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AN OPTIMISTIC-PESSIMISTIC DEA MODEL BASED ON GAME CROSS EFFICIENCY APPROACH

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Abstract. The ranking of the decision making units (DMUs) is an essential problem in data envelopment analysis (DEA). Numerous approaches have been proposed for fully ranking of units. Majority of these methods consider DMUs with optimistic approach, whereas their weaknesses are ignored. In this study, for fully ranking of the units, a modified optimistic–pessimistic approach, which is based on game cross efficiency idea is proposed. The proposed game like iterative optimistic-pessimistic DEA procedure calculates the efficiency scores according to weaknesses and strengths of units and is based on non-cooperative game. This study extends the optimistic-pessimistic DEA approach to obtain robust rank values for DMUs. The proposed approach yields Nash equilibrium solution, thus overcomes the problem of non-uniqueness of the DEA optimal weights that can possibly reduce the usefulness of cross efficiency. Finally, in order to verify the validity of the proposed model and to show the practicability of algorithm, we apply a real-world example for selection of industrial R&D projects. The proposed model can increase the discriminating power of DMUs and can fully rank the DMUs.

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1. Introduction

Data envelopment analysis (DEA) is a performance evaluation method based on linear programming to measure the relative efficiency of a set of decision making units (DMUs) with multiple inputs-outputs. DEA developed by Charnes *et al.* [2] has gained substantial development, become an essential managerial tool for evaluating the performance of systems, and found significant applications in various real areas [7, 40]. The classical DEA models have some advantages; however, they are very often unable to distinguish DMUs, i.e. cannot rank efficient DMUs. The ranking of the DMUs is a very important problem in DEA literature. To overcome this inability in discriminating the efficient DMUs, several methods, such as cross efficiency (hereafter, CE), were proposed. CE method, first proposed by Sexton *et al.* [26] and further improved by Doyle and Green [5, 6], is a very popular ranking method. It provides a peer-appraisal assessing for each DMU with the weights of all the DMUs instead of its own weights only. The main difficulty with the CE evaluation is that the optimal weights obtained by the classical DEA model may not be unique, which may lead to different CE scores and

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ranking for the same DMU. To tackle this difficulty, which may reduce the usefulness of CE evaluations, Doyle and Green [5] pointed out a possible solution that may incorporate alternative secondary goals to the selection of weights among the alternative optimal solution sets. For a DMU, they proposed benevolent and aggressive secondary models to select the optimal weights.

Liang *et al.* [15] introduced a number of alternative secondary goals for the CE evaluation by extending the CE model of Doyle and Green [5]. They proposed three benevolent secondary DEA models with a concept of the ideal point. Liang *et al.* [16] proposed a game CE model based on approach of game theory. This game theory based on CE model treats each DMU as a player that seeks to maximize its own score without deteriorating the CE scores of other DMUs. Wang and Chin [28] presented some secondary goals for the CE evaluation. They changed the target efficiency from the ideal point 1 to the classic (Charnes-Cooper-Rhodes) efficiency. Wu *et al.* [33] eliminated the average assumption, using the concept of Shannon entropy in determining the ultimate cross efficiency, and proposed the entropy model for cross efficiency evaluation. Ramón *et al.* [23] introduced a CE evaluation that focuses on the selection of the weights profiles. Wang and Chin [29] proposed a neutral model with which each DMU determines the weights only from its own point of view without considering their impact on the other DMUs and reduces the number of zero weights effectively for outputs. Moreover, Wang *et al.* [31] introduced a weight determination DEA model, which simultaneously reduces the number of inputs and outputs with zero weights without being aggressive or benevolent. Wang *et al.* [32] introduced four neutral DEA models for cross efficiency evaluation by introducing an ideal DMU (IDMU), which is defined as the unit consuming the least inputs to produce the most outputs, and an anti-ideal DMU (ADMU), which utilizes the most inputs and produces the least outputs. Jahanshahloo *et al.* [10] suggested a secondary goal based on symmetric weights choice for cross efficiency evaluation. Wu *et al.* [35] proposed an approach based on information entropy theory instead of the mean of CE scores. Wang and Chin [30] put forward the use of ordered weighted averaging (OWA) operator weights for cross efficiency evaluation. Ramon *et al.* [24] examined the selection of the profiles of weights to be used in cross efficiency evaluations and proposed a weight selection model called "peer-restricted" cross efficiency evaluation to avoid zero weights ignoring the profiles of weights of the inefficient DMUs. Örkcü and Bal [22] introduced a goal programming method into CE models which includes concepts such as classical DEA, minmax and minsum efficiency criteria. Lim [17] suggested new aggressive and benevolent formulations of cross efficiency evaluation by minimizing (or maximizing) the cross efficiency of the best (or worst) peer DMU. Additionally, Wu *et al.* [34] and Contreras [3] proposed secondary models to optimize the rank position of the evaluated DMU. Wu *et al.* [36] suggested a weight-balanced DEA CE model reducing the differences in the weighted inputs and weighted outputs, which cannot only guarantee the maximum efficiency of DMU under evaluation. Alcaraz *et al.* [1] addressed multiple optimal weight problems in CE evaluations and proposed a method that yields a ranking range determined by the best and the worst rankings that each DMU can attain. They pointed out their approach that could be a useful tool to analyse the stability of the rankings that the cross efficiency yields. Ruiz [25] introduced a CE model that uses non-oriented measures of efficiency based on directional distance functions (DDF). Oukil and Amin [21] handled the cross efficiency evaluation with rewards individual appreciativeness and used an OWA operator to calculate the aggregated efficiency scores. Wu *et al.* [37] proposed a series of new secondary goal models. Khodabakhsh and Aryavas [12] suggested a CE evaluation method based on an optimistic-pessimistic approach, considering both the least preferred and the most preferred of DMUs. Liu *et al.* [18] developed an iterative algorithm to achieve an aggressive game cross efficiency evaluation combining the aggressive and the game cross efficiency evaluations proposed by Liang *et al.* [16]. Liu [19] considered cross efficiency intervals, which take the aggressive and benevolent formulations into account at the same time, and their variances for ranking DMUs. For calculating the optimal balanced cross-efficiency, Li *et al.* [13] proposed a game-like iterative approach. Yang and Emrouznejad [39] developed a performance index based on efficient and anti-efficient frontiers in without explicit input (DEA–WEI) models for ranking problem.

The classical DEA models do not consider the weakness of the DMUs. Khodabakhshi and Aryavash [11] proposed a DEA model that evaluates the DMUs under an optimistic-pessimistic approach. By means of this model, the minimum and maximum possible efficiency scores of DMUs can be computed. Khodabakhshi and

Aryavash [12] presented a new cross efficiency method based on the optimistic-pessimistic approach. This model can determine a unique set of optimal weights for each DMU under consideration and can fully rank DMUs without needing any secondary goal. Their ideas were inspirational for our study in developing optimistic–pessimistic approach model for fully ranking of the DMUs.

In many DEA applications, some forms of direct or indirect competition may exist among the DMUs under study. An academic applying for research grants is in competition with other academics. Participants in organized sporting events such as the Olympic games constitute competitive DMUs. Arguably, many DEA analyses of banks and of bank branches are all examples of indirect if not direct competition. These same observations might be applied to organizations such as hospitals and schools that operate in tight financial situations. Sun *et al.* [27] examined the efficiency of infrastructure investment and the fund allocation of Chinese provincial capital cities using game-cross efficiency approach. In this application, the cities were considered as DMUs and it was assumed that there are competitions existing among the cities. Essid *et al.* [8] proposed a novel framework to portfolio selection based on a combination between the maverick index and DEA game cross-efficiency approach. In their analysis, each financial asset is viewed as a player competing for investment funds through boosting its ranking compared to its opponents. Thus, a set of unique Nash equilibrium DEA scores to shares was provided. Xie *et al.* [38] adopted a DEA game cross-efficiency model to assess the environmental performance of China's power generation sector under dual control of generation and carbon emissions. Xie *et al.* [38] put the competitiveness into consideration in evaluating the performance of the generation sector in terms of game theory. Li *et al.* [13] developed a DEA-game cross efficiency approach to generate a unique and fair allocation plan by explicitly considering both competition and cooperation relationships among DMUs in the fixed cost allocation problem. In Li *et al.* [13] approach, each DMU was considered as a player and a super-additive characteristic function was defined for coalitions of DMUs.

In this study, DMUs are evaluated according to both weaknesses and strengths of DMUs from the perspective of a noncooperative game. This study, based on the game cross efficiency DEA approach, extends the optimistic-pessimistic DEA approach to obtain robust rank values for DMUs. Maintaining original optimistic-pessimistic scores for each DMU, the proposed approach yields Nash equilibrium solution, thus overcomes the problem of non-uniqueness of the DEA optimal weights that can possibly reduce the usefulness of cross efficiency.

The remainder of this paper was organized as follows. The classical and optimistic-pessimistic cross efficiency models presented in Section 2. Proposed optimistic-pessimistic approach based on game cross efficiency presented in Section 3. Section 4 examines an illustrative numerical example and a real example to show potential applications of the proposed method. Finally, some conclusions are given in Section 5.

2. Cross efficiency evaluation based on optimistic-pessimistic approach

There are n DMUs to be evaluated in terms of m inputs and s outputs. Let x_{ij} $(i = 1, \ldots, m)$ and y_{rj} $(r = 1, \ldots, s)$ represent the input and output values of DMU_j $(j = 1, \ldots, n)$, respectively. Then, the efficiency of DMU_k can be calculated as

$$\theta_k = \sum_{r=1}^{s} u_{rk} y_{rk} \Big/ \sum_{i=1}^{m} v_{ik} x_{ik}$$

where, for DMU_k $(k = 1, \ldots, n)$, v_{ik} and u_{rk} are the input and output weights assigned to ith input and rth output, respectively. The classical CCR model can be expressed for DMU_k under evaluation by

$$\max \theta_k = \sum_{r=1}^{s} u_{rk} y_{rk}$$

s.t.

$$\sum_{i=1}^{m} v_{ik} x_{ik} = 1 \tag{2.1}$$

$$\sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leq 0, \quad j = 1, 2, \ldots, n$$
$$u_{rk}, v_{ik} \geq 0, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m$$

Let u^*_{rk} $(r = 1, \ldots, s)$ an v^*_{ik} $(i = 1, \ldots, m)$ be an optimal solution to the model (2.1) for DMU_k. DMU_k is efficient if and only if $\theta^*_k = 1$. As such, $\theta_{jk} = \sum\limits_{r=1}^{s} u^*_{rk} y_{rj} / \sum\limits_{i=1}^{m} v^*_{ik} x_{ij}$ is referred to as a cross efficiency value of the DMU_j $(j = 1, \ldots, n)$ according to favourable weights of DMU_k and reflects the peer-evaluation of DMU_j. Element θ_{jk} in the matrix is the efficiency value of DMU_j under the weight of DMU_k. The elements on the diagonal refer to self-evaluated efficiency values of $\text{DMU}_k (\theta^*_k = \sum_{r=1}^{s} u^*_{rk} y_{rk})$. The cross efficiency value of DMU_j is the mean value of all θ_{jk}, that is, $\bar{\theta}_j = \frac{1}{n} \sum_{k=1}^{n} \theta_{jk}$ $(j = 1, \ldots, n)$. The model (2.1) may have alternative multiple optimal solutions. Thus, non-uniqueness of input and output weights would damage the use of cross efficiency evaluation.

The optimistic-pessimistic DEA model of Khodabakhshi and Aryavash [12] is as follows:

$$\begin{aligned}
&\text{min or max } \theta_k = \sum_{r=1}^{s} u_{rk} y_{rk} \\
&\text{s.t.} \\
&\sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\
&\sum_{i=1}^{m} w_{ij} x_{ij} - \sum_{r=1}^{s} u_{rk} y_{rj} = 0, \quad j = 1, 2, \ldots, n. \\
&\sum_{j=1}^{n} w_{ij} - v_{ik} = 0, \quad i = 1, 2, \ldots, m, \\
&u_{rk}, v_{ik}, w_{ij} \geq 0, \quad r = 1, \ldots, s; i = 1, \ldots, m, \quad j = 1, 2, \ldots, n
\end{aligned} \tag{2.2}$$

To obtain optimistic-pessimistic score of DMUs, model (2.2) is run two times according to objective function: θ_k is minimized to determine its minimum value whereas θ_k is maximized to determine its maximum value for DMU_k. The minimizing and maximizing the model (2.2) indicates the pessimistic and optimistic viewpoints, respectively. The unfavourable optimal weights u^P_{rk}. and v^P_{ik} for DMU_k can be determined by minimizing the model (2.2) and therefore the pessimistic cross efficiency score of DMU_j can be evaluated as follows:

$$\theta^P_{jk} = \sum_{r=1}^{s} u^P_{rk} y_{rj} \Big/ \sum_{i=1}^{m} v^P_{ik} x_{ij}, \quad j = 1, \ldots, n$$

The overall pessimistic score of DMU_j is calculated as follows:

$$\theta^p_j = \frac{1}{n} \sum_{k=1}^{n} \theta^p_{jk}, \quad j = 1, \ldots, n. \tag{2.3}$$

Similarly, by maximizing the model (2.2), the favourable optimal weights u^O_{rk} and v^O_{ik} for DMU_k can be determined and therefore the optimistic cross efficiency score of DMU_j can be evaluated as follows:

$$\theta^O_{jk} = \sum_{r=1}^{s} u^O_{rk} y_{rj} \Big/ \sum_{i=1}^{m} v^O_{ik} x_{ij}, \quad j = 1, \ldots, n$$

The overall optimistic score of DMU_j is calculated as follows:

$$\theta_j^O = \frac{1}{n}\sum_{k=1}^{n}\theta_{jk}^O, \quad j = 1,\ldots,n. \tag{2.4}$$

The final optimistic-pessimistic cross efficiency score of DMU_j is calculated as the average of the pessimistic and optimistic cross efficiency scores:

$$\theta_j = \frac{\theta_j^P + \theta_j^O}{2}, \quad j = 1,\ldots,n.$$

3. Proposed optimistic-pessimistic DEA model based on game cross efficiency approach

In original game cross efficiency approach, each DMU is viewed as a player that seeks to maximize its own efficiency, without deteriorating the CE of each of the other DMUs. In this study, the game cross efficiency idea was used in two ways as pessimistic and optimistic. From a pessimistic point of view, each DMU seeks to minimize its own efficiency while the pessimistic cross efficiency of each of the other DMUs does not remain below their average pessimistic scores. In other words, the pessimistic game cross efficiency model aims to minimize the efficiency score of under evaluation DMU while allowing the all DMUs to get a larger score than the average pessimistic (worst) scores. This situation is similar to the *minimax* regret criterion in the game theory.

For the pessimistic view, we consider the following linear programming model for DMU_k:

$$\begin{aligned}
&\min\ \theta_k = \sum_{r=1}^{s} u_{rk}^d y_{rk}\\
&\text{s.t.}\\
&\sum_{i=1}^{m} v_{ik}^d x_{ik} = 1\\
&\sum_{i=1}^{m} w_{ij}x_{ij} - \sum_{r=1}^{s} u_{rk}^d y_{rj} = 0, \quad j = 1,2,\ldots,n\\
&\sum_{j=1}^{n} w_{ij} - v_{ik}^d = 0, \quad i = 1,2,\ldots,m,\\
&\alpha_d \sum_{i=1}^{m} v_{ik}^d x_{id} - \sum_{r=1}^{s} u_{rk}^d y_{rd} \le 0, \quad d = 1,2,\ldots,n\\
&u_{rk}^d, v_{ik}^d, w_{ij} \ge 0, \quad r = 1,\ldots,s;\ i = 1,\ldots,m, \quad j = 1,2,\ldots,n
\end{aligned} \tag{3.1}$$

where, $\alpha_d \le 1$ is a parameter, which initially takes the value given by the average original pessimistic cross efficiency of DMU_d ($\alpha_d = \alpha_d^P$). This α_d becomes the best (average) pessimistic game-cross efficiency score when the algorithm converges. For each DMU_k, model (2.5) is solved n times, once for each $d = 1,2,\ldots,n$. Model (2.5) minimizes the efficiency of DMU_k under the condition that the efficiency of a given DMU_d, $\sum_{r=1}^{s} u_{rk}^d y_{rd} / \sum_{i=1}^{m} v_{ik}^d x_{id}$, is not smaller than a given value of pessimistic $\alpha_d = \alpha_d^P$ score. The constraint $\alpha_d \sum_{i=1}^{m} v_{ik}^d x_{id} - \sum_{r=1}^{s} u_{rk}^d y_{rd} \le 0$ allows to improve the efficiency scores of other units. This model investigates whether the DMU_k unit may have a worse score under the constraint that the DMU_k and other units have an efficiency score bigger than the average α_d^P value.

Let u_{rk}^{d*} $(r = 1, \ldots, s)$ an v_{ik}^{d*} $(i = 1, \ldots, m)$ be optimal solution to the model (2.5) for DMU_k. Since $\sum_{i=1}^{m} v_{ik}^{d} x_{ik} = 1$, the average game pessimistic cross efficiency for DMU_k is calculated as

$$\alpha_k^P = \frac{1}{n}\sum_{d=1}^{n}\sum_{r=1}^{s} u_{rk}^{d*} y_{rk}.$$

We follow a iterative procedure like Liang *et al.* [16] in both average game pessimistic and game optimistic cross efficiency scores. For game pessimistic approach, the algorithm begins with the conventional pessimistic cross-efficiency score as developed in (2.3), and for each DMU_d, solves model (2.5) for each k , using this as the initial α_d. This process is repeated for every d, and the average of the objective function values of (2.5) becomes the new α_d. When the difference between the previous and the following α_d is less than ε in every iterations, the algorithm terminates. The iterative procedure for the pessimistic view can be summarized as follows:

Step 1: Solve model (2.2) and obtain a set of pessimistic optimal relative weights for each DMU_d $(d = 1, \ldots, n)$. Use the (2.3) and obtain pessimistic cross eficiency scores for each DMU_d. Let $t = 1$ and $\alpha_d = \alpha_d^P$.
Step 2: Solve model (2.5). Let

$$\alpha_k^{P,2} = \frac{1}{n}\sum_{d=1}^{n}\sum_{r=1}^{s} u_{rk}^{d*}\left(\alpha_d^P\right) y_{rk},$$

or in a general format

$$\alpha_k^{P,t+1} = \frac{1}{n}\sum_{d=1}^{n}\sum_{r=1}^{s} u_{rk}^{d*}\left(\alpha_d^{P,t}\right) y_{rk}.$$

where $u_{rk}^{d*}(\alpha_d^{P,t})$ represents optimal value of model (2.5) when and $\alpha_d = \alpha_d^{P,t}$.
Step 3: If $|\alpha_k^{P,t+1} - \alpha_k^{P,t}| < \varepsilon$ for all k, then stop. $\alpha_d^{P,t+1}$ is the best average game pessimistic cross efficiency given to DMU_d. Otherwise, if $|\alpha_k^{P,t+1} - \alpha_k^{P,t}| \geq \varepsilon$ for some k $\quad (k = 1, \ldots, n)$, ε is a specified small positive value (it can be set 0.0001), then let $\alpha_d = \alpha_d^{P,t+1}$ and go to Step 2.

The optimistic view is similar to the original game idea. Each DMU seeks to maximize its own efficiency while the optimistic cross efficiency of each of the other DMUs do not remain below their optimistic scores. For the optimistic view, the following linear programming model can be considered for each DMU_k:

$$\begin{aligned}
&\max\ \theta_k = \sum_{r=1}^{s} u_{rk}^{d} y_{rk}\\
&\text{s.t.}\\
&\sum_{i=1}^{m} v_{ik}^{d} x_{ik} = 1\\
&\sum_{i=1}^{m} w_{ij} x_{ij} - \sum_{r=1}^{s} u_{rk}^{d} y_{rj} = 0, \quad j = 1, 2, \ldots, n\\
&\sum_{j=1}^{n} w_{ij} - v_{ik}^{d} = 0, \quad i = 1, 2, \ldots, m,\\
&\alpha_d \sum_{i=1}^{m} v_{ik}^{d} x_{id} - \sum_{r=1}^{s} u_{rk}^{d} y_{rd} \leq 0, \quad d = 1, 2, \ldots, n\\
&u_{rk}^{d}, v_{ik}^{d}, w_{ij} \geq 0, \quad r = 1, \quad , s;\ i = 1, \ldots, m, \quad j = 1, 2, \ldots, n
\end{aligned} \tag{3.2}$$

where, $\alpha_d \leq 1$ is a parameter. For optimistic view in the model (2.6), α_d initially takes the value given by the average original optimistic cross efficiency of DMU_d $(\alpha_d = \alpha_d^O)$.

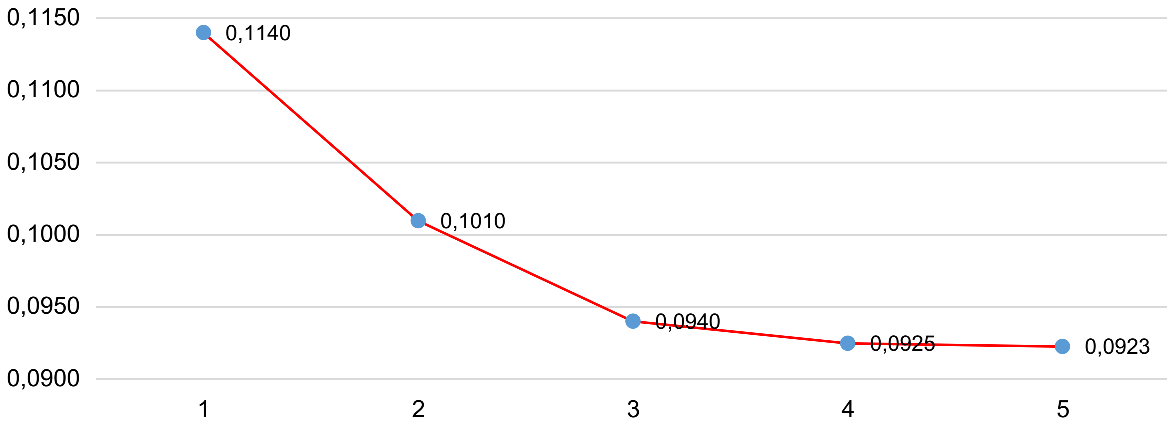

FIGURE 1. Achieving the best game pessimistic cross efficiency for the second of these seven academic units.

Like all other units, the efficiency score for the under evaluation unit (DMU_k) is kept by the $\sum_{r=1}^{s} u_{rk}^{d} y_{rk} / \sum_{i=1}^{m} v_{ik}^{d} x_{ik} \geq \alpha_k$ constraint and in this constraint α_k value is the average maximum value (α_k^O) obtained from the Khodabakhsh and Aryavas [12] model for DMU_k. We want to maximize the efficiency score ($\sum_{r=1}^{s} u_{rk}^{d} y_{rk}$) of DMU_k while guaranteeing that the same unit's efficiency score is greater than the average optimistic cross efficiency score. In other words, we investigate whether the DMU_k unit has an even better score while the DMU_k unit and other units have an efficiency score greater than the average α_d^O values. In the light of these thoughts, it could be concluded that the model given with (2.6) is suitable to *maximax* (maximum of the maximum) decision rule. A similar situation is valid for Liang *et al.* [16]'s game-cross efficiency model. In addition, both Liang *et al.* [16]'s game-cross efficiency model and the game-based optimistic approach proposed in this article are similar to the logic of the benevolent cross efficiency.

When the algorithm converges, this α_d becomes the best (average) optimistic game-cross efficiency score. To see how this happens, the problem of investigation of performance of the seven academic departments of a university,which is one of the well-known benchmark example for the fully ranking problem in the literature, should be considered. The data set for this example with three inputs and three outputs is given in the Appendix [12]. For example, when the proposed algorithm converges for the second of these seven academic units, it begins with the 0.1140 classical pessimistic cross efficiency scores, and when the proposed algorithm is terminated, it reaches the final 0.0923 game pessimistic cross efficiency scores. Figure 1 shows the detailed solution process for related DMU in terms of pessimistic view, including the situation in each iteration. Moreover, for all DMUs, Figure 2 shows that the proposed algorithm finds the game pessimistic cross-efficiency scores for the seven DMUs. In the pessimistic case, efficiency scores are a decreasing sequence and the sequence converges to Nash equilibrium similar to Liu *et al.* [18]'s aggressive game cross efficiency approach. Liu *et al.* [18] presented an aggressive game cross efficiency model using pessimistic efficiency.

Model (2.6) maximizes the efficiency of DMU_k under the condition that the efficiency of a given DMU_d, $\sum_{r=1}^{s} u_r^d y_{rd} / \sum_{i=1}^{m} v_i^d x_{id}$, is not less than a given value of optimistic $\alpha_d = \alpha_d^O$ score.

Let u_{rk}^{d*} $(r = 1, \ldots, s)$ an v_{ik}^{d*} $(i = 1, \ldots, m)$ be the optimal solution to the model (2.6) for DMU_k. Since $\sum_{i=1}^{m} v_{ik}^{d} x_{ik} = 1$, the average game optimistic cross efficiency for DMU_k is calculated as $\alpha_k^O = \frac{1}{n} \sum_{d=1}^{n} \sum_{r=1}^{s} u_{rk}^{d*} y_{rk}$.

For game optimistic approach, the algorithm begins with the conventional optimistic cross-efficiency score as developed in (2.4), and for each DMU_d, solves model (2.6) for each k , using this as the initial α_d. This process is repeated for every d, and the average of the objective function values of (2.4) becomes the new optimistic α_d. When the difference between the previous and the following α_d is less than ε in every iterations, the algorithm

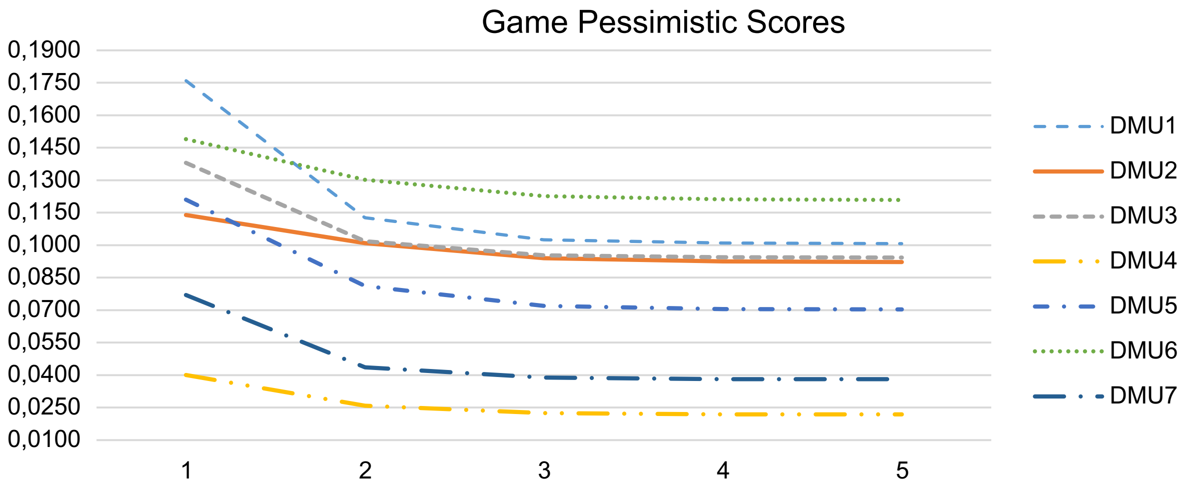

FIGURE 2. The pessimistic game cross-efficiency calculation for DMUs.

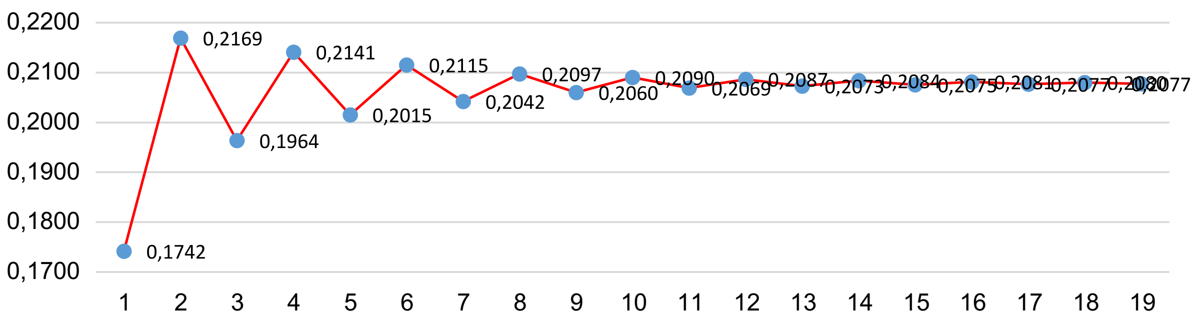

FIGURE 3. Achieving the best game optimistic cross efficiency for the second of these seven academic units.

terminates. The iterative procedure for the optimistic view can be tracked similar to pessimistic view. The final game optimistic-pessimistic cross efficiency score of DMU_k is calculated as the average of the game pessimistic and game optimistic cross efficiency scores:

$$\alpha_k^{OP} = \frac{\alpha_k^P + \alpha_k^O}{2}, \quad k = 1, \ldots, n.$$

Considering the previous example for optimistic view, when the proposed algorithm converges for the second of these seven academic units, it begins with the 0.1742 classical optimistic cross efficiency scores, and when the proposed algorithm is terminated, it reaches the final 0.2077 game optimistic cross efficiency scores. Figure 3 shows the detailed solution process for related DMU in terms of optimistic view, including the situation in each iteration. Furthermore, the average of these game optimistic and pessimistic cross efficiency scores gives the final cross efficiency score of the proposed algorithm for DMU_2 as 0.153

Figure 4 shows that after 19 iterations, the proposed algorithm finds the game optimistic cross-efficiency scores for the seven DMUs.

As can be seen from Figures 3 and 4, the optimistic game cross efficiency score increases when iteration (t) becomes an even number and decreases when iteration (t) becomes an odd number similar to Liang *et al.* [16]'s game-cross efficiency model.

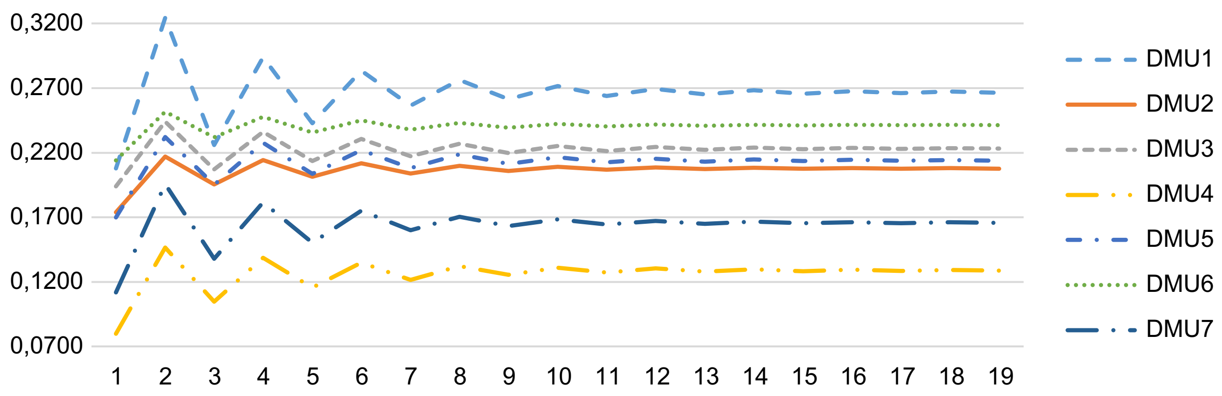

FIGURE 4. The optimistic game cross-efficiency calculation for DMUs.

TABLE 1. Results of the models.

DMU	CCR	Optimistic-pessimistic approach				Proposed approach			
	Score	θ_j^P	θ_j^O	$\theta_j^{\text{opt-pest}}$	[Rank]	α_k^P	α_k^O	α_k^{OP}	[Rank]
1	1	0.176	0.208	0.192	[1]	0.101	0.266	0.184	[1]
2	1	0.114	0.174	0.144	[5]	0.092	0.208	0.150	[4]
3	1	0.138	0.194	0.166	[3]	0.094	0.223	0.159	[3]
4	0.820	0.040	0.080	0.060	[7]	0.022	0.129	0.076	[7]
5	1	0.121	0.170	0.146	[4]	0.070	0.214	0.142	[5]
6	1	0.149	0.214	0.181	[2]	0.121	0.241	0.181	[2]
7	1	0.077	0.112	0.094	[6]	0.038	0.166	0.102	[6]

Table 1 includes the self-efficiency scores obtained by CCR, optimistic-pessimistic cross efficiency scores, and proposed approach scores for the seven academic departments of the university. Numbers in square brackets are the ranks of the corresponding efficiencies in each type of models.

It can be understood in Table 1 that according to CCR model, except DMU4, all DMUs are obtained as efficient. The proposed approach has greater discriminating power than the basic CCR self-evaluation method and can fully rank the DMUs similar to optimistic-pessimistic model.

The proposed optimistic-pessimistic approach based on game cross efficiency DEA model and Khodabakhsh and Aryavas's [12] optimistic-pessimistic approach are very similar in terms of efficiency scores. Between the proposed approach and Khodabakhsh and Aryavas's optimistic-pessimistic approach, Spearman's rank correlation test [4], with the statistic of $r_s = 0.964$ shows that under the significance level $\alpha = 0.01$ there is a powerful correlation in the same direction between the efficiency ranking values of the DMUs obtained by two models. Hence, it can be statistically concluded that the proposed and Khodabakhsh and Aryavas's models assign similar rank values to DMUs.

4. Real-world numerical example

In this section, in order to further show the usefulness and the practicability of the proposed optimistic-pessimistic DEA approach based on game cross efficiency model, a real-world data from well-known literature was used. In addition to the proposed approach, the results of the previous cross-efficiency methods were also given. The real-world data is about the selection of 37 R&D projects, which was suggested by Oral *et al.* [20] and also studied by Green *et al.* [9], Liang *et al.* [16], Wu *et al.* [37], and Li *et al.* [19]. There are 37 projects

TABLE 2. Results of the models for the R&D projects data.

DMU Project	CCR		Opt-Pess		Proposed	
	Score	[Rank]	Score	[Rank]	Score	[Rank]
1	0.654	[12]	0.0328	[6]	0.0304	[7]
2	0.551	[21]	0.0288	[12]	0.0274	[11]
3	0.336	[37]	0.0107	[37]	0.0100	[37]
4	0.528	[24]	0.0200	[31]	0.0182	[30]
5	0.506	[28]	0.0246	[20]	0.0218	[20]
6	0.615	[16]	0.0217	[25]	0.0197	[28]
7	0.506	[29]	0.0210	[28]	0.0171	[34]
8	0.420	[33]	0.0201	[30]	0.0201	[27]
9	0.518	[25]	0.0256	[17]	0.0218	[21]
10	0.543	[23]	0.0235	[23]	0.0225	[19]
11	0.562	[19]	0.0255	[18]	0.0251	[17]
12	0.552	[20]	0.0229	[24]	0.0216	[22]
13	0.505	[30]	0.0239	[22]	0.0213	[23]
14	0.654	[13]	0.0295	[11]	0.0280	[10]
15	0.652	[14]	0.0251	[19]	0.0209	[25]
16	0.854	[4]	0.0374	[2]	0.0374	[2]
17	1	[1]	0.0883	[1]	0.0686	[1]
18	0.762	[7]	0.0282	[13]	0.0256	[14]
19	0.518	[26]	0.0213	[26]	0.0208	[26]
20	0.352	[35]	0.0156	[36]	0.0158	[36]
21	0.602	[17]	0.0299	[10]	0.0289	[9]
22	0.507	[27]	0.0280	[14]	0.0251	[16]
23	0.675	[10]	0.0345	[5]	0.0332	[5]
24	0.500	[31]	0.0277	[15]	0.0254	[15]
25	0.402	[34]	0.0178	[34]	0.0181	[31]
26	0.663	[11]	0.0321	[7]	0.0316	[6]
27	0.742	[8]	0.0358	[4]	0.0355	[3]
28	0.348	[36]	0.0199	[32]	0.0180	[32]
29	0.578	[18]	0.0264	[16]	0.0260	[13]
30	0.550	[22]	0.0213	[27]	0.0196	[29]
31	0.946	[3]	0.0303	[9]	0.0272	[12]
32	0.639	[15]	0.0197	[33]	0.0173	[33]
33	0.430	[32]	0.0169	[35]	0.0159	[35]
34	0.797	[5]	0.0207	[29]	0.0210	[24]
35	1	[2]	0.0369	[3]	0.0338	[4]
36	0.771	[6]	0.0311	[8]	0.0291	[8]
37	0.739	[9]	0.0245	[21]	0.0247	[18]

(DMUs) and each project is characterized by five outputs and only one input. For detailed information about this R&D project selection, Oral *et al.* [20] can be tracked and the detailed data are shown in Appendix.

Table 2 includes the self-efficiency scores obtained by CCR, optimistic-pessimistic cross efficiency scores, and proposed approach scores. Numbers in square brackets are the ranks of the corresponding efficiencies in each type of models.

According to the classical CCR results, two projects (17 and 35) are identified as efficient. Between the proposed approach and Khodabakhsh and Aryavas's optimistic-pessimistic approach, Spearman's correlation coefficient $r_s = 0.975$ shows that under the significance level $\alpha = 0.01$ the proposed and Khodabakhsh and Aryavas's models assign similar rank values to DMUs. Furthermore, Liu *et al.* [14] applied a new balanced

Table 3. Cross-efficiency and project selection results for different methods.

Project	Proposed	OP	O	G	L	W	Budget	Proposed	OP	O	G	L	W
17	0.0686	0.0883	1.0000	0.9750	0.9987	0.9677	32.1	Yes	Yes	Yes	Yes	Yes	Yes
35	0.0338	0.0369	0.9999	1.0000	1.0000	0.9772	36	Yes	Yes	Yes	Yes	Yes	Yes
16	0.0374	0.0374	0.7181	0.7800	0.8162	0.7568	35.4	Yes	Yes	Yes	Yes	Yes	Yes
27	0.0355	0.0358	0.6997	0.7120	0.7287	0.6949	57.1	Yes	Yes	Yes	Yes	Yes	Yes
23	0.0332	0.0345	0.6754	0.6550	0.6696	0.6412	75.6	Yes	Yes	Yes	Yes	Yes	Yes
31	0.0272	0.0303	0.8226	0.8660	0.9078	0.8356	44.6	Yes	Yes	Yes	Yes	Yes	Yes
1	0.0304	0.0328	0.6543	0.6140	0.6332	0.6004	84.2	Yes	Yes	Yes	Yes	Yes	Yes
26	0.0316	0.0321	0.6323	0.6320	0.6504	0.6163	69.3	Yes	Yes	Yes	Yes	Yes	Yes
36	0.0291	0.0311	0.7598	0.7590	0.7671	0.7402	64.1	Yes	Yes	Yes	Yes	Yes	Yes
21	0.0289	0.0299	0.5481	0.5650	0.5843	0.5502	74.4	Yes	Yes	Yes	Yes	Yes	Yes
14	0.0280	0.0295	0.6539	0.6110	0.6292	0.5967	95	Yes	Yes	Yes	Yes	Yes	Yes
18	0.0256	0.0282	0.6858	0.7150	0.7373	0.6930	46.7	Yes	Yes	Yes	Yes	Yes	Yes
2	0.0274	0.0288	0.5512	0.5190	0.5343	0.5080	90	Yes	Yes				
22	0.0251	0.0280	0.4607	0.4720	0.4895	0.4613	82.1	Yes	Yes				
24	0.0254	0.0277	0.4766	0.4760	0.4905	0.4651	92.3						
29	0.0260	0.0264	0.5550	0.5590	0.5710	0.5454	72	Yes	Yes		Yes		Yes
37	0.0247	0.0245	0.7391	0.6840	0.7050	0.6639	66.4			Yes	Yes	Yes	Yes
11	0.0251	0.0255	0.5280	0.5380	0.5542	0.5307	76.5						
9	0.0218	0.0256	0.5177	0.4440	0.4735	0.4319	95.9						
15	0.0209	0.0251	0.6153	0.5370	0.5829	0.5194	83.8			Yes		Yes	
5	0.0218	0.0246	0.5064	0.4570	0.4788	0.4468	75.4						
10	0.0225	0.0235	0.5058	0.5240	0.5353	0.5119	77.5						
12	0.0216	0.0229	0.5027	0.5300	0.5422	0.5153	47.5			Yes	Yes	Yes	Yes
34	0.0210	0.0207	0.7766	0.6990	0.7373	0.6728	28	Yes	Yes	Yes	Yes	Yes	Yes
13	0.0213	0.0239	0.5045	0.4660	0.4898	0.4533	58.5						
19	0.0208	0.0213	0.4625	0.4840	0.5015	0.4708	78.6						
6	0.0197	0.0217	0.6148	0.5280	0.5609	0.5101	90						
30	0.0196	0.0213	0.5285	0.5390	0.5446	0.5242	82.9						
32	0.0173	0.0197	0.6184	0.6060	0.6209	0.5882	54.5			Yes	Yes	Yes	Yes
7	0.0171	0.0210	0.5060	0.4360	0.4639	0.4226	87.4						
8	0.0201	0.0201	0.4156	0.4090	0.4168	0.4004	88.8						
4	0.0182	0.0200	0.5106	0.4570	0.4860	0.4436	67.5						
28	0.0180	0.0199	0.3300	0.3310	0.3412	0.3233	80						
25	0.0181	0.0178	0.3193	0.3590	0.3799	0.3465	68.5						
33	0.0159	0.0169	0.3916	0.4040	0.4188	0.3919	52.7						
20	0.0158	0.0156	0.2581	0.3070	0.3294	0.2951	54.1						
3	0.0100	0.0107	0.1729	0.2590	0.2388	0.1697	50.2						
							Budget Sum	986.6	986.6	994.7	982.9	994.7	982.9

cross-efficiency evaluation approach to selection of 37 R&D projects and compared their results with studies of Green *et al.* [9], Liang *et al.* [16], and Wu *et al.* [37]. The correlation scores among cross-efficiency values generated by these four methods are about 0.970 and are significant at the 0.01 level.

Table 3 exhibits the cross efficiency evaluation and project selection results for proposed methods and other methods in the literature. These R&D projects are selected by maximizing the weighted efficiency with the budget as the weights under the condition that the sum of the financial budget cannot exceed 1000 (Li *et al.* [14]).

According to our optimistic-pessimistic based on game like iterative method results, the projects of {17, 35, 16, 27, 23, 31, 1, 26, 36, 21, 14, 18, 2, 22} and {29, 34} should be chosen. Note that by decreasing the efficiency scores of 37 R&D projects {24} should be chosen, but we use {29, 34} instead of {24}. This is because when we choose projects until {22}, the used budget is 886.6 and the available budget is only 123.4, while if {24} is selected, the available budget is only 21.1, which prevents any additional project selection. If we select {29, 34} instead of {22}, it can make full use of the financial budget more effectiveness by reaching a used budget of 986.76.

The cross efficiency and selection results of Green *et al.* [9], Liang *et al.* [15, 16], Wu *et al.* [37], Li *et al.* [13], and Khodabakhsh and Aryavas' optimistic-pessimistic [12] are also provided in Table 3. For simplification of denotation and with the representation of Li *et al.* [13], Green *et al.* [9], Liang *et al.* [15, 16], Wu *et al.* [37],

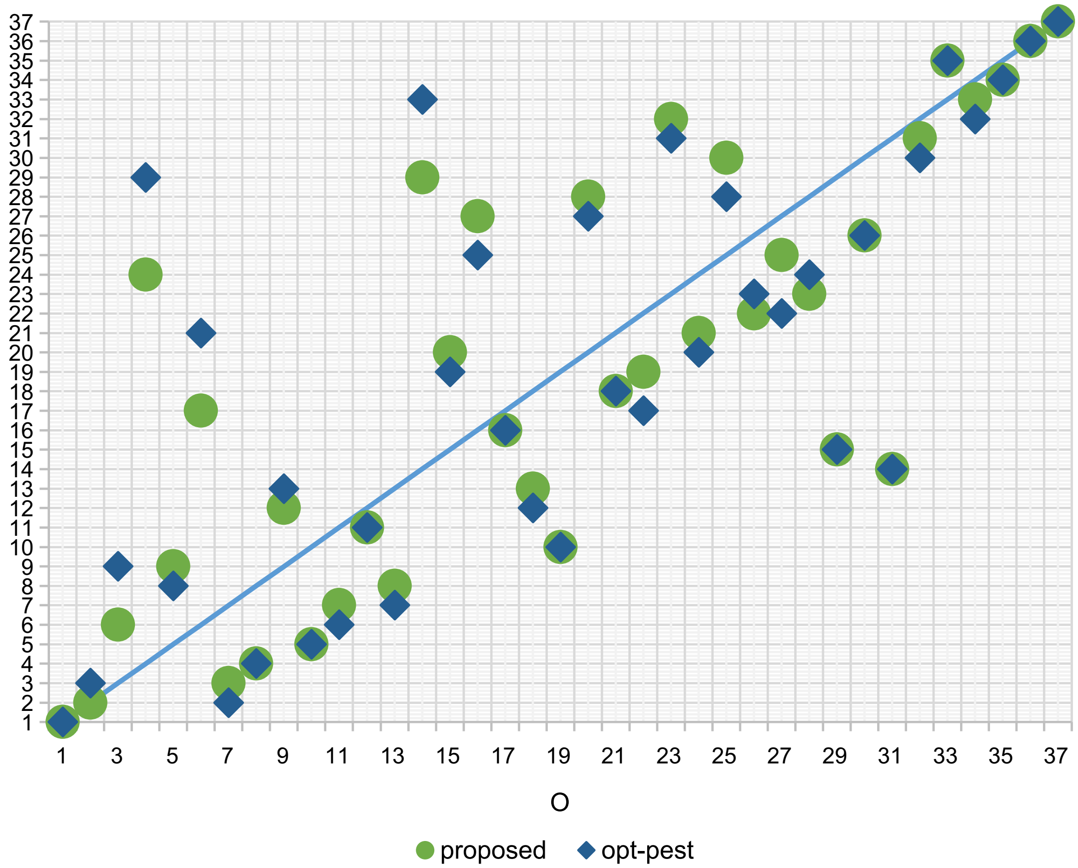

Figure 5. Relationship between the proposed method- Li *et al.* [13] approach and optimistic-pessimistic method-Li *et al.* [13] approach.

Li *et al.* [13], Khodabakhsh and Aryavas' optimistic-pessimistic [12], and our proposed model are denoted as O, G, L, W, OP, and PR, respectively. The sum budget of selected projects of our approach and Khodabakhsh and Aryavas' optimistic-pessimistic approach [12] reach 986.6. Methods O, G, L, and W used 994.7, 982.9, 994.7, and 982.9, respectively. Although our method and Khodabakhsh and Aryavas' optimistic-pessimistic method choose the same projects, the correlation of our method with the methods in the literature is higher than the correlation of Khodabakhsh and Aryavas' optimistic-pessimistic method with the methods in the literature. Among the proposed approach and O, G, L, and W methods, Spearman's correlation coefficient $r_s = 0.726$, 0.784, 0.768, and 0.793, respectively. A Spearman rank correlation test [4], associated p-value much less than 0.01, shows that these sets of efficiencies are highly correlated. However, these correlations were $r_s = 0.722$, 0.768, 0.757, and 0.774, respectively for the Khodabakhsh and Aryavas' optimistic-pessimistic method.

Figure 5 illustrates the accordance of the ranking values obtained by the proposed method and the Khodabakhsh and Aryavas' optimistic-pessimistic method with the Li *et al.* [13] method. As can be seen from Figure 5 that the results of the proposed model are more consistent with the model by Li *et al.* [13] when compared with Khodabakhsh and Aryavas' optimistic-pessimistic method. The results of the proposed model are all very close to the 45-degree line compared to classical optimistic-pessimistic method. The 45-degree line shows the results of ranking obtained by the Li *et al.* [13] model. Similarly, Figure 6 illustrates the relationship between the proposed method and Khodabakhsh and Aryavas' optimistic-pessimistic method and Wu *et al.* [37] method. The results

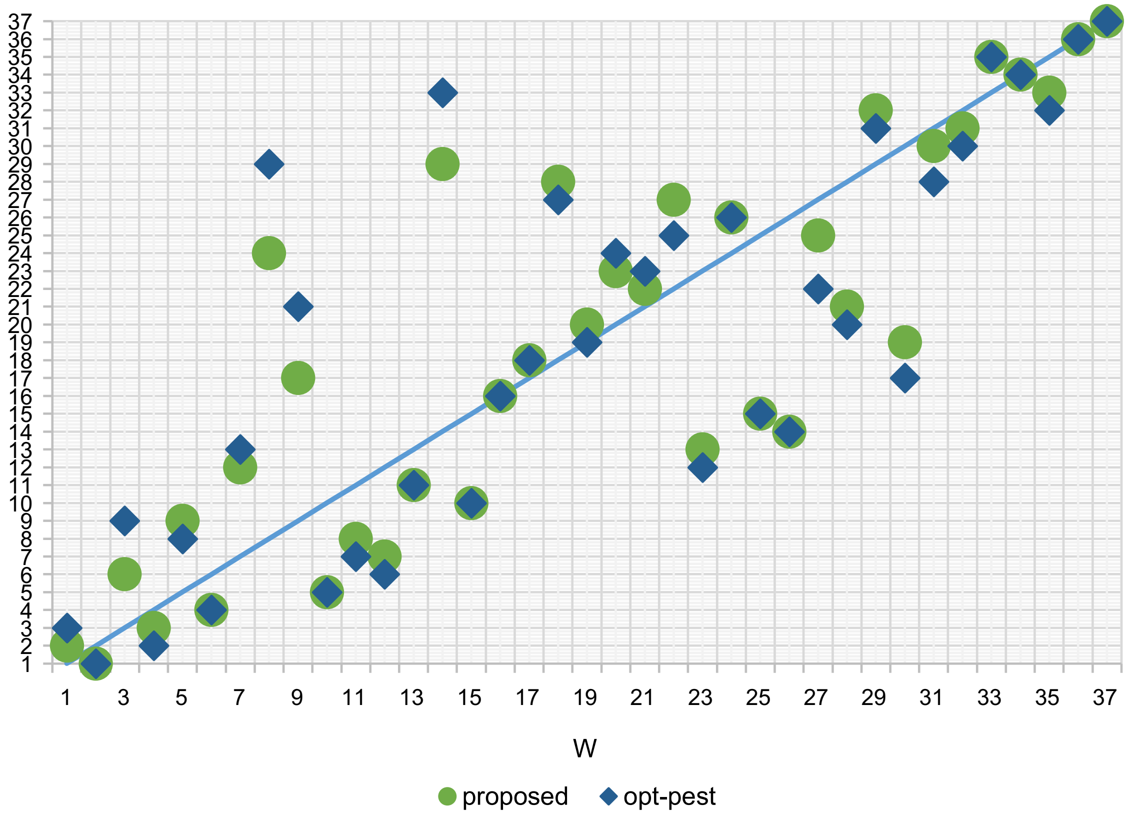

FIGURE 6. Relationship between the proposed method- Wu *et al.* [37] approach and optimistic-pessimistic method-Wu *et al.* [37] approach.

of the proposed model are all very close to the 45-degree line with compared to classical optimistic-pessimistic method. The 45-degree line shows the results of ranking obtained by the Wu *et al.* [37] model for the Figure 6. From Figures 5 and 6, it can be concluded that proposed method is more compatible with the methods of Li *et al.* [13] and Wu *et al.* [37] in terms of the ranking values of DMUs than the Khodabakhsh and Aryavas's optimistic-pessimistic method.

As a result, the contribution of our approach lies in the fact that in the light of optimistic and pessimistic ideas presented in this article we can find a solution which is actually a Nash equilibrium and which is not affected by the multiple optimal solutions in the traditional optimistic-pessimistic model. The optimistic and pessimistic initial scores were changed, however the same scores were achieved. The optimistic and pessimistic game cross-efficiencies scores are a Nash equilibrium solution. The optimistic game cross efficiency score increases when iteration (t) becomes an even number and decreases when iteration (t) becomes an odd number similar to Liang *et al.* [16]'s game-cross efficiency model. The pessimistic game cross efficiency score is a decreasing sequence which converges to Nash equilibrium similar to Liu *et al.* [18]'s aggressive game cross efficiency approach. Thus, the results and decisions based upon optimistic-pessimistic cross efficiency analysis and game theory are reliable.

5. Conclusions

Cross-efficiency evaluation is an effective approach for performance evaluation that achieves strong discrimination and therefore is particularly useful for ranking DMUs. In cross-efficiency evaluation, the optimal DEA weights for a given DMU are also used to assess the other DMUs. The main difficulty with the cross-efficiency evaluation is the possible existence of alternate optima for the DEA weights, which may lead to different cross-efficiency scores, and consequently to different rankings of units, depending on the choice that each DMU makes. In this study, based on game cross efficiency idea, an optimistic-pessimistic DEA model is proposed for estimating cross efficiency scores that considers both weaknesses and strengths of DMUs. The validity of the proposed approach has been tested and verified with both numerical example and real-world illustrative example taken from the well-known literature. The obtained results showed that the proposed approach has an improved discrimination power. The proposed cross efficiency evaluation in this paper is in the form of constant returns to scale (CRS). For that reason, it can be extended to both input and output-oriented forms with variables returns to scale (VRS).

Appendix A.

Table A.1. Data of seven departments in a university.

DMU	The number of undergraduate students y_1	The number graduate sdents y_2 papers y_3	The number of of post research staff x_1	The number of academic	The academic staff salaries in thousands of pounds x_2	The support staff salaries in thousands of pounds x_3
1	60	35	17	12	400	20
2	139	41	40	19	750	70
3	225	68	75	42	1500	70
4	90	12	17	15	600	100
5	253	145	130	45	2000	250
6	132	45	45	19	730	50
7	305	159	97	41	2350	600

Table A.2. Data of 37 R&D projects.

DMU	Indirect economic contribution y_1	Direct economic contribution y_2	Technical contribution y_3	Social contribution y_4	Scientific contribution y_5	The financial budget x_1
1	67.53	70.82	62.64	44.91	46.28	84.2
2	58.94	62.86	57.47	42.84	45.64	90
3	22.27	9.68	6.73	10.99	5.92	50.2
4	47.32	47.05	21.75	20.82	19.64	67.5
5	48.96	48.48	34.9	32.73	26.21	75.4
6	58.88	77.16	35.42	29.11	26.08	90
7	50.1	58.2	36.12	32.46	18.9	87.4
8	47.46	49.54	46.89	24.54	36.35	88.8
9	55.26	61.09	38.93	47.71	29.47	95.9
10	52.4	55.09	53.45	19.52	46.57	77.5
11	55.13	55.54	55.13	23.36	46.31	76.5
12	32.09	34.04	33.57	10.6	29.36	47.5
13	27.49	39	34.51	21.25	25.74	58.5
14	77.17	83.35	60.01	41.37	51.91	95
15	72	68.32	25.84	36.64	25.84	83.8
16	39.74	34.54	38.01	15.79	33.06	35.4

Table A.2. continued.

DMU	Indirect economic contribution y_1	Direct economic contribution y_2	Technical contribution y_3	Social contribution y_4	Scientific contribution y_5	The financial budget x_1
17	38.5	28.65	51.18	59.59	48.82	32.1
18	41.23	47.18	40.01	10.18	38.86	46.7
19	53.02	51.34	42.48	17.42	46.3	78.6
20	19.91	18.98	25.49	8.66	27.04	54.1
21	50.96	53.56	55.47	30.23	54.72	74.4
22	53.36	46.47	49.72	36.53	50.44	82.1
23	61.6	66.59	64.54	39.1	51.12	75.6
24	52.56	55.11	57.58	39.69	56.49	92.3
25	31.22	29.84	33.08	13.27	36.75	68.5
26	54.64	58.05	60.03	31.16	46.71	69.3
27	50.4	53.58	53.06	26.68	48.85	57.1
28	30.76	32.45	36.63	25.45	34.79	80
29	48.97	54.97	51.52	23.02	45.75	72
30	59.68	63.78	54.8	15.94	44.04	82.9
31	48.28	55.58	53.3	7.61	36.74	44.6
32	39.78	51.69	35.1	5.3	29.57	54.5
33	24.93	29.72	28.72	8.38	23.45	52.7
34	22.32	33.12	18.94	4.03	9.58	28
35	48.83	53.41	40.82	10.45	33.72	36
36	61.45	70.22	58.26	19.53	49.33	64.1
37	57.78	72.1	43.83	16.14	31.32	66.4

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