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IMPACT OF PRICING STRUCTURE ON SUPPLY CHAIN COORDINATION WITH COOPERATIVE ADVERTISING

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Abstract. This paper develops a game-theoretic model in a two-echelon supply chain composed of one manufacturer and two retailers to study the effect of pricing structure and cooperative-advertising decisions on the supply chain coordination performance. In the proposed model, different pricing structures are analyzed and then, two types of pricing structure in supply chain coordination mechanisms are presented, in addition to considering four possible scenarios for pricing structure. For the first two scenarios, retailers determine the retail prices, while in the other two ones, the sales price is set by the manufacturer. Therefore, the retailers are obliged to comply with this rule. The manufacturer-Stackelberg and the cooperative games are formulated for each scenario by considering key assumptions associated with advertising expenditures to maintain the potential demand size. This paper also presents some analytical results and determines the equilibrium of the models for each scenario. Finally, a numerical analysis is conducted to illustrate the impact of pricing structure on the optimal decision variables and the profit of the supply chain members.

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1. Introduction

A supply chain is a system of companies processing the raw materials to finished products reached to customers, it also involves multiple echelons where every echelon consists of one or more players, from suppliers to retailers acting their own jobs. In this regard it should be noted all of the supply chain members are assumed rational which implies that each player has his/her own objective of maximizing the revenue by choosing the best strategy, consequently. Sometimes, the optimal strategy chosen by one player may conflict with the others' choices of optimal strategies. As a result, it leads to conflicts among the preferences or utilities of the members of the supply chain. In common, it can be said that a structure of independent and rational members yields a lower profit than a cooperative supply chain, where the objective is to maximize the supply chain profit function. In this connection, many studies have been accomplished to show the improvements in the channel profit using supply chain coordination mechanisms.

Regarding supply chain coordination, cooperative advertising (CA) plays a major role in marketing plans. CA is a financial arrangement that occurred between a manufacturer and a retailer where the manufacturer

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cooperates in local advertising expenditures of the retailer. In the recent years, there has been an increasing interest among researchers on cooperative advertising studies and investment.

Additionally, pricing decisions are clearly another fundamental measures which can affect the total profit in a supply chain since price is one of the four primary variables in the marketing mix that managers use to develop a marketing plan. At any participation rate of CA, pricing normally includes decisions for the manufacturer's wholesale price as well as the retail price. Simultaneous decisions on CA and pricing strategies in supply chains coordination studies attract many researchers' attentions.

Expanding the employment of dual sales channels and setting on-line sales prices by manufacturers and, on the other hand, the increasing negotiation power of retailers make it necessary to investigate retail price decision-maker and pricing structure. Thus, the above-named issues inspire us to explore the subsequent problems. (3.1) Which firm does it have to determine the retail price? (3.2) Which pricing structure does it work better for the manufacturer, the retailer, and consumers? (3.3) How can joint CA and pricing lead to supply chain coordination in different scenarios?

To answer these questions, we consider a two-echelon supply chain in which a manufacturer sells a product to the two competing retailers. We study the impact of pricing structure decisions on the supply chain performance and propose two types of pricing structures in supply chain coordination and four possible scenarios for decision making of the firms. The rest of this paper is organized as follows. Section 2 briefly reviews the related literature. Section 3 introduces a pricing and cooperative advertising model formulation of the problem. Section 4 presents the manufacturer Stackelberg equilibrium for each scenario. Section 5 investigates the corresponding pricing and cooperative advertising issues in the centralized setting for each scenario. Section 6 gives a numerical example to investigate scenarios in centralized and decentralized situations. And later the maximal profits, decision variables, and sensitivity analyses are presented. Section 7, finally concludes the paper and presented managerial insights and future research directions.

2. Literature review

Berger [3] was the first researcher who studied cooperative advertising topics, mathematically in a manufacturer-retailer supply chain. Since then, other researchers have extended different aspects of Berger's work. In this field, the existing studies are various in terms of their model structures being time-dependent (dynamic games) [8, 17] or not (static games) [1, 6, 14, 16]. The majority of literature have used static models and investigated cooperative advertising in two-sided monopoly circumstances, where a manufacturer sells the product to a single retailer. The main conclusion of these studies is that cooperative advertising can be effective for increasing retailer's advertising and reference price, expanding the amount of demand and finally increasing profits for all channel members. For an excellent review on cooperative advertising in supply chain management, we refer the interested readers to Aust and Buscher [2] and Jorgensen and Zaccour [7], who provided two comprehensive analyses of cooperative advertising literature.

Several studies in the literature have focused on supply chain with one manufacturer and one retailer who can prevent other better insights, the reason is that in the real world in a supply chain, a manufacturer usually deals with two or more retailers. Karray and Zaccour [11] studied a market with two manufacturers and two retailers, in which they considered only advertising decision variables. In another study, Wang *et al.* [15] considered a marketing channel with one manufacturer and two competing retailers. Similarly, Karray and Amin [10] considered a channel where a manufacturer sells his product to the market through two competing retailers. They showed that the levels of pricing and advertising competition in the marketplace consequentially affect the efficiency of cooperative advertising in coordinating the channel. Liu *et al.* [12] considered two-manufacturer two-retailer supply chain and evaluated the efficiency of cooperative advertising. They found that these plans do not benefit the supply chain if they lead to a significant decrease in the channel members' unit margins.

In a supply chain with two competing manufacturers, Chakraborty *et al.* [4] investigated a situation that manufacturers sell their products through a common retailer. In their model, the demand that the retailer is faced was considered stochastic and dependent on the retail prices as well. Chiang *et al.* [5] studied the direct

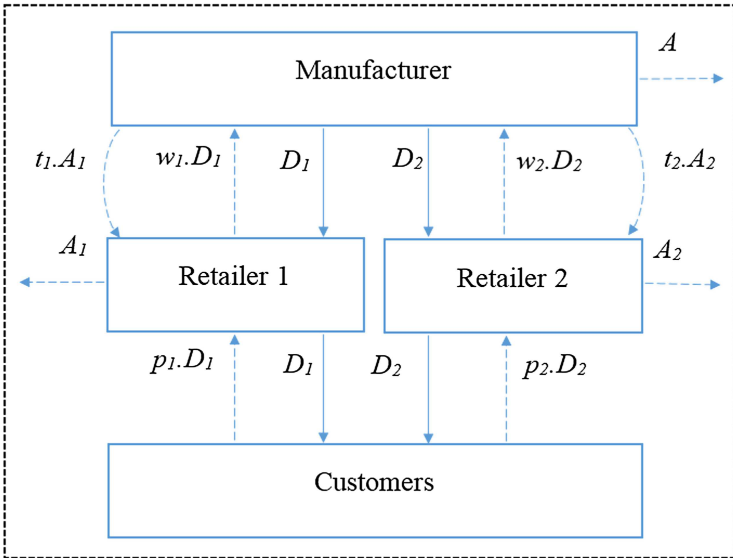

FIGURE 1. Supply chain.

channel allows a manufacturer to impose retailer's pricing behavior. It induced the retailer to the price that the manufacturer determines for the online channel. Zhao *et al.* [18] provided an important understanding of the effects of the market power structures on pricing strategies by considering a dual-channel supply chain with a complementary product. Luo *et al.* [13] examined a supply chain with the monopolistic retailer and duopolistic manufacturers with differentiated brands. Both horizontal and vertical competitions were considered through a study of different power structure combinations. They found that for the channel members, being first to declare the pricing decision results in lower profit.

These findings point three important features. First, according to the usage expansion of the dual channel and changes in traditional networks, we need to analyze different pricing structures in the supply chain. Second, simultaneous analysis of cooperative advertising and pricing decisions in the supply chain could be more interesting and efficient. Third, the manufacturer usually collaborates with more than one retailer, who are often different in the market, initial demand, etc. Based on these significant matters, the current paper builds on these three instructions from the literature.

Although nearly half of the CA literature articles have included pricing decisions and some articles have incorporated a dual-channel structure into their model, no article has so far analyzed the impact of pricing structure on the supply chain. This paper contributes to the literature as follows. The stud first investigates to analyze different pricing structures in the supply chain. The main objective of this paper is to learn how pricing structure affect the decision of retail price and members' profit. To reach this aim, we model two echelon supply chain with one manufacturer and two retailers, and then analyze the proposed model in four scenarios with Manufacturer Stackelberg and cooperative games, and numerical analysis is also presented.

3. MODEL FORMULATION

In this section, the problem and related assumptions, as well as notations used in this research, are defined. Then the demand functions are introduced, and their development in line with channel structure in leader-follower game models are presented. Consider a supply chain that consists of a single manufacturer and two retailers with different power and initial market, denoted as player M, R_i (while $i = 1, 2$), respectively (see Fig. 1). In the following, four scenarios are introduced, and the Manufacturer-Stackelberg game is analytically

Table 1. Scenarios.

	p determiner	
w condition	Manufacturer	Retailer
The same w	MSS	RDS
Different w	MSD	RDD

investigated. In the first scenario, the retailer determines the retail price. Generally, each retailer may offer a different price to the market. Therefore, retail prices are different. However, the manufacturer may consider the wholesale price to be the same or different scenarios.

Therefore, in the first scenario (RDS) retailers determine retail prices. In the mentioned scenario, retail prices are different while wholesale prices are the same. In the second scenario (RDD), retailers determine retail prices, and thus, retail prices are different, and wholesalers also offer various prices to the retailers. In the next two scenarios, the retail price is determined by the manufacturer. Therefore, P is the retail price of a product determined by the manufacturer, and both retailers should supply the product at the same price. In the third scenario (MSS), the manufacturer sets the retail price equally for each retailer and offers the same wholesale price to both. While in the fourth scenario (MSD) the manufacturer determines the fixed retail price, the wholesale prices are considered different for the two retailers. Manufacturer controls wholesale price $w(wi)$, global advertising expenditures A and cooperative advertising participation rates t_i. Each retailer sets his own local advertising expenditure A_i. We examine pricing decisions in a different pricing structure. In the first two scenarios, the manufacturer also determines constant retail price p (MSD, MDD), and in in the next ones, two retailers determine retail price (p_i) (RDS, RDD). So problem variables, are shown in Table 1. Before presenting the model, some key notations are detailed as follows: The demanded quantity for each retailer depends on both pricing and advertising efforts. The demand function is influenced by retail prices as well as local and global advertising expenditures. In the demand function selection, we follow [9] and choose a demand model that is linearly influenced by price and increases in nonlinear trend by advertising. In this model, the retailers' demand function decreases in price independent of other retailers' pricing. Furthermore, the demand for retailer i increases with local advertising effort and manufacturer global brand advertising. The demand functions for manufacturer and retailers are shown in (3.1) and (3.2) (Tabs. 2 and 3).

$$D = \sum_{i=1}^{n} \gamma_i(\alpha - \beta(w_i + m_i) + A + kA_i) \tag{3.1}$$

$$D_i = \gamma_i((\alpha - \beta(w_i + m_i) + A + kA_i), \quad i = 1, 2. \tag{3.2}$$

Considering above assumptions for profit functions of manufacturer and retailers are shown in (3.3)–(3.5).

$$\Pi_M = \sum_{i=1}^{2} w_i\gamma_i(\alpha - \beta(w_i + m_i) + A + kA_i) - \sum_{i=1}^{2} t_i \cdot A_i^2 - A^2 \tag{3.3}$$

$$\Pi_{R_i} = m_i\gamma_i(\alpha - \beta(w_i + m_i) + A + kA_i) - (1 - t_i)A_i^2, \quad i = 1, 2 \tag{3.4}$$

$$\Pi_{\text{SC}} = \Pi_M + \sum_{i=1}^{2} \Pi_{R_i} = \sum_{i=1}^{2} p_i\gamma_i(\alpha - \beta \cdot p_i + A + k \cdot A_i) - \sum_{i=1}^{2} A_i^2 - A^2. \tag{3.5}$$

In this section, parameters and variables are introduced. In the next section, four scenarios are modeled, and then the optimal equilibrium of manufacturer Stackelberg games are analyzed.

TABLE 2. Decision variables.

$p(p_i)$	Retail price (of retailer i)
$w(w_i)$	Wholesale price (for retailer i)
$m(m_i)$	Profit margin (of retailer i)
A	Global advertising expenditures
A_i	Local advertising expenditures of retailer i
t_i	Cooperative Advertising participation rate for retailer i

TABLE 3. Notations.

Π_M	The manufacturer's profit
Π_{R_i}	The retailer ith's profit
Π_S	Total channel's profit
$D_i(p_i, A_i, A)$	The retailer ith's demand function (depend on National Branding, Cooperative promotion, and retail price)
γ_i	A distinct part of the Initial base demand for retailer i
α	The constant part of the Initial base demand
β	The intensity of the saturation effect
k	effect of local advertising in consist of global advertising

4. Manufacturer Stackelberg game models

In this section, we model the decision process of supply chain members in four pricing structures, with the manufacturer as the leader and the retailers as the followers. We assume that the manufacturer holds the channel leadership, *i.e.*, he considers retailers' reactions and takes his actions in pricing, global advertising and suggests participation rate to the retailers and also the reaction of the following retailers is taken into consideration. After that, the retailers have to set their own decision variables including local advertising expenditure and maybe retail price. To determine the Stackelberg equilibrium by backward induction, we first solve the retailer i's optimal problem when the manufacturer's decision variables are given. The mathematical model of each scenario is presented and solved as follows.

4.1. Retailer determiner with Different retail price and the same wholesale price (RDS)

In this scenario, the manufacturer determines the same wholesale price for both retailers (w), global advertising cost (A) and cooperation rates (t_i). While the retailer i determines the cost of local advertising (A_i) and his margin of profit (m_i). Therefore, retail prices (p_i) will be different in this scenario ($p_i = w + m_i$). Hence, the models of this scenario will be in the form of (4.1) and (4.2).

$$\Pi_M(w_i, A, t_i) = w \cdot \sum_{i=1}^{n} \gamma_i(\alpha - \beta(w + m_i) + A + k \cdot A_i) - \sum_{i=1}^{n} t_i \cdot A_i^2 - A^2 \tag{4.1}$$

$$\Pi_{R_i}(m_i, A_i) = m_i \gamma_i(\alpha - \beta(w + m_i) + A + k \cdot A_i) - (1 - t_i)A_i^2, \quad i = 1, 2. \tag{4.2}$$

Theorem 4.1. *Objective function (4.1) is a concave function with respect to w, A and t_i. And objective function (4.2) is a concave function with respect to m_i and A_i*

Proposition 4.2. *Due to the Theorem 4.1 we can set the partial first order derivatives $\sigma\Pi_{R_i}/\sigma m_i$ and $\sigma\Pi_{R_i}/\sigma A_i$ to zero and by solving the resulted system of equations, the optimal variables for each retailer is* *obtained as follows,*

$$m_i = \frac{2(1-t_i)(\alpha - \beta w + A)}{4\beta(1-t_i) - \gamma_i k^2}, \qquad i = 1,2 \tag{4.3}$$

$$A_i = \frac{\gamma_i k(\alpha - \beta w + A)}{4\beta(1-t_i) - \gamma_i k^2}, \qquad i = 1,2. \tag{4.4}$$

Proposition 4.3. *Due to the Theorem 4.1 we can set the partial first order derivatives $\sigma\Pi_M/\sigma w$, $\sigma\Pi_M/\sigma A_i$ and $\sigma\Pi_M/\sigma t_i$ to zero and by solving the resulted system of equations, the optimal variables for Manufacturer is obtained as follows,*

$$w = \frac{\alpha}{\beta}\ \frac{64\beta^2\sum\gamma_i - 12\beta k^2\sum\gamma_i^2 - 32\beta k^2\prod\gamma_i + 3k^4\sum\gamma_i^2\gamma_{3-i}}{128\beta^2\sum\gamma_i 4\beta(9k^2+4)\sum\gamma_i^2 - 32\beta(2k^2+1)\prod\gamma_i + (9k^4+8k^2)\sum\gamma_i^2\gamma_{3-i}} \tag{4.5}$$

$$A = \frac{\alpha}{\beta}\ \frac{4\beta\sum\gamma_i - 2k^2\prod\gamma_i}{128\beta^2\sum\gamma_i 4\beta(9k^2+4)\sum\gamma_i^2 - 32\beta(2k^2+1)\prod\gamma_i + (9k^4+8k^2)\sum\gamma_i^2\gamma_{3-i}} \tag{4.6}$$

$$t_i = \frac{(\gamma_i k^2 - 4\beta)}{4\beta} - \frac{w(\gamma_i k^2 - 4\beta)}{(\alpha + A + \beta w)}, \quad i = 1,\ldots,n. \tag{4.7}$$

4.2. Retailer determiner with Different retail price and Different wholesale price (RDD)

In this scenario, the manufacturer offers different wholesale prices to each retailer (w_i), sets global advertising costs (A) and cooperation rates (t_i), while the retailer i determines the cost of local advertising (A_i) and margin of profit (m_i) so retail prices will be different in this scenario ($p_i = w + m_i$). Therefore, the models of this scenario will be in the form of (4.8) and (4.9).

$$\Pi_M(w_i, A, t_i) = \sum_{i=1}^{n} w_i \cdot \gamma_i(\alpha - \beta(w_i + m_i) + A + k\cdot A_i) - \sum_{i=1}^{n} t_i, \cdot A_i^2 - A^2 \tag{4.8}$$

$$\Pi_{R_i}(m_i, A_i) = m_i\gamma_i(\alpha - \beta(w_i + m_i) + A + k\cdot A_i) - (1-t_i)A_i^2, \quad i = 1,2. \tag{4.9}$$

Theorem 4.4. *Objective function (4.8) is a concave function with respect to w_i, A and t_i. And objective function (4.9) is a concave function with respect to m_i and A_i Proof. See the Appendix A.*

Proposition 4.5. *Due to the Theorem 4.4, we can set the partial first order derivatives $\sigma\Pi_{R_i}/\sigma m_i$ and $\sigma\Pi_{R_i}/\sigma A_i$ to zero and by solving the resulted system of equations, the optimal variables for each retailer is obtained as follows,*

$$m_i = \frac{2(1-t_i)(\alpha + A - \beta w_i)}{4\beta(1-t_i) - k^2\gamma_i}, \qquad i = 1,2 \tag{4.10}$$

$$A_i = \frac{k\gamma_i(\alpha + A - \beta w_i)}{4\beta(1-t_i) - k^2\gamma_i}, \qquad i = 1,2. \tag{4.11}$$

Proposition 4.6. *Due to the Theorem 4.4 we can set the partial first order derivatives $\sigma\Pi_M/\sigma w_i$, $\sigma\Pi_M/\sigma A$ and $\sigma\Pi_M/\sigma t_i$ to zero and by solving the resulted system of equations, the optimal variables for Manufacturer is obtained as follows,*

$$w_i = \frac{27\alpha k^4\gamma_i\gamma_{3-i} - 96\alpha\beta k^4\gamma_i - 144\alpha\beta k^2\gamma_{3-i} + 512\alpha\beta^2}{81k^4\beta\gamma_i\gamma_{3-i} + 72k^2\beta\gamma_i\gamma_{3-i} + 1024\beta^3 - 288\beta^2k^2\sum\gamma_i - 128\beta^2\sum\gamma_i}, \quad i = 1,2 \tag{4.12}$$

$$A = \frac{128\alpha\beta\sum\gamma_i - 72\alpha k^2\gamma_i\gamma_{3-i}}{81k^4\beta\gamma_i\gamma_{3-i} + 72k^2\beta\gamma_i\gamma_{3-i} + 1024\beta^3 - 288\beta^2k^2\sum\gamma_i - 128\beta^2\sum\gamma_i} \tag{4.13}$$

$$t_i = \frac{(\gamma_i k^2 - 4\beta)}{4\beta} - \frac{w_i(\gamma_i k^2 - 4\beta)}{(\alpha + A + \beta w_i)}, \quad i = 1,2. \tag{4.14}$$

4.3. Manufacturer determiner with the Same retail price and the Same wholesale price (MSS)

In this scenario, the retailer i supplies the product to the customer at a price set by the manufacturer and determines only the amount of local advertising costs. The manufacturer determines the same wholesale price (w) and the same retail price (p). So, he sets the constant margin of the retailer profit ($m = p - w$). He sets global advertising costs (A) and cooperation rates (t_i). Therefore, the models of this scenario will be in the form of (4.15) and (4.16)

$$\Pi_M(m, w, A, t_i) = w \cdot \sum_{i=1}^{n} \gamma_i(\alpha - \beta(w + m) + A + k \cdot A_i) - \sum_{i=1}^{n} t_i \cdot A_i^2 - A^2 \tag{4.15}$$

$$\Pi_{R_i}(A_i) = m_i\gamma_i(\alpha - \beta(w + m) + A + k.A_i) - (1 - t_i)A_i^2, \quad i = 1, 2. \tag{4.16}$$

Theorem 4.7. *Objective function (4.15) is a concave function with respect to w,m, A and t_i. And objective function (4.16) is a concave function with respect to A_i*

Proof. See the Appendix A. □

Proposition 4.8. *Due to the Theorem 4.7 we can set the partial first order derivative $\sigma\Pi_{R_i}/\sigma A_i$ to zero, and by solving the resulted system of equations, the optimal variables for each retailer are obtained as follows,*

$$A_i = \frac{2\alpha\gamma_i k(4\beta\sum\gamma_i - k^2\sum\gamma_i^2)}{16\beta^2\sum\gamma_i - 4\beta k^2\sum\gamma_i^2 - k^2\sum\gamma_i^3 - k^2\prod\gamma_i\sum\gamma_i}, \quad i = 1, 2. \tag{4.17}$$

Proposition 4.9. *Due to the Theorem 4.7 we can set the partial first order derivatives $\sigma\Pi_M/\sigma w$, $\sigma\Pi_M/\sigma m$, $\sigma\Pi_M/\sigma A$ and $\sigma\Pi_M/\sigma t_i$ to zero and by solving the resulted system of equations, the optimal variables for Manufacturer are obtained as follows,*

$$w = \frac{2\alpha k^2\sum\gamma_i^2}{16\beta^2\sum\gamma_i - 4\beta k^2\sum\gamma_i^3 - k^2\prod\gamma_i\sum\gamma_i} \tag{4.18}$$

$$m = \frac{16\alpha\beta\sum\gamma_i - 4\alpha k^2\sum\gamma_i^2}{16\beta^2\sum\gamma_i - 4\beta k^2\sum\gamma_i^2 - k^2\sum\gamma_i^3 - k^2\prod\gamma_i\sum\gamma_i} \tag{4.19}$$

$$A = \frac{\alpha k^2(\sum\gamma_i^3 + \prod\gamma_i\sum\gamma_i)}{16\beta^2\sum\gamma_i - 4\beta k^2\sum\gamma_i^2 - k^2\sum\gamma_i^3 - k^2\prod\gamma_i\sum\gamma_i} \tag{4.20}$$

$$t_i = 0, \quad i = 1, 2. \tag{4.21}$$

4.4. Manufacturer determiner with the Same retail price and Different wholesale price (MSD)

In this scenario, the retailer i supplies the product at a price set by the manufacturer to the customer and determines only the amount of local advertising costs. The manufacturer determines global advertising costs (A), cooperation rates (t_i) and the same retail price (p). Considering the different wholesale price, the manufacturer offers a different margin for retailers ($m_i = p - w_i$). Therefore, the models of this scenario will be in the form of (4.22) and (4.23)

$$\Pi_M(p, w_i, A, t_i) = \sum_{i=1}^{n} w_i \cdot \gamma_i(\alpha - \beta p + A + k \cdot A_i) - \sum_{i=1}^{n} t_i \cdot A_i^2 - A^2 \tag{4.22}$$

$$\Pi_{R_i}(A_i) = (p - w_i)\gamma_i(\alpha - \beta p + A + k \cdot A_i) - (1 - t_i)A_i^2, \quad i = 1, 2. \tag{4.23}$$

Theorem 4.10. *Objective function (4.22) is a concave function with respect to p, w_i, A and t_i And objective function (4.23) is concave with respect to A_i.*

Proof. See the Appendix A. □

Proposition 4.11. *Due to the Theorem 4.10 we can set the partial first order derivatives $\sigma\Pi_{R_i}/\sigma A_i$ to zero, and by solving the resulted system of equations, the optimal variables for each retailer are obtained as follows,*

$$A_i = \frac{k\gamma_i(p - w_i)}{2(1 - t_i)}, \quad i = 1, 2. \tag{4.24}$$

Proposition 4.12. *Due to the Theorem 4.10 we can set the partial first order derivatives $\sigma\Pi_M/\sigma w_i$, $\sigma\Pi_M/\sigma m$, $\sigma\Pi_M/\sigma A$ and $\sigma\Pi_M/\sigma t_i$ to zero and by solving the resulted system of equations, the optimal variables for Manufacturer is obtained as follows,*

$$w_i = \frac{2\alpha k^2\gamma_i^2 + 2\alpha k^2\gamma_i\gamma_{3-i} + 16\alpha\beta\gamma_{3-i} - 16\alpha\beta\gamma_i}{32\beta^2\gamma_i - 4\beta k^2\gamma_i^2 - 4\beta k^2\gamma_i\gamma_{3-i} - k^2\gamma_i^3 - 2k^2\gamma_i^2\gamma_{3-i} - k^2\gamma_{3-i}^2\gamma_i} \tag{4.25}$$

$$p = \frac{32\alpha\beta - 2\alpha k^2\sum\gamma_i}{32\beta^2 - 4\beta k^2\sum\gamma_i - k^2\sum\gamma_i^2 - 2k^2\prod\gamma_i} \tag{4.26}$$

$$A = \frac{\alpha k^2(\sum\gamma_i)^2}{32\beta^2 - 4\beta k^2\sum\gamma_i - k^2\sum\gamma_i^2 - 2k^2\prod\gamma_i} \tag{4.27}$$

$$t_i = 0, \quad i = 1, 2. \tag{4.28}$$

5. Cooperative games

In the case of cooperation, it is assumed that supply chain members do not try to maximize their own profits, but the whole channel profit. It is also assumed that the information can be shared between the channel members. In this situation, any national or local issues facing the manufacturer or retailers is coordinated so that it will be possible that players' individual strategies and utilities are not defined in the cooperation. This necessitates the application of other methods like bargaining games to obtain a fair distribution of utility between the coordinated players.

As the agreement on wholesale price is something which should be drowned between channel members (Manufacturer and retailers), it will not affect the function of the coordinated supply chain which leads to the same problem for RDD and RDS scenarios (so-called RD). Also, the coordinated supply chain will be the same for MSS and MSD scenarios (so-called MS)

5.1. Retailer determiner with the different retail price (RD)

Consider now a situation where all members are prepared to cooperate and pursue the optimal decisions. Therefore the model of cooperative RD will be in the form of (5.1)

$$\prod_s(p_i, A, A_i) = \sum_{i=1}^{2} p_i \cdot \gamma_i(\alpha - \beta p_i + A + k \cdot A_i) - \sum_{i=1}^{2} A_i^2 - A^2. \tag{5.1}$$

Theorem 5.1. *Supply chain objective (5.1) is a concave function with respect to variables p_i, A, A_i.*

Proof. See the Appendix A. □

Proposition 5.2. *Due to the Theorem 5.3 we can set the partial first order derivatives $\sigma\Pi_S/\sigma A$ $\sigma\Pi_S/\sigma p_i$ and $\sigma\Pi_S/\sigma A_i$ to zero and by solving the resulted system of equations, the optimal variables for each retailer is obtained as follows,*

$$p_i = \frac{2\alpha(k^2\gamma_{3-i} - 4\beta)}{4\beta(\gamma_1+\gamma_2)(1+k^2) - k^2(2+k^2)\gamma_1\gamma_2 - 16\beta^2}, \quad i = 1,2 \tag{5.2}$$

$$A_i = \frac{\alpha k\gamma_i(k^2\gamma_{3-i} - 4\beta)}{4\beta(\gamma_1+\gamma_2)(1+k^2) - k^2(2+k^2)\gamma_1\gamma_2 - 16\beta^2}, \quad i = 1,2 \tag{5.3}$$

$$A = \frac{2\alpha(k^2\gamma_1\gamma_2 - 2\beta(\gamma_1+\gamma_2))}{4\beta(\gamma_1+\gamma_2)(1+k^2) - k^2(2+k^2)\gamma_1\gamma_2 - 16\beta^2}. \tag{5.4}$$

5.2. Manufacturer determiner with the same retail price (MS)

Consider now a situation where all members are prepared to cooperate and pursue the optimal decisions. Therefore the model of cooperative MS will be in the form of (5.5)

$$\prod_s(p, A, A_i) = \sum_{i=1}^{2} p_i \cdot \gamma_i(\alpha - \beta p_i + A + k\cdot A_i) - \sum_{i=1}^{2} A_i^2 - A^2. \tag{5.5}$$

Theorem 5.3. *Supply chain objective (5.5) is a concave function with respect to variables p, A, A_i.*

Proof. See the Appendix A. □

Proposition 5.4. *Due to the Theorem 4.4 we can set the partial first order derivatives $\sigma\Pi_S/\sigma A$, $\sigma\Pi_S/\sigma p$ and $\sigma\Pi_S/\sigma A_i$ to zero and by solving the resulted system of equations, the optimal variables for each retailer is obtained as follows,*

$$p = \frac{2\alpha(\gamma_i+\gamma_2)}{4\beta(\gamma_1+\gamma_2) - (\gamma_1^2+\gamma_2^2)(1+k^2) - 2\gamma_1\gamma_2} \tag{5.6}$$

$$A_i = \frac{\alpha k\gamma_i(\gamma_1+\gamma_2)}{4\beta(\gamma_1+\gamma_2) - (\gamma_1^2+\gamma_2^2)(1+k^2) - 2\gamma_1\gamma_2}, \quad i = 1,2 \tag{5.7}$$

$$A = \frac{\alpha(\gamma_i+\gamma_2)^2}{4\beta(\gamma_1+\gamma_2) - (\gamma_1^2+\gamma_2^2)(1+k^2) - 2\gamma_1\gamma_2}. \tag{5.8}$$

6. Numerical example

In the previous sections, we considered two different forms of supply chain behavior. Though we were able to determine the two equilibriums analytically, the resulted expressions are too complicated for a meaningful interpretation. Hence, we apply a Numerical example to get insights into the effects of multiple retailers in supply chains and also retail prices, advertising expenditures, and profits in the framework of the two examined games. We formulate each of these problems as nonlinear programming problems and solve using Matlab12 (Tabs. 4 and 5).

In RDS scenario we may have different participation rate with respect to parameters. The effects of price sensitivity β on participation rate t are illustrated in Figure 2. We can see that an increase of β is accompanied by a more similar participation rates. Figure 3 compares the resulting participation rate in the initial demand of retailer 1 ($\gamma1$) (Tabs. 6 and 7).

Figures 4–6 display the players' profit function in two scenarios with respect to the β. Another important parameter is the initial demand of retailers (γ_i). Figures 7–9 show that first retailer's initial demand (γ_i) affect the optimal supply chain profits.

Table 4. Numerical example parameters.

Parameter	γ_1	γ_2	α	β	k
Value	73.16	59.99	10 000	88	1.1

Table 5. Numerical example.

	Model:	RDS	RDD	MSS	MSD
	Π_{R_1}	11 642 781	11 365 171	10 711 834	15 604 056
Manufacturer	Π_{R_2}	8 740 519	9 010 093	4 854 826	2 112 774
Stackelberg	Π_M	34 378 436	34 386 176	7 382 668	4 945 466
	Π_S	54 761 736	54 761 736	22 949 328	22 662 296
Cooperative game	Π_S^*	96 994 952	96 994 952	96 382 024	96 382 024

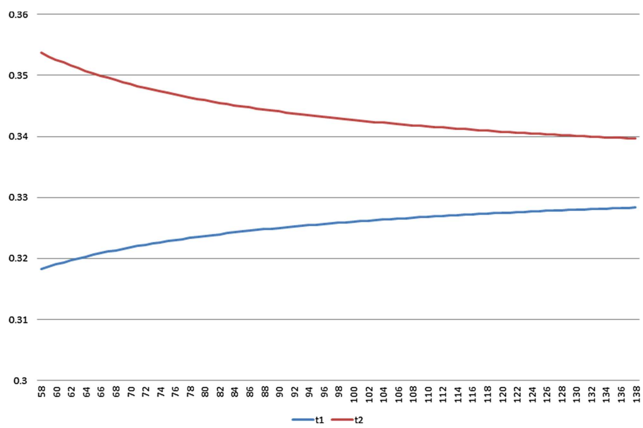

Figure 2. Participation rates with respect to β (RDS).

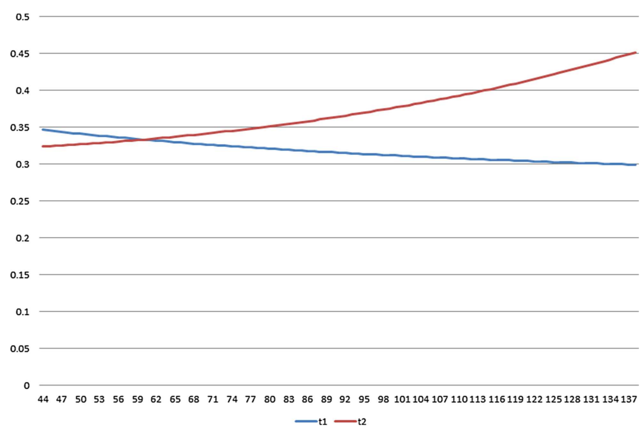

Figure 3. Participation rates with respect to $\gamma1$ (RDS).

Table 6. Pricing variables.

	Parameter	RDS	RDD	MSS	MSD
Manufacturer	p_1	139.01	139.64	146.99	147.48
Stackelberg	p_2	134.48	133.75	146.99	147.48
Cooperative	p_1^*	149.54	149.54	145.45	145.45
game	p_2^*	141.01	141.01	145.45	145.45

Table 7. Advertising variables.

	Parameter	RDS	RDD	MSS	MSD
Manufacturer Stackelberg	t_1	0.3248	0.33	0	0
	t_2	0.3444	0.33	0	0
	A_1	3199	3214	5159	5752
	A_2	2473	2460	4230	3622
	A	3437	3439	1283	1265
Cooperative game	A_1^*	6017	6017	5853	5853
	A_2^*	4652	4652	4799	4799
	A^*	9700	9700	9683	9683

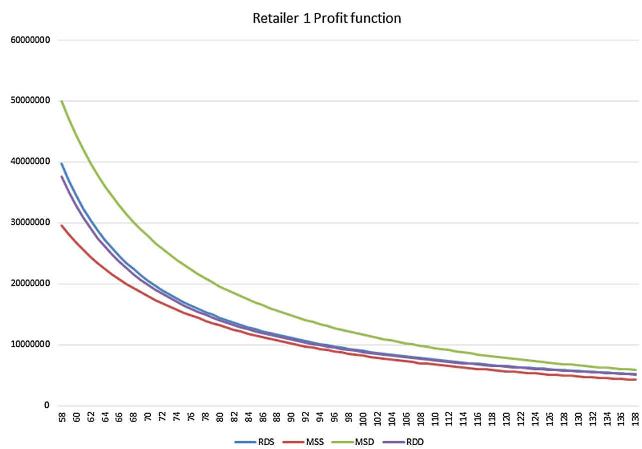

Figure 4. Retailer 1's profit functions with respect to β.

7. Conclusion and managerial insight

Can pricing structure affect supply chain member's profit? Our model gives a novel and strategic response to this question. Our model investigates the strategic effect of the pricing structure on the performances of all members. We have used a game-theoretical model to show how pricing decision in different situation affects supply chain profit.

As an important consideration, when a manufacturer decides whether to determine or relax retail price, he has a profitability significant impact on supply chain. Considering that price determiner can also influence all members' profit, this paper has investigated the cooperative advertising problem by taking pricing effect into the consideration.

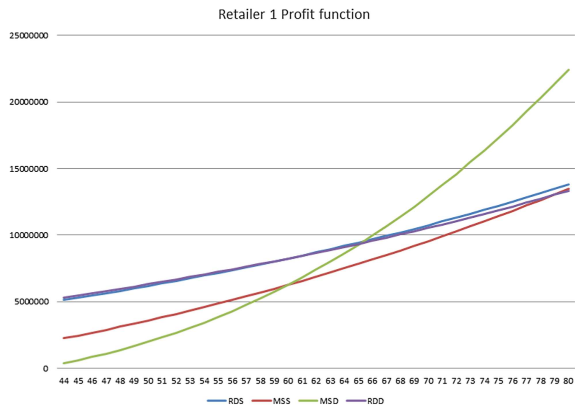

Figure 5. Retailer 1's profit functions with respect to γ_1.

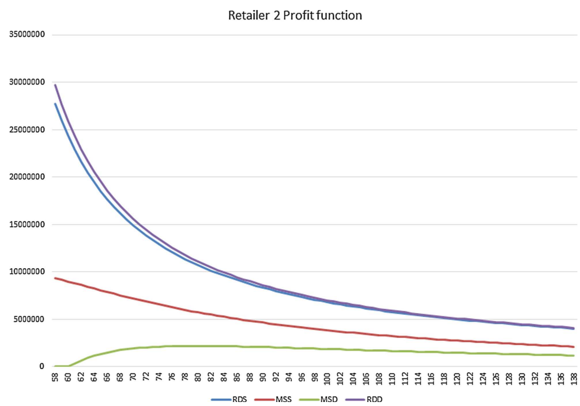

Figure 6. Retailer 2's profit functions with respect to β.

The main results of this paper include the following: In scenarios where the retail price is determined identically by the manufacturer (MSD, MSS), under decentralized conditions, manufacturer profits are sharply reduced. Therefore, it's best to set retail prices by retailers. Contrarily, if the manufacturer decides to set retail price own and obliges retailers to comply with this rule, it's better for him to provide the same wholesale price to retailers and use the MMS scenario. Because in an MSD scenario by offering lower prices to the larger retailer, the weaker retailer will be eliminated, and then, manufacturer profits will also decrease. In any case, the weaker retailer (retailer 2) prefer to set retail prices on their own. The conditions that the manufacturer would offer the same retail price and different wholesale prices (MSD) will be in the interest of the larger retailer (retailer 1). As expected, the overall supply chain profit in centralized conditions is more than decentralized. Notice that the profits of the centralized supply chain members are higher in RD scenario in comparison with MS. In RD* scenarios manufacturer profit will be higher when he considers different wholesale prices (RDD). Within the members of the retailer echelon, the profit of the larger retailer (retailer 1) will be reduced and the weaker retailer's profit will be added.

There are several possible valuable extensions of this work include the following. Firstly, the system that consists of advertising and retail price competition can bring the issue closer to the real world. Therefore, if two retailers are active in a common marketplace, their pricing and advertising effort effect on each other so that an additional improvement would be a change in the sales response function. The demand function of

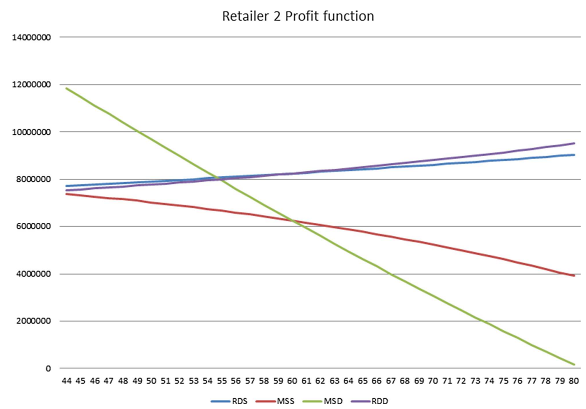

FIGURE 7. Retailer 2's profit functions with respect to γ_1.

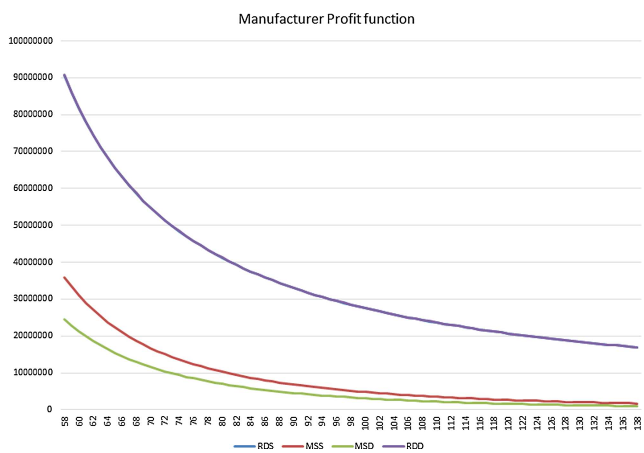

FIGURE 8. Manufacturer's profit functions with respect to β.

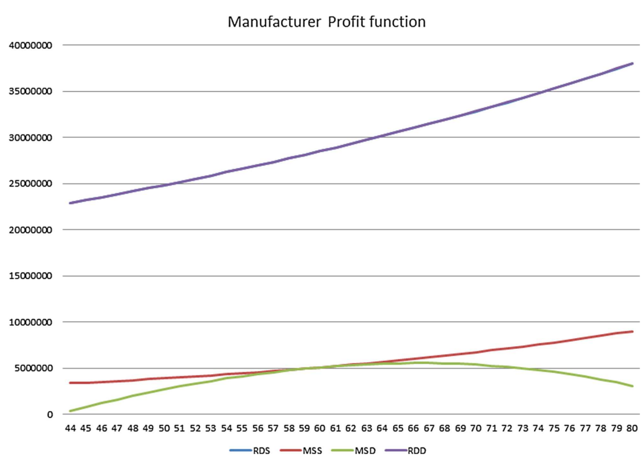

FIGURE 9. Manufacturer's profit functions with respect to γ_1.

each retailer in this study is independent of other ones. Moreover, the effect of pricing and advertising on a competitor's demand function would be a valuable extension. Secondly, extensions of supply chain structure could be considered to investigate the real world situations, dual channel, a coalition of members and online retailer are valuable extensions of the channel structure. Finally, we assume monopolistic manufacturer in the supply chain, and we have taken only downstream competition situation. The consideration of alternative product and both stream competitions can be another important extension of this model.

Appendix A.

Proof of Theorem 4.1. To proof the optimality of the solutions of channel members in RDS, we calculate the Hessian matrix for each profit function, at first for the manufacturer, the second order partial derivatives are calculated, and the Hessian matrix is as follows:

$$H_M = \begin{pmatrix} \frac{\partial^2 \prod_M}{\partial A^2} & \frac{\partial^2 \prod_M}{\partial A \partial w} & \frac{\partial^2 \prod_M}{\partial A \partial t_1} & \frac{\partial^2 \prod_M}{\partial A \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial w \partial A} & \frac{\partial^2 \prod_M}{\partial w^2} & \frac{\partial^2 \prod_M}{\partial w \partial t_1} & \frac{\partial^2 \prod_M}{\partial w \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial t_1 \partial w} & \frac{\partial^2 \prod_M}{\partial t_1 \partial w} & \frac{\partial^2 \prod_M}{\partial t_1^2} & \frac{\partial^2 \prod_M}{\partial t_1 \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial t_2 \partial w} & \frac{\partial^2 \prod_M}{\partial t_2 \partial w} & \frac{\partial^2 \prod_M}{\partial t_2 \partial t_1} & \frac{\partial^2 \prod_M}{\partial t_2^2} \end{pmatrix} = \begin{pmatrix} -2 & \gamma_1+\gamma_2 & 0 & 0 \\ \gamma_1+\gamma_2 & -2\beta(\gamma_1+\gamma_2) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The first principal minor of is negative $H_M^1 = -2$. The second principal minor of H_M is $H_M^2 = (\gamma_1+\gamma_2)(4\beta - (\gamma_1+\gamma_2))$ and is positive if $4\beta \geq \gamma_1+\gamma_2$. And also we have $H_M^3 = 0$ and $H_M^4 = 0$. So, the principal minors of H_M have alternating algebraic signs at the solution. It means that is negative semi-definite and the profit of Manufacturer is concave at this solution, which is a local maximum. And the Hessian matrix for each retailer is as follows:

$$H_{R_i} = \begin{pmatrix} \frac{\partial^2 \prod_{R_i}}{\partial A_i^2} & \frac{\partial^2 \prod_{R_i}}{\partial A_i \partial m_i} \\ \frac{\partial^2 \prod_{R_i}}{\partial m \partial A_i} & \frac{\partial^2 \prod_{R_i}}{\partial m_i^2} \end{pmatrix} = \begin{pmatrix} -2(1-t_i) & k\gamma_i \\ k\gamma_i & -2\beta\gamma_i \end{pmatrix}.$$

The first principal minor of is negative $H_{R_i}^1 = -2(1-t_i)$ because we have $0 \leq t_i \leq 1$. The second principal minor of H_{R_i} is $H_{R_i}^2 = 4\beta\gamma_i(1-t_i) - k^2\gamma_i^2$ and is positive if $4\beta(1-t_i) \geq k^2\gamma_i$ so, the principal minors of H_{R_i} have alternating algebraic signs at the solution. It means that is negative definite and the profit of retailer is concave at this solution, which is a local maximum. □

Proof of Theorem 4.4. To proof the optimality of the solutions of channel members in RDD, we calculate the Hessian matrix for each profit function, at first for the manufacturer, the second order partial derivatives are calculated, and the Hessian matrix is as follows:

$$H_M = \begin{pmatrix} \frac{\partial^2 \prod_M}{\partial A_i^2} & \frac{\partial^2 \prod_M}{\partial A \partial w_1} & \frac{\partial^2 \prod_M}{\partial A \partial w_2} & \frac{\partial^2 \prod_M}{\partial A \partial t_1} & \frac{\partial^2 \prod_M}{\partial A \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial w_1 \partial A} & \frac{\partial^2 \prod_M}{\partial w_1^2} & \frac{\partial^2 \prod_M}{\partial w_1 \partial w_2} & \frac{\partial^2 \prod_M}{\partial w_1 \partial t_1} & \frac{\partial^2 \prod_M}{\partial w_1 \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial w_2 \partial A} & \frac{\partial^2 \prod_M}{\partial w_2 \partial w_1} & \frac{\partial^2 \prod_M}{\partial w_2^2} & \frac{\partial^2 \prod_M}{\partial w_2 \partial t_1} & \frac{\partial^2 \prod_M}{\partial w_2 \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial t_1 \partial A} & \frac{\partial^2 \prod_M}{\partial t_1 \partial w_1} & \frac{\partial^2 \prod_M}{\partial t_1 \partial w_2} & \frac{\partial^2 \prod_M}{\partial t_1^2} & \frac{\partial^2 \prod_M}{\partial t_1 \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial t_2 \partial A} & \frac{\partial^2 \prod_M}{\partial t_2 \partial w_1} & \frac{\partial^2 \prod_M}{\partial t_2 \partial w_2} & \frac{\partial^2 \prod_M}{\partial t_2 \partial t_1} & \frac{\partial^2 \prod_M}{\partial t_2^2} \end{pmatrix} = \begin{pmatrix} -2 & \gamma_1 & \gamma_2 & 0 & 0 \\ \gamma_1 & -2\beta\gamma_1 & 0 & 0 & 0 \\ \gamma_2 & 0 & -2\beta\gamma_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The first principal minor of is negative $H_M^1 = -2$. The second principal minor of H_M is $H_M^2 = \gamma_1(4\beta - \gamma_1)$ and is positive if $4\beta \geq \gamma_1$ and also we have $H_M^3 = 2\beta\gamma_1\gamma_2(\gamma_1+\gamma_2-4\beta)$ and is negative if $4\beta \geq \gamma_1+\gamma_2$. The next principal minor is $H_M^4 = 0$ and $H_M^5 = 0$. So, the principal minors of H_M have alternating algebraic signs at the solution. It means that is negative semi-definite and the profit of Manufacturer is concave at this solution, which is a local maximum. And the Hessian matrix for each retailer is as follows:

$$H_{R_i} = \begin{pmatrix} \frac{\partial^2 \prod_{R_i}}{\partial A_i^2} & \frac{\partial^2 \prod_{R_i}}{\partial A_i \partial m_i} \\ \frac{\partial^2 \prod_{R_i}}{\partial m \partial A_i} & \frac{\partial^2 \prod_{R_i}}{\partial m_i^2} \end{pmatrix} = \begin{pmatrix} -2(1-t_i) & k\gamma_i \\ k\gamma_i & -2\beta\gamma_i \end{pmatrix}.$$

The first principal minor of is negative $H^1_{R_i} = -2(1-t_i)$. The second principal minor of H_M is $H^2_{R_i} = 4\beta\gamma_i(1-t_i) - k^2\gamma_i^2$ and is positive if $4\beta(1-t_i) \geq k^2\gamma_i$ so, the principal minors of H_{R_i} have alternating algebraic signs at the solution. It means that is negative definite and the profit of retailer is concave at this solution, which is a local maximum. □

Proof of Theorem 4.7. To proof the optimality of the solutions of channel members in MSS, we calculate the Hessian matrix for each profit function, at first for the manufacturer, the second order partial derivatives are calculated, and the Hessian matrix is as follows:

$$H_M = \begin{pmatrix} \frac{\partial^2 \prod_M}{\partial A^2} & \frac{\partial^2 \prod_M}{\partial A \partial w} & \frac{\partial^2 \prod_M}{\partial A \partial w} & \frac{\partial^2 \prod_M}{\partial A \partial t_1} & \frac{\partial^2 \prod_M}{\partial A \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial w \partial A} & \frac{\partial^2 \prod_M}{\partial w^2} & \frac{\partial^2 \prod_M}{\partial w \partial m} & \frac{\partial^2 \prod_M}{\partial w \partial t_1} & \frac{\partial^2 \prod_M}{\partial w \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial m \partial A} & \frac{\partial^2 \prod_M}{\partial m \partial w} & \frac{\partial^2 \prod_M}{\partial m^2} & \frac{\partial^2 \prod_M}{\partial m \partial t_1} & \frac{\partial^2 \prod_M}{\partial m \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial t_1 \partial A} & \frac{\partial^2 \prod_M}{\partial t_1 \partial w} & \frac{\partial^2 \prod_M}{\partial t_1 \partial m} & \frac{\partial^2 \prod_M}{\partial t_1^2} & \frac{\partial^2 \prod_M}{\partial t_1 \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial t_2 \partial A} & \frac{\partial^2 \prod_M}{\partial t_2 \partial w} & \frac{\partial^2 \prod_M}{\partial t_2 \partial m} & \frac{\partial^2 \prod_M}{\partial t_2 \partial t_1} & \frac{\partial^2 \prod_M}{\partial t_2^2} \end{pmatrix} = \begin{pmatrix} -2 & \sum\gamma_i & \gamma_2 & 0 & 0 \\ \sum\gamma_i & -2\beta\sum\gamma_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The first principal minor of is negative $H^1_M = -2$. The second principal minor of H_M is $H^2_M = \sum\gamma_i(4\beta - \sum\gamma_i)$ and is positive if $4\beta \geq \sum\gamma_i$ and also we have $H^3_M = 0$ $H^4_M = 0$ $H^5_M = 0$ so, the principal minors of H_M have alternating algebraic signs at the solution. It means that is negative semi-definite and the profit of Manufacturer is concave at this solution, which is a local maximum. Given that H_{R_i} have one variable and second derivative of H_{R_i} is negative, so the profit of Retailers is concave at this solution $H^1_{R_i} = -2(1-t_i)$. □

Proof of Theorem 4.10. To proof the optimality of the solutions of channel members in MSD, we calculate the Hessian matrix for each profit function, at first for the manufacturer, the second order partial derivatives are calculated, and the Hessian matrix is as follows:

$$H_M = \begin{pmatrix} \frac{\partial^2 \prod_M}{\partial A^2} & \frac{\partial^2 \prod_M}{\partial A \partial w_1} & \frac{\partial^2 \prod_M}{\partial A \partial w_2} & \frac{\partial^2 \prod_M}{\partial A \partial p} & \frac{\partial^2 \prod_M}{\partial A \partial t_1} & \frac{\partial^2 \prod_M}{\partial A \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial w_1 \partial A} & \frac{\partial^2 \prod_M}{\partial w_1^2} & \frac{\partial^2 \prod_M}{\partial w_1 \partial w_2} & \frac{\partial^2 \prod_M}{\partial w_1 \partial p} & \frac{\partial^2 \prod_M}{\partial w_1 \partial t_1} & \frac{\partial^2 \prod_M}{\partial w_1 \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial w_2 \partial A} & \frac{\partial^2 \prod_M}{\partial w_2 \partial w_1} & \frac{\partial^2 \prod_M}{\partial w_2^2} & \frac{\partial^2 \prod_M}{\partial w_2 \partial p} & \frac{\partial^2 \prod_M}{\partial w_2 \partial t_1} & \frac{\partial^2 \prod_M}{\partial w_2 \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial p \partial A} & \frac{\partial^2 \prod_M}{\partial p \partial w_1} & \frac{\partial^2 \prod_M}{\partial p \partial w_2} & \frac{\partial^2 \prod_M}{\partial p^2} & \frac{\partial^2 \prod_M}{\partial p \partial t_1} & \frac{\partial^2 \prod_M}{\partial p \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial t_1 \partial A} & \frac{\partial^2 \prod_M}{\partial t_1 \partial w_1} & \frac{\partial^2 \prod_M}{\partial t_1 \partial w_2} & \frac{\partial^2 \prod_M}{\partial t_1 \partial p} & \frac{\partial^2 \prod_M}{\partial t_1^2} & \frac{\partial^2 \prod_M}{\partial t_1 \partial t_2} \\ \frac{\partial^2 \prod_M}{\partial t_2 \partial A} & \frac{\partial^2 \prod_M}{\partial t_2 \partial w_1} & \frac{\partial^2 \prod_M}{\partial t_2 \partial w_2} & \frac{\partial^2 \prod_M}{\partial t_2 \partial p} & \frac{\partial^2 \prod_M}{\partial t_2^2 \partial t_1} & \frac{\partial^2 \prod_M}{\partial t_2^2} \end{pmatrix} = \begin{pmatrix} -2 & \gamma_1 & \gamma_2 & 0 & 0 \\ \gamma_1 & -2\beta\gamma_1^2 & 0 & 0 & 0 \\ \gamma_2 & 0 & -2\gamma_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The first principal minor of is negative $H^1_M = -2$. The second principal minor of H_M is $H^2_M = 3\gamma_1^3$ and the third one is $H^3_M = -4\gamma_1^2\gamma_2^2$. And also we have $H^4_M = 0$, $H^5_M = 0$ and $H^6_M = 0$ so, the principal minors of H_M have alternating algebraic signs at the solution. It means that is negative semi-definite and the profit of Manufacturer is concave at this solution, which is a local maximum. Given that H_{R_i} have one variable and second derivative of H_{R_i} is negative, therefore the profit of Retailers is concave at this solution $H^1_{R_i} = -2(1-t_i)$. □

Proof of Theorem 5.1. To proof the optimality of the solutions of centralized supply chain profit function, we calculate the Hessian matrix for RD cooperative game's profit function, the second order partial derivatives are calculated, and the Hessian matrix is as follows:

$$H_{\mathrm{SC}}=\begin{pmatrix}\frac{\partial^2\prod_{\mathrm{SC}}}{\partial A^2} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A\partial A_1} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A\partial A_2} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A\partial p_1} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A\partial p_2}\\ \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_1\partial A} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_1^2} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_1\partial A_2} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_1\partial p_1} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_1\partial P_2}\\ \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_2\partial A} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_2\partial A_1} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_2^2} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_2\partial p_1} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_2\partial p_2}\\ \frac{\partial^2\prod_{\mathrm{SC}}}{\partial p_1\partial A} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial p_1\partial A_1} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial p_1\partial A_2} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial p_1^2} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial p_1\partial p_2}\\ \frac{\partial^2\prod_{\mathrm{SC}}}{\partial p_2\partial A} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial p_2\partial A_1} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial p_2\partial A_2} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial p_2\partial p_1} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial p_2^2}\end{pmatrix}=\begin{pmatrix}-2 & 0 & 0 & \gamma_1 & \gamma_2\\ 0 & -2 & 0 & K\gamma_1 & 0\\ 0 & 0 & -2 & 0 & 0\\ \gamma_1 & K\gamma_1 & 0 & -2\beta\gamma_1 & 0\\ \gamma_2 & 0 & 0 & 0 & -2\beta\gamma_2\end{pmatrix}.$$

The first principal minor of is negative $H^1_{\mathrm{SC}}=-2$. The next principal minors of H_{SC} is $H^2_{\mathrm{SC}}=4$ and $H^3_{\mathrm{SC}}=-8$. And also we have $H^4_{\mathrm{SC}}=4\gamma_1(4\beta-(1+k)\gamma_1)$ $H^5_{\mathrm{SC}}=4\gamma_1(4\beta-(1+k)\gamma_1)$ so, the principal minors of H_{SC} have alternating algebraic signs at the solution. It means that is negative definite and the profit of supply chain is concave at this solution, which is a local maximum. □

Proof of Theorem 5.3. To proof the optimality of the solutions of centralized supply chain profit function, we calculate the Hessian matrix for MS cooperative game's profit function, the second order partial derivatives are calculated, and the Hessian matrix is as follows:

$$H_{\mathrm{SC}}=\begin{pmatrix}\frac{\partial^2\prod_{\mathrm{SC}}}{\partial A^2} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A\partial A_1} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A\partial A_2} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A\partial p}\\ \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_1\partial A} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_1^2} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_1\partial A_2} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_1\partial p}\\ \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_2\partial A} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_2\partial A_1} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_2^2} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial A_2\partial p}\\ \frac{\partial^2\prod_{\mathrm{SC}}}{\partial p\partial A} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial p\partial A_1} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial p\partial A_2} & \frac{\partial^2\prod_{\mathrm{SC}}}{\partial p^2}\end{pmatrix}=\begin{pmatrix}-2\beta\sum\gamma_i & \sum\gamma_i & k\gamma_1 & k\gamma_2\\ \sum\gamma_i & -2 & 0 & 0\\ k\gamma_1 & 0 & -2 & 0\\ k\gamma_2 & 0 & 0 & -2\end{pmatrix}.$$

The first principal minor of is negative $H^1_{\mathrm{SC}}=-2\beta\sum\gamma_i$. The second principal minors of H_{SC} is $H^2_{\mathrm{SC}}=4\beta\sum\gamma_i-(\sum\gamma_i)^2$ and is positive if $4\beta\geq\sum\gamma_i$. The third principal minor is $H^3_{\mathrm{SC}}=-8\beta\sum\gamma_i+2k^2\gamma_1^2+2(\sum\gamma_i)^2$, and it is negative if $4\beta\geq\sum\gamma_i+k^2\gamma_1^2/\sum\gamma_i$. And also we have $H^4_{\mathrm{SC}}=-2\beta\sum\gamma_i(-8)-\sum\gamma_i(4\sum\gamma_i)+k\gamma_1(-4k\gamma_1)-k\gamma_2(4k\gamma_2)$ and is positive if $4\beta\geq\sum\gamma_i+k^2(\gamma_1^2+\gamma_2^2)/\sum\gamma_i$ so, the principal minors of H_{SC} have alternating algebraic signs at the solution. It means that is negative definite and the profit of supply chain is concave at this solution, which is a local maximum. □

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