RAIRO-Oper. Res. 54 (2020) 597–614
https://doi.org/10.1051/ro/2019117

RAIRO Operations Research
www.rairo-ro.org

FORECASTING STOCK MARKET PRICE BY USING FUZZIFIED CHOQUET INTEGRAL BASED FUZZY MEASURES WITH GENETIC ALGORITHM FOR PARAMETER OPTIMIZATION

Shanoli Samui Pal and Samarjit Kar*

Abstract. In this paper, fuzzified Choquet integral and fuzzy-valued integrand with respect to separate measures like fuzzy measure, signed fuzzy measure and intuitionistic fuzzy measure are used to develop regression model for forecasting. Fuzzified Choquet integral is used to build a regression model for forecasting time series with multiple attributes as predictor attributes. Linear regression based forecasting models are suffering from low accuracy and unable to approximate the non-linearity in time series. Whereas Choquet integral can be used as a general non-linear regression model with respect to non classical measures. In the Choquet integral based regression model parameters are optimized by using a real coded genetic algorithm (GA). In these forecasting models, fuzzified integrands denote the participation of an individual attribute or a group of attributes to predict the current situation. Here, more generalized Choquet integral, *i.e.*, fuzzified Choquet integral is used in case of non-linear time series forecasting models. Three different real stock exchange data are used to predict the time series forecasting model. It is observed that the accuracy of prediction models highly depends on the non-linearity of the time series.

Mathematics Subject Classification. 91B84, 46A55, 28E10, 68W50.

Received May 12, 2017. Accepted December 13, 2019.

1. Introduction

Stock market is unpredictable as there are several complex factors influencing its ups and downs. Therefore the trend of the series is also affected by those factors and by their non-linear relationship. In the stock market forecasting, technical analysis is one of the traditional methods applied by investors for decision making. There are some other statistical methods such as autoregressive conditional heteroscedasticity (ARCH) model [10], generalized ARCH (GARCH) model [4], autoregressive moving average (ARMA) model [5], autoregressive integrated moving average (ARIMA) model [5]. All these models are different types of regression models assuming some mathematical distribution, those distributions are not always followed by realistic stock market time series data [3, 42].

Nowadays, several soft computing approaches like evolutionary algorithms, artificial neural networks, fuzzy logic, rough set theory and their hybridization have been developed and perform well in forecasting of stock

Keywords. Time series forecasting, fuzzified Choquet integral, fuzzy measure, signed fuzzy measure, intuitionistic fuzzy measure, genetic algorithm.

Department of Mathematics, NIT Durgapur, Durgapur 713209, West Bengal, India.
*Corresponding author: kar_s_k@yahoo.com

markets. In [9, 16], authors proposed hybrid time series forecasting models based on neural network and fuzzy time series. A hybridized genetic algorithm and neural network model had been developed to predict stock price index [27]. Caia [7] proposed a hybrid GA model based on fuzzy time series and genetic algorithm (FTSGA) on TAIEX as experimental data set and concluded that the model improved the accuracy. Teoh *et al.* [36] proposed a hybrid model based on multi-order fuzzy time series by using rough sets theory to obtain fuzzy logical relationship from time series and an adaptive expectation model to improve forecasting accuracy on TAIEX and National Association of Securities Dealers Automated Quotations (NASDAQ) experimental data sets. Pai and Lin [29] developed a hybrid ARIMA and support vector machines model in stock price forecasting. Lahmiri [19–21] proposed several hybrid models based on different data decompoisition techniques like variational mode decompoisition, wavelet transform and applied on stock prices to develop predictive models. Lahmiri and Boukadoum [25] presented a new ensemble system based on continuous wavelet transform (CWT) to analyze stock returns, backpropagation neural network (BPNN) to process CWT-based coefficients, and particle swarm optimization (PSO) to adjust the weights and biases of BPNN. There are some recent works on stock market time series data. Deep learning technique has been used in predicting digital currencies, like Bitcoin, Digital Cash and Ripple [24], an ensemble of neural networks (NN) coupled with particle swarm intelligence for parameter optimization has been used in the technical analysis information fusion [22], singular spectrum analysis (SSA) and support vector regression (SVR) coupled with particle swarm optimization (PSO) has been implemented for intraday stock price prediction [23], neural network weight adjustment using zSlices-based generalized type-2 fuzzy set has been applied in predicting closing price index of Shenzhen stock exchange, closing price index of Shanghai stock exchange [32], fuzzy transformation and neural network with back propagation learning has been used in stock market closing price index [30], data discretization using fuzzy statistics and rule generation by rough set theory has been used in stock market time series forecasting [33], an improved fuzzy time series model for unequal interval length using genetic algorithm has been applied on BSE sensex time series and Shenzhen stock exchange data [31].

Choquet integral [8] mostly applied on real-valued data. To extend the domain, fuzzified Choquet integral had been introduced and it has been used in data mining as a non-linear regression tool, classifications and decision-making. Yang *et al.* [41] proposed a fuzzified Choquet integral with interval-valued integrand (CIII) based regression model on temperature prediction. They used double genetic algorithm (GA) to estimate the regression coefficients as well as the values of signed fuzzy measure. Wang *et al.* [37] proposed non-linear non-negative multi-regressions based on Choquet integral, as a generalization of the traditional linear multi-regression tool. An evolutionary algorithm, GA had been used to adjust the unknown parameters in their work. Yang *et al.* [40] proposed fuzzy numbers and fuzzification of the Choquet integral to deal with the linguistic attributes in data sets. They provided a detailed description of fuzzified Choquet integral with fuzzy integrand. Their discussion covered the Choquet integral with respect to fuzzy measure and signed fuzzy measure.

In this paper, our objective is to develop the forecasting models for stock market time series data which are multi-attributed and non-linear in nature. Here, we use fuzzified Choquet integral with respect to fuzzy measure, signed fuzzy measure and intuitionistic fuzzy measure. Fuzzified Choquet integral with respect to fuzzy measure has been applied as non-linear regression tool [37] and fuzzified Choquet integral with respect to signed fuzzy measure has been applied [41] on interval valued and crisp data. In this paper, fuzzified Choquet integral with respect to all three measure is applied on crisp data. Here, attributes are stock exchange basic indexes such as high price, low price and closing price. We have tried to find, is there any relation among them to predict closing price. A result comparison is given on three stock exchanges such as Bombay stock exchange (BSE), New York stock exchange (NYSE) and Taiwan stock exchange corporation (TAIEX) separately in training and testing data sets. It is observed that their prediction capability highly depends on data. At the same time, Choquet integral based on fuzzy measure (FM) and intuitionistic fuzzy measure (IFM) gives better results comparison to Choquet integral based on signed fuzzy measure (SFM). A comparison of the proposed approach and neural network with backpropagation learning is also shown. In this work, the intuitionistic fuzzy measure has been considered as one of the non classical measure in Choquet integral based regression model. We have compared the results of two data sets BSE and TAIEX with the works where intuitionistic fuzzy time series models have been proposed [18, 38]. But, we can remind about the purposes of the approaches. Purpose of approaches like ours, is to find participation of predicting attributes to decide the objective attributes. Whereas purposes of fuzzy time series are forecasting time series with uncertainty due to noise, missing data, etc.

The remainder of this paper is organized as follows. First, preliminaries on Choquet integral has been discussed in Section 2, which covers Choquet integral with real-valued integrand, fuzzified Choquet integral with fuzzy measure, signed fuzzy measure and intuitionistic fuzzy measure in its consequent sub-sections. Regression model based on fuzzified Choquet integral is discussed in Section 3. Proposed method has been described in Section 4. Brief data descriptions are given in Section 5. Results and discussion are presented in Section 6. Finally, the conclusions and future research directions are presented in Section 7.

2. Preliminaries

In this section, we discuss about Choquet integral and generalized Choquet integral. The original concept of Choquet integral [8] supports real-valued integrand and it works as non-linear aggregation tool in data mining for real numbers. But, it is not capable to deal with qualitative data/linguistic data, etc. This shortcoming can be overcome by generalized Choquet integral which is extended to fuzzy domain for imprecise information like linguistic data.

2.1. Choquet integral with real-valued integrand

Let $f : X \to (-\infty, \infty)$ be a real-valued function. The Choquet integral of f can be defined as:

$$\int f \,\mathrm{d}\mu \;=\; \int_0^\infty \mu(F_\alpha)\,\mathrm{d}\alpha, \tag{2.1}$$

where f is a non-negative real-valued function on $(X, \mathcal{P}(X))$ such that $F_\alpha = \{x | f(x) \geq \alpha, x \in X\} \in \mathcal{P}(X)$ for any $\alpha \in [0, \infty)$, $F_\alpha \in [0, \infty)$ is the α-cut set of f.

To calculate the value of integral (2.1), the values of f at $X = \{x_1, x_2, \ldots, x_N\}$ should be rearranged in non-decreasing order, *i.e.*, $f(x'_1) \leq f(x'_2) \leq \ldots \leq f(x'_N)$, where $(x'_1, x'_2, \ldots, x'_N)$ is a permutation or rearrangement of $(x_1, x_2, \ldots, x_N)$. Then, evaluate the value of the expression

$$\int f \,\mathrm{d}\mu \;=\; \sum_{i=1}^{n} \left[f(x'_i) \;-\; f(x'_{i-1})\right] \mu\left(x'_i, x'_{i+1}, \ldots, x'_N\right), \tag{2.2}$$

where $f(x'_0) = 0$.

2.2. Fuzzified Choquet integral with three different measures

The concept of Choquet integral is extended to fuzzy domain as fuzzified Choquet integral to measure the linguistic information. Three different fuzzy measures are described below.

2.2.1. Fuzzy measure

Let F be a collection of subsets of a nonempty set X, when X is finite it can be power set of X, $\mathcal{P}(X)$ with $\phi \in F$ and $X \in F$. A non-negative monotone set function, μ, is a mapping from F to $[0, \infty)$ satisfying the conditions:

$$\mu(\phi) \;=\; 0, \tag{2.3}$$
$$A \subset B \Rightarrow \mu(A) \leq \mu(B), \forall A, B \in F \tag{2.4}$$

μ is regular if $\mu(X) = 1$.

Table 1. Example.

$\mu(\phi)$	0
$\mu(\{x_1\})$	0.20
$\mu(\{x_2\})$	0.10
$\mu(\{x_3\})$	0.40
$\mu(\{x_1, x_2\})$	0.34
$\mu(\{x_1, x_3\})$	0.75
$\mu(\{x_2, x_3\})$	0.58
$\mu(\{x_1, x_2, x_3\})$	1

In most of the real life problem, like our proposed work, predictor data points X of time series are finite, so continuity of μ is not required. Such a set function is called a fuzzy measure [37]. This set function is non-additive in nature which means the joint participation of predictor attributes may be more or less than the sum of their individual participation to predict the current situation.

Choquet integral is one of the non-linear integrals that is able to aggregate non-negative monotone set function to replace the additive measure for data mining. It can be defined as in equation (2.1) and after rearrangement of f at $X = \{x_1, x_2, \ldots, x_N\}$, the equation (2.2) is evaluated to obtain the integral value.

An example of using non-negative monotone set function and the Choquet integral as an aggregation tool in data mining is given below.

Example 2.1 ([37])**.** To detect an object that may be a tank or an armored personnel carrier (APC), three different means, x_1, x_2 and x_3 are adopted, and they are regarded as information sources, that is, $X = \{x_1, x_2, x_3\}$. Suppose that the importance of degrees (the values of non-negative monotone set function μ) of these means and their combinations are in Table 1.

Also suppose that, through means x_1, x_2 and x_3, the marginal evaluations (information) for tank hypothesis, denoted by f_1, f_2 and f_3, respectively, which are obtained as $f_1 = f(x_1) = 0.85$, $f_2 = f(x_2) = 0.21$, $f_3 = f(x_3) = 0.90$.

The Choquet integral with respect to μ can be used to obtain a comprehensive assessment for the tank hypothesis. Its value is calculated as follows:

Let $x'_1 = x_2$, $x'_2 = x_1$, $x'_3 = x_3$.

Then $f(x'_1) = 0.21$, $f(x'_2) = 0.85$, $f(x'_3) = 0.90$, and

$$\begin{aligned}\int f\,\mathrm{d}\mu &= \sum_{i=1}^{3} \left[f(x'_i) \ - \ f\left(x'_{i-1}\right)\right] \mu\left(x'_i, x'_{i+1}, \ldots, x'_N\right) \\ &= \ 0.21 * 1 + (0.85 - 0.21) * 0.75 \\ &\quad + (0.90 - 0.85) * 0.40 \\ &= \ 0.71.\end{aligned}$$

Set function μ is restricted to be non-negative in case of fuzzy measure, which may represent limitations in real problems, such as in financial, sociological applications where negative values may exist. Thus a more generalized fuzzy measure, signed fuzzy measure has been proposed [40, 41], which is discussed below in brief.

2.2.2. *Signed fuzzy measure*

Let F be a collection of subsets of a nonempty set X, when X is finite it can be power set of $(X, \mathcal{P}(X))$ with $\phi \in F$ and $X \in F$. A nonadditive and non-monotonic set function, μ, is a mapping from F to $[0, \infty)$ is called signed fuzzy measure if $\mu(\phi) = 0$. Signed fuzzy measure μ is called a generalized fuzzy measure if it is naon-negative, *i.e.*,

$$\mu(A) \geq 0, \forall A \in F.$$

It is non-additive in nature which means the joint participation of predictor attributes may be more or less than the sum of their individual participation to predict the current situation.

Choquet integral of f at $X = \{x_1, x_2, \ldots, x_N\}$ can be defined as:

$$\int f \,\mathrm{d}\mu \;=\; \int_{-\infty}^{0} [\mu(F_\alpha) - \mu(X)] \,\mathrm{d}\alpha + \int_{0}^{\infty} \mu(F_\alpha) \,\mathrm{d}\alpha, \tag{2.5}$$

where f is a measurable function on $(X, \mathcal{P}(X))$ such that $F_\alpha = \{x | f(x) \geq \alpha, x \in X\} \in \mathcal{P}(X)$ for any $\alpha \in [-\infty, \infty)$, $F_\alpha \in [0, \infty)$ is the α-cut set of f, if both of above Riemann integrals exist and at least one of them has finite value.

To calculate the value of integration, the values of f at $X = \{x_1, x_2, \ldots, x_N\}$ should be rearranged in non-decreasing order, *i.e.*, $f(x'_1) \leq f(x'_2) \ldots \leq f(x'_N)$, where $(x'_1, x'_2, \ldots, x'_N)$ is a permutation or rearrangement of $(x_1, x_2, \ldots, x_N)$. Then, evaluate the value of the expression

$$\int f \,\mathrm{d}\mu \;=\; \sum_{i=1}^{n} \left[f(x'_i) \;-\; f(x'_{i-1}) \right] \mu \left(x'_i, x'_{i+1}, \ldots, x'_N \right), \tag{2.6}$$

where $f(x'_0) = 0$.

An example of using non-negative non-monotonic set function is given below.

Example 2.2 ([41])**.** Suppose that $X = \{x_1, x_2\}$, $\mu_1 = \mu(\{x_1\}) = 2$, $\mu_2 = \mu(\{x_2\}) = 3$ and $\mu_3 = \mu(\{X\}) = 1$. Then, μ is a signed fuzzy measure, not a fuzzy measure.

The set function μ is non-additive monotonic in case of fuzzy measure and non-additive, non-monotonic in case of signed fuzzy measure. The property monotonic of fuzzy measure can be replaced by intuitionistic approach where the monotonic relation within all μ are redefined as follows.

2.2.3. Intuitionistic fuzzy measure

The fuzzy set [43] assigns a membership degree to each of its element and non-membership is automatically the degree equal to one minus the assigned membership degree. Atanassov [1] proposed an extended work of fuzzy sets to intuitionistic fuzzy sets, which assigns a membership degree and a non-membership degree to each of its element with given condition that their sum does not exceed one.

Intuitionistic fuzzy set A of a universe of discourse $X = \{x_1, x_2, \ldots, x_N\}$ can be defined as

$$A \;=\; < x_i, \mu_A(x_i), \nu_A(x_i) > | x_i \in X,$$

which assigns a membership degree $\mu_A(x_i) \in [0, 1]$ and a non-membership degree $\nu_A(x_i) \in [0, 1]$ to each of its element under the condition

$$0 \leq \mu_A(x_i) + \nu_A(x_i) \leq 1, \forall x_i \in X.$$

An intuitionistic fuzzy value (IFV) [39] is the ordered pair $\alpha(x_i) = (\mu_\alpha(x_i), \nu_\alpha(x_i))$ where $\mu_\alpha(x_i) \in [0, 1]$, $\nu_\alpha(x_i) \in [0, 1]$, $\mu_\alpha(x_i) + \nu_\alpha(x_i) \leq 1$.

Let $\alpha(x_i)$ and $\alpha(x_j)$ be two IFVs. Then score value can be defined as $s(\alpha(x_i)) = \mu_\alpha(x_i) - \nu_\alpha(x_i)$ and $s(\alpha(x_j)) = \mu_\alpha(x_j) - \nu_\alpha(x_j)$, respectively. Accuracy of them can be defined as $h(\alpha(x_i)) = \mu_\alpha(x_i) + \nu_\alpha(x_i)$ and

$h(\alpha(x_j)) = \mu_\alpha(x_j) + \nu_\alpha(x_j)$, respectively.

$$\begin{gathered}\text{If } s(\alpha(x_i)) < s(\alpha(x_j)), \text{then } \alpha(x_i) \text{ is smaller}\\ \text{than } \alpha(x_j), \textit{i.e.}, \alpha(x_i) < \alpha(x_j),\\ \text{If } s(\alpha(x_i)) = s(\alpha(x_j)), \text{then}\\ \text{If } h(\alpha(x_i)) = h(\alpha(x_j)), \text{ then } \alpha(x_i) \text{ is same as } \alpha(x_j),\\ \text{If } h(\alpha(x_i)) < h(\alpha(x_j)), \text{ then } \alpha(x_i) \text{ is smaller}\\ \text{than } \alpha(x_j), \;\textit{i.e.}, \;\alpha(x_i) < \alpha(x_j).\end{gathered} \tag{2.7}$$

Let F be a collection of subsets of a nonempty set X, when X is finite it can be power set of X, $\mathcal{P}(X)$ with $\phi \in F$ and $X \in F$. A non-negative monotone set function, μ represent the degree of participation and ν represent the degree of non-participation, are mapping from F to $[0, 1)$ satisfying the conditions:

$$\mu(\phi) = 0, \; \mu(X) = 1, \; \nu(\phi) = 1, \; \nu(X) = 0 \tag{2.8}$$

$$A \subset B \Rightarrow \mu(A) \leq \mu(B), \forall A, B \in F \tag{2.9}$$

depending on the scores and accuracy of A and B as defined previously in equation (2.7).

Here μ is non-additive monotonic and also non-negative.

Choquet integral of f at $X = \{x_1, x_2, \ldots, x_N\}$ can be defined as:

$$\int f \, \mathrm{d}\mu = \int_0^\infty \mu(F_\alpha) \, \mathrm{d}\alpha, \tag{2.10}$$

where f is a non-negative measurable function on $(X, \mathcal{P}(X))$ such that $F_\alpha = \{x | f(x) \geq \alpha, x \in X\} \in \mathcal{P}(X)$ for any $\alpha \in [0, \infty)$, $F_\alpha \in [0, \infty)$ is the α-cut set of f.

To calculate the value of integral, the values of f at $X = \{x_1, x_2, \ldots, x_N\}$ should be rearranged in non-decreasing order, *i.e.*, $f(x'_1) \leq f(x'_2) \ldots \leq f(x'_N)$, where $(x'_1, x'_2, \ldots, x'_N)$ is a permutation or rearrangement of $(x_1, x_2, \ldots, x_N)$. Then, evaluate the value of the expression

$$\int f \, \mathrm{d}\mu = \sum_{i=1}^{n} \left[f(x'_i) - f(x'_{i-1}) \right] \mu \left(x'_i, x'_{i+1}, \ldots, x'_N \right), \tag{2.11}$$

where $f(x'_0) = 0$.

Another integral operator, Sugeno integral is defined based on Sugeno fuzzy measure [34] which is defined below:

2.2.4. Sugeno fuzzy measure

A Sugeno fuzzy measure is a function $\mu : F \rightarrow [0, 1]$ such that

$$\mu(\phi) = 0. \tag{2.12}$$

If $A \subseteq B$ then $\mu(A) \leq \mu(B)$, If $A_n \in F$ and $A_1 \subseteq A_2 \subseteq A_3 \subseteq A_4 \ldots$ then

$$\lim_{n\to\infty} \mu(A_n) = \mu \left(\lim_{n\to\infty} A_n \right). \tag{2.13}$$

3. Fuzzified Choquet integral based regression model

Forecasting models are nothing but regression models representing relationship between predictive and objective attributes. Objective attributes are the current situation which can be estimated by using historical data points. Fuzzified Choquet integral [37] is a non-linear regression tool defined as fuzzy measure and has been used to represent the non-linear relationship between the attributes. Fuzzified Choquet integral with signed fuzzy measure [41] had been utilized in temperature prediction to build Choquet Integral with Interval Integrand (CIII), which was also a regression tool on homogeneous fuzzy data.

In this paper, we are trying to find forecasting model through regression analysis. Let $X = \{x_1, x_2, \ldots, x_N\}$ are N data points in time series. Let, w be the window size. To predict the tth time point objective attribute, $(t-w-1)$th and $(t-w)$th time point predictive attributes are used and pw are the number of predictive attributes where p is the different types of predictive attributes and w is the window size. We have developed the forecasting model to estimate the 3rd data point of closing price by using previous two data points from high, low and closing price of stock exchange time series as these are basic indexes of stock market data. Here, $p = 3$ and $w = 2$. In such way, consecutive w number of historical data from p attributes are used to estimate current objective attribute. Regression model [41] can be constructed as follows:

$$\hat{y} = c + \int (a.\hat{f} + b)\,\mathrm{d}\mu \tag{3.1}$$

where

$\hat{\boldsymbol{y}}$: estimated value of the objective attribute,

$\hat{\boldsymbol{f}}$: are the functional values at $\{x_1, x_2, \ldots, x_N\}$, in our model those are time series data,

$\boldsymbol{\mu}$: fuzzy measure satisfying $\mu(\phi) = 0, \mu(X) = 1, A \leq B \Rightarrow \mu(A) \leq \mu(B), \forall A, B \in \mathcal{P}(X)$, signed fuzzy measure satisfying $\mu(\phi) = 0, \mu(A) \geq 0, \forall A \in \mathcal{P}(X)$ and intuitionistic fuzzy measure described in Section 2.2.3,

a: real-valued function defined on X are scaling parameter,

b: real-valued function defined on X are shifting parameter,

c: adjusting constant.

According to the models, there are $2^n + 2n$ unknown parameters of which $2n + 1$ are regression coefficients and $2^n - 1$ are the values of fuzzy measure to estimate. To reduce the number of parameters, we use the technique as in [41] to normalize the functional values. In such way, we can neglect the shifting coefficients and adjusting constant because after normalization the values of all attributes are referred to the origin. So, number of parameters reduces to $n + 2^n - 1$.

4. Proposed method

In this proposed approach we try to find a prediction model where fuzzified Choquet integral works as a regression tool. Three different set functions are used for Choquet integral fuzzy measures or participation functions.

Yang *et al.* [41] developed CIII regression tool for temperature prediction on meteorological data. They used interval data and signed fuzzy measure.

In this paper, we have used normalized time series data points as integrand along with fuzzified Choquet integral where participation function is fuzzy measure, signed fuzzy measure and intuitionistic fuzzy measure in three different models.

To get the normalized time series, data points are divided by the maximum upper bounds within which all the training and testing data points lie. We are using the regression model given below:

$$\hat{y} = \int a \cdot \hat{f}\,\mathrm{d}\mu \tag{4.1}$$

where $n + 2^n - 1$ parameters need to be estimated,

$$\int f\,\mathrm{d}\mu \;=\; \sum_{i=1}^{n} \left[f(x_i') \;-\; f(x_{i-1}')\right] \mu\left(x_i', x_{i+1}', \ldots, x_N'\right), \tag{4.2}$$

where $f(x_0') = 0$, the values of f at $X = \{x_1, x_2, \ldots, x_N\}$ should be rearranged in non-decreasing order, *i.e.*, $f(x_1') \leq f(x_2') \ldots \leq f(x_N')$, where $(x_1', x_2', \ldots, x_N')$ is a permutation or rearrangement of $(x_1, x_2, \ldots, x_N)$.

A brief description is given here to elaborate the methodology. An example is already discussed in fuzzy measure section. The process is similar. We are considering 9 data points, say, $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9$ and corresponding time series values, say, $f(x_1)$, $f(x_2)$, $f(x_3)$, $f(x_4)$, $f(x_5)$, $f(x_6)$, $f(x_7)$, $f(x_8)$, $f(x_9)$. For $w = 2$, number of predictive attributes is 6. So, we have $f(x_1)$ is the first high price, $f(x_2)$ is the first low price, $f(x_3)$ is the first closing price, $f(x_4)$ is the second high price, $f(x_5)$ is the second low price, $f(x_6)$ is the second closing price, $f(x_7)$ is the third high price, $f(x_8)$ is the third low price, $f(x_9)$ is the third closing price. $f(x_1)$, $f(x_2)$, $f(x_3)$, $f(x_4)$, $f(x_5)$, $f(x_6)$ are predictive attributes, when third closing price is the objective attribute. Again $f(x_4)$, $f(x_5)$, $f(x_6)$, $f(x_7)$, $f(x_8)$, $f(x_9)$ are predictive attributes, when forth closing price is the objective attribute. Now, $f(x_1)$, $f(x_2)$, $f(x_3)$, $f(x_4)$, $f(x_5)$, $f(x_6)$ is rearranged in non-decreasing order to get the new arrangement, say, $f(x'_1)$, $f(x'_2)$, $f(x'_3)$, $f(x'_4)$, $f(x'_5)$, $f(x'_6)$ whereas $(x'_1, x'_2, \ldots, x'_6)$ is a permutation or rearrangement of $(x_1, x_2, \ldots, x_6)$. GA gives us the optimized values of regression coefficients as and values of participation, say μs. So, approximation of third closing price, say f_{A_3} is calculated as:

$$\begin{aligned} f_{A_3} = a_1 * (f(x'_1) - f(x'_0)) * \mu(x'_1, x'_2, \ldots, x'_6) \\ + a_2 * (f(x'_2) - f(x'_1)) * \mu(x'_2, \ldots, x'_6) + \ldots \\ + a_6 * (f(x'_6) - f(x'_5)) * \mu(x'_6) \end{aligned} \tag{4.3}$$

where $f(x'_0) = 0$. Next, $f(x_4)$, $f(x_5)$, $f(x_6)$, $f(x_7)$, $f(x_8)$, $f(x_9)$ is rearranged in non-decreasing order to get the new arrangement, say, $f(x'_1)$, $f(x'_2)$, $f(x'_3)$, $f(x'_4)$, $f(x'_5)$, $f(x'_6)$ whereas $(x'_1, x'_2, \ldots, x'_6)$ is a permutation or rearrangement of $(x_4, x_5, \ldots, x_9)$. Optimized values of regression coefficients as and values of participation, say μs are used to approximate of forth closing price, say f_{A_4} as given below:

$$\begin{aligned} f_{A_4} = a_1 * (f(x'_1) - f(x'_0)) * \mu(x'_1, x'_2, \ldots, x'_6) \\ + a_2 * (f(x'_2) - f(x'_1)) * \mu(x'_2, \ldots, x'_6) + \ldots \\ + a_6 * (f(x'_6) - f(x'_5)) * \mu(x'_6) \end{aligned} \tag{4.4}$$

where $f(x'_0) = 0$. Similar process has been followed for more than 9 data points.

μ is the fuzzy measure, with non-additive monotonicity and non-negativity, $\mu(\phi) = 0, \mu(X) = 1$, $A \subset B \Rightarrow \mu(A) \leq \mu(B), \forall A, B \in F$ in first model.

In the second model, μ is signed fuzzy measure, with non-additivity and non-negativity, $\mu(\phi) = 0$, $\mu(A) \geq 0, \forall A \in F$.

In the third model, μ is the intuitionistic fuzzy measure where participation and non-participation of an element are considered and score and accuracy values like as intuitionistic fuzzy set are used to order μ. Then, μ are made monotonic.

In Section 3, it is explained that we need to estimate $n+2^n-1$ number of parameters. GA is a well established algorithm with many modifications and many application, whereas there are several existing or newly proposed evolutionary algorithms, efficiencies of whose are more or less same in respect of solution quality, but may be some of them are computationally efficient [15]. In our work, we compare the results of fuzzified Choquet integral with fuzzified integrand with fuzzy measure and intuitionistic fuzzy measure with existing work on fuzzified Choquet integral with fuzzified integrand with signed fuzzy measure [41] where GA had been used as parameter optimization tool. So, we use the same as optimization algorithm. Following section describes some important nomenclatures of genetic algorithm.

4.1. Genetic algorithm (GA)

Genetic Algorithm is a search optimization algorithm inspired by Darwin's law of survival of fittest [12]. It mimics the process of natural evolution. It uses operators inspired by biological processes of Inheritance, Mutation, Selection and Crossover. The solutions are represented as chromosomes. Each chromosome is composed of values that can either be the binary, real or floating point in representation. By using an objective function, the chromosomes are mapped from variable (genotypic) space to objective (phenotypic) space. A set of solutions defines the population of a generation. The population evolves as more fit solutions (chromosomes) substitutes less fit solutions. This way GA gradually nears to optimal solutions.

Here, GA is used to estimate $n+2^n-1$ number of parameters. Initially, fixed number of population is created in first generation. Each chromosome has $n+2^n-1$ number of genes. To estimate the regression coefficients a s, random numbers following uniform distribution $U(-2,2)$ and μ s, random numbers following uniform distribution $U(0,1)$ are considered. In each of the three models $\mu(\phi)=0$. To reorder μ s, we are using steps as in [37]. If we arrange $\mathcal{P}(f(X))$ as, $E_0=\phi$, $E_1=\{f(x_1)\}$, $E_2=\{f(x_2)\}$, $E_3=\{f(x_1),f(x_2)\}$, $E_4=\{f(x_3)\}$, $E_5=\{f(x_1),f(x_3)\}$, $E_6=\{f(x_2),f(x_3)\}$, $E_7=\{f(x_1),f(x_2),f(x_3)\}$, $E_8=\{x_4\}$, $E_9=\{f(x_1),f(x_4)\}$, $\ldots f(x_1),f(x_2),f(x_3),f(x_4),\ldots$ are predictive attributes. For each $E_k,\ k=2$ to (2^n-2), check each $E_h,\ h=1$ to $(k-1)$, whether $E_h\subset E_k$ with $\mu(E_h)>\mu(E_k)$. If yes, exchange their values.

There are three main operators selection, crossover and mutation which control the performance of GA. Crossover and mutation operator are used to explore and exploit the solution space using parent chromosomes of population and produce child chromosomes. Table 2 shows some of the initial random values for the regression parameters for a particular population in case of fuzzy measure in fuzzified Choquet integral for BSE data.

In this study, the genetic operators have been used to solve the proposed problem which are discussed below.

- *Selection*: sum of squared error (SSE) is used as fitness function. According to the minimum SSE value chromosomes are ranked, best two directly go into the next generation.

- *Crossover*: two consecutive ranked chromosomes work as parents. Accordingly a single point is selected randomly from these chromosomes they exchange their gene from that position and create new children which goes into the next generation.

- *Mutation*: mutation rate is 0.01 in our models. So, those genes are selected randomly from any chromosomes, except the best two, and mutated by getting random values following uniform distribution $U(-2,2)$ for $a_i,\ \{i=1,\ldots,n\}$ and $U(0,1)$ for $\mu_j,\ \{j=1,\ldots,2^n-1\}$, where n is the number of predicting attributes.

In such way, some fixed number of generation is repeated and lastly, we get the estimated values for those $n+2^n-1$ number of parameters. Using those estimated values, test data sets are observed and root mean square error (RMSE) is calculated between original and predicted values of time series.

5. Data description

Stock market data are multi-attributed in nature and stock market prediction depends on several factors so it is quite difficult to build forecasting models for itself. As there are several factors for the ups and downs of the stock market, the pattern is non-linear in nature. In our work, basic indexes of stock exchange time series data such as high price, low price and closing price are used to develop the models. Following three different stock exchange time series data are collected as training and testing data sets.

- *Bombay stock exchange (BSE)*: BSE's popular equity index – the S&P BSE SENSEX- is India's mostly tracked stock market benchmark index. There are several work *i.e.*, stock market analysis, forecasting models on BSE data [11, 17]. In this paper, training data set, collected daily from January to December, 2014 (241 data points) and testing data set, collected daily from January to April, 2015 (84 data points) are used [6].

- *New York stock exchange (NYSE)*: NYSE is American stock exchange, hugely used in several works [13, 26]. In our work, training data set, collected daily from January to December, 2014 (252 data points) and testing data set, collected daily from January to April, 2015 (82 data points) are used [28].

- *Taiwan stock exchange corporation (TAIEX)*: TAIEX is Taiwan stock exchange benchmark index. It is also hugely used in several works [2, 7, 14, 36]. In our work, training data set, collected daily from January to

Table 2. Initial values of some of $(n + 2^n - 1)$ coefficients in case of fuzzy measure (FM) in fuzzified Choquet integral, for $n = 6$ on BSE time series from Jan to Dec, 2014.

Coefficients	FM	Coefficients	FM
a_1	0.739194	μ_9	0.460309
a_2	–1.669729	μ_{10}	0.773016
a_3	1.921260	μ_{11}	0.579176
a_4	0.291052	μ_{12}	0.863850
a_5	–1.457822	μ_{13}	0.773016
a_6	–1.871264	μ_{14}	0.863850
μ_0	0.000000	–	–
μ_1	0.138082	–	–
μ_2	0.285365	–	–
μ_3	0.199938	μ_{58}	0.918992
μ_4	0.285365	μ_{59}	0.943652
μ_5	0.576553	μ_{60}	0.918992
μ_6	0.773016	μ_{61}	0.943652
μ_7	0.576553	μ_{62}	0.918992
μ_8	0.579176	μ_{63}	1.000000

December, 2014 (247 data points) and testing data set, collected daily from January to April, 2015 (81 data points) are used [35].

6. Result and discussion

In this work, window size w denotes previous w days data, which are used to predict the following day. Here, we consider the window size $w = 2$ that means the previous two days high price, low price and closing price are used to predict the closing price of the following day. So, there are six predictive attributes and one objective attribute in our models. Recent pasts help in future prediction, so, a small w value is considered here. It can be more than that or can be one.

Data are normalized by dividing with possible maximum bounds of each data sets. In case of training data sets on the mentioned period, maximum bounds are 30 000 (BSE), 12 000 (NYSE), 9600 (TAIEX), respectively. For testing data sets, maximum bounds for three data sets are 29 700 (BSE), 12 000 (NYSE) and 10 000 (TAIEX), respectively. Real coded GA is used to estimate the regression coefficients and values of different fuzzy measures. There are total $n+2^n-1$ coefficients and values to estimate three different models on three data sets BSE, NYSE and TAIEX. The estimated coefficients and values are shown for three data sets in three tables, Tables 3, 4 and 5, respectively. We consider the population size as 32 and the number of generation as 4000 for the three models. Mutation rate is 0.01. Root mean square error (RMSE) is determined to measure the error metric in all models. Average forecasting error (AFE) has been also calculated as another error metric for fuzzy measure (FM), signed fuzzy measure (SFM), and intuitionistic fuzzy measure (IFM) (three measures are used in fuzzified Choquet integral). Expression of AFE is given below.

$$\text{AFE}(\%) \;=\; \frac{1}{n}\sum_{i=1}^{n}\frac{|\text{actual}_i - \text{forecasted}_i|}{\text{actual}_i} \times 100\%. \tag{6.1}$$

Figures 1–3 depicted the original and predicted time series of BSE, NYSE and TAIEX training data sets respectively. Figures 4–6 depicted the original and predicted time series of BSE, NYSE and TAIEX testing data sets respectively.

TABLE 3. Estimated values of $(n + 2^n - 1)$ coefficients in case of fuzzy measure (FM), signed fuzzy measure (SFM), and intuitionistic fuzzy measure (IFM) (three measures are used in fuzzified Choquet integral), for $n = 6$ on BSE time series from Jan to Dec, 2014.

Coefficients	FM	SFM	IFM	Coefficients	FM	SFM	IFM
a_1	1.001850	0.997722	1.001786	μ_{29}	0.826733	0.316854	0.883147
a_2	1.020738	1.999913	1.010067	μ_{30}	0.925647	0.986097	0.883147
a_3	–0.1596056	1.173825	–0.162312	μ_{31}	0.939286	0.598963	0.942181
a_4	1.819574	1.819680	1.674141	μ_{32}	0.266828	0.613451	0.123118
a_5	1.917177	1.912547	1.799764	μ_{33}	0.451697	0.289407	0.417008
a_6	1.403884	0.260558	1.484512	μ_{34}	0.307593	0.437006	0.335642
μ_0	0.000000	0.000000	0.000000	μ_{35}	0.620910	0.475107	0.526025
μ_1	0.000035	0.300474	0.000014	μ_{36}	0.656003	0.790286	0.709087
μ_2	0.000582	0.266130	0.000197	μ_{37}	0.689078	0.912565	0.686800
μ_3	0.003961	0.815374	0.003843	μ_{38}	0.767673	0.518612	0.802742
μ_4	0.024842	0.509317	0.003630	μ_{39}	0.782944	0.385196	0.784369
μ_5	0.003961	0.000259	0.003843	μ_{40}	0.826733	0.847749	0.882758
μ_6	0.024842	0.564478	0.029209	μ_{41}	0.806418	0.316900	0.866230
μ_7	0.027561	0.211735	0.055120	μ_{42}	0.826733	0.093871	0.882758
μ_8	0.027561	0.510747	0.027877	μ_{43}	0.826733	0.195280	0.882758
μ_9	0.027561	0.369897	0.055120	μ_{44}	0.826733	0.764383	0.882758
μ_{10}	0.027561	0.110082	0.055120	μ_{45}	0.826733	0.420881	0.882758
μ_{11}	0.027561	0.860815	0.055120	μ_{46}	0.826733	0.666690	0.883147
μ_{12}	0.120934	0.409111	0.055120	μ_{47}	0.826733	0.414863	0.882758
μ_{13}	0.027561	0.000034	0.055120	μ_{48}	0.826733	0.976745	0.883147
μ_{14}	0.468303	0.015003	0.296834	μ_{49}	0.826733	0.392907	0.883147
μ_{15}	0.826733	0.092553	0.883147	μ_{50}	0.826733	0.677480	0.883147
μ_{16}	0.057002	0.728482	0.087866	μ_{51}	0.826733	0.421734	0.883147
μ_{17}	0.150681	0.052609	0.117208	μ_{52}	0.826733	0.414863	0.908513
μ_{18}	0.178600	0.808626	0.092374	μ_{53}	0.826733	0.897801	0.883147
μ_{19}	0.306197	0.745554	0.201281	μ_{54}	0.947962	0.392907	0.923137
μ_{20}	0.374579	0.726202	0.364465	μ_{55}	0.939947	0.677480	0.916199
μ_{21}	0.337001	0.096452	0.426883	μ_{56}	0.9507505	0.421734	0.980335
μ_{22}	0.552278	0.541576	0.690733	μ_{57}	0.959481	0.816385	0.942977
μ_{23}	0.557365	0.015182	0.717555	μ_{58}	0.981284	0.498486	0.992273
μ_{24}	0.579004	0.872365	0.717618	μ_{59}	0.980498	0.997820	0.992273
μ_{25}	0.761509	0.787605	0.736485	μ_{60}	0.995567	0.250788	0.994644
μ_{26}	0.783150	0.785216	0.834082	μ_{61}	0.989781	0.449020	0.997925
μ_{27}	0.826733	0.339131	0.883147	μ_{62}	0.999625	0.268684	0.999591
μ_{28}	0.826733	0.531680	0.883147	μ_{63}	1.000000	0.989173	1.000000

RMSE in every 4000 generations on three data sets are shown in Figures 7–9. It is observed that after some generation all of them converge.

Tables 6 and 7 show the RMSE of training and testing data sets of three stock market data. Bold face shows the best result among them. It can be observed that, the result does not remarkably fluctuate for three different fuzzy measure. In Table 6, fuzzy measure gives a better result for BSE and NYSE where as signed measure gives better for TAIEX. In case of testing data sets, fuzzy measure gives less error for BSE and TAIEX data sets and intuitionistic fuzzy measure gives less error for NYSE and which are shown in Table 7.

Proposed approach and neural network with backpropagation learning are used on multi-attributed stock market time series forecasting. Here, BPNN is used with six nodes in the input layer, four nodes in the hidden layer and one node in the output layer. Two nodes take input from high price, another two nodes take input

TABLE 4. Estimated values of $(n + 2^n - 1)$ coefficients in case of fuzzy measure (FM), signed fuzzy measure (SFM), and intuitionistic fuzzy measure (IFM) (three measures are used in fuzzified Choquet integral), for $n = 6$ on NYSE time series from Jan to Dec, 2014.

Coefficients	FM	SFM	IFM	Coefficients	FM	SFM	IFM
a_1	1.000137	0.999520	1.000172	μ_{29}	0.903234	0.882722	0.811177
a_2	0.879961	1.999018	0.877917	μ_{30}	0.867404	0.986097	0.811177
a_3	1.999036	1.999653	1.999846	μ_{31}	0.946338	0.598963	0.920361
a_4	1.251068	1.896727	1.268682	μ_{32}	0.162922	0.613451	0.087012
a_5	1.977279	1.827485	1.998530	μ_{33}	0.428875	0.289407	0.150244
a_6	–1.995239	0.086653	–1.999920	μ_{34}	0.266828	0.437006	0.123118
μ_0	0.000000	0.000000	0.000000	μ_{35}	0.486976	0.475107	0.201281
μ_1	0.000035	0.181489	0.000745	μ_{36}	0.486976	0.956441	0.483266
μ_2	0.001071	0.940567	0.002226	μ_{37}	0.486976	0.912565	0.489943
μ_3	0.001071	0.815374	0.003321	μ_{38}	0.518612	0.518612	0.483266
μ_4	0.001193	0.509317	0.004677	μ_{39}	0.625401	0.411527	0.544009
μ_5	0.001071	0.122467	0.003321	μ_{40}	0.612626	0.847749	0.609309
μ_6	0.015593	0.564478	0.010832	μ_{41}	0.626180	0.316900	0.576049
μ_7	0.001193	0.306232	0.011305	μ_{42}	0.625777	0.093871	0.622615
μ_8	0.017286	0.000156	0.021595	μ_{43}	0.732067	0.195280	0.639615
μ_9	0.017286	0.192188	0.021595	μ_{44}	0.783505	0.764383	0.702773
μ_{10}	0.017286	0.110082	0.021595	μ_{45}	0.851378	0.111579	0.804137
μ_{11}	0.017286	0.230791	0.021595	μ_{46}	0.827418	0.349170	0.793967
μ_{12}	0.017286	0.409111	0.021595	μ_{47}	0.855331	0.959169	0.857573
μ_{13}	0.017286	0.046987	0.021595	μ_{48}	0.860509	0.556586	0.862823
μ_{14}	0.468303	0.015003	0.033656	μ_{49}	0.860509	0.159394	0.866676
μ_{15}	0.028714	0.000254	0.021595	μ_{50}	0.903234	0.420881	0.866676
μ_{16}	0.109985	0.667571	0.045682	μ_{51}	0.910705	0.999926	0.908219
μ_{17}	0.044649	0.052609	0.058937	μ_{52}	0.910705	0.414863	0.908219
μ_{18}	0.177244	0.808626	0.075525	μ_{53}	0.952414	0.999963	0.922752
μ_{19}	0.279695	0.621286	0.200602	μ_{54}	0.933616	0.392907	0.922752
μ_{20}	0.276470	0.726202	0.161438	μ_{55}	0.952414	0.677480	0.922752
μ_{21}	0.429052	0.096452	0.298964	μ_{56}	0.952414	0.421734	0.922752
μ_{22}	0.337001	0.541576	0.290616	μ_{57}	0.975203	0.816385	0.922752
μ_{23}	0.488228	0.015182	0.450685	μ_{58}	0.952414	0.347522	0.922752
μ_{24}	0.493976	0.872365	0.320299	μ_{59}	0.984693	0.997820	0.939273
μ_{25}	0.689595	0.787605	0.622615	μ_{60}	0.988721	0.250788	0.953315
μ_{26}	0.720983	0.785216	0.622615	μ_{61}	0.999916	0.005843	0.999823
μ_{27}	0.813654	0.339131	0.736485	μ_{62}	0.999994	0.268684	0.999714
μ_{28}	0.796959	0.531680	0.717968	μ_{63}	1.000000	0.989173	1.000000

from low price and the remaining two nodes take input from closing price out of six nodes in input layer. The neural network is trained with historical data and RMSE are shown in Table 6. We can say from the result comparisons that proposed approaches work well comparison to neural network with backpropagation learning. As we are concluding from here, we do not test it with testing data sets.

Table 8 shows the execution time of the regression model using fuzzified Choquet integral with all three measures and BPNN. From the table, it has been observed that the computation time of our model is quite high as it has used population based algorithm to find the regression parameters comparison to BPNN based forecasting model. Calculated AFEs for regression model using fuzzified Choquet integral with all three measures on BSE, NYSE and TAIEX are given in Table 9.

TABLE 5. Estimated values of $(n + 2^n - 1)$ coefficients in case of fuzzy measure (FM), signed fuzzy measure (SFM), and intuitionistic fuzzy measure (IFM) (three measures are used in fuzzified Choquet integral), for $n = 6$ on TAIEX time series from Jan to Dec, 2014.

Coefficients	FM	SFM	IFM	Coefficients	FM	SFM	IFM
a_1	1.005114	1.004078	1.004903	μ_{29}	0.812369	0.316854	0.930620
a_2	0.062318	1.997224	0.071716	μ_{30}	0.903234	0.986097	0.925602
a_3	0.935566	1.951227	0.972023	μ_{31}	0.966087	0.598963	0.942181
a_4	0.563145	1.014179	0.627733	μ_{32}	0.329202	0.613451	0.118832
a_5	1.099774	1.335693	1.165907	μ_{33}	0.451697	0.289407	0.240272
a_6	–1.995239	–1.864760	–1.999920	μ_{34}	0.857989	0.437006	0.150244
μ_0	0.000000	0.000000	0.000000	μ_{35}	0.806418	0.475107	0.302554
μ_1	0.028482	0.000098	0.016145	μ_{36}	0.938083	0.887022	0.887786
μ_2	0.012417	0.266130	0.016145	μ_{37}	0.877036	0.912565	0.602239
μ_3	0.028482	0.815374	0.016145	μ_{38}	0.938083	0.518612	0.887786
μ_4	0.028482	0.509317	0.016145	μ_{39}	0.938410	0.385196	0.887786
μ_5	0.028482	0.000062	0.016145	μ_{40}	0.942302	0.847749	0.887786
μ_6	0.030094	0.564478	0.018121	μ_{41}	0.942302	0.316900	0.887786
μ_7	0.028482	0.000020	0.016145	μ_{42}	0.942302	0.093871	0.918650
μ_8	0.050183	0.153060	0.032904	μ_{43}	0.942302	0.195280	0.908000
μ_9	0.028482	0.000016	0.016145	μ_{44}	0.942302	0.764383	0.918650
μ_{10}	0.050183	0.110082	0.032904	μ_{45}	0.942302	0.111579	0.973116
μ_{11}	0.050183	0.860815	0.032904	μ_{46}	0.967387	0.349170	0.950932
μ_{12}	0.050183	0.409111	0.032904	μ_{47}	0.975235	0.956354	0.983556
μ_{13}	0.050183	0.000876	0.032904	μ_{48}	0.975235	0.556586	0.976101
μ_{14}	0.235785	0.015003	0.094166	μ_{49}	0.975235	0.159394	0.983556
μ_{15}	0.050183	0.000065	0.032904	μ_{50}	0.975235	0.420881	0.983556
μ_{16}	0.129973	0.728482	0.059415	μ_{51}	0.975235	0.811393	0.983556
μ_{17}	0.050515	0.052609	0.123118	μ_{52}	0.975235	0.414863	0.983556
μ_{18}	0.207790	0.808626	0.075525	μ_{53}	0.975235	0.526401	0.983556
μ_{19}	0.320103	0.745554	0.201281	μ_{54}	0.975235	0.392907	0.983556
μ_{20}	0.410539	0.726202	0.152240	μ_{55}	0.975235	0.677480	0.983556
μ_{21}	0.510363	0.096452	0.364465	μ_{56}	0.975235	0.421734	0.983556
μ_{22}	0.468303	0.541576	0.457297	μ_{57}	0.976302	0.816385	0.983556
μ_{23}	0.526025	0.015182	0.717555	μ_{58}	0.975235	0.052287	0.983556
μ_{24}	0.526025	0.872365	0.489943	μ_{59}	0.984562	0.997820	0.983556
μ_{25}	0.527146	0.787605	0.736485	μ_{60}	0.989717	0.250788	0.993348
μ_{26}	0.527146	0.785216	0.761509	μ_{61}	0.999954	0.005843	0.999948
μ_{27}	0.618874	0.339131	0.896676	μ_{62}	0.999994	0.268684	0.999871
μ_{28}	0.814572	0.531680	0.825861	μ_{63}	1.000000	0.989173	1.000000

Table 10 shows the calculated RMSE and AFE of intuitionistic fuzzy time series [18,38] for BSE and TAIEX for different time period. Though the perspective of the models are different, it can be concluded that RMSEs 131.28 and 43.23 are smaller than 192.38 and 68.60, which is obtained in our proposed approach.

Time complexity of an algorithm depends on the execution time of each statement of the code. Though BPNN takes a quite small time to execute, but it is not computationally efficient. Whereas fuzzified Choquet integral with different fuzzy measures shows good results taking more time than BPNN. As GA is used in proposed approach and it has its parameters like number of generations and population size, together with the number of μs, as in a block of code those three parameters are used in separate for-loops to evaluate the expression like (4.3) or (4.4). The time complexity depends on those three parameters. Therefore, the time complexity is $\mathcal{O}(gP\mu)$, where g is the number of generations, P is the population size and μ is the number of measure values.

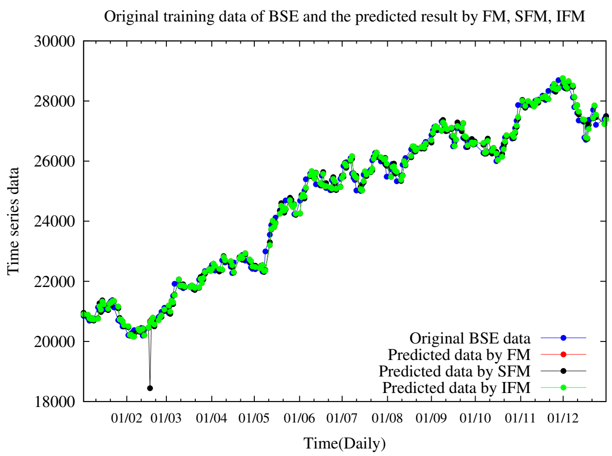

FIGURE 1. BSE training data from Jan to Dec, 2014.

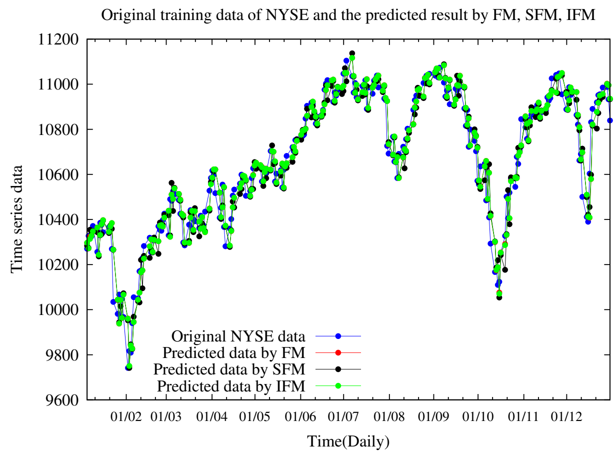

FIGURE 2. NYSE training data from Jan to Dec, 2014.

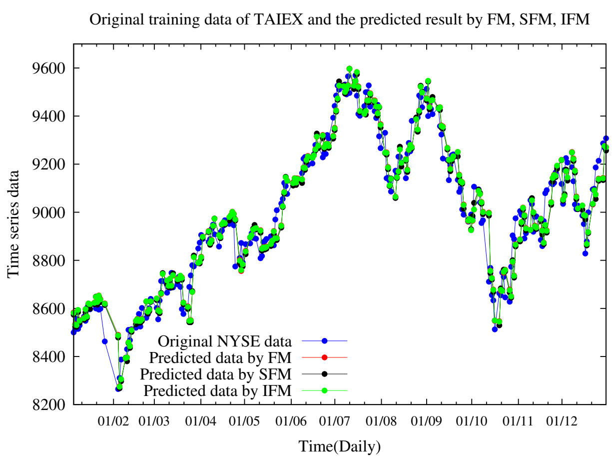

FIGURE 3. TAIEX training data from Jan to Dec, 2014.

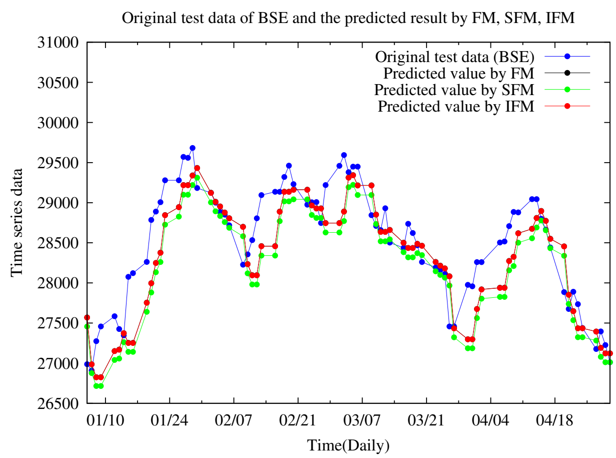

FIGURE 4. BSE test data from Jan to April, 2015.

7. CONCLUSION

In this study, we have developed three different time series models based on fuzzified Choquet integral with respect to fuzzy measure, signed fuzzy measure and intuitionistic fuzzy measure on stock exchange data sets from BSE, NYSE and TAIEX. A comparative study is given in the result and discussion section.

The results have been verified in stock exchange time series prediction with respect to three different measure. Since, stock exchange time series are non-linear in nature, from result on testing data sets we can say accuracy of prediction models highly depends on the non-linearity of the time series.

In future, intuitionistic fuzzy measure can be improved as intuitionistic fuzzy sets for more improved prediction include participation and non-participation of attribute. Except that, different fuzzy measure in the sense of higher order fuzzy set/fuzzy type-2 can also be developed. Another aspect of the proposed approach is in its application where we need to decide the participation of predictive attributes to predict the objective attributes. It can be utilized in autonomous transportation system to make a comfort, safe and timely travel planning.

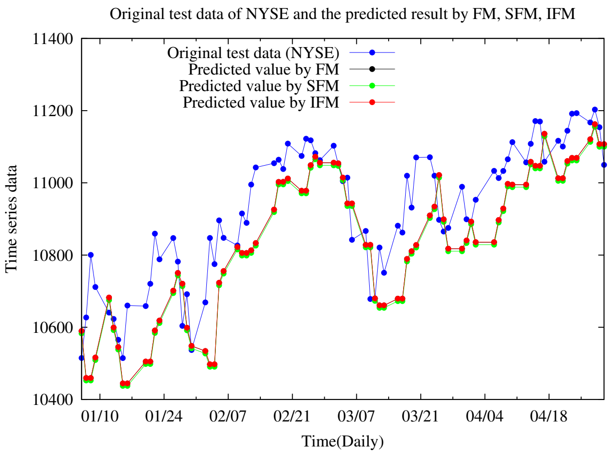

FIGURE 5. NYSE test data from Jan to April, 2015.

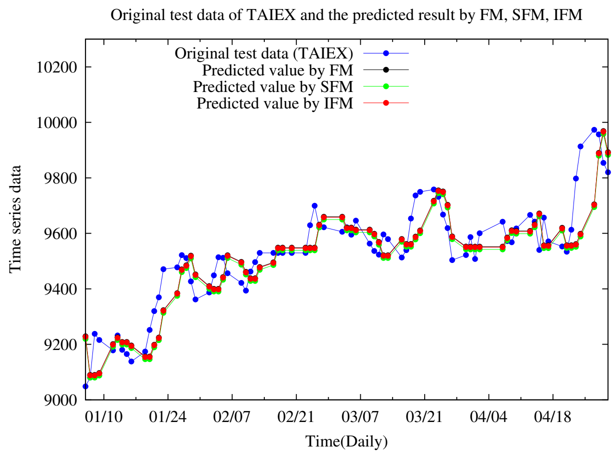

FIGURE 6. TAIEX test data from Jan to April, 2015.

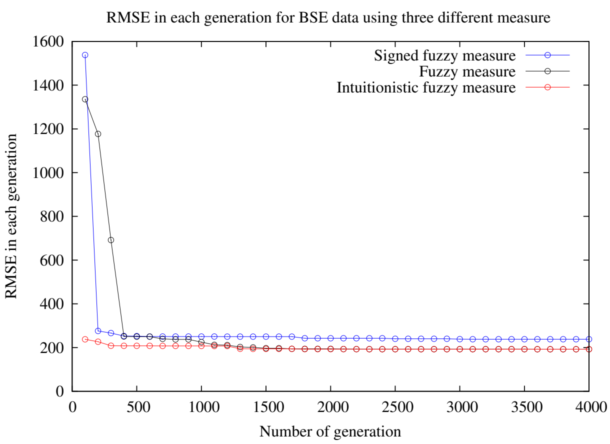

FIGURE 7. RMSE in each generation of BSE training data from Jan to Dec, 2014.

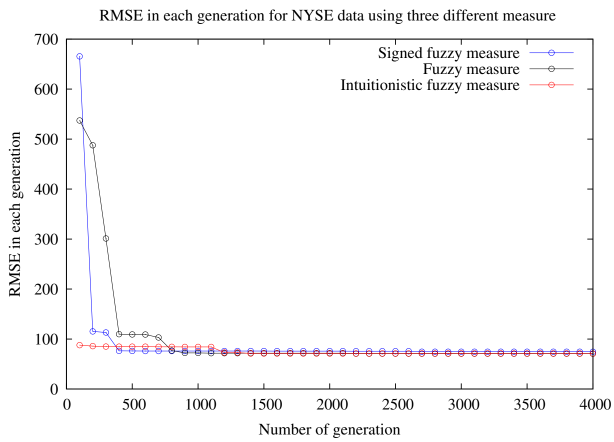

FIGURE 8. RMSE in each generation of NYSE training data from Jan to Dec, 2014.

TABLE 6. RMSE of training data set for three time series BSE, NYSE, TAIEX from Jan to Dec, 2014 by using three different measures.

	Fuzzy measure (FM)	Signed FM	Intuitionistic FM	BPNN
BSE	**192.38**	238.16	192.63	3550.31
NYSE	**70.89**	74.59	70.91	403.91
TAIEX	68.60	**68.04**	68.44	476.39

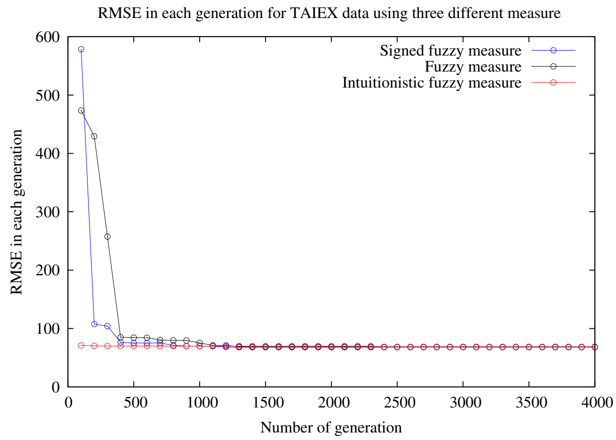

FIGURE 9. RMSE in each generation of TAIEX training data from Jan to Dec, 2014.

TABLE 7. RMSE of testing data set for three time series BSE, NYSE, TAIEX from Jan to April, 2015 by using three different measures.

	FM	SFM	IFM
BSE	**388.31**	454.39	389.16
NYSE	132.74	137.24	**132.49**
TAIEX	**89.74**	92.19	90.16

TABLE 8. Execution time of training data set for three time series BSE, NYSE, TAIEX from Jan to Dec, 2014 by using three different measures.

	FM	SFM	IFM	BPNN
BSE	5m13.500s	6m8.000s	6m8.675s	0.470s
NYSE	5m23.056s	6m25.431s	6m17.546s	0.243s
TAIEX	5m9.437s	6m11.176s	6m8.778s	0.235s

TABLE 9. AFE of training data set for three time series BSE, NYSE, TAIEX from Jan to Dec, 2014 by using three different measures.

	FM	SFM	IFM
BSE	0.60%	0.64%	0.60%
NYSE	0.50%	0.54%	0.50%
TAIEX	0.58%	0.58%	0.58%

TABLE 10. RMSE and AFE of training data set for two time series BSE, TAIEX for different time period by using intuitionistic fuzzy time series.

	RMSE	AFE
BSE	131.28 [18]	6.307% [18]
TAIEX	43.23 [38]	0.51% [38]

Compliance with ethical standards

Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgements. The first author would like to thank DST INSPIRE, India for their help and supports to sustain the work.

References

[1] K. Atanassov, Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **20** (1986) 87–96.
[2] K. Bisht and S. Kumar, Intuitionistic fuzzy set-based computational method for financial time series forecasting. *Fuzzy Inf. Eng.* **10** (2018) 307–323.
[3] K. Bisht and S. Kumar, Hesitant fuzzy set based computational method for financial time series forecasting. *Granular Comput.* **4** (2019) 655–669.
[4] T. Bollerslev, Generalized autoregressive conditional heteroscedasticity. *J. Econom.* **31** (1986) 307–327.
[5] G. Box and G. Jenkins, Time Series Analysis: Forecasting and Control. Holden-Day, San Francisco (1976).
[6] BSE data set. http://in.finance.yahoo.com/q/hp?s=B̂SESN.
[7] Q. Caia, D. Zhanga, B. Wua and S.C.H. Leung, A novel stock forecasting model based on fuzzy time series and genetic algorithm. *Proc. Comput. Sci.* **18** (2013) 1155–1162.
[8] G. Choquet, Theory of capacities. *Ann. Inst. Fourier* **5** (1954) 131–295.
[9] E. Egrioglu, E. Bas, C.H. Aladag and U. Yolcu, Probabilistic fuzzy time series method based on artificial neural network. *Am. J. Intell. Syst.* **6** (2016) 42–47.
[10] R.F. Engle, Autoregressive conditional heteroscedasticity with estimator of the varience of United Kingdom inflation. *Econometrica* **50** (1982) 987–1008.
[11] S.S. Gangwar and S. Kumar, Probabilistic and intuitionistic fuzzy sets–based method for fuzzy time series forecasting. *Cybern. Syst. Int. J.* **45** (2014) 349–361.
[12] D.E. Goldberg, Genetic Algorithms in Search, Optimization, and Machine Learning. Addison-Wesley, Reading, MA (1989).
[13] C.W.J. Granger, Forecasting stock market prices: lessons for forecasters. *Int. J. Forecasting* **8** (1992) 3–13.
[14] K.K. Gupta and S. Kumar, A novel high-order fuzzy time series forecasting method based on probabilistic fuzzy sets. *Granular Comput.* **4** (2019) 699–713.
[15] R. Hassan, B. Cohanim, O. de Weck and G. Venter, A comparison of particle swarm optimization and the genetic algorithm. In: Proceedings of the 1st AIAA Multidisciplinary Design Optimization Specialist Conference (2005) 18–21.
[16] K. Hurang and H.K. Yu, The application of neural networks to forecast fuzzy time series. *Phys. A: Stat. Mech. Appl.* **363** (2006) 481–491.
[17] B.P. Joshi and S. Kumar, Intuitionistic fuzzy sets based method for fuzzy time series forecasting. *Cybern. Syst. Int. J.* **43** (2012) 34–47.
[18] S. Kumar and S.S. Gangwar, Intuitionistic fuzzy time series: an approach for handling non-determinism in time series forecasting. *IEEE Trans. Fuzzy Syst.* **24** (2016) 1270–1281.
[19] S. Lahmiri, Wavelet low- and high-frequency components as features for predicting stock prices with backpropagation neural networks. *J. King Saud Univ. – Comput. Inf. Sci.* **26** (2014) 218–227.
[20] S. Lahmiri, Intraday stock price forecasting based on variational mode decomposition. *J. Comput. Sci.* **12** (2016) 23–27.
[21] S. Lahmiri, A variational mode decompoisition approach for analysis and forecasting of economic and financial time series. *Expert Syst.: App. Int. J.* **55** (2016) 268–273.
[22] S. Lahmiri, A technical analysis information fusion approach for stock price analysis and modeling. *Fluct. Noise Lett.* **17** (2018) 1850007.
[23] S. Lahmiri, Minute-ahead stock price forecasting based on singular spectrum analysis and support vector regression. *Appl. Math. Comput.* **320** (2018) 444–451.
[24] S. Lahmiri and S. Bekiros, Cryptocurrency forecasting with deep learning chaotic neural networks. *Chaos Solitons Fractals* **118** (2019) 35–40.
[25] S. Lahmiri and M. Boukadoum, Intelligent ensemble forecasting system of stock market fluctuations based on symetric and asymetric wavelet functions. *Fluct. Noise Lett.* **14** (2015) 1550033.
[26] W. Leigh, R. Purvis and J.M. Ragusa, Forecasting the NYSE composite index with technical analysis, pattern recognizer, neural network, and genetic algorithm: a case study in romantic decision support. *Decis. Support Syst.* **32** (2002) 361–377.
[27] C. Nikolopoulos and P. Fellrath, A hybrid expert system for investment advising. *Expert Syst.* **11** (1994) 245–250.
[28] NYSE data set. http://finance.yahoo.com/q/hp?s=N̂YA+Historical+Prices.
[29] P.F. Pai and C.S. Lin, A hybrid ARIMA and support vector machines model in stock price forecasting. *Omega* **33** (2005) 497–505.
[30] S.S. Pal and S. Kar, Time series forecasting using fuzzy transformation and neural network with back propagation learning. *J. Intell. Fuzzy Syst.* **33** (2017) 467–477.

[31] S.S. Pal and S. Kar, Fuzzy time series model for unequal interval length using genetic algorithm. *Inf. Technol. Appl. Math. Adv. Intell. Syst. Comput.* **699** (2018) 205–216.

[32] S.S. Pal and S. Kar, A hybridized forecasting method based on weight adjustment of neural network using generalized type-2 fuzzy set. *Int. J. Fuzzy Syst.* **21** (2019) 308–320.

[33] S.S. Pal and S. Kar, Time series forecasting for stock market prediction through data discretization and rule generation by rough set theory. *Math. Comput. Simul.* **162** (2019) 18–30.

[34] M. Sugeno, *Theory of fuzzy integrals and its applications*. Ph.D. thesis, Tokyo Institute of Technology (1974).

[35] TAIEX data set. Available at: http://finance.yahoo.com/q/hp?s=T̂WII+Historical+Prices.

[36] H.J. Teoh, T.L. Chen, C.H. Cheng and H.-H. Chu, A hybrid multi-order fuzzy time series for forecasting stock markets. *Expert Syst. App.* **36** (2009) 7888–7897.

[37] Z. Wang, K.S. Leung, M.L. Wong, J. Fang and K. Xu, Nonlinear non-negative multiregressions based on Choquet integrals. *Int. J. Approximate Reasoning* **25** (2000) 71–87.

[38] Y. Wang, Y. Lei, X. Fan and Y. Wang, Intuitionistic fuzzy time series forecasting model based on intuitionistic fuzzy reasoning. *Math. Prob. Eng.* **2016** (2016) 5035160.

[39] Z. Xu, Choquet integrals of weighted intuitionistic fuzzy information. *Inf. Sci.* **180** (2010) 726–736.

[40] R. Yang, Z. Wang, P.A. Heng and K.S. Leung, Fuzzy numbers and fuzzification of the Choquet integral. *Fuzzy Sets Syst.* **15** (2005) 95–113.

[41] R. Yang, Z. Wang, P.A. Heng and K.-S. Leung, Fuzzified Choquet integral with a fuzzy-valued integrand and its application on temperature prediction. *IEEE Trans. Syst. Man Cybern. – Part B: Cybern.* **38** (2008) 367–380.

[42] U. Yolcu, E. Bas and E. Egrioglu, A new fuzzy inference system for time series forecasting and obtaining the probabilistic forecasts via subsampling block bootstrap. *J. Intell. Fuzzy Syst.* **35** (2018) 2349–2358.

[43] L.A. Zadeh, Fuzzy sets. *Inform. Control* **8** (1965) 338–353.