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STRATEGIC SOURCING UNDER RECALL LOSS SHARING AND PRODUCT QUALITY INVESTMENT

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Abstract. Product recall has been a widespread practical operation risk in the production outsourcing. To remit even avoid product recall risk, this paper considers a two-echelon supply chain where the original equipment manufacturer (OEM) orders a critical component from one or two contract manufacturers (CMs) and uses it to produce finished product with potential quality defect. The CMs can decide investment level to reduce defect possibility and share recall loss with the OEM once product recall is implemented. When the recall loss sharing rate is fixed, the OEM may adopt the single sourcing strategy or the dual sourcing strategy which depends on the recall loss sharing rate. Moreover, if the sharing rate is relatively small, the single sourcing strategy is an optimal choice for the OEM. However, when the recall loss sharing rate is determined by the OEM, she prefers to adopt the dual sourcing strategy. Meanwhile, an increase of the recall loss sharing rate may not force the CM to improve product quality. By the numerical analysis, if the marginal recall loss is large or the wholesale price is relatively small, the OEM and the CMs can reach a win-win scenario. Finally, we examine an extension in which the CMs have pricing ability on wholesale price, and the result shows that the OEM can not obtain a cost-reduction benefit under the dual sourcing strategy.

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1. Introduction

It has been a common practice that brand-name companies outsource the production of their products to CMs. Conventional wisdom holds that outsourcing is a pervasive feature of supply chain and has contributed significantly to the growth of global economy. However, the outsourcing decision can be far more complex in practice. On the one hand, outsourcing has many advantages to the brand-name companies, such as saving operation and capital cost, strengthening the firms' core competencies in R&D and marketing. On the other hand, outsourcing has also caused inevitable impacts, for instance, the lacking of flexibility of manufacturing adjustment, and increasing recall risk because of possible deficiency in product quality management [18].

In practice, the dual sourcing strategy also has inherent flaws such as amplifying recall risk. For example, the iPhone 6s (Plus) uses the A9 processors from Samsung and TSMC, respectively. The quality difference on battery life incurs Apple's fans resentment and shouting returns [36]. Similarly, Galaxy Note 7's battery comes

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from two vendors: Samsung's own SDI subsidiary and Amperex Technology (ATL). However, Galaxy Note 7 using SDI-manufactured battery brings the overheating issues and has been recalled in the global [1]. As the previous examples show, the potential recall risk causes corresponding production loss for the CM. After the Galaxy Note 7 battery exploding, Samsung SDI may loss around $16 million which is tied down by supplying of defect component in the third quarter of 2016 [33]. Consequently, Samsung Electronics and Samsung SDI need to suffer recall loss of Note 7 together as the cause of supplying defect batteries. Moreover, for the detail recall loss allocation scheme, some studies introduce the partial allocation contract to allocate the external recall loss [6]. However, compared with the partial allocation contract, the fixed allocation contract has been commonly adopted, in which the recall loss is allocated at the fixed sharing rate. It is therefore interesting to compare the performance of a partial allocation rate case against that of a fixed sharing rate case in our study.

In addition, facing recall loss due to defect components, component suppliers have been taking actions to improve their components quality performance. After the battery explosion event, for instance, Samsung SDI has planned on investing about $128 million to improve battery safety. In terms of the production of OLED displays, LG Display invests $2.6 billion to improve component performance and increase iphone reliability [15]. These examples indicate the quality improvement of key component makes the OEM's sourcing strategy choice become a challenge. To address this challenge, there is an expanding body of literature considering reliability improvement (more see, *e.g.* [12,21,43]). In particular, they assume that the production or component process is unreliable in terms of the quantity of qualified output. To reduce recall risk of sold product, the supplier should have incentive on quality investment to improve component performance. With consideration of this motivation, this paper will propose an endogenous quality improvement setting to explore its effect on the OEM's sourcing strategy choice.

In order to study the OEM's strategies preferences on procurement outsourcing, we suppose that there are two sourcing strategies (single sourcing and dual sourcing) for the OEM under different quality improvement incentive cases. Specifically, we attempt to solve the following questions: (i) What is the optimal sourcing strategy from the OEM's perspective, and which external and internal conditions does it depend on? (ii) How would the supply chain partners' optimal decisions and profits change under different sourcing strategies? And on a deeper level, what is the role of the feature of market condition in influencing the change? (iii) How does the endogenous decision on the recall loss sharing rate influence the OEM's profit and product defect rate? In particular, whether the endogenous transformation will affect the OEM's sourcing strategy preference?

To address these questions, we consider a two-echelon supply chain where the original equipment manufacturer (OEM, she) orders a critical component (or potentially a finished product) from outside contract manufacturers (CMs) and uses it to produce a product for the end market. With the single and the dual sourcing strategies, the product may have recall risk but can be improved by the CM(s)' quality investment. Furthermore, we analyze the two quality improvement cases (the fixed sharing rate and the partial sharing rate), and characterize the OEM's optimal sourcing strategy.

When the recall loss sharing rate is exogenous, the OEM may adopt the single sourcing strategy or the dual sourcing strategy which depends on the recall loss sharing rate. Specifically, when the sharing rate is small, the OEM prefers to adopt the single sourcing strategy. Otherwise, the dual sourcing strategy is superior to the single sourcing strategy. Meanwhile, the OEM achieves higher sale avenue and lower recall loss under the single sourcing strategy than that under the dual sourcing strategy. Therefore, the single sourcing strategy may be an optimal choice for the OEM.

When the recall loss sharing rate is endogenous, the OEM prefers to adopt the dual sourcing strategy. Due to the endogenous recall loss sharing rate, the OEM generally controls the whole supply chain and extracts other partners' profit. That is to say, the OEM may stretch the CM's profit seriously while the latter has to obey the former contract clause. Intuitively, the OEM may prefer to adopt the dual sourcing strategy since the two CMs' competition decreases their control power on the supply chain. As an extension, we consider the wholesale price is endogenous, and find that the OEM pays higher wholesale price under the dual sourcing strategy than that under the single sourcing strategy. This result suggests the OEM can not obtain the cost-reduction benefit under the dual sourcing strategy.

In addition, under the endogenous recall loss sharing rate, the marginal recall loss (the wholesale price) has a positive (negative) effect on the quality investment level and the recall loss sharing rate. Interestingly, we find an increase of the recall loss sharing rate may not motivate the CM to improve product quality. When the sharing rate beard by the CM becomes small, the OEM increases order quantity as a strategic response. Therefore, the CM is willing to add quality investment to decrease his recall loss. Moreover, facing increasing marginal recall loss, the order quantity first increases then decreases under the single sourcing strategy, but under the other strategy, it first decreases then increases and back decreases. With a decrease of the wholesale price, the order quantity first increases then reaches a ceiling. For the OEM's profit, it decreases in the marginal recall loss and the wholesale price. In terms of the CM's profit, it first increases then decreases in the marginal recall loss under the single sourcing strategy, but it first decreases then increases and back decreases under the dual sourcing strategy. In addition, when the marginal recall loss is large or the wholesale price is relatively small, the OEM and the CMs can reach a win-win scenario.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the model setting. The optimal sourcing strategies for the fixed sharing rate and the partial sharing rate are presented in Sections 4 and 5, respectively. In section 6, we derive some sensitivity analysis and analyze the impact of sourcing strategy on optimal decisions and profits of supply chain members. Section 7 presents a model extension where we consider the wholesale price is endogenous. Section 8 concludes the paper. All proofs are given in the Appendix A.

2. Literature review

Our study is primarily related to three broad streams of literature. The first stream focuses on the procurement outsourcing in the supply chain. The second stream examines the sourcing strategy under uncertain environment. The last stream addresses the product's quality improvement in the presence of price competition.

The first stream of literature is related to the procurement outsourcing in the production operation. Early studies focus on the effect of production outsourcing on cost reduction [37]. In consideration of scale economies, Cachon and Harker [5] show firms can benefit from outsouring strategy that mitigates price competition. When supply chain partners have opportunities to invest in cost reduction, Gilbert *et al.* [16] show a external supplier can dampen competition between the OEMs. In addition, a number of studies which examine procurement outsoucing issues also consider other factors, such as technology spillovers, service quality or value-added services [7,27–29]. Nevertheless, these papers only consider the competition between the OEMs, and our focus is upstream competition between the CMs. In recent studies, Chen *et al.* [9] consider a more complex competition structure, namely, a CM not only is the OEM's upstream partner but also is the latter's downstream competitor under the private procurement cost for the CM. Following a similar supply chain structure, Wang *et al.* [39] drive optimal quantity timing equilibrium by considering three Cournot game sequences. With simple quantity discount and price-only contracts, Kayis *et al.* [24] focus on information asymmetry regarding suppliers' production costs to examine a manufacturer's optimal procurement strategy (control *vs.* delegation). To illustrate the impact of the OEMs competition on their optimal procurement strategy, Bolandifar *et al.* [2] adopt the quantity discount pricing form. In a similar pricing form, Dong *et al.* [11] examine optimal component procurement strategy with private discount ability information in which an OEM can either control component procurement or delegate this task to the CM. Our study differs from these studies in three critical aspects. First, we analyze the strategic response of the down-tier supplier (namely, the CM). Specifically, the CM could improve the product's defect rate by quality investment. Second, we focus on the OEM's sourcing strategy by examining two possible recall loss sharing cases. Third, we describe the quality deficit by product recall which can be recognised after sale rather than before sale.

This paper is also related to the supply chain literature in strategic sourcing choices under uncertain environment. In the early studies, Mishra and Tadikamalla [30] compare single sourcing and dual sourcing and find that single-sourcing with lot streaming can result in better performance than dual sourcing. Burke *et al.* [3] analyze the supplier selection and quantity allocation decisions under demand uncertainty. Then they characterize conditions under which single-sourcing and multiple-sourcing strategies are optimal. Moreover, Burke *et al.* [4] extend the analysis to an uncertain supplier reliability setting in an environment with stochastic demand. Yu *et al.* [42] develop a two-stage supply chain in which two suppliers who are geographically different and offer two wholesale prices provide a needed critical part for the manufacturer's final product. They compare the performance of these two sourcing strategies and identify the critical factors governing the choice between the two sourcing alternatives. In a subsequent paper, the effect of learning on the supplier selection problem is investigated by Glock [17]. He *et al.* [19] drive the optimal procurement allocation strategy in the presence of supply disruption risks. Based on the special cost characteristics, Heese [20] uses the generalized Nash bargaining framework to study a manufacturer's sourcing decision problem by considering bilateral negotiations and learning effect. Moreover, Silbermayr and Minner [34] extend the models in [20] by developing a stochastic dynamic model in which a buyer can use dual sourcing strategy and purchase from two potential suppliers to satisfy a constant deterministic demand. Motivated by scale economies with the total output produced, Jain and Hazra [23] formulate a simultaneous game model where a buyer allocates its sourcing requirements among two asymmetric suppliers. To explore the strategic sourcing choice under capacity constraint and asymmetric cost information, Huang *et al.* [22] consider a buyer who procures a critical component from two outside suppliers for the final product. Kumar *et al.* [25] study two competing retailers engaged in competition and determine the cutoff probabilities for optimal sourcing strategies for the retailer. In a co-opetitive supply chain, Niu *et al.* [32] investigate the OEM's sourcing strategy choice and find she always prefers dual sourcing even though one supplier is unreliable. However, the above literatures only consider the retalier's sourcing choices but not suppliers' strategic response by improving product quality. In addition, different from these studies which without enough consideration of the superiority of single sourcing, we propose a more practical case where the CMs can determine the quality investment level to improve defect product given the consideration of order quantity from the OEM.

Finally, there is a growing literature on product quality choice in the price competition setting. Tagaras and Lee [35], as the early study, measure the quality level of incoming materials and final product by a probability that the material or product is functional operation. With a dynamic decision setting, Mukhopadhyay and Kouvelis [31] propose differential game model where two firms decide quality and price of their respective product in a continuous time. Then Wang and Yang [38] adopt a two-stage noncooperative game in which two firms decide respective product quality in the first stage. Using a similar competition supply chain, Choudhary *et al.* [10] develop a duopoly model where two firms choose different product quality and compete in quality and price. Moreover, our paper is most closely to the literature on quality improvement by investment incentive of supply chain members. In a yield uncertainty environment, Zhu *et al.* [43] consider a supply chain where both a buyer and a supplier need to invest in quality improvement under different quality cost sharing contracts. They demonstrate the buyer is more likely to obtain high-quality product by allocating a higher order to the supplier. By elaborating improvement with product quality, Chao *et al.* [6] examine the implications of product recall cost sharing formats on quality improvement effort of the supplier. From the view of cost information about service quality, Zhu and Mukhopadhyay [44] develop a single-to-single model to drive the optimal quality level for the supplier under the full information and the asymmetric information, respectively. In the decentralised supply chain, Xiao *et al.* [40] study a quality coordination problem consisting a manufacturer and a retailer with a revenue-sharing contract. As a significant contribution, they develop a dynamic game theoretic model and consider consumers' response to waiting time. Based a similar contract, El Ouardighi [14] explores the potential coordinating power of revenue-sharing contracts in supply quality management. As an expansion, Yan [41] investigates a joint pricing and product quality decisions problem in a decentralized supply chain and compares different contract formats. In development, Dong *et al.* [13] assume the brand owner and the supplier can exert efforts to improve the quality of the manufacturing process and the quality of the sourced component, respectively. And quality improvement efforts of both parties are private actions but not verifiable and contractible. However, a significant difference between our work and other literatures comes from the focus of study context in this stream. Most papers on quality improvement focus on the firm's pricing decision and quality choice, while we examine the effect of quality improvement level on the OEM's sourcing strategy choice with different recall loss sharing cases.

3. The model

Consider a two-echelon supply chain where the original equipment manufacturer (OEM) orders a critical component (or potentially a finished product) from the two outside contract manufacturers (CMs) and uses it to produce product in the terminal market. For tractability, the OEM and the CMs, which are risk neutral and maximize their own expected profits, are represented as the subscripts o and c, respectively.

After the product is sold to the consumer, during its using period, it may fail to perform its function if any one of its components fail to do so. Therefore, we define the defect rate, which is the proportion of defect units failing to meet consumers' satisfactions relative to the order quantity the CM delivered to the OEM, to characterize the quality of products. Clearly, this implies that the larger the defect rate, the lower the level of quality. Specifically, the CM's initial product defect rate is denoted by $\overline{\lambda} \leqslant 1$, which can be reduced by quality investment such as technical assistance and employees training. In essential, the quality investment promotes the yield rate and decreases the defect product quantity [21]. Given the level e_i of CM i's quality investment, CM i's defect rate reduces from the initial level $\overline{\lambda}$ to an improved level $\lambda_i = \overline{\lambda} - e_i$, where $0 \leqslant \lambda_i \leqslant \overline{\lambda}$, $i = 1, 2$. And when the CM i's quality investment level is e_i, then he occurs an invest-related cost $\frac{\eta_i}{2} e_i^2$, where $\eta_i > 0$ denotes the investment cost parameter to improve quality. Moreover, under the single sourcing strategy, the symbols of the CM's order quantity, investment level and cost parameter simplify q, e and $\eta > 0$, respectively.

The defect product can incur the recall cost comprising of replacement costs, labor costs, and other service costs, and we refer to it henceforth as recall loss m. Based on the business practice, the recall loss should be shared between the OEM and CMs, and thereby the sharing rate is θ. Without loss of generality, the OEM's and the CMs' unit loss are $(1-\theta)m$ and θm, respectively. Note that $\theta > 1$ implies the CM bears extra loss and the OEM benefits from product recall based on penalty clause of outsourcing production contract. For the ordered product, the OEM needs to pay unit transfer payment w to CMs. Note that we assume the CMs offer same wholesale price w to the OEM, and this assumption has practical sense which has been extensively used [8]. In an extension, we also analyze the situation in which the CMs can determine the wholesale price simultaneously, and find that our main findings are robust.

Assume that market demand is price sensitive and the inverse demand function is given by $p = a - bq$, where q is the total available-to-sell quantity, parameter a denotes the market potential and $b > 0$ is the quantity sensitivity parameter. This paper studies both monopoly setting and duopoly setting. In the monopoly setting, the inverse demand function is given by

$$p(q) = a - bq,$$

where q is the monopolist's production. In the duopoly setting, we model the competition among CMs and then apply the Cournot model. Then the inverse demand function for the OEM is assumed as

$$p(q_1, q_2) = a - b(q_1 + q_2),$$

where q_1 and q_2 are the available-to-sell quantities by CM 1 and CM 2, respectively.

The used product will be returned after consumers find the product is defect, thus the defect rate will be one of the most important factors in affecting the market demand. Therefore, the OEM has incentive to make quality investment for the CM to reduce the defect rate and to gain a competitive advantage in quality over its rival. The main problems the OEM faces are how many products to order under different sourcing strategies, and whether to determine recall loss sharing rate.

To investigate how the recall loss sharing rate affects the OEM's sourcing strategies, we consider a fixed sharing rate case and a partial sharing case, respectively. Specifically, under the fixed sharing rate case, the OEM assumes a fixed percentage of the total recall loss based industry standards, and the OEM decides the sharing rate under the partial sharing rate case. Therefore, Table 1 summarizes four cases for simplified expression. Throughout the paper, we use the following notation. Let θ^{R}, q^{R} and π_o^{R} be the OEM's optimal sharing rate, order quantity and profit, respectively, for case $\mathrm{R} \in \{\mathrm{FS}, \mathrm{FD}, \mathrm{PS}, \mathrm{PD}\}$. Similarly, e^{R} and π_c^{R} present the CM's optimal quality investment level and profit for case R.

Table 1. The cases under different sourcing strategies.

	Single sourcing	Dual sourcing
The fixed sharing rate	FS	FD
The partial sharing rate	PS	PD

4. Sourcing strategies with fixed sharing rate

In this section, we focus on the optimal sourcing strategy for the OEM with a fixed sharing rate. Specifically, the external failure cost between the CM and the OEM is allocated at the fixed sharing rate. In addition, we assume that both the OEM and the CM are fully informed about the component's initial failure rates (*i.e.*, the initial state of the component quality prior to quality improvement effort). This usually occurs when the CM and the OEM have been working together for several years, and therefore both parties have a good idea of the component's process capabilities.

4.1. Single sourcing strategy

Under the single sourcing strategy (namely, case FS), the OEM only orders component from one CM, and assembles products for selling to market. Formally, the sequence of events is as follows: (i) the OEM chooses order quantity from the CM; (ii) the CM chooses related quality investment level at a fixed recall loss sharing rate; (iii) expected demand is realized after the product has been delivered from the CM to the OEM. When the OEM adopts the single sourcing strategy, the profit function is

$$\pi_o(q) = q(a - bq) - (1-\theta)(\overline{\lambda} - e)mq - wq, \tag{4.1}$$

where the first term is the realized revenue, the second term is the cost share incurred for recall loss, and the last term is the transfer payment to the CM. Note that w is assumed to exogenous, because there are usually many third-party manufacturers who would like to compete for the outsource business. So there is not much flexibility for the manufacturer to negotiate w with the OEM. Correspondingly, the CM's profit can be written as

$$\pi_c(e) = wq - \theta(\overline{\lambda} - e)mq - \frac{\eta}{2}e^2, \tag{4.2}$$

where the first term is the transfer payment from the OEM, the second term is the cost share incurred for recall loss, and the last term is invest-related cost.

Therefore, the mathematical formulation of Stackelberg game under the single sourcing can be given as follows,

$$\begin{aligned} &\max_q \pi_o(q|e^*) \\ &\text{s.t.} \quad e^* = \arg\max_{0\leqslant e\leqslant\overline{\lambda}} \pi_c(e), \\ &\qquad\quad \pi_c(e) \geqslant 0. \end{aligned}$$

In the following, we will develop model and solve it backward. The following lemma states the CM's best response after taking into account the OEM's order quantity.

Lemma 4.1. *Given the OEM's order quantity, the CM's best response on quality improvement level is as follows,*

$$e^* = \frac{\theta mq}{\eta}\cdot$$

Intuitively, the CM sustains penalty incentive from the recall loss m and the sharing rate θ due to quality defect. Namely, the two parameters are higher, the CM makes more quality investment to reduce component defect probability. Besides, order quantity is a positive incentive for the CM. That is, more order quantity, more quality investment level offered by the CM. And the CM also incurs internal negation incentive because of investment cost. In a word, the CM would not prefer making quality improvement investment with an increase in the investment cost.

The OEM, as the leader of the game, will anticipate the CM's best response when making the decision on the order quantity q. With the CM's best response e^*, the OEM's profit can be rewritten as,

$$\pi_o = -\Big(b - \frac{1}{\eta}\theta(1-\theta)m^2\Big)q^2 + (a - w - \overline{\lambda}m(1-\theta))q. \tag{4.3}$$

The next proposition gives the unique solution for q that maximizes the OEM's profit.

Proposition 4.2. *Under the single sourcing strategy, when* $\theta \leqslant \frac{(a-w+\overline{\lambda}m)-\sqrt{(a-w+\overline{\lambda}m)^2-8b\eta\overline{\lambda}^2}}{2\overline{\lambda}m}$*, the CM's optimal investment level*

$$e^{\rm FS} = \theta m \frac{a - w - (1-\theta)\overline{\lambda}m}{2\left[b\eta - \theta(1-\theta)m^2\right]},$$

and the OEM's optimal order quantity

$$q^{\rm FS} = \eta \frac{a - w - (1-\theta)\overline{\lambda}m}{2\left[b\eta - \theta(1-\theta)m^2\right]}.$$

In this proposition, the condition of $b\eta - \theta(1-\theta)m^2 > 0$ is easy to meet based on practical business. In the economic market, firms recall their own products frequently due to quality defect under complete legal system. Then consumers are accustomed to this behaviour of products recall and not suspicious of the firms' reputation. Consequently, the recall loss of firms is limited and mainly derived from defect products. Otherwise, firms will be bankrupt once products recall happens. The following proposition shows the effects of recall loss sharing rate on optimal order quantity and investment level.

Proposition 4.3. *When the CM's the sharing rate of defect product recall loss is less than a threshold value, i.e.,* $\theta \leqslant \widehat{\theta}_1$*, then order quantity increase in the sharing rate, i.e.,* $\frac{{\rm d}q^{\rm FS}}{{\rm d}\theta} \geqslant 0$*; similarly, when* $\theta \leqslant \widehat{\theta}_2$*, then* $\frac{{\rm d}e^{\rm FS}}{{\rm d}\theta} \geqslant 0$.

According to this proposition, we find, under a small sharing rate, with an increase in the sharing rate, the OEM offers more order quantity to the CM ($\theta \leqslant \widehat{\theta}_1$), and she also gives more quality investment for improving product quality ($\theta \leqslant \widehat{\theta}_2$). Naturally, these results suggest that the order quantity and the quality investment level are unimodal in sharing rate. In other words, the OEM has a most preferred sharing rate, and she has no incentives to further add order quantity or increase quality investment level, if the rate still can.

Moreover, plugging the optimal order quantity into the OEM's profit function, we can achieve the OEM's optimal profit as follows,

$$\pi_o^{\rm FS} = \frac{\eta}{4}\frac{(a - w - \overline{\lambda}m(1-\theta))^2}{b\eta - \theta(1-\theta)m^2}. \tag{4.4}$$

In the following, we also obtain similar result about the effect of sharing rate on the OEM's profit which summarized by the following proposition.

Proposition 4.4. *When the CM's the sharing rate of defect product recall loss is less than a threshold value, i.e.,* $0 < \theta \leqslant \widehat{\theta}_3$*, then the OEM's optimal profit increases in the sharing rate, i.e.,* $\frac{{\rm d}\pi_o^{\rm FS}}{{\rm d}\theta} \geqslant 0$.

Similarly, when the sharing rate is small, the OEM can earn more profit with an increase in the sharing rate. That is to say, bearing lesser recall loss makes the OEM order more quantity, and the CM could provide higher quality investment due to heavier pressure of the recall loss. Consequently, the OEM can obtain more profit. Meanwhile, the CM also reduces the recall loss by improving product quality. However, the sharing rate may damage the CM's profit when the rate becomes large. This is because the high sharing rate could make the CM have less incentive to provide necessary quality investment due to the less profit from the quality improvement business.

4.2. Dual sourcing strategy

Under the dual sourcing strategy (namely, case FD), the OEM will choose order quantity q_i from the CM i, $i = 1, 2$. Formally, the sequence of events is as follows: (i) the OEM chooses order quantity from the two CMs respectively; (ii) the CMs choose related quality investment levels simultaneously at a fixed recall loss sharing rate; (iii) expected demand is realized after the product has been delivered from the CMs to the OEM. Therefore, the OEM's profit function under the dual sourcing strategy can be written as follows,

$$\Pi_o(q_1, q_2) = (q_1 + q_2)\left[a - b(q_1 + q_2)\right] - (1-\theta)(\overline{\lambda} - e_1)mq_1 - (1-\theta)(\overline{\lambda} - e_2)mq_2 - w(q_1 + q_2), \tag{4.5}$$

where the first term $(q_1 + q_2)\left[a - b(q_1 + q_2)\right]$ is the realized revenue, the second term $(1-\theta)(\overline{\lambda} - e_1)mq_1 + (1-\theta)(\overline{\lambda} - e_2)mq_2$ is the cost share incurred for recall loss, and the last term $w(q_1 + q_2)$ is the transfer payment to the CM 1 and 2. Similarly with the single sourcing strategy, the transfer payments for the CM 1 and 2 are exogenous. Besides, we also assume that the components from the CM 1 and 2 have same initial quality level $\overline{\lambda}$. There are some reasonable interpretations for this assumption. First, in the outsource industry, the OEM usually provides the components quality requirement further the detailed design. Second, the two CMs have mature manufacturer technology and use alike raw material for the components. Correspondingly, the CM 1's profit can be written as follows

$$\Pi_{c1}(e_1) = wq_1 - \theta(\overline{\lambda} - e_1)mq_1 - \frac{\eta_1}{2}e_1^2, \tag{4.6}$$

and the CM 2's profit is

$$\Pi_{c2}(e_2) = wq_2 - \theta(\overline{\lambda} - e_2)mq_2 - \frac{\eta_2}{2}e_2^2, \tag{4.7}$$

where the first term is the transfer payment from the OEM, the second term is the cost share incurred for recall loss, and the last term is invest-related cost for the CM 1 and 2, respectively.

Therefore, the mathematical formulation of Stackelberg game under the dual sourcing can be given as follows,

$$\begin{aligned}
\max_{q_1,q_2}\ & \Pi_o(q_1, q_2 | e_1^*, e_2^*) \\
\text{s.t.}\ \ & e_1^* = \arg\max_{0\leqslant e_1\leqslant \overline{\lambda}} \Pi_{c1}(e_1), \\
& e_2^* = \arg\max_{0\leqslant e_2\leqslant \overline{\lambda}} \Pi_{c2}(e_2), \\
& \Pi_{c1}(e_1) \geqslant 0,\ \Pi_{c2}(e_2) \geqslant 0.
\end{aligned}$$

In the following, we develop model and solve it backward. The following lemma states the two CMs' best response after taking into account the OEM's quantity decision.

Lemma 4.5. *Given the OEM's order quantity, the CMs' best response on quality improvement level is as follows,*

$$e_1^* = \frac{\theta m q_1}{\eta_1},\ e_2^* = \frac{\theta m q_2}{\eta_2}.$$

Due to the heterogeneity of the two CMs, the findings about the lemma are different from the single sourcing strategy. In the dual sourcing strategy, the two CMs have different cost for quality improvement. Consequently, when one CM makes optimal response for quality investment level, he only considers the cost for quality improvement of himself, but does not consider the other cost. Meanwhile, the two CMs sustain penalty incentive from recall loss m and sharing rate θ due to quality defect. Apparently, the recall loss and the sharing rate are mutually complementary. Therefore, the two parameters are higher, the CMs make more quality investment to reduce component defect probability. Besides, order quantities are positive incentive for the CMs. More order quantity, more quality investment level offered by the CMs. And the CMs also incur internal negative incentive because of investment cost. That is to say, the CMs would not prefer making quality improvement investment with an increase in the investment cost. Note that, although the two CMs' optimal response about quality investment levels only depends on their own order quantity, their competition is embodied in quantity by demand function substantially. This result signifies the CM does not consider another CM's order quantity when he decides investment level. However, with an increase in another CM's order quantity, the CM can achieve less order quantity and further decrease investment level.

The OEM, as the leader of the game, will anticipate the two CMs' best response when making the decision on the order quantities q_1 and q_2. With the two CMs' best response e_1^* and e_2^*, the OEM's profit can be rewritten as:

$$\begin{aligned}\Pi_o = & -\Big(b-\frac{1}{\eta_1}\theta(1-\theta)m^2\Big)q_1^2+(a-w-\overline{\lambda}m(1-\theta))q_1-2bq_1q_2\\ & -\Big(b-\frac{1}{\eta_2}\theta(1-\theta)m^2\Big)q_2^2+(a-w-\overline{\lambda}m(1-\theta))q_2.\end{aligned} \tag{4.8}$$

The next proposition gives the unique solution for q_1 and q_2 that will maximize the OEM's profit.

Proposition 4.6. *Under the dual sourcing strategy, if* $1<\theta\leqslant\frac{(a-w+\overline{\lambda}m)-\sqrt{(a-w+\overline{\lambda}m)^2-8b(\eta_1+\eta_2)\overline{\lambda}^2}}{2\overline{\lambda}m}$, *the CMs' optimal investment levels are*

$$e_1^{\mathrm{FD}}=e_2^{\mathrm{FD}}=\frac{\theta m}{2}\frac{a-w-(1-\theta)\overline{\lambda}m}{b(\eta_1+\eta_2)-\theta(1-\theta)m^2},$$

respectively. And the OEM's optimal order quantity allocations are

$$q_1^{\mathrm{FD}}=\frac{\eta_1}{2}\frac{a-w-(1-\theta)\overline{\lambda}m}{b(\eta_1+\eta_2)-\theta(1-\theta)m^2},\quad q_2^{\mathrm{FD}}=\frac{\eta_2}{2}\frac{a-w-(1-\theta)\overline{\lambda}m}{b(\eta_1+\eta_2)-\theta(1-\theta)m^2}.$$

Otherwise, the OEM prefers to procure component only from CM 1 and this case degenerates into the single sourcing strategy.

Proposition 4.6 shows that the CMs make same quality investment although they are heterogeneous. And there are some interesting findings under specific conditions. For instance, if the CMs do not bear quality recall loss at all, or the product does not have recall loss, *i.e.*, $\theta=0$ or $m=0$, then the CMs do not offer quality investment, but achieve order quantity. At the same time, when the CMs bear all recall loss, they could make quality improvement investment but not consider quality recall unit loss m. Next, plugging the optimal order quantity into the OEM's profit function, we can achieve the OEM's optimal profit as follows,

$$\pi_o^{\mathrm{FD}}=\frac{\eta_1+\eta_2}{4}\frac{(a-w-\overline{\lambda}m(1-\theta))^2}{b(\eta_1+\eta_2)-\theta(1-\theta)m^2}. \tag{4.9}$$

Note that the OEM procures only component from the CM 1 when $\theta\leqslant 1$. That is to say, the single sourcing strategy becomes the optimal choice for the OEM. This is because if the OEM needs to bear the recall loss, she prefers to offer all orders to one CM with high efficiency. Consequently, the CM has more incentive to improve product quality, then the marginal recall loss is reduced for the OEM and the CM simultaneously. This result is counterintuitive given that the exact literature shows that the dual sourcing strategy is optimal choice for the OEM in the presence of the uncertain demand/supply under outsourcing sourcing. In business practice, for a monopolist buyer, the dual sourcing brings value by reducing the variability in market output, and thus reducing the market output inefficiency caused by the random yield. However, these literatures don't consider the product defect possibility and the CM's strategic response on quality improvement. In fact, the OEM may source product or key component by single sourcing or dual sourcing in practice. Therefore, in our paper, we find counterintuitive results by taking account of the CM(s)' quality investment decision to improve the product's quality level.

To analyze the effect of sharing rate on the OEM's profit, the corresponding results are summarized by the following proposition (to better focus on the role of the dual sourcing strategy, the inequation of $\theta > 1$ is assumed).

Proposition 4.7. *When the sharing rate of defect product recall loss the CM bears is less than a threshold, namely, $\theta \leqslant \widehat{\theta}_4$, then $\frac{\mathrm{d}q_i^{\mathrm{FD}}}{\mathrm{d}\theta} \geqslant 0$; similarly, when $\theta \leqslant \widehat{\theta}_5$, then $\frac{\mathrm{d}e_i^{\mathrm{FD}}}{\mathrm{d}\theta} \geqslant 0$; when $\theta \leqslant \widehat{\theta}_6$, then $\frac{\mathrm{d}\pi_o^{\mathrm{FD}}}{\mathrm{d}\theta} \geqslant 0$, $i = 1, 2$.*

The Proposition 4.7 shows the effect of the sharing rate of recall loss on the OEM's order quantity and the CMs' quality investment levels. Specifically, when the CMs bear less sharing proportion of recall loss, *i.e.*, $\widehat{\theta}_4$ or $\widehat{\theta}_5$, the OEM could distribute more order quantity or CMs make more quality investment levels with an increase in the sharing rate the CMs bear. However, when the sharing rate of recall loss is more to enlarge, the CMs have to decrease quality investment level due to pressure of high quality recall loss. Furthermore, because of the high product defect rate, the OEM has to decrease order quantity to CMs. Consequently, the each CM's profit first increases then decreases in the sharing rate of recall loss. Interestingly, the OEM's profit increases in the sharing rate all the time. This is because the OEM can adjust order quantity to obtain high marginal profit and further achieve high total profit.

Proposition 4.8. *When the two CMs are homogeneous (i.e., $\eta_1 = \eta_2 = \eta$) and the OEM has a recall benefit (i.e., $\theta > 1$), compared with the single sourcing, the OEM orders more product with the higher defect rate under the dual sourcing, i.e., $q^{\mathrm{FS}} < q_1^{\mathrm{FD}} + q_2^{\mathrm{FD}}$, $e^{\mathrm{FS}} > e_i^{\mathrm{FD}}$ $(i = 1, 2)$, and the dual sourcing is optimal choice for the OEM, i.e., $\pi_o^{\mathrm{FS}} < \pi_o^{\mathrm{FD}}$.*

With two homogeneous CMs, the above proposition also provides the comparison of the OEM's order quantity, the CM's quality investment level and profit under the single and the dual sourcing strategy when $\theta > 1$. In this case, compared with the single sourcing strategy, the OEM can make more order quantity and the CMs can make lower quality investment level under the dual sourcing strategy. Furthermore, the OEM obtains higher profit under the dual sourcing strategy than that under the single sourcing strategy. That is to say, the OEM prefers to adopt the dual sourcing strategy. This is an intuitive result because the existed literatures show dual sourcing strategy has more advantage than single sourcing strategy under uncertain environment. However, in this paper, our result depends on the sharing rate of recall loss heavily. Although the delivered product has lower quality under the dual sourcing strategy than that under the single sourcing strategy (because competition between the two CMs makes them achieve low order quantity), the OEM obtains product-recall benefit from CMs who have excessive loss-taking ($\theta > 1$). However, when the OEM also bears certain recall loss ($\theta \leqslant 1$), the CMs make low quality improvement investment, then the produced product has high defect rate. Therefore, the OEM does not have incentive to adopt the dual sourcing strategy.

5. Sourcing strategies with partial sharing rate

In the previous section, we have analyzed the OEM's optimal sourcing strategy when the recall loss sharing rate is fixed. In this section, we consider another case where the sharing rate is partial between the OEM and the CMs. Specifically, the sharing rate is determined by the OEM as a leader in the Stackelberg game. Note that the case with partial sharing rate is different from that with fixed sharing rate, which allocates the external recall loss between the CMs and the OEM at a fixed sharing rate. We will analyze the effect of the OEM's the sharing rate decision on herself sourcing strategy choice.

5.1. Single sourcing strategy

When the OEM adopts the single sourcing strategy (namely, case PS), she only orders component from one CM and assembles products for selling to market. Under the PS case, the sequence of events is as follows: (i) the OEM orders product or key component quantity from the CM and decides recall loss sharing rate simultaneously; (ii) the CM then chooses related quality investment level after the recall loss sharing rate has been determined; (iii) after the product has been delivered from the CM to the OEM, the consumer demand is realized. In this case, she will order q units to maximize the expected profit:

$$\bar{\pi}_o(q,\theta) = q(a-bq) - (1-\theta)(\overline{\lambda}-e)mq - wq, \tag{5.1}$$

where the first term $q(a-bq)$ is the realized revenue, the second term $(1-\theta)(\overline{\lambda}-e)mq$ is the cost share incurred for recall loss, and the last term wq is the transfer payment to the CM. Similarly with the above section, the exogenous transfer payment w still holds on due to competition of third-party manufacturers. Note that the recall risk sharing rate becomes the OEM's decision variable under the partial sharing rate case. This is a reasonable supposition when the OEM can decide the recall loss sharing rate. Correspondingly, the CM's profit can be written as,

$$\bar{\pi}_c(e) = wq - \theta(\overline{\lambda}-e)mq - \frac{\eta}{2}e^2, \tag{5.2}$$

where the first term is the transfer payment from the OEM, the second term is the cost share incurred for recall loss, and the last term is invest-related cost.

Based on the sequence of the events under the PS case, the optimization problem faced by the OEM is defined as,

$$\begin{aligned} \max_{q,\theta}\ & \bar{\pi}_o(q,\theta|e^*) \\ \text{s.t.}\ \ & e^* = \arg\max_{0\leqslant e\leqslant\overline{\lambda}} \pi_c(e), \\ & \pi_c(e) \geqslant 0. \end{aligned}$$

Because the problem is depicted as Stackelberg game, we would develop model and solve it backward. Because the optimal response on order quantity is depicted in Lemma 4.1, we do not character the optimal response any more in this case. Next, the solved result is summarized directly by the following proposition.

Proposition 5.1. *Under the single sourcing strategy, the optimal order quantity is*

$$q^{\mathrm{PS}} = \eta\frac{2(a-w)-\overline{\lambda}m}{4b\eta - m^2}\cdot$$

And the CM's recall loss sharing rate and optimal investment level are

$$\theta^{\mathrm{PS}} = \frac{1}{2}\left(1+\frac{\overline{\lambda}(4b\eta-m^2)}{m(2(a-w)-m\overline{\lambda})}\right),\ e^{\mathrm{PS}} = \frac{\overline{\lambda}}{2}\left(1+\frac{m(2(a-w)-m\overline{\lambda})}{\overline{\lambda}(4b\eta-m^2)}\right),$$

respectively.

Proposition 5.1 shows that the OEM forces the CM to bear more than half of recall loss ($\theta > \frac{1}{2}$) and the percentage decreases with order quantity. Whereas once $e^{\mathrm{PS}} = \overline{\lambda}$, then $\theta^{\mathrm{PS}} = 1$. This is because an increase in the order quantity may motivate the CM to invest heavily in quality improvement. Therefore, the whole supply chain achieves a extreme situation. That is to say, the OEM offers most order quantity to the CM and let him bear all quality-related recall loss. Meanwhile, the CM would make highest quality investment level so that the product has highest quality and does not occur recall loss. The reason behinds this result is that the OEM can decide recall loss sharing rate and has stronger control power on recall loss allocation with the CM. In this case, the quality defect problem could be solved because the CM invests for quality improvement as much as possible.

5.2. Dual sourcing strategy

Here we analyze the OEM's profit under the dual sourcing strategy (namely, case PD). When the OEM adopts the strategy, she will choose order quantity q_i from the CM i, $i = 1, 2$. The sequence of events under this sourcing strategy is as follows: (i) the OEM chooses order quantity and recall loss sharing rate from the two CMs respectively; (ii) the two CMs determine related quality investment levels simultaneously; (iii) after the product has been delivered from the two CMs to the OEM, the consumer demand is realized. If the OEM sources product or key component from two CMs, he will order related quantity to maximize the expected profit,

$$\overline{\Pi}_o(q_1, q_2, \theta) = (q_1 + q_2)\left[a - b(q_1 + q_2)\right] - (1-\theta)(\overline{\lambda} - e_1)mq_1 - (1-\theta)(\overline{\lambda} - e_2)mq_2 - w(q_1 + q_2), \quad (5.3)$$

where the first term $(q_1 + q_2)\left[a - b(q_1 + q_2)\right]$ is the realized revenue, the second term $(1-\theta)(\overline{\lambda} - e_1)mq_1 + (1-\theta)(\overline{\lambda} - e_2)mq_2$ is the cost share incurred for recall loss, and the last term $w(q_1 + q_2)$ is the transfer payment to the two CMs. Correspondingly, the CM 1's profit can be written as,

$$\overline{\Pi}_{c1}(e_1) = wq_1 - \theta(\overline{\lambda} - e_1)mq_1 - \frac{\eta_1}{2}e_1^2. \quad (5.4)$$

Similarly, the CM 2's profit function is

$$\overline{\Pi}_{c2}(e_2) = wq_2 - \theta(\overline{\lambda} - e_2)mq_2 - \frac{\eta_2}{2}e_2^2, \quad (5.5)$$

where the first term is the transfer payment from the OEM, the second term is the cost share incurred for recall loss, and the last term is invest-related cost. Without loss of generality, $\eta_1 \leqslant \eta_2$ is assumed to reflect the CMs' heterogeneity.

Based on the sequence of the events under the PD case, the optimization problem faced by the OEM is defined as,

$$\begin{aligned} \max_{q_1, q_2, \theta} \quad & \overline{\Pi}_o(q_1, q_2, \theta | e_1^*, e_2^*) \\ \text{s.t.} \quad & e_1^* = \arg\max_{0 \leqslant e_1 \leqslant \overline{\lambda}} \pi_{c1}(e_1), \\ & e_2^* = \arg\max_{0 \leqslant e_2 \leqslant \overline{\lambda}} \pi_{c2}(e_2), \\ & \pi_{c1}(e_1) \geqslant 0, \ \pi_{c2}(e_2) \geqslant 0. \end{aligned}$$

Because the problem is depicted as Stackelberg game, we would develop model and solve it backward. The solved result is summarized as follows.

Proposition 5.2. *Under the dual sourcing strategy, the OEM's optimal order quantities for two CMs are*

$$q_i^{\mathrm{PD}} = \eta_i \frac{a - w - \overline{\lambda}m}{2b(\eta_1 + \eta_2) - m^2},$$

and the CMs' recall loss sharing rates and optimal investment levels are

$$\theta^{\mathrm{PD}} = \frac{1}{2}\left(1 + \frac{\overline{\lambda}(2b(\eta_1 + \eta_2) - m^2)}{m(a - w - \overline{\lambda}m)}\right), \ e_i^{\mathrm{PD}} = \frac{\overline{\lambda}}{2}\left(1 + \frac{m(a - w - \overline{\lambda}m)}{\overline{\lambda}(2b(\eta_1 + \eta_2) - m^2)}\right).$$

According to the Proposition 5.2, we find similar result with the Proposition 5.1. Specifically, the OEM offers most order quantity to two CMs when the latter tries their best to improve product quality (namely, when $e^{\text{PD}} = \overline{\lambda}$, then $\theta^{\text{PD}} = 1$). Apparently, the whole supply chain achieves an extreme situation in which the two CMs bear just all recall loss due to product recall. In addition, there are also some differences between the two sourcing strategies. For instance, under the dual sourcing strategy, the two CMs have lower control power on the sharing rate than that under the single sourcing strategy. This is because the OEM splits all order quantity to the two CMs equally. However, the two CMs still determine same quality investment levels to provide product with perfect quality. These changes mean the two CMs have lower profits than that under the single sourcing strategy. Furthermore, Proposition 5.3 indicates the indifference of sourcing strategies choices for the OEM under the partial sharing rate case.

Proposition 5.3. *With the same marginal production cost for the CMs, the OEM prefers the dual sourcing strategy.*

Different from the case of fixed sharing rate, as shown in Proposition 5.3, the OEM prefers to adopt the dual sourcing strategy when the CMs have the same production efficiency. This result occurs because the right to make decision on recall loss sharing rate decreases the OEM's recall loss even brings positive revenue for her, *i.e.*, $\theta \geqslant 1$. Consequently, the OEM benefits from a serious product recall under the dual sourcing strategy. However, some parameters such as marginal recall loss and wholesale price affect the decisions and profits of the OEM and the CMs, and this investigation could be demonstrated by numerical examples in Section 6. In addition, from the CM(s)' perspective, he achieves low order quantity and profit under the dual sourcing strategy because the competition between the CMs decreases their control power on recall loss allocation. Note that the CM with low investment efficiency may have low profit so that he has to reject the cooperation with the OEM, and this possibility suggests dual sourcing strategy becomes single sourcing strategy and the another CM would be a monopolist. However, to keep with the competition setting, the OEM may distribute more order to the CM with costly investment expense.

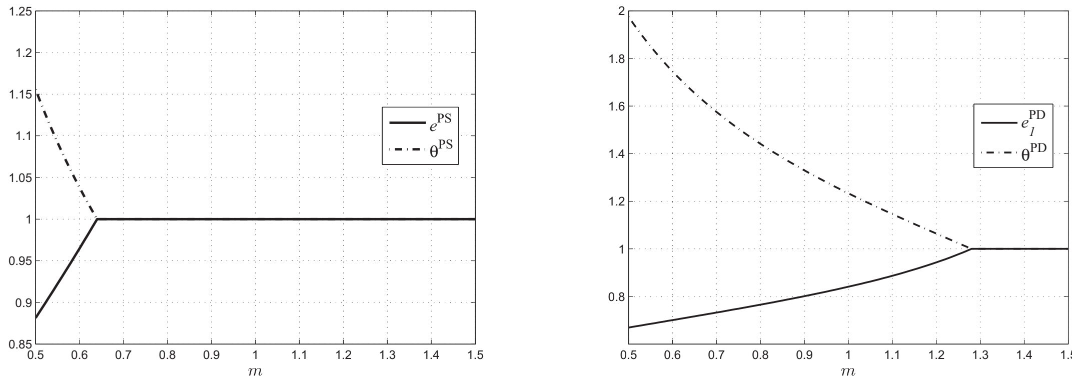
6. Numerical examples

In this section, we use several numerical examples to analyze the effect of different sourcing strategies on optimal decisions, and to seek the optimal profit. Meanwhile, we also analyze the effect of the cost for quality improvement on sourcing strategies choice. In particular, parameter settings that we used are provided as follows: $a = 4$, $w = 1.5$, $\overline{\lambda} = 1$, $m = 1$, $b = 0.8$ and $\eta = \eta_1 = \eta_2 = 1$. In the following, all decisions and the profits of the OEM and the CMs are investigated based on the use of different parameter settings. Along this line, when the value for one or more of the parameters is changed, the others remain unchanged as described above.

6.1. The effect on optimal decisions

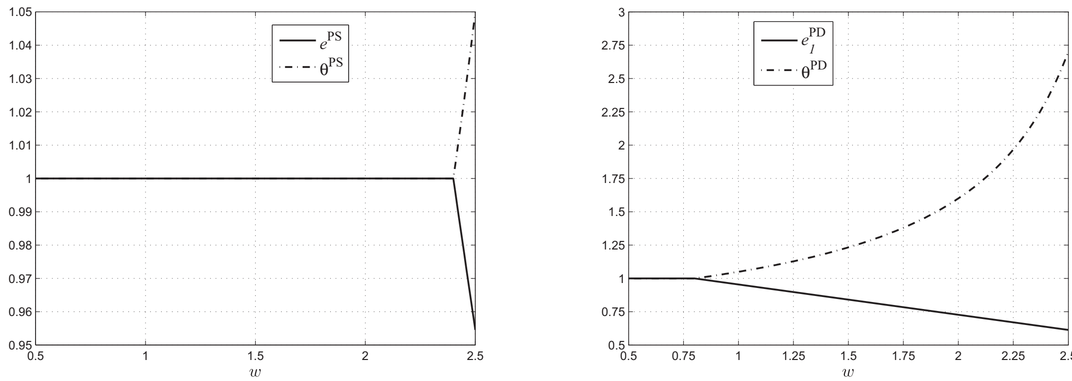
Now we analyze the effect of the marginal recall loss and wholesale price on the decision of the OEM under different sourcing strategies. First, the effect of the marginal recall loss on quality investment level and recall loss sharing rate will be analyzed by numerical example under specific parameters.

In the single sourcing strategy, as Figure 1a shows, the quality investment level (recall loss sharing rate) first increases (decreases) then reaches a highest (lowest) value, *i.e.*, $\overline{\lambda}$ (1), with an increase in the marginal recall loss. That is to say, the marginal recall loss has diverse influence on the quality investment level and the recall loss sharing rate. In particular, facing the pressure of increasing marginal recall loss, the CM increases quality investment level to decrease the possibility of product recall. Consequently, as a recognition mechanism for the CM with high quality investment level, the OEM decreases the recall loss sharing rate on him accordingly. However, the quality investment level reaches a ceiling when the marginal recall loss is overlarge. Accordingly, the OEM only requires the CM to bear all recall loss (*i.e.*, $\theta^{\text{PS}} = 1$). In the Figure 1b, under the dual sourcing strategy, we also obtain same results but the point to reach a ceiling of the quality investment level is later than



(a) (b)

FIGURE 1. The effect of marginal recall loss on quality investment level and recall loss sharing rate under different sourcing strategies.



(a) (b)

FIGURE 2. The effect of wholesale price on quality investment level and recall loss sharing rate under different sourcing strategies.

that of the single sourcing strategy. This is because the each CM obtains less order quantity under the dual sourcing strategy and further has few incentives to invest product quality.

Facing increasing wholesale price, the decisions of both the OEM and the CM have different changes with facing the marginal recall loss, as shown in the Figure 2. For example, in the Figure 2a, with a decrease in the wholesale price, the quality investment level (recall loss sharing rate) first increases (decreases) then reaches a highest (lowest) value, *i.e.*, $\overline{\lambda}$ (1). In fact, a decrease of the wholesale price forces the CM to strengthen its investment level and avoid unnecessary recall loss. Meanwhile, the OEM can split more revenue by increasing wholesale price, so she decreases the recall loss sharing rate to enhance the CM's willing to increase quality investment level. However, with a further decrease in the wholesale price, the quality investment level can reach a ceiling and the OEM only requires the CM to bear all recall loss ($\theta^{\mathrm{PS}} = 1$), accordingly. Similarly, with a decrease in the wholesale price, the quality investment level and the recall loss sharing rate first reach a critical value under the single sourcing strategy than that of the dual sourcing strategy. This result implies that an increase in the marginal recall loss and a decrease in the wholesale price have the same effects on the OEM's and the CM's optimal decisions.

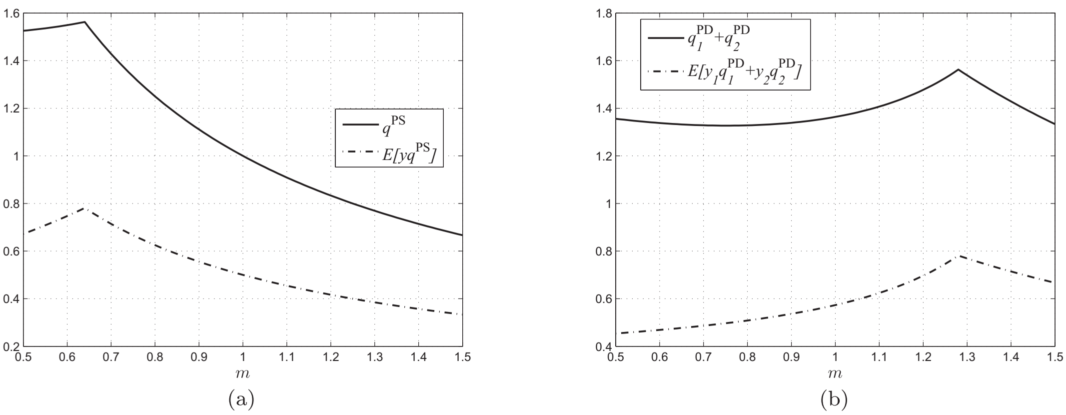


FIGURE 3. The effect of recall loss on quantity and investment level under different sourcing strategies.

Figure 3a shows that order quantity first increases and then decreases as the marginal recall loss becomes more serious (m increases). This occurs because the marginal recall loss incentives the CM to bear more investment effort, and the OEM can benefit it from increasing order quantity. However, as the marginal recall loss increases, the optimal quality investment level reaches a ceiling eventually. Consequently, the OEM only incurs more profit loss with a increase in the marginal recall loss. Note that, when m is close to zero, the increase of the expected order quantity is more rapidly than the increase of the order quantity, since the CM also increases the quality investment level at the same time. Interestingly, under the dual sourcing strategy, the order quantity first decreases then increases and back decreases with an increase in the marginal recall loss in the Figure 3b. To understand this pattern, note that the marginal recall loss not only changes the OEM's recall loss sharing rate, but also affects the CM's quality investment level. When m is close to zero, the CM is more motivated to invest in product quality and the OEM decreases the recall loss sharing rate on the CM accordingly. Thus, the OEM can decrease order quantity to balance market sale quantity in the narrow range of the marginal recall loss. However, as the increase of the m, the OEM has to add order quantity to ensure qualified product whose quantity is relatively sufficient. Furthermore, when the quality investment level reaches a ceiling, the OEM cannot benefit by increasing order quantity, but faces the pressure of increasing marginal recall loss. Consequently, the OEM decreases order quantity eventually. In terms of qualified product quantity, the marginal recall loss has same effect on it under different sourcing strategy. Differently, the ceiling of the m is first reached under the single sourcing and then under the dual sourcing. This finding suggests that the competition between the CMs delays the arrival of reaching the ceiling.

Moreover, as shown in Figure 4, with a decrease in the wholesale price, the order quantity and the qualified product quantity first increase then reach a ceiling under each sourcing strategy. This is intuitive because the OEM obtains more profit by increasing order quantity when the wholesale price decreases. However, the increase is limited because there is a ceiling on the quality investment level. In addition, we find that the ceiling is started earlier under the single sourcing than that under the dual sourcing. This is because the competition between the CMs splits their order quantity. Consequently, facing decreasing wholesale price, the two CMs have less incentive to increase their quality investment level, and the ceiling of the investment level is started later under the dual sourcing strategy.

6.2. The effect of sourcing strategies on optimal profit

In the previous subsection, we consider the effect of the marginal recall loss and the wholesale price on the OEM's and the CM's decisions under different sourcing strategies. In the following, the OEM's and the CM's

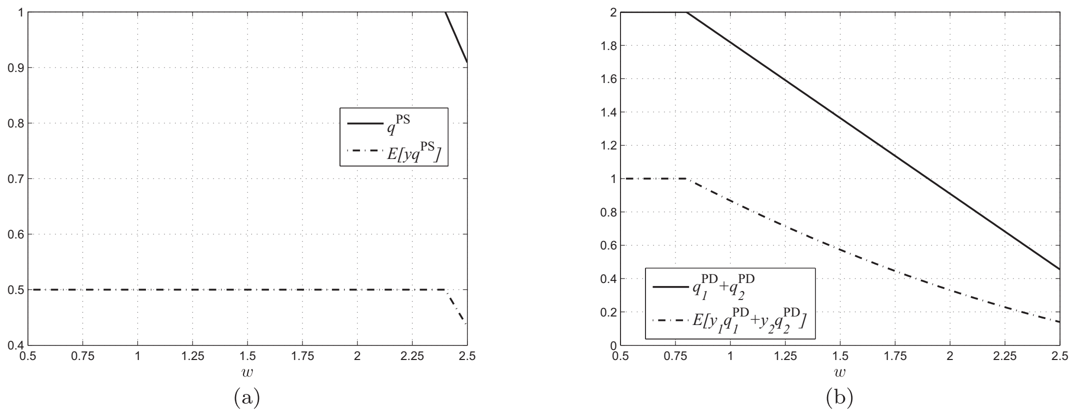
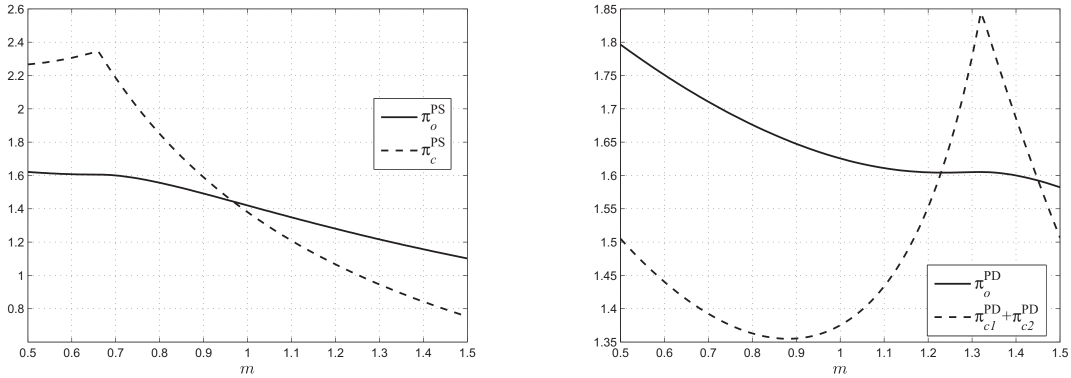


FIGURE 4. The effect of loss share level on quantity and investment level under different sourcing strategies.

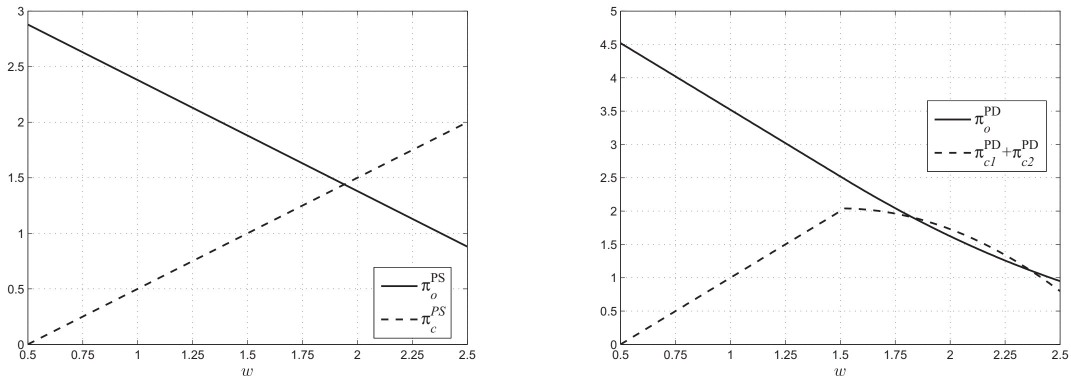
profits will be analyzed under single and dual sourcing strategies, respectively. For the single sourcing strategy, from Figure 5a, the OEM's profit decreases in the marginal recall loss all the way but first slowly then rapidly. Essentially, the marginal recall loss plays a key role. Specifically, the increasing of the m forces the CM to increase quality investment level and decreases the probability of product recall although it also causes large marginal recall loss. When the m is close to zero, the quality investment level increases in the m, thus the OEM's profit decreases slowly. However, with the further increase of the m, the quality investment level no longer increases, and then the OEM's profit decreases rapidly. Differently, the CM's profit first increases then decreases in the m. This is because the OEM can decrease the recall loss sharing rate with the increase of the m when it is relatively small, thus the CM decreases recall loss. Under the dual sourcing strategy, as shown in Figure 5b, the changes of the OEM's and the CM's profit on the m become complicate. To be specific, the OEM's profit decreases in the m but first rapidly then slowly and back rapidly. Note that the OEM obtains less profit when the m is relatively small because the CMs split the OEM's order quantity and have less motivation to increase quality investment level. Consequently, the OEM decreases recall loss sharing rate slowly on the CM and obtains profit slowly. And the two CMs' profit first decreases then increases and back decreases with an increase in the m. Note that the OEM decreases order quantity with an increase in the m from the start. Therefore, the CM's profit first decreases. When the m is moderate, due to the overlapping of increasing investment level and decreasing recall loss sharing rate, the CM's profit increases in the m. But with the further increase in the m, the marginal recall loss is significant and the CM's profit begins to decrease rapidly. For the sourcing strategy choice of the supply chain numbers, the OEM and the CMs can reach a win-win scenario when the marginal recall loss is relatively large.

According to Figure 6a, the OEM's profit decreases and the CM's profit increases in the wholesale price. This result is intuitive because the wholesale price benefits (damages) for the CMs (OEM). However, under the dual sourcing strategy, the CMs' profit first increases then decreases with an increase in the wholesale price in Figure 6b. Note that with the wholesale price increases, the OEM decreases order quantity due to the low marginal revenue. Consequently, the CM's profit also decreases when the wholesale price is relatively large. Moreover, in the single sourcing strategy, the CM's profit may be higher than the OEM's when the wholesale price is relatively large, it is intuitive. However, in the dual sourcing strategy, the CMs' profit may be lower than the OEM's profit when the wholesale price is relatively large. In addition, when the wholesale price is relatively small, the OEM and the CMs can reach a win-win scenario.



(a) (b)

FIGURE 5. The effect of loss share level on the profits under different sourcing strategies.



(a) (b)

FIGURE 6. The effect of loss share level on the profits under different sourcing strategies.

7. Extension with endogenous wholesale price

In the preceding analysis, we assumed that in the business between OEM and CMs, the wholesale price for order production w is exogenously fixed. A natural extension of this assumption is to study how the endogenous wholesale price impacts the OEM's sourcing strategy choice. In this section, we consider w to be an endogenous variable that is determined by CMs. In addition, we only study the OEM's sourcing strategy choice under fixed sharing rate because the results under the other case are simple and no further analysis significance.

When the CMs determine the wholesale price with fixed sharing rate for defect product recall loss, the sequence of events is as follows: (i) the CM (or CMs) determines the component wholesale price under single sourcing (or dual sourcing); (ii) the OEM chooses order quantity from the CM(s); (iii) the CM(s) chooses related quality investment level at a fixed recall loss sharing rate; (iv) expected demand is realized after the product has been delivered from the CM(s) to the OEM. By backwards induction, we derive the analytical solutions, and the detail on the optimal wholesale price, order quantity and quality investment level is included in the Appendix A. The following proposition compares the CMs' wholesale price under different sourcing strategy.

Proposition 7.1. *With the endogenous wholesale price, when the CMs have same marginal quality investment cost, i.e.,* $\eta = \eta_1 = \eta_2$*, we have* $w_i^{\mathrm{FD}} \geqslant w^{\mathrm{FS}}$*. And under specific condition,* $\pi_o^{\mathrm{FS}} \geqslant \pi_o^{\mathrm{FD}}$*.*

In general, as the superiority of the dual sourcing strategy, the CMs may offer low wholesale price due to order quantity competition between them, and the OEM obtains a cost-reduction benefit. However, in our paper, the CM charges a higher wholesale price under the dual sourcing than that under the single sourcing. This result implies that the OEM bears heavy order quantity cost due to upstream competition from CMs who are homogeneous. Note that the each CM receives lower order quantity and makes a lower quality investment level under the dual sourcing than that under the single sourcing based on the result of Proposition 4.8. Owing to the recall loss sharing rate and the CMs' first-mover advantages, the CMs prefer to raise price to offset potential recall loss under the dual sourcing, especially when they share a higher recall loss proportion (The specific condition please see the proof in the Appendix A). Consequently, the OEM prefers to adopt the single sourcing when the CMs can determine the wholesale price and have same quality investment cost. Moreover, we find this result is consistent with results derived from the above case where the wholesale price is exogenous.

8. Conclusion

In this paper, we consider a supply chain in which an OEM outsources her production to a CM or two CMs depending on different sourcing strategies. Based on the business operation, we assume the product has quality problem but can be improved by the CM(s)' quality investment. Meanwhile, we also consider two recall loss allocation settings, namely, the fixed sharing rate and the partial sharing rate. Under the two cases, we characterize the OEM's optimal sourcing strategy.

With the fixed recall loss sharing rate, the OEM may adopt the single sourcing strategy or the dual sourcing strategy which depends on the recall loss sharing rate. Specifically, when the sharing rate is small, the OEM prefers to adopt the single sourcing strategy. Otherwise, the dual sourcing strategy is superior to the dual sourcing strategy. Intuitively, under the dual sourcing strategy, the OEM would split all order quantity to the respective CM. Consequently, the each CM lacks the incentive to improve product's quality by quality investment. This implies the dual sourcing strategy may be not optimal choice for the OEM. However, when the recall loss sharing rate is larger than one, the OEM benefits from the product recall by shifting the recall loss and further extorting a penalty to the CM. Naturally, the OEM prefers to adopt the dual sourcing strategy.

When the recall loss sharing rate is endogenous, the OEM is more likely to adopt the dual sourcing strategy. Moreover, once the OEM decides recall loss sharing rate, she has stronger control power on sharing rate than that of the CM(s). Therefore, the OEM transfers the recall loss to the CM and further obtains additional revenue by asking for overloaded penalty. Although this result may be impractical, we still find some management insights based on the previous finding. For example, if the OEM has strong control power on sharing rate, then she generally controls the whole supply chain and extracts other partners' profit. Intuitively, the OEM may prefer to adopt the dual sourcing because the competition of the two CMs decreases their control power on the sharing rate. In addition, as an extension, we consider the wholesale price is endogenous, and the OEM pays higher wholesale price under the dual sourcing than that under the single sourcing. This result implies the OEM can not obtain the cost-reduction benefit under the dual sourcing.

By numerical examples, we investigate the effects of some exogenous parameters on the OEM and the CM when the recall loss sharing rate is endogenous. To be specific, with an increase in the marginal recall loss (or a decrease in the wholesale price), the quality investment level (the recall loss sharing rate) increases (decreases) then reaches a ceiling eventually. Interestingly, we find the CM is more likely to increase quality investment with a decrease in the recall loss sharing rate. With a decrease in the wholesale price, the order quantity first increases then reaches a ceiling. For the OEM's profit, it decreases in the marginal recall loss and the wholesale price. In addition, the CM's profit first increases then decreases in the marginal recall loss under the single sourcing strategy, but it first decreases then increases and back decreases under the dual sourcing strategy. With an increase in the wholesale price, the CM's profit increases under the single sourcing strategy, but further decreases when the wholesale price is relatively large under the dual sourcing strategy. In summary, when the marginal recall loss is large or the wholesale price is relatively small, the OEM and the CMs can reach a win-win scenario.

Finally, we discuss some limitations of the current study and suggest future research directions. First, we focus only on the sourcing strategy choice of one monopolist and do not consider the setting of multi OEMs. With the OEMs' competition setting, each firm's sourcing strategy may depend on its competitor's sourcing strategy. Second, we assume the product's defect rate is uncertain, but the product's supply is also uncertain in practice. Intuitively, some of our insights would change if the OEM trades off supply uncertainty and recall loss risk. Therefore, further research may attempt to better understand how competition of OEMs and trade off between supply uncertainty and recall loss risk affect the OEM's soucring strategy choices.

Appendix A.

Proof of Theorem 4.1. First, taking the first and second derivatives of the function (4.2) with respect to e, we get that

$$\frac{\mathrm{d}\pi_c}{\mathrm{d}e} = \theta m q - \eta e, \tag{A.1}$$

$$\frac{\mathrm{d}^2\pi_c}{\mathrm{d}e^2} = -\eta < 0. \tag{A.2}$$

Apparently, π_c is concave in e. We solve the $\frac{\mathrm{d}\pi_c}{\mathrm{d}e} = 0$ to get the CM's best response on quality investment level.

Proof of Proposition 4.2. First, taking the first and second derivatives of the function (4.3) with respect to q, we get that

$$\frac{\mathrm{d}\pi_o}{\mathrm{d}q} = -2\Big(b - \frac{1}{\eta}\theta(1-\theta)m^2\Big)q + (a - w - \overline{\lambda}m(1-\theta)), \tag{A.3}$$

$$\frac{\mathrm{d}^2\pi_o}{\mathrm{d}q^2} = -2\Big(b - \frac{1}{\eta}\theta(1-\theta)m^2\Big) < 0. \tag{A.4}$$

Apparently, π_o is concave in q when $b\eta - \theta(1-\theta)m^2 > 0$. We solve the $\frac{\mathrm{d}\pi_o}{\mathrm{d}q} = 0$ to get the OEM's optimal order quantity. Accordingly, substituting the OEM's optimal order quantity into the CM's best response on quality improvement level, then the optimal quality improvement level can be obtained.

In addition, to ensure that the optimal quality investment level is lower than the initial level ($e^{\mathrm{FS}} \leqslant \overline{\lambda}$), we have

$$\begin{aligned} e^{\mathrm{FS}} - \overline{\lambda} &= \frac{\theta m(a - w - (1-\theta)\overline{\lambda}m) - 2\overline{\lambda}(b\eta - \theta(1-\theta)m^2)}{2(b\eta - \theta(1-\theta)m^2)} \\ &= \frac{-\overline{\lambda}m^2\theta^2 + (a - w + \overline{\lambda}m)m\theta - 2\overline{\lambda}b\eta}{2(b\eta - \theta(1-\theta)m^2)} \leqslant 0. \end{aligned} \tag{A.5}$$

Because the denominator $2(b\eta - theta(1-\theta)m^2)$ is positive, next we only seek a threshold to satisfy $-\overline{\lambda}m^2\theta^2 + (a - w + \overline{\lambda}m)m\theta - 2\overline{\lambda}b\eta = 0$. By the first order condition with respect to θ, we get two solutions, $\frac{(a-w+\overline{\lambda}m) - \sqrt{(a-w+\overline{\lambda}m)^2 - 8\overline{\lambda}^2 b\eta}}{2\overline{\lambda}m^2}$ and $\frac{(a-w+\overline{\lambda}m) + \sqrt{(a-w+\overline{\lambda}m)^2 - 8\overline{\lambda}^2 b\eta}}{2\overline{\lambda}m^2}$. Note that $b\eta - \theta(1-\theta)m^2 > 0$, the latter can be omitted. Thus when $\theta \leqslant \frac{(a-w+\overline{\lambda}m) - \sqrt{(a-w+\overline{\lambda}m)^2 - 8\overline{\lambda}^2 b\eta}}{2\overline{\lambda}m^2}$, the optimal internal solutions are existed.

Proof of Proposition 4.3. First, taking the first derivative of the optimal order quantity with respect to θ, we get that

$$\begin{aligned}\frac{\mathrm{d}q_o^{\mathrm{FS}}}{\mathrm{d}\theta} &= \frac{\eta m}{2}\frac{\overline{\lambda}(b\eta-\theta(1-\theta)m^2)+(a-w-\overline{\lambda}m(1-\theta))(1-2\theta)m}{(b\eta-\theta(1-\theta)m^2)^2}\\ &= \frac{\eta m}{2(b\eta-\theta(1-\theta)m^2)^2}(\overline{\lambda}b\eta-m(\overline{\lambda}m(1-\theta)^2+(1-2\theta)(a-w)))\\ &= \frac{\eta m}{2(b\eta-\theta(1-\theta)m^2)^2}(\overline{\lambda}b\eta-m(\overline{\lambda}m\theta^2-2(a-w+\overline{\lambda}m)\theta+(a-w+\overline{\lambda}m))).\end{aligned}$$

Apparently, the sign of $\frac{\mathrm{d}q_o^{\mathrm{FS}}}{\mathrm{d}\theta}$ depends on the sign of $\overline{\lambda}b\eta-m(\overline{\lambda}m\theta^2-2(a-w+\overline{\lambda}m)\theta+(a-w+\overline{\lambda}m)))$. We solve the $\frac{\mathrm{d}q_o^{\mathrm{FS}}}{\mathrm{d}\theta}=0$ to get the threshold of θ,

$$\widehat{\theta}_1=\frac{1}{\overline{\lambda}m}\left((a-w+\overline{\lambda}m)-\sqrt{(a-w+\overline{\lambda}m)+\overline{\lambda}^2b\eta}\right). \tag{A.6}$$

Naturally, when $\theta\leqslant\widehat{\theta}_1$, then $\frac{\mathrm{d}q_o^{\mathrm{FS}}}{\mathrm{d}\theta}\geqslant 0$, otherwise, $\frac{\mathrm{d}q_o^{\mathrm{FS}}}{\mathrm{d}\theta}<0$.

Accordingly, taking the first derivative of the optimal quality improvement level with respect to θ, we get that

$$\begin{aligned}\frac{\mathrm{d}e_o^{\mathrm{FS}}}{\mathrm{d}\theta} &= \frac{m}{2}\frac{(a-w-\overline{\lambda}m(1-2\theta))(b\eta-\theta(1-\theta)m^2)+\theta(1-2\theta)m^2(a-w-\overline{\lambda}m(1-\theta))}{(b\eta-\theta(1-\theta)m^2)^2}\\ &= \frac{m}{2(b\eta-\theta(1-\theta)m^2)^2}((a-w-\overline{\lambda}m)b\eta-m(m(a-w)\theta^2-2\overline{\lambda}b\eta\theta))\\ &= \frac{m}{2(b\eta-\theta(1-\theta)m^2)^2}(-m^2(a-w)\theta^2+2\overline{\lambda}mb\eta\theta+(a-w-\overline{\lambda}m)b\eta).\end{aligned}$$

Apparently, the sign of $\frac{\mathrm{d}e^{\mathrm{FS}}}{\mathrm{d}\theta}$ depends on the sign of $-m^2(a-w)\theta^2+2\overline{\lambda}mb\eta\theta+(a-w-\overline{\lambda}m)b\eta$. We solve the $\frac{\mathrm{d}e^{\mathrm{FS}}}{\mathrm{d}\theta}=0$ to get the threshold of θ,

$$\widehat{\theta}_2=\frac{\overline{\lambda}b\eta+\sqrt{(\overline{\lambda}b\eta)^2+b\eta(a-w)(a-w-\overline{\lambda}m)}}{m(a-w)}. \tag{A.7}$$

Naturally, when $\theta\leqslant\widehat{\theta}_2$, then $\frac{\mathrm{d}e^{\mathrm{FS}}}{\mathrm{d}\theta}\geqslant 0$, otherwise, $\frac{\mathrm{d}e^{\mathrm{FS}}}{\mathrm{d}\theta}<0$.

Proof of Proposition 4.4. First, taking the first derivative of the function (4.4) with respect to θ, we get that

$$\begin{aligned}\frac{\mathrm{d}\pi_o^{\mathrm{FS}}}{\mathrm{d}\theta} &= \frac{2(a-w-\overline{\lambda}m(1-\theta))\overline{\lambda}m(b\eta-\theta(1-\theta)m^2)+(a-w-\overline{\lambda}m(1-\theta))^2(1-2\theta)m^2}{(b\eta-\theta(1-\theta)m^2)^2}\\ &= \frac{m(a-w-\overline{\lambda}m(1-\theta))}{(b\eta-\theta(1-\theta)m^2)^2}(2\overline{\lambda}(b\eta-\theta(1-\theta)m^2)+m(1-2\theta)(a-w-\overline{\lambda}m(1-\theta)))\\ &= \frac{m(a-w-\overline{\lambda}m(1-\theta))}{(b\eta-\theta(1-\theta)m^2)^2}(2\overline{\lambda}b\eta+m(a-w)(1-2\theta)-\overline{\lambda}m^2(1-\theta))\\ &= \frac{m(a-w-\overline{\lambda}m(1-\theta))}{(b\eta-\theta(1-\theta)m^2)^2}(2\overline{\lambda}b\eta+m(a-w)-\overline{\lambda}m^2-\theta m(2(a-w)-\overline{\lambda}m)).\end{aligned} \tag{A.8}$$

Apparently, the sign of $\frac{\mathrm{d}\pi_o}{\mathrm{d}\theta}$ depends on the sign of $2\overline{\lambda}b\eta+m(a-w)-\overline{\lambda}m^2-\theta m(2(a-w)-\overline{\lambda}m)$. We solve the $\frac{\mathrm{d}\pi_o}{\mathrm{d}\theta}=0$ to get the threshold of θ,

$$\widehat{\theta}_3=\frac{1}{2}+\overline{\lambda}\frac{4b\eta-m^2}{2m(2(a-w)-\overline{\lambda}m)}. \tag{A.9}$$

Naturally, when $\theta \leqslant \widehat{\theta}_3$, then $\frac{\mathrm{d}\pi_o^{\mathrm{FS}}}{\mathrm{d}\theta} \geqslant 0$.

Proof of Theorem 4.5. First, taking the first and second derivatives of the functions (4.6) and (4.7) with respect to e_1 and e_2, we get that

$$\frac{\mathrm{d}\pi_{c1}}{\mathrm{d}e_1} = \theta m q_1 - \eta_1 e_1, \tag{A.10}$$

$$\frac{\mathrm{d}\pi_{c2}}{\mathrm{d}e_2} = \theta m q_2 - \eta_2 e_2, \tag{A.11}$$

$$\frac{\mathrm{d}^2\pi_{c1}}{\mathrm{d}e_1^2} = -\eta_1 < 0, \tag{A.12}$$

$$\frac{\mathrm{d}^2\pi_{c2}}{\mathrm{d}e_2^2} = -\eta_2 < 0. \tag{A.13}$$

Apparently, π_{ci} is concave in e_i, $i = 1, 2$. We solve the $\frac{\mathrm{d}\pi_{ci}}{\mathrm{d}e_i} = 0$ to get the CM i's best response on quality investment level.

Proof of Proposition 4.6. First, taking the first and second derivatives of the function (4.8) with respect to q_1 and q_2, we get that

$$\frac{\partial \Pi_o}{\partial q_1} = -2\Big(b - \frac{1}{\eta_1}\theta(1-\theta)m^2\Big)q_1 + (a - w - \overline{\lambda}m(1-\theta)) - 2bq_2, \tag{A.14}$$

$$\frac{\partial \Pi_o}{\partial q_2} = -2\Big(b - \frac{1}{\eta_2}\theta(1-\theta)m^2\Big)q_2 + (a - w - \overline{\lambda}m(1-\theta)) - 2bq_1, \tag{A.15}$$

$$\frac{\partial^2 \Pi_o}{\partial q_1^2} = -2\Big(b - \frac{1}{\eta_1}\theta(1-\theta)m^2\Big), \tag{A.16}$$

$$\frac{\partial^2 \Pi_o}{\partial q_2^2} = -2\Big(b - \frac{1}{\eta_2}\theta(1-\theta)m^2\Big), \tag{A.17}$$

$$\frac{\partial^2 \Pi_o}{\partial q_1 \partial q_2} = -2b. \tag{A.18}$$

Note that $\frac{\partial^2 \Pi_o}{\partial q_1^2} < 0$, and $\frac{\partial^2 \Pi_o}{\partial q_1^2}\frac{\partial^2 \Pi_o}{\partial q_2^2} - \big(\frac{\partial^2 \Pi_o}{\partial q_1 \partial q_2}\big)^2 > 0$ when $\theta > 1$, the Hessian matrix is negative definite. Apparently, Π_o is jointly concave in q_1 and q_2. We solve the $\frac{\partial \Pi_o}{\partial q_i} = 0$ to get the OEM's optimal order quantity. Accordingly, substituting the OEM's optimal order quantity into the CM's best response on quality improvement level, then the optimal quality improvement level can be also obtained. In addition, to ensure that the optimal quality investment level is lower than the initial level ($e_i^{\mathrm{FS}} \leqslant \overline{\lambda}$) for the CM i, it needs a similar threshold to satisfy the inequation. Due to the same process with the Proposition 4.2 on the threshold, and we omit the solving solution proof.

However, when $\theta \leqslant 1$, the above critical point is not optimal value for the OEM, then the optimal point may be boundary optimal solution. Now we turn to solve the boundary optimal solution. Intuitively, $q_1 = q_2 = 0$ makes the OEM obtain a zero profit. And when $q_1 = 0$ or $q_2 = 0$, the OEM's optimal solution and profit are same with that under single sourcing, namely, $q_i^* = \frac{\eta_i}{2}\frac{a-w-\overline{\lambda}m(1-\theta)}{b\eta_i - \theta(1-\theta)m^2}$, $\pi_o^* = \frac{\eta_i}{4}\frac{(a-w-\overline{\lambda}m(1-\theta))^2}{b\eta_i - \theta(1-\theta)m^2}$. Because $\eta_1 \leqslant \eta_2$, then the OEM can only source component from the CM 1 to maximize profits.

Proof of Proposition 4.7. Due to the CMs' homogeneity, we only proof the CM 1's sensitivity analysis. First, taking the first derivative of the optimal order quantity with respect to θ, we get that

$$\frac{\mathrm{d}q_1^{\mathrm{FD}}}{\mathrm{d}\theta} = \frac{\eta_1 m}{2}\frac{\overline{\lambda}(b(\eta_1+\eta_2) - \theta(1-\theta)m^2) + (a - w - \overline{\lambda}m(1-\theta))(1-2\theta)m}{(b(\eta_1+\eta_2) - \theta(1-\theta)m^2)^2}$$

$$= \frac{\eta_1 m}{2(b(\eta_1+\eta_2)-\theta(1-\theta)m^2)^2}(\overline{\lambda}b(\eta_1+\eta_2)-m(\overline{\lambda}m(1-\theta)^2+(1-2\theta)(a-w)))$$
$$= \frac{\eta_1 m}{2(b(\eta_1+\eta_2)-\theta(1-\theta)m^2)^2}(\overline{\lambda}b(\eta_1+\eta_2)-m(\overline{\lambda}m\theta^2-2(a-w+\overline{\lambda}m)\theta+(a-w+\overline{\lambda}m))).$$

Apparently, the sign of $\frac{\mathrm{d}q_1^{\mathrm{FD}}}{\mathrm{d}\theta}$ depends on the sign of $\overline{\lambda}b(\eta_1+\eta_2)-m(\overline{\lambda}m\theta^2-2(a-w+\overline{\lambda}m)\theta+(a-w+\overline{\lambda}m)))$. We solve the $\frac{\mathrm{d}q_1^{\mathrm{FD}}}{\mathrm{d}\theta}=0$ to get the threshold of θ,

$$\widehat{\theta}_4 = \frac{1}{\overline{\lambda}m}((a-w+\overline{\lambda}m)+\sqrt{(a-w+\overline{\lambda}m)+\overline{\lambda}^2 b(\eta_1+\eta_2)}). \tag{A.19}$$

Naturally, when $\theta \leqslant \widehat{\theta}_4$, then $\frac{\mathrm{d}q_o^{\mathrm{FD}}}{\mathrm{d}\theta} \geqslant 0$, otherwise, $\frac{\mathrm{d}q_o^{\mathrm{FD}}}{\mathrm{d}\theta} < 0$.

Accordingly, taking the first derivative of the optimal quality improvement level with respect to θ, we get that

$$\frac{\mathrm{d}e_1^{\mathrm{FD}}}{\mathrm{d}\theta} = \frac{m}{2}\frac{(a-w-\overline{\lambda}m(1-2\theta))(b(\eta_1+\eta_2)-\theta(1-\theta)m^2)+\theta(1-2\theta)m^2(a-w-\overline{\lambda}m(1-\theta))}{(b(\eta_1+\eta_2)-\theta(1-\theta)m^2)^2}$$
$$= \frac{m}{2(b(\eta_1+\eta_2)-\theta(1-\theta)m^2)^2}((a-w-\overline{\lambda}m)b(\eta_1+\eta_2)-m(m(a-w)\theta^2-2\overline{\lambda}b(\eta_1+\eta_2)\theta))$$
$$= \frac{m}{2(b(\eta_1+\eta_2)-\theta(1-\theta)m^2)^2}(-m^2(a-w)\theta^2+2\overline{\lambda}mb(\eta_1+\eta_2)\theta+(a-w-\overline{\lambda}m)b(\eta_1+\eta_2)).$$

Apparently, the sign of $\frac{\mathrm{d}e_1^{\mathrm{FD}}}{\mathrm{d}\theta}$ depends on the sign of $-m^2(a-w)\theta^2+2\overline{\lambda}mb(\eta_1+\eta_2)\theta+(a-w-\overline{\lambda}m)b(\eta_1+\eta_2)$. We solve the $\frac{\mathrm{d}e_1^{\mathrm{FD}}}{\mathrm{d}\theta}=0$ to get the threshold of θ,

$$\widehat{\theta}_5 = \frac{\overline{\lambda}b(\eta_1+\eta_2)+\sqrt{(\overline{\lambda}b(\eta_1+\eta_2))^2+b(\eta_1+\eta_2)(a-w)(a-w-\overline{\lambda}m)}}{m(a-w)}. \tag{A.20}$$

Naturally, when $\theta \leqslant \widehat{\theta}_5$, then $\frac{\mathrm{d}e_1^{\mathrm{FD}}}{\mathrm{d}\theta} \geqslant 0$, otherwise, $\frac{\mathrm{d}e_1^{\mathrm{FD}}}{\mathrm{d}\theta} < 0$.

Finally, taking the first derivative of the function (4.4) with respect to θ, we get that

$$\frac{\mathrm{d}\Pi_o^{\mathrm{FD}}}{\mathrm{d}\theta} = \frac{\eta_1+\eta_2}{4}\frac{2(a-w-\overline{\lambda}m(1-\theta))\overline{\lambda}m(b(\eta_1+\eta_2)-\theta(1-\theta)m^2)+(a-w-\overline{\lambda}m(1-\theta))^2(1-2\theta)m^2}{(b(\eta_1+\eta_2)-\theta(1-\theta)m^2)^2}$$
$$= \frac{\eta_1+\eta_2}{4}\frac{m(a-w-\overline{\lambda}m(1-\theta))}{(b(\eta_1+\eta_2)-\theta(1-\theta)m^2)^2}(2\overline{\lambda}(b(\eta_1+\eta_2)-\theta(1-\theta)m^2)+m(1-2\theta)(a-w-\overline{\lambda}m(1-\theta)))$$
$$= \frac{\eta_1+\eta_2}{4}\frac{m(a-w-\overline{\lambda}m(1-\theta))}{(b(\eta_1+\eta_2)-\theta(1-\theta)m^2)^2}(2\overline{\lambda}b(\eta_1+\eta_2)+m(a-w)(1-2\theta)-\overline{\lambda}m^2(1-\theta))$$
$$= \frac{\eta_1+\eta_2}{4}\frac{m(a-w-\overline{\lambda}m(1-\theta))}{(b(\eta_1+\eta_2)-\theta(1-\theta)m^2)^2}(2\overline{\lambda}b(\eta_1+\eta_2)+m(a-w)-\overline{\lambda}m^2-\theta m(2(a-w)-\overline{\lambda}m)). \tag{A.21}$$

Apparently, the sign of $\frac{\mathrm{d}\Pi_o^{\mathrm{FD}}}{\mathrm{d}\theta}$ depends on the sign of $2\overline{\lambda}b(\eta_1+\eta_2)+m(a-w)-\overline{\lambda}m^2-\theta m(2(a-w)-\overline{\lambda}m)$. We solve the $\frac{\mathrm{d}\Pi_o^{\mathrm{FD}}}{\mathrm{d}\theta}=0$ to get the threshold of θ,

$$\widehat{\theta}_6 = \frac{1}{2}+\overline{\lambda}\frac{4b(\eta_1+\eta_2)-m^2}{2m(2(a-w)-\overline{\lambda}m)}. \tag{A.22}$$

Naturally, when $\theta \leqslant \widehat{\theta}_6$, then $\frac{\mathrm{d}\Pi_o^{\mathrm{FD}}}{\mathrm{d}\theta} \geqslant 0$, otherwise, $\frac{\mathrm{d}\Pi_o^{\mathrm{FD}}}{\mathrm{d}\theta} < 0$.

Proof of Proposition 4.8. In the proposition, we need to compare the optimal solutions of order quantity and quality improvement and the OEM's optimal profit under the two sourcing strategies. For tractability, the CMs are assumed as homogeneous ones, namely, they have same marginal quality investment cost. For the comparison of optimal order quantity, we have

$$\begin{aligned} q^{\mathrm{FS}}-(q_1^{\mathrm{FD}}+q_1^{\mathrm{FD}}) &= \eta\frac{a-w-(1-\theta)\overline{\lambda}m}{2(b\eta-\theta(1-\theta)m^2)}-\eta\frac{a-w-(1-\theta)\overline{\lambda}m}{2b\eta-\theta(1-\theta)m^2} \\ &= \eta(a-w-(1-\theta)\overline{\lambda}m)\left(\frac{1}{2(b\eta-\theta(1-\theta)m^2)}-\frac{1}{2b\eta-\theta(1-\theta)m^2}\right) \\ &= \eta(a-w-(1-\theta)\overline{\lambda}m)\frac{\theta(1-\theta)m^2}{2(b\eta-\theta(1-\theta)m^2)(2b\eta-\theta(1-\theta)m^2)} \\ &< 0. \end{aligned} \tag{A.23}$$

And for the quality improvement level, we have

$$\begin{aligned} e^{\mathrm{FS}}-e_i^{\mathrm{FD}} &= \frac{\theta m}{2}\frac{a-w-(1-\theta)\overline{\lambda}m}{b\eta-\theta(1-\theta)m^2}-\frac{\theta m}{2}\frac{a-w-(1-\theta)\overline{\lambda}m}{2b\eta-\theta(1-\theta)m^2} \\ &= \frac{1}{2}\theta m(a-w-(1-\theta)\overline{\lambda}m)\left(\frac{1}{b\eta-\theta(1-\theta)m^2}-\frac{1}{2b\eta-\theta(1-\theta)m^2}\right) \\ &= \frac{1}{2}\theta m(a-w-(1-\theta)\overline{\lambda}m)\frac{b\eta}{(b\eta-\theta(1-\theta)m^2)(2b\eta-\theta(1-\theta)m^2)} \\ &> 0. \end{aligned} \tag{A.24}$$

Finally, for the optimal profit under different sourcing strategy, we can find

$$\begin{aligned} \pi_o^{\mathrm{FS}}-\pi_o^{\mathrm{FD}} &= \frac{\eta}{2}\frac{(a-w-(1-\theta)\overline{\lambda}m)^2}{2(b\eta-\theta(1-\theta)m^2)}-\frac{\eta}{2}\frac{(a-w-(1-\theta)\overline{\lambda}m)^2}{2b\eta-\theta(1-\theta)m^2} \\ &= \frac{\eta}{2}(a-w-(1-\theta)\overline{\lambda}m)^2\left(\frac{1}{2(b\eta-\theta(1-\theta)m^2)}-\frac{1}{2b\eta-\theta(1-\theta)m^2}\right) \\ &= \frac{\eta}{2}(a-w-(1-\theta)\overline{\lambda}m)^2\frac{\theta(1-\theta)m^2}{2(b\eta-\theta(1-\theta)m^2)(2b\eta-\theta(1-\theta)m^2)} \\ &< 0. \end{aligned} \tag{A.25}$$

Proof of Proposition 5.1. Because the second stage on the CM's quality investment level decision is same with that FS case, then we analyze the first stage on the order quantity and the sharing rate decisions directly. Based on the function (4.3), taking the first and second derivatives with respect to q and θ respectively, we get that,

$$\frac{\partial\overline{\pi}_o}{\partial q} = -2\Big(b-\frac{1}{\eta}\theta(1-\theta)m^2\Big)q+(a-w-\overline{\lambda}m(1-\theta)), \tag{A.26}$$

$$\frac{\partial^2\overline{\pi}_o}{\partial q^2} = -2\Big(b-\frac{1}{\eta}\theta(1-\theta)m^2\Big), \tag{A.27}$$

$$\frac{\partial\overline{\pi}_o}{\partial\theta} = \frac{1}{\eta}(1-2\theta)m^2q^2+\overline{\lambda}mq, \tag{A.28}$$

$$\frac{\partial^2\overline{\pi}_o}{\partial\theta^2} = -\frac{2}{\eta}m^2q^2, \tag{A.29}$$

$$\frac{\partial^2\overline{\pi}_o}{\partial q\partial\theta} = \frac{2}{\eta}(1-2\theta)m^2q+\overline{\lambda}m. \tag{A.30}$$

Because $\frac{\partial \overline{\pi}_o}{\partial q} < 0$, and $M = \frac{\partial^2 \overline{\pi}_o}{\partial q^2}\frac{\partial^2 \overline{\pi}_o}{\partial \theta^2} - \left(\frac{\partial^2 \overline{\pi}_o}{\partial q \partial \theta}\right)^2 > 0$, the Hessian matrix is negative definite, and then $\overline{\pi}_o$ is jointly concave in q and θ. By the two first-order conditions, we have

$$q^{\mathrm{PS}} = \eta \frac{2(a-w) - \overline{\lambda} m}{4b\eta - m^2}, \tag{A.31}$$

$$\theta^{\mathrm{PS}} = \frac{1}{2} + \frac{\overline{\lambda}\eta}{2mq^{\mathrm{PS}}}, \tag{A.32}$$

$$e^{\mathrm{PS}} = \frac{\theta^{\mathrm{PS}} m q^{\mathrm{PS}}}{\eta}\cdot \tag{A.33}$$

Thus the above optimal solutions are the supply chain members' optimal decision and can be written as follows,

$$\theta^{\mathrm{PS}} = \frac{1}{2}\left(1 + \frac{\overline{\lambda}(4b\eta - m^2)}{m(2(a-w) - m\overline{\lambda})}\right), \tag{A.34}$$

$$e^{\mathrm{PS}} = \frac{\overline{\lambda}}{2}\left(1 + \frac{m(2(a-w) - m\overline{\lambda})}{\overline{\lambda}(4b\eta - m^2)}\right). \tag{A.35}$$

And the OEM's optimal profit is

$$\pi_o^{\mathrm{PS}} = \frac{\eta}{4b\eta - m^2}\left((a - w - \overline{\lambda} m)(a-w) + b\eta\overline{\lambda}^2\right). \tag{A.36}$$

Note that when $e^{\mathrm{PS}} \geqslant \overline{\lambda}$, then we can have $\theta^{\mathrm{PS}} \geqslant 1$.

Proof of Proposition 5.2. According to the result of Proposition 5.1, we surmise, in this case, the optimal solutions are boundary solutions. Therefore, based on the function (4.8), taking the first derivative with respect to q and θ respectively, we get that,

$$\frac{\partial \overline{\Pi}_o}{\partial q_1} = -2\left(b - \frac{1}{\eta_1}\theta(1-\theta)m^2\right)q_1 + (a - w - \overline{\lambda} m(1-\theta)) - 2bq_2, \tag{A.37}$$

$$\frac{\partial \overline{\Pi}_o}{\partial q_2} = -2\left(b - \frac{1}{\eta_2}\theta(1-\theta)m^2\right)q_2 + (a - w - \overline{\lambda} m(1-\theta)) - 2bq_1, \tag{A.38}$$

$$\frac{\partial \overline{\Pi}_o}{\partial \theta} = \frac{1}{\eta_1}(1-2\theta)m^2 q_1^2 + \frac{1}{\eta_2}(1-2\theta)m^2 q_2^2 + \overline{\lambda} m(q_1 + q_2), \tag{A.39}$$

$$\frac{\partial^2 \overline{\Pi}_o}{\partial q_1^2} = -2\left(b - \frac{1}{\eta_1}\theta(1-\theta)m^2\right), \tag{A.40}$$

$$\frac{\partial^2 \overline{\Pi}_o}{\partial q_2^2} = -2\left(b - \frac{1}{\eta_2}\theta(1-\theta)m^2\right), \tag{A.41}$$

$$\frac{\partial^2 \overline{\Pi}_o}{\partial \theta^2} = -2m^2\left(\frac{q_1^2}{\eta_1} + \frac{q_2^2}{\eta_2}\right), \tag{A.42}$$

$$\frac{\partial^2 \overline{\Pi}_o}{\partial q_1 \partial q_2} = -2b, \tag{A.43}$$

$$\frac{\partial^2 \overline{\Pi}_o}{\partial q_1 \partial \theta} = \frac{2}{\eta_1}(1-2\theta)m^2 q_1 + \overline{\lambda} m, \tag{A.44}$$

$$\frac{\partial^2 \overline{\Pi}_o}{\partial q_2 \partial \theta} = \frac{2}{\eta_2}(1-2\theta)m^2 q_2 + \overline{\lambda} m. \tag{A.45}$$

Note that,

$$M_1 = \frac{\partial^2 \overline{\Pi}_o}{\partial q_1^2} = -2\Big(b - \frac{1}{\eta_1}\theta(1-\theta)m^2\Big) < 0, \tag{A.46}$$

$$M_2 = \frac{\partial^2 \overline{\Pi}_o}{\partial q_1^2}\frac{\partial^2 \overline{\Pi}_o}{\partial q_2^2} - \left(\frac{\partial^2 \overline{\Pi}_o}{\partial q_1 \partial q_2}\right)^2 = 4\Big(b - \frac{1}{\eta_1}\theta(1-\theta)m^2\Big)\Big(b - \frac{1}{\eta_2}\theta(1-\theta)m^2\Big) - 4b^2 > 0, \tag{A.47}$$

$$\begin{aligned} M_3 &= \begin{vmatrix} -2\Big(b - \frac{1}{\eta_1}\theta(1-\theta)m^2\Big) & -2b & \frac{2}{\eta_1}(1-2\theta)m^2 q_1 + \overline{\lambda} m \\ -2b & -2\Big(b - \frac{1}{\eta_2}\theta(1-\theta)m^2\Big) & \frac{2}{\eta_2}(1-2\theta)m^2 q_2 + \overline{\lambda} m \\ \frac{2}{\eta_1}(1-2\theta)m^2 q_1 + \overline{\lambda} m & \frac{2}{\eta_2}(1-2\theta)m^2 q_2 + \overline{\lambda} m & -2m^2\left(\frac{q_1^2}{\eta_1} + \frac{q_2^2}{\eta_2}\right) \end{vmatrix} \\ &= -4b\left(\frac{2}{\eta_1}(1-2\theta)m^2 q_1 + \overline{\lambda} m\right)\left(\frac{2}{\eta_2}(1-2\theta)m^2 q_2 + \overline{\lambda} m\right) \\ &\quad + 2\left(b - \frac{1}{\eta_2}\theta(1-\theta)m^2\right)\left(\frac{2}{\eta_1}(1-2\theta)m^2 q_1 + \overline{\lambda} m\right)^2 \\ &\quad + 2\left(b - \frac{1}{\eta_1}\theta(1-\theta)m^2\right)\left(\frac{2}{\eta_2}(1-2\theta)m^2 q_2 + \overline{\lambda} m\right)^2 \\ &\quad - 8m^2\left(\frac{q_1^2}{\eta_1} + \frac{q_2^2}{\eta_2}\right)\left(\left(b - \frac{1}{\eta_1}\theta(1-\theta)m^2\right)\left(b - \frac{1}{\eta_2}\theta(1-\theta)m^2\right) - b^2\right) < 0. \end{aligned} \tag{A.48}$$

Intuitively, the Hessian matrix is negative definite, and then $\overline{\Pi}_o$ is jointly concave in q_1, q_2 and θ. By the three first-order conditions, we have

$$q_1^{\mathrm{PD}} = \eta_1 \frac{a - w - \overline{\lambda} m}{2b(\eta_1 + \eta_2) - m^2}, \tag{A.49}$$

$$q_2^{\mathrm{PD}} = \eta_2 \frac{a - w - \overline{\lambda} m}{2b(\eta_1 + \eta_2) - m^2}, \tag{A.50}$$

$$\theta^{\mathrm{PD}} = \frac{1}{2} + \frac{1}{2}\frac{\overline{\lambda}(2b(\eta_1 + \eta_2) - m^2)}{m(a - w - \overline{\lambda} m)}, \tag{A.51}$$

$$e_i^{\mathrm{PD}} = \frac{\overline{\lambda}}{2} + \frac{\overline{\lambda}}{2}\frac{m(a - w - \overline{\lambda} m)}{\overline{\lambda}(2b(\eta_1 + \eta_2) - m^2)}. \tag{A.52}$$

And the OEM's optimal profit is

$$\pi_o^{\mathrm{PD}} = \frac{2\eta}{(4b\eta - m^2)^2}\Big((2b\eta - m^2)\big((a-w)(a - w - \overline{\lambda} m) + 2b\eta\overline{\lambda}^2\big) + \frac{1}{4}m^2(a-w)^2\Big). \tag{A.53}$$

Note that when $e_i^{\mathrm{PD}} \geqslant \overline{\lambda}$, then we can have $\theta^{\mathrm{PD}} \geqslant 1$.

Proof of Proposition 5.3. According to Propositions 4.8 and 5.1, we can obtain the difference between the OEM's optimal profits under single and dual sourcing strategies,

$$\pi_o^{\mathrm{PS}} - \pi_o^{\mathrm{PD}} = \frac{\eta}{(4b\eta - m^2)^2}\left(\frac{1}{2}m^2(a-w)(a - w - 2\overline{\lambda} m) - b\eta\overline{\lambda}^2(4b\eta - 3m^2)\right). \tag{A.54}$$

Apparently, the OEM's sourcing strategy choice depends on the sign of $\frac{1}{2}m^2(a-w)(a-w-2\overline{\lambda}m) - b\eta\overline{\lambda}^2(4b\eta - 3m^2)$. Note that $e^{\mathrm{PS}} \leqslant \overline{\lambda}$, $e_i^{\mathrm{PD}} \leqslant \overline{\lambda}$, we can proof the $\frac{1}{2}m^2(a-w)(a-w-2\overline{\lambda}m) - b\eta\overline{\lambda}^2(4b\eta - 3m^2) < 0$, namely, the OEM prefers the dual sourcing strategy.

Proof of Proposition 7.1. First, we solve the equilibrium result of the OEM and the CM under the single sourcing case. Because the sub-equilibrium of third-stage and second-stage (*i.e.*, investment level game and order quantity game) have been obtained, using the result of Proposition 4.2 and the OEM's profit after substituted the best responses on investment level and order quantity (*i.e.*, Eq. (4.4)), now we are ready to derive the optimal wholesale price of the first-stage game.

Given the best response on quality investment level and order quantity, we can rewrite the CM's profit as follows

$$\begin{aligned}\pi_c &= wq - \theta(\overline{\lambda} - e)mq - \frac{\eta}{2}e^2\\ &= (w-\theta m)q + \theta emq - \frac{\eta}{2}e^2\\ &= (w-\theta m)\eta\frac{a-w-(1-\theta)\overline{\lambda}m}{2(b\eta-\theta(1-\theta)m^2)} + \frac{\eta}{2}\theta^2m^2\left(\frac{a-w-(1-\theta)\overline{\lambda}m}{2(b\eta-\theta(1-\theta)m^2)}\right)^2. \end{aligned} \tag{A.55}$$

Then taking the first and second derivatives of the CM's optimal profit with respect to w, we get that

$$\begin{aligned}\frac{\mathrm{d}\pi_c}{\mathrm{d}w} &= \eta\left(\frac{a-w-(1-\theta)\overline{\lambda}m+\theta m}{2(b\eta-\theta(1-\theta)m^2)} - \theta^2m^2\frac{a-w-(1-\theta)\overline{\lambda}m}{4(b\eta-\theta(1-\theta)m^2)^2}\right)\\ &= \frac{\eta(2(a-2w-(1-\theta)\overline{\lambda}m+\theta m)(b\eta-\theta(1-\theta)m^2)-\theta^2m^2(a-w-(1-\theta)\overline{\lambda}m))}{4(b\eta-\theta(1-\theta)m^2)^2}\\ &= \frac{\eta(2(a-(1-2\theta)\overline{\lambda}m)(b\eta-\theta(1-\theta)m^2)-\theta^2m^2(a-(1-\theta)\overline{\lambda}m)-(4b\eta-\theta(4-3\theta)m^2)w)}{4(b\eta-\theta(1-\theta)m^2)^2}, \end{aligned} \tag{A.56}$$

$$\frac{\mathrm{d}^2\pi_c}{\mathrm{d}w^2} = -\frac{\eta(4b\eta-\theta(4-3\theta)m^2)}{4(b\eta-\theta(1-\theta)m^2)^2}. \tag{A.57}$$

Apparently, π_c is concave in w. We solve the $\frac{\mathrm{d}\pi_o}{\mathrm{d}w}=0$ to get the CM's optimal wholesale price, namely, $w^{\mathrm{FS}} = \frac{2(a-(1-2\theta)\overline{\lambda}m)(b\eta-\theta(1-\theta)m^2)-\theta^2m^2(a-(1-\theta)\overline{\lambda}m)}{4b\eta-\theta(4-3\theta)m^2}$. Similarly, we also obtain the optimal wholesale price under the dual sourcing strategy, *i.e.*, $w_i^{\mathrm{FD}} = \frac{2(a-(1-2\theta)\overline{\lambda}m)(b(\eta_1+\eta_2)-\theta(1-\theta)m^2)-\theta^2m^2(a-(1-\theta)\overline{\lambda}m)}{4b(\eta_1+\eta_2)-\theta(4-3\theta)m^2}$. Note that the CMs have same optimal wholesale price due to their symmetry.

Next, when the CMs have the same marginal production cost, we compare the CM's wholesale price under different sourcing strategy and get that

$$w_i^{\mathrm{FD}} - w^{\mathrm{FS}} = 2b\eta\theta^2m^2(a-\overline{\lambda}m) > 0. \tag{A.58}$$

Moreover, the difference of the OEM's optimal profits under the single sourcing and the dual sourcing is

$$\begin{aligned}\pi_o^{\mathrm{FS}}-\pi_o^{\mathrm{FD}} = \frac{4\theta m^2(a-\overline{\lambda}m)(2b\eta-\theta(1-\theta)m^2)}{(8b\eta-\theta(4-3\theta)m^2)^2(4b\eta-\theta(4-3\theta)m^2)^2}&\Big(\big(a-\overline{\lambda}m\big)\big(2b\eta-\theta(1-\theta)m^2\big)\big(4b\eta-\theta(4-3\theta)m^2\big)^2\\ &- b\eta\theta\big(a-\overline{\lambda}m\big)\big(16b^2\eta^2-b\eta\theta(46-39\theta)m^2+2\theta^2(3-2\theta)(4-3\theta)m^4\big)\\ &+ 2b\eta\theta^3\overline{\lambda}m^3\big(6b\eta(1-\theta)-\theta^2(4-3\theta)m^2\big)\Big). \end{aligned} \tag{A.59}$$

Apparently, when $\big(a-\overline{\lambda}m\big)\big(2b\eta-\theta(1-\theta)m^2\big)\big(4b\eta-\theta(4-3\theta)m^2\big)^2 - b\eta\theta\big(a-\overline{\lambda}m\big)\big(16b^2\eta^2-b\eta\theta(46-39\theta)m^2+2\theta^2(3-2\theta)(4-3\theta)m^4\big)+2b\eta\theta^3\overline{\lambda}m^3\big(6b\eta(1-\theta)-\theta^2(4-3\theta)m^2\big) > 0$, the OEM prefers the single sourcing strategy, otherwise, the dual sourcing becomes an optimal choice.

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