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# INTERACTIVE MULTIOBJECTIVE DEA TARGET SETTING USING LEXICOGRAPHIC DDF

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**Abstract.** In this paper, a new interactive multiobjective target setting approach based on lexicographic directional distance function (DDF) method is proposed. Lexicographic DDF computes efficient targets along a specified directional vector. The interactive multiobjective optimization approach consists in several iteration cycles in each of which the Decision Making Unit (DMU) is presented a fixed number of efficient targets computed corresponding to different directional vectors. If the DMU finds one of them promising, the directional vectors tried in the next iteration are generated close to the promising one, thus focusing the exploration of the efficient frontier on the promising area. In any iteration the DMU may choose to finish the exploration of the current region and restart the process to probe a new region. The interactive process ends when the DMU finds its most preferred solution (MPS).

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## 1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric technique that can be used for assessing the relative efficiency of a set of Decision Making Units (DMU). The DMUs must be homogeneous, *i.e.* comparable, consume the same inputs and produce the same outputs. The specific amount of inputs consumed and outputs produced by each DMU are known and they are used by DEA to infer the Production Possibility Set (PPS) that contains all the feasible operating points. The non-dominated subset of the PPS is the efficient frontier. The DMUs that lie on the efficient frontier are termed efficient DMUs while the inefficient DMUs are projected onto the efficient frontier using a certain DEA model (radial, non-radial, hyperbolic, slacks-based, directional vector-based, etc.).

Therefore, the two main aims of existing DEA models are: (1) determining an efficiency score that measures how far an inefficient DMU is from the efficient frontier, and (2) computing an efficient target (a.k.a. benchmark) that indicates the improvements to be made by the DMU in the different input and output dimensions. Note that improvements mean reductions in the case of input variables and increases in the case of output variables.

*Keywords.* DEA, target setting, interactive multiobjective optimization, lexicographic directional distance function, most preferred solution.

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Note also that the two aims above are interrelated in the sense that the distance of a DMU to the efficient frontier depends on the target computed (see *e.g.* [27]).

There are some DEA approaches that emphasize the target setting aim more than the efficiency scoring one. That is the case, for example, of those DEA approaches that search for the least distant target (*e.g.* [1–3]) or compute a stepwise benchmarking path that divides the total improvement effort so that it does not have to be made all in one step (*e.g.* [24, 28, 29]).

It is clear also that DEA target setting can be approached as a multiobjective linear programming (MOLP) problem and there are, in fact, many multiobjective optimization DEA (MODEA) approaches. A standard MODEA approach, for example, is the reference point model [18] which for any given reference point (a.k.a. aspiration level vector) can find a point on the efficient frontier. Another standard approach is the multi-objective model for ratio optimization (MORO) and with dominance (MORO-D) proposed in Estellita Lins *et al.* [9]. The latter also proposes the MOLP model for target optimization (MOTO) which directly optimizes the target inputs and outputs instead of their corresponding multiplicative ratios. Some MODEA approaches use *a priori* articulation of preferences (*e.g.* [37, 45]) while others sample the whole Pareto frontier so that the DMU can select the most preferred target from the set of efficient targets computed (*e.g.* [14, 25]). Value efficiency [15, 20, 21] is different as its aim is to compute the overall efficiency of the DMUs once the DMU has located her MPS in the efficient frontier, which is the step we consider in this paper.

In any case, the MODEA approaches that are more relevant for this paper are those that use interactive multiobjective optimization (IMO). Some of them use well-known IMO approaches such as the step method (STEM) [4], the Geoffrion–Dyer–Feinberg method (G–D–F) [10], the Zionts–Wallenius method (Z–W) [46], the Wierzbicki method (WM) [39], the interactive weighted Tchebycheff method (IWT) [35], the satisfying trade-off method (STOM) [33], the Pareto race (PR) [19] or the gradient projection method (GPM) [41]. A summarized description of these methods is included as supplementary material.

As regards IMO DEA approaches, they are summarized and compared in Table 1. Korhonen *et al.* [22] and Estellita Lins *et al.* [9], for example, propose using PR. In a very interesting paper, Wong *et al.* [40] compares five of the above methods (STEM, G–D–F, WM, IWT and STOM) on the efficiency assessment of UK retail banks. Yang *et al.* [43, 44] and Malekmohammadi *et al.* [31] use GPM on the same dataset. Hosseinzadeh Lotfi *et al.* [16,17] apply Z–W to the output-oriented and the non-oriented (a.k.a. combined-oriented) MODEA model, respectively. Ebrahimnejad and Tavana [8] apply STOM to the NATO enlargement problem using an output-oriented MODEA model and considering desirable and undesirable outputs. Tavana *et al.* [36] use STOM to derive trade-offs from the DMU and use the minmax reference point model (which they show its equivalent to a combined-oriented DEA model). Note that Tavana *et al.* [36] use a slacks-maximization phase II to compute efficient targets (since conventional DDF targets are just weak efficient).

Apart from using well-known IMO approaches in DEA other authors have proposed specific IMO methods. Thus, in a seminal paper Golany [12] proposed an interactive MOLP procedure for output-oriented MODEA target setting. Post and Spronk [34] propose an Interactive DEA (IDEA) procedure that involves computing, independently along each input and output dimension, the maximum performance improvement from a feasible given aspiration level vector followed by a readjustment of the aspiration level of a certain dimension according to the preferences of the DMU and respecting the computed improvement boundary. And Lozano and Villa [29] propose using an interactive multiobjective approach that uses AHP in an interactive way to derive weights for performing trade-offs between the different inputs and output dimensions.

But perhaps the interactive MODEA approach more relevant for this research is the one proposed in Bogetoft and Nielsen [5], which is based on the directional distance function (DDF) DEA model [6]. They propose an internet-based software tool that allows the user to try different directional vectors and, from these trials, progressively articulate their preferences. The choices of the directional vectors can come from weights, aspiration points or constraints on performance levels. The approach is rather flexible but not very structured. In addition, the DDF model used only guarantees weak efficiency.

DDF is an often-used DEA model that is simple, flexible, easy to interpret and very effective in representing the improvement preferences for projecting a DMU. However, the approach proposed in this paper does not

Table 1. Comparison of interactive DEA approaches.

| Interactive DEA approach | IMO method | Main features |
|---|---|---|
| Golany [12] | DEA-specific | Input levels given; target output value computed; $(k+1)$ solutions is presented to the DMU in each iteration;weights for the different output dimensions adjusted in each iteration |
| Post *et al.* [34] | Interactive Data Envelopment Analysis | DEA and interactive multiple goal programming; production frontier units and PPS interior units |
| Korhonen *et al.* [22] | Modified Pareto race method | Compare radial projection target with MPS target; technology change simulated by random changes of the efficient frontier. |
| Estellita Lins *et al.* [9] | Pareto Race method, Multi-objective simplex method | Multi-objective model for ratio optimization (MORO); discretionary and non-discretionary variables; MORO with dominance (MORO-D); output reductions and input increments allowed; weights indifference region |
| Bogetoft and Nielsen [5] | DEA-specific | Internet based benchmarking software; DDF model in any given direction; methods for determining initial directions suggested; results shown graphically |
| Wong *et al.* [40] | G–D–F method, Wierzbicki method, STEM method, Tchebychev method, STOM method | Output-oriented CCR and BCC model; no prior judgement required; PROMOIN software; three fixed priority goal rules as the termination criterion of all the methods; methods compared based on five criteria (each with a given weight): the DM's confidence, ease of understanding, ease of elicitation, computational charge and No. of iterations |
| Yang *et al.* [43] | Reference point method, Gradient projection method | Output oriented CCR and BCC models transformed to minimax MOLP formulation; individual and group preferences |
| Lozano and Villa [29] | DEA-specific | AHP-based; maximize weighted sum of proportional improvements inputs and outputs for which improvement is desired; inputs and outputs for which no change is allowed; inputs and outputs for which worsening is allowed |
| Hosseinzadeh Lofti *et al.* [16] | Zionts–Wallenius method | Output-oriented CCR and BCC model transformed to min-ordering (max-min) MOLP formulation |
| Hosseinzadeh Lofti *et al.* [17] | Zionts–Wallenius method | Combined-oriented CCR and BCC model (DDF) transformed to min-ordering (minimax) MOLP formulation |
| Malekmohammadi *et al.* [31] | Reference point method, Gradient projection method | Aggregate model; maximize weighted sum of total inputs and outputs improvements all DMUs projected simultaneously; aggregated mode transformed to minimax MOLP formulation |

Table 1. continued.

| Interactive DEA approach | IMO method | Main features |
|---|---|---|
| Ebrahimnejad and Hossein-zadeh Lofti [7] | Zionts–Wallenius method | Combined-oriented CCR model (DDF) transformed to weighted minimax MOLP formulation |
| Ebrahimnejad and Tavana [8] | STOM method | Output-oriented BCC model transformed to minimax MOLP formulation; desirable and undesirable outputs |
| Yang and Xu [42] | Gradient projection method | Input-oriented DEA model transformed to minimax MOLP formulation |
| Tavana *et al.* [36] | STOM method | Combined-oriented approach transformed to the minimax MOLP formulation; DDF plus slacks maximization phase II; desirable and undesirable variables; uncontrollable variables. |
| Moradi Dalini and Noura [32] | Reference point method | Data envelopment scenario analysis (DESA) model; imprecise DEA model transformed to minimax MOLP formulation |
| Gerami [11] | STEM method | Non-radial and component-based efficiency and value efficiency scores and corresponding benchmarks; no computational complexity and inaccurate determination of the value function issues; infeasibility under certain conditions |
| Proposed approach | DEA-specific | Lexicographic DDF; three parameters (number of targets to present in each iteration, interval width reduction factor, the separation threshold reduction factor); directional efficiency score for each target |

use conventional DDF. Instead, it uses a Lexicographic DDF method, which is somewhat more complex but guarantees reaching the efficient frontier (see [26]). A simpler alternative to the Lexicographic DDF method is to use DDF followed by a slacks-maximization phase (see [36]). However, the additive character of the objective function of such slacks-maximization model allows for alternative optima. In addition, the Lexicographic DDF approach uses the information provided by the directional vector all along the projection process.

The IMO approach proposed in this paper involves a structured process that resembles the IWT of Steuer and Choo [35], which is a simple and easy to understand iterative approach, with a nice graphical interpretation. Thus, in the proposed approach, the modification of the projection direction carried out in each iteration based on the feedback from the DMU is relatively simple and does not require that the DMU classifies the different input and output dimensions into three groups (those that should be improve, those that should be maintained and those that should be relaxed). Instead, in the proposed approach, same as in Steuer and Choo [35], the updated projection directions are generated randomly around the one selected by the DMU as most promising among those considered in the previous cycle. However, in the proposed approach, instead of the augmented weighted Tchebycheff model used in the IWT approach of Steuer and Choo [35] for computing an efficient projection from the ideal or a super-ideal point, the projection is computed from inside, *i.e.* from the own DMU, and using a lexicographic DDF approach. The lexicographic DDF approach was proposed in Lozano and Soltani [26] for DEA target setting when the DMU can articulate its preference *a priori*. It is an extension of the lexicographic radial approach of Korhonen *et al.* [23] to any given directional vector. Although the lexicographic DDF uses a directional vector, it has the advantage over conventional DDF that it guarantees an efficient target, not just weak efficient. In addition, lexicographic DDF benefits from the graphical interpretation of DDF.

The contributions of the paper are, therefore, twofold: it is the first IMO DEA approach that uses an iterative direction-sampling approach analogous to the method of Steuer and Choo [35] but projecting not from an outside operating point like the ideal point but from the own DMU. Also, it computes the targets using the lexicographic DDF method [26], which is more complex but also more effective than conventional DDF.

The structure of the paper is the following. In Section 2 the IWT method of Steuer and Choo [35] and the lexicographic DDF approach of Lozano and Soltani [26] are briefly reviewed. In Section 3 the proposed interactive lexicographic DDF (ILD) approach is described and illustrated. In Section 4 an application to a shipping lines case study is presented. Section 5 summarizes and concludes.

## 2. Preliminaries

Let us denote by $z \in \Omega \subset \mathbb{R}^n$ the vector of feasible decision variables and $(f_1(z), f_2(z), \ldots, f_Q(z))$ the different objective functions which let us assume are to maximized. Let us also denote $\left(f_1^*, f_2^*, \ldots, f_Q^*\right)$ the corresponding ideal point whose components are the optimal value of each objective function computed considering each of the objective functions individually, *i.e.* $f_q^* = \mathrm{Max}_{z\in\Omega}\ f_q(z)$.

Given any non-negative weight vector $(w_1, w_2, \ldots, w_Q)$ and a small constant $\varepsilon > 0$ the augmented weighted Tchebycheff method consists in solving

$$\underset{z\in\Omega}{\mathrm{Min}}\ \underset{q}{\mathrm{Max}}\ w_q \cdot \left(f_q^* - f_q(z)\right) + \varepsilon \cdot \sum_{q=1}^{Q} \left(f_q^* - f_q(z)\right) \tag{2.1}$$

which is equivalent to this formulation

$$\begin{aligned} &\mathrm{Min}\ \ \mu\ - \varepsilon \cdot \sum_{q=1}^{Q} f_q(z) \\ &\text{s.t.} \\ &w_q \cdot \left(f_q^* - f_q(z)\right) \le \mu \quad \forall q \\ &z \in \Omega. \end{aligned} \tag{2.2}$$

Graphically, the computed solution generally corresponds to the intersection of the ray that departs from the ideal point with direction $\left(-\frac{1}{w_1}, -\frac{1}{w_2}, \cdots, -\frac{1}{w_Q}\right)$ and the Pareto front. This model guarantees that a Pareto optimal solution is computed. Moreover, any Pareto optimal solution can be computed in this way by just chosen an appropriate weight vector. The idea behind the IWT is to try different weight vectors (and hence different projection directions) showing the corresponding Pareto optimal solutions to the DMU trying to find a MPS. This is done in a structured way whose details are described in the Supplementary material file and that basically involves shrinking the cone of weight vectors in order to focus attention on a specific region of the Pareto frontier which the DMU thinks is promising.

Let
$j = 1, 2, \ldots, n$ index on DMUs.
$i = 1, 2, \ldots, m$ index on inputs.
$k = 1, 2, \ldots, s$ index on outputs.
$x_{ij}$ amount of input $i$ consumed by DMU $j$.
$y_{kj}$ amount of output $k$ produced by DMU $j$.

In the MODEA case, the objective functions are $\left(-\sum_{j=1}^{n} \lambda_j x_{1j}, \ldots, -\sum_{j=1}^{n} \lambda_j x_{mj}, \sum_{j=1}^{n} \lambda_j y_{1j} \ldots, \sum_{j=1}^{n} \lambda_j y_{sj}\right)$. The decision variables are $(\lambda_1, \lambda_2, \ldots, \lambda_n)$ and the feasible region depends on the DEA technology considered, *e.g.* $\Omega^{\mathrm{VRS}} = \left\{\boldsymbol{\lambda} \in \mathbb{R}^n : \lambda \ge 0 \quad \sum_{j=1}^{n} \lambda_j = 1\right\}$. If we are computing the target for a certain DMU 0, it may be desirable that the target weakly dominates the observed DMU. In that case the feasible region would be

$$\Omega_0^{\text{VRS}} = \left\{ \boldsymbol{\lambda} \in \mathbb{R}^n : \lambda \geq 0 \quad \sum_{j=1}^{n} \lambda_j = 1 \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{i0} \ \forall i \quad \sum_{j=1}^{n} \lambda_j y_{kj} \geq y_{k0} \ \forall k \right\}. \tag{2.3}$$

A key feature of IWT is that the projection is done from the ideal-point (or any point that dominates it) and, therefore, from outside the PPS. The approach proposed in Bogetoft and Nielsen [5] is the opposite and follows the traditional DEA perspective of projecting from the observed DMU 0 using a directional vector $g = (g^x, g^y) \in \mathbb{R}^{m+s}$. In order to compute a target that weakly dominates DMU 0 it is required that $g \geq 0$. The conventional DDF model computes the maximum feasible step size $\beta$ along the direction $(-g^x, g^y)$ solving

$$\begin{array}{ll}
\text{Max} \quad \beta & \\
\text{s.t.} & \\
\sum_j \lambda_j x_{ij} \leq x_{i0} - \beta \cdot g_i^x & \forall i \\
\sum_j \lambda_j y_{kj} \geq y_{k0} + \beta \cdot g_k^y & \forall k \\
\sum_j \lambda_j = 1 & \\
\lambda_j \geq 0 & \forall j.
\end{array} \tag{2.4}$$

A problem with this conventional DDF model is that it computes a weakly efficient target. To remedy that, Lozano and Soltani [26] have proposed a lexicographic DDF approach which moves along the given direction $(-g^x, g^y)$ as much as possible and when a certain input or output variable cannot be improved (*i.e.* a weakly efficient target is found) the corresponding variable remains fixed at that value and the procedure continues by setting the corresponding component of $(-g^x, g^y)$ to zero. In that way, we can arrive at an efficient target guided by the given directional vector to the last. The lexicographic DDF is computationally more expensive than conventional DDF as every time a weak efficient target is found it needs to solve a simple mixed integer linear program (MILP) model to identify the input and output dimensions that can still keep on improving. When no input or output variable can be further improved it means that the efficient frontier has been reached and the lexicographic DDF method stops. Note that a similar lexicographic approach has been proposed by Korhonen *et al.* [23] for a specific direction, namely the combined-radial DEA model.

Finally, from the target $(x^*, y^*)$ computed by the lexicographic DDF method, a directional efficiency score of DMU 0 can be computed as

$$\eta_0 = \frac{1 - \frac{1}{m} \sum_i \frac{x_{i0} - x_i^*}{x_{i0}}}{1 + \frac{1}{s} \sum_k \frac{y_k^* - y_{k0}}{y_{k0}}}. \tag{2.5}$$

## 3. Proposed interactive lexicographic DDF approach

The proposed ILD approach involves a number of cycles in each of which $P$ efficient targets are presented to the DMU whose target is to be determined (labelled DMU 0 and henceforth called "the DMU"). The value of $P$ is one of the few parameters of the method and is limited not so much by the computational effort required but by the cognitive stress put on the DMU when evaluating the trade-offs involved in that number of solutions. Note that, in general, each solution is a multidimensional vector which makes using a graphical representation of the solutions (*e.g.* using parallel coordinates) highly recommendable.

Before the interactive process for projecting DMU 0 starts, it is convenient to normalize the data. Assuming that $x_0 > 0$ , $y_0 > 0$ the normalized data can be denoted as

$$\begin{array}{ll}
\hat{x}_{ij} = \dfrac{x_{ij}}{x_{i0}} & \forall i \forall j \\
\hat{y}_{kj} = \dfrac{y_{kj}}{y_{k0}} & \forall k \forall j.
\end{array} \tag{3.1}$$

Note that the above normalization is such that $(\hat{x}_0, \hat{y}_0) = (1, 1, \ldots, 1)$.

Next the maximum improvement along each input and output dimension should be computed. This requires solving the following $m+s$ models

$$\begin{aligned}
&\hat{x}_{i0}^{\min} = \text{Min} \quad \hat{x}_i\\
&\text{s.t.}\\
&\textstyle\sum_j \lambda_j \hat{x}_{i'j} \le \hat{x}_{i'0} \qquad \forall i' \ne i\\
&\textstyle\sum_j \lambda_j \hat{x}_{ij} \le \hat{x}_i\\
&\textstyle\sum_j \lambda_j \hat{y}_{kj} \ge \hat{y}_{k0} \qquad \forall k\\
&\textstyle\sum_j \lambda_j = 1\\
&\lambda_j \ge 0 \qquad \forall j
\end{aligned} \tag{3.2}$$

$$\begin{aligned}
&\hat{y}_{k0}^{\max} = \text{Max} \quad \hat{y}_k\\
&\text{s.t.}\\
&\textstyle\sum_j \lambda_j \hat{x}_{ij} \le \hat{x}_{i0} \qquad \forall i\\
&\textstyle\sum_j \lambda_j \hat{y}_{k'j} \ge \hat{y}_{k'0} \qquad \forall k' \ne k\\
&\textstyle\sum_j \lambda_j \hat{y}_{kj} \ge \hat{y}_k\\
&\textstyle\sum_j \lambda_j = 1\\
&\lambda_j \ge 0 \qquad \forall j.
\end{aligned} \tag{3.3}$$

Similar models are solved also in Post and Spronk [34] to bound the improvements along each dimension that are feasible from a given aspiration level vector. However, while in the interactive method of Post and Spronk [34] these models must be solved each time the aspiration level vector is readjusted, in the proposed approach it is done only once, since the lexicographic DDF projections are made always from the observed DMU 0.

From the solutions to (3.2) and (3.3) the maximum feasible improvement for each variable is determined as

$$\begin{aligned}
R_{i0}^x &= \hat{x}_{i0} - \hat{x}_{i0}^{\min} = 1 - \hat{x}_{i0}^{\min} \qquad \forall i\\
R_{k0}^y &= \hat{y}_{k0}^{\max} - \hat{y}_{k0} = \hat{y}_{k0}^{\max} - 1 \qquad \forall k.
\end{aligned} \tag{3.4}$$

Note that solving (3.2) and (3.3) also allows detecting whether the DMU is efficient, which occurs if and only if $\hat{x}_{i0}^{\min} = 1\ \forall i$ , $\hat{y}_{k0}^{\max} = 1\ \forall k$, or, equivalently, $R_{i0}^x = R_{k0}^y = 0\ \ \forall i \forall k$. It is assumed that if the DMU is efficient then its target is itself and there is no need to use the proposed interactive lexicographic DDF approach.

Assuming in what follows that the DMU is inefficient, then its corresponding $R_{0i}^x$ and $R_{0k}^y$ values are used to generate the directional vectors that the lexicographic DDF method will use in each cycle. Let us denote by $g^{t,p} = (g^{x,t,p}, g^{y,t,p}) \quad p = 1, 2, \ldots, P$ the $P$ directional vectors generated in cycle $t$. Those directional vectors are generated using

$$\begin{aligned}
g_i^{x,t,p} &= \hat{\alpha}_i^{x,t,p} \cdot R_{i0}^x \quad \forall i\\
g_k^{y,t,p} &= \hat{\alpha}_k^{y,t,p} \cdot R_{k0}^y \quad \forall k
\end{aligned} \tag{3.5}$$

where $\hat{\alpha}_i^{x,t,p}$ and $\hat{\alpha}_k^{y,t,p}$ are normalized random coefficients, *i.e.*

$$\sum_i \hat{\alpha}_i^{x,t,p} + \sum_k \hat{\alpha}_k^{y,t,p} = 1 \quad \forall t \forall p. \tag{3.6}$$

These coefficients can be computed in two steps. First, some random coefficients are generated from a uniform distribution on bounded intervals whose interval width $\Delta_t$ changes from one cycle to another

$$\begin{aligned}
\alpha_i^{x,t,p} &\in U\left[\text{lower}_i^{x,t}, \text{upper}_i^{x,t}\right] \quad \forall i \forall t \forall p\\
\alpha_k^{y,t,p} &\in U\left[\text{lower}_k^{y,t}, \text{upper}_k^{y,t}\right] \quad \forall k \forall t \forall p
\end{aligned} \tag{3.7}$$

where

$$\begin{aligned}
&\text{If } \bar{\alpha}_i^{x,t} - \frac{1}{2}\cdot\Delta_t < 0 \Rightarrow \text{lower}_i^{x,t} = 0 \quad \text{upper}_i^{x,t} = \Delta_t \\
&\text{If } \bar{\alpha}_i^{x,t} + \frac{1}{2}\cdot\Delta_t > 1 \Rightarrow \text{lower}_i^{x,t} = 1-\Delta_t \quad \text{upper}_i^{x,t} = 1 \\
&\text{Otherwise lower}_i^{x,t} = \bar{\alpha}_i^{x,t} - \frac{1}{2}\cdot\Delta_t \quad \text{upper}_i^{x,t} = \bar{\alpha}_i^{x,t} + \frac{1}{2}\cdot\Delta_t \\
&\text{If } \bar{\alpha}_k^{y,t} - \frac{1}{2}\cdot\Delta_t < 0 \Rightarrow \text{lower}_k^{y,t} = 0 \quad \text{upper}_k^{y,t} = \Delta_t \\
&\text{If } \bar{\alpha}_k^{y,t} + \frac{1}{2}\cdot\Delta_t > 1 \Rightarrow \text{lower}_k^{y,t} = 1-\Delta_t \quad \text{upper}_k^{y,t} = 1 \\
&\text{Otherwise lower}_k^{y,t} = \bar{\alpha}_k^{y,t} - \frac{1}{2}\cdot\Delta_t \quad \text{upper}_k^{y,t} = \bar{\alpha}_k^{y,t} + \frac{1}{2}\cdot\Delta_t.
\end{aligned} \tag{3.8}$$

In the second step, these random coefficients are normalized

$$\begin{aligned}
\hat{\alpha}_i^{x,t,p} &= \frac{\alpha_i^{x,t,p}}{\sum_i \alpha_i^{x,t,p} + \sum_k \alpha_k^{y,t,p}} \quad \forall i \forall t \forall p \\
\hat{\alpha}_k^{y,t,p} &= \frac{\alpha_k^{y,t,p}}{\sum_i \alpha_i^{x,t,p} + \sum_k \alpha_k^{y,t,p}} \quad \forall k \forall t \forall p.
\end{aligned} \tag{3.9}$$

In the first cycle, since the DMU has not indicated yet any preference, the values $\bar{\alpha}_i^{x,1} = \bar{\alpha}_k^{y,1} = 0.5 \ \ \forall i \forall k$ are used and the interval width is $\Delta_1 = 1$, which means that, in the first cycle,

$$\begin{aligned}
\alpha_i^{x,1,p} &\in U[0,1] \quad \forall i \forall p \\
\alpha_k^{y,1,p} &\in U[0,1] \quad \forall k \forall p.
\end{aligned} \tag{3.10}$$

In any subsequent cycle $t$, the values $\bar{\alpha}_i^{x,t}$ and $\bar{\alpha}_k^{y,t}$ will correspond to the coefficients that led to the promising target selected by the DMU among the $P$ efficient targets presented to her in the previous cycle $t-1$. Thus, if in the previous cycle, a certain efficient target $\left(x^{t-1,p}, y^{t-1,p}\right)$ is selected to intensify the search around that region, we set

$$\begin{aligned}
\bar{\alpha}_i^{x,t} &= \hat{\alpha}_i^{x,t-1,p} \quad \forall i \\
\bar{\alpha}_k^{y,t} &= \hat{\alpha}_k^{y,t-1,p} \quad \forall k.
\end{aligned} \tag{3.11}$$

Also, to increase the focus of the search around $\bar{\alpha}_i^{x,t}$ and $\bar{\alpha}_k^{y,t}$, the width interval for the next iteration is reduced by an interval width reduction factor $\delta$, *i.e.* $\Delta_t = \delta \cdot \Delta_{t-1}$.

Alternatively, the DMU can opt for not selecting any of the targets presented to her in the previous cycle $t-1$. In that case, in the next iteration the search is restarted so that $\bar{\alpha}_i^{x,t} = \bar{\alpha}_k^{y,t} = 0.5 \ \ \forall i \forall k$ and $\Delta_t = 1$ again. Note that the width reduction factor $\delta$ is another parameter of the method and is related with the speed with which the search intensification on a specific region is sought. This could even be changed from one cycle to another although, in principle, a fixed value is assumed. This parameter, as well as parameter $P$, can vary though, depending upon the DMU being projected.

Since the $P$ directional vectors in a given cycle are generated randomly, it may happen that they fall too close. To avoid presenting the DMU with rather similar efficient targets, especially in the first cycle, when no specific region of the efficient frontier has been selected for a more fine-grained exploration, we propose to establish a separation threshold so that if in a certain cycle $t$ the efficient target computed by lexicographic DDF for a certain directional vector $g^{t,p} = (g^{x,t,p}, g^{y,t,p})$ is closer than the threshold $\varepsilon_t$ to any of the other $p-1$ efficient targets computed previously in that cycle then it is discarded and a new random directional vector is generated. Specifically, given two different efficient targets $(\hat{x}^{t,p}, \hat{y}^{t,p})$ and $(\hat{x}^{t,p\prime}, \hat{y}^{t,p\prime})$, they are considered to be too close if

$$\left\| \left(\hat{x}^{t,p}, \hat{y}^{t,p}\right) - \left(\hat{x}^{t,p\prime}, \hat{y}^{t,p\prime}\right) \right\|_2 \le \varepsilon_t. \tag{3.12}$$

In the first iteration (and in each iteration in which the search is restarted) the separation threshold is set to $\varepsilon_1 = \frac{1}{P\cdot(P-1)}$. And, in each cycle, once the DMU selects one of the computed efficient targets as promising, the separation threshold for the next cycle is reduced by a separation threshold reduction factor $\mu$, *i.e.*

$$\varepsilon_t = \mu \cdot \varepsilon_{t-1}. \tag{3.13}$$

Note that, in the first cycle after the search process is restarted and in order to increase the effectiveness of the process, the separation threshold of each new efficient target computed in that first cycle is checked not only with respect to the previous efficient targets of that cycle but also with respect to the $P$ efficient targets that the DMU discarded in the previous cycle (which was the reason to restart the search in the first place). Note also that if, in a given cycle, the DMU selects one of the presented efficient targets as promising, then in the next cycle only $P-1$ new efficient targets need to be computed, as the one selected as promising is kept for reference in that new cycle (with index $p=1$).

As a summary of the process, Figure 1 shows a flow diagram of the proposed ILD approach.

## 4. Illustration

Consider the small dataset shown in Table 2. It corresponds to six DMUs that consume two inputs and produce one output. The table also shows the ideal points corresponding to each DMU $x_i^{\min} = \hat{x}_i^{\min} \cdot x_{i0}$ and $y_k^{\max} = \hat{y}_k^{\max} \cdot y_{k0}$ and the maximum feasible improvements (3.4) computed from the solutions of models (3.2) and (3.3). It can be seen that all DMUs except DMU F are efficient. Hence, we will use the proposed ILD approach to find the MPS for that DMU. The number of targets to present to the DMU in each cycle was set to $P=5$ and the interval width factor and the separation reduction factor were set to $\delta = 0.5$ and $\mu = 0.5$, respectively. Although, as indicated in the previous section, for projecting a DMU its observed inputs and outputs are used to normalize the data and the computations are done with the normalized data, the input and output targets shown in what follows correspond to the absolute (*i.e.* un-normalized) values.

Table 3 shows, for cycle $t=1$, the different $\hat{\alpha}^{t,p} = (\hat{\alpha}^{x,t,p}, \hat{\alpha}^{y,t,p})$ generated and, for each one, the different steps of the lexicographic DDF projection. For each efficient target computed by the lexicographic DDF method, the Euclidean distance with respect to the previously computed targets are also shown. Recall that, in the first iteration, the interval width used to generate the $\hat{\alpha}^{t,p}$ vectors is $\Delta_1 = 1$ and that they are centred around $\bar{\alpha}^1 = \left(\bar{\alpha}_1^{x,1}, \bar{\alpha}_2^{x,1}, \bar{\alpha}_1^{y,1}\right) = (0.5 \;\; 0.5 \;\; 0.5)$. The separation threshold for the first iteration is $\varepsilon_1 = 1/P\cdot(P-1) = 1/20 = 0.05$. Figure 2 shows the graphical representation of the five trajectories followed by the lexicographic DDF method. Note that in three of them the lexicographic DDF projection took just one step, requiring two steps in the other two cases.

Table 3 also shows, for each of the $P$ efficient targets presented to the DMU, the corresponding directional efficiency scores. That information may be useful for the DMU to gauge the effort (in terms of input reductions and output increases) required for attaining each of the presented efficient targets. To more fully appreciate the amount of input reductions and output increases and the trade-offs involved in each of the presented efficient targets the DMU can be provided with the parallel coordinates representation shown in Figure 3. In that parallel coordinates chart, the upper limit for each of the two inputs corresponds to the observed input consumption of the DMU, while the lower limit corresponds to the minimum feasible value $x_{i0}^{\min}$. Similarly, for the output, the lower limit is the observed amount of output of the DMU while the upper limit is the maximum feasible value $y_{k0}^{\max}$, computed using (3.3). It can be noted that four of the targets presented to the DMU in this first cycle imply reducing input 1 much more than input 2, two of them (those corresponding to $p=2$ and $p=5$) with a small output increase and the other two ($p=1$ and $p=3$) with a large output increase. Target $p=4$ is different, as it implies reducing input 2 more than input 1 with a small output increase. Overall, target $p=5$ seems to have a better balance between input reductions and output increase. Therefore, after analyzing the information provided to the DMU in the first cycle, *i.e.* the efficient targets with their corresponding directional efficiency scores, the DMU chooses $\left(x^{1,5}, y^{1,5}\right) = (7.45 \;\; 8.29 \;\; 11.37)$ as most promising.

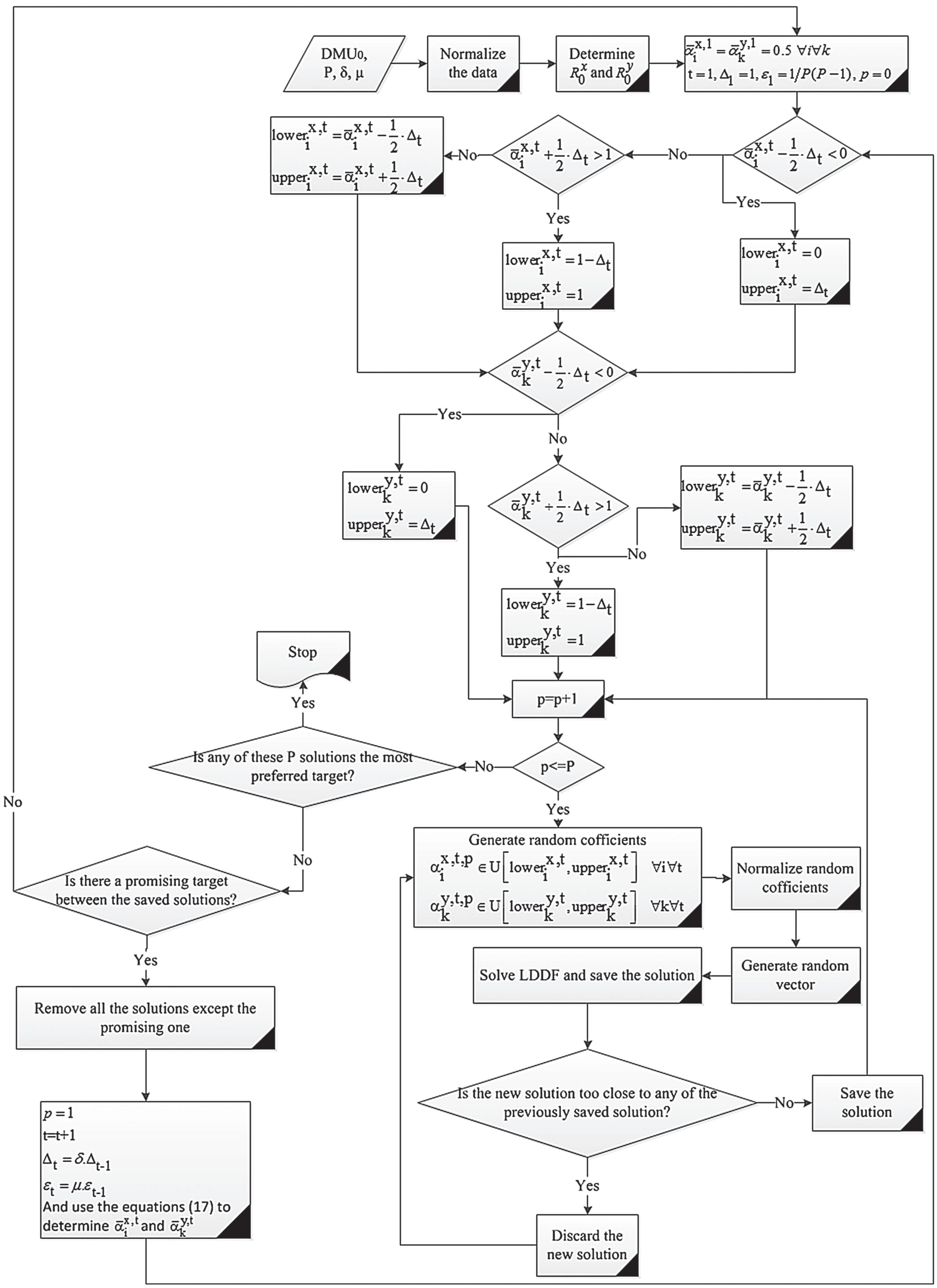


FIGURE 1. Flow diagram of proposed ILD approach.

TABLE 2. Dataset for illustrative example.

| DMU | $x_1$ | $x_2$ | $y$ | $x_1^{\min}$ | $x_2^{\min}$ | $y^{\max}$ | $\mathbb{R}_1^x$ | $\mathbb{R}_2^x$ | $\mathbb{R}^y$ |
|---|---|---|---|---|---|---|---|---|---|
| A | 8 | 12 | 14 | 8 | 12 | 14 | 0 | 0 | 0 |
| B | 4 | 10 | 10 | 4 | 10 | 10 | 0 | 0 | 0 |
| C | 5 | 5 | 8 | 5 | 5 | 8 | 0 | 0 | 0 |
| D | 10 | 4 | 10 | 10 | 4 | 10 | 0 | 0 | 0 |
| E | 12 | 6 | 12 | 12 | 6 | 12 | 0 | 0 | 0 |
| F | 12 | 12 | 10 | 4 | 4 | 14 | 8 | 8 | 4 |

TABLE 3. Detailed results of cycle $t = 1$.

| | | $t = 1,\ \bar{\alpha}^1 = (0.5, 0.5, 0.5),\ \Delta_1 = 1,\ \varepsilon_1 = 1/P(P-1) = 0.05$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $p$ | $\alpha$ | LexDDF Step | $x_1$ | $x_2$ | $y$ | $D_{p1}$ | $D_{p2}$ | $D_{p3}$ | $D_{p4}$ | Directional Eff. Score |
| 0 | – | – | 12.00 | 12.00 | 10.00 | – | – | – | – | – |
| 1 | (0.01,0.21,0.78) | 1 | 11.98 | 10.15 | 13.38 | – | – | – | – | – |
| | | 2 | 9.23 | 10.15 | 13.38 | – | – | – | – | 0.603 |
| 2 | (0.68,0.17,0.14) | 1 | 4.87 | 10.19 | 10.74 | 0.450 | – | – | – | 0.584 |
| 3 | (0.22,0.00,0.78) | 1 | 9.80 | 11.97 | 13.99 | – | – | – | – | – |
| | | 2 | 8.02 | 11.97 | 13.99 | 0.192 | 0.444 | – | – | 0.595 |
| 4 | (0.23,0.47,0.30) | 1 | 9.49 | 7.00 | 11.59 | 0.318 | 0.475 | 0.494 | – | 0.592 |
| **5** | **(0.41,0.34,0.25)** | **1** | **7.45** | **8.29** | **11.37** | **0.294** | **0.274** | **0.406** | **0.202** | **0.577** |

**Notes.** Bold row indicates the target selected as most promising.

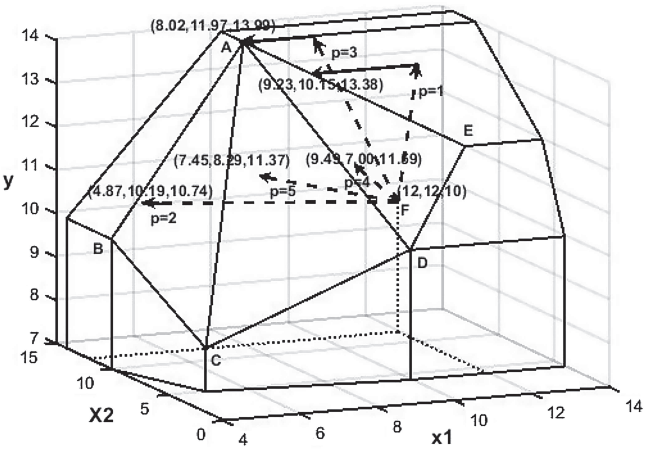


FIGURE 2. Visualization of lexicographic DDF projections of cycle $t = 1$ for DMU F.

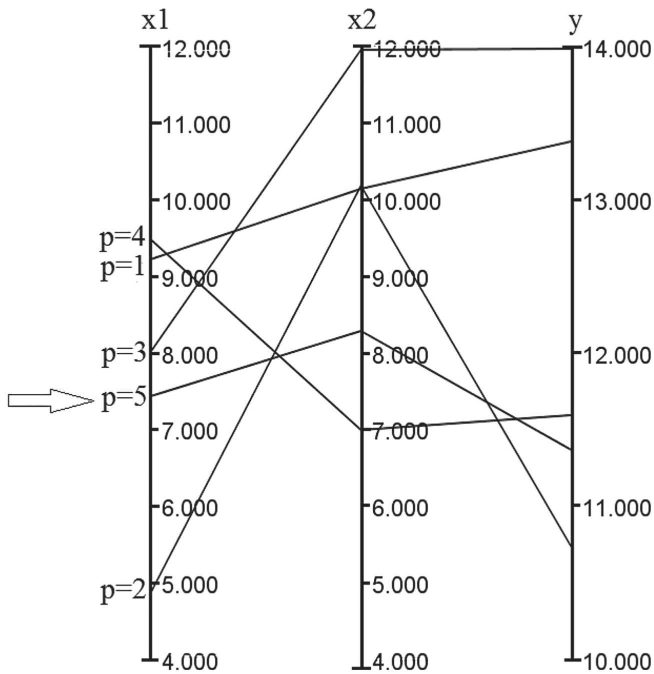


FIGURE 3. Parallel coordinates of the efficient targets of cycle $t = 1$ for DMU F (arrow indicates the target selected as most promising).

TABLE 4. Detailed results of cycle $t = 2$.

| $t = 2, \bar{\alpha}^2 = (0.41, 0.34, 0.25), \Delta_2 = 0.5, \varepsilon_2 = 0.025$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $p$ | $\alpha$ | LexDDF Step | $x_1$ | $x_2$ | $y$ | $D_{p1}$ | $D_{p2}$ | $D_{p3}$ | $D_{p4}$ | Directional Eff. Score |
| 0 | – | – | 12.00 | 12.00 | 10.00 | – | – | – | – | – |
| 1 | (0.41,0.34,0.25) | 1 | 7.45 | 8.29 | 11.37 | – | – | – | – | 0.577 |
| 2 | (0.38,0.15,0.47) | 1 | 7.65 | 10.26 | 12.72 | 0.213 | – | – | – | 0.587 |
| 3 | (0.42,0.33,0.25) | 1 | 7.43 | 8.35 | 11.39 | 0.006 | 0.208 | – | – | – |
| **3** | **(0.52,0.20,0.28)** | **1** | **6.28** | **9.74** | **11.52** | **0.156** | **0.170** | **–** | **–** | **0.580** |
| 4 | (0.60,0.34,0.06) | 1 | 5.64 | 8.37 | 10.31 | 0.185 | 0.333 | 0.175 | – | 0.567 |
| 5 | (0.50,0.36,0.14) | 1 | 6.55 | 8.06 | 10.75 | 0.099 | 0.284 | 0.162 | 0.092 | 0.566 |

**Notes.** Bold row indicates the target selected as most promising.

In the second cycle $t = 2$, the target selected in the previous cycle is kept and four new targets are computed using random $\hat{\alpha}^{t,p}$ vectors generated as per (3.7) and (3.9) with $\bar{\alpha}^2 = \alpha^{1,5} = (0.41\ 0.34\ 0.25)$ and $\Delta_2 = \delta \cdot \Delta_1 = 0.5 \cdot 1 = 0.5$. The separation threshold also decreases to $\varepsilon_2 = \mu \cdot \varepsilon_1 = 0.5 \cdot 0.05 = 0.025$. Table 4 and Figures 4 and 5 show the corresponding efficient targets computed by the lexicographic DDF method in this cycle. As mentioned above, only four of the five targets are new since the target selected in the previous cycle is carried over to this cycle. Note that it happened once that the efficient target computed for a certain $\hat{\alpha}^{2,p}$ vector failed to pass the separation threshold filter and hence it was discarded and a new $\hat{\alpha}^{2,p}$ was generated. Note also that, although the efficient targets presented to the DMU in this second cycle represent different trade-offs

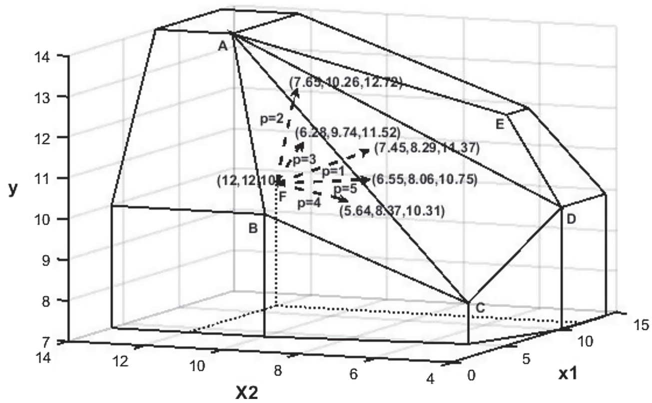


FIGURE 4. Visualization of lexicographic DDF projections of cycle $t = 2$ for DMU F.

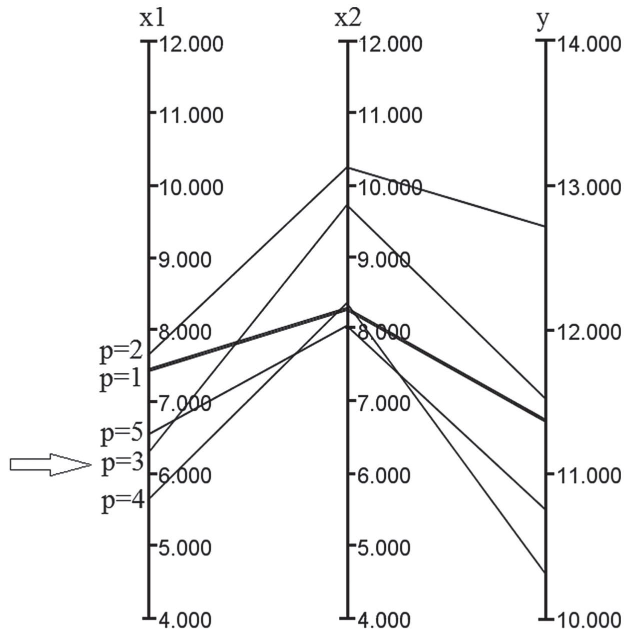


FIGURE 5. Parallel coordinates of the efficient targets of cycle $t = 2$ for DMU F (a thicker line indicates that the target is carried over from the previous cycle, arrow indicates the target selected as most promising).

in terms of their inputs and output, the overall dispersion of the targets is smaller than in the previous cycle. This is due to the reduction of the interval width of the $\hat{\alpha}^{t,p}$ vectors.

After analyzing the information provided to the DMU in cycle $t = 2$, the DMU chooses $\left(x^{2,3}, y^{2,3}\right) = (6.28\ \ 9.74\ \ 11.52)$ as most promising. Therefore, a third search-intensifying cycle $t = 3$ is initiated, keeping the selected target and computing four new targets using random $\hat{\alpha}^{t,p}$ vectors generated with $\bar{\alpha}^3 = \hat{\alpha}^{2,3} = (0.52\ \ 0.20\ \ 0.28)$ and $\Delta_3 = \delta \cdot \Delta_2 = 0.5 \times 0.5 = 0.25$. The separation threshold also decreases, $\varepsilon_3 = \mu \cdot \varepsilon_2 = 0.5 \times 0.025 = 0.0125$. Table 5 and Figures 6 and 7 show the corresponding efficient targets computed by the lexicographic DDF method in this cycle.

TABLE 5. Detailed results of cycle $t = 3$.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $t = 3, \bar{\alpha}^3 = (0.52, 0.20, 0.28), \Delta_3 = 0.25, \varepsilon_3 = 0.0125$ | | | | | | | | | | |
| p | $\alpha$ | LexDDF Step | $x_1$ | $x_2$ | $y$ | $D_{p1}$ | $D_{p2}$ | $D_{p3}$ | $D_{p4}$ | Directional Eff. Score |
| 0 | – | – | 12.00 | 12.00 | 10.00 | – | – | – | – | – |
| 1 | (0.52,0.20,0.28) | 1 | 6.28 | 9.74 | 11.37 | – | – | – | – | 0.580 |
| 2 | (0.54,0.27,0.19) | 1 | 6.11 | 9.06 | 11.03 | 0.076 | – | – | – | 0.573 |
| 3 | (0.65,0.15,0.20) | 1 | 5.15 | 10.42 | 11.06 | 0.119 | 0.139 | – | – | 0.586 |
| **4** | **(0.37,0.28,0.35)** | **1** | **7.85** | **8.84** | **11.93** | **0.157** | **0.172** | **0.275** | **–** | **0.583** |
| 5 | (0.49,0.19,0.32) | 1 | 6.56 | 9.90 | 11.80 | 0.039 | 0.111 | 0.145 | 0.140 | 0.581 |

**Notes.** Bold row indicates the target selected as MPS.

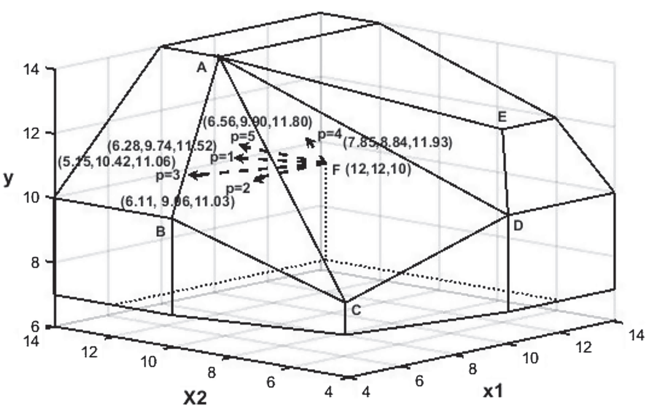


FIGURE 6. Visualization of lexicographic DDF projections of cycle $t = 3$ for DMU F.

Note that in this cycle the computed targets present small variations. The DMU selects target $\left(x^{3,4}, y^{3,4}\right) =$ (7.85 8.84 11.93) as the MPS and the procedure stops.

## 5. Case study

In this section, the proposed ILD approach is applied to the container shipping lines (CSLs) in Gutiérrez *et al.* [13]. These CSL consume three inputs (Labor, number of ships and fleet capacity, measured in Twenty-foot equivalent units, TEU) and produce two outputs: number of containers carried (in TEU) and turnover (in million USD). The dataset contains 18 DMUs, of which eight are efficient. This can be detected after computing the corresponding maximum feasible improvements of the different variables $R_{0i}^x$ and $R_{0k}^y$.

In what follows, we present the results of the proposed interactive multiobjective process for determining the targets of second largest DMU, namely MSC. Table 6 shows the corresponding normalized data. The normalization of the data is important as the inputs and outputs generally have different scales. In addition, because of the specific normalization used in (3.1), the normalized targets and the maximum feasible improvements of the different variables $R_{0i}^x$ and $R_{0k}^y$ all can be interpreted as relative inputs and outputs improvements with respect to the observed values for DMU 0. The computed targets can be un-normalized at any time by just multiplying them by $x_{i0}$ and $y_{k0}$.

The number of efficient targets to present to the DMU was set to $P = 5$ and the interval width factor and the separation reduction factor were set to $\delta = 0.5$ and $\mu = 0.5$, respectively. Table 7 shows the efficient targets presented to the DMU in each of the first four cycles, together with their corresponding directional

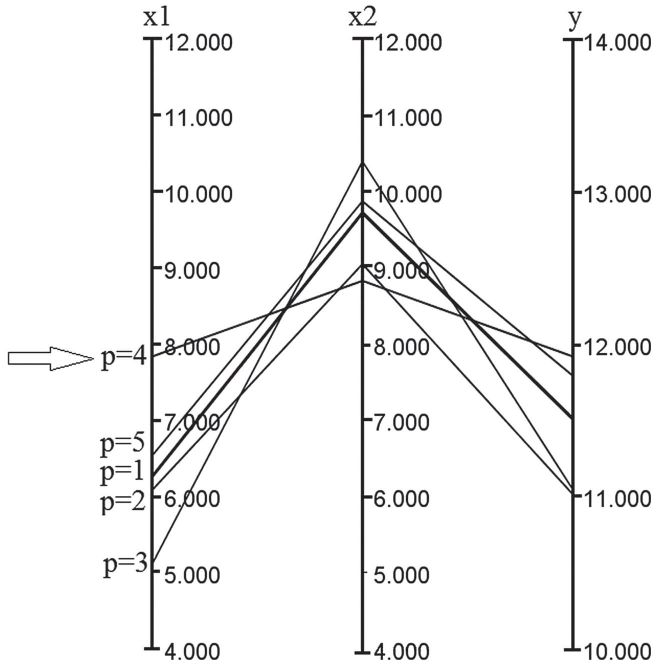


FIGURE 7. Parallel coordinates of the efficient targets of cycle $t = 3$ for DMU F (a thicker line indicates that the target is carried over from the previous cycle, arrow indicates the target selected as MPS).

TABLE 6. Normalized data for DMU MSC.

| | Inputs | | | Outputs | |
|---|---|---|---|---|---|
| DMU | Employees | # ships | Fleet capacity (TEU) | Containers carried (TEU) | Turnover |
| Maersk | 1.289 | 1.078 | 1.157 | 1.341 | 1.815 |
| MSC | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| CMA_CGM | 0.895 | 0.712 | 0.623 | 0.766 | 0.964 |
| Hapag_Lloyd | 0.351 | 0.281 | 0.304 | 0.451 | 0.563 |
| COSCON | 3.768 | 0.356 | 0.324 | 0.505 | 0.392 |
| Evergreen_Line | 0.218 | 0.419 | 0.391 | 0.565 | 0.246 |
| APL | 0.262 | 0.323 | 0.349 | 0.479 | 0.499 |
| CSCL | 0.227 | 0.303 | 0.302 | 0.651 | 0.281 |
| OOCL | 0.408 | 0.160 | 0.196 | 0.404 | 0.395 |
| CSAV | 0.367 | 0.163 | 0.128 | 0.174 | 0.275 |
| MOL | 0.527 | 0.226 | 0.228 | 0.294 | 0.445 |
| NYK | 0.085 | 0.193 | 0.236 | 0.345 | 0.450 |
| Hamburg_Süd | 0.252 | 0.226 | 0.190 | 0.224 | 0.406 |
| K_Line | 0.375 | 0.231 | 0.221 | 0.299 | 0.998 |
| YML | 0.221 | 0.206 | 0.215 | 0.270 | 0.267 |
| HMM | 0.107 | 0.130 | 0.169 | 0.244 | 0.478 |
| Wan_Hai | 0.040 | 0.158 | 0.081 | 0.261 | 0.145 |
| Delmas | 0.038 | 0.158 | 0.060 | 0.067 | 0.161 |

Table 7. Normalized efficient targets presented to DMU MSC.

$t = 1, \bar{\alpha}^1 = (\bar{\alpha}^{x,1}, \bar{\alpha}^{y,1}) = (0.5, 0.5, 0.5, 0.5, 0.5), \Delta_1 = 1, \varepsilon_1 = 0.050$

| $p$ | $\alpha$ | Employees | #ships | Fleet capacity | Containers carried | Turnover | $D_{p1}$ | $D_{p2}$ | $D_{p3}$ | $D_{p4}$ | Directional Eff. Score |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | (0.25, 0.39, 0.08, 0.14, 0.14) | 0.836 | 0.746 | 0.791 | 1.045 | 1.160 | – | – | – | – | 0.718 |
| 2 | (0.43, 0.18, 0.11, 0.03, 0.25) | 0.836 | 0.746 | 0.800 | 1.008 | 1.226 | 0.076 | – | – | – | 0.711 |
| 3 | (0.29, 0.17, 0.29, 0.22, 0.04) | 0.867 | 0.770 | 0.817 | 1.067 | 1.204 | 0.067 | 0.075 | – | – | 0.720 |
| 4 | (0.12, 0.07, 0.30, 0.31, 0.20) | 0.905 | 0.798 | 0.848 | 1.092 | 1.260 | 0.151 | 0.134 | 0.084 | – | 0.723 |
| 5 | (0.08, 0.22, 0.23, 0.09, 0.37) | 0.957 | 0.808 | 0.867 | 1.032 | 1.457 | 0.335 | 0.277 | 0.278 | 0.213 | 0.705 |

$t = 2, \bar{\alpha}^2 = (\bar{\alpha}^{x,2}, \bar{\alpha}^{y,2}) = (0.5, 0.5, 0.5, 0.5, 0.5), \Delta_2 = 1, \varepsilon_2 = 0.050$

| $p$ | $\alpha$ | Employees | #ships | Fleet capacity | Containers carried | Turnover | Distance to the points in step $t = 1$ | | | | | $D_{p1}$ | $D_{p2}$ | $D_{p3}$ | $D_{p4}$ | Directional Eff. Score |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | $D_{p1}$ | $D_{p2}$ | $D_{p3}$ | $D_{p4}$ | $D_{p5}$ | | | | | |
| 1 | (0.22, 0.47, 0.20, 0.05, 0.06) | 0.788 | 0.712 | 0.753 | 1.015 | 1.091 | 0.103 | 0.155 | 0.171 | 0.254 | 0.430 | – | – | – | – | 0.713 |
| 2 | (0.33, 0.00, 0.36, 0.05, 0.26) | 0.856 | 0.760 | 0.823 | 1.014 | 1.269 | 0.120 | 0.055 | 0.086 | 0.104 | 0.224 | 0.209 | – | – | – | 0.712 |
| 3 | (0.08, 0.27, 0.25, 0.13, 0.26) | 0.923 | 0.798 | 0.838 | 1.050 | 1.345 | 0.216 | 0.166 | 0.156 | 0.097 | 0.122 | 0.314 | 0.115 | – | – | 0.712 |
| **4** | **(0.15, 0.10, 0.13, 0.36, 0.25)** | **0.930** | **0.816** | **0.868** | **1.108** | **1.296** | **0.206** | **0.183** | **0.138** | **0.054** | **0.180** | **0.308** | **0.143** | **0.084** | – | **0.725** |
| 5 | (0.17, 0.18, 0.30, 0.10, 0.25) | 0.899 | 0.772 | 0.819 | 1.034 | 1.310 | 0.167 | 0.113 | 0.116 | 0.086 | 0.169 | 0.262 | 0.064 | 0.056 | 0.105 | 0.708 |

$t = 3, \bar{\alpha}^1 = (\bar{\alpha}^{x,1}, \bar{\alpha}^{y,1}) = (0.15, 0.10, 0.13, 0.36, 0.25), \Delta_3 = 0.5, \varepsilon_3 = 0.025$

| $p$ | $\alpha$ | Employees | #ships | Fleet capacity | Containers carried | Turnover | $D_{p1}$ | $D_{p2}$ | $D_{p3}$ | $D_{p4}$ | Directional Eff. Score |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | (0.15, 0.10, 0.13, 0.36, 0.25) | 0.930 | 0.816 | 0.868 | 1.108 | 1.296 | – | – | – | – | 0.725 |
| 2 | (0.34, 0.00, 0.14, 0.20, 0.32) | 0.879 | 0.778 | 0.833 | 1.047 | 1.270 | 0.124 | – | – | – | 0.716 |
| **3** | **(0.12, 0.28, 0.07, 0.29, 0.23)** | **0.934** | **0.810** | **0.862** | **1.101** | **1.291** | **0.095** | **0.091** | – | – | **0.726** |
| 4 | (0.11, 0.21, 0.14, 0.18, 0.36) | 0.945 | 0.824 | 0.895 | 1.058 | 1.420 | 0.158 | 0.181 | 0.141 | – | 0.717 |
| 5 | (0.41, 0.20, 0.09, 0.22, 0.09) | 0.847 | 0.755 | 0.800 | 1.054 | 1.175 | 0.196 | 0.108 | 0.173 | 0.288 | 0.781 |

$t = 4, \bar{\alpha}^1 = (\bar{\alpha}^{x,1}, \bar{\alpha}^{y,1}) = (0.12, 0.28, 0.07, 0.29, 0.23), \Delta_4 = 0.25, \varepsilon_4 = 0.0125$

| $p$ | $\alpha$ | Employees | #ships | Fleet capacity | Containers carried | Turnover | $\mathrm{D}_{p1}$ | $\mathrm{D}_{p2}$ | $\mathrm{D}_{p3}$ | $\mathrm{D}_{p4}$ | Directional Eff. Score |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | (0.12, 0.28, 0.07, 0.29, 0.23) | 0.934 | 0.810 | 0.862 | 1.101 | 1.291 | – | – | – | – | 0.726 |
| 2 | (0.22, 0.16, 0.15, 0.27, 0.21) | 0.895 | 0.798 | 0.839 | 1.085 | 1.246 | 0.112 | – | – | – | 0.725 |
| 3 | (0.24, 0.26, 0.07, 0.23, 0.19) | 0.881 | 0.780 | 0.828 | 1.074 | 1.227 | 0.131 | 0.034 | – | – | 0.721 |
| **4** | **(0.06, 0.28, 0.04, 0.42, 0.20)** | **0.958** | **0.836** | **0.890** | **1.126** | **1.335** | **0.120** | **0.132** | **0.165** | – | **0.727** |
| 5 | (0.01, 0.28, 0.20, 0.24, 0.27) | 0.981 | 0.809 | 0.878 | 1.083 | 1.346 | 0.144 | 0.138 | 0.166 | 0.058 | 0.732 |

**Notes.** Bold row indicates the target selected as most promising or, for $t = 4$, as MPS.

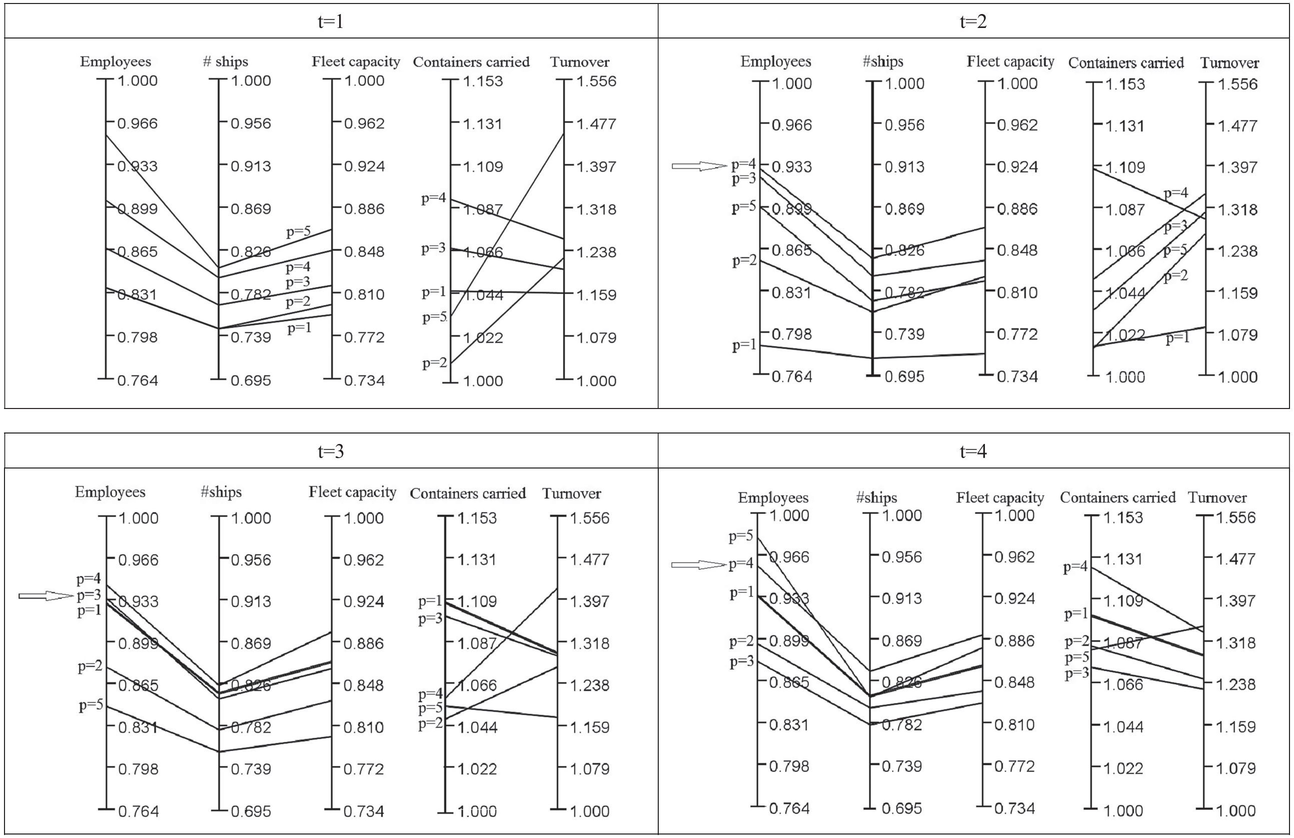


FIGURE 8. Parallel coordinates of normalized efficient targets presented to DMU MSC in the different cycles (a thicker line indicates that the target is carried over from the previous cycle, arrow indicates the target selected as most promising or, for $t = 4$, as MPS).

efficiency score. Recall that the directional efficiency score measures how far overall that efficient target is from the observed DMU and thus the magnitude of the effort (*i.e.* the difficulty) of attaining that target. Thus, the larger the directional efficiency score, the closer the target, and *vice versa*, the lower the directional efficiency score, the farther away the target.

Note that, in the first cycle, the DMU did not select any of the five efficient targets presented to it and, hence, the search was restarted in cycle $t = 2$. In this case, the new targets not only must not be close to each other but also to the unpromising targets of the previous iteration. In any case, in cycle $t = 2$, and assessing, with the help of the parallel coordinates shown in Figure 8, the corresponding trade-offs between the different input and output dimensions, the DMU chose the target $\left(\hat{x}^{2,4}, \hat{y}^{2,4}\right) = (0.930\ \ 0.816\ \ 0.868\ \ 1.108\ \ 1.296)$, whose directional efficiency score was 0.725. This target was carried over to cycle $t = 3$, relabeling it as $\left(\hat{x}^{3,1}, \hat{y}^{3,1}\right)$. In that cycle that target plus four new efficient targets, generated with a shrunk random vector cone and allowing less separation between the targets, were presented to the DMU. In cycle $t = 3$, the DMU selected as most promising target $\left(\hat{x}^{3,3}, \hat{y}^{3,3}\right) = (0.934\ \ 0.810\ \ 0.862\ \ 1.101\ \ 1.291)$, with a directional efficiency score of 0.726. This led to a new search-intensifying cycle, in which both the amplitude of the random vector cone and the separation threshold were further reduced, leading to a set of less disperse efficient targets. Since, in this cycle $t = 4$, the DMU declared one of the computed efficient targets, namely $\left(\hat{x}^{4,4}, \hat{y}^{4,4}\right) = (0.958\ \ 0.836\ \ 0.890\ \ 1.126\ \ 1.335)$, as the MPS, the interactive process stopped. The directional efficiency associated to the final MPS target is 0.727.

## 6. Conclusions

In this paper, a new interactive multiobjective optimization approach for DEA target setting has been proposed. It delineates a structured process that in each iteration cycle presents to the DMU a fixed number of efficient targets. These targets are computed using a lexicographic DDF method with the corresponding directional vectors generated randomly but in a controlled manner. In particular, when the DMU selects one of the computed targets as most promising, the search intensifies in that region of the efficient frontier probing it in a more fine-grained fashion. Also, to make the most out of the limited number of targets that are presented to the DMU in each cycle measures are taken to prevent presenting targets that are too similar. This is done imposing a separation threshold that decreases in each successive search-intensifying cycle. In any cycle, if the DMU does not find any of the presented targets promising, the process backtracks and restarts the search so that any region of the efficient frontier can again be explored.

The method has three parameters, which can be varied depending on the DMU being projected. These parameters are the number of targets to present in each iteration ($P$), the interval width reduction factor ($\delta$) and the separation threshold reduction factor ($\mu$). To help the DMU choose between the different efficient targets parallel coordinates or any other similar visualization aid is highly recommended, given the multidimensional character of the computed targets. The directional efficiency score associated to each efficient target indicates how ambitious (in terms of inputs reduction and output increase) that target is and hence the effort required for attaining it. This information can also be helpful for the DMU when choosing between the different efficient targets.

The proposed ILD approach has been applied to 18 container shipping lines. In particular, the results of the proposed interactive process for one of the inefficient DMUs have been presented and commented. The information provided to the DMU at each step of the process has proven rather useful for the DMU for progressively articulating her preferences as regards the trade-offs between the different inputs and outputs.

Overall, the proposed approach achieves a balance between simplicity and flexibility. It benefits greatly from its graphical interpretation, making it intuitive and easy to understand. Although the iterative character of the lexicographic DDF approach makes it more complex than the conventional DDF, it guarantees that computed targets are efficient.

As a continuation of this research, it would be desirable to extend the proposed ILD approach to a more general setting, like the one in Tavana *et al.* [36], which considers desirable and undesirable variables as well as uncontrollable variables. Devising ways to handle zero and negative data is also a worthy endeavor.

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