RAIRO-Oper. Res. 54 (2020) 1685–1701
https://doi.org/10.1051/ro/2019100

RAIRO Operations Research
www.rairo-ro.org

OPTIMAL REPLENISHMENT DECISION FOR RETAILERS WITH VARIABLE DEMAND FOR DETERIORATING PRODUCTS UNDER A TRADE-CREDIT POLICY

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Abstract. In this study one obtained the optimal decision of a retailer for the replenishment rate with selling-price and credit-period dependent demand to maximize the profit. A time-varying deterioration rate was considered for those products. A credit-period was offered by the retailer to the end customer to settle the whole payments. The aim of the model was to obtain the maximum profit for the retailer based model. A solution methodology with an algorithm was used to obtain the global optimum profit. An illustrative numerical example was given to test the practical applicability of the model. Numerical study indicated that the profit was at a maximum when the permissible delay-period for payment offered by the suppliers was lies between the permissible delay-time, and the cycle time, offered by the retailer.

Mathematics Subject Classification. 90B05, 90B06.

Received December 13, 2018. Accepted October 30, 2019.

1. Introduction

In real world businesses, demand is a function of the selling price and credit-period instead of being constant. Nowadays, researchers focus on real world applications in their retailer based supply chain research. Many industries have improved their sales and profit *via* share marketing and trade-credit financing policies. In practice, many retailers/customers do not want to pay large amounts of money at one time for a product. For more sales with reduced on hand inventory stock, suppliers/retailers allow a fixed amount of time for payment without any fine. In general, the holding costs are reduced due to this permissible-delay because the amount of capital investment in the stock is reduced in the time interval of the permissible period. The retailer can accelerate revenue *via* share market investment or banking business during the delay-period (*i.e.*, credit-period).

Keywords. Retailing, trade credit, deterioration, selling-price depended demand.

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Research on trade-credit has been conducted for many years, Goyal [16] first discussed the effects of the trade-credit period on the optimal inventory policy. Although Goyal's [16] model was extended by Chand and Ward [6] under the assumptions of the classical economic order quantity model, which obtained a different result. An ordering policy for deteriorating items was proposed by Aggarwal and Jaggi [2], in which permissible delay-in-payments plays a vital role. Similarly, Jamal *et al.* [24] proposed an ordering policy for deteriorating items with allowable shortages and permissible delays in payments. Hwang and Shinn [22] developed a retailer pricing and lot sizing policy for exponentially deteriorating products with a permissible delay-in-payments. Chu *et al.* [9] discussed the economic order quantity of deteriorating items under permissible delay-in-payments. Chung [10] generalized a theorem for the determination of economic order quantity with a permissible delay-in-payments. Jamal *et al.* [25] calculated an optimal payment time for a retailer under permitted delay-in-payments by the wholesaler. An inventory model with imperfect production was proposed by Sarkar [28], in which demand was considered as stock-dependent with delay-in-payments. In the same year, an inventory model was developed by Sarkar [29], in which, deterioration depended on time. Variable deterioration with the presence of a trade-credit for fixed lifetime products was considered by Sarkar *et al.* [36]. Recently, Tiwari *et al.* [42] discussed a trade-credit policy and a partial backordering strategy for a green product production system. Several researchers have developed different types of retailer based inventory models that consider delay-in-payments, but demand is dependent on the credit-period and selling-price, thus, still there is a gap in this research area.

Several types of deterioration for a two-echelon supply chain model were developed by Sarkar [30]. In modern business environment, a single vendor may fulfill the demand of several customers [13]. Thus, a model of a single-vendor and multiple buyers is a realistic approach these days. In the literature, Goyal [15] first optimized the joint cost for a single buyer and single vendor. This research was extended by Banerjee [5] with a lot-for-lot strategy. Goyal [17] extended Banerjee's [5] model again by considering single-setup, multi-delivery (SSMD). Recently, Dey *et al.* [14] developed an integrated-inventory model, in which the demand depends on the selling price of the products. A two-echelon supply chain model for deteriorating items was developed by Sarkar *et al.* [37]. In this model, they used some investments to reduce setup costs and improve the process quality. Different research models ([14,27], etc.) along with selling price dependant demand were developed by several researchers. However, a retailer based model for deteriorating items with variable demand, in which demand depends on the selling-price and the credit-period has still not been considered by any researcher. Thus, a pioneering attempt was taken to cover this research gap in the model proposed in this study.

Chang and Dye [8] generalized an inventory model for deteriorating items with partial backlogging and permissible delay-in-payments. Two more benefits of trade-credit were illustrated by Teng [38]. First, trade-credit attracted more attention of new customers, who considered the trade-credit policy to be a type of price reduction; and second, it caused a reduction in outstanding sales. Abad and Jaggi [1] used a joint approach to set the unit price and the length of the credit-period for sellers when the end demand was price-sensitive. Arcelus *et al.* [4] computed retailers pricing, credit, and inventory policies for deteriorating items in response to temporary price/credit incentives. Huang [20] discussed a model for optimum retailer ordering policies in the EOQ model with trade credit financing. Chang [7] developed an EOQ model with deteriorating items and inflation when the supplier credits were linked with order quantity. However, the strategy of granting credit terms adds an additional dimension of default risk to the supplier, as discussed by Teng *et al.* [40]. The optimal ordering policy in DCF analysis of deteriorating items when trade credit depended on the order quantity was discussed by Chung and Liao [11]. Ho *et al.* [19] designed an inventory model under two-levels of trade credit and limited storage space derived without derivatives. The variable demand depended on the selling-price in a production model, which was considered by Sarkar *et al.* [33]. Huang [21] developed an economic order quantity under conditionally permissible delays-in-payments. Ho *et al.* [19] examined optimal pricing, shipment and payment policies for an integrated supplier–buyer inventory model with two-part trade-credit. The deterioration rate was considered to be variable by Sarkar and Sarkar [33]. They considered a probabilistic deterioration rate in the EMQ model. Jaggi *et al.* [23] presented retailer's optimal replenishment decisions with credit-linked demand under permissible delay-in-payments. Liao [26] proposed an EOQ model with non-instantaneous receipt and exponentially deteriorating items with a two-level trade-credit policy. Teng and Chang [39] developed an optimal

Table 1. Author(s) contribution table.

Author(s)	Retailing	Deterioration	Selling-price dependent demand	Credit -period
Aggarwal and Jaggi [2]		√		√
Arcelus *et al.* [4]	√			√
Chung [10]		√		√
Chu *et al.* [9]		√		√
Dey *et al.* [14]			√	
Goyal [16]				√
Huang [20]				√
Jaggi *et al.* [23]	√			√
Jamal *et al.* [24]	√	√		√
Sarkar [28]				√
Sarkar *et al.* [35]				√
Thangam and Uthayakumar [41]	√		√	√
This model	√	√	√	√

replenishment policy in a production model with two-levels of trade-credit policy. Tsao [43] calculated retailer's optimal ordering and discounting policies with advance sales discounts and trade credits. Similarly, Thangam and Uthayakumar [41] derived a model with selling-price and credit-period dependent demand. The concept of delays-in-payment for imperfect production processes and deteriorating items was developed by Sarkar [28] and Sarkar *et al.* [35]. Several researchers have developed different order quantity or production models for deteriorating items, with trade-credit but a retailer based model for deteriorating items in which the demand depends on the product selling-price and credit-period has still not been considered in the existing literature. This research gap is filled by present research model.

The contributions of different author(s) to this research field are shown in Table 1. From Table 1, it is clear that most researchers have developed a supply chain model along with the effects of the credit-period. However, a retailer based model for deteriorating items, in which the demand is dependent on the product selling-price and credit-period has still not been considered in any of the existing literature. Thus, an initial attempt was taken in this research to fulfill the research gap.

The impacts of both the selling-price and credit-period on retailer's demand is very important in retailer based management for perishable items in a two-level trade-credit policy. The marginal effects of the credit-period on sales are proportional to the unrealized potential of the market demand. Thus, the retailer's demand becomes a function of both the selling-price and credit-period. An economic production quantity (EPQ) model for perishable items is developed, in which the retailer's demand is a function of both the credit-period and selling-price with two-level trade-credit financing.

In this model, the following matters are addressed briefly: (1) The retailer's demand is a function of both the selling-price and credit-period; (2) The retailer's trade-credit-period (M) offered by the supplier is not necessarily longer than the customer's trade credit period (N) offered by the retailer; (3) The replenishment rate is finite; (4) The items being sold are perishable; and (5) A two-level trade-credit financing is adopted instead of single level trade credit financing between the supplier and retailer. To maximize the retailer's profits, one has to determine the optimal credit-period (N^*), selling-price (s^*), and replenishment time (T^*).

In the next section, the assumptions and notations related to this study are presented. In Section 3, the model formulated by considering the possible costs and revenues is shown. Section 4 shows that the optimal replenishment policy not only exists but is also unique and the optimal conditions are derived to find the optimal selling price. In Section 5, several numerical examples are presented to illustrate this theory.

2. Problem definition, notation, and assumptions

The problem definition of this model along with the notations and assumptions are defined in this section.

2.1. Problem definition

The retailer's optimal decision for replenishment derived in this study was when customer's demand depended on the credit-period and selling-price of the item. Items deteriorated at an exponential rate and were a function of on-hand inventory. Retailers were also giving a credit-period to the customers to maximize their profit. The production rate was finite and was always greater than the demand due to there being no shortages as well as the lead time being negligible in this model. Finally, a gross profit was calculated, which was maximized along with the optimal values of selling-price for the item, delay-period and cycle time.

2.2. Notation

The notations and assumptions used in this model are noted below.

Decision variables	
s	unit selling-price per item of good quality ($/unit)
N	permissible delay-period in payments for the customer, offered by the retailer (days)
T	cycle time (days)
Parameters	
$\lambda(s, N)$	annual demand, as a function of both s and N
A	ordering cost per order ($/order)
h	holding cost per unit per unit time excluding interest charges ($/unit/unit time)
c	unit purchasing cost per item ($/item)
M	permissible delay-period in payments for the retailer, offered by the supplier
P	annual replenishment rate
L	maximum lifetime of products
θ	deterioration rate, where $0 \leq \theta < 1$
t_1	time at which the production stops in a cycle
$I(t)$	inventory level at time t, where $0 \leq t \leq T$
I_e	interest earned ($/year)
I_k	interest charged by the supplier ($/year)
$\mathrm{TP}(s, T, N)$	annual total profit ($/cycle)

λ is used in place of $\lambda(s, N)$ throughout this paper.

2.3. Assumptions

The following assumptions were considered when developing the model.

(1) The demand $\lambda(s, N)$ is a marginally increasing function with respect to N and downward sloping function of price s. The production rate is finite and $P > \lambda$. The gross profit $(s - c)\lambda(s, N)$ is concave.
(2) The rate of deterioration of a product varies with time. It follows a time-dependent function $\theta = \frac{1}{1+L-t}$; where L is the maximum lifetime of the product and when $t \to L$; $\theta \to 1$, *i.e.*, 100% deterioration at maximum lifetime (see [28]).
(3) Before the settlement of an account, the retailer can use sales revenue to earn the interest with an annual rate I_e up to the end of period M. At time $t = M$, the credit is settled and the retailer starts to pay the interest at rate I_k for the items in stock.

(4) The retailer offers a credit-period N for each of his customers to settle the account. The time horizon is infinite. The inventory holding cost is charged only on the amount of undecayed stock. Shortages are not allowed and lead time is negligible.

3. Mathematical model

For a given selling-price $s > 0$, the marginal effect of the credit-period on sales is proportional to the unrealized potential of market demand without any delay. Under this assumption, the demand can be defined in the following two ways. First, the demand $\lambda(s, N)$ is represented by the following partial differential equation,

$$\frac{\partial \lambda(s,N)}{\partial N} = r[\alpha(s) - \lambda(s,N)]. \tag{3.1}$$

Equation (3.1) can be rewritten as follows:

$$\frac{\partial \lambda(s,N)}{\partial N} + r\lambda(s,N) = r\alpha(s). \tag{3.2}$$

Integrating equation (3.2) and using initial condition $N = 0$, $\lambda(s,N) = \beta(s)$ gives

$$\lambda(s,N) = \alpha(s) - [\alpha(s) - \beta(s)]\mathrm{e}^{-rN}. \tag{3.3}$$

where $\alpha(s)$ is the maximum demand over the planning horizon, when the selling-price is s, and r is the saturation of demand where $0 \leq r < 1$. Second, $\lambda(s,N)$ can also be represented by the following difference equation:

$$\lambda(s, N+1) - \lambda(s,N) = r[\alpha(s) - \lambda(s,N)]. \tag{3.4}$$

From the above equations (3.3) and (3.4), one can obtain

$$\lambda(s,N) = \alpha(s) - [\alpha(s) - \beta(s)]\mathrm{e}^{-rN}. \tag{3.5}$$

$$\text{and } \lambda(s,N) = \alpha(s)[1 - (1-r)^N] + \beta(s)(1-r)^N. \tag{3.6}$$

The inventory level $I(t)$ (see Fig. 1) at time t satisfies the following differential equations:

when the production is in the interval of $0 \leq t \leq t_1$, then the inventory satisfies the following differential equation

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \theta I(t) = P - \lambda(s,N) \text{ if } 0 \leq t \leq t_1, \text{ where } \theta = \frac{1}{1+L-t}\cdot \tag{3.7}$$

Using the value of the deterioration rate θ, equation (3.7) can be rewritten as follows

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \frac{1}{1+L-t}I(t) = P - \lambda(s,N). \tag{3.8}$$

Integrating equation (3.8) and using the initial conditions of $t = 0$, and $I(0) = 0$, one can obtain the following equation:

$$I(t) = (P - \lambda(s,N))\,(1+L-t)\log\left(\frac{1+L}{1+L-t}\right)\cdot \tag{3.9}$$

Again, while the system is in the interval of $t_1 \leq t \leq T$, then the inventory satisfies the following differential equation

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \theta I(t) = -\lambda(s,N) \text{ if } t_1 \leq t \leq T, \text{ where } \theta = \frac{1}{1+L-t}\cdot \tag{3.10}$$

Using value of the deterioration rate θ in equation (3.10), one can obtain the following:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \frac{1}{1+L-t}I(t) = -\lambda(s,N). \tag{3.11}$$

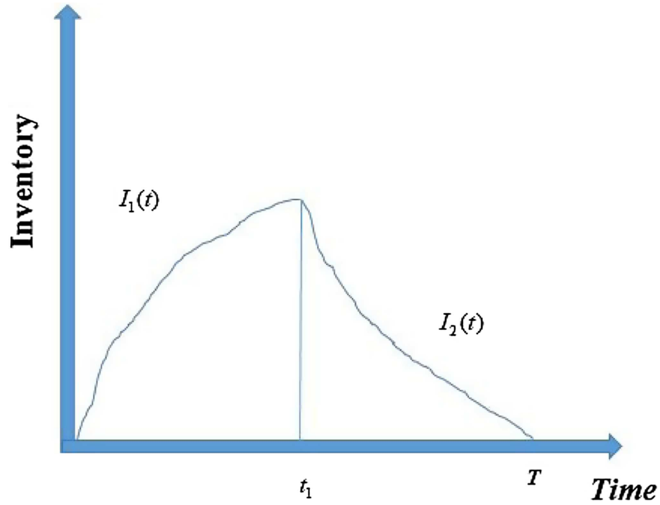

FIGURE 1. Graphical representation of inventory system.

Integrating equation (3.11) results the following equation

$$I(t) = (1 + L - t)\lambda(s, N) \log\left(\frac{1 + L - t}{1 + L - T}\right) \text{ (using the condition } I(T) = 0). \tag{3.12}$$

Thus, the inventory in $[0, T]$ is given by

$$I(t) = \begin{Bmatrix} I_1(t) & \text{if} & 0 \leq t \leq t_1 \\ I_2(t) & \text{if} & t_1 \leq t \leq T \end{Bmatrix} \tag{3.13}$$

where,

$$I_1(t) = (P - \lambda(s, N))(1 + L - t) \log\left(\frac{1 + L}{1 + L - t}\right) \tag{3.14}$$

$$I_2(t) = (1 + L - t)\lambda(s, N) \log\left(\frac{1 + L - t}{1 + L - T}\right). \tag{3.15}$$

By the given condition $I_1(t) = I_2(t)$, at $t = t_1$, following equation is obtained:

$$t_1 = (1 + L)\left[1 - \mathrm{e}^{-\frac{\lambda(s,N)}{P} \log \frac{(1+L)}{(1+L-T)}}\right]. \tag{3.16}$$

Ordering Cost (OC). For a smooth business, the ordering cost plays a vital role. There is a cost for ordering products, known as the ordering cost. The ordering cost per cycle for the system is given as follows:

$$\mathrm{OC} = \frac{A}{T}.$$

Holding Cost (HC). The products, which are ordered by vendor/retailer, have to be hold in the showroom or a store room. There are some costs associated with storing and holding products, known as holding costs.

Thus, the annual stock holding cost (excluding interest charges) per cycle is as follows:

$$\begin{aligned}\mathrm{HC} &= \frac{h}{T}\int_0^T I(t)\,\mathrm{d}t \\ &= \frac{h}{2T}(P-\lambda(s,N))\log(1+L)[(1+L)^2-(1+L-t_1)^2] \\ &\quad + \frac{h}{T}(P-\lambda(s,N))\left[\frac{(1+L-t_1)^2}{2}\log(1+L-t_1)-\frac{(1+L-t_1)^2}{4}\right. \\ &\quad \left. -\frac{(1+L)^2}{2}\log(1+L)+\frac{(1+L)^2}{4}\right]-\frac{h}{T}\lambda(s,N)\left[\frac{(1+L-T)^2}{2}\log(1+L-T)\right. \\ &\quad \left. -\frac{(1+L-T)^2}{4}-\frac{(1+L-t_1)^2}{2}\log(1+L-t_1)+\frac{(1+L-t_1)^2}{4}\right] \\ &\quad + \frac{h}{2T}\lambda(s,N)\log(1+L-T)[(1+L-T)^2-(1+L-t_1)^2]. \end{aligned} \tag{3.17}$$

Deteriorating Cost (DC). The deterioration plays a major role for any deterioration production system. Some extra costs will be added in the total cost due to deterioration. Thus, the annual deteriorating cost per cycle is given as follows:

$$\mathrm{DC} = \frac{cPt_1}{T} + c\lambda(s,n).$$

Sales Profit (SP). Every industry cares about its own profits and the annual sales profit is given by $(s-c)\lambda(s,n)$.

Interest Payable (IP). Based on the values of T, N, and M there are the following three cases to be considered:

Case I. $N \le M \le T+N$.

$$\begin{aligned}\mathrm{IP} &= cI_k\frac{\lambda(s,N)}{\theta T}\int_M^{T+N}\left(\mathrm{e}^{\theta(T+N-t)}-1\right)\mathrm{d}t \\ &= cI_k\frac{\lambda(s,N)}{T}(1+L-t)^2\left[\mathrm{e}^{\frac{(T+N-M)}{(1+L-t)}}-\frac{(T+N-M)}{(1+L-t)}-1\right]. \end{aligned} \tag{3.18}$$

Case II. $N \le T+N \le M$.

In this case, there is no interest payable by the retailer.

Case III. $M \le N \le T+N$.

$$\begin{aligned}\mathrm{IP} &= \frac{cI_k}{T}\left[\int_M^N Pt_1\,\mathrm{d}t+\int_N^{T+N}\frac{\lambda(s,N)}{\theta(\mathrm{e}^{\theta(T+N-t)}-1)\,\mathrm{d}t}\right] \\ &= cI_k\frac{\lambda(s,N)}{T}(1+L-t)^2\left[\left(\frac{1}{1+L-t}\right)^2 Pt_1(N-M)\right. \\ &\quad \left. + \lambda(s,N)\left(\mathrm{e}^{\frac{T}{(1+L-t)}}-\frac{T}{1+L-t}-1\right)\right]. \end{aligned} \tag{3.19}$$

Interest Earned (IE). Three cases are also considered for interest earned:

Case I. $N \le M \le T+N$ (see Fig. 2).

Therefore, the annual interest earned is given as follows:

$$\mathrm{IE} = \frac{sI_e\lambda(s,N)(M-N)^2}{2T}. \tag{3.20}$$

Case II. $N \le T+N \le M$ (see Fig. 3).

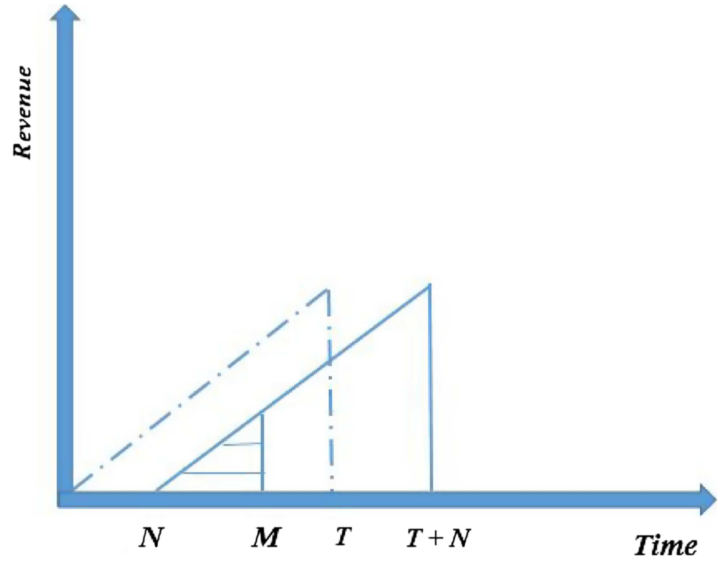

FIGURE 2. Retailer's interest earned when $N \leq M \leq T + N$.

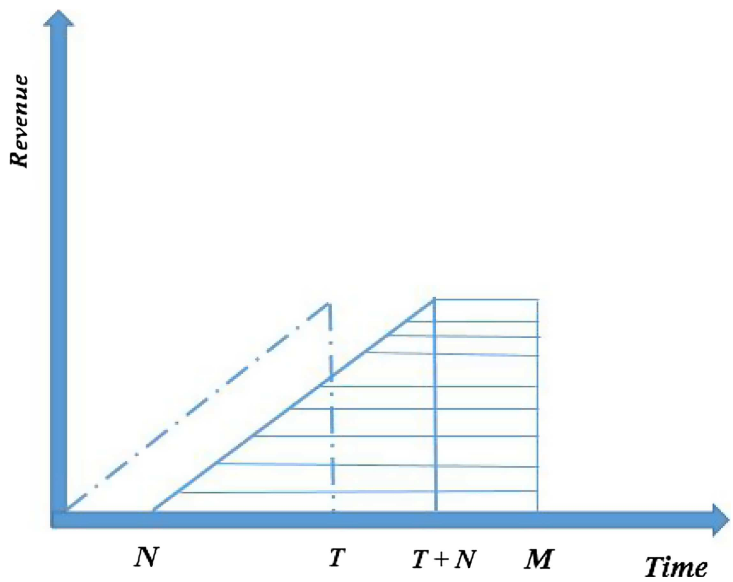

FIGURE 3. Retailer's interest earned when $N \leq T + N \leq M$.

The annual interest earned is

$$\begin{aligned} \text{IE} &= \frac{sI_e}{T}\left[\frac{\lambda(s,N)T^2}{2} + \lambda(s,N)T(M-T-N)\right] \\ &= sI_e\lambda(s,N)\left[M - N - \frac{T}{2}\right]. \end{aligned} \tag{3.21}$$

Case III. $M \leq N \leq T + N$.

There is no interest earned by the retailer.

Then, the total annual profit of the retailer can be obtained as follows:

$$\mathrm{TP}(s,T,N) = \mathrm{SP} - \mathrm{OC} - \mathrm{HC} - \mathrm{DC} - \mathrm{IP} + \mathrm{IE}.$$

From the arguments noted above, the annual profit is given as follows:

$$\mathrm{TP}(s,T,N) = \left\{\begin{array}{lll} \mathrm{TP}_1(s,T,N), & \text{if} & N \le M \le T+N \\ \mathrm{TP}_2(s,T,N), & \text{if} & N \le T+N \le M \\ \mathrm{TP}_3(s,T,N), & \text{if} & M \le N \le T+N \end{array}\right\} \tag{3.22}$$

where

$$\mathrm{TP}_1(s,T,N) = s\lambda(s,N) - \frac{\alpha_1}{T} \tag{3.23}$$

$$\mathrm{TP}_2(s,T,N) = s\lambda(s,N) - \frac{\alpha_2}{T} + sI_e\lambda(s,N)\left[M - N - \frac{T}{2}\right] \tag{3.24}$$

$$\mathrm{TP}_3(s,T,N) = (s-c)\lambda(s,N) - \frac{\alpha_3}{T}\cdot \tag{3.25}$$

The values of α_1, α_2 and, α_3 are given in Appendix A.

4. Solution methodology

To obtain the critical points, the first order partial derivatives with respect to the decision variables are separately equal to 0. Therefore, for this problem, the mathematically necessary and sufficient conditions are detailed below:

Necessary condition. To find the critical point that is the 1st order partial derivatives of zero. *i.e.*,

$$\frac{\partial \mathrm{TP}_i}{\partial s} = 0,\ \frac{\partial \mathrm{TP}_i}{\partial N} = 0,\ \frac{\partial \mathrm{TP}_i}{\partial T} = 0. \quad \text{where, } i = 1,2,3$$

the following equations are used,

$$\frac{\partial \mathrm{TP}_1}{\partial s} = \lambda + s\lambda'(s,N) + \frac{\beta_1}{T} \tag{4.1}$$

$$\frac{\partial \mathrm{TP}_1}{\partial N} = (s)\lambda'(s,N) + \frac{\beta_2}{T} \tag{4.2}$$

$$\frac{\partial \mathrm{TP}_1}{\partial T} = \frac{\beta_3}{T^2} + \frac{\beta_4}{T} \tag{4.3}$$

$$\frac{\partial \mathrm{TP}_2}{\partial s} = s\lambda'(s,N) + \frac{\beta_5}{T} + \left(\lambda(s,N) + s\lambda'(s,N)\right) I_e \left(M - N - \frac{T}{2}\right) \tag{4.4}$$

$$\frac{\partial \mathrm{TP}_2}{\partial N} = (s-c)\lambda'(s,N) + \frac{\beta_5}{T} + sI_e\lambda'(s,N)\left(M - N - \frac{T}{2}\right) - sI_e\lambda(s,N) \tag{4.5}$$

$$\frac{\partial \mathrm{TP}_2}{\partial T} = \frac{\beta_6}{T^2} + \frac{\beta_4}{T} - \frac{1}{2}sI_c\lambda(s,N) \tag{4.6}$$

$$\frac{\partial \mathrm{TP}_3}{\partial s} = (s-c)\lambda'(s,N) + \frac{\beta_7}{T} \tag{4.7}$$

$$\frac{\partial \mathrm{TP}_3}{\partial N} = (s-c)\lambda'(s,N) + \frac{\beta_8}{T} \tag{4.8}$$

$$\frac{\partial \mathrm{TP}_3}{\partial T} = \frac{\beta_9}{T^2} + \frac{\beta_{10}}{T}\cdot \tag{4.9}$$

See Appendix A, for the values of $\beta_1 - \beta_{10}$.

The following procedure states the optimality of the decision variable.

Any function that is defined in an open interval (a, b) is concave if for any two points $x, y \in (a, b)$ and each ζ, $0 \leq \zeta \leq 1$, the following is satisfied:

$$f(\zeta x + (1-\zeta)y) \geq \zeta f(x) + (1-\zeta) f(y).$$

If a continuous function h is defined in a closed interval $[a, b]$ as its domain and $h(a), h(b)$ are opposite in sign, then there exists a point p within $[a, b]$, such that $h(p) = 0$. (by the Intermediate value theorem)

Lemma 4.1. *If a function $h(t)$ is continuous on (a, b) and differentiable with a non increasing function (i.e., $\frac{\mathrm{d}h}{\mathrm{d}t}$ is non-increasing) then h is concave.*

Proof. See Appendix B. □

Case 1. $N \leq M \leq T + N$

For given values of s and N, the first derivative of TP_1 with respect to T was given earlier in this report. The optimal value can be solved from equation $\frac{dTP_1}{dT} = 0$, it is easy to show that

$$\begin{aligned}
\frac{\partial^2 \mathrm{TP}_1}{\partial T^2} = -\Bigg[& h(P - \lambda(s,N)) \left[\frac{1}{2}(1+L-T)^2 \log(1+L-T) - \frac{1}{4}(1+L-T)^2 \right. \\
& \left. + \frac{1}{2}(1+L-t_1)^2 \log(1+L-t_1) + \frac{1}{4}(1+L-t_1)^2 \right] \\
& + h\lambda(s,N) \log(1+L-T)(1+L-T) \\
& + \frac{1}{2} h\lambda(s,N) [\log(1+L-T)][(1+L-T)^2 - (1+L-t_1)^2][-2-2t+2T] \\
& + \frac{1}{2} h\lambda(s,N) \log(1+L-T)[(1+L-T)^2 - (1+L-t_1)^2] \\
& + cPt_1 + \frac{1}{2} sI_c \lambda(s,N)(M-N)^2 \Bigg] < 0. \qquad (4.10)
\end{aligned}$$

As, $\frac{\partial^2 \mathrm{TP}_1}{\partial T^2} < 0$, the function TP_1 is non-increasing on $(0, \infty)$ and the Lemma TP_1 is a concave function on $(0, \infty)$. At zero, the following is true.

$$\begin{aligned}
\mathrm{TP}_1(0) = & \left(A - \frac{SI_e \lambda(s,N)(M-N)^2}{2} \right) \\
& - cI_k \lambda(s,N)(1+L-t)^2 \left[\mathrm{e}^{\frac{(T+N-M)}{(1+L-t)}} - \frac{(T+N-M)}{(1+L-t)} - 1 \right]. \qquad (4.11)
\end{aligned}$$

Since, $\mathrm{e}^{\frac{(T+N-M)}{(1+L-t)}} > \frac{(T+N-M)}{(1+L-t)} - 1$, $\mathrm{TP}_1(0) > 0$, if $\left(A - \frac{SI_e \lambda(s,N)(M-N)^2}{2} \right) > 0$. $\mathrm{TP}_1 = -\infty < 0$ as T tends to ∞, and $\mathrm{TP}_1(0) > 0$.

Thus, by the intermediate value theorem, the result is unique as well as optimum.

By the similar arguments and using the following algorithms, one can obtain other decision variables s and N along with optimal profit.

Algorithm. The following algorithm was developed to find the optimal value of s and N.

Step 1. Fixed $m = 1$.Using the equation $(s - c)\frac{\partial \lambda}{\partial s} + \lambda(s) = 0$, solve s_j considering all input parameters.

Step 2. Using the value of s_j and the above noted arguments of equations (4.10) and (4.11), find T_{im} (i=1,2,3) and let $s_{im} = s_j$.

Step 3. Using the value of T solve the equations (4.1), (4.4), and (4.7) *i.e.*, $\frac{\partial \mathrm{TP}_i}{\partial s}=0$ for s. Find s_i^* such that $\frac{\partial^2 \mathrm{TP}_i}{\partial s^2}$ at $(T=T_{im}, s+s_i^*)$ and let $s_{i,m+1}=s_i^*$.

Step 4. Solve the equations (4.3), (4.6), and (4.9) *i.e.*, $\frac{\partial \mathrm{TP}_i}{\partial T}=0$ for T by using the value $s=s_{i,m+1}$ and let the solution be $T_{i,m+1}$.

Step 5. If for a pre assing small $\epsilon>0$ $|T_{im}-T_{im+1}|<\epsilon$ and $|s_{im}-s_{im+1}|<\epsilon$, then $s^*=s_{im+1}$ and $T^*=T_{im+1}$. Otherwise $m=m+1$, go to step 3.

5. Numerical example

In this section, numerical examples are discussed to demonstrate the applicability of this model and the optimum results are compared with the results of some existing literature to show the benefit of this model. The parametric values for the numerical results are taken from Sarkar and Sarkar's [32] model and Thangam and Uthayakumar's [41] model, and by using Mathematica 9.0, one can find the following numerical values.

Example 5.1. The demand is $\alpha(s)-[\alpha(s)-\beta(s)]\mathrm{e}^{-rN}$ and the parametric values are as follows: ordering cost $A=\$1000$/order, holding cost $h=\$4.5$/unit/unit time, unit purchasing cost $c=\$15$/unit, permissible delay-period $M=30$ days, replenishment rate $P=700$ units, interest earned $I_e=25\%$ per year, interest charged $I_k=15\%$ per year, lifetime $L=100$ days, $\alpha(s)=80-1.21s$, $\beta(s)=30-1.21s$, and $r=0.5$.

Then using the above noted data, one can obtain the following optimum result which is given in Table 2. From Table 2, it is found that the profit is maximized for Case I, when the optimum selling-price (s^*) is \$18.76 /unit, the permissible delay-period (N^*) is 30.77 days, and the optimum cycle time (T^*) is 39.90 days. The total profit in this case is \$493.69, which is more beneficial than that of Thangam and Uthayakumar's [41] model.

Example 5.2. When the demand is $\alpha(s)[1-(1-r)^N]+\beta(s)(1-r)^N$ and the parametric values are the same as Example 5.1.

Then by using the above data, one can obtain the following optimum result, which is given in Table 3. From Table 3, it is found that the profit is maximized for Case I, when the optimum selling-price (s^*) is \$19.75/unit, the permissible delay-period (N^*) is 31.28 day, and the optimum cycle time (T^*) is 41.31 days. In this case, the optimal profit is \$520.53 which is also superior to Thangam and Uthayakumar's [41] model.

Table 2. Optimum values of Example 5.1 for profit and decision variables.

	Profit (\$)	Selling price s (\$)	Permissible delay N (days)	Cycle time T (days)
TP_1	**493.69**	**18.76**	**30.77**	**39.90**
TP_2	410.55	17.00	20.21	37.13
TP_3	459.69	27.45	5.00	31.53

Table 3. Optimum values of Example 5.2 for profit and decision variables.

	Profit (\$)	Selling price s^* (\$)	Permissible delay N^* (days)	Cycle time T^* (days)
TP_1	**520.53**	**19.75**	**31.28**	**41.31**
TP_2	483.00	18.33	23.78	37.80
TP_3	490.54	16.51	4.17	27.03

6. Sensitivity analysis

From the numerical experiment, one can determine that the profit was maximized for Case I, for which sensitivity analysis for the key parameters was conducted and the results are presented in Table 4. The effects of key parameters are graphically shown in Figure 4. From the sensitivity analysis shown in Table 4, one can obtain the following insights.

(1) Ordering cost is slightly affected in the total profit calculation. The total system profit is bound to decrease when the ordering cost is high.
(2) To hold the inventory a holding cost is needed, which is more effective. An increase in holding cost leads the system profit upwards, whereas, profit is decreased if the holding cost is reduced, which is an interesting findings in this research.

Table 4. Sensitivity analysis for key parameter.

Parameters	Changes (in %)	TP_1	Parameters	Changes (in %)	TP_1
	−50%	+02.14		−50%	+27.50
	−25%	+01.07		−25%	+07.13
A	+25%	−01.08	L	+25%	−03.68
	+50%	−02.17		+50%	−05.93
	−50%	−32.94		−50%	+0.16
	−25%	−16.47		−25%	+0.08
h	+25%	+16.47	I_e	+25%	−0.08
	+50%	+32.94		+50%	−0.16
	−50%	−26.26		−50%	−85.09
	−25%	−13.13		−50%	−72.72
c	+25%	+13.13	I_k	+25%	−21.29
	+50%	+26.26		+50%	−32.15
	−50%	+31.39		−50%	+0.39
	−25%	+15.59		−25%	+0.13
M	+25%	−15.73	P	+25%	−0.08
	+50%	−31.93		+50%	−0.13

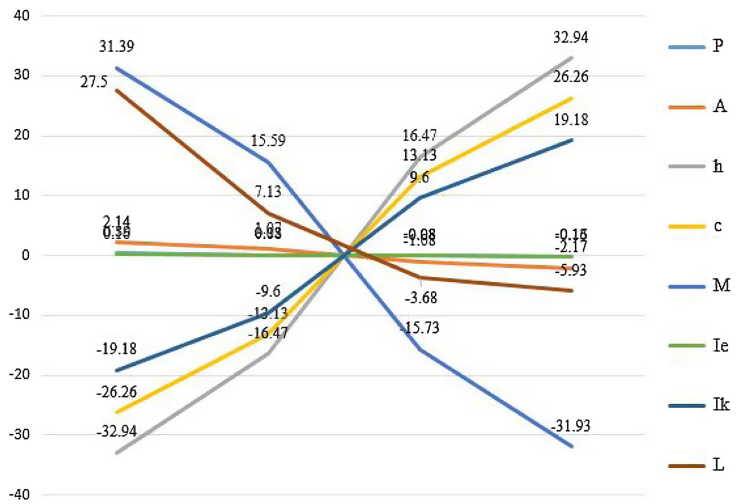

Figure 4. Graphical representation of the effects on total cost relative to changes in the parametric values.

(3) A change in delay-period and charged interest is always harmful to the total system profit, which is clear from the sensitivity analysis, shown in Table 4.
(4) The lifetime of the product also greatly affects the total profit. An increase in the maximum lifetime of the products reduces the total system profit, whereas, a decrease in the lifetime of the product increases the total system profit.
(5) The replenishment rate and the earned interest I_e have a slight effect on the total system profit.

7. Conclusions

Based on the result of the present study, if any industry wishes to adopt a new policy to increase their profit, then increasing the credit-period is the best policy. The convergence between operational problems and financial tools is given by the credit-period. The latter is mostly based on a private valuation of profits and benefits associated with the trade-credit policy. A retailer based model was formulated for perishable items under a two-level trade-credit policy with the assumption that the market demand was sensitive to both the selling-price and the credit-period offered by the retailer. A solution procedure was described to find the optimal solution, and a lemma and an algorithm were developed. Finally, the total system profit was maximized along with the optimum values of the selling-price, cycle-time and delay-period. The profit of the first case was the global optimal profit among all of the cases. It was found that increasing the permissible-delay increased the profit more than that of the other cases. The limitation of this model is that no shortages and no lead time are considered. The model can be developed by considering environmental issues (see [3]) as well as by considering complementary products or assembled products. This model can also be extended by considering two types of trade credits: one is earlier informed and the other is after purchasing. To keep the brand image of products, an inspection (see [31]) and a warranty can also be added to this model (see [18]). In real life situations, demand is uncertain and, thus, a safety stock or safety period can also be considered (see [14]) in this model, thus adding more realistic dimensions to this research.

Appendix A.

The values of γ, $\alpha_1, \alpha_2, \alpha_3$, δ, and β_1–β_{10} are presented as follows:

$$\gamma = A + \Bigg[\frac{h}{2}(P-\lambda(s,N))\log(1+L)\left[(1+L)^2-(1+L-t_1)^2\right]$$
$$- h\,(P-\lambda(s,N))\left[\frac{(1+L-t_1)^2}{2}\log(1+L-t_1)-\frac{(1+L-t_1)^2}{4}\right.$$
$$\left.-\frac{(1+L)^2}{2}\log(1+L)+\frac{(1+L)^2}{4}\right]+h\lambda(s,N)\left[\frac{(1+L-T)^2}{2}\log(1+L-T)\right.$$
$$\left.-\frac{(1+L-T)^2}{4}-\frac{(1+L-t_1)^2}{2}\log(1+L-t_1)+\frac{(1+L-t_1)^2}{4}\right]$$
$$-\frac{h}{2}\lambda(s,N)\log(1+L-T)[(1+L-T)^2-(1+L-t_1)^2]\Bigg]$$
$$\alpha_1 = \gamma + cPt_1 + cI_k\lambda(s,N)(1+L-t)^2\left[\mathrm{e}^{\frac{(T+N-M)}{(1+L-t)}}-\frac{(T+N-M)}{(1+L-t)}-1\right]$$
$$-\frac{sI_e\lambda(s,N)(M-N)^2}{2},$$
$$\alpha_2 = \gamma + cPt_1$$

$$\alpha_3 = \gamma + cI_k\lambda(s,N)(1+L-t)^2\left[(\frac{1}{1+L-t})^2 Pt_1(N-M) + \lambda(s,N)\left(\mathrm{e}^{\frac{T}{(1+L-t)}} - \frac{T}{1+L-t} - 1\right)\right]$$

$$\delta = \frac{h\lambda'(s,N)\log(1+L)[(1+L)^2-(1+L-t_1)]}{2} - h\lambda'(s,N)\left[\frac{1}{2}(1+L-t_1)^2\log(1+L-t_1) - \frac{(1+L-t_1)^2}{4} - \frac{1}{2}(1+L)^2\log(1+L) + \frac{(1+L)^2}{4}\right] - h\lambda'(s,N)\left[\frac{1}{2}(1+L-T)^2\log(1+L-T) - \frac{(1+L-T)^2}{4} - \frac{1}{2}(1+L-t_1)^2\log(1+L-t_1) + \frac{(1+L-t_1)^2}{4}\right] + \frac{h\lambda'(s,N)\log(1+L-T)[(1+L-T^2)-(1+L-t_1)^2]}{2}$$

where, $$\lambda'(s,N) = -\frac{s_{\alpha\max}-s}{(s-s_{\alpha\min})^2} - \frac{1}{s-s_{\alpha\min}} - \mathrm{e}^{-Nr}\left(-\frac{s_{\alpha\max}-s}{(s-s_{\alpha\min})^2} - \frac{1}{s-s_{\alpha\min}} + \frac{s_{\beta\max}-s}{s-s_{\beta\min}} + \frac{1}{s-s_{\beta\min}}\right)$$

$$\beta_1 = \delta - cI_k\lambda'(s,N)(1+L-t)^2\left[\mathrm{e}^{\frac{(T+N-M)}{(1+L-t)}} - \frac{(T+N-M)}{(1+L-t)} - 1\right] + \frac{sI_e\lambda'(s,N)(M-N)^2}{2} + \frac{I_e\lambda(s,N)(M-N)^2}{2}$$

$$\beta_2 = \delta + \frac{sI_e\lambda'(s,N)(M-N)^2}{2} - sI_e\lambda'(s,N)(M-N)$$

$$\beta_3 = \Bigg[A + \frac{h(P-\lambda(s,N))\log(1+L)[(1+L)^2-(1+L-t_1)^2]}{2} - h(P-\lambda(s,N))\Big[\frac{1}{2}(1+L-t_1)^2\log(1+L-t_1) - \frac{1}{4}(1+L-t_1)^2 - \frac{1}{2}(1+L)^2\log(1+L) + \frac{1}{4}(1+L)^2\Big] + h(P-\lambda(s,N))\Big[\frac{1}{2}(1+L-T)^2\log(1+L-T) - \frac{1}{4}(1+L-T)^2 - \frac{1}{2}(1+L-t_1)^2\log(1+L-t_1) + \frac{1}{4}(1+L-t_1)^2\Big] - \frac{h\lambda(s,N)\log(1+L-T)[(1+L-T)^2-(1+L-t_1)^2]}{2} + cPt_1 - \frac{sI_c\lambda(s,N)(M-N)^2}{2}\Bigg]$$

$$\beta_4 = h\lambda(s,N)\log(1+L-T)(1+L-T) + \frac{1}{2}h\lambda(s,N)[\log(1+L-T)][(1+L-T)^2-(1+L-t_1)^2][-2-2t+2T]$$

$$\beta_5 = \beta_2 - \frac{sI_e\lambda'(s,N)(M-N)^2}{2} + sI_e\lambda'(s,N)(M-N)$$

$$\beta_6 = \beta_3 + \frac{sI_c\lambda(s,N)(M-N)^2}{2} + \frac{1}{2}h\lambda(s,N)[\log(1+L-T)][(1+L-T)^2-(1+L-t_1)^2][-2-2t+2T]$$

$$\beta_7 = \beta_5 - cI_k\lambda'(s,N)Pt_1(N-M)$$
$$\beta_8 = \beta_7 - cI_k\lambda(s,N)Pt_1$$
$$\beta_9 = \beta_6 + cI_k\lambda(s,N)Pt_1(N-M)$$
$$\beta_{10} = \beta_4 + \lambda(s,N)\log(1+L-T)(1+L-T).$$

Appendix B. Proof of Lemma 4.1

Proof of Lemma 4.1. Let a function h be defined on a closed interval $[0,1]$ as follows:

$$h(t) = f(ty + (1-t)x) - tf(y) - (1-t)f(x)$$

where, $a < x < y < b$.

The main theme is to show that h is non-negative on [0,1]. Since h is continuous and $h(0) = h(1) = 0$ then,

$$\frac{\mathrm{d}h(t)}{\mathrm{d}t} = (y-x)\frac{\mathrm{d}f}{\mathrm{d}t} - f(y) + f(x)$$

when $t + g > t$, one can obtain the following equation:

$$\frac{\mathrm{d}h(t+g)}{\mathrm{d}t} - \frac{\mathrm{d}h(t)}{\mathrm{d}t} = (y-x)\left[\frac{\mathrm{d}f(t+g)}{\mathrm{d}t} - \frac{\mathrm{d}f(t)}{\mathrm{d}t}\right].$$

Since, $\frac{\mathrm{d}f}{\mathrm{d}t}$ is a non-increasing function

$$\frac{\mathrm{d}f(t+g)}{\mathrm{d}t} - \frac{\mathrm{d}f(t)}{\mathrm{d}t} < 0.$$

then, in the closed interval $[0,1]$, $\frac{\mathrm{d}h(t)}{\mathrm{d}t}$ is non-increasing. Let the function h be minimum at the point $p \in [0,1]$. If $p = 1$, $h(t) \geq h(1) = 0$ on $[0,1]$. Since h takes a local minimum at p, then $\frac{\mathrm{d}h(p)}{\mathrm{d}t} \geq 0$. $\frac{\mathrm{d}h}{\mathrm{d}t}$ is non-increasing, so $\frac{\mathrm{d}h}{\mathrm{d}t} \geq 0$ on $[0,p]$. By a similar argument, it can be shown that h is non-decreasing on $[0,p]$, then the minimum of h in $[0,1]$ is non-negative and $h \geq 0$ is on $[0,1]$. □

Acknowledgements. This work has supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (2019R1A2C1088255).

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