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A GEOMETRIC PROGRAMMING APPROACH FOR A VENDOR MANAGED INVENTORY OF A MULTIRETAILER MULTI-ITEM EPQ MODEL

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Abstract. Due to the uncertain situations of the world, considering inventory management in a stochastic environment gains a lot of interest. In this paper, we propose a multi-item economic production quantity (EPQ) model with a shortage for a single-vendor, multi-retailer supply chain under vendor managed inventory (VMI) policy in a stochastic environment. Three stochastic constraints are developed in the model. Geometric programming (GP) approach is employed to find the optimal solution of the nonlinear stochastic programming problem to minimize the mean-variance of the total inventory cost of the system. Since the problem is in the Signomial form, first, an algorithm is used to convert the model into the standard GP form. The performance of the addressed model and the solving method are evaluated based on computational experiments and sensitivity analysis. A case study in an Iranian furniture supply chain is conducted to show the applicability of the proposed model and 17.78% improvement in terms of total cost is gained.

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1. Introduction

Fulfilling customers' demand plays a crucial role in the success of organizations. In supply chain management (SCM), a series of organizations integrate and cooperate with a specific end goal to enhance the competitive capacities of the whole chain [1]. Due to the uncertainty in the business paradigm, individual businesses no longer compete solely but rather work together as a supply chain, make the need and necessity for effective decision support systems [2]. Vendor managed inventory (VMI) is one of the collaboration mechanism that gathered a lot of interest recently and has been adopted by the successful retail businesses such as Wal-Mart, JC Penney, Dillard Department stores, Intel and Shell [3]. Initially, VMI originated in the retail industry to conquer some of the problems such as the amount of the retail storage space, the measure of inventory to be kept on hand, the inventory obsolescence, and the logistics of returning items [4,5]. Reduction in costs, inventory requirements, improvement in the customer service and a dramatic reduction in the bullwhip effect in SCs are some of the advantages of implementing VMI [6,7]. Several authors dedicated their works to the study of VMI

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partnership in supply chain management. Magee [8] described an early theoretical framework of VMI regarding the question of who should have the authority to control the inventories. However, the concept has only become widespread during the 1990s.

Dong and Xu's study estimated the effect of VMI in the long term and short term on supply chain's members within the EOQ framework. They proved that VMI always leads to a higher buyer's profit. In the long term, VMI is a successful production network technique that can increase the supplier's profit compared to the short run [3]. Using the same assumptions as Dong and Xu's, Yao *et al.* [9] presented an EOQ analytical model that explores how key logistics parameters, most remarkably ordering costs and inventory carrying charges, can affect the benefits which derived from VMI. Pasandideh *et al.* [10] extended an analytical model to investigate the impact of important supply chain parameters on the cost savings obtained from investigation of vendor-managed inventory. They developed their inventory model in the light of EOQ where the shortage is backlogged and examines their model with and without the implementation of VMI. Pasandideh *et al.* [11] investigated a VMI problem based on EOQ model, with the shortage. They proposed a genetic algorithm (GA) based heuristic to solve the model. Pasandideh *et al.* [12] investigated the vendor-managed inventory problem of a single-vendor single-buyer supply chain framework. The multi-product EPQ model with shortage was considered and a genetic algorithm (GA) based heuristic was proposed to solve the model. Mokhtari and Rezvan [13] proposed a single-buyer multi-product VMI model for a supply chain problem, with the use of a production-inventory system in which shortage was allowed and partially backordered. They proposed a decomposition based analytical approach to solve the model. Recently, Alfarez *et al.* considered a VMI and a consignment stock (CS) partnership and assumed that the products contain a given amount of defective units. They proposed three mathematical models for VMI-CS system, traditional and integrated system [14]. Ramrakhyani *et al.* considered VMI system with consignment inventory policy. They presented eight inventory SC models and compered the profit function of supplier and manufacturer in different environments to show the fruitfulness of the SCM system in a manufacturing industry [15]. Pasandideh *et al.* modeled the joint replenishment problem for a two-level supply chain under VMI policy. The objective of their work was to find the optimal number of order in both traditional and VMI policy. They applied a metaheuristic approach to solve their model [16].

In the literature, the performance measure of channels and supply chains are either maximizing the expected profit or minimizing the predictable cost [17]. However, the importance of the expected objective function profoundly depends on the associated variance. For example, if the target is minimizing the cost, the expected cost is naturally a performance measure. However, if the variance of the cost is large, the chance of deviating from the expected cost will be high. If the supply chain does not operate under the same status for a long run, using the expected measure as the only objective is insufficient. Moreover, each decision makers may consider a different degree of risk aversion; therefore, find an effective way to incorporate the risk aversion into an appropriate and implementable decision context is of great value [17].

The mean-variance formulation, which is a fundamental theory of risk management, was introduced by Markowitz in the 1950s. When it comes to studying decision-making problems with risk concerns, the mean-variance (MV) approach and the Von Neumann–Morgenstern utility (VNMU) approach are two well-established methodologies. Due to limitations in the VNMU application, the MV approach is accomplished to provide an implementable and useful solution [17]. Although both VNMU and MV approaches have great importance and popularity, their applications in supply chain management are not investigated thoroughly; works of Lau [18], Bassok and Nagarajan [19], Buzacott *et al.* [20], and Gan *et al.* [21], are some examples in the supply chain management area. To the best of the authors' knowledge, no study investigates VMI with MV performance measure. However, under stochastic conditions, Kiesmuller and Broekmeulen [22] investigated the advantage of VMI in a stochastic multi-product system dealing with slow-moving items. Lee and Ren [23] investigated the impact of exchange rate uncertainty in a single-vendor VMI system with stochastic demand and determined the optimal replenishment policy under these conditions.

The rest of the paper is organized as follows. In Section 2, the problem definition and the assumptions are presented. In Section 3, the problem is formulated into a non-linear stochastic programming model. A Geometric Programming (GP) approach would be used to solve the problem in Section 4. In order to demonstrate the performance of the proposed approach, computational experiments, sensitivity analysis and solution method comparison are presented in Section 5. A case study is proposed in Section 6. Finally, Section 7 concludes the paper with a summary and future direction.

2. Problem statement

In the traditional supply chain, unlike the VMI policy, the vendor observes the customer's demand indirectly and the retailer appears to be the "leader" in this relationship. Not having any responsibility for holding the products, the vendor only takes the order quantity from the retailer and satisfies the demand by making the necessary deliveries. After the implementation of VMI, the retailer no longer manages its inventory system and the vendor's information system directly receives consumer's demand. Moreover, the vendor is responsible for order setup and holding cost; therefore, the vendor has a combined inventory [3]. Unlike the former system, the vendor and the retailer act as a single unit in a VMI system. They work based on an agreement which is the main idea of VMI and admitted by both parties. According to this agreement, the vendor establishes and manages the inventory control policies. Therefore, in this article, it is assumed that the vendor pays the ordering and holding costs on behalf of retailers and retailers pay no cost.

This research has been motivated by the work of Pasandideh *et al.* [12]. We address a single-vendor multi-item multi-constraint EPQ model with the shortage in the form of backorder for an SC under the VMI policy in this research. The EPQ model is utilized with practical instances of finite production rate for this problem. Moreover, the lead-time is less than a day can be neglected and the selling prices are constant during the planning horizon. However, unlike the model of Pasandideh *et al.* [12] a multi-retailer system is assumed and to bring the model more applicable to the real world issues, additional contractual agreements between the vendor and the retailers including constraints on the storage capacity, the number of orders, and the available budget are considered in stochastic form. To make the model more applicable, the ordering fixed cost of the vendor, the ordering fixed cost of retailers, the holding cost, and the fixed backorder cost are assumed stochastic. In addition, a mean-variance analysis of the problem is considered. To the best of the authors' knowledge no study has been investigated VMI with MV performance measure. The purpose of the model is to determine the optimal order quantities along with the maximum backorder levels of the product in a cycle, so that the total cost of the system is minimized while the constraints are fulfilled. Therefore, geometric programming (GP) approach is employed. Since the model contains Signomial terms and is not in the standard GP form, an algorithm is proposed first to make the model in the standard GP form, and then solve the model efficiently. For further validation, similar numerical examples are solved with both GP approach in GGPLAB solver of MATLAB software and BARRON solver of GAMS software, and the results are compared to each other.

2.1. Notations

The following notations are used to model the problem.

For $i = 1, 2, \ldots, r$ and $j = 1, 2, \ldots, n$ define the parameters and variables of the model as:

- r Number of retailers
- n Number of products
- O_j Fixed ordering cost of vendor per order of the jth product
- O_{ij} Fixed ordering cost of retailer i per order of the jth product
- d_{ij} Annual demand for retailer i for product j
- d_j Vendor's total annual demand of product j ($d_j = \sum_{i=1}^{r} d_{ij}$)
- P_j Production rate of product j in each period
- π Fixed backorder cost per unit (not depending on the time)
- $\hat{\pi}$ Fixed backorder cost per unit per time unit
- h_j Holding cost of product j per unit ($h_j = \hat{i}C_j$)

$\hat{i}$	Fixed interest rate (rate of the holding cost that is not dependent on the time)
C_j	Retailer's procurement cost per unit of product j
s_j	Space occupied by each unit of product j
S	Vendor's available storage capacity for all products
N	Total number of orders for all products in each cycle
B	Total available budget in each cycle
α	The probability of violating each of stochastic constraints
q_{ij}	Order quantity of retailer i for product j in a cycle
q_j	Order quantity of product j in a cycle ($q_j = \sum_{i=1}^{r} q_{ij}$)
b_{ij}	Backorder level of retailer i for product j
b_j	Maximum backorder level of product j in a cycle ($b_j = \sum_{i=1}^{r} b_{ij}$)
$\mathrm{TC}_{\mathrm{VMI}}$	Total inventory costs of the VMI supply chain
$\mathrm{KR}_{\mathrm{VMI}}$	Retailer's inventory cost after utilizing the VMI system
$\mathrm{KV}_{\mathrm{VMI}}$	Vendor's inventory cost after VMI

3. Model development

In this section, the model will be proposed. As mentioned before, after the implementation of VMI, the vendor takes the responsibility of managing the inventory levels, order quantities, and lead-time. We assume that the vendor pays the ordering and holding cost on behalf of the buyer [24]. Therefore, unlike the traditional systems, the retailer will pay no cost.

$$\mathrm{KR}_{\mathrm{VMI}} = 0. \tag{3.1}$$

According to Pasandideh *et al.* [12], after both parties accept the VMI contract, the inventory cost of both the retailer and the vendor, and therefore the total inventory costs of the whole integrated chain, are calculated as follows:

$$\begin{aligned} \mathrm{TC}_{\mathrm{VMI}} &= \mathrm{KR}_{\mathrm{VMI}} + \mathrm{KV}_{\mathrm{VMI}} \\ &= \sum_{j=1}^{n}\left(\frac{d_j O_j}{q_j}\right) + \sum_{i=1}^{r}\sum_{j=1}^{n}\left(\frac{d_{ij}O_{ij}}{q_{ij}} + \frac{\hat{\pi}_{ij}+h_j}{2\rho_j q_{ij}} b_{ij}^2 - b_{ij}h_j + \frac{\rho_j q_{ij} h_j}{2}\right)\cdot \end{aligned} \tag{3.2}$$

To make the model more applicable, the ordering fixed cost of the vendor, the ordering fixed cost of retailers, the holding cost, and the fixed backorder cost are assumed stochastic. Considering a mean-variance analysis for the problem, the objective function would be:

$$\begin{aligned} E(\mathrm{TC}_{\mathrm{VMI}}) &= E(\mathrm{KR}_{\mathrm{VMI}}) + E(\mathrm{KV}_{\mathrm{VMI}}) \\ &= \sum_{j=1}^{n}\left(\frac{d_j E(O_j)}{q_j}\right) + \sum_{i=1}^{r}\sum_{j=1}^{n}\left(\frac{d_{ij}E(O_{ij})}{q_{ij}} + \frac{E(\hat{\pi}_{ij})+E(h_j)}{2\rho_j q_{ij}} b_{ij}^2 - b_{ij}E(h_j) + \frac{\rho_j q_{ij} E(h_j)}{2}\right) \end{aligned} \tag{3.3}$$

$$\begin{aligned} \mathrm{Var}(\mathrm{TC}_{\mathrm{VMI}}) &= \mathrm{Var}(\mathrm{KR}_{\mathrm{VMI}}) + \mathrm{Var}(\mathrm{KV}_{\mathrm{VMI}}) \\ &= \sum_{j=1}^{n}\left(\left(\frac{d_j}{q_j}\right)^2 \mathrm{var}(O_j)\right) \\ &\quad + \sum_{i=1}^{r}\sum_{j=1}^{n}\left(\left(\frac{d_{ij}}{q_{ij}}\right)^2 \mathrm{var}(O_{ij}) + \frac{b_{ij}^4}{4\rho_j^2 q_j^2}(\mathrm{var}(\hat{\pi}_{ij}) + \mathrm{var}(h_j)) + b_{ij}^2 \mathrm{var}(h_j) + \frac{\rho_j^2 q_{ij}^2}{4}\mathrm{var}(h_j)\right). \end{aligned} \tag{3.4}$$

As mentioned before to make the model closer to reality, three constraints including the available storage capacity, the total number of order for all items, and the available budget are assumed in a stochastic form in the model.

The capacity of the vendor's warehouse space to store the items is limited to S with a probability greater than α and since the average inventory of the jth item is $(q_j - b_j)$, the space constraint will be:

$$P\left(\sum_{j=1}^{n} s_j(q_j\rho_j - b_j) \leq S\right) \geq \alpha. \tag{3.5}$$

Moreover, the total number of order for all items is limited to N with a probability greater than α.

$$P\left(\sum_{i=1}^{r}\sum_{j=1}^{n} \frac{d_{ij}}{q_{ij}} \leq N\right) \geq \alpha. \tag{3.6}$$

The total available budget is limited to B with a probability greater than α.

$$P\left(\sum_{j=1}^{n} C_j q_j \leq B\right) \geq \alpha. \tag{3.7}$$

Assuming a normal distribution with a mean μ and variance σ^2, the constraints would be:

$$\sum_{j=1}^{n} s_j(q_j\rho_j - b_j) \leq \mu_S - Z_\alpha\sigma_S \tag{3.8}$$

$$\sum_{i=1}^{r}\sum_{j=1}^{n} \frac{d_{ij}}{q_{ij}} \leq \mu_N - Z_\alpha\sigma_N \tag{3.9}$$

$$\sum_{j=1}^{n} C_j q_j \leq \mu_B - Z_\alpha\sigma_B. \tag{3.10}$$

According to the above mentioned the multi-item multi-retailer EPQ model under VMI policy and in a stochastic environment could be obtained as:

$$\begin{aligned}\text{Min}\, E(\text{TC}_{\text{VMI}}) = &\sum_{j=1}^{n}\left(\frac{d_j E(O_j)}{q_j}\right)\\ &+ \sum_{i=1}^{r}\sum_{j=1}^{n}\left(\frac{d_{ij}E(O_{ij})}{q_{ij}} + \frac{E(\hat{\pi}_{ij}) + E(h_j)}{2\rho_j q_{ij}} b_{ij}^2 - b_{ij}E(h_j) + \frac{\rho_j q_{ij} E(h_j)}{2}\right) \qquad (3.11)\\ \text{Min}\,\text{Var}(\text{TC}_{\text{VMI}}) = &\sum_{j=1}^{n}\left(\left(\frac{d_j}{q_j}\right)^2 \text{var}(O_j)\right)\\ &+ \sum_{i=1}^{r}\sum_{j=1}^{n}\left(\left(\frac{d_{ij}}{q_{ij}}\right)^2 \text{var}(O_{ij}) + \frac{b_{ij}^4}{4\rho_j^2 q_j^2}\Big(\text{var}(\widehat{\pi}_{ij}) + \text{var}(h_j)\Big) + b_{ij}^2 \text{var}(h_j)\right.\\ &\left.+ \frac{\rho_j^2 q_{ij}^2}{4}\text{var}(h_j)\right). \qquad (3.12)\end{aligned}$$

Subject to satisfying:

$$\sum_{j=1}^{n} s_j(q_j\rho_j - b_j) \le \mu_S - Z_\alpha \sigma_S$$

$$\sum_{i=1}^{r}\sum_{j=1}^{n} \frac{d_{ij}}{q_{ij}} \le \mu_N - Z_\alpha \sigma_N$$

$$\rho_j = 1 - \frac{d_j}{p_j} \quad \forall\ j \tag{3.13}$$

$$h_j = \hat{i}C_j \quad \forall\ j \tag{3.14}$$

$$q_j = \sum_{i=1}^{r} q_{ij} \quad \forall\ j \tag{3.15}$$

$$b_j = \sum_{i=1}^{r} b_{ij} \quad \forall\ j \tag{3.16}$$

$$q_{ij} \ge 0 \tag{3.17}$$

$$b_{ij} \ge 0. \tag{3.18}$$

The goal is to determine the values of the order quantities (q_{ij}, q_j) and the maximum backorder level (b_{ij}, b_j) in a cycle so that the total cost of the supply chain under the VMI system is minimized while the constraints are fulfilled.

4. Solution approach

Most decision makers are risk averse. As it is mentioned before, two popular approaches for handling risk are Markowitz mean-variance model [25], and von Neuman Morgenstern expected utility model [26]. In mean-variance approach, the risk is equated with the variance. Assuming the variance as a surrogate for risk, the expected value for a given level of risk is minimized. An appropriate choice for the objective function would be the mean plus a constant (λ) times the variance [27]. Therefore, the objective function of this problem becomes:

$$\begin{aligned} \text{Min}\ & \left(\sum_{j=1}^{n}\left(\frac{d_j E(O_j)}{q_j}\right) + \sum_{i=1}^{r}\sum_{j=1}^{n}\left(\frac{d_{ij}E(O_{ij})}{q_{ij}} + \frac{E(\hat{\pi}_{ij}) + E(h_j)}{2\rho_j q_{ij}} b_{ij}^2 - b_{ij}E(h_j) + \frac{\rho_j q_{ij} E(h_j)}{2}\right)\right) \\ & + \lambda\left(\sum_{j=1}^{n}\left(\left(\frac{d_j}{q_j}\right)^2 \text{var}(O_j)\right)\right) + \sum_{i=1}^{r}\sum_{j=1}^{n}\left(\left(\frac{d_{ij}}{q_{ij}}\right)^2 \text{var}(O_{ij}) + \frac{b_{ij}^4}{4\rho_j^2 q_j^2}\left(\text{var}\left(\widehat{\pi}_{ij}\right) + \text{var}(h_j)\right)\right. \\ & \left. + b_{ij}^2 \text{var}(h_j) + \frac{\rho_j^2 q_{ij}^2}{4}\text{var}(h_j)\right). \end{aligned} \tag{4.1}$$

The proposed formulation is a nonlinear programming model. In this article, geometric programming (GP) approach is utilized for solving the problem. Although geometric programming restricts the form of the objective and constraint functions, it gives extremely efficient and reliable solution methods even for large-scale problems [28].

Geometric programming (GP) term was presented by Duffin, Peterson, and Zener in 1967 in their book "geometric programming: Theory and Application" [29]. A Geometric Programing (GP) is a kind of mathematical optimization problem. The objective and constraint functions in GP have a specific form.

Solving a GP problem requires two steps, first, detecting the feasibility and second detecting whether the constraints are in the standard form of GP or not, these steps are known as phase 1.

A geometric program is an optimization problem of the form:

$$\begin{array}{l} \min f_0(x) \\ \text{subject to:} \\ f_i(x) \leq 1, \quad i = 1, \ldots, m, \\ g_i(x) = 1, \quad i = 1, \ldots, p, \end{array} \tag{4.2}$$

where f_i are posynomial functions, g_i are monomials, and x_i are the optimization variables. The problem (4.2) is a geometric program in standard form. In a standard GP problem, the objective function must be posynomial and minimized. The equality restrictions can only have the form of monomial equal to one, and the inequality ones must be in the form of a posynomial less than or equal to one. There is an implied restriction that the variables are positive.

In some cases, the objective and constraint functions present posynomial functions contain negative coefficient; this type of problems belongs to signomial programming [28]. Unlike posynomials, signomial geometric programming (SGP) problems are not globally convex therefore, non-linear optimization problems, which contains signomials, are generally solved harder, but signomial optimization problems are more realistic for real-world non-linear problems.

4.1. Converting the model into standard GP form

This paper takes advantage of the global optimization approach proposed by Xu [30] for solving SGP problems. Some transformation and convexification strategies, which was suggested by Xu [30], are applied to convert the original problem into a sequence of standard geometric programming problems that can be solved to reach a global solution.

It is obvious that the first objective function and the first constraint contain negative terms and therefore they are not in the standard GP form, so; the proposed model is in the form of SGP.

$$\begin{aligned} \text{Min } E(\text{TC}_{\text{VMI}}) = & \sum_{j=1}^{n} \left(\frac{d_j E(O_j)}{q_j} \right) \\ & + \sum_{i=1}^{r} \sum_{j=1}^{n} \left(\frac{d_{ij} E(O_{ij})}{q_{ij}} + \frac{E(\hat{\pi}_{ij}) + E(h_j)}{2\rho_j q_{ij}} b_{ij}^2 - b_{ij} E(h_j) + \frac{\rho_j q_{ij} E(h_j)}{2} \right) + \Phi \\ \text{Min Var}(\text{TC}_{\text{VMI}}) = & \sum_{j=1}^{n} \left(\left(\frac{d_j}{q_j} \right)^2 \text{var}(O_j) \right) \\ & + \sum_{i=1}^{r} \sum_{j=1}^{n} \left(\left(\frac{d_{ij}}{q_{ij}} \right)^2 \text{var}(O_{ij}) + \frac{b_{ij}^4}{4\rho_j^2 q_j^2} (\text{var}(\widehat{\pi}_{ij}) + \text{var}(h_j)) + b_{ij}^2 \text{var}(h_j) \right. \\ & \left. + \frac{\rho_j^2 q_{ij}^2}{4} \text{var}(h_j) \right). \end{aligned} \tag{4.3}$$

Subject to satisfying:

$$\sum_{j=1}^{n} s_j (q_j \rho_j - b_j)(\mu_S - Z_\alpha \sigma_S)^{-1} \leq 1 \tag{4.4}$$

$$\sum_{i=1}^{r} \sum_{j=1}^{n} \left(\frac{d_{ij}}{q_{ij}} \right) (\mu_N - Z_\alpha \sigma_N)^{-1} \leq 1 \tag{4.5}$$

$$\sum_{j=1}^{n} (C_j q_j)(\mu_B - Z_\alpha \sigma_B)^{-1} \leq 1 \tag{4.6}$$

and constraints number (3.13)–(3.18).

Where $\Phi > 0$ is a sufficiently large value to keep equation (4.3) always positive.

Then, an additional variable x_0 is used to create a linear objective function. The first objective function is transformed as the first constraint and the rest of constraints are rearrange into quotient form. Since the second objective function does not contain any negative term, it remains without any change.

$$\text{Min } x_0 \tag{4.7}$$

$$\begin{aligned}\text{Min } \ \operatorname{Var}(\mathrm{TC}_{\mathrm{VMI}}) = &\sum_{j=1}^{n}\left(\left(\frac{d_j}{q_j}\right)^2 \operatorname{var}(O_j)\right)\\ &+\sum_{i=1}^{r}\sum_{j=1}^{n}\left(\left(\frac{d_{ij}}{q_{ij}}\right)^2 \operatorname{var}(O_{ij}) + \frac{b_{ij}^4}{4\rho_j^2 q_j^2}\left(\operatorname{var}(\widehat{\pi}_{ij}) + \operatorname{var}(h_j)\right) + b_{ij}^2 \operatorname{var}(h_j)\right.\\ &\left.+\ \frac{\rho_j^2 q_{ij}^2}{4}\operatorname{var}(h_j)\right).\end{aligned}$$

Subject to satisfying:

$$\frac{\left(\sum\limits_{j=1}^{n}\left(\frac{d_j E(O_j)}{q_j}\right) + \sum\limits_{i=1}^{r}\sum\limits_{j=1}^{n}\left(\frac{d_{ij}E(O_{ij})}{q_{ij}} + \frac{E(\hat{\pi}_{ij})+E(h_j)}{2\rho_j q_{ij}} b_{ij}^2 + \frac{\rho_j q_{ij} E(h_j)}{2}\right)\right) + \Phi}{\sum\limits_{i=1}^{r}\sum\limits_{j=1}^{n}(b_{ij}E(h_j)) + x_0} \leq 1 \tag{4.8}$$

$$\frac{\sum\limits_{j=1}^{n}(s_j(q_j\rho_j))}{\left(\sum\limits_{j=1}^{n} b_j\right) + 1}(\mu_S - Z_\alpha\sigma_S)^{-1} \leq 1 \tag{4.9}$$

and constraints number (3.13)–(3.18), (4.5), and (4.6).

Constraints (4.8) and (4.9) are not still allowable in standard GP form. To tackle this difficulty, denominators of these constraints are approximated with monomial functions, but their posynomial numerators remain unchanged. The required monomial approximation can be calculated using the following arithmetic geometric mean approximation [30].

With the use of the posynomial function $g(x) = \sum_v u_v(x)$ with $u_v(x)$ being the monomial terms, the following expression is computed:

$$g(x) \geq \hat{g}(x) = \prod_v \left(\frac{u_v(x)}{\alpha_v(y)}\right)^{\alpha_v(y)} \tag{4.10}$$

where the parameters $\alpha_v(y)$ can be obtained by computing

$$\alpha_v(y) = \frac{u_v(y)}{g(y)}, \quad \forall\ v \tag{4.11}$$

where y is a fixed point with $y > 0$. It can be easily verified that $\hat{g}(y)$ is the best local monomial approximation of $g(x)$ near y [28]. Applying the presented monomial approximation method to each denominator of constraints

(4.8) and (4.9) we get the following optimization problem:

$$\text{Min } x_0$$

$$\begin{aligned}\text{Min }\ \text{Var}(\text{TC}_{\text{VMI}}) = &\sum_{j=1}^{n}\left(\left(\frac{d_j}{q_j}\right)^2 \text{var}(O_j)\right)\\ &+\sum_{i=1}^{r}\sum_{j=1}^{n}\left(\left(\frac{d_{ij}}{q_{ij}}\right)^2 \text{var}(O_{ij}) + \frac{b_{ij}^4}{4\rho_j^2 q_j^2}(\text{var}(\widehat{\pi}_{ij}) + \text{var}(h_j)) + b_{ij}^2 \text{var}(h_j)\right.\\ &\left.+\ \frac{\rho_j^2 q_{ij}^2}{4}\text{var}(h_j)\right).\end{aligned}$$

Subject to satisfying:

$$\frac{\left(\sum_{j=1}^{n}\left(\frac{d_j E(O_j)}{q_j}\right) + \sum_{i=1}^{r}\sum_{j=1}^{n}\left(\frac{d_{ij}E(O_{ij})}{q_{ij}} + \frac{E(\hat{\pi}_{ij})+E(h_j)}{2\rho_j q_{ij}} b_{ij}^2 + \frac{\rho_j q_{ij} E(h_j)}{2}\right)\right) + \Phi}{\hat{f}_0^-(b_{ij}, x_0)} \le 1 \qquad (4.12)$$

$$\frac{\sum_{j=1}^{n}(s_j(q_j\rho_j))}{\hat{f}_{12}^-(b_j)}(\mu_S - Z_\alpha\sigma_S)^{-1} \le 1 \qquad (4.13)$$

and constraints number (3.13)–(3.16), (4.5), and (4.6).

In this problem, $\hat{f}_0^-(b_{ij}, x_0)$ and $\hat{f}_{12}^-(b_j)$ are the corresponding monomial approximation of posynomial functions $\sum_{i=1}^{r}\sum_{j=1}^{n}(b_{ij}E(h_j)) + x_0$ and $(\sum_{j=1}^{n} b_j) + 1$ respectively. They have the following formulations:

$$\hat{f}_0^-(b_{ij}, x_0) = \prod_{i=1}^{r}\prod_{j=1}^{n}\left(\frac{b_{ij}E(h_j)}{\alpha_{ij}}\right)^{\alpha_{ij}}\left(\frac{x_0}{\alpha_x}\right)^{\alpha_x} \qquad (4.14)$$

$$\hat{f}_{12}^-(b_j) = \prod_{j=1}^{n}\left(\frac{b_j}{\alpha_j}\right)^{\alpha_j}\left(\frac{1}{\alpha_y}\right) \qquad (4.15)$$

where α_{ij}, α_x, α_j, and α_y can be computed by using (4.11) as follows:

$$\alpha_{ij} = \frac{(b_{ij}E(h_j))^{(r)}}{\left(\sum_{i=1}^{r}\sum_{j=1}^{n}(b_{ij}E(h_j)) + x_0\right)^{(r)}} \quad \forall j \qquad (4.16)$$

$$\alpha_x = \frac{x_0^{(r)}}{\left(\sum_{i=1}^{r}\sum_{j=1}^{n}(b_{ij}E(h_j)) + x_0\right)^{(r)}} \quad \forall j \qquad (4.17)$$

$$\alpha_j = \frac{b_j^{(r)}}{\left(\left(\sum_{j=1}^{n} b_j\right) + 1\right)^{(r)}} \quad \forall j \qquad (4.18)$$

$$\alpha_y = \frac{1}{\left(\left(\sum_{j=1}^{n} b_j\right) + 1\right)^{(r)}} \quad \forall j. \qquad (4.19)$$

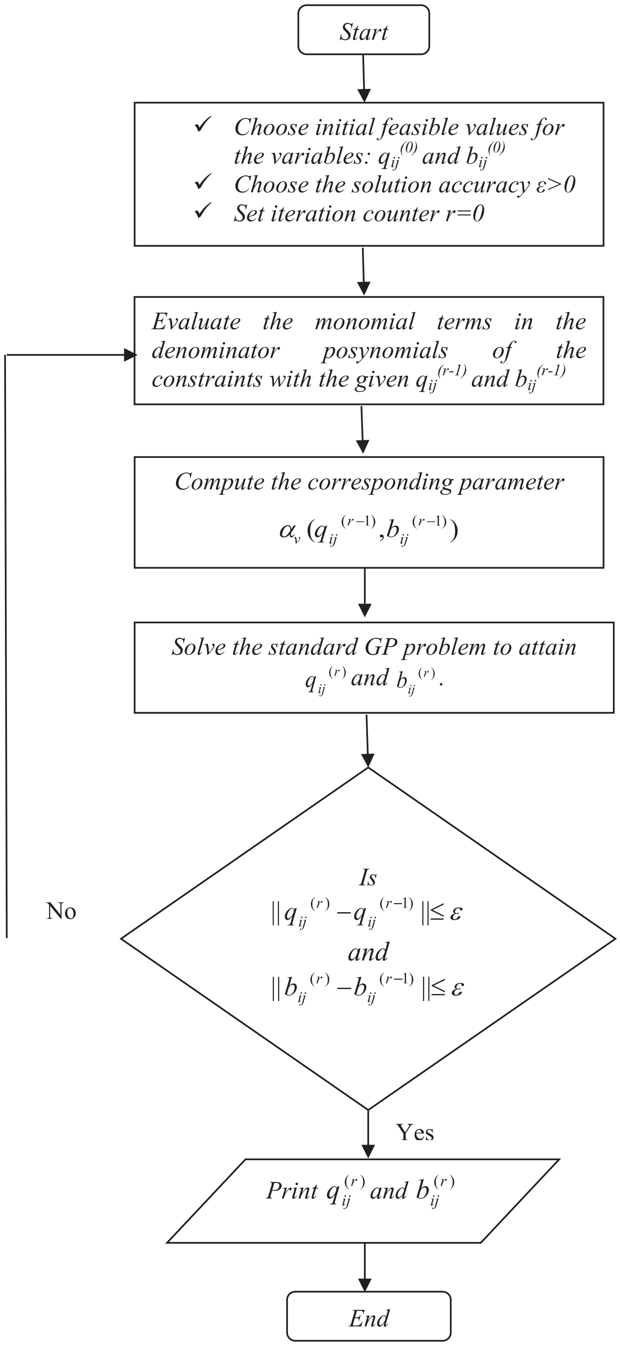

FIGURE 1. The basic steps of the GP algorithm.

Without loss of generaity constraints (3.17) and (3.18) will change to $q_{ij} > 0$ and $b_{ij} > 0$ to fit the model to the standard form. With these changes, the proposed model is in the standard GP form. Therefore, it can be solved efficiently [28]. Now we use the iterative algorithm proposed by Xu [30] to reach the optimal solution of the problem. The basic steps of the algorithm are given as a flowchart (Fig. 1).

The iterative solution of problem converges to a point satisfying the KKT conditions of the original SGP problem [31].

According to the presented iterative method, the optimal solution of the SGP problem can be found efficiently.

5. Computational experiments and Sensitivity analysis

In order to demonstrate the performance of the proposed procedure and to study its performances, computational experiments are given in this section. Since no benchmark is available, random data as shown in Table 1 is used for numerical examples. For each parameter, an interval with lower and upper bound is considered and for each examples data are generated from this interval. It should be noted that, for all examples $\hat{\imath} = 0.3$, $\varepsilon = 10^{-4}$, and $\lambda = 0.4$. The mean and variance value for stochastic parameters are calculated first.

In addition, test problems with the different number of retailers and products are solved with both BARRON solver of GAMS software and also GP approach coded in GGPLAB solver of MATLAB software. The objective function value and CPU time will be compared. All the test problems are solved on a personal computer with Intel corei5-5200U processor having 2.20 GHz CPU and 8 GB RAM. Furthermore, the GP algorithm is coded using MATLAB R2014a software.

As mentioned before, in order to solve the multi-objective mean-variance model the ultimate objective function is considered the mean plus a constant (λ) times the variance. Therefore, equation (4.1) is considered as the ultimate objective function of the model. Then, the problem will be solved with the help of a commercial solver. Since the model is a nonlinear programming, the solver cannot guarantee to reach the optimal solution.

Table 1. The range of data for numerical examples.

Parameter	Interval
d_{ij}	$\sim U$ [15–25]
A_j	$\sim U$ [6–10]
A_{ij}	$\sim U$ [1–5]
$\hat{\pi}_{ij}$	$\sim U$ [2–5]
C_j	$\sim U$ [35–45]
P_j	$\sim U$ [60–120]
f_j	$\sim U$ [1–5]
F	$\sim U$ [500–900]
M	$\sim U$ [600–1100]
X	$\sim U$ [1500–2500]

Table 2. The total cost comparison of GP and GAMS.

Example	Retailers	Products	Total cost ($)		Variance	Improvement (%)
	i	j	BARRON	GP/MATLAB		
1	2	2	80.826	64.899	12.904	19.705
2	2	3	210.498	177.283	37.194	15.778
3	2	4	266.951	216.523	55.988	18.890
4	3	2	183.879	138.409	27.924	24.727
5	3	3	248.203	209.187	37.243	15.719
6	3	4	475.545	383.370	71.147	19.382
7	4	2	244.349	190.720	39.847	21.947
8	4	3	457.489	357.948	57.559	21.758
9	4	4	781.543	649.784	93.744	16.858

Table 3. The CPU time comparison of GP and GAMS.

Example	Retailers	Products	CPU time (s)	
	i	j	BARRON	GP
1	2	2	2.12	3.10
2	2	3	3.34	7.67
3	2	4	4.77	15.32
4	3	2	5.67	10.73
5	3	3	6.51	17.95
6	3	4	6.97	21.36
7	4	2	7.32	19.70
8	4	3	7.72	22.02
9	4	4	8.12	25.92

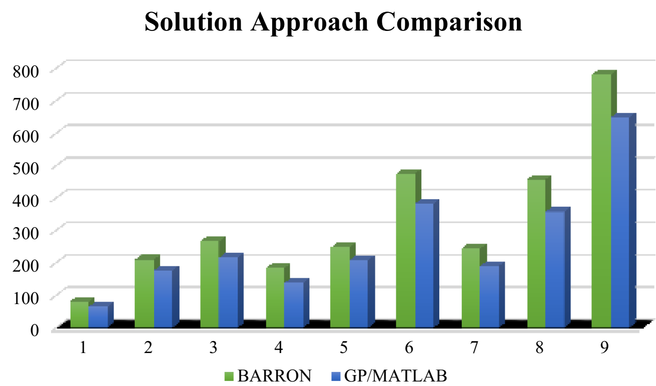

Figure 2. The total cost comparison trend of GP and GAMS.

Therefore, GP approach is conducted to find the optimal solution. In addition, to compare the proposed GP approach, results obtained from GP approach in GGPLAB solver of MATLAB software are compared with the results from BARON solver of GAMS software in terms of objective function value and CPU time. To measure risks the variance of the total costs are reported, too. This information is tabulated below (Tabs. 2 and 3).

It is clear that GP approach outperforms GAMS in term of total cost; however, GP obtains more CPU time than BARRON does. For the sake of brevity, the solution approaches comparison is demonstrated in Figure 2.

For further validation, sensitivity analyses are done on some important parameters of the objective function as well as constraints, so the accuracy of the model will be discussed. For this purpose, the objective function parameters from 10 to 100% of the initial value will be increased and the accuracy of the model will be checked.

It is obvious that the fixed ordering cost is paid each time we have an order therefore with increase in the vendor's fixed ordering cost in order to decrease the number of times this cost is paid, the order quantity in each order is increased. In addition, due to the increase in the vendor's fixed ordering cost the optimal value of the objective function would be increased (Figs. 3 and 4).

When the holding cost increases, willingness for storing will decrease and that leads to an increase in the backorder level. Similar to the increase in other cost parameters, an increase in holding cost makes the optimal value of the objective function increased (Figs. 5 and 6).

As expected, the increase in the total available budget makes the whole chain more capable of producing, so the order quantity will be increased (Fig. 7).

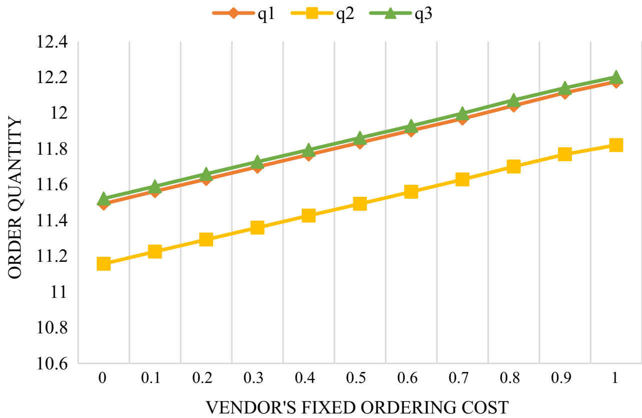

FIGURE 3. Increasing in order quantity with increasing in vendors fixed ordering cost.

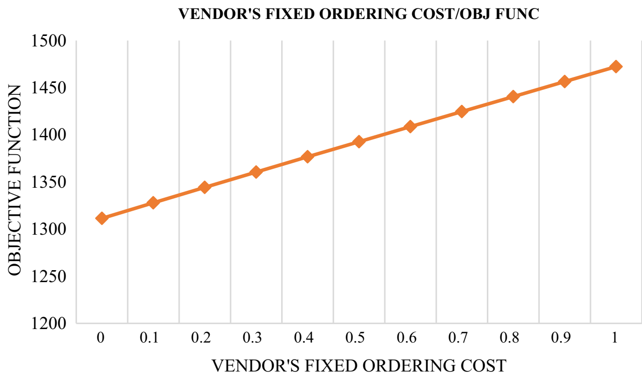

FIGURE 4. Increasing in objective function value with increasing in vendors fixed ordering cost.

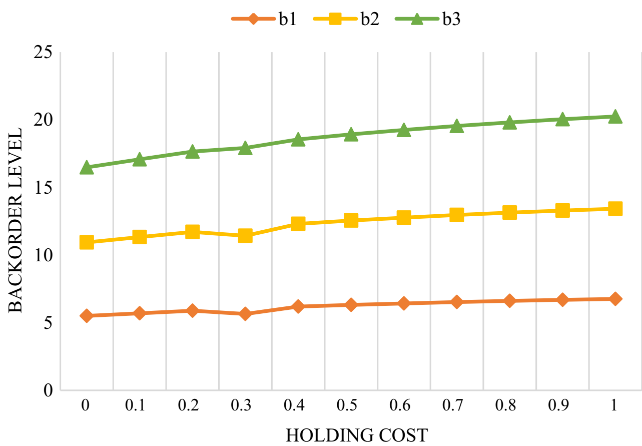

FIGURE 5. Increasing in backorder level with increasing in holding cost.

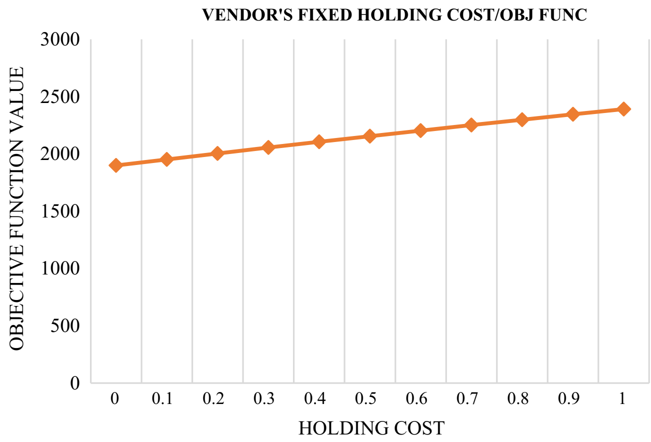

Figure 6. Increasing in objective function value with increasing in holding cost.

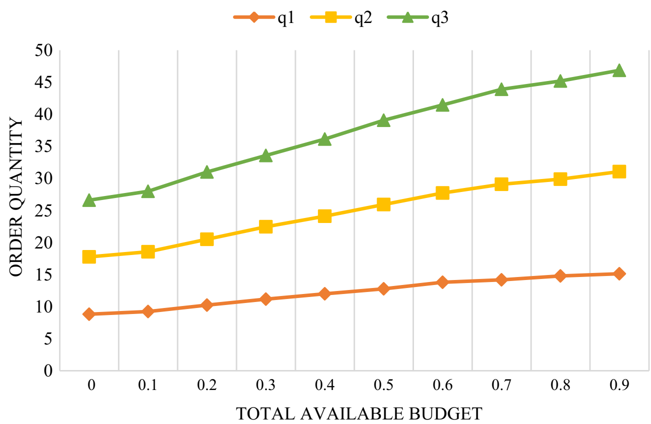

Figure 7. Increasing in order quantity with increasing in the total available budget.

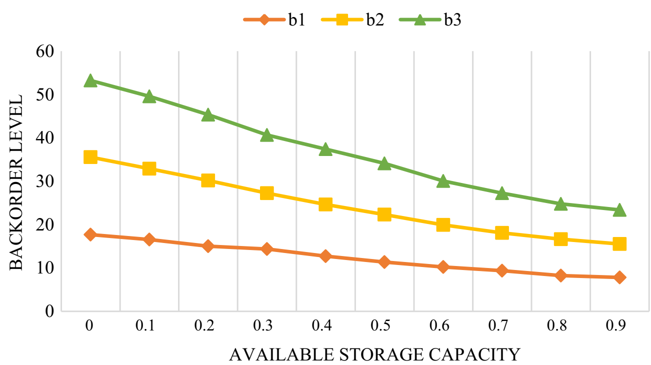

Figure 8. Decreasing in backorder level with increasing in available storage capacity.

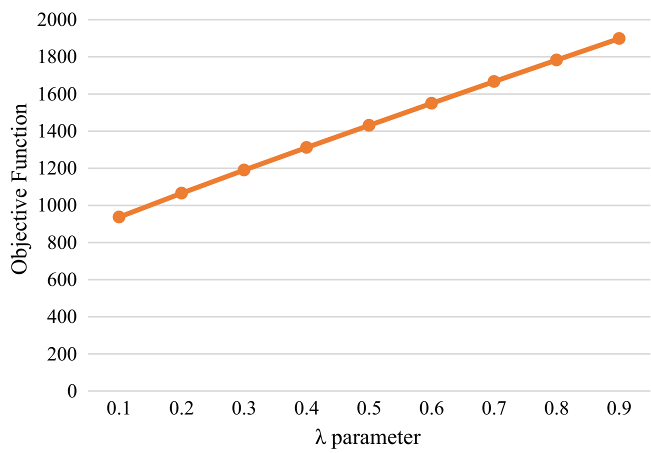

FIGURE 9. Increasing in the objective function value with increasing in the λ parameter.

Increasing in the storage capacity enables the system to store more and that leads to a decrease in the backorder level (Fig. 8).

The value of the λ parameter will be increased to 90% of the initial value. As it is expected with increases in the variance coefficient, the variance value increases and the objective function value becomes worse (Fig. 9).

These expected behaviors show the accuracy of the proposed model.

6. Case study

To validate the proposed model a real-life example has been solved using the model. The company of this study is a business that originated in 1980 in the city of Jajrood. The company, which is called Veniz, has a furniture production factory and acts as the supplier (vendor) of the chain and it interacts with three furniture galleries (retailers). However, they cooperate in a traditional supply chain and the company only takes the order quantities from galleries and satisfies the demand by making the necessary deliveries.

Data that are obtained from interviews with the plant manager and the corresponding values of mean and variance are tabulated below. It should be noted that the cost value for the year 2017 was 5123$ which is equal to 281 765 000 Rials (Tabs. 4–8).

As mentioned before, all the costs information were received and put in the proposed model. Table 9 demonstrates the results of this computation.

The cost value gained by the model is 4211.7$ which is improved by 17.78%.

TABLE 4. The retailers' and vendor's demand.

d_{ij}	$j=1$	$j=2$	$j=3$
$i=1$	400	420	500
$i=2$	340	520	310
$i=3$	400	350	450
d_j	1140	1290	1260

TABLE 5. Retailers' ordering fixed cost.

A_{ij}	$j=1$	$j=2$	$j=3$
$i=1$	80	76	78
$i=2$	75	77	75
$i=3$	80	78	79

TABLE 6. The fixed backorder cost per unit per time unit.

π_{ij}	$j=1$	$j=2$	$j=3$
$i=1$	3	5	4
$i=2$	4	4	2
$i=3$	5	3	3

TABLE 7. The rest of the parameter for solving the problem.

	$j=1$	$j=2$	$j=3$
A_j	362	365	363
C_j	40	39	43
P_j	1300	1450	1400
f_j	3	5	4

TABLE 8. The mean and variance value of each stochastic parameter.

$E(A_{ij})$	77.55
$\text{var}(A_{ij})$	3.77
$E(A_j)$	363.66
$\text{var}(A_j)$	2.33
$E(\pi_{ij})$	3.66
$\text{var}(\pi_{ij})$	1
$E(h_j)$	12.26
$\text{var}(h_j)$	0.39
$E(M)$	2000
$\text{var}(M)$	27.5
$E(X)$	500 000
$\text{var}(X)$	16.66
$E(F)$	10 000
$\text{var}(F)$	16.66

7. CONCLUSION

This research has been motivated by the work of Pasandideh *et al.* [12]. However, compared to that model, the presented model is more applicable. While Pasandideh *et al.* [12] considered a single-retailer multi-product SC under VMI policy and solve their model with GA, in this research, a multi-item EPQ model with the shortage in the form of backorder was considered for a single-vendor, multi-retailer supply chain under VMI contractual agreement. Some parameters were assumed stochastic in order to make the model more applicable to the real

TABLE 9. Output obtained from VMI strategy.

	$j = 1$	$j = 2$	$j = 3$
q_j	636.747	714.641	741.446
b_j	60.284	60.659	57.034
Cost value ($)		4211.7	
Variance		141.49	

world issues. In addition, three constraints including storage capacity, number of orders and available budget were considered in stochastic form. Geometric programming (GP) approach was employed to find the optimal solution of the nonlinear stochastic programming problem with the objective of minimizing the mean-variance of the total inventory cost of the system. Since the problem is in the Signomial form, by using the procedure and algorithm proposed by Xu [30], first, the model was converted into the standard GP form and then the optimal value of the problem was reached. To evaluate the performance of the addressed model and the solving method test problems with different number of retailers and items were solved with both BARRON solver of GAMS software and also GP approach in MATLAB software. As a result, GP approach outperformed GAMS in terms of the minimum total cost; however, GP obtained more CPU time than GAMS. Sensitivity analyses were done on some important parameters of the objective function as well as constraints, so the accuracy of the model has been discussed. Moreover, to demonstrate the applicability of the proposed methodology, a case study in an Iranian furniture supply chain was conducted and 17.78% improvement in terms of total cost was gained.

For future researches in this area, we recommend the following:

(a) The problem can be solved with meta-heuristic algorithms.
(b) Inflations can be considered.
(c) In addition to backorders, lost sales can also be assumed for shortages.
(d) The economic order quantity (EOQ) model can also be utilized.

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REFERENCES

[1] R.Z. Farahani and M. Elahipanah, A genetic algorithm to optimize the total cost and service level for just-in-time distribution in a supply chain. *Int. J. Prod. Econ.* **111** (2008) 229–243.
[2] V. Schmid, K.F. Doerner and G. Laporte, Rich routing problems arising in supply chain management. *Eur. J. Oper. Res.* **224** (2013) 435–448.
[3] Y. Dong and K. Xu, A supply chain model of vendor managed inventory. *Transp. Res. Part E: Logist. Transp. Rev.* **38** (2002) 75–95.
[4] A. Blatherwick, Vendor-managed inventory: fashion fad or important supply chain strategy? *Supply Chain Manage.: Int. J.* **3** (1998) 10–11.
[5] G. Cachon and M. Fisher, Campbell soup's continuous replenishment program: evaluation and enhanced inventory decision rules. *Prod. Oper. Manage.* **6** (1997) 266–276.
[6] M.L. Fox, Integrating vendor-managed inventory into supply chain decision-making. In: International Conference Proceedings-American Production and Inventory Control Society (1996).
[7] S. Hohmann and S. Zelewski, Effects of vendor-managed inventory on the bullwhip effect. In: Management Innovations for Intelligent Supply Chains. IGI Global (2013) 167–183.
[8] J.F. Magee, Production Planning and Inventory Control. McGraw-Hill (1958).
[9] Y. Yao, P.T. Evers and M.E. Dresner, Supply chain integration in vendor-managed inventory. *Decis. Support Syst.* **43** (2007) 663–674.
[10] S.H.R. Pasandideh, S.T.A. Niaki and A. Roozbeh Nia, An investigation of vendor-managed inventory application in supply chain: the EOQ model with shortage. *Int. J. Adv. Manuf. Technol.* **49** (2010) 329–339.
[11] S.H.R. Pasandideh, S.T.A. Niaki and A.R. Nia, A genetic algorithm for vendor managed inventory control system of multi-product multi-constraint economic order quantity model. *Expert Syst. App.* **38** (2011) 2708–2716.

[12] S.H.R. Pasandideh, S.T.A. Niaki and J.A. Yeganeh, A parameter-tuned genetic algorithm for multi-product economic production quantity model with space constraint, discrete delivery orders and shortages. *Adv. Eng. Softw.* **41** (2010) 306–314.

[13] H. Mokhtari and M.T. Rezvan, A single-supplier, multi-buyer, multi-product VMI production-inventory system under partial backordering. *Oper. Res.* (2017) 1–21.

[14] H.K. Alfares and A.M. Attia, A supply chainmodel with vendor-managed inventory, consignment, and quality inspection errors. *Int. J. Prod. Res.* **55** (2017) 5706–5727.

[15] R.K. Ramrakhyani, M.S. Tootooni and N. Nagarur, Analysis of a vendor managed consignment inventory system with kan-ban withdrawals and payment delays. *Ind. Syst. Eng. Rev.* **6** (2018) 41–61.

[16] S.H.R. Pasandideh, S.T.A. Niaki and P. Ahmadi, Vendor-managed inventory in the joint replenishment problem of a multi-product single-supplier multiple-retailer supply chain: a teacher-learner-based optimization algorithm. *J. Modell. Manage.* **13** (2018) 156–178.

[17] T.-M. Choi, D. Li and H. Yan, Mean–variance analysis of a single supplier and retailer supply chain under a returns policy. *Eur. J. Oper. Res.* **184** (2008) 356–376.

[18] H.-S. Lau, The newsboy problem under alternative optimization objectives. *J. Oper. Res. Soc.* (1980) 525–535.

[19] Y. Bassok and M. Nagarajan, Contracting Under Risk Preferences. University of Southern California, Los Angeles, CA (2004).

[20] J. Buzacott, H. Yan and H. Zhang, Risk analysis of commitment–option contracts with forecast updates. *IIE Trans.* **43** (2011) 415–431.

[21] X. Gan, S.P. Sethi and H. Yan, Channel coordination with a risk-neutral supplier and a downside-risk-averse retailer. *Prod. Oper. Manage.* **14** (2005) 80–89.

[22] G.P. Kiesmüller and R.A.C.M. Broekmeulen, The benefit of VMI strategies in a stochastic multi-product serial two echelon system. *Comput. Oper. Res.* **37** (2010) 406–416.

[23] J.-Y. Lee and L. Ren, Vendor-managed inventory in a global environment with exchange rate uncertainty. *Int. J. Prod. Econ.* **130** (2011) 169–174.

[24] J. Razmi, R.H. Rad and M.S. Sangari, Developing a two-echelon mathematical model for a vendor-managed inventory (VMI) system. *Int. J. Adv. Manuf. Technol.* **48** (2010) 773–783.

[25] H. Markowitz, Portfolio Selection: Efficient Diversification of Investments. Cowles Foundation Monograph No. 16. John Wiley & Sons, Inc., New York, NY (1959).

[26] J. Von Neuman, Theory of Games and Economic Behavior. Princeton University Press, Princeton, NJ (1953).

[27] J.M. Mulvey, R.J. Vanderbei and S.A. Zenios, Robust optimization of large-scale systems. *Oper. Res.* **43** (1995) 264–281.

[28] S. Boyd, S.J. Kim, L. Vandenberghe and A. Hassibi, A tutorial on geometric programming. *Optim. Eng.* **8** (2007) 67.

[29] R.J. Duffin, E.L. Peterson and C.M. Zener, Geometric programming: theory and application (1967).

[30] G. Xu, Global optimization of signomial geometric programming problems. *Eur. J. Oper. Res.* **233** (2014) 500–510.

[31] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge University Press, Cambridge (2004).