i
5	R_k	Total refining capacity in thousand tons, at refinery k
6	r_k	Sour crude refining capacity in thousand tons, at refinery k
7	G_q	Tonnage in thousands, of product from q types of crude oil ($q = 1, 2$)
8	P_q	Gross refinery margin ($) from product G_q
9	T_s	Carrying capacity in thousands of tons of vessel class s
10	X_{ijs}	Number of class s vessels used on route (i, j) at full load
11	C_{ijs}	Voyage cost ($) of vessel class s on route (i, j)
12	L_j	Tonnage in thousands, of sweet crude oil unloaded at port j
13	H_j	Tonnage in thousands, of sour crude oil unloaded at port j
14	$L_{jk,r}$	Tonnage in thousands, of sweet crude oil from port j to refinery k by railway
15	$H_{jk,r}$	Tonnage in thousands, of sour crude oil from port j to refinery k by railway
16	$L_{jk,p}$	Tonnage in thousands, of sweet crude oil from port j to refinery k by pipeline
17	$H_{jk,p}$	Tonnage in thousands, of sour crude oil from port j to refinery k by pipeline
18	I_{jk}	Indicator 0/1 variable indicating if pipeline exists from port j to refinery k
19	C_{jk}	Pipeline cost of transporting one thousand tonnes from port j to refinery k
20	M_{jk}	Capacity of pipeline section (j, k)

the sour crude, it is not produced in large quantities. This cannot be decided without considering the binding constraints, which may restrict the production of the more profitable product. Again, linear programming is required to decide this issue.

Maximize objective function:

$$\begin{aligned}\sum_{q=1}^{2} P_q G_q - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s} C_{ijs}X_{ijs} - \sum_{j=1}^{m}\sum_{k=1}^{p} I_{jk}C_{jk}L_{jk,r}\\ - \sum_{j=1}^{m}\sum_{k=1}^{p} I_{jk}C_{jk}L_{jk,p} - \sum_{j=1}^{m}\sum_{k=1}^{p} I_{jk}C_{jk}H_{jk,r}\\ - \sum_{j=1}^{m}\sum_{k=1}^{p} I_{jk}C_{jk}H_{jk,p}.\end{aligned} \tag{4.1}$$

Supply constraints:

$$\sum_{k=1}^{s}\sum_{j=1}^{n} T_k X_{ijk} \le a_i, \quad i = 1, \ldots, m. \tag{4.2}$$

Transshipment constraints:

$$\sum_{l=1}^{s}\sum_{i=1}^{m} T_l X_{ijl} \le M_{jk}, \quad j = 1, \ldots, n,\ k = 1, \ldots, p. \tag{4.3}$$

Refining capacity constraints:

$$\sum_{j=1}^{n} L_{jk,r} + \sum_{j=1}^{n} L_{jk,p} \leq R_k, \quad k = 1, \ldots, p \tag{4.4}$$

$$\sum_{j=1}^{n} H_{jk,r} + \sum_{j=1}^{n} H_{jk,p} \leq r_k, \quad k = 1, \ldots, p \tag{4.5}$$

$$\sum_{j=1}^{n} L_{jk,r} + \sum_{j=1}^{n} L_{jk,p} + \sum_{j=1}^{n} H_{jk,r} + \sum_{j=1}^{n} H_{jk,p} \leq R_k, \quad k = 1, \ldots, p. \tag{4.6}$$

Flow disaggregation constraints:

$$\sum_{k=1}^{s} \sum_{i=1}^{m} T_k X_{ijk} - H_j = 0, \quad j = 1, \ldots, n \tag{4.7}$$

$$\sum_{k=1}^{s} \sum_{i=1}^{m} T_k X_{ijk} - L_j = 0, \quad j = 1, \ldots, n \tag{4.8}$$

$$L_j - \sum_{j=1}^{n} \sum_{k=1}^{p} L_{jk,r} - \sum_{j=1}^{n} \sum_{k=1}^{p} L_{jk,p} = 0, \quad j = 1, \ldots, n \tag{4.9}$$

$$H_j - \sum_{j=1}^{n} \sum_{k=1}^{p} H_{jk,r} - \sum_{j=1}^{n} \sum_{k=1}^{p} H_{jk,p} = 0, \quad j = 1, \ldots, n. \tag{4.10}$$

Blending constraints:

$$G_1 - 0.95 \sum_{j=1}^{n} L_j = 0 \tag{4.11}$$

$$G_2 - 0.95 \sum_{j=1}^{n} H_j = 0. \tag{4.12}$$

Constraints declared in equation (4.2), are supply constraints and refer to the supply of crude at the production sites. The general form of the supply constraint, is a sum-product of the capacity of the vessels and the number of voyages made by the vessel. We have m of these constraints. Constraints in equation (4.3), are the transshipment constraints at the Indian ports, of which np are included in the general model. Constraints in equations (4.4)–(4.6), are the refining capacity constraints at the three refineries. There are $3p$ of these constraints. They require a detailed explanation; in the standard integer programming, additional capacity as fixed charges, are switched on if required, using binary variables. With flexible capacity we need to use a capping or upper bound constraint on one type of capacity. We do not know *a priori*, how much sour crude processing capacity will be required at each refinery. Since the processing is by the tanker load, we can partition the number of tankers loads into sour crude loads, and sweet crude loads, and have upper bound constraints for the two types of loads and one for the total capacity. The refining capacity constraints are in groups of three, and describe, sweet crude refining capacity, sour crude refining capacity and total refining capacity at each refinery respectively. In each set of three constraints, the first constraint, for example equation (4.4), ensures that the low sulphur crude oil piped to the refinery from the two port terminals is less than the refinery capacity. The second constraint equation (4.5), ensures that the high sulphur crude oil piped to the refinery from the two port terminals does not exceed the refinery's capacity to process sour crude oil. The third constraint in equation (4.6), is required to ensure that the sum of low sulphur crude oil and high sulphur crude oil piped to the refinery does not exceed the total refinery capacity. This constraint is not redundant, and cannot be obtained by adding the other two. This is because the sour refining capacity is given as a fraction of total refining

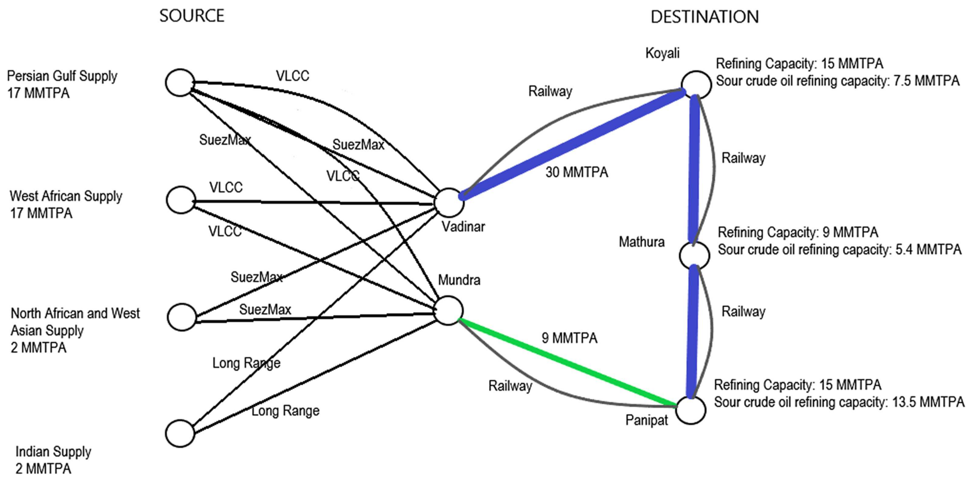

FIGURE 2. Transshipment network.

capacity. Constraints in equations (4.7)–(4.10), are required to disaggregate the bulk volumes of low sulphur crude oil and high sulphur crude oil from the two ports into specific parcels assigned to each refinery, either by pipeline, or by railway line. These constraints are required because the parcels travel different distances along the pipeline to the three refineries, and the costing in the objective function, is per unit kilometer. Constraints in equations (4.11) and (4.12), relate the production levels of the two products to the blended inputs. The coefficients in these two blending constraints are 0.95 and indicate that up to 95% of the raw material is used up to create the products G_1, and G_2.

Figure 2 displays the network for the concrete instance of the problem. The tonnage shipped to each refinery is known in parcels of 270 TMT, 135 TMT, and 60 TMT, since these are the carrying capacities of the vessels used. The freight costs are given in Table 1 in page 1092. The marine transportation costs are in US dollars, and the pipeline and railway transportation costs are in Indian Rupees. We converted the pipeline and railway transportation costs to USD by dividing by 70, which is the approximate exchange rate between the two currencies. The mixed integer objective function is formulated as in equation (4.13). As specified for the general transshipment model, the choice of objective function is maximizing gross refinery margin (GRM) subject to various constraints. The constraints can be grouped as supply constraints, transshipment node constraints, refining capacity constraints, flow disaggregation constraints, and blending constraints. The additional constraints, are required to deal with flexible refining capacity and fleet heterogeneity. The objective function reflects the profit from the gross refinery margin and the loss from the cost of transportation by ships and the pipeline from the ports to the refineries. All the variables used in the problem definition, are given in Table 3. Some of the variable names are different from the general model, to keep the notation simple. In the formulation below, all costing and profiting is in unit dollars, and all volumes are in thousand metric tonnes (TMT).

Maximize objective function:

$$\begin{aligned}
&\$58\,400\,G_1 + \$47\,450\,G_2 - \$2\,000\,000\,V_1 - \$2\,000\,000\,V_2 - \$3\,500\,000\,V_3 - \$3\,500\,000\,V_4 \\
&- \$1\,500\,000\,S_1 - \$1\,500\,000\,S_2 - \$2\,000\,000\,S_3 - \$2\,000\,000\,S_4 \\
&- \$400\,000\,\mathrm{LR}_1 - \$400\,000\,\mathrm{LR}_2 \\
&- \$10\,851\,L_{V1,r} - \$27\,694\,L_{V2,r} - \$31\,160\,L_{V3,r} - \$10\,851\,H_{V1,r} - \$27\,694\,H_{V2,r} - \$31\,160\,H_{V3,r} \\
&- \$27\,157\,L_{M3,r} - \$30\,623\,L_{M2,r} - \$47\,466\,L_{M1,r} - \$27\,157\,H_{M3,r} - \$30\,623\,H_{M2,r} - \$47\,466\,H_{M1,r} \\
&- \$8139\,L_{V1,p} - \$20\,771\,L_{V2,p} - \$23\,370\,L_{V3,p} - \$8139\,H_{V1,p} - \$20\,771\,H_{V2,p} - \$23\,370\,H_{V3,p} \\
&- \$20\,368\,L_{M3,p} - \$20\,368\,H_{M3,p}.
\end{aligned} \tag{4.13}$$

Table 3. Variables defined in concrete case.

Serial no.	Variable name	Variable definition
1	G_1 and G_2	TMTs of product from two types of crude oil
2	V_1 and V_2	VLCC from Persian Gulf to Vadinar/Salaya and Mundra
3	V_3 and V_4	VLCC from West Africa to Vadinar/Salaya and Mundra
4	S_1 and S_2	Suezmax from Persian Gulf to Vadinar/Salaya and Mundra
5	S_3 and S_4	Suezmax from N. Africa/W. Asia to Vadinar/Salaya and Mundra
6	LR_1 and LR_2	LR from Bombay High to Vadinar/Salaya and Mundra
7	L_V	TMT of sweet crude oil at Vadinar/Salaya terminal
8	H_V	TMT of sour crude oil at Vadinar/Salaya terminal
9	L_M	TMT of sweet crude oil delivered to Mundra terminal
10	H_M	TMT of sour crude oil delivered to Mundra terminal
11	$L_{V1,r}$	Sweet crude oil from Vadinar to Koyali by rail
12	$L_{V2,r}$	Sweet crude oil from Vadinar to Mathura by rail
13	$L_{V3,r}$	sweet crude oil from Vadinar to Panipat by rail
14	$H_{V1,r}$	Sour crude oil from Vadinar to Koyali by rail
15	$H_{V2,r}$	Sour crude oil from Vadinar to Mathura by rail
16	$H_{V3,r}$	Sour crude oil from Vadinar to Panipat by rail
17	$L_{M1,r}$	Sweet crude oil from Mundra to Koyali by rail
18	$L_{M2,r}$	Sweet crude oil from Mundra to Mathura by rail
19	$L_{M3,r}$	Sweet crude oil from Mundra to Panipat by rail
20	$H_{M1,r}$	Sour crude oil from Mundra to Koyali by rail
21	$H_{M2,r}$	Sour crude oil from Mundra to Mathura by rail
22	$H_{M3,r}$	Sour crude oil from Mundra to Panipat by rail
23	$L_{V1,p}$	Sweet crude oil from Vadinar to Koyali by pipeline
24	$L_{V2,p}$	Sweet crude oil from Vadinar to Mathura by pipeline
25	$L_{V3,p}$	sweet crude oil from Vadinar to Panipat by pipeline
26	$H_{V1,p}$	Sour crude oil from Vadinar to Koyali by pipeline
27	$H_{V2,p}$	Sour crude oil from Vadinar to Mathura by pipeline
28	$H_{V3,p}$	Sour crude oil from Vadinar to Panipat by pipeline
29	$L_{M1,p}$	Sweet crude oil from Mundra to Koyali by pipeline
30	$L_{M2,p}$	Sweet crude oil from Mundra to Mathura by pipeline
31	$L_{M3,p}$	Sweet crude oil from Mundra to Panipat by pipeline
32	$H_{M1,p}$	Sour crude oil from Mundra to Koyali by pipeline
33	$H_{M2,p}$	Sour crude oil from Mundra to Mathura by pipeline
34	$H_{M3,p}$	Sour crude oil from Mundra to Panipat by pipeline

Supply constraints:

$$270\,V_1 + 270\,V_2 + 135\,S_1 + 135\,S_2 \leq 15\,000 \tag{4.14}$$
$$270\,V_3 + 270\,V_4 \leq 17\,000 \tag{4.15}$$
$$135\,S_3 + 135\,S_4 \leq 2000 \tag{4.16}$$
$$60\,\mathrm{LR}_1 + 60\,\mathrm{LR}_2 \leq 2000. \tag{4.17}$$

Trans-shipment constraints:

$$270\,V_1 + 135\,S_1 + 270\,V_3 + 135\,S_3 + 60\,\mathrm{LR}_1 \leq 30\,000 \tag{4.18}$$
$$270\,V_2 + 270\,V_4 + 135\,S_2 + 135\,S_4 + 60\,\mathrm{LR}_2 \leq 9000. \tag{4.19}$$

Refining capacity constraints:

$$L_{V1,r} + L_{M1,r} + L_{V1,p} + L_{M1,p} \leq 15\,000 \tag{4.20}$$
$$H_{V1,r} + H_{M1,r} + H_{V1,p} + H_{M1,p} \leq 7500 \tag{4.21}$$
$$L_{V1,r} + L_{M1,r} + L_{V1,p} + L_{M1,p} + H_{V1,r} + H_{M1,r} + H_{V1,p} + H_{M1,p} \leq 15\,000 \tag{4.22}$$
$$L_{V2,r} + L_{M2,r} + L_{V2,p} + L_{M2,p} \leq 9000 \tag{4.23}$$
$$H_{V2,r} + H_{M2,r} + H_{V2,p} + H_{M2,p} \leq 5400 \tag{4.24}$$
$$L_{V2,r} + L_{M2,r} + L_{V2,p} + L_{M2,p} + H_{V2,r} + H_{M2,r} + H_{V2,p} + H_{M2,p} \leq 9000 \tag{4.25}$$
$$L_{V3,r} + L_{M3,r} + L_{V3,p} + L_{M3,p} \leq 15\,000 \tag{4.26}$$
$$H_{V3,r} + H_{M3,r} + H_{V3,p} + H_{M3,p} \leq 13\,500 \tag{4.27}$$
$$L_{V3,r} + L_{M3,r} + L_{V3,p} + L_{M3,p} + H_{V3,r} + H_{M3,r} + H_{V3,p} + H_{M3,p} \leq 15\,000. \tag{4.28}$$

Flow disaggregation constraints:

$$270\,V_1 + 135\,S_1 - H_V = 0 \tag{4.29}$$
$$270\,V_2 + 135\,S_2 - H_M = 0 \tag{4.30}$$
$$270\,V_3 + 135\,S_3 + 60\,\mathrm{LR}_1 - L_V = 0 \tag{4.31}$$
$$270\,V_4 + 135\,S_4 + 60\,\mathrm{LR}_2 - L_M = 0 \tag{4.32}$$
$$L_V - L_{V1,r} - L_{V1,p} - L_{V2,r} - L_{V2,p} - L_{V3,r} - L_{V3,p} = 0 \tag{4.33}$$
$$L_M - L_{M1,r} - L_{M1,p} - L_{M2,r} - L_{M2,p} - L_{M3,r} - L_{M3,p} = 0 \tag{4.34}$$
$$H_V - H_{V1,r} - H_{V1,p} - H_{V2,r} - H_{V2,p} - H_{V3,r} - H_{V3,p} = 0 \tag{4.35}$$
$$H_M - H_{M1,r} - H_{M1,p} - H_{M2,r} - H_{M2,p} - H_{M3,r} - H_{M3,p} = 0. \tag{4.36}$$

Blending constraints:

$$G_1 - 0.95\,L_V - 0.95\,L_M = 0 \tag{4.37}$$
$$G_2 - 0.95\,H_V - 0.95\,H_M = 0. \tag{4.38}$$

5. Computational results

The sweet crude yields the higher profit margin, so one would expect that producing more units of premium gasoline is best. In favour of the contrary opinion, we have seen in many product mix cases, that one may profit, to produce more ordinary gasoline, though its profit margin is less, because of resource levels or processing capacity or the nature of the binding constraints. Therefore, according to LP theory, we cannot give higher priority to the sweet crude, just because of the higher profit margin. Solving the integer version of the problem, using Frontline's Solver, we get a maximum Gross Profit of \$895 572 537.5, or a figure that approaches about 900 million dollars, this by producing 19 580 TMT of high-grade gasoline and 14 236 TMT of lower grade gasoline. In barrel terms this is 143 million bbl. of the high grade product, and 104 million bbl. of the other product. Therefore in this product mix problem, it is better to manufacture more of the product with the higher contribution. Second, in this solution, almost all the supply at the Persian Gulf, West Africa, North Africa, West Asia and Bombay High are procured. In scaled-up models with many suppliers, one would not be in a position to purchase all the available supply, and the decision support of the MILP model will be appreciated more.

The shipping pattern is as follows: $V_1 = 23$, $V_2 = 32$, $V_3 = 62$, $S_2 = 1$, $S_3 = 14$, $\mathrm{LR}_1 = 33$, $L_V = 20\,610$, $H_V = 6210$, $L_M = 0$, $H_M = 8775$. Referring to Table 3, we can decode this as 55 VLCC tanker loads from the Persian Gulf, and 62 VLCC tanker loads from West Africa. The other significantly large shipment is *via* 15 SuezMax from North Africa and West Asia. $\mathrm{LR}_1 = 33$ indicates that all of the offshore production at Bombay High is shipped to Vadinar. The railway is not used on any segment of the network, as the pipeline is much

TABLE 4. Sensitivity analysis of production to margin for product 1.

		Gross	Refinery	Margin 2			
	G_1	20 000	30 000	40 000	50 000	60 000	70 000
Gross	20 000	1881	1881	1881	1881	1881	1881
Refinery	30 000	14 193	7139.25	7139.25	7139.25	7139.25	7139.25
Margin 1	40 000	19 579.5	19 579.5	17 784	17 784	17 784	17 784
	50 000	19 579.5	19 579.5	19 579.5	19 579.5	19 579.5	19 579.5
	60 000	19 579.5	19 579.5	19 579.5	19 579.5	19 579.5	19 579.5
	70 000	19 579.5	19 579.5	19 579.5	19 579.5	19 579.5	19 579.5

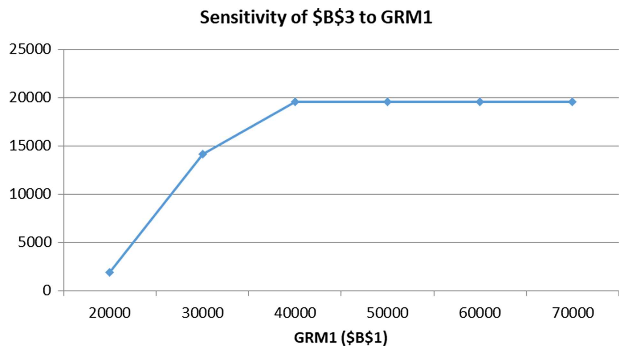

FIGURE 3. Sensitivity of first product with respect to its margin.

more economical. Both pipelines are used. The optimal pipeline flows are as $L_{V1,p} = 12\,000$, $L_{V2,p} = 7515$, $L_{V3,p} = 1095$. This means that out of the 20 610 TMT of sweet crude oil reaching Vadinar, 12 000 TMT are sent to Koyali, 7515 TMT to Mathura, and 1095 TMT to Panipat *via* the Salaya–Mathura pipeline. Out of the 6210 TMT of sour crude oil reaching Vadinar, nothing is sent to Koyali, 1485 TMT sent to Mathura, and 4725 TMTs sent to Panipat. The Mundra pipeline is exclusively for sour crude oil to Panipat and 8775 TMTs are sent through it.

As far as utilization is concerned, 80% of the total refining capacity at Koyali, is used for refining the sweet crude oil. Interestingly, none of the sour crude oil is processed at Koyali. At Mathura, 83.5% of the total capacity is used for refining sweet crude oil, and the balance of 16.5% is used for refining sour crude oil. Mathura refinery is thus utilized 100%. At the Panipat refinery 90% of the total capacity is utilized for refining 13 500 TMT of high sulphur crude oil. Out of this 13 500 TMT, 5725 TMT is sent from Vadinar port, and the remaining 8775 TMT is from Mundra port. From the remaining 10% capacity, 7.3% is used for refining the 1095 TMT of sweet crude oil transported from Vadinar.

Sensitivity Analysis was done using Solver Table from Palisade Software. We varied both the Gross Refinery Margins over a wide range of values, and noted the effect on the product mix. Tables 4 and 5 below show the production level for each product as the two profit margin coefficients vary in the range of (20 000;70 000). Figures 3 and 4 show that the production levels stabilize to a fixed optimal value without trend, as the refinery margin increases. This same information can be seen in the tables, that have the advantage of displaying the precise numerical values.

The various increments of the first product G_1's profit margin are along the left most column, and the various increments of the second product G_2's profit margin are along the top row. One can see in Table 4, that at the lowest profit margin of the first product, that is at the level of $20 000 and for all the increments of profit for the second product, the production of G_1 is 1881 TMT. When the profit margin of G_1 increases to 30 000 the

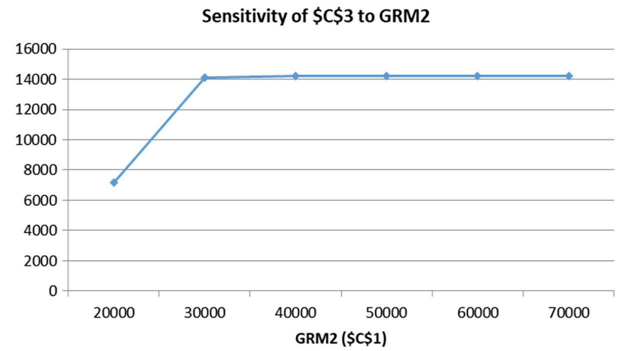

FIGURE 4. Sensitivity of second product with respect to its margin.

TABLE 5. Sensitivity analysis of production to margin for product 2.

		Gross	Refinery	Margin 2			
	G_2	20 000	30 000	40 000	50 000	60 000	70 000
Gross	20 000	7182	14 107.5	14 235.75	14 235.75	14 235.75	14 235.75
Refinery	30 000	0	14 107.5	14 235.75	14 235.75	14 235.75	14 235.75
Margin 1	40 000	0	11 542.5	14 235.75	14 235.75	14 235.75	14 235.75
	50 000	0	11 286	14 107.5	14 235.75	14 235.75	14 235.75
	60 000	0	11 286	14 107.5	14 235.75	14 235.75	14 235.75
	70 000	0	11 286	14 107.5	14 235.75	14 235.75	14 235.75

production is 14 193 TMT for GRM2 level of \$20 000 and \$7139.25 for increasing increments of GRM2. For the \$40 000 profit margin level of G_1, the production level of G_1 is 19 579.5 TMT at the \$20 000 and \$30 000 level of GRM2, but falls to 17 784 TMT for all the remaining increments of GRM1. As the profit margin for product 1, crosses \$50 000, the production of G_1 stabilizes at 19 579.5 TMT for all levels of profit for the second product.

Table 5, which is for the second product has to be read column by column; for each increment of GRM2, we look at the production level, at all increments of the first product. At the \$20 000 level of GRM2, there is only 7182 TMT production for GRM1 level of \$20 000. At the \$30 000 level of GRM2, there is 14 107.5 TMT produced at the \$20 000 level and \$30 000 level, but the production drops to 11 542.5 TMT at the \$40 000 level, and still further to 11 286 TMT at the \$50 000 and \$60 000 level. At the \$40 000 level of GRM2, the production of G_2 is 14 235.75 TMT at the lower levels of GRM1, but drops to 14 107.5 TMT at the higher levels. For GRM2 at the values of \$50 000, \$60 000, and \$070 000, the production is stable at 14 235.75 TMT at all levels of GRM1. Similar solver table analysis has also been done for supply levels at the production sites, and for refinery capacity.

6. CONCLUSIONS

Though the objective function value at the end of the modelling and algorithmic solution, indicates nothing to worry about in the financial dimension, the oil & gas sector managers, needs to be vigilant. Due the threat of depleting fossil fuel reserves, the petroleum sector has been driven to more difficult raw materials such as shale gas and heavy crude. A second threat is technological obsolescence, due to the progressive acceptance of renewable energy and the electric vehicle in the environmental act of reducing the carbon footprint. The demand for petroleum products is high and inelastic in many parts of the world, where the usage does not yet match the industrialised West; the business is sustainable in these countries. The policy changes of suppliers and transporters that occur from year to year, must be noted, in the existing mathematical model of transportation and transshipment. The model needs to be revised as per the changes in the technological coefficients, prices, resource levels, and other parameters.

This mathematical model at present looks at the transportation problem at the annual level. The program solution reflects the reality of the supply chain in terms of cost and profit related decisions. Sensitivity analysis has informed decisions about which refinery capacity needs to be augmented and which do not. Most importantly, the model can be scaled up easily in terms of number of suppliers, types of vessels, transshipment nodes, number of refineries, and number of pipelines. For a large integrated company, this is very important. This model can be used by other petroleum companies, in emerging economies. Logistics cost can be varied to see the effects on selection. In this scenario, 117 very large tankers are used for the sea-routes and Suezmax are not used very much. If that many large tankers are not available, the petroleum company can compare alternative carriers with minimal remodelling of the constraints. The shipping requirement information present in the solution alerts transporters, to the precise requirement in the year and they can mobilise their fleet accordingly. Other scenarios, for which it can be useful are:

(1) new suppliers
(2) different crude oil options
(3) new ports or terminals
(4) reduced freight rates
(5) new pipelines
(6) expanded refining capacity
(7) A third commodity type such as heavy crude, or shale gas.

To be modelled in the future, are the scheduling constraints at the shore tanks and pumping stations. The storage tank capacities are known, but the loading and unloading of the tanks need to be synchronized with the schedules of the arriving vessels. This is to maintain the inventory between lower and upper bounds, which otherwise becomes expensive to hold. The data used so far, is specific only to the petroleum company. If the transporter schedules are incorporated into such a tactical model, it can be converted into a dynamic operational model, that accounts for monthly, weekly, and even daily events. A more sophisticated model will be built, as the dialogue with the company managers, and marine transporters progresses.

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