RAIRO-Oper. Res. 54 (2020) 1573–1592
https://doi.org/10.1051/ro/2019115

RAIRO Operations Research
www.rairo-ro.org

OPTIMAL DECISIONS ON FENCING, PRICING, AND SELECTION OF PROCESS MEAN IN IMPERFECTLY SEGMENTED MARKETS

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Abstract. This paper integrates the selection of a process mean, production and marketing decisions at a firm's level. We discussed a manufacturing firm's problem that integrates its manufacturing decisions on production quantities and selection of a process mean with marketing decisions. The marketing decisions include setting prices, and the fencing investment to mitigate the effect of demand leakages between market segments. The manufacturing firm yields products of varied quality based on a single quality characteristic (*e.g.*, amount of fill). The firm operates in a monopoly, and manufacturing process is assumed to follow a normal distribution, and therefore, it produces multi-grade (class) products distinct in their single quality characteristic. Depending upon the quality characteristic, a product with quality characteristic equal to greater than the upper specification limit is classified as grade 1 product, and sold in primary market at a full price. When the quality characteristic falls between the lower and the upper specification limits, it is referred to as a grade 2 product, and sold in a secondary market at a discounted price. Any product with a quality characteristic lower than the lower specification limit is reworked at an additional cost. A 100% error-free inspection is conducted to segregate the products at a negligible cost. Unlike many related studies in literature, this research proposes a novel integration of the pricing and production quantity decisions along with the process targeting in the two markets with pricing decision in the presence of demand leakages due to cross-elasticity. Furthermore, it is assumed that the firm can mitigate the demand leakage at an additional investment on improving fencing. Thus, the firm's optimal decision would also include the pricing in each market segment, and fencing investment along with its decision on the production quantity for each product class. Mathematical models are developed to address this problem assuming the price-dependent stochastic demand. Structural properties of these models are explored and efficient heuristic solution methodologies are developed. Later, we also developed models when the stochastic demand information is only partially known, and proposed Harmony Search algorithm on the problem. Numerical experimentation is reported to highlight the importance of the proposed integrated framework and the impact of the problem related parameters on a firm's profitability and its integrated optimal control decisions on selection of a process mean, pricing, production quantity, and fencing investment.

Mathematics Subject Classification. 90B05, 90B30, 91B25.

Received August 17, 2018. Accepted December 13, 2019.

Keywords. Process mean, stochastic demand, pricing, demand leakage, partial demand information, structural properties, heuristics, harmony search.

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1. Introduction

The problem of the selection of a process mean has attracted much attention from academic researchers and industry practitioners alike. This is due to the large number of industrial processes that require determining the optimal selection of a process mean [4, 18, 19, 21, 36, 38]. These industrial processes include canning, milling, grinding, plating among others [12, 13]. An optimal selection of a process mean is essential to minimize costs [10, 38]. For example, most manufacturers in US have selected the process mean at a higher level resulting in a higher conformance but leading to an additional giveaway cost and reducing profitability [9, 35]. Alternatively, a tight (lower) level of process mean would have saved the giveaway cost but would have resulted in more number of nonconforming and re-worked products. Thus, optimal selection of a process mean enables exercising a control that balances the two extremes.

Despite being well-researched, the problem of mean selection for a manufacturing process is largely analyzed as a single process production planning problem and decision is to determine the best selection of a process mean while other parameters are assumed to be exogenous (fixed) with an objective of profit maximization [12, 13, 21, 35, 38]. Since the single manufacturing process is stochastic, it yields the products of varied quality characteristics which are segregated based on a single quality characteristic. Often the products are classified into two or more categories that correspond to the market specification (needs) or sale conditions. These markets are more commonly recognized by distinguishable prices where the corresponding products are offered. In most studies, the prices are assumed to be exogenous and therefore the firms only decide on the production decision, *i.e.*, the selection of a process mean. Nevertheless, the pricing is a paramount factor that drives the demand in most marketplaces [8, 30, 44]. Price differentiation is one of the most promising tool in science of profit maximization, the *revenue management* (RM). Regardless of the market segmentation strategy, the market segments can experience demand leakage effects which are more commonly referred to as cannibalization. Once a firm decides to split its market share into multiple markets using a sale condition or price differentiation, it must safeguard these market segments by fencing these to benefit from the market segmentation. Several studies have revealed that heterogeneity is the prime cause for demand leakages [2, 26, 33, 35, 37, 48, 49].

There is a growing interest to integrate the marketing and production decisions at a firm's level. Also, there is a need for coordination models that integrate the manufacturing firm's production decisions (process mean, production quantities) with its market decisions (pricing in each market, market segmentation, and fencing investment). This paper enhances the existing knowledge body in the area of selection of a process mean by developing an integrated framework for determining optimal process mean, along with optimal pricing and production decisions, while taking customers' behaviors into account. An essential market behavior commonly known as heterogeneity is tackled in this research by mimicking the demand leakage effects. We have also proposed a mechanism to mitigate these effects using fencing scheme which incurred a cost. This research along with the control decisions related to process mean, pricing and production, also estimates the optimal investment for fencing the existing market segments. To accomplish the proposed integrated framework, novel mathematical models are proposed for joint determination of pricing, production, selection of a process mean, and fencing investment that mitigates demand leakages when a manufacturing firm experiences price-dependent stochastic demand in an imperfectly segmented markets with full information of the demand distribution. Later, we revisited the problem when the information on price-dependent stochastic demand is partially known and utilize a distribution-free approach for developing a lower bound estimate on the expected profits including the worst possible revenues for the firm in a single selling period.

The rest of the paper is organized as follows: Section 2 presents a brief literature review and positions this research among the contemporary published works. A mathematical model is developed in Section 3. While extending this model, the case of partial demand information is analyzed using the distribution-free approach and the updated model is presented in Section 4. In Section 5, the cost of fencing is modeled and incorporated into the mathematical models developed in Sections 3 and 4. Section 7 discusses the results of numerical simulation study. Finally, the conclusion, limitations, and future work suggestions are outlined out in Section 8.

2. Literature review

This research interfaces two main domain areas: the selection of a process mean problem from production planning and the integration of revenue and pricing optimization tools for profitability augmentation. A brief discussion of the two domains follows.

The problem of economic selection of the process mean is among the ones with rich research history since the seminal work of Springer [43]. Later, a number of studies have contributed to this area of research [4,10,11,18,19,21,22,24,45]. The developments in selection of a process mean research find applications in several processes such as canning, electroplating, milling, grinding, and others [16,18,23,38]. Typically in a selection of a process mean, a firm sets specification limits for the quality characteristics of the products it manufactures. These specifications are set in consideration with the manufacturing process capability and the market needs. The quality characteristics for a manufactured product may include, but not limited to weight,thickness, length among other attributes [10,16,35,36,38,46]. The products manufactured are segregated depending upon their quality characteristics, and are offered in distinct markets using differentiated pricing, sale conditions, or the combination of both. Hong *et al.* [20] addressed the problem of selling multi-grade (class) products in distinct markets at differentiated exogenous (fixed) prices. The study only determined the optimal selection of a process mean. Among contemporary studies, Chen [5] studied the design of process mean, standard deviation, and sampling limits based on Burr distribution. Chen and Chou [6] also proposed a model for joint determination of specification limits, process mean, and economics production (manufacturing) quantity. The consideration of more than one objectives in addition to profitability maximization has also attracted the attention of researchers. Duffuaa and El-Ga'aly [12] have proposed a multi-objective model for selection of a process mean for exogenous prices with 100% error-free inspection. Later, Duffuaa and El-Ga'aly [13] extended their work in Duffuaa and El-Ga'aly [12] when a sampling plan is used instead. The impact of error with 100% inspection and the error in sampling plan based inspection is explored in Duffuaa and El-Ga'aly [14,15], respectively.

Undoubtedly, interfacing pricing decision with the selection of process mean and production decisions is a recent development. This aspect enables integrating production decisions with the marketing decisions at a firm's level. As discussed earlier, most studies have determined the process mean only for an exogenous (fixed) price. Raza and Turiac [35] were the first to interface the selection of a process mean decision with RM tools of price differentiation with demand leakage effects. Later, in an extension to Raza and Turiac [35] a multi-objective framework was presented in Raza *et al.* [36,37]. But, both of these studies have considered a fixed exogenous demand leakage effect, and a price differentiation decision to segment a perceived single market share. In contrast to earlier studies, this paper suggests a comprehensive approach to optimal fencing investment decision for diminishing the leakage effects between pre-existing market along with pricing, production, and the selection of a process mean decisions. This work contributes in extending a study reported in Zhang *et al.* [49] for an application to industries with operations such as canning, milling, grinding, plating and others. This paper also models the lower bound on the expected revenue (profit) including the worst possible profit, the manufacturing firm is likely to attain given the information on the price-dependent stochastic demand is partially known, *i.e.*, distribution is unknown. For this situation, the use of this distribution-free approach based on Scarf [40]'s rule is suggested which was extended in Gallego and Moon [17], and Raza [31].

3. Model development

We develop a novel mathematical model that enables the joint determination of firm's manufacturing decisions with its pricing and market segmentation decisions using the tools from Revenue Management (RM). It is assumed that the firm operates in a monopoly and sells two products which are distinguishable based on a single quality characteristic, ξ. A number of assumptions to facilitate the development of analytical model and achieve tractability are listed in Section 3.2. the list of symbols used in developing model are provided in Table 1 (see, Sect. 3.1).

Next, it is assumed that firm (manufacturer) yields products using a single process (*e.g.*, canning). The products have a single quality characteristic, ξ, which is a random variable since the manufacturing (canning) process follows normal distribution with a probability distribution, $f(\xi)$, and the cumulative probability distribution, $F(\xi)$. The firm's manufacturing decision is selection a process mean, μ, where the process variability, σ is exogenous and fixed. Given a selection of a process mean, μ, the products of diverse quality characteristics are produced. The firm carries out a 100% error-free inspection of all products at a negligible cost. Since the process is random, the quality characteristic of a product, ξ, is also a random outcome. Depending upon the quality characteristic, ξ, the firm segregates the products into three categories: (i) grade 1, when the quality characteristic, ξ, is at least up to upper specification limit, ζ_1, which means that $\xi \geq \zeta_1$; (ii) grade 2, when the quality characteristic, ξ is less than upper specification limit, ζ_1, but it is at least up to the lower specification limit, ζ_2, which means, $\zeta_2 \leq \xi \leq \zeta_1$; and (iii) nonconforming when the quality characteristic, ξ, in the product is less than lower specification limit, ζ_2 such that, $\xi < \zeta_2$.

3.1. Notations

TABLE 1. Notations

Parameters:	
σ	Process variability, *i.e.*, standard deviation
ζ_1	Upper specification limit for class (grade) 1 products
ζ_2	Lower specification limit for class (grade) 2 products
a	Fixed production cost per unit item
b	Variable production cost per unit item
$\kappa_i = \kappa_i(\mu)$	Expected cost per unit for product class (grade) i, $\forall\, i = \{1, 2, 3\}$
$f(\xi)$	Probability distribution of quality characteristic ξ yielded from the process
$F(\xi)$	Cumulative probability distribution of quality characteristic ξ yielded from the process
$\Delta_1 = \dfrac{(\zeta_1 - \mu)}{\sigma}$	Standard normal transformation of ζ_1
$\Delta_2 = \dfrac{(\zeta_2 - \mu)}{\sigma}$	Standard normal transformation of ζ_2
$\emptyset(\Delta_i)$	Standard normal probability distribution of $\Delta_i, \forall\, i = \{1, 2\}$
$\varrho(\Delta_i)$	Standard normal cumulative probability distribution of $\Delta_i, \forall\, i = \{1, 2\}$
α	Maximum perceived cumulative price-dependent deterministic demand (market share)
β	Price sensitivity of the price-dependent deterministic demand
g	Expected giveaway cost [19,35,38], $g \geq 0$
$d_1 = d_1(p_1, p_2, \gamma)$	Price-dependent deterministic demand in full price market segment
$d_2 = d_2(p_1, p_2, \gamma)$	Price-dependent deterministic demand for discount price market segment
x_i	Stochastic (price-independent) demand factor for product class (grade) i, $\forall\, i = \{1, 2\}$
C	Total yield capacity
χ_i	Holding cost per unit of an excess inventory of product class $i, \forall i = \{1, 2\}$
ω_i	Shortage cost per unit of an unmet inventory of product class $i, \forall i = \{1, 2\}$
D_i	Stochastic price dependent demand for product class i, $\forall\, i = \{1, 2\}$
μ_{x_i}	Mean of the stochastic demand factor, x_i, i, $\forall\, i = \{1, 2\}$
σ_{x_i}	Standard deviation of the stochastic demand factor, x_i, i, $\forall\, i = \{1, 2\}$
$\phi_i(x_i)$	Probability distribution function for price-dependent

TABLE 1. Continued.

Parameters:	
	stochastic demand of product class $i, \forall i = \{1, 2\}$
$\Phi_i(x_i)$	Cumulative probability distribution function for price-dependent stochastic demand in product class $i, \forall i = \{1, 2\}$
$\pi = \pi(\mu, p_1, p_2, q_1, q_2)$	Total profit to the manufacturing firm
$E(\cdot)$	Expected value of a parameter
"~"	Accent used for the case when distribution is unknown and distribution-free approach is used
Decision variables:	
p_i	Price for product class i, $\forall\, i = \{1, 2\}$
q_i	Production quantity for product class, $q_i \geq 0$, i, $\forall\, i = \{1, 2\}$
γ	deterministic demand leakage rate per unit price difference, $0 \leq \gamma \leq \overline{\gamma}$
μ	process mean, $\zeta_2 \leq \mu \leq \zeta_1$
τ	Number of nonconforming products reworked, $\tau = C - \sum_{i=1}^{2} q_i \geq 0$

3.2. Assumptions

The following *assumptions* are incorporated in order to support the analytical tractability of the model formulation.

(1) The firm operates in a monopoly and sells its products in a single selling period.
(2) Products are classified on a single quality characteristic, ξ.
(3) In the continuous manufacturing process, the products yielded are subject to 100% error-free inspection which perfectly classifies these products into three categories corresponding to primary, secondary market, and re-worked items.
(4) The products are sold in two market segments based on the quality characteristic.
(5) The price-dependent stochastic demand in each market segment is observed simultaneously in the single selling period and firm decides prior to observing the demand.
(6) Market segments are imperfect, and demand leakage is in both directions from full (higher) market segment to the discount (lower) price market segment and vice versa. However, the demand leakage is deterministic and related to the price differential.
(7) Demand leakage could be mitigated using one time investment at the start of the selling period.

Assumptions 1 through 4 are commonly observed in the process mean determination literature. These assumptions can be observed in Roan *et al.* [38], Hariga and Al-Fawzan [19], and Duffuaa and El-Ga'aly [12,13]. Whereas, assumptions 5 through 7 are widely considered in the research that interface pricing and production decisions Zhang *et al.* [49], Raza and Turiac [35], and Raza *et al.* [36].

Next, these products are distinguished in their quality characteristics, and therefore, are sold in the market at differentiated prices. Similar to Zhang *et al.* [49], it is assumed that there are two existing market segments, $i, \forall\, i = \{1, 2\}$. Product class (grade) 1 is sold into market segment 1 which is regarded as full price market segment. Whereas, product class 2 is offered into the discounted market segment 2. The full price market segment, that is market segment 1, has market share, α_1 with a price sensitivity, β_1. Similarly, market segment 2 has market share, α_2 and the corresponding price sensitivity, β_2. It assumed that the price-dependent deterministic demand, $u_i(p_i)$, $\forall\, i = \{1, 2\}$, is linear, $u_i = u_i(p_i) = \alpha_i - \beta_i\, p_i$, $\forall\, i = \{1, 2\}$. u_i, $\forall\, i = \{1, 2\}$ follows increasing price elasticity [8]. The demand leakage effect is assumed to operate from the full price market segment (segment 1) to the discounted price market segment (segment 2). There is a total of, $\gamma(p_1 - p_2)$ demand that is leaked from the full price market segment to the discounted price. Thus, the adjusted price-dependent deterministic demand, y_i, $\forall\, i = \{1, 2\}$ is modeled as:

$$d_1 = \alpha_1 - \beta_1 p_1 - \gamma(p_1 - p_2) \tag{3.1}$$

$$d_2 = \alpha_2 - \beta_2 p_2 + \gamma(p_1 - p_2). \tag{3.2}$$

In equations (3.1) and (3.2), it is obvious that, when $0 \le \gamma \le \overline{\gamma}$ such that $\beta_i >> \gamma, \forall\, i = \{1, 2\}$, we would have, $p_1 \ge p_2$. The maximum price for full price market segment 1 is, $\overline{p}_i = \dfrac{\alpha_i}{\beta_i}, \forall\, i = \{1, 2\}$. Also, the amount of demand leakage is zero, when $\gamma = 0$, irrespective of the price differential, $p_1 - p_2$. Nevertheless, it most applications, $\gamma > 0$ is assumed to be exogenous (fixed), otherwise, γ can be reduced at an investment for fencing the market segments to mitigate the demand leakages between market segments [48,49]. A less likely scenario is when the price difference is zero, *i.e.*, $p_1 - p_2 = 0$, the market segments are indifferent and are not differentiated with distinct prices, but rather the products are sold in a single market.

The firm experiences the price-dependent stochastic demand, D_i, $\forall\, i = \{1, 2\}$ in market segment, i. This study assumes most widely used *additive modeling* approach [8,29,35]. In additive modeling approach, D_i, $\forall\, i = \{1, 2\}$ has two components. One is from the price-dependent deterministic (risk-less) demand $d_i, \forall\, i = \{1, 2\}$ whereas, the other component of the demand is the stochastic price independent demand factor, x_i. Using additive modeling [8, 29, 35] framework we have:

$$D_i = d_i + x_i,\ \forall\, i = \{1, 2\} \tag{3.3}$$

In equation (3.3), the price independent demand additive factor x_i, $\forall\, i = \{1, 2\}$, is assumed to have a probability distribution function, $\phi_i(x_i)$, and a cumulative probability distribution function, $\Phi_i(x_i)$. Both $\phi_i(x_i)$, and $\Phi_i(x_i)$ are assumed continuous, twice differentiable and following an increasing failure rate [29, 47]. These characteristics are widely observed in commonly studied distributions such as Uniform, Normal and Lognormal [29, 47]. x_i is bounded in $[\underline{x}_i, \overline{x}_i]$ and the expectation of x_i is $\mu_{x_i} = 0, \forall\, i = \{1, 2\}$, with standard deviation $\sigma_{x_i} > 0$. Since the firm experiences the price-dependent stochastic demand, D_i in each market segment, i, it costs the firm a holding cost, χ_i, to stock an item of class i which is unsold in market segment, i. Similarly, ω_i is the shortage penalty per unit experienced by the firm for any unsatisfied demand of product class i in the corresponding market segment, i.

In equation (3.4), a mathematical formulation is suggested for the total profit, π perceived by the firm. We remark here that π is a random variable and $E(\pi)$ is the expectation of π (see, [12, 13, 35, 39]). In the mathematical formulation for π in equation (3.4), the price-differential dependent demand leakage coefficient, γ is exogenous (fixed), however, later it will also be computed against an optimal investment of fencing similar to a study in [49]. However, in the following we present a formulation with an exogenous, demand leakage coefficient, γ:

$$\pi = \begin{cases} p_1 \min\{q_1, D_1\} - \chi_1[q_1 - D_1]^+ - \omega_1[D_1 - q_1]^+ - \kappa(\xi)\, q_1 - b\, q_1(\zeta - \zeta_1) & \xi \ge \zeta_1; \\ p_2 \min\{q_2, D_2\} - \chi_2[q_2 - D_2]^+ - \omega_2[D_2 - q_2]^+ - \kappa(\xi)\, q_2, & \zeta_2 \le \xi < \zeta_1; \\ E(\pi) - r\,\tau - \tau\ \kappa(\xi), & \xi < \zeta_2. \end{cases} \tag{3.4}$$

In equation (3.4), the total (net) profit for the firm is formulated. Again recall here π is a random variable, whose expectation is, $E(\pi)$ which is derived in forthcoming equation (3.9). Since the firm experiences price dependent stochastic demand in each market segment. The formulation relies on the modeling the newsvendor problem with pricing along with shortage and holding costs in each of the two market segment [8,33,35]. The first term is the expected profit from grade 1 products, the second term is the expected profit from grade 2 products, and last term is the expected profit (often a loss) from non-conforming products after rework. Recall here that price-dependent adjusted demands, D_i, $\forall\, i = \{1, 2\}$, also contain the demand leakages effects, $\gamma(p_1 - p_2)$, which may also be regarded as cross-elasticity [49]. Recall that following some earlier studies (see [7, 32, 47]), we can use the following relationships to simplify the profit function built in equation (3.9):

$$\min\{q_i, D_i\} = q_i - E_{x_i}[q_i - D_i]^+, \qquad \forall i = \{1, 2\} \tag{3.5}$$

$$E_{x_i}\,[D_i - q_i]^+ = d_i - q_i + E_{x_i}[q_i - D_i]^+, \qquad \forall i = \{1, 2\}. \tag{3.6}$$

Also using earlier studies (see, [8,33,35]), we obtain the expected leftover/excess inventory in equation (3.7).

$$E_{x_i}\left[q_i - D_i\right]^+ = \int_{\underline{x}_i}^{q_i-d_i} \Phi_i(x_i)\,\partial\, x_i,\ \forall i = \{1,2\} \tag{3.7}$$

Using the relationships outlined in equations (3.5)–(3.6), equation (3.4) can be re-written as follows:

$$\pi = \begin{cases} (p_1+\omega_1-\kappa(\xi))\,q_1 - \omega_1\,d_1 - (p_1+\omega_1+\chi_1)\displaystyle\int_{\underline{x}_1}^{q_1-d_1}\Phi_1(x_1)\partial\,x_1 - b\,q_1(\xi-\zeta_1), & \xi \geq \zeta_1;\\ (p_2+\omega_2-\kappa(\xi))\,q_2 - \omega_2\,d_2 - (p_2+\omega_2+\chi_2)\displaystyle\int_{\underline{x}_2}^{q_2-d_2}\Phi_2(x_2)\partial\,x_2, & \zeta_2 \leq \xi < \zeta_1;\\ E(\pi) - r\,\tau - \tau\ \kappa(\xi), & \xi < \zeta_2. \end{cases} \tag{3.8}$$

Next, we explore equation (3.8) further to derive the mathematical expression for the expected profit, $E(\pi)$. We have followed the approach used in [35] to obtain equation (3.9)

$$\begin{aligned} E(\pi) = &\int_{\zeta_1}^{\infty}\left((p_1+\omega_1-\kappa(\xi))\,q_1 - \omega_1\,d_1 - (p_1+\omega_1+\chi_1)\int_{\underline{x}_1}^{q_1-d_1}\Phi_1(\xi_1)\partial\,x_1 - b\,q_1(\xi-\zeta_1)\right) f(\xi)\,\partial\,\xi\\ &+\int_{\zeta_2}^{\zeta_1}\left((p_2+\omega_2-\kappa(\xi))\,q_2 - \omega_2\,d_2 - (p_2+k_2+\chi_2)\int_{\underline{x}_2}^{q_2-d_2}\Phi_2(x_2)\partial\,x_2\right) f(\xi)\,\partial\,\xi\\ &+\int_{-\infty}^{\zeta_2}(E(\pi)-r\,\tau-\tau\ \kappa(\xi))\ f(\xi)\,\partial\,\xi. \end{aligned} \tag{3.9}$$

With some re-arrangements. Thus, equation (3.9) can be re-written as:

$$\begin{aligned} E(\pi) = &\left((p_1+\omega_1)\,q_1 - \omega_1\,d_1 - (p_1+\omega_1+\chi_1)\int_{\underline{x}_1}^{q_1-d_1}\Phi_1(x_1)\partial\,x_1\right)\int_{\zeta_1}^{\infty} f(\xi)\partial\,\xi - q_1\int_{\zeta_1}^{\infty}\kappa(\xi)f(\xi)\partial\,\xi\\ &- b\,q_1\int_{\zeta_1}^{\infty}(x-\zeta_1)\,f(\xi)\,\partial\,\xi + \left((p_2+\omega_2)\,q_2 - \omega_2\,d_2 - (p_2+\omega_2+\chi_2)\int_{\underline{x}_2}^{q_2-d_2}\Phi_2(x_2)\partial\,x_2\right)\\ &\times\int_{\zeta_2}^{\zeta_1} f(\xi)\,\partial\,\xi - q_2\int_{\zeta_2}^{\zeta_1}\kappa(\xi)f(\xi)\partial\,\xi + (E(\pi)-r\,\tau)\int_{-\infty}^{\zeta_2} f(\xi)\,\partial\,\xi - \tau\int_{-\infty}^{\zeta_2}\kappa(\xi)f(\xi)\partial\,\xi. \end{aligned} \tag{3.10}$$

In equation (3.10), we adopt some simplifications such that: $1-\varrho(\Delta_1) = \int_{\zeta_1}^{\infty} f(\xi)\partial\,\xi$, $\varrho(\Delta_1)-\varrho(\Delta_2) = \int_{\zeta_2}^{\zeta_1} f(\xi)\partial\,\xi$, $\varrho(\Delta_2) = \int_{-\infty}^{\zeta_2} f(\xi)\partial\,\xi$, $\kappa_1 = \int_{\zeta_1}^{\infty}\kappa(\xi)\partial\,\xi$, $\kappa_2 = \int_{\zeta_2}^{\zeta_1}\kappa(\xi)\partial\,\xi$, $\kappa_3 = \int_{-\infty}^{\zeta_2}\kappa(\xi)\partial\,\xi$, and $g = \int_{\zeta_1}^{\infty}(x-\zeta_1)f(\xi)\partial\,\xi$.

It is also assumed that the direct unit production cost for the finished product is a linear function of the quality characteristic, ξ; many previous studies have followed this framework (*e.g.*, [4,19,41]). Thus, the production cost per unit at quality characteristic, ξ, is $\kappa(\xi) = a + b\,\xi$, where a is the fixed production cost and b is the cost of obtaining a specific quality characteristic for one unit of finished product. Therefore, the expected unit production cost of class 1 products is given by κ_1 as follows:

$$\begin{aligned} \kappa_1 &= \int_{\zeta_1}^{\infty}\kappa(\xi)f(\xi)\partial\,\xi\\ &= \int_{\zeta_1}^{\infty}(a+b\,\xi)f(\xi)\,\partial\,\xi\\ &= (a+b\mu)\,(1-\varrho(\Delta_1)) + \sigma\,b\,\emptyset(\Delta_1). \end{aligned} \tag{3.11}$$

In equation (3.11), $\emptyset(\Delta_1) = \frac{1}{\sqrt{2\pi}}\,e^{-\frac{1}{2}\Delta_1^2}$.

Next, in order to determine the giveaway cost [12, 13, 19, 35, 38] we first determine the expected additional quality characteristic, g, that is given away when a process mean is set to a value, μ. We can write:

$$\begin{aligned} g &= \int_{\zeta_1}^{\infty} (x - \zeta_1)\, f(\xi)\partial\, \xi \\ &= E(\xi|\xi \geq \zeta_1)\,(1 - \varrho(\Delta_1)) \\ &= \left(\mu + \sigma \frac{\emptyset(\Delta_1)}{1 - \varrho(\Delta_1)}\right)(1 - \varrho(\Delta_1)) \\ &= \mu\,(1 - \varrho(\Delta_1)) + \sigma\emptyset(\Delta_1). \end{aligned} \tag{3.12}$$

Notice here that equation (3.12) gives the expected quality characteristic of giveaway. As b is the cost per unit quality of characteristic [19], therefore, the expected giveaway cost per unit would be $b\,g$. The expected production cost for grade 2 item, $\kappa_2 = \int_{\zeta_2}^{\zeta_1} \kappa(\xi)\, f(\xi)\partial\, \xi$ is determined similarly as follows:

$$\begin{aligned} \kappa_2 &= \int_{\zeta_2}^{\zeta_1} \kappa(\xi) f(\xi)\, \partial\, x \\ &= \int_{\zeta_2}^{\zeta_1} (a + b\,\xi) f(\xi)\, \partial\, \xi \\ &= (a + b\mu)\,(\varrho(\Delta_1) - \varrho(\Delta_2)) + \sigma b\,(\emptyset(\Delta_2) - \emptyset(\Delta_1)) \end{aligned} \tag{3.13}$$

where in equation (3.13), $\varrho(\Delta_1) - \varrho(\Delta_2) = \displaystyle\int_{\zeta_2}^{\zeta_1} f(\xi) d\,\xi$, $\emptyset(\Delta_1) = \dfrac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}\Delta_1^2}$, $\emptyset(\Delta_2) = \dfrac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}\Delta_2^2}$, $\Delta_1 = \dfrac{\zeta_1 - \mu}{\sigma}$ and $\Delta_2 = \dfrac{\zeta_2 - \mu}{\sigma}$.

Lastly, the expected production of a non-conforming item, $\kappa_3 = \int_{-\infty}^{\zeta_2} \kappa(\xi)\partial\, \xi$ is determined in the following equation:

$$\begin{aligned} \kappa_3 &= \int_{-\infty}^{\zeta_2} (a + b\,\xi) f(\xi)\, \partial\, \xi \\ &= (a + b\mu) f(\Delta_2) - \sigma\, b\, f(\Delta_2). \end{aligned} \tag{3.14}$$

Notice here that a firm has capacity, C. Given that, if the process mean is set to μ, the total expected number of nonconforming items would be, $\tau = C\, \int_{-\infty}^{\zeta_2} f(\xi)\partial\, \xi = C\, \varrho(\Delta_2) \leq C - (q_1 + q_2)$.

Finally, after substituting the values derived the simplified expected profit function, $E(\pi)$ would be:

$$\begin{aligned} E(\pi) = \frac{1}{1 - \varrho(\Delta_2)} \Bigg\{ &\left((p_1 + \omega_1)\, q_1 - \omega_1\, d_1 - (p_1 + \omega_1 + \chi_1) \int_{\underline{x}_1}^{q_1 - d_1} \Phi_1(x_1)\partial\, x_1 \right) (1 - \varrho(\Delta_1)) - \kappa_1\, q_1 - b\, g\, q_1 \\ &+ \left((p_2 + \omega_2)\, q_2 - \omega_2\, d_2 - (p_2 + \omega_2 + \chi_2) \int_{\underline{x}_2}^{q_2 - d_2} \Phi_2(x_2)\partial\, x_2 \right) (\varrho(\Delta_1) - \varrho(\Delta_2)) - \kappa_2\, q_2 \\ &- (r\, \varrho(\Delta_2) + \kappa_3)\, \varrho(\Delta_2)\, C \Bigg\}. \end{aligned} \tag{3.15}$$

The firm's optimization problem would be:

$$\text{P}: \max E(\pi) \tag{3.16}$$

subject to:

$$p_1 - p_2 \geq 0 \tag{3.17}$$

$$(1 - \varrho(\Delta_1))\, C - q_1 \geq 0 \tag{3.18}$$

$$(\varrho(\Delta_1) - \varrho(\Delta_2))\, C - q_2 \geq 0 \tag{3.19}$$

$$(1 - \varrho(\Delta_2))\, C - \sum_{i=1}^{2} q_i \geq 0 \tag{3.20}$$

$$\mu - \zeta_2 \geq 0 \tag{3.21}$$

The problem P, is nonlinear program subject to nonlinear constraints. Here, the control decisions are μ, p_1, p_2, q_1, and q_2. The expected number of reworked items, τ, are computed such that, $\tau = C-(q_1+q_2)$ based on the assumption of 100% inspection. In problem P, we maximize the expected profit, $E(\pi)$ while observing constraints in equations (3.17)–(3.21). Constraint in equation (3.17) ensures the price in the primary market at least the same or higher compared to the price in the secondary market. Equation (3.18) limits the products that could be manufactured and later sold in the primary (full price) market. Likewise, equation (3.18) enables determining quantities that are to be produced for secondary (discounted price) market. Similarly, equation (3.20) limits the total production of items for primary and secondary market, $\sum_{i=1}^{2} q_i$ must not exceed the allocated capacity, $(1 - \varrho(\Delta_2))\, C$. Lastly, it is desirable to set the process mean at least or more than the lower specification limit, ζ_2 to ensure least nonconforming items being manufactured.

3.3. Structural properties

The structural properties of the problem P, are analyzed here in order to develop an efficient heuristic based solution approach. Later, these properties are planned to be subsequently incorporated in developing direct search meta-heuristics, such as Harmony Search (HS) meta-heuristic [35]. Earlier related studies in [19], [9], and [10] have developed similar efficient heuristic solution procedures for process mean selection problem but for exogenous price under no capacity limitations. Here, we are able to find an optimal control for $(\mu^*, p_1^*, p_2^*, q_1^*, q_2^*)$ using these structural properties. Once these controls are determined for a given demand leakage rate, γ then, the optimal fencing investment cost, $G(\gamma^*)$ is computed using HS meta-heuristic which is modelled in Section 5. Our approach is consistent with a related study in Zhang *et al.* [49]. This hybridization scheme significantly improves the performance of the HS meta-heuristic.

To this end, firstly being a nonlinear program P, can also be addressed using Karush Kuhn Tucker (KKT) optimality conditions [3]. However, the present complexity only enables us to conduct a sequential analysis which may be used in developing an efficient *heuristic* procedure. A considerable number of recent studies have adopted this approach [34, 42, 49].

Proposition 1. *In* P*, given a process mean, μ, and pricing decisions, $\upsilon, p_i\, \forall i = \{1, 2\}$, the optimal production decisions under no capacity constraints are:*

$$q_1^* = d_1 + \Phi_1^{-1}(\rho_1) \tag{3.22}$$

$$q_2^* = d_2 + \Phi_2^{-1}(\rho_2) \tag{3.23}$$

where, $\rho_1 = \dfrac{(p_1 + \omega_1)(1 - \varrho(\Delta_1)) - \kappa_1 - bg}{(1 - \varrho(\Delta_1))(p_1 + \omega_1 + \chi_1)}$, *and* $\rho_2 = \dfrac{(p_2 + \omega_2)(\varrho(\Delta_1) - \varrho(\Delta_2)) - \kappa_2}{(\varrho(\Delta_1) - \varrho(\Delta_2))(p_2 + \omega_2 + \chi_2)}$.

Proof. See Appendix A. □

Using the optimal decisions on production quantities (q_i, $\forall\, i = \{1, 2\}$, and z), we can reduce the the problem P to P′. P′ is obtained by ignoring the capacity and production related constraints including limitation on process (target) mean in equations (3.18)–(3.21). Thus, P′ only contains pricing decisions (p_i, $\forall\, i = \{1, 2\}$, υ) and it is a constrained problem represented in equations (3.24) and (3.25).

$$\mathrm{P}': \; E(\boldsymbol{\pi}) \tag{3.24}$$

$$\text{subject to:}$$

$$p_1 - p_2 \geq 0. \tag{3.25}$$

While ignoring the constraint equation (3.25), and substituting, un-constrained production quantities, q_i, $\forall\, i = \{1, 2\}$ from equations (3.22) and (3.23) yields:

$$\begin{aligned} E(\boldsymbol{\pi}) = \frac{1}{1-\varrho(\Delta_2)} \Bigg\{ &\left((p_1+\omega_1)(d_1+\Phi_1^{-1}(\rho_1)) - \omega_1 d_1 - (p_1+\omega_1+\chi_1) \int_{\underline{x}_1}^{\Phi_1^{-1}(\rho_1)} \Phi_1(x_1)\partial x_1 \right) (1-\varrho(\Delta_1)) \\ &- (\kappa_1 + bg)(d_1+\Phi_1^{-1}(\rho_1)) + \Bigg((p_2+\omega_2)(d_2+\Phi_2^{-1}(\rho_2)) - \omega_2 d_2 - (p_2+\omega_2+\chi_2) \\ &\times \int_{\underline{x}_2}^{\Phi_2^{-1}(\rho_2)} \Phi_2(x_2)\partial x_2 \Bigg) (\varrho(\Delta_1)-\varrho(\Delta_2)) - \kappa_2(d_2+\Phi_2^{-1}(\rho_2)) - (r\varrho(\Delta_2)+\kappa_3)\varrho(\Delta_2)C \Bigg\}. \end{aligned} \tag{3.26}$$

In the forthcoming analysis, we drop the expectation operand, $E(\cdot)$. Using the first order optimality condition on pricing decisions, results. Recall here that P′, given process mean, μ, and un-capacitated optimal production decisions, q_i, $\forall\, i = 1, 2$.

$$\begin{aligned} \frac{\partial \pi}{\partial p_1} = (1-\varrho(\Delta_1)) \Bigg\{ &d_1 + \Phi_1^{-1}(\rho_1) + (p_1-\omega_1)\left(\frac{\partial d_1}{\partial p_1} + \frac{\partial \Phi_1^{-1}(\rho_1)}{\partial p_1}\right) - \omega_1 \frac{\partial d_1}{\partial p_1} \\ &- \int_{\underline{x}_1}^{\Phi_1^{-1}(\rho_1)} \Phi_1(x_1\partial x_1) + (p_1+\omega_1+\chi_1)\rho_1 \frac{\partial \Phi_2^{-1}(\rho_1)}{\partial p_1} \Bigg\} \\ &- (\kappa_1+bg)\left(\frac{\partial d_1}{\partial p_1} + \frac{\partial \Phi_1^{-1}(\rho_1)}{\partial p_1}\right) + (\varrho(\Delta_1)-\varrho(\Delta_2)) \left\{ (p_2+\omega_2)\frac{\partial d_2}{\partial p_1} - \omega_2 \frac{\partial d_2}{\partial p_1} \right\} \\ &- \kappa_2 \frac{\partial d_2}{\partial p_1} = 0 \end{aligned} \tag{3.27}$$

$$\begin{aligned} \frac{\partial \pi}{\partial p_2} = (1-\varrho(\Delta_1)) &\left\{ (p_1-\omega_1)\frac{\partial d_1}{\partial p_2} - \omega_1 \frac{\partial d_1}{\partial p_2} \right\} - (\kappa_1+bg)\frac{\partial d_1}{\partial p_2} + (\varrho(\Delta_1)-\varrho(\Delta_2)) \Bigg\{ d_2 + \Phi_2^{-1}(\rho_2) \\ &+ (p_2-\omega_2)\left(\frac{\partial d_2}{\partial p_2} + \frac{\partial \Phi_2^{-1}(\rho_2)}{\partial p_2}\right) - \omega_2 \frac{\partial d_2}{\partial p_2} - \int_{\underline{x}_2}^{\Phi_2^{-1}(\rho_2)} \Phi_2(x_2)\partial x_2 - (p_2+\omega_2+\chi_2)\rho_2 \frac{\partial \Phi_2^{-1}(\rho_2)}{\partial p_2} \Bigg\} \\ &- \kappa_2 \left(\frac{\partial d_2}{\partial p_2} + \frac{\partial \Phi_2^{-1}(\rho_2)}{\partial p_2}\right) = 0 \end{aligned} \tag{3.28}$$

where in equations (3.27) and (3.28), $d_1 = \alpha_1 - \beta_1 p_1 - \gamma(p_1 - p_2)$, $d_2 = \alpha_2 - \beta_2 p_2 + \gamma(p_1 - p_2)$, $\dfrac{\partial d_1}{\partial p_1} = -(\beta_1 + \gamma)$, $\dfrac{\partial d_1}{\partial p_2} = \gamma$, $\dfrac{\partial d_2}{\partial p_1} = -(\beta_1 + \gamma)$, and $\dfrac{\partial d_2}{\partial p_2} = -(\beta_2 + \gamma)$ Also,

$$\frac{\partial \Phi_1^{-1}(\rho_1)}{\partial p_1} = \frac{\chi_1(1-\varrho(\Delta_1))^2 - (\kappa_1 + bg)(1-\varrho(\Delta_1))}{((1-\varrho(\Delta_1))(p_1+\omega_1+\chi_1))^2} \tag{3.29}$$

$$\frac{\partial \Phi_2^{-1}(\rho_1)}{\partial p_2} = \frac{\chi_2(\varrho(\Delta_1)-\varrho(\Delta_2))^2 - \kappa_2(\varrho(\Delta_1)-\varrho(\Delta_2))}{((\varrho(\Delta_1)-\varrho(\Delta_2))(p_2+\omega_2+\chi_2))^2} \tag{3.30}$$

$$\rho_1 = \frac{(p_1+\omega_1)(1-\varrho(\Delta_1)) - \kappa_1 - bg}{(1-\varrho(\Delta_1))(p_1+\omega_1+\chi_1)} \tag{3.31}$$

$$\rho_2 = \frac{(p_2+\omega_2)(\varrho(\Delta_1)-\varrho(\Delta_2)) - \kappa_2}{(\varrho(\Delta_1)-\varrho(\Delta_2))(p_2+\omega_2+\chi_2)}. \tag{3.32}$$

Also $\dfrac{\partial \Phi_2^{-1}(\rho_1)}{\partial p_2} = 0$, $\dfrac{\partial \Phi_2^{-1}(\rho_2)}{\partial p_1} = 0$.

These structural properties are utilized in developing an efficient heuristic solution algorithm. The algorithm combines the piece-wise optimal conditions noted earlier with a bounded search using golden section method [3]. Any constraint violations are handled using one of the approaches suggested in Michalewicz [27].

3.4. Heuristic solution algorithm

- **Step 1.** Input parameters, α, β, a, b, σ, ω_1, ω_2, χ_1, χ_2, θ, $\psi = 10$, and $M = 10\,000$.
- **Step 2.** Set, $k = 1$; Let, $\underline{\mu} = \zeta_2$, $\overline{\mu} = \zeta_2 + 6\,\sigma$, $R(k) = \dfrac{1}{\psi^{-k}}$, ${}^{(k)}\mu \sim \mathrm{U}[\underline{\mu}, \overline{\mu}]$, $V^* = -M$.
- **Step 3.** Let, $\mu \leftarrow {}^{(k)}\mu$.
  - *Step 3a.* Compute, κ_i, $\forall\, i = \{1,2,3\}$
  - *Step 3b.* Solve equations (3.27)–(3.28) numerically to determine prices, p_1, and p_2 simultaneously.
  - *Step 3c.* Determine q_1, q_2, and w using Proposition 1.
  - *Step 3d.* Compute, Value function, V using equation (3.33).

$$V = V({}^{(k)}\mu) = \boldsymbol{\pi} - R(k)\Bigg\{[p_2 - p_1]^+ + [q_1 - (1-\varrho(\Delta_1))\,C]^+ + [q_2 - (\varrho(\Delta_1) - \varrho(\Delta_2))\,C]^+ + \left[\sum_{i=1}^{2} q_i - (1-\varrho(\Delta_2))\,C\right]^+ + [\zeta_2 - \mu]^+\Bigg\}. \tag{3.33}$$

- **Step 4.** ${}^{(k)}\mu_{l_1} = {}^{(k)}\underline{\mu} + (1-0.618)\,({}^{(k)}\overline{\mu} - {}^{(k)}\underline{\mu})$, ${}^{(k)}\mu_{l_2} = {}^{(k)}\overline{\mu} + 0.618\,({}^{(k)}\overline{\mu} - {}^{(k)}\underline{\mu})$.
- **Step 5.** Repeat Step 3 (including all sub-steps) for each, ${}^{(k)}\mu_{l_1}$, ${}^{(k)}\mu_{l_2}$; Compute the corresponding the value functions, $V({}^{(k)}\mu_{l_1})$, $V({}^{(k)}\mu_{l_2})$ respectively.
- **Step 6.**
  - *Step 6a.* If $V({}^{(k)}\mu_{l_1}) > V({}^{(k)}\mu_{l_2})$, then ${}^{(k+1)}\underline{\mu} = {}^{(k)}\underline{\mu}$, ${}^{(k+1)}\overline{\mu} = {}^{(k)}\mu_{l_2}$.
  - *Step 6b.* If $V({}^{(k)}\mu_{l_2}) \leq V({}^{(k)}\mu)$, then ${}^{(k+1)}\underline{\mu} = {}^{(k)}\mu_{l_1}$, ${}^{(k+1)}\overline{\mu} = {}^{(k)}\overline{\mu}$.
- **Step 7.** If $|\,{}^{(k)}\overline{\mu} - {}^{(k)}\underline{\mu}| \geq 10^{-3}$ OR $|V^* - V({}^{(k)}\mu)| \geq 10^{-3}$, then, $k = k+1$, and go to Step 3.
- **Step 8.** $\mu^* \in [\,{}^{(k)}\underline{\mu},\ {}^{(k)}\overline{\mu}]$, $q_i^* = {}^{(k)}q_i$, $\forall\, i = \{1,2\}$, $w^* = {}^{(k)}w$, $p_i^* = {}^{(k)}p_i$, $\forall\, i = \{1,2\}$. Also, $\boldsymbol{\pi}^* = E(\boldsymbol{\pi}^*)$. Terminate.

4. The case of partial demand information

In this section, we extend the problem P, formulated in equations (3.16)–(3.21) to a situation in which the manufacturing firm may only have partial information about the price-dependent stochastic demand for each market segment, D_i, $\forall\, i = \{1,2\}$. Here it is assumed that the distribution for price-dependent stochastic demand,

D_i, $\forall\, i = \{1, 2\}$ is unknown. However, the simple price-dependent stochastic demand related parameters such as mean, μ_{x_i} and standard deviation σ_{x_i} are known. To address this situation, the use of the distribution-free (DF) approach is suggested. DF is primarily based on [40]'s rule which more recently used is a number of studies (see, [17,31,32]). We recall equation (3.14) outlines π, the total revenue (a random variable) for a firm as established earlier in equation (3.4).

$$\pi = \begin{cases} p_1 \min\{q_1, D_1\} - \chi_1 [q_1 - D_1]^+ - \omega_1 [D_1 - q_1]^+ - \kappa(\xi)\, q_1 - b\, q_1 (x - \zeta_1) & \xi \geq \zeta_1; \\ p_2 \min\{q_2, D_2\} - \chi_2 [q_2 - D_2]^+ - \omega_2 [D_2 - q_2]^+ - \kappa(\xi)\, q_2, & \zeta_2 \leq \xi < \zeta_1; \\ E(\pi) - r\,\tau - \tau\ \kappa(\xi), & \xi < \zeta_2. \end{cases} \tag{4.1}$$

It is worth noticing here that in equation (4.1), the distribution knowledge about the price-dependent stochastic demand for each market segment is needed. As such due to missing demand information, exact estimation of $E_{x_i}[q_i - D_i]^+$, $\forall\, i = \{1, 2\}$ is not possible. Nevertheless, we use distribution-free approach rooted in [40]'s rule to develop a lower bound on, $E_{x_i}[q_i - D_i]^+, \forall\, i = \{1, 2\}$. Using results in [31], we obtained:

$$E_{x_i}[q_i - D_i]^+ \leq \frac{(\tilde{q}_i - \tilde{d}_i) + \sqrt{(\tilde{q}_i - \tilde{d}_i)^2 + \sigma_{x_i}^2}}{2}, \quad \forall\, i = 1, 2. \tag{4.2}$$

By substituting $E_{x_i}[q_i - D_i]^+$ obtained in equation (4.2) into equation (4.1) and after making use of simplifications carried out in Section 3, we have obtained the following mathematical expression for the expected profit, $E(\tilde{\pi})$ in equation (4.3). We remark here that $E(\tilde{\pi})$ formulated in equation (3.15), is lower bound estimate on $E(\pi)$ which is reported in equation (4.3) as follows:

$$\begin{aligned} E(\tilde{\pi}) = \frac{1}{1 - \varrho(\tilde{\Delta}_2)} \Bigg\{ & \left((\tilde{p}_1 + \omega_1)\, \tilde{q}_1 - \omega_1\, \tilde{d}_1 - (\tilde{p}_1 + \omega_1 + \chi_1)\, \frac{(\tilde{q}_1 - \tilde{d}_1) + \sqrt{(\tilde{q}_1 - \tilde{d}_1)^2 + \sigma_{x_1}^2}}{2} \right) \\ & \times \left(1 - \varrho(\tilde{\Delta}_1)\right) - \tilde{\kappa}_1\, \tilde{q}_1 - b\, \tilde{g}\, \tilde{q}_1 \\ & + \left((\tilde{p}_2 + \omega_2)\, \tilde{q}_2 - \omega_2\, \tilde{d}_2 - (\tilde{p}_2 + \omega_2 + \chi_2)\, \frac{(\tilde{q}_2 - \tilde{d}_2) + \sqrt{(\tilde{q}_2 - \tilde{d}_2)^2 + \sigma_{x_2}^2}}{2} \right) \left(\varrho(\tilde{\Delta}_1) - \varrho(\tilde{\Delta}_2) \right) \\ & - \tilde{\kappa}_2\, \tilde{q}_2 - (r\, \varrho(\tilde{\Delta}_2) + \tilde{\kappa}_3)\, \varrho(\tilde{\Delta}_2)\, C \Bigg\}. \end{aligned} \tag{4.3}$$

In equation (4.3), $\tilde{\pi} = E(\tilde{\pi})$ is maximized *via* a control decision on $\tilde{\mu}$, $\tilde{p}_1$, $\tilde{p}_2$, $\tilde{q}_1$, and $\tilde{q}_2$. The number of reworked items are, $\tilde{w} = C - (\tilde{q}_1 + \tilde{q}_2)$. Thus, the revised optimization problem $\tilde{\mathrm{P}}$, would be:

$$\tilde{\mathrm{P}}: \ \max E(\tilde{\pi}) \tag{4.4}$$

subject to:

$$\tilde{p}_1 - \tilde{p}_2 \geq 0 \tag{4.5}$$

$$(1 - \varrho(\tilde{\Delta}_1))\, C - \tilde{q}_1 \geq 0 \tag{4.6}$$

$$(\varrho(\tilde{\Delta}_1) - \varrho(\tilde{\Delta}_2))\, C - \tilde{q}_2 \geq 0 \tag{4.7}$$

$$(1 - \varrho(\tilde{\Delta}_2))\, C - \sum_{i=1}^{2} \tilde{q}_i \geq 0 \tag{4.8}$$

$$\tilde{\mu} - \zeta_2 \geq 0. \tag{4.9}$$

We recall here that, the formulation of problem $\tilde{\mathrm{P}}$, reported in equations (4.4)–(4.9) is almost identical with a previously formulated problem P, in equations (3.16)–(3.21). Mainly these differ in control decisions, and to point it out, we have used an *accent*, "$\tilde{}$" to distinguish the decisions $(\mu, p_1, p_2, q_1, q_2)$ for problem P, from the corresponding decisions when the distribution is unknown, for the case of distribution-free problem $\tilde{\mathrm{P}}$, with decisions $(\tilde{\mu}, \tilde{p}_1, \tilde{p}_2, \tilde{q}_1, \tilde{q}_2)$.

Unlike the earlier problem P, the problem here, $\tilde{\mathrm{P}}$ brings considerable challenges in terms of determining directional derivatives. To overcome this shortcoming, we have suggested the use of a direction search method rooted in *Harmony search* (HS) meta-heuristic. We have constructed and implemented the Harmony Search meta-heuristic developed earlier for a related problem in [35]. To maintain the brevity of this paper, the implementation procedure of HS for the present problem, $\tilde{\mathrm{P}}$ is not presented here. The implementation of HS uses the same parametric settings suggested in Raza and Turiac [35] who have implemented HS onto a related problem.

5. Fencing investment modeling

As identified earlier that in most studies market segmentation has been considered perfect and more importantly the segmentation is achieved at no additional investment. However, in practice most market segmentation strategies require an additional effort (which is often an investment) by a firm to ensure a fencing. Zhang *et al.* [49] has recently considered fencing cost (investment) incurred to a firm and therefore market segmentation needs an additional cost for maintaining the fences. An increased investment on fencing is expected to improve a firm's ability to mitigate the demand leakages between market segments. Zhang *et al.* [49] suggested two simple modeling approaches: (i) linear fencing cost (ii) non-linear fencing cost. Both of these are discussed in the following.

5.1. Linear fencing cost

In linear fencing cost, the total cost of fencing is linear function of demand leakage rate per unit price differential, γ_0 such that

$$G(\gamma) = G_0 - g_0\,\gamma, \quad \forall\, 0 \le \gamma \le \overline{\gamma} \tag{5.1}$$

where in equation (5.1), G_0 is the fixed cost of having a perfect fence (no demand leakage), g_0 is the incremental cost per unit γ, and $\overline{\gamma}$ is the maximum allowable demand leakage rate per unit price difference. As a special case, in equation (5.1), $\overline{\gamma} = \dfrac{G_0}{g_0}$. Thus, γ is a control variable for a fencing cost (investment), $G(\gamma)$.

5.2. Nonlinear fencing cost

In case nonlinear fencing cost model, the total cost of fencing is formulated as a function of γ and represented as:

$$G(\gamma) = \frac{G_0}{g_0 + \gamma}\cdot \tag{5.2}$$

In equation (5.2), when $\gamma = 0$ implied that the firm able to perfectly fence the market segments. In this situation, total fencing cost would be, $G(\gamma = 0) = \dfrac{G_0}{g_0}$. An interesting situation can also be realized, when the perfect fencing cannot be achieved when $g = 0$.

In this situation,

$$G(\gamma) = \frac{G_0}{\gamma}\cdot \tag{5.3}$$

Notice in equation (5.3), as $\gamma \to 0$, then, $G(\gamma) \to \infty$. This guarantees that perfect market segmentation is not possible.

6. Revenue maximization with fencing investment

The expected revenue (profit) models developed earlier in Sections 3 and 4 are tailored by incorporating the fencing cost, $G(\gamma)$.

When the distribution of the price-dependent stochastic demand is known subtract the fencing cost investment, $G(\gamma)$ into equation (3.15) and obtained a revised expected revenue (profit), $E(\Pi)$ presented in equation (6.1).

$$E(\Pi) = E(\pi) - G(\gamma). \tag{6.1}$$

Recall, unlike equation (3.15), in equation (6.1), the decision variables also include γ in addition to decisions on pricing, p_1, p_2, production quantities, q_1, q_2, and the process selection mean, μ. For convenience, we can drop function arguments and expectation operator. Thus, $\Pi = E(\Pi)$, $\pi = E(\pi)$, and $G = G(\gamma)$ in equation (6.1).

Likewise, for the case when the distribution of the price-dependent stochastic demand in each market segment in unknown, we obtain:

$$E(\tilde{\Pi}) = E(\tilde{\pi}) - G(\tilde{\gamma}). \tag{6.2}$$

Where in equation (6.2), $E(\tilde{\pi})$ is obtained from equation (4.3). Again in equation (6.2) for brevity, $\tilde{\Pi} = E(\tilde{\Pi})$, $\tilde{\pi} = E(\tilde{\pi})$, and $\tilde{G} = G(\tilde{\gamma})$. Decisions include: $\tilde{\mu}$, $\tilde{p}_1$, $\tilde{p}_2$, $\tilde{q}_1$, $\tilde{q}_2$, and $\tilde{\gamma}$.

Addition of fencing investment, $G(\gamma)$ for perfect demand information, and $G(\tilde{\gamma})$ for partial demand information, results augmented complexity into the respective problems. The selection of the fencing investment cost function in Section 5 often yields optimal fencing decisions, either to perfectly fence with a maximum investment for fencing, otherwise do not fence (see, [49]). The proposed heuristic solution approach along with the Harmony search meta-heuristic is implemented using MATLAB [25].

7. Numerical study

In this section, a numerical study is reported with the models developed. All numerical studies are carried out on an Intel Core i7-3520M processor 2.67 GHz and 4 GB RAM, with Windows 7, 32-bit operating system. The numerical study assumes several problems related input parameters that were also being considered in relevant literature in the field and these problem related parameters are shown in Table 2. The price-dependent stochastic demand in each market segment, $x_i, \forall\, i = \{1, 2\}$, is assumed to have followed the uniform distribution, such that, $x_i \sim \mathrm{U}\left[\mu_{x_i} - \sqrt{3}\,\sigma_{x_i}, \mu_{x_i} + \sqrt{3}\,\sigma_{x_i}\right]$, $\forall\, i = \{1, 2\}$, also $\mu_{x_i} = 0, \forall\, i = \{1, 2\}$ (see [28,31]).

Table 2. Parameter selection for numerical experiment.

Parameter	Value(s)
C	1000
ζ_1	9
ζ_2	11
a	4
b	0.25
σ	3
r	$0.25 \times \kappa_1$
α_1	500
β_1	0.3
α_2	1000
β_2	1
G_0	1000
g_0	{2,3,4,6,7,10}
σ_{x_i}	{5,10,15}

TABLE 3. Linear fencing with distribution known.

σ_x	g_0	μ^*	p_1^*	p_2^*	q_1^*	q_2^*	G^*	$E(\Pi^*)$
5		8.253	837.695	620.200	257.164	381.738		218 679.769
10	2	8.273	837.697	621.742	265.637	381.919		217 905.518
15		8.292	837.699	623.106	274.108	382.081		217 127.698
5		8.253	837.695	620.200	257.164	381.738		218 679.770
10	3	8.273	837.697	621.742	265.637	381.919		217 905.518
15		8.292	837.699	623.106	274.108	382.081		217 127.698
5		8.253	837.695	620.200	257.164	381.738		218 679.770
10	4	8.273	837.697	621.742	265.637	381.919	1000	217 905.518
15		8.292	837.699	623.106	274.108	382.081		217 127.698
5		8.253	837.695	620.200	257.164	381.738		218 679.770
10	6	8.273	837.697	621.742	265.637	381.919		217 905.518
15		8.292	837.699	623.106	274.108	382.081		217 127.698
5		8.253	837.695	620.200	257.164	381.738		218 679.770
10	10	8.273	837.697	621.742	265.637	381.919		217 905.518
15		8.292	837.699	623.106	274.108	382.081		217 127.698

TABLE 4. Distribution free case with linear fencing.

σ_x	g_0	$\tilde{\mu}^*$	$\tilde{p}_1^*$	$\tilde{p}_2^*$	$\tilde{\gamma}^*$	$\tilde{q}_1^*$	$\tilde{p}_1^*$	$\tilde{G}^*$	$E(\tilde{\Pi}^*)$	EVAI	% deviation
5		8.272	839.210	646.583	0	282.201	381.906	1000	213 292.1	5387.716	0.46
10	2	8.303	839.136	620.041	0	285.858	382.170	1000	217 318.2	587.348	0.27
15		8.123	839.631	668.676	0	265.730	380.153	1000	211 272.2	5855.526	0.27
5		8.251	837.261	619.447	0	272.476	381.710	1000	218 392.2	287.597	0.13
10	3	8.303	839.136	620.041	0	285.858	382.170	1000	217 318.2	587.348	0.27
15		8.370	841.594	620.572	0	293.447	382.594	1000	216 191.9	935.802	0.43
5		8.251	837.261	619.447	0	272.476	381.710	1000	218 392.2	287.597	0.13
10	4	8.303	839.136	620.041	0	285.858	382.170	1000	217 318.2	587.348	0.27
15		8.374	838.533	621.650	0	302.406	373.270	1000	202 045.8	5081.95	0.29
5		8.251	837.261	619.447	0	272.476	381.710	1000	218 392.2	287.597	0.13
10	6	8.303	839.136	620.041	0	285.858	382.170	1000	217 318.2	587.348	0.27
15		8.370	841.594	620.572	0	293.447	382.594	1000	216 191.9	935.802	0.43
5		8.251	837.261	619.447	0	272.476	381.710	1000	218 392.2	287.597	0.13
10	10	8.303	839.136	620.041	0	285.858	382.170	1000	217 318.2	587.348	0.27
15		8.370	841.594	620.572	0	293.447	382.594	1000	216 191.9	935.802	0.43

7.1. Negligible fencing cost

In this case, we assume that the market segment is achieved at a negligible cost, but the segmentation could be perfect and imperfect. An imperfect segmentation is attributed when $\gamma > 0$ and cannot be controlled at an additional investment. Simulation results are reported in Table 3, from this, we can observe that as process variability, σ increases, the optimal control decision causes the process mean, μ^* to increase. This behavior is largely observed as most manufacturers tend to giveaway additional material fill to avoid scrap and rework costs [10, 41]. The difference between the optimal prices, $p_1^* - p_2^*$ is also observed diminishing as the process variability, σ increases. Thus, we can conclude that the price difference, $p_1^* - p_2^*$, decreases as the process variability, σ, increases. The expected optimal revenue shows a diminishing trend with an increase in process variability, σ.

Table 5. Known distribution with non-linear fencing.

σ_x	g_0	μ^*	p_1^*	p_2^*	q_1^*	q_2^*	G^*	$E(\Pi^*)$
5		8.233	837.692	620.395	257.166	381.534	500	219 436.5
10	2	8.253	837.694	621.912	265.638	381.733	500	218 659.5
15		8.272	837.696	623.254	274.110	381.912	500	217 879.1
5		7.771	697.482	697.338	228.710	372.666	166.667	214 937.6
10	6	8.239	837.691	622.034	265.639	381.599	166.667	219 163.8
15		8.259	837.693	623.362	274.111	381.791	166.667	218 381.5
5		8.218	837.689	620.544	257.167	381.377	142.857	219 978.8
10	7	8.238	837.691	622.043	265.639	381.590	142.857	219 199.8
15		8.258	837.693	623.369	274.111	381.782	142.857	218 417.4

Table 6. Distribution free case with non-linear fencing.

σ_x	g_0	$\tilde{\mu}^*$	$\tilde{p}_1^*$	$\tilde{p}_2^*$	$\tilde{\gamma}^*$	$\tilde{q}_1^*$	$\tilde{q}_2^*$	$\tilde{G}^*$	$E(\tilde{\Pi}^*)$	EVAI	% deviation
5	2	8.230	837.257	619.647	0	272.491	381.504	500	219 149.3	287.196	0.13
10	2	8.290	839.608	620.139	0	284.373	382.066	500	218 067.5	591.991	0.27
15	2	8.358	842.096	620.632	0	292.044	382.529	500	216 932.3	946.753	0.43
5	6	8.538	837.675	618.675	0	266.609	381.437	167.766	204 185	594.32	0.28
10	6	8.282	839.942	620.207	0	283.401	381.994	166.667	218 568.1	595.655	0.27
15	6	8.350	842.446	620.674	0	291.121	382.483	166.667	217 426.8	954.684	0.44
5	7	8.216	837.255	619.802	0	272.502	381.345	142.857	219 691.9	286.903	0.13
10	7	8.281	839.966	620.212	0	283.332	381.989	142.857	218 603.9	595.935	0.27
15	7	8.349	842.471	620.677	0	291.055	382.480	142.857	217 462.1	955.270	0.44

When the price-dependent stochastic demand variability $\sigma_x = \sigma_{x_i}, \forall\, i = \{1,2\}$ increases, for a given process variability, σ, the expected revenue, $E(\Pi^*)$, the process mean, μ^* and the price difference, $p_1^* - p_2^*$, are observed diminishing. In a contrast, when demand leakage rate per unit price difference, γ, increases, the process mean, μ^*, price values, p_1^*,p_2^*, rises whereas the price difference, $p_1^* - p_2^*$, and expected revenue, $E(\Pi^*)$, are showing a declining tendency. For instance, with no demand leakage and demand leakage rate, *i.e.*, $\gamma = 0$, price-dependent stochastic demand variability, $\sigma_{x_i} = 3, \forall\, i = 1,2$, when process variability, σ increases its value from 2 to 5, the optimal expected revenue, $E(\Pi^*)$ is reduced from \$2402.3134 to \$1146.8869. Also, when process variability, σ, increases from 2 to 5 with the negligible fencing cost, at no demand leakage, *i.e.*, $\gamma = 0$, the optimal process mean, μ^* increases. The observation from the Table 3 is that when $\sigma = 2$, and price-dependent stochastic demand variability, $\sigma_x = \sigma_{x_i} = 3, \forall\, i = \{1,2\}$, the demand leakage rate per unit price difference, γ, increases from 0 to 0.001, whereas at the same occasion, the optimal revenue, $E(\Pi^*)$ decreases from \$2402.31 to \$2396.42 respectively which is a very marginal change.

7.2. Linear fencing

In this experimentation, $G_0 = 1000$, whereas, the simulation study tests to compute optimal fencing (cost) investment, γ^* for a set of input parameters, $G_0 = 1000$, $g_0 = \{2,3,4,6,10\}$ in addition to the parameters used in Section 7.1. Notice here that at an fencing investment cost, $G_0 = 1000$, the market segments are perfectly fenced, mainly due to linear modeling framework employed. Thus, it is observed is that in all the experiments demand leakage rate per unit price difference (γ) leans towards zero with a maximum investment for fencing, *i.e.*, $G^* = G(\gamma^*) = 1000$, with $\gamma^* = 0$. Table 3 reports detailed results of numerical experimentation. This table mainly studies the impact of process variability, σ on optimal process mean, μ^*, the fencing investment decision,

γ^*, along with pricing, p_1^*, p_2^* and production quantities, q_1^*, q_2^*, decisions. As the process variability increases, the process mean increases. For instance, when $\sigma_x = \sigma_{x_i} = 3, \forall\, i = \{1, 2\}$, as the process variability increases from 2 to 5, the price-dependent stochastic demand variability, the optimal process mean, μ^* increases. The optimal expected revenue, $E(\Pi^*)$, decreases by 65.99% when the process variability (σ) increases from 2 to 5. For a given process variability, the increment in price-dependent stochastic demand standard error, σ_{x_i}, $\forall\, i = \{1, 2\}$, results in diminishing optimal expected revenue, $E(\Pi^*)$, and the corresponding price difference, $p_1^* - p_2^*$, and the optimal process mean, μ^*, diminishes. The firm opts to invest maximum, $G_0 = 1000$, to achieve perfect market segmentation, $\gamma^* = 0$ that leads to zero demand leakages between two market segmentation at the maximum investment, $G_0 = 1000$.

7.3. Non-linear fencing

Similar to a numerical study reported with linear fencing cost in Section 7.2, here we carried out numerical experimentation with non-linear fencing cost model developed earlier in Section 5. Values experimented are $g_0 = \{2, 6, 7\}$ for nonlinear cost parameter. Table 5 reports the optimal fencing decision when the distribution of the price-dependent stochastic demand in each market segment is known and have followed uniform distribution. Whereas, the corresponding analysis is reported in Table 6, when the distribution of the price-dependent stochastic demand is unknown and therefore the distribution-free approach based on Scarf [40]'s rule is implemented.

It can be clearly seen that γ^* also approaches to zero resulting in a perfect market segmentation *via* maximum investment for fencing. This behavior is found for both cases, when demand distribution is known, as well as, when the distribution is unknown. The optimal process mean, μ^*, increases as the variability of price-dependent stochastic demand, $\sigma_x = \sigma_{x_i}, \forall\, i = \{1, 2\}$ increases. Indeed, an increase in the price-dependent stochastic demand variability results diminishing optimal expected revenues in the case when distribution is known, and when it is unknown alike that is, $E(\Pi^*)$, and $E(\tilde{\Pi}^*)$ respectively. Here a new notion, Expected Value of Additional Information (EVAI) is also computed and reported in Table 6. Gallego and Moon [17] utilized this as performance measure to assess the competitiveness of the distribution-free approach on a newsvendor problem with an exogenous price. Then, this performance measure is widely tested on a number of studies [1, 28, 31, 32]. Mathematically, $\text{EVAI} = \Pi(\mu^*, p_1^*, p_2^*, q_1^*, q_2^*, \gamma^*) - \Pi(\tilde{\mu}^*, \tilde{p}_1^*, \tilde{p}_2^*, \tilde{q}_1^*, \tilde{q}_2^*, \tilde{\gamma}^*)$. This translates to the expected revenue gains the firm is likely to receive, if it were to know the demand distribution perfectly, and therefore would have made a decision $\mu^*, p_1^*, p_2^*, q_1^*, q_2^*, \gamma^*$ instead, $\tilde{\mu}^*, \tilde{p}_1^*, \tilde{p}_2^*, \tilde{q}_1^*, \tilde{q}_2^*, \tilde{\gamma}^*$. In this numerical study, an increase in the demand variability, σ_{x_i}, $\forall\, i = \{1, 2\}$, results in a higher EVAI. This implies when demand variability is higher, the knowledge of demand distribution could protect a firm better from revenue losses.

8. Conclusion, limitations, and future work suggestions:

This paper addresses the problem of interfacing a manufacturing process firm's pricing and production (inventory) decisions with its selection of a process mean. It is assumed that the existing market segments are imperfect and therefore demand leakages are experienced from one market segment to another. The fencing scheme is proposed at an investment (cost) incurred to firm for mitigating the demand leakages. Furthermore, it is believed that the demand experienced by the firm in each market segment is price-dependent stochastic with a demand leakage that can be controlled by an investment on fencing. Later, the case of the price-dependent stochastic with unknown distribution is also explored. Mathematical models are developed for a firm's optimal decisions on selection of a process mean, pricing and production decisions on each market segment, along with the fencing investment cost to mitigate demand leakages. The mathematical model is re-visited for the case when distribution of the price-dependent stochastic demand is unknown and the distribution-free approach is utilized for formulation. A detailed numerical study is reported to investigate the impact of demand leakages and fencing strategies on the firm's profitability.

Although, the paper contributes to the existing literature by developing novel models, the work has some limitations. The work presented mainly assumes monopoly, therefore, the leakages, or the cannibalization effect the firm may experience in a competition are not considered. The model developed assumed perfect manufacturing process, in which, there are no failures. Also the process does not deteriorate, and therefore the products of perfect quality are yielded. The quality variation is only due to process variability which is assumed to be normally behaved. In reality, the process distribution may not be normal, and also due to machine tool wear, the process is expected to deteriorate. Additionally, due to machine malfunctioning the products are required to be assessed and regardless of the assessment plan (*e.g.*, 100% inspection, or sampling plans) there are errors incurred in classification and it also costs to manufacturer in assessing products. Another limitation of this study was it considered only a single process and quality attribute for the products. In practice manufacturing a product may require a number of operations or processes, therefore, one may experience the problem of selecting a number of process means (or tolerances) as opposed to a single selection of a process mean. Future studies can be designed to address some of the aforementioned limitations and relax some of these listed assumptions to make mathematical models more realistic in addressing real-life process industry problems.

Appendix A.

A.1. Proof of Proposition 1

Referring to Section 3, since pricing decision are exogenous (fixed), and there are no capacity related constraints assumed. Thus, we only optimize the revenue function in equation (A.1) in the following.

$$E(\pi) = \frac{1}{1-\varrho(\Delta_2)} \Bigg\{ \left((p_1+\omega_1)\, q_1 - \omega_1\, d_1 - (p_1+\omega_1+\chi_1) \int_{\underline{x}_1}^{q_1-d_1} \Phi_1(x_1)\partial\, x_1 \right) (1-\varrho(\Delta_1)) - \kappa_1\, q_1 - b\, g\, q_1$$
$$+ \left((p_2+\omega_2)\, q_2 - \omega_2\, d_2 - (p_2+\omega_2+\chi_2) \int_{\underline{x}_2}^{q_2-d_2} \Phi_2(x_2)\partial\, x_2 \right)$$
$$\times\, (\varrho(\Delta_1)-\varrho(\Delta_2)) - \kappa_2\, q_2 - (r\,\varrho(\Delta_2)+\kappa_3)\,\varrho(\Delta_2)\, C \Bigg\}. \tag{A.1}$$

Consequently, in order to determine the optimal production quantities, q_i^*, $\forall i = \{1, 2\}$, the first order optimality condition is utilized. Thus

$$\frac{\partial\pi}{\partial q_1} = (p_1+\omega_1-(p_1+\omega_1+\chi_1)\Phi_1(q_1-d_1))\,(1-\varrho(\Delta_1)) - \kappa_1 - b\,g = 0 \tag{A.2}$$
$$\frac{\partial\pi}{\partial q_2} = (p_2+\omega_2-(p_2+\omega_2+\chi_2)\Phi_2(q_2-d_2))\,(\varrho(\Delta_1)-\varrho(\Delta_2)) - \kappa_2 = 0. \tag{A.3}$$

Finding the optimal order quantity, q_i, $\forall i = \{1, 2\}$.

$$q_1 = d_1 + \Phi_1^{-1}\left(\frac{(p_1+\omega_1)(1-\varrho(\Delta_1)) - \kappa_1 - b\,g}{(1-\varrho(\Delta_1)\,(p_1+\omega_1+\chi_1)}\right) \tag{A.4}$$
$$q_2 = d_2 + \Phi_2^{-1}\left(\frac{(p_2+\omega_2)(\varrho(\Delta_1)-\varrho(\Delta_2)) - \kappa_2}{(\varrho(\Delta_1)-\varrho(\Delta_2))\,(p_2+\omega_2+\chi_2)}\right). \tag{A.5}$$

Using some simplifications, we get the terms similar to critical fractiles (see, [7, 29, 34]), $\rho_1 = \frac{(p_1+\omega_1)(1-\varrho(\Delta_1))-\kappa_1-b\,g}{(1-\varrho(\Delta_1)\,(p_1+\omega_1+\chi_1)}$, and $\rho_2 = \frac{(p_2+\omega_2)(\varrho(\Delta_1)-\varrho(\Delta_2))-\kappa_2}{(\varrho(\Delta_1)-\varrho(\Delta_2))\,(p_2+\omega_2+\chi_2)}$

$$q_1 = d_1 + \Phi_1^{-1}(\rho_1) \tag{A.6}$$
$$q_2 = d_2 + \Phi_2^{-1}(\rho_2). \tag{A.7}$$

where in equations (A.6) and (A.7), we have $d_1 = \alpha - \beta_1\, p_1 - \gamma(p_1 - p_2)$, $d_2 = \alpha_2 - \beta_2\, p_2 + \gamma(p_1 - p_2)$.

Conflict of interest statement

No conflicts of interest to be declared.

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