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# OPTIMAL CONTROL OF AN INVENTORY SYSTEM UNDER WHOLE SALE PRICE CHANGES

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**Abstract.** Nowadays business owners use lots of incentive schemes to make customers buy more products. In this paper optimal ordering policy for customers is obtained when the manufacturer increases the purchasing price or temporary decreases it. Offering a special sale from the manufacturer is probabilistic and shortage occurs as partial backlogging. In this paper, the initial level of inventory when the purchasing price changes is not equal to zero. With respect to the assumptions, the amount of special order quantity, the shortage quantity, and the expected total saving from making an special order is optimized for the customer. The optimal amount of decision variables are obtained by maximizing the expected total saving function and a closed-form solution is derived. Several numerical examples are solved and sensitivity analysis is performed to prove the applicability of the proposed model. Finally, the impact of some parameters of the model including the demand, the probability of making a special order, the future prices, and the initial inventory is investigated. Optimal ordering policy for the customers is obtained in cases when an announced price increase occurs and when the prices temporarily decrease.

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## 1. Introduction

In business environment, sometimes the manufacturers increase the selling price for a variety of reasons. For instance, when the quality of goods improves or the lead time decreases, it is possible that the manufacturers increase the prices because of added services in a specific time in the future and these new prices will remain after occurrence. On the other hand, sometimes the manufacturers or the wholesalers reduce their selling prices to sell more products in a short period of time. Obviously, lowered prices are temporary and will return to the actual prices after a while. These changes in prices can affect the customer's behavior and profit that can be gained from selling the products.

Moreover, in both claimed situations there is a possibility that the manufacturers let the customers or retailers to make a special order which can be beneficial for both stakeholders. In previous inventory models making a special sale assumed to be the right of customers. In other words, the manufacturer always allows them to

*Keywords.* Inventory control, special sale, known price increase, partial backordering, uncertain special order.

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make a special order. However, in this paper, offering a special order from the manufacturer in a specific time is probabilistic.

Known price increase along with special sale are well-developed in many aspects with different assumptions by prior studies. Probably, Naddor [17] and Brown [3] can be mentioned among the first works which has initiated this field of research. Later, Arcelus *et al.* [2] compared the pros and cons of the two mostly used payment reduction plans namely: discounts on the purchasing price and having delay in payment. Sharma [31] investigated an economic production quantity (EPQ) model when purchasing price increase/decrease and there is an opportunity to make a special order prior to price changes. In this model, shortage is allowed as partial backlogging. Cardenas-Barron *et al.* [4] investigated the optimal ordering policy in an economic order quantity (EOQ) model with considering backordered shortage when the purchasing prices temporarily decreases. Taleizadeh *et al.* [33] optimized ordering policy in an EOQ model with partial backordering shortage when the manufacturer temporarily decreases the purchasing prices. Pal *et al.* [21] developed a deterministic, multi item inventory model when the demand for products depends on the purchasing price and the manufacturer offers discount to the buyers. Yu and Hsu [39] determined the economic ordering policy when known price increase occurs for imperfect products when shortage is not allowed. In this paper, the buyer is allowed to make a special order prior to price increment. Furthermore, Taleizadeh and Pentico [32] determined optimal ordering policy for a buyer who faces an announced price increase when shortage is allowed as partial backlogging and the buyer can make a special order prior to price increment. Taleizadeh *et al.* [34] developed an EOQ model for perishable products when back ordered shortage is allowed and the supplier temporarily reduces the purchasing price and let the customer make a special order. Yang and Ouyang [38] determined optimal special order quantity for a customer in an environment where the purchasing price has an impact on the demand rate and an announced price increase occurs. Chung *et al.* [6] investigated a model for deteriorating products when the purchasing price increases and there is an opportunity to make a special order prior to price increment. Sarkar *et al.* [29] developed an EOQ to find the optimal ordering policy for a case when the supplier permit delay payment and temporarily decrease the prices. In this model, they assumed that the shortage is not allowed and the demand is time-dependent. Sarkar and Saren [27] studied a model where the manufacturer offers a full trade-credit to the retailers but retailers offers partial trade-credit to the customers. Sarkar *et al.* [28] developed an inventory control model with variable setup costs and unequal lot sizes by considering environmental issues. Pal *et al.* [23] developed an inventory model for a two-echelon supply chain when the demand for the products depend on the price, quality, promotions of the retailer. They exploited Stackelberg game to further analyze the behavior of different firms in the supply chain. Ouyang *et al.* [19] developed an inventory control model for deteriorating items when the supplier decides to increase the prices in a specific time in the future and the customer can make a special order prior to occurrence of price increment. Xia [37] investigated a retailer's optimal ordering and pricing decision toward temporary decrement in purchasing prices. Kim and Sarkar [13] investigated the optimal ordering policy for a multi-stage production system where the ordering cost depends on the lead time. Sarkar *et al.* [26] developed a model to minimize the costs by deciding on setup costs, quality of the products, and lot sizes. Sarkar *et al.* [30] developed a closed-loop model to control carbon emission in a three-layer supply chain. In another work, Ahmed and Sarkar [1] focused on inventory decision-making problems with considering environmental concerns. For imperfect products, Sarkar *et al.* [25] developed a model with environmental concerns. In this paper, they assumed that the manufacturer offers a trade-credit to the customer. Ganguly *et al.* [9] exploited Stackelberg game to study the effect of different factors such as lead time and premium price on the inventory policy with respect to environmental concerns. Noh *et al.* [18] developed a production inventory control model for a case when the demand depends on the purchasing price and advertisement. They exploited Stackelberg game to solve the model under coordination of the stakeholders. Dey *et al.* [7] studied an inventory control model when the demand depends on the purchasing price with considering discrete setup cost and environmental issues. Iqbal and Sarkar [10] investigated an inventory control model for a time-dependent deteriorating products with considering environmental issues.

In another line of research, some of the researchers have focused on inventory control modes with adding uncertainty to different parameters in the model. Considering this, Ertogral and Rahim [8] assumed that the replenishment intervals are independent and identically distributed random variable and shortage is partial backlogging. Chiang [5] developed an inventory control model when the manufacturer's visit intervals are stochastic and shortage is not allowed. Karimi-Nasab and Konstantaras [11] developed an inventory control model when the buyer can make a special order and inventory replenishment follows a specific probability distribution function. Maihami and Karimi [15] developed a model for deteriorating products to obtain the best policy for both pricing and replenishment when demand is stochastic and depends on purchasing prices. Muthuraman *et al.* [16] studied a continuous inventory model with stochastic lead time and demand. Pal *et al.* [22] investigated pricing and ordering policy in a two-echelon supply chain considering that the products can be defected with a random rate and the defected items can be reworked and sold. Li *et al.* [14] obtained an optimal policy when there is some machining tools with probabilistic lifespan and obtained maximum allowable life span. Karimi-Nasab and Wee [12] developed an inventory control model when the time between two consecutive replenishments is probabilistic and formulated the model when it follows truncated exponential distribution. Taleizadeh *et al.* [35] investigated an inventory control model to optimize the ordering policy when an announced price increase occurs and the customer can make a special order prior to price increment. They assumed that the replenishment intervals is probabilistic and solved the model exponential and uniform cases. Taleizadeh *et al.* [36] developed an inventory control model when the manufacturer permits delay in payment to persuade the customer to purchase more products. They assumed that the replenishment time is probabilistic and the shortage occurs as partial backordering. Pal and Adhikari [20] studied a production inventory control model for imperfect items when the imperfect items can be remanufactured, and a random proportion of the products are imperfect in each cycle.

Considering the foregoing, Table 1 represents the assumptions and contributions of prior studies and the position of the current work.

This paper assumes that the materialization of special order is stochastic in two types of purchasing price changes. First, when the purchasing price increases permanently and second, when the purchasing prices decrease only for short period of time. In both cases, there is possibility that manufacturer let customer to make a special order. Therefore, the novelty of this study is in two points. First, this paper studies price increase and decrease simultaneously which is rarely investigated in prior studies. Second, making a special order is not always possible and it is probabilistic. To the best of our knowledge, this study is the first one that considers both assumptions simultaneously. The remainder of this paper is organized as follows. In Section 2, a problem and assumptions are presented. Section 3 attempts to formulate the problem and to obtain the optimal values for decision variables of the model. In Section 4, numerical examples and sensitivity analysis are performed to demonstrate the impact of fluctuations in different parameters, and finally in Section 5, a conclusion is made with respect to the obtained results.

## 2. Problem definition

Assume that there is a situation in which an announced price increase or temporary price decrease is going to occur at a particular time in near future, $t_S$, to $C_K$ when it increases and $C_S$ in when it decreases. The manufacturer lets the customers to make a special order with the probability of $p$, it is clear that with the probability of $1 - p$, the manufacturer will not offer a special sale. In the first case, with the probability of $p$, at the time of $t_S$, the manufacturer lets the customers to make a special order at the normal price, and claims that the rest of orders will replenish at new prices which are higher than the present prices. However, in the second case the manufacturer lets the customers to make a special order at decreased prices only for a short period of time. In this inventory system, customers can face shortage due to partial backordering, and $\alpha$ percent of shortage will be backordered while the rest of shortage will be a lost sale. The amount of inventory on hand at the time of $t_S$ is $q_S$ which influences the amount of the expected total saving function. The customer follows an EOQ policy both before and after the price variation, while at the time of price changes, $t_S$, it should make decision according to the level of inventory and the expected total saving function. In this paper two possible scenarios can occur:

(a) The manufacturer offers a special sale.

TABLE 1. A skeletal examination of the related papers.

| References | Shortage | | | | Special sale | Known Price Increase | Probabilistic replenishment intervals or lead time | Deterioration products | Decision variables | | | Objective function | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Backordered | Lost Sale | Partial Backordering | No shortage | | | | | Order Quantity | Cycle Length | Replenishment-up-to level | Convex | Concave |
| Cardenas-Barron *et al.* [4] | * | | | | * | | | | * | | | * | |
| Taleizadeh *et al.* [33] | | | * | | * | | | | | * | | | * |
| Yu and Hsu [39] | | | | * | | * | | * | * | | | * | |
| Taleizadeh and Pentico [32] | | | * | | | * | | | | * | | | * |
| Yang and Ouyang [38] | | | | * | | * | | | | * | | | * |
| Taleizadeh *et al.* [34] | * | | | | * | | | * | | * | | | * |
| Chung *et al.* [6] | | | | * | | * | | * | | * | | | * |
| Karimi-Nasab and Konstantaras [11] | | | * | | * | | | | | | * | | * |
| Ouyang *et al.* [19] | | | | * | | * | | * | | * | | | * |
| Taleizadeh *et al.* [35] | | | * | | | * | * | | | | * | | * |
| Taleizadeh *et al.* [36] | | | * | | | | * | | * | | | | * |
| This paper | | | * | | * | * | | | * | | | | * |

(b) The manufacturer does not offer special sale.

If the manufacturer does not offer a special sale, it is clear that the customer do not make a special order, but if special order materializes, the expected total saving function have to be written to calculate the amount of profit which customers will gain by the advantage of taking a special order. In this situation the customer wants to know how much products he should purchase as a special order to maximize the expected total saving. It should be mentioned that the words customer, buyer, and retailer are used interchangeably in this study.

### Assumptions

(1) The inventory level at $t_S$ is not necessarily equal to zero. The initial inventory of the customer is assumed to be $q_s$. This assumption has an impact on the amount of the expected total saving of the customer.
(2) The shortage is allowed and it is partially backordered. It is assumed that $\alpha$ of the shortage will be backordered.
(3) The manufacturer let the customer to make a special order with the probability of $p$ at time $t_S$. This assumption helps the better represent an environment where the manufacturer offers special order probabilistically.
(4) The unit purchasing price in the first case will increase to $C_K$ and in the second one it decreases to $C_S$ at time $t_S$. This assumption leads to considering two possible cases in the model.
(5) The unit holding cost is a proportion of the unit purchasing cost. Since for more expensive products, commonly there is a higher holding cost, this assumption makes the model more realistic.

## 3. Model formulation

The following notations are used to model the problem.

*Parameters*

- $D$ Demand rate (units/year).
- $C$ Regular unit purchasing price (in dollars).
- $C_S$ The special sale prices (in dollars).
- $C_K$ Future unit purchasing price (in dollars).
- $h$ Inventory holding cost per unit per time period ($h = iC$), (\$/unit/year).
- $h_K$ Inventory holding cost per unit per time period when price increased ($h = iC_k$), (\$/unit/year).
- $p$ Probability that the manufacturer offers a special sale.
- $q_s$ The initial amount of inventory when the price increases or special sale happens.
- $Q$ The order quantity before the price increase occurs.
- $\pi$ Backordered cost per unit per time period (\$/unit/year).
- $\pi'$ Lost sale cost per unit in normal price (\$/unit).
- $\alpha$ Proportion of shortage that will backordered (in percentage).

*Decision variables*

- $Q_S$ Number of items replenished as a special order (unit/order).
- $b_S$ Shortage quantity per cycle for special order period (units).

*Other variables*

- ETS Expected total saving (in dollars)
- $\text{Cost}_N$ The costs which occurs when the customer does not make a special order.
- $\text{Cost}_S$ The costs which occurs when the customer makes a special order.
- $b$ Shortage quantity per cycle before the purchasing price increases (unit).
- $b_K$ Shortage quantity per cycle after the purchasing price increases (unit).
- (*) Indicate the optimal value.

### 3.1. Modelling

In the classical inventory control model, EOQ, a one-time special ordering opportunity does not consider the profit function since the profit function is in an infinitive time horizon while the special order occurs in a finite time horizon. However, it seems that a firm can benefit from a one-time discount that occurs in a finite time horizon. In this paper, a function is defined to measure the total saving from making a special order at time $t_S$, if it materializes. This function measures the differences between the costs of two possible scenarios:

(1) The manufacturer offers a special sale and the customer make a special order.
(2) The manufacturer offers a special sale and the customer does not make a special order.

While in a deterministic environment order quantity and time of price fluctuations are clear from the beginning, in stochastic environment several scenarios may occur. In this paper two possible scenarios are denoted by $S$, in which the special ordering opportunity materializes and the buyer makes a special order, and $N$ in which the special ordering opportunity again materializes but the customer does not make a special order. Each scenario yields different costs, time planning and ordering policy.

The amount of inventory on hand at time $t_S$ can change the special order quantity if it materializes. To analyze the impact of this factor on the order quantity, this amount is shown by $q_S$. The partial backordering inventory policy is considered. According to this assumption, $\alpha$ percent of shortage will be backordered in every cycle. The whole amount of shortage is indicated by $b$ and intuitively, $b_S$ for the special order period, and $b_K$ for the case when the purchasing price increases.

Two cases are made in this paper to explore the best ordering policy when the purchasing price increases or decreases. In the first case, the purchasing price increases and the best ordering policy for retailer is achieved. In the second case, the purchasing price decreases for just a short period of time and the manufacturer may let the customers to make a special order by the probability of $p$.

*Case 1: Purchasing price increase permanently*

When an announcement in price increase will happen in the future, the amount of order quantity will decrease since the holding cost per unit increases. The purchasing prices increase permanently at $t_S$ and at the same time, customers have this chance to make a special order with amount of $Q_S$. As stated before, shortage occurs as partial backordering. According to the explanation made in the previous section, the proposed model in first case is illustrated in Figure 1.

The costs in normal purchasing system, when the buyer does not make a special order, contains the costs of $q_S$ units, which exists in the system at time $t_S$, and the costs occur when the purchasing price increases. For $q_S$ units, there is a holding cost for both cases, and if the customer does not make a special order, there is also a shortage cost for both backordered and lost sale units. The amount of shortage and holding costs can obtain by the areas in Figure 1. Based on this facts, the costs for $q_S$ units will be

$$\left(\frac{hq_S^2}{2D}\right) + \alpha\pi\left(\frac{b^2}{2D}\right) + (1-\alpha)\,\pi' b. \tag{3.1}$$

Also when the price increases, the costs consist of the ordering cost, the purchasing cost, the holding cost and finally backordered and lost sale costs. It is clear that from time $t_S$ to $t_S + \frac{Q_S}{D}$, the customer can make $\left(\frac{Q_S}{Q_K} - \frac{q_S}{D}\right)$ unit of orders, which must be multiplied by the costs of one cycle. With the new prices, these costs can be written as follows

$$\left(\frac{Q_S}{Q_K} - \frac{q_S}{D}\right)\left[A + C_K Q_K + h_K\left(\frac{(Q_K - b_K)^2}{2D}\right) + \alpha\pi\left(\frac{b_K^2}{2D}\right) + (1-\alpha)\,\pi' b_K\right]. \tag{3.2}$$

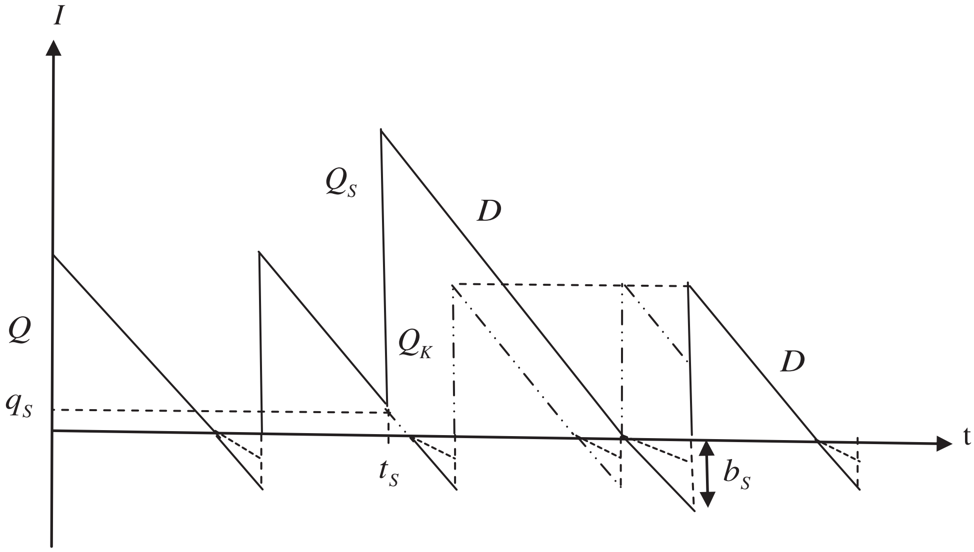


FIGURE 1. The inventory system scheme when special order materializes and the prices increase forever after $t_S$.

In order to obtain the best ordering policy, the costs should be calculated from time $t_S$ to $t_S + \frac{Q_S}{D}$ in both scenarios. So the accumulative costs, when the buyer does not make a special order is

$$\begin{aligned}\mathrm{Cost}_N = &\left(\frac{hq_S^2}{2D}\right) + \alpha\pi\left(\frac{b^2}{2D}\right) + (1-\alpha)\,\pi' b \\ &+ \left(\frac{Q_S}{Q_K} - \frac{q_S}{D}\right)\left[A + C_K Q_K + h_K\left(\frac{(Q_K - b_K)^2}{2D}\right) + \alpha\pi\left(\frac{b_K^2}{2D}\right) + (1-\alpha)\,\pi' b_K\right]. \end{aligned} \tag{3.3}$$

As mentioned earlier, for the $q_S$ units which exists at time $t_S$ in the system, there are not any shortage costs and the holding cost for these units is like the holding cost in equation (3.1). So the accumulated costs, when the buyer makes a special order can be written as

$$\mathrm{Cost}_S = \left(\frac{hq_S^2}{2D}\right) + \left[A + CQ_S + h\left(\frac{(Q_S - b_S)^2}{2D}\right) + \alpha\pi\left(\frac{b_S^2}{2D}\right) + (1-\alpha)\,\pi' b_S\right]. \tag{3.4}$$

The expected total saving can be obtained by calculating the differences between the costs in normal purchasing system, when the special order materializes, and when the buyer makes a special order; so it is

$$\mathrm{ETS} = \mathrm{Cost}_N - \mathrm{Cost}_S. \tag{3.5}$$

After replacing the values, the expected total saving between two possible scenarios will be

$$\begin{aligned}\mathrm{ETS}^1(Q_S, b_S) = p &\left[\begin{array}{l}\left(\frac{hq_S^2}{2D}\right) + \alpha\pi\left(\frac{b^2}{2D}\right) + (1-\alpha)\,\pi' b \\ + \left(\frac{Q_S}{Q_K} - \frac{q_S}{D}\right)\left[A + C_K Q_K + h_K\left(\frac{(Q_K - b_K)^2}{2D}\right) + \alpha\pi\left(\frac{b_K^2}{2D}\right) + (1-\alpha)\,\pi' b_K\right]\end{array}\right] \\ &- p\left[\left(\frac{hq_S^2}{2D}\right) + \left[A + CQ_S + h\left(\frac{(Q_S - b_S)^2}{2D}\right) + \alpha\pi\left(\frac{b_S^2}{2D}\right) + (1-\alpha)\,\pi' b_S\right]\right]\end{aligned} \tag{3.6}$$

which is a function of two decision variables $Q_S$ and $b_S$. It can be shown that $\mathrm{ETS}^1(Q_S, b_S)$ is a concave function in $Q_S$ and $b_S$ (see Appendix A). The following theorem yields to the solution of the problem.

**Theorem 3.1.** *The profit is maximized at* $b_S^* = \frac{hpQ_S-(1-\alpha)p\pi' D}{h+\alpha\pi}$ *where*

$$Q_S^* = \frac{\frac{D}{hQ_K}\left[A + C_K Q_K + h_K\left(\frac{(Q_K-b_K)^2}{2D}\right) + \alpha\pi\left(\frac{b_K^2}{2D}\right) + (1-\alpha)\,\pi' b_K\right] - \frac{p(1-\alpha)\pi' D}{(h+\alpha\pi)} - \frac{CD}{h}}{1-\frac{hp}{h+\alpha\pi}}.$$

*Proof.* From Appendix A, $\partial \mathrm{ETS}^1(Q_S, b_S)/\partial Q_s = 0$ leads to

$$Q_S^* = b_S - \frac{CD}{h} + \frac{D}{hQ_K}\left[A + C_K Q_K + h_K\left(\frac{(Q_K-b_K)^2}{2D}\right) + \alpha\pi\left(\frac{b_K^2}{2D}\right) + (1-\alpha)\,\pi' b_K\right].$$

Likewise, from $\partial \mathrm{ETS}^1(Q_S, b_S)/\partial b_s = 0$, $b_S^* = \frac{hpQ_S-(1-\alpha)p\pi' D}{h+\alpha\pi}$ which, upon putting into the expression of $Q_S^*$, yields the results. □

*Case 2: Purchasing price decrease for a short time period*

Using the same approach as in Case 1, the costs of normal ordering policy can be calculated as

$$\begin{aligned}\mathrm{Cost}_N = {} & \left(\frac{hq_S^2}{2D}\right) + \alpha\pi\left(\frac{b^2}{2D}\right) + (1-\alpha)\,\pi' b \\ & + \left(\frac{Q_S}{Q} - \frac{q_S}{D}\right)\left[A + CQ + h\left(\frac{(Q-b)^2}{2D}\right) + \alpha\pi\left(\frac{b^2}{2D}\right) + (1-\alpha)\,\pi' b\right].\end{aligned} \tag{3.7}$$

The accumulated costs when the manufacturer lets the retailer make a special order by less prices with the same approach in the previous case will be as follows

$$\mathrm{Cost}_S = \left(\frac{hq_S^2}{2D}\right) + \left[A + C_S Q_S + h_S\left(\frac{(Q_S-b_S)^2}{2D}\right) + \alpha\pi\left(\frac{b_S^2}{2D}\right) + (1-\alpha)\,\pi' b_S\right]. \tag{3.8}$$

According to the explanations made in prior sections, the model of the second case is denoted in Figure 2. In Figure 2, purchasing prices decrease for a short time and manufacturer let customers to make a special order at $t_S$ with amount of $Q_S$. Shortage is partial backordering and the amount of inventory at the beginning is $q_S$.

The optimal values for the amount of order quantity and shortage can be derived from the expected total saving function. This function will be obtainedas in the previous section and likewise, the differences between the costs of having a normal ordering policy and making special order can be obtained when it materializes. So the expected total saving will be

$$\mathrm{ETS} = p\left[\mathrm{Cost}_N - \mathrm{Cost}_S\right]. \tag{3.9}$$

By replacing the amounts of $\mathrm{Cost}_N$ and $\mathrm{Cost}_S$, the expected total saving will be as follows

$$\mathrm{ETS}^2(Q_S, b_S) = p\left[\begin{array}{l}\left(\frac{hq_S^2}{2D}\right) + \alpha\pi\left(\frac{b^2}{2D}\right) + (1-\alpha)\,\pi' b \\ + \left(\frac{Q_S}{Q} - \frac{q_S}{D}\right)\left[A + CQ + h\left(\frac{(Q-b)^2}{2D}\right) + \alpha\pi\left(\frac{b^2}{2D}\right) + (1-\alpha)\,\pi' b\right]\end{array}\right] \tag{3.10}$$

$$- p\left[\left(\frac{hq_S^2}{2D}\right) + \left[A + C_S Q_S + h_S\left(\frac{(Q_S-b_S)^2}{2D}\right) + \alpha\pi\left(\frac{b_S^2}{2D}\right) + (1-\alpha)\,\pi' b_S\right]\right]. \tag{3.11}$$

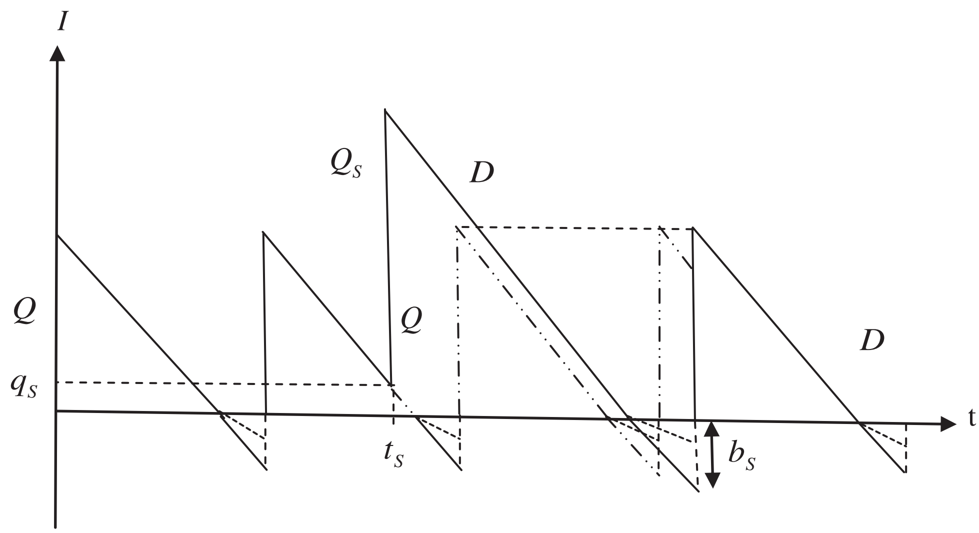


FIGURE 2. The inventory system scheme when special order materialize and the prices decrease for a short time period.

As in Case 1, it can also be shown like Appendix A that ETS in (A.4) is also concave and the optimal solution to it can be obtained as

$$b_S^* = \frac{h_S p Q_S - (1-\alpha)\, p\pi' D}{h_S + \alpha\pi} \tag{3.12}$$

and

$$Q_S^* = \frac{\frac{D}{h_S Q}\left[A + CQ + h\left(\frac{(Q-b)^2}{2D}\right) + \alpha\pi\left(\frac{b^2}{2D}\right) + (1-\alpha)\,\pi' b\right] - \frac{p(1-\alpha)\pi' D}{(h_S+\alpha\pi)} - \frac{C_S D}{h_S}}{1 - \frac{h_S p}{h_S+\alpha\pi}}. \tag{3.13}$$

## 4. Numerical tests

A dairy producer sells its products to retailers with deterministic demand. In the beginning, purchasing prices for customers are fixed. However, after a while, in the first scenario, the supplier decides to increases the purchasing prices at a predetermined time ($t_S$) to $C_K$. This occurs due to improvement made in the quality of some dairy products such as milk. At this point, there is a possibility ($p$) that the retailers are able to make a special order. By making special order, retailers can save some money before the prices increment. In the second scenario, on the contrary, the dairy producer, with the probability of $p$, decides to temporarily decrease the prices of products for the customers to $C_S$ (\$), and let them to make a special order. This occurs because some of the products are produced more than what is should. As a result, the manufacturer must sell these products under a lower price and retailers can gain some benefit.

Considering the aforementioned details, several examples are designed and solved to show the applicability of the proposed model. According to Pentico and Drake [24], under a condition, when $\beta \geq 1 - \sqrt{\frac{2AD}{D\pi'^2}}$, the partial backordering policy can lead to an optimal solution. All of the examples in Tables 2 and 3 are designed under this condition. The value of parameters of the model in Cases 1 and 2 are based on the opinion of the experts of the diary industries and are $C = \$100$/unit, $q_S = 15$ units, $\pi = \$20$/unit/year, $A = \$200$/order. Other necessary inputs to solve model are represented in Tables 2 and 3, separately.

The numerical results represented in Tables 2 and 3 indicates that under fluctuation of different parameters, what decisions should be made to ensure that the gained profit is optimal. Table 2 refers to the case when an

Table 2. The values of parameters and the optimal solutions of the first case.

| Specific parameters | | | | | | Results | | |
|---|---|---|---|---|---|---|---|---|
| $p$ | $C_K$ (\$) | $\alpha$ | $\pi'$ | $D$ (units) | $i$ (%) | $Q_S^*$ (units) | $b_S^*$ (units) | $\text{ETS}^1(Q_S, b_S)$ (\$) |
| 0.2 | 140 | 0.85 | 20 | 200 | 15 | 668.64 | 58.94 | 3052.90 |
| 0.6 | 120 | 0.90 | 30 | 220 | 20 | 399.17 | 115.63 | 2953.70 |
| 0.2 | 140 | 0.85 | 20 | 240 | 15 | 795.22 | 70.05 | 3653.60 |
| 0.6 | 120 | 0.90 | 30 | 260 | 20 | 464.69 | 134.43 | 3497.20 |
| 0.2 | 140 | 0.85 | 20 | 280 | 15 | 921.18 | 81.11 | 4244.70 |
| 0.6 | 120 | 0.90 | 30 | 300 | 20 | 529.65 | 153.05 | 4024.00 |
| 0.2 | 140 | 0.85 | 20 | 320 | 15 | 1046.60 | 92.12 | 4828.30 |
| 0.6 | 120 | 0.90 | 30 | 340 | 20 | 594.14 | 171.52 | 4537.60 |
| 0.2 | 140 | 0.85 | 20 | 360 | 15 | 1171.70 | 103.10 | 5405.90 |
| 0.6 | 120 | 0.90 | 30 | 380 | 20 | 658.23 | 189.86 | 5040.80 |
| 0.2 | 140 | 0.85 | 20 | 400 | 15 | 1296.30 | 114.03 | 5978.50 |
| 0.6 | 120 | 0.90 | 30 | 420 | 20 | 721.95 | 208.09 | 5535.40 |

Table 3. The values of parameters and the optimal solutions of the second case.

| Specific parameters | | | | | | Results | | |
|---|---|---|---|---|---|---|---|---|
| $p$ | $C_S$ (\$) | $\alpha$ | $\pi'$ | $D$ (units) | $i$ (%) | $Q_S^*$ (units) | $b_S^*$ (units) | $\text{ETS}^2(Q_S, b_S)$ (\$) |
| 0.2 | 80 | 0.85 | 20 | 200 | 15 | 451.7 | 33.24 | 1021.80 |
| 0.6 | 60 | 0.90 | 30 | 220 | 20 | 1079.0 | 245.76 | 15 706.00 |
| 0.2 | 80 | 0.85 | 20 | 240 | 15 | 533.8 | 39.21 | 1125.80 |
| 0.6 | 60 | 0.90 | 30 | 260 | 20 | 1264.9 | 287.98 | 17 227.00 |
| 0.2 | 80 | 0.85 | 20 | 280 | 15 | 615.1 | 45.12 | 1332.90 |
| 0.6 | 60 | 0.90 | 30 | 300 | 20 | 1450.1 | 330.02 | 19 885.00 |
| 0.2 | 80 | 0.85 | 20 | 320 | 15 | 695.9 | 50.97 | 1533.40 |
| 0.6 | 60 | 0.90 | 30 | 340 | 20 | 1634.5 | 371.88 | 22 512.00 |
| 0.2 | 80 | 0.85 | 20 | 360 | 15 | 776.1 | 56.78 | 1729.00 |
| 0.6 | 60 | 0.90 | 30 | 380 | 20 | 1818.3 | 413.59 | 25 114.00 |
| 0.2 | 80 | 0.85 | 20 | 400 | 15 | 855.9 | 62.55 | 1920.70 |
| 0.6 | 60 | 0.9 | 30 | 420 | 20 | 2001.6 | 455.18 | 27 696.00 |

announced price increase occurs. Besides, Table 3 represents the results for the case when the manufacturer temporarily decreases the purchasing prices. In both cases, the probability of making a special order by the retailer is shown by $p$. It can be realized that when the probability of making a special order increase, the expected special order quantity and the expected total saving increase exponentially in the second case. However, the impact of this probability on the expected special order quantity and the expected total saving is negligible in the first case.

## 5. Sensitivity analysis

In order to investigate the effect of some parameters on special order quantity, amount of shortage and the expected total saving, the sensitivity analysis is performed below. The initial amount of parameters in the model are $\alpha = 0.9$, $D = 200$ unit/year, $C = \$100$/unit, $C_S = \$80$/unit, $C_K = \$140$/unit, $p = 0.5$, $q_S = 15$ unit, $\pi = \$20$/unit/year, $\pi' = \$30$/unit, $i = 0.2$, $A = \$200$/order. The results of two cases witch are modeled before,

Table 4. The results of sensitivity analysis for the case with price increases.

| Parameter | Percentage of change in parameter | Value | | | Percentage of changes | | |
|---|---|---|---|---|---|---|---|
| | | $Q_S^*$ (units) | $b_S^*$ (units) | $\mathrm{ETS}^1(Q_S, b_S)$ ($ ) | $Q_S^*$ (units) | $b_S^*$ (units) | $\mathrm{ETS}^1(Q_S, b_S)$ ($) |
| $D$(units) | +75 | 1051.2 | 262.8 | 13 162 | +70.3 | +70.0 | +70.9 |
| | +50 | 907.4 | 226.9 | 11 373 | +47.0 | +46.8 | +47.7 |
| | +25 | 762.9 | 190.9 | 9556 | +23.6 | +23.5 | +24.1 |
| | 0 | 617.4 | 154.6 | 7703 | 0 | 0 | 0 |
| | −25 | 470.8 | 118.0 | 5798 | −23.7 | −23.7 | −24.7 |
| | −50 | 322.4 | 80.9 | 3799 | −47.8 | −47.7 | −50.7 |
| | −75 | 170.7 | 42.9 | 1558 | −72.4 | −72.3 | −79.8 |
| $p$ | +75 | 832.3 | 369.5 | 17 184 | +34.8 | +139 | +123.1 |
| | +50 | 745.1 | 262.3 | 13 520 | +20.7 | +69.7 | +75.5 |
| | +25 | 675.0 | 212.2 | 10 592 | +9.3 | +37.3 | +37.5 |
| | 0 | 617.4 | 154.6 | 7703 | 0 | 0 | 0 |
| | −25 | 569.3 | 106.4 | 5207 | −7.8 | −31.2 | −32.4 |
| | −50 | 528.4 | 65.6 | 3109 | −14.4 | −57.6 | −59.6 |
| | −75 | 493.3 | 30.48 | 1383 | −20.1 | −80.3 | −82.0 |
| $C_K$($) | +21.43 | 1029.1 | 262.9 | 22 266 | +66.7 | +70.1 | +189.1 |
| | +14.29 | 892.0 | 226.8 | 16 598 | +44.5 | +46.7 | +115.5 |
| | +7.14 | 754.8 | 190.7 | 11 743 | +22.3 | +23.4 | +52.5 |
| | 0 | 617.4 | 154.6 | 7703 | 0 | 0 | 0 |
| | −7.14 | 479.9 | 118.4 | 4477 | −22.3 | −23.4 | −41.9 |
| | −14.29 | 342.3 | 82.2 | 2071 | −44.6 | −46.8 | −73.1 |
| | −21.43 | 204.4 | 45.9 | 486 | −66.9 | −70.3 | −93.7 |
| $q_S$ (units) | +75 | 617.4 | 154.6 | 7384 | 0 | 0 | −4.1 |
| | +50 | 617.4 | 154.6 | 7490 | 0 | 0 | −2.8 |
| | +25 | 617.4 | 154.6 | 7596 | 0 | 0 | −1.4 |
| | 0 | 617.4 | 154.6 | 7703 | 0 | 0 | 0 |
| | −25 | 617.4 | 154.6 | 7809 | 0 | 0 | +1.4 |
| | −50 | 617.4 | 154.6 | 7915 | 0 | 0 | +2.8 |
| | −75 | 617.4 | 154.6 | 8022 | 0 | 0 | +4.1 |

are shown in Tables 4 and 5 respectively. Similar to Section 3, all the examples in Tables 4 and 5 are designed to satisfy the condition in Pentico and Drake [24].

According to Tables 4 and 5, it can be realized that thespecial order quantity, the shortage and the expected total saving are more affected by demand rate than other parameters. Figures 3–6 are made according to Tables 3 and 4 to denote the changes.

It is clear from Tables 4 and 5 and Figure 3, when the demand increases, the amount of special order quantity, the amount of shortage and the expected total saving increases too. The amount of the increment in the values of the mentioned three metrics are greater in the first case compared to the second case. It seems that in both cases (first and second one), the amount of special order quantity, the amount of shortage and the expected total saving are highly sensitive to the demand. According to Tables 4 and 5, the amount of the special order quantity and the shortage in the first case (when the known price increase happens) are more sensitive to demand rate than in the second case (when special sale happens). It can be noted that according to Table 5 (second case), the expected total saving increase with a slightly higher rate compared to the amount of order quantity and the shortage.Further, the negative effect of demand decrement is more than the effect of demand increment in both cases. Therefore, the retailer should be more concerned when the demand for products starts to decrease.

TABLE 5. The results of sensitivity analysis for the case with special sale.

| Parameter | Percentage of change in parameters | Value | | | Percentage of change | | |
|---|---|---|---|---|---|---|---|
| | | $Q_S^*$ (units) | $b_S^*$ (units) | $\text{ETS}^2\,(Q_S, b_S)$ ($ ) | $Q_S^*$ (units) | $b_S^*$ (units) | $\text{ETS}^2\,(Q_S, b_S)$ ($) |
| | +75 | 681.0 | 144.8 | 4293 | +66.9 | +66.1 | +71 |
| | +50 | 590.9 | 125.8 | 3717 | +44.8 | +44.3 | +48.1 |
| | +25 | 500.0 | 106.6 | 3126 | +22.5 | +22.2 | +24.5 |
| $D$(units) | 0 | 408.1 | 87.2 | 2511 | 0 | 0 | 0 |
| | −25 | 314.8 | 67.5 | 1861 | −22.9 | −22.6 | −25.9 |
| | −50 | 219.5 | 47.2 | 1145 | −46.2 | −45.9 | −54.4 |
| | −75 | 120.4 | 26.1 | 255 | −70.5 | −70.1 | −90 |
| | +75 | 519.3 | 198.4 | 5349 | +27.2 | +127.5 | +113.1 |
| | +50 | 475.5 | 154.6 | 4362 | +16.5 | +77.3 | +73.7 |
| | +25 | 439.0 | 118.1 | 3398 | +7.6 | +35.4 | +35.3 |
| $p$ | 0 | 408.1 | 87.2 | 2511 | 0 | 0 | 0 |
| | −25 | 381.6 | 60.7 | 1724 | −6.5 | −30.4 | −31.3 |
| | −50 | 358.7 | 37.8 | 1045 | −12.1 | −56.7 | −58.4 |
| | −75 | 338.6 | 17.7 | 472 | −17.0 | −79.7 | −81.2 |
| | +18.75 | 140.2 | 27.9 | 23 | −65.6 | −68 | −99.1 |
| | +12.5 | 221.1 | 46.9 | 564 | −45.8 | −46.2 | −77.5 |
| | +6.25 | 309.9 | 66.7 | 1380 | −24.1 | −23.5 | −45 |
| $C_S$ ($) | 0 | 408.1 | 87.2 | 2511 | 0 | 0 | 0 |
| | −6.25 | 517.5 | 108.5 | 4006 | +26.8 | +24.4 | +59.6 |
| | −12.5 | 640.3 | 130.7 | 5929 | +56.9 | +49.9 | +136.1 |
| | −18.75 | 779.5 | 153.8 | 8361 | +91.0 | +76.4 | +233 |
| | +75 | 408.1 | 87.2 | 2263 | 0 | 0 | −9.9 |
| | +50 | 408.1 | 87.2 | 2348 | 0 | 0 | −6.6 |
| | +25 | 408.1 | 87.2 | 2428 | 0 | 0 | −3.3 |
| $q_S$(units) | 0 | 408.1 | 87.2 | 2511 | 0 | 0 | 0 |
| | −25 | 408.1 | 87.2 | 2593 | 0 | 0 | +3.3 |
| | −50 | 408.1 | 87.2 | 2676 | 0 | 0 | +6.6 |
| | −75 | 408.1 | 87.2 | 2758 | 0 | 0 | +9.9 |

From Tables 4 and 5 and Figure 4, when the probability that the manufacturer offer a special sale increases, the amount of special sale, the amount of shortage and the expected total saving increases too and they are in a direct relationship. It is understandable that these metrics are more sensitive to this probability in the first case compared the second one. It is considerable from the percentage of change in the expected values of the special order quantity, shortage and total saving that the proposed model is highly sensitive to this probability. In contrast with the demand changes, the absolute effect of the increment in the probability of making a special sale is more than its decrement on the total saving, the special order quantity and the shortage in both cases (Tabs. 4 and 5). However, it should be noted that the effect of fluctuation of this probability on the special order quantity is less than the shortage and the total saving in both cases. It worth mentioning that the retailer can potentially save much more by persuading the manufacturer to enhance the probability of letting the retailer to make a special order.

As it is shown in Table 4 and Figure 5, when the future purchasing price increases, in the first case, the amount of special order quantity, the amount of shortage and the expected total saving increase too. From the amount of changes, it is understandable that these three metrics of the model are highly sensitive to future purchasing price.Furthermore, it can be realized from Table 4 and Figure 5 that when the future price increases with higher

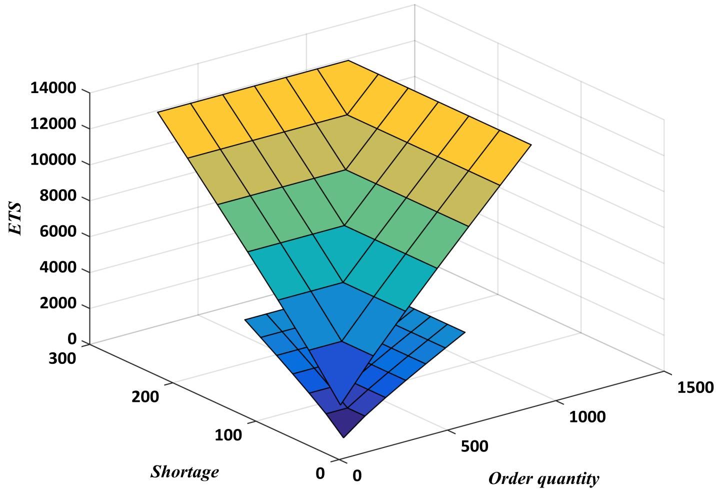


FIGURE 3. Effect of increasing of $D$ on special order quantity, shortage and the expected total save.

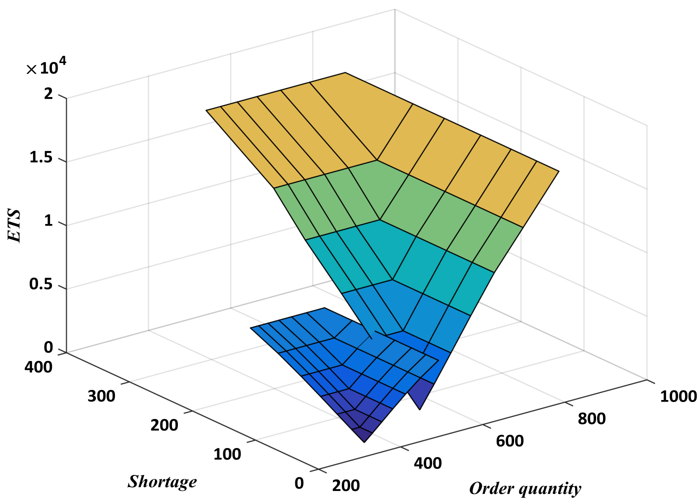


FIGURE 4. Effect of increasing of $p$ on special order quantity, shortage and the expected total save.

proportions, the total saving by making a special order enhance exponentially. In other words, the retailer can save much more money by making a special order when the future prices enhances with higher percentages. In addition, in the second case, the effect of special sale price on the amount of special order quantity, the amount of shortage and the expected total saving is investigated in Tables 5 and Figure 6. There is an indirect relationship between the special sale price and the amount of the three metrics of the model. In other words, when the sale prices increase, the three metrics of the model decrease. In this situation, with relatively higher discounts, the retailer can expect more saving from making an special order. It worth to mention that the positive effect of smaller special sale prices is relatively more than the negative effect of higher sale prices on the three metrics of the model. For instance, the expected total saving of the retailer increase by 233% when the special sale price decrease by 18.75%. However, the expected total saving of the customer decrease by 99.1% when the special sale price increase by 18.75%. In this situation, with high savings from making a special order at lower special sale

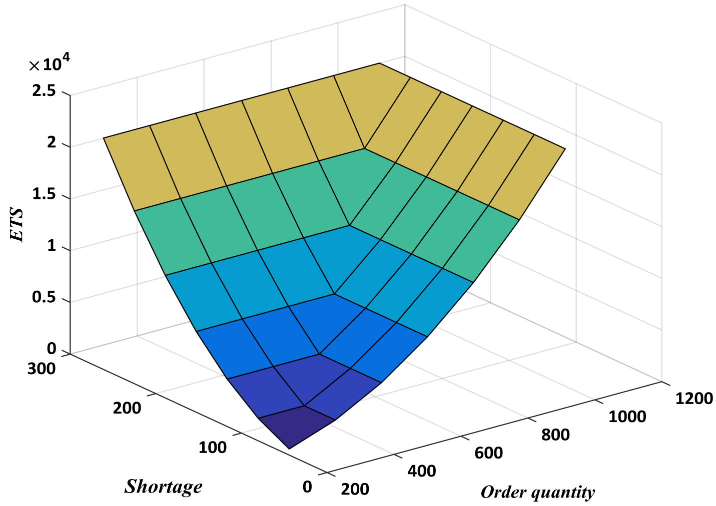


FIGURE 5. Effect of increasing of $C_K$ on special order quantity, shortage and the expected total save.

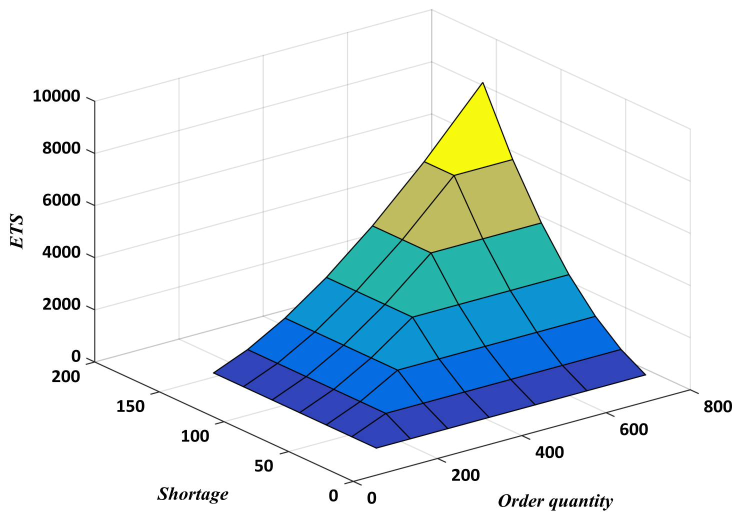


FIGURE 6. Effect of decreasing of $C_S$ on special order quantity, shortage and the expected total save.

price, the retailer should entice the manufacturer to lower the prices by for example offering that they purchase more goods if the manufacturer offers more discount.

From Tables 4 and 5 and Figure 7, it is comprehensible that the expected total saving is in an indirect relationship with the initial inventory when the price increases or special sale happens. When the initial inventory increases, the expected total saving from making a special order decreases. Also it can be realized that this parameter has no effect on the amount of special order quantity and shortage.Therefore, we did not depict a figure to represent their relationship.This is because the initial inventory is a constant value in the total saving function and it becomes zero when we take the derivatives to find optimal order quantity.According to Tables 4 and 5, it can be realized that the amount of expected total saving is greater in the first case compared to the

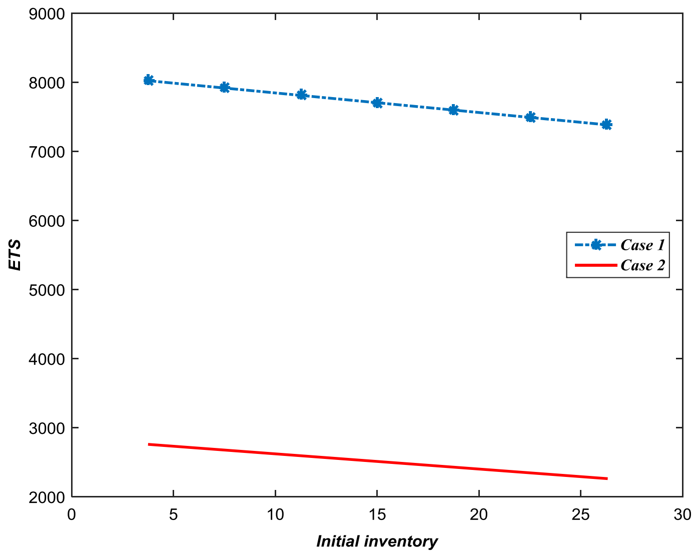


FIGURE 7. Effect of $q_S$ on the expected total save.

second one. In addition, the sensitivity of the second case is more to the changes in the initial inventory in comparison with the firs case. In other words, when the special sale occurs, the percentage of the changes in the expected total saving becomes more salient compared to the case when an announced price increase happens. However, the effect of the initial inventory on the expected total saving is relatively less than other investigated parameters (demand, probability of making special order, and prices).

## 6. Conclusion

In this paper the optimal ordering policy when there is an uncertainty in offering the especial order from supplier is investigated under two types of price changes: when an announced price increase happens and when temporary price decrease occurs. Shortage is partially backlogged and the level of inventory at the time of possible special materialization is not necessarily equal to zero. It is assumed that the manufacturer let the customer know how and when the price changes. Based on the fact that price increases or decreases, two different cases are developed in this paper. The difference between ordering a special quantity and not is named as the expected total saving function. In the mathematical model, the total saving function is maximized with respect to two decision variables and the optimal amount of the special order quantity and the shortage is obtained for each two cases. In order to show the applicability of the proposed model and to study the effect of some parameters such as demand, the probability that the manufacturer offers a special order, future purchasing price and the initial inventory when the price increases or special sale occurs, some numerical example are solved and sensitivity analysis is performed.

It was found that the initial inventory when the price increases or special sale happens has less impact on the expected total saving than the other three parameters. The results of this study can potentially be beneficial for the retailers which are working in an environment where the purchasing prices fluctuate more frequently. Furthermore, by considering that the occurrence of the special order is probabilistic, this study can help the decision makers in an environment with high level of uncertainty. Both stakeholders of the supply chain (manufacturer and retailer) can benefit from the results by investing on more important parameters. It is shown that the effect of the demand, the probability of making a special order, and increased or special sale prices are considerable on the total saving, the special order quantity, and the shortage. As a result, one should not neglect these impacts in a competitive environment. Future research may direct to deal other system variables such probabilistic demand with lead time and variation in the delivery mechanisms and their respective costs.

## Appendix A. Concavity of $ETS(Q_S, b_S)$

The first derivative $\partial\mathrm{ETS}(Q_S, b_S)/\partial Q_s$ from equation (3.6) leads to

$$\frac{p}{Q_K}\left[A + C_K Q_K + h_K\left(\frac{(Q_K - b_K)^2}{2D}\right) + \alpha\pi\left(\frac{b_K^2}{2D}\right) + (1-\alpha)\,\pi' b_K\right] - Cp - hp\frac{Q_S - b_S}{D} = 0 \tag{A.1}$$

from which

$$Q_S^* = b_S - \frac{CD}{h} + \frac{D}{hQ_K}\left[A + C_K Q_K + h_K\left(\frac{(Q_K - b_K)^2}{2D}\right) + \alpha\pi\left(\frac{b_K^2}{2D}\right) + (1-\alpha)\,\pi' b_K\right]. \tag{A.2}$$

Similarly,

$$\partial\mathrm{ETS}(Q_S, b_S)/\partial b_s = hp\frac{Q_S - b_S}{D} - \alpha p\pi\frac{b_S}{D} - (1-\alpha)\,\pi' p = 0. \tag{A.3}$$

Yields

$$b_S^* = \frac{hpQ_S - (1-\alpha)\,p\pi' D}{h + \alpha\pi} \tag{A.4}$$

$$H\left[Q_S, b_S\right] = \begin{vmatrix} \dfrac{\partial^2\mathrm{ETS}}{\partial^2 Q_S} & \dfrac{\partial^2\mathrm{ETS}}{\partial b_S \partial Q_S} \\ \dfrac{\partial^2\mathrm{ETS}}{\partial Q_S \partial b_S} & \dfrac{\partial^2\mathrm{ETS}}{\partial^2 b_S} \end{vmatrix} = \begin{vmatrix} -\dfrac{hp}{D} & \dfrac{hp}{D} \\ \dfrac{hp}{D} & -\dfrac{hp}{D} - \dfrac{\alpha p\pi}{D} \end{vmatrix} = -\frac{hp}{D}\left(Q_S - b_S\right)^2 - \frac{\alpha p b_S^2 \pi}{D} \le 0. \tag{A.5}$$

Since the Hessian $H\left[Q_S, b_S\right] \quad \le 0$, the profit function $\mathrm{ETS}(Q_S, b_S)$ in equation (3.6) is concave. So the stationary point will be the maximum value of the function.

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