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DYNAMIC COOPERATIVE ADVERTISING STRATEGY IN OAO SUPPLY CHAIN WITH CUSTOMER RETURN

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Abstract. Nowadays many manufacturers are increasingly adopting their own online direct channel and the offline retail channel to sell their products as the quick development of e-commerce and third party logistics. To gain more and more market share, the manufacturer and the retailer implement unconditional return strategy, which does not affect secondary sales. We build a differential game model for the optimal advertising and the optimal advertising cost sharing proportion for centralized and decentralized OAO (Online and Offline) supply chain considering customer returns rates. We further analyze how the returns rates affect the optimal decisions of the manufacturer and the retailer. The results show that the returns rates, the brand reputation and the influence factors of retail channel goodwill on demand of online direct channel strongly influence the optimal advertising decisions. Furthermore, the retailer does not support for the manufacturer advertising efforts in Stackelberg game. Compared with the centralized OAO supply chain, the decentralized system results in channel inefficiency. To coordinate the channels, we design a two-way advertising cost-sharing contract. By this contract, each member of the supply chain reaches a win-win situation and is willing to cooperate. Numerical studies verify the conclusions of this paper.

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1. Introduction

With the popularization of e-commerce, shopping methods have changed from a single traditional retail channel to a consumer mode combining traditional channel and online direct channel. The rise of shopping online has reduced the market share of traditional retail channels, which has attracted the attention of traditional channel retailers and manufacturers. Combining the traditional retail channel with the online direct channel can help them to keep competitive in the market. Considering the advantages of the online channel (such as price, non-regional restriction, the lower cost and so on), manufacturers have established their own official flagship stores on major online platforms (such as JD.com, Tmall.com, Suning.com and Dangdang.com) to directly sell their products to consumers, for the purpose of gaining more market share and keeping competitive. As the online direct and traditional retail channel sell the identical product, the competition between manufacturer and retailer

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becomes more and more intense, and the channel conflict is increasingly aggravating. Cooperative advertising has an important effect on channel promotion, and a successful advertising cooperation between manufacturers and retailers is an efficient tool to weaken the channel conflict. In order to alleviate the competition and conflict between channels, the manufacturer and the retailer implement cooperative advertising strategy. Additionally in a highly competitive market environment, the returns policies of the online and retail channels play an important role in the business market. The returns policy is widely used by firms as a strategic tool for gaining more market share and profit. Today, more and more sellers promise to allow consumers to return the products unconditionally within the stipulated time if it does not affect its secondary sales. Many Taobao sellers allow consumers to return the product in seven days without reason. The traditional retailers in large shopping malls also allow consumers to return the products within the specified time. So after the purchase, when customers find that if the product misfit their preferences or they dislike the products within a specified time, they will return the products back. After the sellers receive the returned products, they will fully refund the money to customers. Besides, allowing return is also a common business strategy to win customers in the OAO supply chain environment. The increasing customer returns may have a negative impact on the product and the advertising has a positive impact on the product. Investigating how the players implement the dynamic cooperative advertising strategy with customer returns taken into consideration is significant for an OAO supply chain.

The existing literatures mainly study the supply chain with one manufacturer and one retailer. The manufacturer undertakes a certain proportion of retailer's advertising expense, which encourages a retailer to advertise more for product. As the market share and profits of retailer increase, the conflicts between online channel and retail channel are relieved. Considering the retailing power shifting from manufacturers to retailers, Huang *et al.* [5] develop three models to probe into the efficiency of vertical cooperative advertising. Later Huang *et al.* [6] focus on researching optimal advertising strategy selection of manufacturer and retailer in cooperative game. Results indicate that compared with the leader-follower structure, profit of the system is higher under any partnership advertising scheme. Yue *et al.* [18] research the cooperative advertisement coordination in a two-level manufacturer-retailer supply chain with a price-sensitive market, considering the price deduction providing by manufacturer. Yang *et al.* [17] take the retailer's fairness concern into consideration and go into its impact. They find that the fairness concerns can influence the advertising effort and participation rate. What's more, when a retailer from unfairness concerns to fair-minded concerns, the profit of each member will achieve a Pareto improvement. In distribution channels consisting of one manufacturer and one retailer, Xie *et al.* [13] presume that the advertising level and the retail price influence the demand. They analyze the cooperative advertising and pricing strategies, and discover that the entire profits will be improved under the cooperative game. Aust *et al.* [1] study the optimal cooperative advertising and pricing issues for manufacturers and retailers under the four different cooperation degree games, and conclude that the advertising investment in the two-part cooperative advertising model is the largest and the retail price is lower. SeyedEsfahani *et al.* [12] research the profit of the supply chain system under different cooperation degrees of traditional channel. They find that: the higher the degree of cooperation, the greater the profit is Javid and Hoseinpour [8] study the cooperative advertising coordination decisions on a manufacturer-retailer supply chain. The manufacturer invests national advertisement while the retailer invests local advertising. By analyzing, they find if the manufacturer and the retailer take action simultaneously and independently, the manufacturer does not share the local-advertising cost of retailer. While manufacturer may share the local-advertising cost of retailer in the leader-follower game; meanwhile the sharing rate of cooperation positively depends upon the manufacturer's marginal profit, however that is negatively depends on the marginal profit of the retailer.

Some scholars have studied the dynamic cooperative advertising by introducing the goodwill function, and considered the impact of goodwill on demand. Xu and Zou [14] propose a dynamic model for perishable items, and calculate the optimal advertising and inventory levels using the Pontryagin's maximum theorem. Taking not only the participation ratio but also the accrual rate into account, Zhang *et al.* [19] develop a dynamic model to investigate the influence of the accrual rate to cooperative advertising decisions. Zhang *et al.* discover that the increase in participation rate will not always increase the retailer's advertising efforts due to the accrual rate; Furthermore, manufacturer and retailer benefit by a high accrual rate. Zhang *et al.* [20] consider that advertisement investment impacts consumers' reference price, and explore the cooperative advertising coordination strategies under the condition that reference price affects demand. Based on the Nerlove-Arrow model, He *et al.* [4] establish a dynamic model in a competitive environment for the supply chain, which concludes multiple manufacturers and a single retailer, and give the optimal advertising strategies for each supply chain member. To study the decision of the member enterprises in the supply chain under the restriction of capacity, Yang [16] establishes two models. The results show the profit of the supply chain will not change no matter the manufacturers and retailers opt for cooperating in the case of a small capacity or not. Chen *et al.* [2] examine the cooperative advertising problem in a dual-channel supply chain in which the channel conflicts between manufacturer's electronic channel and traditional channel. They find that: the cooperative advertising decisions of supply chain members not only correlated with the marginal profit of the channels but also correlated with the impact of retailer's local advertising effort on electronic channel. Zhang *et al.* [22] research the cooperative advertising of a dual-supply chain under product brand differences. Compared with the Stackelberg game, the advertising input of traditional channel products becomes larger and the advertising input of electronic direct channel products becomes smaller under the centralized case. Furthermore, a cost-sharing policy for product advertising effort is put forward to making the dual-channel supply chain coordinated. Chen *et al.* [3] research the dual-channel supply chain under product differentiation, in which the manufacturer sells his product in electronic direct channel but the retailer sells the manufacturer's product and another manufacturer's product in traditional channel. They discuss the impact of parameters of Internet channel acceptance and product differentiation on the members and the profit of the supply chain system. Zhang *et al.* [21] establish a differential game model for a two-echelon supply chain deteriorating items to study the characterization of pricing, advertising and production strategies and the effect of cooperative advertising on the performance of the supply chain. Huang *et al.* [7] establish three strategic models for the supply chain concluding one manufacturer and two competing retailers to research the cooperative promotion. Huang *et al.* discover that both the manufacturer and the supply chain constantly prefer the retailer with same strategies, in spite of the truth that retailers do not necessarily persist in the same strategies.

Although there are many literatures study the advertising cooperation strategies of supply chain, they do not take the customer return to consideration. Nowadays, allowing consumers to return the products within the stipulated time is a common business strategy. In this paper, considering the consumers returns, we study the cooperative advertising strategy in an OAO supply chain, which is composed of a manufacturer and a retailer. Through his owned direct channel and an independent retail channel, the manufacturer sells his product to customers. Not only the manufacturer but also the retailer allow consumers to return goods unconditionally within the stipulated time if it does not affect its secondary sales (*i.e.* false failure returns). The advertising and the returns rates can affect the brand goodwill of product. In addition, the advertising and the brand goodwill affect the demand of each channel. In this paper, we build a stochastic differential game model for centralized and decentralized OAO supply chain when both the manufacturer and the retailer implement unconditional return strategy. We study the dynamic cooperative advertising strategy in the OAO supply chain and examine how the customer returns rates affect the optimal cooperative advertising and the advertising cost sharing proportion. By comparing the total profit of the supply chain under the centralized and decentralized decision, we propose a contract (*i.e.* two-way advertising cost-sharing contract) to coordinate the supply chain.

The rest part is as below. Section 2 presents the problem description and the modeling framework. Section 3 derives and compares the manufacturer and retailer's decisions. Section 4 designs a two-way advertising cost-sharing contract. Section 5 uses numerical experiments to prove the conclusions of this paper. Section 6 is the conclusion of the paper and suggests topics for future research.

2. Problem description and model framework

2.1. Problem description

We focus on the OAO supply chain made of a manufacturer and a retailer. The manufacturer sells a single product by his owned direct channel and retail channel simultaneously. Customers can acquire the product either from the direct channel or from the retail channel. There exists competition between the channels. To gain more and more market share, both the manufacturer and the retailer allow consumers to return goods unconditionally within a certain period of time if it does not affect its secondary sales. Considering the returned product has no quality problem and can be resold as normal, so we do not consider the salvage value problem in this paper. The manufacturer and retailer can then resell the returned products to other customers by repacking. $\rho_e(t)$ represents the customer returns rate of the direct channel at time t and $\rho_r(t)$ represents the customer returns rate of the retail channel at time t. Under the OAO supply chain, we not only take the advertising advertised for the direct channel by the manufacturer and that advertised for the traditional channel by the retailer into account, but also take the manufacturer advertising for traditional channel into consideration. Let $A_{me}(t)$ denote the manufacturer's advertisement for the direct channel. Denote $A_{rr}(t)$ as the retailer's advertisement for the retail channel. $A_{mr}(t)$ represents the manufacturer's advertisement in the retail channel. In addition, we take $t_1(t)$, $t_2(t)$ severally represent the sharing proportion that manufacturer is willing to share for retailer's traditional advertising cost and the sharing rate that the retailer is willing to participate in the manufacturer's traditional advertising cost. On this condition, we will build and analyze our model.

2.2. Model assumptions

Assumption 2.1. *The quadratic advertising cost functions have been adopted in Jørgensen et al. [9], Zhang et al. [20], Salma Karray et al. [11], Zhang and chen [22] and many others. The advertising costs are described as follows:*

$$C(A_i(t)) = \frac{\mu}{2}A_i^2(t), \quad i \in \{mr, rr, me\} \tag{2.1}$$

where, μ is advertising cost coefficient.

Assumption 2.2. *When consumer returns the product, the cost of returned product will occur. We assume that the cost of returned product is proportional to sales volume and the return cost of each channel is all borne by the manufacturer, so the cost functions of returned product can be given by the following equation:*

$$C(\rho_i(t)) = c_i\rho_i(t)d_i(t), \quad i \in \{r, e\} \tag{2.2}$$

where $c_r(>0)$ and $c_e(>0)$ separately represent the unit cost of the returned product from the retail channel and the direct channel. Additionally, the return cost c_i mainly includes the transaction cost, the time cost and shipping fee.

Assumption 2.3. *Utilizing the goodwill functions in Zhang and Chen [22], Karray and Martín-Herrán [10] and many others, we further consider that the customer returns has a negative impact on brand goodwill and adopt the modified Nerlove-Arrow model to describe changes in goodwill. We assume the changing of the goodwill can be described by the following dynamic equation:*

$$\frac{\mathrm{d}G_r(t)}{\mathrm{d}t} = \lambda(A_{mr}(t) + A_{rr}(t)) - \theta_1\rho_r(t) - \delta G_r(t) \tag{2.3}$$

$$\frac{\mathrm{d}G_e(\mathrm{t})}{\mathrm{d}t} = \lambda A_{me}(t) - \theta_2\rho_e(t) - \delta G_e(t) \tag{2.4}$$

$$G(0) = G_0$$

where $G_r(t)$ is the accumulated goodwill from retailing channel's product over time t, and $G_e(t)$ is the accumulated goodwill from direct channel product over time t. $G_0 > 0$ is the initial value of goodwill. $\delta(\delta > 0)$ is dynamic diminishing rate of goodwill, and $\lambda(\lambda > 0)$ is the influence coefficient of advertising on the accumulation of product goodwill. $\theta_i(i = e, r)$ is the influence factor of customer returns rate on goodwill.

2.3. Model establishment

Utilizing the demand function in Zhang *et al.* [20], Zhang and Chen [22] and many others, the demand functions of two channels are furtherly modified as the following equations:

$$d_e(t) = \alpha_2 G_r(t) + \beta_2 G_e(t) + \tau_1 A_{me}(t) - \tau_2 A_{mr}(t) - \tau_3 A_{rr}(t) \tag{2.5}$$

$$d_r(t) = \alpha_1 G_r(t) - \beta_1 G_e(t) + \phi_1 A_{mr}(t) + \phi_2 A_{rr}(t) - \phi_3 A_{me}(t) \tag{2.6}$$

where, $\alpha_1(\alpha_1 > 0)$ is the influence factor of $G_r(t)$ for $d_r(t)$, and $\alpha_2(\alpha_2 > 0)$ is the positive spillover factor of the goodwill of retailing channel to the demand of electronic channel. $\beta_1(\beta_1 > 0)$ is the influence factor of $G_e(t)$ for $d_r(t)$, and $\beta_2(\beta_2 > 0)$ is the influence factor of $G_e(t)$ for $d_e(t)$. $\tau_1(\tau_1 > 0)$ is the influence factor of $A_{me}(t)$ for $d_e(t)$, $\tau_2(\tau_2 > 0)$ is the influence factor of $A_{mr}(t)$ for $d_e(t)$, and $\tau_3(\tau_3 > 0)$ is the influence factor of $A_{rr}(t)$ for $d_e(t)$. $\phi_1(\phi_1 > 0)$ is the influence factor of $A_{mr}(t)$ for $d_r(t)$, $\phi_2(\phi_2 > 0)$ is the influence factor of $A_{rr}(t)$ for $d_r(t)$, and $\phi_3(\phi_3 > 0)$ is the influence factor of $A_{me}(t)$ for $d_r(t)$.

In this article, we assume the manufacturer and the retailer have the same discount rate ρ. To figure out the optimal cooperative advertising investment strategy under the maximization of profits is the manufacturer and retailer's decision goals. ω_1 is the marginal profit of retailer in retail channel. ω_2, ω_3 are respectively set as the marginal profits of manufacturer in retail and direct channel. In addition, ω_1, ω_2 and ω_3 are regarded as constant during a certain period. Without loss of generality, we assume the marginal cost of sales for both manufacturer and retailer are zero. Based on the assumptions mentioned above, the profit functions for manufacturer and retailer are separately as below

$$J_r = \int_0^\infty \exp^{-\rho t}\{\omega_1(1-\rho_r(t))d_r(t) - (1-t_1(t))C(A_{rr}(t)) - t_2 C(A_{mr}(t))\}\mathrm{d}t \tag{2.7}$$

$$J_m = \int_0^\infty \exp^{-\rho t}\left\{\begin{matrix} \omega_2(1-\rho_r(t))d_r(t) + \omega_3(1-\rho_e(t))d_e(t) - t_1(t)C(A_{rr}(t)) \\ -(1-t_2)C(A_{mr}(t)) - C(A_{me}(t)) - C(\rho_r(t)) - C(\rho_e(t)) \end{matrix}\right\}\mathrm{d}t. \tag{2.8}$$

Considering that the parameters in the model are all time-independent constants and the game for manufacturer and retailer is same at instant of time, the strategy can be restricted as static strategy. In order to simplify the expression, the time t will be omitted in the following contents.

3. Model analysis

In this section, we discuss the centralized OAO supply chain and the decentralized OAO supply chain. In the centralized decision, maximizing the entire profits of the supply chain is the common goal of the manufacturer and the retailer. The manufacturer controls the manufacturer's advertising for direct channel A_{me} and the manufacturer's advertising for retail channel A_{mr}. The retailer's advertising for the traditional channel A_{rr} is controlled by the retailer. They make decisions simultaneously. Then, we study the decentralized decision under the Stackelberg game, led by the manufacturer. In the decentralized situation, both the manufacturer and the retailer make their own decisions in order to maximize their profits.

3.1. The centralized OAO supply chain

We all know that the supply chain always performs best under the centralized control. Under the centralized control, the manufacturer and the retailer can be seen as internal members of a firm. The common goal is to determine their optimal advertising decisions to maximize the total profit. Since the sharing proportions t_1, t_2 are only used to distribute the profit between the retailer and the manufacturer, so t_1 and t_2 are not the decision variables. Only A_{mr}, A_{me} and A_{rr} are decision variables.

Using the superscript c to denote the optimal advertising under the condition. Adding (2.7) to (2.8), the entire profit J_{m+r} can be obtain

$$J_{m+r} = \max_{A_{mr},A_{me},A_{rr}} \int_0^{\infty} \exp^{-\rho t} \left\{ (\omega_1+\omega_2)(1-\rho_r)d_r + \omega_3(1-\rho_e)d_e - \frac{\mu}{2}(A_{mr}^2 + A_{me}^2 + A_{rr}^2) \right\} \mathrm{d}t. \tag{3.1}$$

The optimal profit function must satisfy the following Hamilton-Jacobi-bellman equation

$$\rho V_{m+r}(G_r,G_e) = \max_{A_{mr},A_{me},A_{rr}} \left\{ \begin{array}{l} [(\omega_1+\omega_2)(1-\rho_r) - c_r\rho_r]\,(\alpha_1 G_r - \beta_1 G_e + \phi_1 A_{mr} + \phi_2 A_{rr} \\ -\phi_3 A_{me}) + [\omega_3(1-\rho_e) - c_e\rho_e]\,(\alpha_2 G_r + \beta_2 G_e + \tau_1 A_{me} - \tau_2 A_{mr} \\ -\tau_3 A_{rr}) + \dfrac{\partial V_{m+r}(G_r,G_e)}{\partial G_r}\,[\lambda(A_{mr}+A_{rr}) - \theta_1\rho_r - \delta G_r] \\ +\dfrac{\partial V_{m+r}(G_r,G_e)}{\partial G_e}(\lambda A_{me} - \theta_2\rho_e - \delta G_e) - \frac{\mu}{2}(A_{rr}^2 + A_{mr}^2 + A_{me}^2) \end{array} \right\} \tag{3.2}$$

where $V_{m+r}(G_r,G_e)$ is the optimal profit of the centralized OAO supply chain.

Similarly, set

$$V_{m+r}(G_r,G_e) = C_1 G_r + C_2 G_e + C_3. \tag{3.3}$$

Then substituting formula (3.3) into equation (3.2), then we get:

$$\begin{cases} C_1 = \dfrac{[(\omega_1+\omega_2)(1-\rho_r) - c_r\rho_r]\,\alpha_1 + [\omega_3(1-\rho_e) - c_e\rho_e]\,\alpha_2}{\rho+\delta} \\ C_2 = \dfrac{-\,[(\omega_1+\omega_2)(1-\rho_r) - c_r\rho_r]\,\beta_1 + [\omega_3(1-\rho_e) - c_e\rho_e]\,\beta_2}{\rho+\delta} \end{cases}. \tag{3.4}$$

Calculating the second-order partial derivatives of the right side of equation (3.2) with respect to A_{mr}, A_{me} and A_{rr}, we obtain Hessian matrix

$$H(A_{mr},A_{mer},A_{rr}) = \begin{bmatrix} -\mu & 0 & 0 \\ 0 & -\mu & 0 \\ 0 & 0 & -\mu \end{bmatrix} = -\mu^3 < 0,$$

which indicates that the profit function is strictly concave function with respect to A_{mr}, A_{me} and A_{rr}.

By figuring out the first-order condition for the right side of equation (3.2) with respect to A_{mr}, A_{me} and A_{rr}, we get

$$\begin{cases} A_{mr}^c = \dfrac{[(\omega_1+\omega_2)(1-\rho_r) - c_r\rho_r]\,\phi_1 - [\omega_3(1-\rho_e) - c_e\rho_e]\,\tau_2 + \lambda C_1}{\mu} \\ A_{me}^c = \dfrac{-\,[(\omega_1+\omega_2)(1-\rho_r) - c_r\rho_r]\,\phi_3 + [\omega_3(1-\rho_e) - c_e\rho_e]\,\tau_1 + \lambda C_2}{\mu} \\ A_{rr}^c = \dfrac{[(\omega_1+\omega_2)(1-\rho_r) - c_r\rho_r]\,\phi_2 - [\omega_3(1-\rho_e) - c_e\rho_e]\,\tau_3 + \lambda C_1}{\mu} \end{cases}$$

Theorem 3.1. *Under the centralized decision, the optimal advertising strategies for manufacturer and retailer are given by*

$$\begin{cases} A_{mr}^c = \dfrac{[(\omega_1+\omega_2)(1-\rho_r) - c_r\rho_r]\,\phi_1 - [\omega_3(1-\rho_e) - c_e\rho_e]\,\tau_2 + \lambda C_1}{\mu} \\ A_{me}^c = \dfrac{-\,[(\omega_1+\omega_2)(1-\rho_r) - c_r\rho_r]\,\phi_3 + [\omega_3(1-\rho_e) - c_e\rho_e]\,\tau_1 + \lambda C_2}{\mu} \\ A_{rr}^c = \dfrac{[(\omega_1+\omega_2)(1-\rho_r) - c_r\rho_r]\,\phi_2 - [\omega_3(1-\rho_e) - c_e\rho_e]\,\tau_3 + \lambda C_1}{\mu} \end{cases}$$

where, C_1, C_2 are given in formula (3.4).

To examine how customer returns rates effect the optimal advertising strategy, we differentiate A^c_{me}, A^c_{mr} and A^c_{rr} with respect to ρ_r, ρ_e and obtain the Corollary 3.2.

Corollary 3.2. *In a centralized OAO supply chain, the relationships between the optimal advertisement levels for manufacturer and retailer and returns rates are as follows*

$$(a)\quad \frac{\partial A^c_{me}}{\partial \rho_r} > 0, \frac{\partial A^c_{mr}}{\partial \rho_r} < 0, \frac{\partial A^c_{rr}}{\partial \rho_r} < 0;$$

$$(b)\quad \frac{\partial A^c_{me}}{\partial \rho_e} < 0, \begin{cases} \dfrac{\partial A^c_{mr}}{\partial \rho_e} > 0, \dfrac{\partial A^c_{rr}}{\partial \rho_e} > 0, \text{ if } \alpha_2 < \dfrac{\rho+\delta}{\lambda}\tau_{\min} \\ \dfrac{\partial A^c_{mr}}{\partial \rho_e} < 0, \dfrac{\partial A^c_{rr}}{\partial \rho_e} < 0, \text{ if } \alpha_2 > \dfrac{\rho+\delta}{\lambda}\tau_{\max} \\ \dfrac{\partial A^c_{mr}}{\partial \rho_e} \times \dfrac{\partial A^c_{rr}}{\partial \rho_e} < 0, \text{ if } \dfrac{\rho+\delta}{\lambda}\tau_{\min} < \alpha_2 < \dfrac{\rho+\delta}{\lambda}\tau_{\max} \end{cases}$$

where $\tau_{\min} = \min(\tau_2, \tau_3)$ and $\tau_{\max} = \max(\tau_2, \tau_3)$.

Corollary 3.2(a) indicates that A^c_{me} increases with increasing ρ_r, but A^c_{mr}, A^c_{rr} decrease when ρ_r increases. Because an increase in ρ_r means the return cost in retail channel increasing, the retailer and the manufacturer will advertise less advertising on traditional channel advertising but the manufacturer will advertise more advertising on direct online channel advertising. This means that the advertising will shift to the channel, in which the customer returns rate reduces. Corollary 3.2(b) shows that A^c_{me} decreases with increasing ρ_r, and A^c_{mr}, A^c_{rr} are related to ρ_e, α_2, τ_2 and τ_3. When α_2 is smaller, A^c_{mr}, A^c_{rr} will increase as ρ_e increases. If α_2 is larger and τ_2, τ_3 are smaller, A^c_{mr} and A^c_{rr} are inversely proportional to ρ_e, namely, A^c_{mr} and A^c_{rr} will increase with the reducing ρ_e. In addition, A^c_{mr} will increases and A^c_{rr} will decreases with the reducing ρ_e, or A^c_{mr} will decreases and A^c_{rr} will increases with the reducing ρ_e.

3.2. The decentralized OAO supply chain

Set the superscript $*$ to denote the optimal decision under the decentralized decision. We model the decision process as a sequential, Stackelberg game, with the manufacturer as the leader and the retailer as the follower. The Stackelberg game sequence is as follows: the manufacturer first decides A^*_{me}, A^*_{mr} , t^*_1, then the retailer determines A^*_{rr} and t^*_2. Each members profit in OAO supply chain is in the next:

$$J_r = \int_0^\infty \exp^{-\rho t}\{\omega_1(1-\rho_r)d_r - (1-t_1)C(A_{rr}) - t_2 C(A_{mr})\}\mathrm{d}t \tag{3.5}$$

$$J_m = \int_0^\infty \exp^{-\rho t}\left\{\begin{array}{l}\omega_2(1-\rho_r(t))d_r + \omega_3(1-\rho_e)d_e - t_1 C(A_{rr}) \\ -(1-t_2)C(A_{mr}) - C(A_{me}) - C(\rho_r) - C(\rho_e)\end{array}\right\}\mathrm{d}t. \tag{3.6}$$

Firstly, given A_{me}, A_{mr}, and t_1, we resolve the optimal advertising strategy of retailer. Supposing that $V_r(G_r, G_e)$ is the retailer's optimal profit, then $V_r(G_r, G_e)$ must satisfy the following HJB equation

$$\rho V_r(G_r, G_e) = \max_{A_{rr}, t_2}\left\{\begin{array}{l}\omega_1(1-\rho_r)(\alpha_1 G_r - \beta_1 G_e + \phi_1 A_{mr} + \phi_2 A_{rr} - \phi_3 A_{me}) - \dfrac{\mu(1-t_1)}{2} \\ A^2_{rr} - \dfrac{\mu t_2}{2}A^2_{mr} + \dfrac{\partial V_r(G_r, G_e)}{\partial G_r}[\lambda(A_{mr} + A_{rr}) - \theta_1\rho_r - \delta G_r] \\ +\dfrac{\partial V_r(G_r, G_e)}{\partial G_e}(\lambda A_{me} - \theta_2\rho_e - \delta G_e)\end{array}\right\}. \tag{3.7}$$

Because the second-order partial derivatives of the right side of equation (3.7) with respect to A_{rr} and t_2 are less than zero, we obtain t_2^*, A_{rr}^* through working out the first-order condition for the equation (3.7) with respect to t_2 and A_{rr}.

$$\begin{cases} t_1^* = 0 \\ C_2 = \dfrac{\omega_1(1-\rho_1)\phi_2 + \dfrac{\partial V_r(G_r,G_e)}{\partial G_r}\lambda}{\mu(1-t_1)} \end{cases} \tag{3.8}$$

Substituting (3.8) into equation (3.6). The manufacturer's optimal profit $V_m(G_r, G_e)$ must satisfy the following HJB equation.

$$\rho V_m(G_r,G_e) = \max_{A_{me},A_{mr},t_1} \left\{ \begin{array}{l} [\omega_2(1-\rho_r) - c_r\rho_r](\alpha_1 G_r - \beta_1 G_e + \phi_1 A_{mr} + \phi_2 A_{rr}^* - \phi_3 A_{me}) \\ + [\omega_3(1-\rho_e) - c_e\rho_e](\alpha_2 G_r + \beta_2 G_e + \tau_1 A_{me} - \tau_2 A_{mr} - \tau_3 A_{rr}^*) \\ + \dfrac{\partial V_m(G_r,G_e)}{\partial G_r}[\lambda(A_{mr} + A_{rr}^*) - \theta_1\rho_r - \delta G_r] + \dfrac{\partial V_m(G_r,G_e)}{\partial G_e} \\ (\lambda A_{me} - \theta_2\rho_e - \delta G_e) - \dfrac{\mu}{2}\left[t_1(A_{rr}^*)^2 + (1-t_2^*)A_{mr}^2 + A_{me}^2\right] \end{array} \right\}. \tag{3.9}$$

As the second-order partial derivatives of the right side of equation (3.9) with respect to A_{me} and A_{mr} are less than zero, by working out the first-order condition of the right side of equation (3.9) as for A_{me} and A_{mr}, we obtain

$$\begin{cases} A_{mr}^* = \dfrac{[\omega_1(1-\rho_r) - c_r\rho_r]\phi_1 - [\omega_3(1-\rho_e) - c_e\rho_e]\tau_2 + \lambda\dfrac{\partial V_m(G_r,G_e)}{\partial G_r}}{\mu} \\ A_{me}^* = \dfrac{-[\omega_2(1-\rho_r) - c_r\rho_r]\phi_3 + [\omega_3(1-\rho_e) - c_e\rho_e]\tau_1 + \lambda\dfrac{\partial V_m(G_r,G_e)}{\partial G_e}}{\mu} \end{cases}. \tag{3.10}$$

Denote

$$V_r(G_r,G_e) = M_1 G_r + M_2 G_e + M_3 \tag{3.11}$$
$$V_m(G_r,G_e) = N_1 G_r + N_2 G_e + N_3. \tag{3.12}$$

Substituting formula (3.11), (3.12) into equations (3.7) and (3.9) respectively, we get

$$\rho(M_1 G_r + M_2 G_e + M_3) = \max_{A_{rr},t_2} \left\{ \begin{array}{l} \omega_2(1-\rho_r)(\alpha_1 G_r - \beta_1 G_e + \phi_1 A_{mr} + \phi_2 A_{rr} - \phi_3 A_{me}) \\ -\dfrac{\mu(1-t_1)}{2}A_{rr}^2 - \dfrac{\mu t_2}{2}A_{mr}^2 + M_2[\lambda A_{me} - \theta_2\rho_e - \delta G_e] \\ +M_1[\lambda(A_{me} + A_{rr}) - \theta_1\rho_r - \delta G_r] \end{array} \right\} \tag{3.13}$$

$$\rho(N_1 G_r + N_2 G_e + N_3) = \max_{A_{me},A_{mr},t_1} \left\{ \begin{array}{l} [\omega_2(1-\rho_r) - c_r\rho_r](\alpha_1 G_r - \beta_1 G_e + \phi_1 A_{mr} + \phi_2 A_{rr} \\ -\phi_3 A_{me}) + [\omega_3(1-\rho_e) - c_e\rho_e](\alpha_2 G_r + \beta_2 G_e + \tau_1 A_{me} \\ -\tau_2 A_{mr} - \tau_3 A_{rr}) + N_1[\lambda(A_{mr} + A_{rr}) - \theta_1\rho_r - \delta G_r] \\ +N_2(\lambda A_{me} - \theta_2\rho_e - \delta G_e) - \dfrac{\mu}{2}(t_1 A_{rr}^2 + A_{mr}^2 + A_{me}^2) \end{array} \right\}. \tag{3.14}$$

By resolving equations (3.13) and (3.14), we obtain the following formulas.

$$\begin{cases} M_1 = \dfrac{\omega_1(1-\rho_r)\alpha_1}{\rho+\delta} \\ M_2 = -\dfrac{\omega_1(1-\rho_r)\beta_1}{\rho+\delta} \\ N_1 = \dfrac{\left[\omega_2(1-\rho_r)-c_r\rho_r\right]\alpha_1+\left[\omega_3(1-\rho_e)-c_e\rho_e\right]\alpha_2}{\rho+\delta} \\ N_2 = \dfrac{-\left[\omega_2(1-\rho_r)-c_r\rho_r\right]\beta_1+\left[\omega_3(1-\rho_e)-c_e\rho_e\right]\beta_2}{\rho+\delta} \end{cases}. \tag{3.15}$$

Next, we figure out the optimal advertising sharing proportion t_1.

Substituting (3.8), (3.10) and (3.15) into equation (3.9), we finally get formula (3.16) by solving the first-order condition for t_1.

$$t_1^* = \frac{2X-Y}{2X+Y} \tag{3.16}$$

where,

$$\begin{cases} Y = \omega_1(1-\rho_r)\phi_2+\lambda M_1 \\ X = \left[\omega_2(1-\rho_r)-c_r\rho_r\right]\phi_2-\left[\omega_3(1-\rho_e)-c_e\rho_e\right]\tau_3+\lambda N_1 \end{cases}. \tag{3.17}$$

Finally, settling formula (3.8), (3.10), (3.15) and (3.16), we acquire Theorem 3.3.

Theorem 3.3. *In the Stackelberg game, the optimal advertising and sharing proportion strategy for manufacturer and retailer are*

$$\begin{cases} A_{mr}^* = \dfrac{\left[\omega_2(1-\rho_r)-c_r\rho_r\right]\phi_1-\left[\omega_3(1-\rho_e)-c_e\rho_e\right]\tau_2+\lambda N_1}{\mu} \\ A_{me}^* = \dfrac{-\left[\omega_2(1-\rho_r)-c_r\rho_r\right]\phi_3+\left[\omega_3(1-\rho_e)-c_e\rho_e\right]\tau_1+\lambda N_2}{\mu} \\ A_{rr}^* = \dfrac{2X+Y}{\mu} \\ t_1^* = \begin{cases} \dfrac{2X-Y}{2X+Y}, & \text{if} \quad 2X-Y>0 \\ 0, & \text{if} \quad 2X-Y\le 0 \end{cases} \\ t_2^* = 0 \end{cases}$$

where, M_1, M_2, N_1, N_2 and X, Y are shown in formulas (3.15) and formulas (3.17).

Theorem 3.3 indicates that under the Stackelberg game the retailer does not support the manufacturer advertising, that is $t_2^* = 0$.

Corollary 3.4. *In decentralized OAO supply chain, the effect of customer returns rates on the sharing proportion of optimal advertising is as follows*

$$\begin{cases} \dfrac{\partial t_1^*}{\partial \rho_e} > 0, & \text{if} \quad \alpha_2 < \dfrac{\rho+\delta}{\lambda}\tau_3 \\ \dfrac{\partial t_1^*}{\partial \rho_r} > 0, & \text{if} \quad \alpha_2 > \dfrac{\rho+\delta}{\lambda}(\tau_3+A) \end{cases}$$

where,

$$A = \frac{c_r\left[(\rho+\delta)\phi_2+\lambda\alpha_1\right]}{(\rho+\delta)\left[(1-\rho_e)\omega_3-c_e\rho_e\right]} > 0.$$

Corollary 3.4 shows that the customer returns rates have an impact on the sharing proportion of the manufacturer, and the degree of influence mainly depends on the factors: α_2 and τ_3. When retailer advertising has a smaller impact factor on the electronic channel demand and the goodwill of traditional retail channel has a significant spillover effect on demand of electronic channel, the manufacturer will support less for the retailer advertising with the increasing customer returns rate of electronic channel. However, the manufacturer will support more for the retailer advertising with the increasing customer returns rate of traditional channel, when retailer advertising has a smaller impact factor on the electronic channel demand and the goodwill of traditional retail channel has a significant spillover effect on demand of electronic channel. Therefore, we can make full use of the positive spillover effect of traditional channel goodwill on electronic channel demand to gain more benefits.

Analyzing the effects of the customer return rates on the optimal advertising in Theorem 3.3, we achieve Corollary 3.5.

Corollary 3.5. *In the decentralized OAO supply chain, the effects of the customer return rates on the optimal advertising are as follows*

$$(a)\quad \frac{\partial A_{me}^*}{\partial \rho_r} > 0, \frac{\partial A_{mr}^*}{\partial \rho_r} < 0, \frac{\partial A_{rr}^*}{\partial \rho_r} < 0;$$

$$(b)\quad \frac{\partial A_{me}^*}{\partial \rho_e} < 0, \begin{cases} \dfrac{\partial A_{mr}^*}{\partial \rho_e} > 0, \dfrac{\partial A_{rr}^*}{\partial \rho_e} > 0, \ \text{if}\ \alpha_2 < \dfrac{\rho+\delta}{\lambda}\tau_{\min} \\ \dfrac{\partial A_{mr}^*}{\partial \rho_e} < 0, \dfrac{\partial A_{rr}^*}{\partial \rho_e} < 0, \ \text{if}\ \alpha_2 > \dfrac{\rho+\delta}{\lambda}\tau_{\max} \\ \quad \dfrac{\partial A_{mr}^*}{\partial \rho_e} \times \dfrac{\partial A_{rr}^*}{\partial \rho_e} < 0, \ \text{if}\ \dfrac{\rho+\delta}{\lambda}\tau_{\min} < \alpha_2 < \dfrac{\rho+\delta}{\lambda}\tau_{\max} \end{cases}.$$

where $\tau_{\min} = \min(\tau_2, \tau_3)$ and $\tau_{\max} = \max(\tau_2, \tau_3)$.

Corollary 3.5 shows that under decentralized control decision, the optimal advertising is closely related to ρ_r, ρ_e, α_2, τ_2 and τ_3. Corollary 3.5(a) indicates that with the increasing ρ_r, both A_{mr}^* and A_{rr}^* will reduce, but A_{me}^* will raise. This means advertising levels will shift to channel where customer returns rate becomes smaller. According to (b), we discover that: (1) A_{me}^* will increase with the declining ρ_e. (2) If $\alpha_2 < \frac{\rho+\delta}{\lambda}\tau_{\min}$, manufacturer and retailer will reduce their traditional channel advertising with the decline of customer returns rate in electronic channel. (3) Inversely, if $\alpha_2 > \frac{\rho+\delta}{\lambda}\tau_{\max}$, manufacturer and retailer will increase their traditional channels advertisement with the decreasing of customer returns rate in the electronic channel. (4) Otherwise, if $\frac{\rho+\delta}{\lambda}\tau_{\min} < \alpha_2 < \frac{\rho+\delta}{\lambda}\tau_{\max}$, with the decline in customer returns rate of the online channel, the advertising investments in the traditional channel for manufacturer and retailer are inconsistent.

From Corollary 3.2 and Corollary 3.5, the Corollary 3.6 is derived.

Corollary 3.6. *Whether under centralized or decentralized decision, there are*

$$(a)\quad \frac{\partial A_{me}}{\partial \rho_r} > 0, \frac{\partial A_{mr}}{\partial \rho_r} < 0, \frac{\partial A_{rr}}{\partial \rho_r} < 0;$$

$$(b)\quad \frac{\partial A_{me}}{\partial \rho_e} < 0, \begin{cases} \dfrac{\partial A_{mr}}{\partial \rho_e} > 0, \dfrac{\partial A_{rr}}{\partial \rho_e} > 0, \ \text{if}\ \alpha_2 < \dfrac{\rho+\delta}{\lambda}\tau_{\min} \\ \dfrac{\partial A_{mr}}{\partial \rho_e} < 0, \dfrac{\partial A_{rr}}{\partial \rho_e} < 0, \ \text{if}\ \alpha_2 > \dfrac{\rho+\delta}{\lambda}\tau_{\max} \\ \quad \dfrac{\partial A_{mr}}{\partial \rho_e} \times \dfrac{\partial A_{rr}}{\partial \rho_e} < 0, \ \text{if}\ \dfrac{\rho+\delta}{\lambda}\tau_{\min} < \alpha_2 < \dfrac{\rho+\delta}{\lambda}\tau_{\max} \end{cases}.$$

Corollary 3.6 shows that, whether under the centralized or the decentralized decision, the relationship between the optimal advertising strategy and the two channels' return rates is consistent. The cooperative advertising largely depends on the ratio of the influence factor of traditional channel goodwill on demand of electronic channel to the influence factors of manufacturer's and retailer's traditional channel advertising on demand of electronic channel.

4. Two-way advertising cost-sharing contract

Compared with the OAO centralized decision, the profits of the decentralized OAO supply chain are always lower, which means the decentralized OAO supply chain is inefficient. But, there is a fundamental question which needs to answer. Can we find a contract through which the incentives of the manufacturer and the retailer be aligned, so that they are willing to set their advertising equal to the optimal advertising to maximized supply chain profits? A two-way advertising cost-sharing contract, in which the players' advertising efforts equal to the optimal advertising under the centralized decision, is proposed for tackling the aforementioned problem. The manufacturer supports a fraction of the retailer's adverting for traditional channel, and a part of the manufacturer's advertising cost for traditional channel will be undertaken by the retailer. We assume that $t_1(0 < t_1 < 1)$ represents the proportion the manufacturer is willing to support the advertising advertised to the traditional channel by the retailer, and $t_2(0 < t_2 < 1)$ represents the sharing rate the retailer is willing to undertake the cost of manufacturer's advertising for traditional channel. So, the form of the two-way advertising cost-sharing contract is $(A^c_{mr}, A^c_{me}, A^c_{rr}, t_1, t_2)$, which will makes sure that the OAO supply chain is coordinated, because when each member adopts optimal advertising strategy under centralized control the total profit will increase.

Under the two-way advertising cost-sharing contract, the profits of manufacturer and retailer are as follows:

$$\begin{aligned}
\rho V_m^{\mathrm{DC}}(A^c_{mr}, A^c_{me}, A^c_{rr}, t_1, t_2) &= (\omega_2(1-\rho_r) - c_r\rho_r)d^c_r + (\omega_3(1-\rho_e) - c_e\rho_e)d^c_e \\
&\quad + \frac{[\omega_2(1-\rho_r) - c_r\rho_r]\,\alpha_1 + [\omega_3(1-\rho_e) - c_e\rho_e]\,\alpha_2}{\rho+\delta}(\lambda(A^c_{mr} + A^c_{rr}) - \theta_1\rho_r - \delta G_r) \\
&\quad + \frac{-[\omega_2(1-\rho_r) - c_r\rho_r]\,\beta_1 + [\omega_3(1-\rho_e) - c_e\rho_e]\,\beta_2}{\rho+\delta}(\lambda A^c_{me} - \theta_2\rho_e - \delta G_e) \\
&\quad - \frac{\mu}{2}(A^c_{me})^2 - \frac{\mu t_1}{2}(A^c_{rr})^2 - \frac{\mu(1-t_2)}{2}(A^c_{mr})^2 \\
\rho V_r^{\mathrm{DC}}(A^c_{mr}, A^c_{me}, A^c_{rr}, t_1, t_2) &= \omega_1(1-\rho_r)d^c_r - \frac{\mu(1-t_1)}{2}(A^c_{rr})^2 - \frac{\mu t_2}{2}(A^c_{mr})^2 \\
&\quad + \frac{\omega_1(1-\rho_r)\alpha_1}{\rho+\delta}(\lambda(A^c_{mr} + A^c_{rr}) - \theta_1\rho_r - \delta G_r) - \frac{\omega_1(1-\rho_r)\beta_1}{\rho+\delta}(\lambda A^c_{me} - \theta_2\rho_e - \delta G_e).
\end{aligned}$$

Only if each member's profit is higher than that under the decentralized control, the players will be willing to adopt the two-way advertising cost-sharing contract. According to the contract, the profits of each member must satisfy the following conditions. If the formula (4.1) is satisfied, the manufacturer will accept the two-way advertising cost-sharing contract with pleasure.

$$V_m^{\mathrm{DC}} > V_m^*. \tag{4.1}$$

By the formula (4.1), one can get $\rho V_m^{\mathrm{DC}} > \rho V_m^*$. Therefor the advertising sharing rates (t_1, t_2) must satisfies the formula (4.2):

$$\begin{aligned}
&[\omega_2(1-\rho_r) - c_r\rho_r]\,d^c_r + [\omega_3(1-\rho_e) - c_e\rho_e]\,d^c_e - \frac{\mu}{2}(A^c_{me})^2 - \frac{\mu t_1}{2}(A^c_{rr})^2 - \frac{\mu(1-t_2)}{2}(A^c_{mr})^2 \\
&+ \frac{[\omega_2(1-\rho_r) - c_r\rho_r]\,\alpha_1 + [\omega_3(1-\rho_e) - c_e\rho_e]\,\alpha_2}{\rho+\delta}[\lambda(A^c_{mr} + A^c_{rr}) - \theta_1\rho_r - \delta G_r] \\
&+ \frac{-[\omega_2(1-\rho_r) - c_r\rho_r]\,\beta_1 + [\omega_3(1-\rho_e) - c_e\rho_e]\,\beta_2}{\rho+\delta}[\lambda A^c_{me} - \theta_2\rho_e - \delta G_e] - \rho V_m^* > 0.
\end{aligned} \tag{4.2}$$

Similarly, the retailer is willing to accept the two-way advertising cost-sharing contract if the formula (4.3) is satisfied.

$$V_r^{\mathrm{DC}} > V_r^*. \tag{4.3}$$

That is the advertising sharing rates (t_1, t_2) must satisfy the formula (4.4).

$$\begin{aligned} &\omega_1(1-\rho_r)d_r^c - \frac{\mu(1-t_1)}{2}(A_{rr}^c)^2 - \frac{\mu t_2}{2}(A_{mr}^c)^2 + \frac{\omega_1(1-\rho_r)\alpha_1}{\rho+\delta}[\lambda(A_{mr}^c + A_{rr}^c) - \theta_1\rho_r - \delta G_r] \\ &- \frac{\omega_1(1-\rho_r)\beta_1}{\rho+\delta}[\lambda A_{me}^c - \theta_2\rho_e - \delta G_e] - \rho V_r^* > 0. \end{aligned} \tag{4.4}$$

Theorem 4.1. *In OAO supply chain, there must exist an advertising sharing contract that enables the supply chain to achieve a coordinated state under which each member can accept the advertising decisions under centralized control. Furthermore, the profits of the manufacturer and the retailer all realize Pareto improvements.*

Proof. From formula (4.1) and formula (4.3), we acquire formula (4.5).

$$V_m^{\mathrm{DC}} + V_r^{\mathrm{DC}} > V_m^* + V_r^*. \tag{4.5}$$

Through the formula (4.2) and formula (4.4), we can obtain the formula (4.6).

$$\left\{\begin{aligned} &[(\omega_1+\omega_2)(1-\rho_r) - c_r\rho_r]\, d_r^c + [\omega_3(1-\rho_e) - c_e\rho_e]\, d_e^c - \frac{\mu}{2}(A_{mr}^c)^2 \\ &-\frac{\mu}{2}(A_{me}^c)^2 - \frac{\mu}{2}(A_{rr}^c)^2 + P_1\left[\lambda(A_{mr}^c + A_{rr}^c) - \theta_1\rho_r - \delta G_r\right] + \\ &P_2\left[\lambda A_{me}^c - \theta_2\rho_e - \delta G_e\right] - \rho(V_m^* + V_r^*) \end{aligned}\right\} > 0 \tag{4.6}$$

where,

$$\begin{aligned} P_1 &= \frac{[(\omega_1+\omega_2)(1-\rho_r) - c_r\rho_r]\,\alpha_1 + [\omega_3(1-\rho_e) - c_e\rho_e]\,\alpha_2}{\rho+\delta} \\ P_2 &= \frac{-[(\omega_1+\omega_2)(1-\rho_r) - c_r\rho_r]\,\beta_1 + [\omega_3(1-\rho_e) - c_e\rho_e]\,\beta_2}{\rho+\delta}. \end{aligned}$$

We can further get $\rho(V_{m+r}^{\mathrm{DC}} - V_{m+r}^*) > 0$ from formula (29), which is equivalent to $V_{m+r}^{\mathrm{DC}} > V_{m+r}^*$. When the sharing proportions belong to set $\left\{(t_1, t_2) | 0 < t_1 < 1, 0 < t_2 < 1, V_m^{\mathrm{DC}} > V_m^*, V_r^{\mathrm{DC}} > V_r^*\right\}$, compared with the profit of decentralized decisions, each member will acquire more profit under the centralized decisions, and be willing to accept the two-way advertising cost-sharing contract. This completes the proof of the Theorem 4.1. $\square$

5. Numerical analysis

We numerically examine how the two channels' return rates effect the optimal advertising and sharing ratios. Set $\alpha_1 = 0.5$, $\beta_1 = 0.1$, $\phi_1 = 1$, $\phi_2 = 1$, $\phi_3 = 0.3$, $\beta_2 = 0.5$, $\tau_1 = 1.5$, $\tau_2 = 0.3$, $\tau_3 = 0.4$, $\lambda = 0.3$, $\delta = 0.01$, $\rho = 0.05$, $\omega_1 = 0.5$, $\omega_2 = 0.3$, $\omega_3 = 0.8$, $\mu_1 = 1$, $c_e = 0.1$, $c_r = 0.3$, $\theta_1 = 0.2$, $\theta_2 = 0.1$.

5.1. The impact of customer returns rates on cooperative advertisements under centralized decision

In this subsection, other parameters are kept constant, while the values of the customer return rate of electronic/traditional channel ρ_e/ρ_r and the positive spillover factor of the goodwill of traditional channel product to the demand of electronic channel α_2 are changed. We focus on the impact of customer returns rate of electronic/traditional channel ρ_e/ρ_ron each member's optimal advertising under the centralized decision, as summarized in Figures 1 and 2.

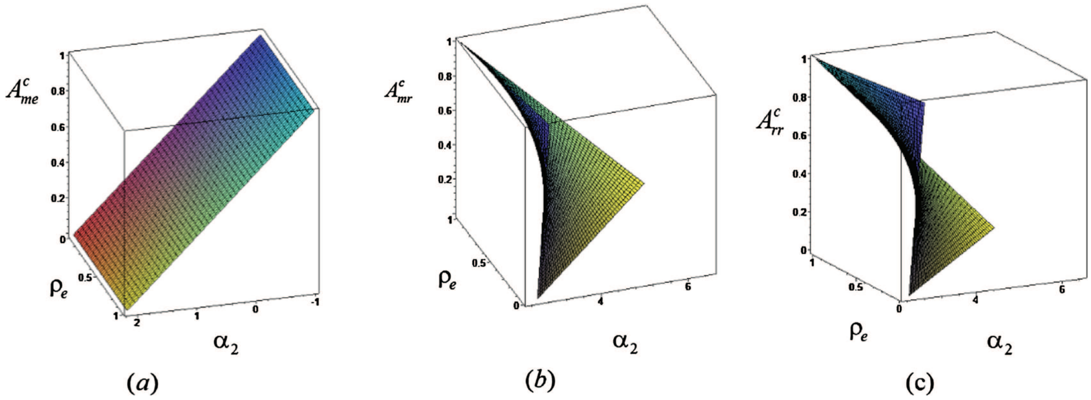

FIGURE 1. The impact of ρ_e on the cooperative advertisements under centralized decision.

When the customer returns rate of traditional channel ρ_r is fixed, we research the impact of the electronic channel's customer returns rate ρ_e on the optimal advertising efforts of each member under centralized decision, as is shown in Figure 1. From Figure 1a, we find that the manufacturer's advertising for electronic channel increase with the electronic channel's customer returns rate decreases, and the manufacturer's advertising for electronic channel is uncorrelated with α_2. Figure 1b and c illustrate that, (1) when α_2 is smaller, the advertisements, that the manufacturer and the retailer advertise to the traditional channel, increase along with the increase in the customer returns rate of direct channel. (2) However, when α_2 is larger, the manufacturer and retailer will all reduce the optimal advertising efforts for traditional channel as the customer returns rate of direct channel increases. (3) In addition, when the positive spillover effect of the goodwill of the traditional channel on demand of the electronic channel is larger enough, the manufacturer will reduce the advertising efforts for traditional channel and even reduce it to zero, and the interesting thing is that the retailer will also reduce her advertising levels for the traditional channel. This is because of that under the centralized decision-making, the advertising efforts will shift from the channel in which the customer returns rate is increasing to the channel in which the customer returns rate is decreasing. Then the total profit of the OAO supply chain system is improved.

Figure 2 illustrates the impact of the traditional channel's customer returns rate (ρ_r) on the optimal advertising efforts of each member under centralized decision, keeping the customer returns rate of electronic channel constant. From Figure 2a, we obtain that manufacturer's advertisement for electronic channel is uncorrelated with α_2, but manufacturer's advertisement for electronic channel is positively correlated with the traditional channel's customer returns rate. When the traditional channel's customer returns rate increases, the manufacturer's advertisement for electronic channel will increase. Figure 2b and c show that with the decreasing the customer return rate for traditional channel, both the advertising of the manufacturer and the retailer for traditional channel will increase. The conclusions from Figure 2 are consistent with Corollary 3.2.

5.2. The impact of customer returns rates on cooperative advertisements under decentralized decision

First, keeping other parameters constant, we explore the effect of electronic channel's customer returns rate ρ_e on the optimal advertisements of each member under decentralized decision by changing ρ_e and α_2, as shown in Figure 3. Similarly, by changing ρ_r and α_2, we research the impact of the traditional channel's customer returns rate ρ_r on the optimal advertisements of each member under decentralized decision, as shown in Figure 4.

From Figure 3a, we find that the advertisement of the manufacturer for electronic channel is irrelevant to α_2, but it negatively correlated with the electronic channel's customer returns rate. That is, the manufacturer's

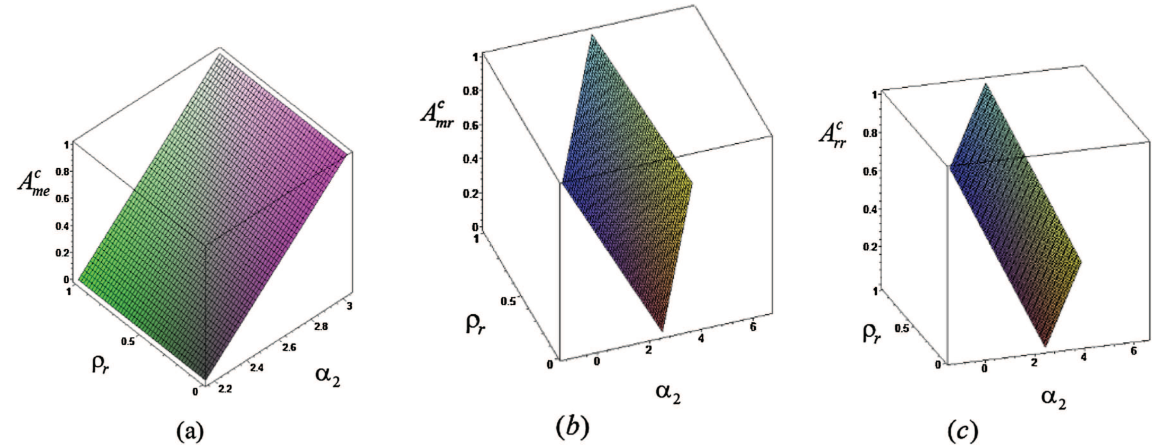

FIGURE 2. The impact of ρ_r on the cooperative advertisements under centralized decision.

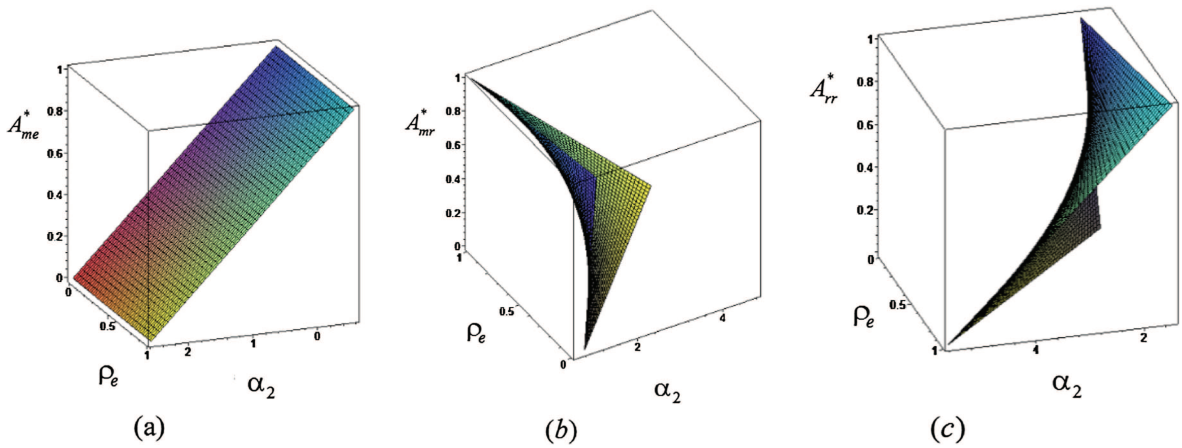

FIGURE 3. The impact of ρ_e on the cooperative advertisements under decentralized decision.

advertisement in electronic channel will increase as the electronic channel's customer return rate decreases. When the electronic channel's customer return rate decreases, the manufacturer will increase the level of advertising in electronic channels. This is consistent with Corollary 3.6. From Figure 3b and c illustrate when α_2 is smaller, the traditional adverting efforts will all increase as the customer returns rate of online channel increases. Inversely, if α_2 is larger, the manufacturer and the retailer's traditional advertising efforts will be cut down with the increase of the online channel's returns rates. In addition, from Figure 3b and c, we also discover that the manufacturer will reduce the traditional advertising and even reduce it to zero if the positive spillover effect of the goodwill of the traditional channel on demand of electronic channel (α_2) is larger. It is interesting that retailer will also reduce the adverting efforts of traditional channel.

Figure 4a shows that the manufacturer's advertisement for electronic channel is irrespective of α_2 but it will increase as the traditional channel's customer returns rate increases. From Figure 4b and c, we discover that the manufacturer and the retailer will advertise more on traditional channel's advertising with the decrease in the customer returns rate of traditional channel. From Figure 4, we also conclude that the advertisement that the manufacturer advertises to the direct channel, is positively related to the customer returns rate in traditional

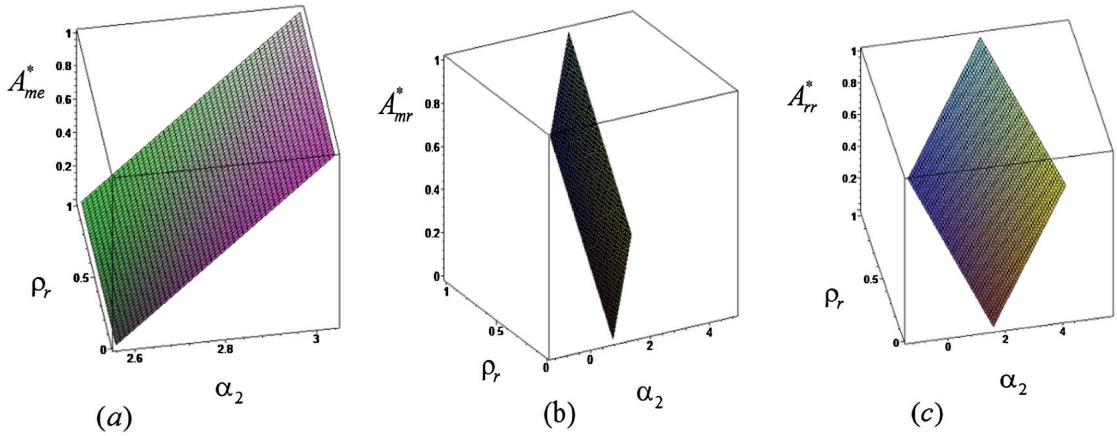

FIGURE 4. The impact of ρ_r on the cooperative advertisements under decentralized decision.

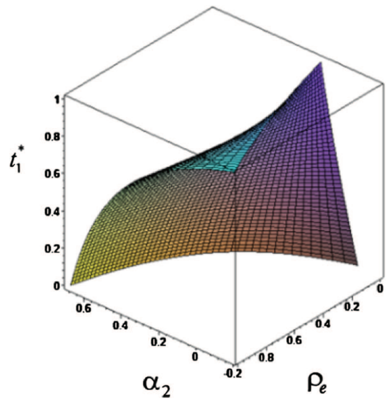

FIGURE 5. The impact of ρ_e on the manufacturer's share rate.

channel, inversely, the advertisements which the manufacturer and the retailer advertise to the traditional channel are all negatively related to the customer returns rate in the traditional channel.

5.3. The impact of customer returns rates on the manufacturer's share rate

As shown in Figure 5, we study the impact of the direct channel's customer returns rate on the manufacturer's share rate by changing the values of ρ_e and α_2. Figure 5 illustrates that if α_2 is smaller, the manufacturer's share rate will increase as the direct channel's customer returns rate increases; However, if α_2 is larger, the manufacturer's share rate will decrease as the direct channel's customer returns rate increases. Figure 6 elucidates the impact of the traditional channel's customer returns rate on the manufacturer's sharing proportion. From Figure 6, we can conclude that when α_2 is smaller, the manufacturer's sharing proportion will increase as the traditional channel's customer returns rate increases; But when α_2 is larger, the manufacturer's sharing proportion will decrease as the traditional channel's customer returns rate increases. The conclusions from Figures 5 and 6 are consistent with Corollary 3.4.

Set $\alpha_2 = 0.1$, $G_r = 2$, $G_e = 1$, according to the parameters assumption above, we obtain the equilibrium advertising of manufacturer and retailer and the total profit in the centralized OAO supply chain and the decentralized OAO supply chain, as well as the manufacturer's share rate under decentralized

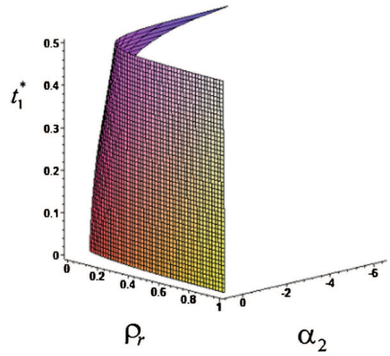

FIGURE 6. The impact of ρ_r on the manufacturer's share rate.

decision. The optimal advertising efforts and total profit are $A_{me}^c = 1.26$, $A_{mr}^c = 2.17$, $A_{rr}^c = 2.66$ and $V_{m+r}^c = 173.59$ under the centralized decision. Under the decentralized decision, the equilibrium advertisements are separately $A_{me}^c = 1.64$, $A_{mr}^c = 1.05$, $A_{rr}^c = 1.83$; The sharing ratio of the manufacturer is $t_1^* = 9.1\%$. What's more, $V_m^* = 78.28$ and $V_r^* = 59.31$ are the profits of manufacturer and retailer respectively. Sharing rates (t_1, t_2) must satisfy the following set in the two-way advertising cost-sharing contract $A = \{(t_1, t_2)|0 < t_1 < 1, 0 < t_2 < 1, 3.54t_1 - 3.68t_2 + 2.27 > 0, 3.68t_2 - 3.54t_1 - 0.47 > 0\}$. As long as (t_1, t_2) belongs to A, the OAO supply chain can be coordinated. From the numerical example above, is not empty. When the share ratios are $t_1 = 0.4$ and $t_2 = 0.6$, the supply chain can reach coordination, because $(0.4, 0.6)\epsilon A$. Similarly, the sharing proportions, which can make the OAO supply chain coordinated, can be determined by using the above method. The specific sharing proportions depend on each player's bargaining ability in OAO supply chain. Compared with the decentralized decision, the total profits of the supply chain under the centralized decision increase by 26.2%. We can get that, in the OAO supply chain, each player is willing to adopt the two-way advertising cost-sharing contract to make co-operative advertising strategies, as the contract can help increase the profitability of the supply chain considering the consumer return.

6. Conclusions

This paper investigates the cooperative advertising strategies of a manufacturer and a retailer under customer return for OAO supply chain. The stochastic game model is used to obtain the equilibrium advertising and advertising share rates under the centralized and decentralized decision-making, based on which we analyze the impact of the return rates on sharing rates and cooperative advertising strategies. Researches show that: (1) The advertising of online channel decreases with the increase in customer returns rate of online channel. (2) The advertisements in traditional channel for manufacturer and retailer are relevant to the positive spillover effect of goodwill on electronic channel in traditional channel as well as return rates. (3) The optimal decision strategy for the retailer in advertising cost sharing proportion is not to undertake the share on retail channel advertising input by the manufacturer. Eventually, we design a two-way advertising cost-sharing contract, which ensures the OAO supply chain coordinated and achieved Pareto improvement. Additionally, this contract can provide a reference when they implement cooperative advertising strategy in the OAO supply chain considering return rates.

We have used stochastic differential game models to derive meaningful conclusions, which facilitate our understandings of the cooperative advertising strategies when the customer return is taken into consideration for the OAO supply chain. The following are some interesting directions on this issue for future research. First, in the current study, we assume the information between members of the supply chain is completed information. Future research may extend to asymmetric information between players. Second, as we assume the online and retail channels sell the identical and single product, another possible extension is to consider that the online channel and offline channel sell different and competing products. Third, in the current study, we study the cooperative advertising between a single manufacturer and a single retailer, so we can naturally extend to study a supply chain with multiple retailers. Finally, another possible extension is to consider the returned products only resold in the secondary-market. All these issues deserve further research.

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References

[1] G. Aust and U. Buscher, Vertical cooperative advertising and pricing decisions in a manufacturer-retailer supply chain: A game-theoretic approach. *Eur. J. Oper. Res.* **223** (2012) 473–482.

[2] G.P. Chen, X.M. Zhang and J. Xiao, Coordination model for cooperative advertising between manufactures and retailers in dual-channel supply chain. *J. Syst. Manag.* **26** (2017) 1168–1175.

[3] G.Y. Chen and N. Liu, Coordination of dual-channel supply chain under product differentiation. *J. Ind. Eng. Manag.* **25** (2011) 239–244.

[4] Y. He, Q.L. Guo, C.X. Wu and X.H. Yue, Cooperative advertising in a supply chain with horizontal competition. *Math. Probl. Eng.* **2013** (2013) 1–16.

[5] Z.M. Huang and S.X. Li, Co-op advertising models in manufacturer-retailer supply chains: A game theory approach. *Eur. J. Oper. Res.* **135** (2001) 527–544.

[6] Z.M. Huang, S.X. Li and V. Mahajan, An analysis of manufacturer-retailer supply chain coordination in cooperative advertising. *Decis. Sci.* **33** (2002) 469–494.

[7] Z.S. Huang, J.J. Nie and J.X. Zhang, Dynamic cooperative promotion models with competing retailers and negative promotional effects on brand image. *Comput. Ind. Eng.* **118** (2018) 291–308.

[8] A.A. Javid and P. Hoseinpour, A game-theoretic analysis for coordinating cooperative advertising in a supply chain. *J. Optim. Theor. Appl.* **149** (2011) 138–150.

[9] S. Jørgensen, S. Taboubi and G. Zaccour, Cooperative advertising in a marketing channel. *J. Optim. Theor. Appl.* **110** (2001) 145–158.

[10] S. Karray and G. Martín-Herrán, A dynamic model for advertising and pricing competition between national and store brands. *Eur. J. Oper. Res.* **193** (2009) 451–467.

[11] S. Karray, G. Martín-Herrán and S.-P. Sigué, Cooperative advertising for competing manufacturers: The impact of longterm promotional effects. *Int. J. Prod. Econ.* **184** (2017) 21–32.

[12] M.M. SeyedEsfahani, M. Biazaran and M. Gharakhani, A game theoretic approach to coordinate pricing and vertical co-op advertising in manufacturer-retailer supply chains. *Eur. J. Oper. Res.* **221** (2011) 263–273.

[13] J.X. Xie and A. Neyret, Co-op advertising and pricing model in manufacturer-retailer supply chains. *Comput. Ind. Eng.* **56** (2009) 1375–1385.

[14] M.H. Xu and X.D. Zuo, An optimal dynamic advertising model with inverse inventory sensitive demand effect for deteriorating items. *J. Syst. Sci. Syst. Eng.* **26** (2017) 593–608.

[15] R.L. Yan, Product brand differentiation and dual-channel store performances of a multi-channel retailer. *Eur. J. Mark.* **44** (2010) 636–642.

[16] F. Yang, Supply chain models with considerations of co-op advertising and capacity constraints. *Open J. Bus. Manag.* **6** (2018) 518–538.

[17] J. Yang, J.X. Xie, X.X. Deng and H.C. Xiong, Cooperative advertising in a distribution channel with fairness concerns. *Eur. J. Oper. Res.* **227** (2013) 401–407.

[18] J.F. Yue, J. Austin, M.C. Wang and Z.M. Huang, Coordination of cooperative advertising in a two-level supply chain when manufacturer offers discount. *Eur. J. Oper. Res.* **168** (2006) 65–85.

[19] J. Zhang, Q.L. Gou, S.S. Li and Z.M. Huang, Cooperative advertising with accrual rate in a dynamic supply chain. *Dyn. Games Appl.* **7** (2017) 112–130.

[20] J. Zhang, Q.L. Gou, L. Liang and Z.M. Huang, Supply chain coordination through cooperative advertising with reference price effect. *Omega* **41** (2013) 345–353.

[21] J.X Zhang, J.Q. Li, L.H. Lu and R. Dai, Supply chain performance for deteriorating items with cooperative advertising. *J. Syst. Sci. Syst. Eng.* **26** (2017) 23–49.

[22] X.M. Zhang and G.P. Chen, Coordination model for cooperative advertising in dual-channel supply chain under product brand differentiation. *J. Ind. Eng. Manag.* **30** (2016) 152–159.