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A NOVEL APPROACH TO SAFETY STOCK MANAGEMENT IN AN INTEGRATED SUPPLY CHAIN WITH CONTROLLABLE LEAD TIME AND ORDERING COST REDUCTION USING PRESENT VALUE

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Abstract. This paper presents a novel approach to safety stock management and investigates the impact of lead time reduction within an integrated vendor–buyer supply chain framework using present value where lead time and ordering cost reductions act dependently. In particular, the cost of the safety stock is determined by adopting a logistic approximation to the standard normal cumulative distribution. The service level is formulated in relation to the dimension of the single shipment, to the average demand of the buyer and to the number of admissible stockouts. We first discuss the case where the lead time and ordering cost reductions with linear function, and then consider the logarithmic functional relationship. Numerical examples including the sensitivity analysis with some managerial insights of system parameters is provided to validate the results of the supply chain models. The main contribution of this paper is introducing various types of ordering cost reduction in Braglia *et al.* (*Appl. Stoc. Mod. Bus. Ind.* **32** (2016) 99–112) by handling a new approach.

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1. Introduction

To improve the competitive capacity of the business, firms tend to become integral part of a supply chain, rather than being single entities. According to this point of view, the development of Joint Economic Lot Size (JELS) models still represents one of the main research topic in the Supply Chain Management (SCM) field.

The remarkable interest in supply chain management related research in the last decade has been due to its significant potential to improve the efficiency of operations and reduce cost. Each individual party in the supply chain can benefit through closer collaboration with other parties and through the integration of various decision processes.

The single vendor single buyer integrated production inventory problem received a lot of attention in recent years. This renewed interest is motivated by the growing focus on supply chain management. Firms are realizing that a more efficient management of inventories across the entire supply chain through better coordination and more cooperation is in the joint benefit of all parties involved. Such collaboration is facilitated by the advances in information technology providing faster and cheaper communication means. Coordination among the members

Keywords. Safety stock management, integrated vendor–buyer supply chain, logistic approximation, ordering cost reductions, present value.

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TABLE 1. Contribution of different authors.

Reference	Safety stock management	Ordering cost reduction	Lead time reduction	Present value
Braglia *et al.* [11]	✓		✓	✓
Liao and Shyu [22]			✓	
Ouyang *et al.* [26]		✓	✓	
Braglia *et al.* [10]	✓			
Gümüs *et al.* [14]	✓			
Braglia and Zavanella [12]	✓			
Jindal and Solanki [19]			✓	✓
Zavanella and Zanoni [40]	✓			
Battini *et al.* [1]	✓			
Proposed model	✓	✓	✓	✓

of a supply chain is an important strategic issue as it effectively maintains inventories across the entire chain, reduces the cost burden, and thereby increases the total profit. Supply chain managers possess all the information relevant to take coordination decisions and the contractual power to have such decisions implemented.

Consignment Stock (CS) is an innovative approach to supply and stock management, based on a strong and continuous collaboration between vendor and buyer to create a "win–win" situation, where both partners have equal gains According to this strategy, the supplier autonomously manages the stock of its own items at the customer warehouse and both decides the dimension of the batches and the time of delivery. In the vendor–buyer relation only the former manages operatively, in an integrated and optimized fashion, the whole stock level of the considered product within the supply chain. Ultimately, the consignment stock concept means that the supplier holds the stock ownership until the customer actually uses it. A comprehensive analysis of the CS policy is provided, *e.g.*, by Valentini and Zavanella [39] and Gümüs *et al.* [14]. An early analytical formulation of the CS was proposed by Braglia and Zavanella [12], who proved the better performances of CS in a stochastic environment (with particular reference to the equal-sized shipments with delayed deliveries case) than the standard JELS model proposed by Hill [16, 17].

Conventional inventory management techniques suggest stocking an inventory level for minimizing the system cost. This technique does not handle risk or the time value of money in the recent highly volatile market situations. In most of the research work, the time value of money was disregarded.

The most important continuous probability distribution used in engineering and science is perhaps the Gaussian distribution. The Gaussian distribution reasonably describes many phenomena that occur in nature. In addition, errors in measurements are extremely well approximated with the Gaussian distribution. The Gaussian distribution finds numerous applications as a limiting distribution. Under certain conditions, the Gaussian distribution provides a good approximation to binomial and hypergeometric distributions. In addition, it appears that the limiting distribution of sample averages is normal. This provides a broad base for statistical inference that proves very valuable in estimation and hypothesis testing.

Supply chain performance in aspect of various market and technical uncertainties is usually deliberated by service level, that is, the expected fraction of demand that the supply chain can satisfy within a predefined allowable delivery time window. Safety stock is imported into supply chains as an important fence against uncertainty in order to provide customers with the promised service level. Although a higher safety stock level guarantees a higher service level, it does increase the supply chain operating cost and thus these levels must be suitably optimized.

In most of the early literature dealing with inventory problems, either using deterministic or probabilistic models, lead time is often viewed as a prescribed constant or a stochastic variable, which therefore, is not subject to control. However, this may not be realistic. Lead time usually consists of the following components: order preparation, order transit, supplier lead time, delivery time and setup time [38]. Further quality improvements and controllable lead time in stochastic environments can be studied through the works such as [13,20,21,23,29–32]. In some cases, lead time can be shortened at an added crashing cost; in other words, it is controllable. By shortening the lead time, we can lower the safety stock, reduce the loss caused by stockout, increase the service level to the customer, and gain the competitive advantages in business. Here a novel approach to safety stock managenment in an integrated supply chain with controllable lead time and ordering cost reduction using present value is to be discussed (Tab. 1).

2. Literature review

Inventory models considering lead time as a decision variable have been developed by several researchers recently. Liao and Shyu [22] first presented a probabilistic inventory model in which the order quantity is predetermined and lead time is a unique decision variable. Ben-Daya and Raouf [2] extended Liao and Shyu's [22] model by considering both lead time and order quantity as decision variables. Ouyang *et al.* [27] generalized Ben-Daya and Raouf's [2] model by allowing shortages with partial back orders.

Moon and Choi [24] and Hariga and Ben-Daya [15] revised Ouyang *et al.*'s [27] model by considering the reorder point as one of the decision variables; they further develop ed a minimax distribution free procedure for the problems. In the above papers [2,15,22,24,27], that focus on deriving the benefits from lead time reduction, the ordering cost is treated as a fixed constant. In a recent article, Ouyang *et al.* [26] relaxed the fixed ordering cost assumption in Moon and Choi [24] and proposed a model to study the effects of lead time and ordering cost reductions.

We note that the lead time and ordering cost reductions in [26] are assumed to act independently, however, this is only one of the possible situations. In practices, the lead time and ordering cost reductions may be related closely; the reduction of lead time may accompany the reduction of ordering cost, and vice versa. For example, the implementation of electronic data interchange (EDI) can reduce both the lead time and ordering cost simultaneously. Therefore, it is more reasonable to assume that lead time and ordering cost reductions act dependently. The purpose of this paper is to study the effect of lead time reduction on continuous review inventory systems with partial back orders. Specifically, we modify Moon and Choi's [24] model to include the cases of the linear and logarithmic relationships between lead time and ordering cost reductions. The objective is to minimize the total related cost by simultaneously optimizing the order quantity, reorder point, and lead time.

Ouyang *et al.* [25] modified Moon and Choi's [24] continuous review inventory model with variable lead time and partial backorders by fuzzifying the backorder rate. A new analytical approach to safety stock management, within single buyer-single vendor framework under VMI with consignment agreement, was presented by Braglia *et al.* [10]. A novel approach to safety stock management in a coordinated supply chain with controllable lead time using present value was presented by Braglia [11]. Priyan and Uthayakumar [28] developed an integrated production-distribution inventory model for a single-vendor single-buyer supply chain system with the consideration of quality inspection errors at the buyer's end, the buyer's warehouse has limited capacity and there is an upper bound on the purchase of products.

Shaikh *et al.* [36] had developed an inventory model according to consideration of price, stock dependent, fully backlogged shortage and inflation. Bhunia *et al.* [3] had developed a price break inventory model for a single deteriorating item with imprecise inventory costs by considering all unit discount policy and variable demand rate dependent on displayed stock level under partially backlogging. Bhunia and Shaikh [4] had dealt with an inventory model, which considers the impact of marketing strategies such as pricing and advertising as well as the displayed inventory level on the demand rate of the system. Bhunia *et al.* [7] had given a memo on stock model with partial backlogging under delay in payments. Shaikh *et al.* [35] developed a two-warehouse inventory model for non-instantaneous deteriorating items with interval-valued inventory costs and stock-dependent demand under inflationary conditions with shortages. Shaikh [33] had done an inventory model for a deteriorating item with selling price and frequency of advertisement dependent demand of an item under the mixed type financial trade credit policy.

Bhunia *et al.* [6] formulated a production-inventory model to investigate the effects of partially integrated production and marketing policy of a manufacturing firm. Shaikh [34] introduced an inventory model for single deteriorating items with two separate storage facilities (own and rented warehouses) due to limited capacity of the existing storage, *i.e.*, own warehouse considering allowable delay in payment. Bhunia *et al.* [8] developed a deterministic inventory model for single deteriorating items with two separate storage facilities (owned and rented warehouse, RW) due to limited capacity of the existing storage (owned warehouse, OW). Bhunia and Shaikh [5] had dealt with an alternative approach for a two-warehouse inventory model for single deteriorating item considering allowable delay in payments with two separate warehouses having different preserving facilities.

Zavanella and Zanoni [40] proposed a research work for the way how a particular VMI policy, known as Consignment Stock (CS), may represent a successful strategy for both the buyer and the supplier. Battini *et al.* [1] proposed an innovative approach to supply and stock management based on a strong and continuous collaboration between vendor and buyer with demand variability, stock-out risk and limited warehouses space. Bowling *et al.* [9] gave a logistic approximation to the cumulative normal distribution. Jindal and Solanki [19] presented a single-vendor single-buyer integrated supply chain inventory models with inflation and time value of money under partial backlogging.

3. Notations and assumptions

3.1. Notations

We need the following notations and assumptions to develop the mathematical model of the proposed model. Additional notations and assumptions will be added up when required.

Decision variables

q Shipment quantity
n Number of shipments per production run from the vendor to the buyer
L Length of the lead time to deliver a shipment from the vendor to the buyer

Parameters

D Average demand rate which is received by the buyer (units/unit time)
P Vendor's production rate (units/unit time)
A_B Buyer's ordering cost per order ($/order)
A_V Vendor's setup cost per setup ($/setup)
V Buyer's variable cost for order handling and receiving ($/order)
F Fixed transportation cost borne by the buyer for each shipment ($/order)
h_V Vendor's stock holding cost per unit per unit time ($/unit/unit time)
H_V Total holding cost at the first cycle of the vendor
h_B Buyer's stock holding cost per unit per unit time ($/unit/unit time)
H_B Inventory holding cost at the first cycle of the buyer
k Number of admissible stockouts per unit time
r Buyer's reorder point
SL Service level
z_{SL} SL – quantile of the standard normal distribution with reference to the service level SL
σ_D Standard deviation of demand rate
ρ Discount rate representing the time value of money
η Inflation rate
i Net constant discount rate of inflation
TC_B Buyer's total cost at the first cycle

PTC_B Buyer's present value of the total cost over an infinite horizon.
TC_V Vendor's total cost at the first cycle.
PTC_V Vendor's present value of the total cost over an infinite horizon.
JTC Expected joint total cost

Random variables

X_D Lead time demand rate of the buyer, a Gaussian random variable

Functions and operators

$f(\cdot)$ Standard normal probability density function (p.d.f)
$F(\cdot)$ Standard normal cumulative density function (c.d.f)

Sets

$\mathbb{R}$ Real numbers
$\mathbb{N}$ Natural numbers

3.2. Assumptions

Some of the strategic characteristics become fundamental and the following hypothesis need to be considered:

(1) The system deals with a single vendor–buyer co-operation with a single product. The vendor and the buyer are in different corporate entities and are also enthusiastic to have an collaboration inventory system. Thus, both the members accept to minimize the integrated expected total cost in the joint strategy. The buyer prefers a continuous review inventory policy and the order is kept whenever the inventory level comes to the reorder point r.
(2) The buyer orders a lot size nq. The vendor produces nq with finite production rate P $(P > D)$ at one setup and ships in quantity q to the buyer over n times. For each shipment, the buyer pays a fixed transportation cost F. The vendor has a setup cost A_V for each production run of size nq, and the buyer has an ordering cost A_B for each order of size nq.
(3) The lead time demand X_D is Gaussian with mean DL and standard deviation $\sigma_D\sqrt{L}$.
(4) The supplier's production rate and the demand rate on the buyer are constant.
(5) The buyer pays transportation and order handling costs.
(6) The production rate is greater than the demand rate.
(7) Safety stock $\mathrm{SS} = z_{\mathrm{SL}}\times$ standard deviation of the lead time demand. Hence, $\mathrm{SS} = z_{\mathrm{SL}} \times \sigma_D\sqrt{L}$.
(8) If SL and z are the service level and the safety factor, respectively we have $\frac{1}{2} \leq \mathrm{SL} < 1$, which is equivalent to the condition $z \geq 0$.
(9) Shortages and backorders are not considered.
(10) The time horizon is infinite.
(11) The net constant discount rate of inflation is given as $i \equiv \rho - \eta > 0$.
(12) The lead time L consists of m mutually independent components. The ith component has a normal duration a_i and carshing cost per unit time c_i. For convenience, we rearrange c_i such that $c_1 < c_2 < \cdots < c_m$. The components of lead time are crashed one at a time starting from the first component because it has the minimum unit crashing cost, and then the second component, and so on. Let $L_0 = \sum_{i=1}^{m} b_i$, and L_i be the length of lead time with components $1, 2, \ldots, i$ crashed to their minimum duration, then L_i can be expressed as $L_i = L_0 - \sum_{j=1}^{i}(b_j - a_j)$, $i = 1, 2, \ldots, m$ and the lead time crashing cost $R(L)$ per cycle is given by $R(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j)$, $L \in [L_i, L_{i-1}]$.

4. A novel stochastic JELS model under the PV criteria

4.1. The safety factor as a function of the shipment quantity

In accordance to Braglia *et al.* [10], the safety stock is determined by taking into an account to the following logistic approximation of the standard normal cumulative distribution.

$$F(x)=\frac{1}{1+e^{-\gamma x}},\qquad x\in\mathbb{R} \tag{1}$$

where $\gamma = 1.702$. Taking inverse for equation (1) we obtain

$$F^{-1}(u)=\gamma^{-1}\ln\left(\frac{u}{1-u}\right),\quad u\in(0,1). \tag{2}$$

Therefore, we can write z_{SL} which is SL-quantile of the standard normal distribution as

$$z_{\mathrm{SL}}=z(q)=\gamma^{-1}\ln\left(\frac{\mathrm{SL}}{1-\mathrm{SL}}\right),\quad \mathrm{SL}\in[1/2,1) \tag{3}$$

and the service level SL can be put in relation to the number of admissible stockouts per unit time k

$$\mathrm{SL}=1-\frac{k}{(Dn/nq)}=1-\frac{kq}{D}\cdot \tag{4}$$

It has to be highlighted that the service level as well as the cost of safety stock are functions only of q and reminding that the lead-time demand is supposed to be a Gaussian random variable. Then the safety factor $z(q)$ and the service level are ostensibly related through the following expression:

$$z(q)=\gamma^{-1}\ln\left(\frac{1-(kq/D)}{1-(1-(kq/D))}\right)=\gamma^{-1}\ln\left(\frac{D}{kq}-1\right). \tag{5}$$

Clearly, it is acceptable to assume $D/k > 1$ as the demand is often bulkier than the number of admissible stockouts per unit time. Thus it is possible to obtain a valid interval for the variable q as

$$0<\frac{kq}{D}\le\frac{1}{2}\ \text{ which implies }\ 0<q\le\frac{D}{2k}\cdot \tag{6}$$

4.2. Buyer's perspective

Now we examine the relevant cost of the buyer. The buyer perceives a replenishment cycle with length equal to q/D. For given $q\in(0,D/2k]$ and $L\in[L_m,L_0]$, the expected net inventory level just before the order arrival is $z(q)\sigma_D\sqrt{L}$, and the expected net inventory at the beginning of the cycle is $q+z(q)\sigma_D\sqrt{L}$, where $z(q)$ is given by equation (5). Therefore, the expected inventory level at $t\in[0,q/D]$ is $z(q)\sigma_D\sqrt{L}-Dt$. Let i be the net constant discount rate of inflation which is formulated as $i\equiv\rho-\eta$. Thus the inventory holding cost for the first cycle at $t\in[0,\frac{q}{D}]$ using the PV approach is written as

$$\begin{aligned}H_B&=nh_B\int_0^{q/D}[q+z(q)\sigma_D\sqrt{L}-Dt]e^{-it}\mathrm{d}t\\&=\frac{nh_B}{i}\left\{\left[q+\frac{\sigma_D\sqrt{L}}{\gamma}\ln\left(\frac{D}{kq}-1\right)\right]\left(1-e^{-iq/D}\right)\right.\\&\quad\left.+\,qe^{-iq/D}-\frac{D}{i}\left(1-e^{-iq/D}\right)\right\}.\end{aligned} \tag{7}$$

The total cost for the buyer at the first cycle is the sum of the ordering cost, variable cost, holding cost and transportation cost. Thus we get the total cost for the first cycle relevant to the buyer as

$$\begin{aligned}\mathrm{TC}_B(q,n,L) = A_B + nF + \frac{nh_B}{i}\left\{\left[q + \frac{\sigma_D\sqrt{L}}{\gamma}\ln\left(\frac{D}{kq}-1\right)\right]\left(1-e^{-iq/D}\right)\right.\\ \left. + qe^{-iq/D} - \frac{D}{i}\left(1-e^{-iq/D}\right)\right\} + nV + nR(L).\end{aligned} \tag{8}$$

Consequently, the present value of the total cost of the buyer over an infinite time horizon can be formulated as

$$\begin{aligned}\mathrm{PTC}_B(q,n,L) = \left(1-e^{-inq/D}\right)^{-1}\left\{A_B + nF + \frac{nh_B}{i}\left\{\left[q + \frac{\sigma_D\sqrt{L}}{\gamma}\ln\left(\frac{D}{kq}-1\right)\right]\right.\right.\\ \left.\left.\times\left(1-e^{-iq/D}\right) + qe^{-iq/D} - \frac{D}{i}\left(1-e^{-iq/D}\right)\right\} + nV + nR(L)\right\}.\end{aligned} \tag{9}$$

4.3. Vendor's perspective

For each production period, q units are produced, and the vendor delivers those inventories to the buyer. After that the vendor will generate only the average inventory level $\frac{q}{D}$ units for delivering the buyer until the inventory level reaches zero. Also the vendor manufactures nq quantity for a lot. Since the cycle length for the vendor is $\frac{mq}{D}$, the average inventory for the vendor can be given as

$$\left[nq\left(\frac{q}{P}+(n-1)\frac{q}{D}\right) - \frac{n^2q^2}{2P} - \frac{q^2}{D}\{1+2+\cdots+(n-1)\}\right]\frac{D}{nq} = \frac{q}{2}\left[(n-1)-(n-2)\frac{D}{P}\right].$$

Therefore the total holding cost for the vendor at the first cycle in the interval $t \in [0, \frac{nq}{D}]$ is expressed as

$$\begin{aligned}H_V &= h_v\int_0^{nq/D}\frac{q}{2}\left[(n-1)-(n-2)\frac{D}{P}\right]e^{-it}\mathrm{d}t\\ &= \frac{h_V}{i}\frac{q}{2}\left[(n-1)-(n-2)\frac{D}{P}\right]\left(1-e^{-inq/D}\right).\end{aligned} \tag{10}$$

Thus the total cost of the vendor at first cycle which is a composition of setup cost and holding cost can be exhibited as

$$\mathrm{TC}_V(q,n) = A_V + \frac{h_v}{i}\frac{q}{2}\left[(n-1)-(n-2)\frac{D}{P}\right]\left(1-e^{-inq/D}\right) \tag{11}$$

Consequently, the present value of the total cost of the vendor over an infinite time horizon can be admitted as

$$\mathrm{PTC}_V(q,n) = \left(1-e^{-inq/D}\right)^{-1}\left\{A_V + \frac{h_v}{i}\frac{q}{2}\left[(n-1)-(n-2)\frac{D}{P}\right]\left(1-e^{-inq/D}\right)\right\}. \tag{12}$$

5. Joint optimization

On considering vendor and buyer cooperation to each other, we framed the expected joint total cost at the first cycle as

$$\begin{aligned}\mathrm{JTC}(q,n,L) &= \mathrm{PTC}_B(q,n,L)+\mathrm{PTC}_V(q,n)\\ &= \left(1-e^{-inq/D}\right)^{-1}\left\{A_B+nF+\frac{nh_B}{i}\left\{\left[q+\frac{\sigma_D\sqrt{L}}{\gamma}\ln\left(\frac{D}{kq}-1\right)\right]\right.\right.\\ &\quad\times\left(1-e^{-iq/D}\right)+qe^{-iq/D}-\left.\frac{D}{i}\left(1-e^{-iq/D}\right)\right\}+nV+nR(L)\\ &\quad+\left.A_V+\frac{h_v}{i}\frac{q}{2}\left[(n-1)-(n-2)\frac{D}{P}\right]\left(1-e^{-inq/D}\right)\right\} \end{aligned} \tag{13}$$

with $q\in(0,D/2k]$, $n\in\mathbb{N}$ and $L\in[L_m,L_0]$.

To establish an exact optimization procedure which is to minimize $\mathrm{JTC}(q,n,L)$ under the constraints $q\in(0,D/2k]$, $n\in\mathbb{N}$ and $L\in[L_m,L_0]$, we seek to define the properties which are satisfied by $\mathrm{JTC}(q,n,L)$.

At first, for fixed (q,n) and $L\in[L_j,L_{j-1}]$, $\mathrm{JTC}(q,n,L)$ is strictly concave in L due to the fact that

$$\begin{aligned}\frac{\partial}{\partial L}\mathrm{JTC}(q,n,L) &= -nc_i+\frac{nh_B}{2i}\frac{\sigma_D}{\gamma\sqrt{L}}\ln\left(\frac{D}{kq}-1\right)\frac{\left(1-e^{-iq/D}\right)}{\left(1-e^{-inq/D}\right)} \quad \text{and}\\ \frac{\partial^2}{\partial L^2}\mathrm{JTC}(q,n,L) &= -\frac{nh_B}{4i}\frac{\sigma_D}{\gamma\sqrt{L}}\ln\left(\frac{D}{kq}-1\right)\frac{\left(1-e^{-iq/D}\right)}{\left(1-e^{-inq/D}\right)}<0.\end{aligned}$$

Therefore, for fixed (q,n) and $L\in[L_j,L_{j-1}]$, the minimum of $\mathrm{JTC}(q,n,L)$ in L lies on the end points of the interval $L\in[L_j,L_{j-1}]$.

Basically, we are granted to write the following equality:

$$\min_{(q,n,L)}\mathrm{JTC}(q,n,L)=\min\left\{\min_{(q,n)}\mathrm{JTC}(q,n,L_j)|j=0,1,\ldots,m\right\}.$$

Secondly, if we relax the integer constraint on n, it is possible to note that $\mathrm{JTC}(q,n,L)$ is strictly convex in n, for fixed (q,L). In fact, putting in evidence the terms that depending on n, substantially $\mathrm{JTC}(q,n,L)$ can be written as follows:

$$\mathrm{JTC}(q,n,L)=C_1\frac{1}{\left(1-e^{-inq/D}\right)}+C_2\frac{n}{\left(1-e^{-inq/D}\right)}+C_3n+C_4 \tag{14}$$

where

$$\begin{aligned}C_1 &= A_B+A_V\\ C_2 &= F+V+R(L)+\frac{nh_B}{i}\left\{\left[q+\frac{\sigma_D\sqrt{L}}{\gamma}\ln\left(\frac{D}{kq}-1\right)\right]\left(1-e^{-iq/D}\right)\right.\\ &\quad+\left.qe^{-iq/D}-\frac{D}{i}\left(1-e^{-iq/D}\right)\right\}\\ C_3 &= \frac{h_Vq}{2i}\left(1-\frac{D}{P}\right)\\ C_4 &= \frac{h_Vq}{2i}\left(\frac{2D}{P}-1\right)\end{aligned}$$

and noticing that

$$\frac{\partial^2}{\partial n^2}\frac{1}{\left(1-e^{-inq/D}\right)}=\left(\frac{qi}{D}\right)^2\frac{e^{inq/D}\left(e^{inq/D}+1\right)}{\left(e^{inq/D}-1\right)^3}>0 \tag{15}$$

and

$$\frac{\partial^2}{\partial n^2}\frac{n}{\left(1-e^{-inq/D}\right)}=\left(\frac{nqi}{D}\right)^2\frac{e^{inq/D}\left(e^{inq/D}+1\right)}{\left(e^{inq/D}-1\right)^3}>0. \tag{16}$$

The equations (15) and (16) suggest that the expected joint total cost is strictly convex in n for (q, L) fixed.

Finally, for fixed (n, L) we relax the constraint on $q \in (0, \xi/2]$, it is possible to note that $\mathrm{JTC}(q, n, L)$ is strictly convex in q, for fixed (n, L). By taking account of the equation (14) we could be able to investigate the convexity of the expected joint total cost $\mathrm{JTC}(q, n, L)$.

Now, for fixed (n, L) we take the derivatives of $\mathrm{JTC}(q, n, L)$ with respect to q and obtain

$$\begin{aligned}\frac{\partial \mathrm{JTC}}{\partial q} &= \frac{-ine^{-inq/D}}{D\left(1-e^{-inq/D}\right)^2}\left\{A_B+A_V+n\left[F+V+R(L)+\frac{nh_B}{i}\left\{\left[q+\frac{\sigma_D\sqrt{L}}{\gamma}\ln\left(\frac{D}{kq}-1\right)\right]\right.\right.\right.\\ &\quad\left.\left.\left.\times\left(1-e^{-iq/D}\right)+qe^{-iq/D}-\frac{D}{i}\left(1-e^{-iq/D}\right)\right\}\right]\right\}+\frac{nh_B}{i}\left\{\left[1-\frac{D\sigma_D\sqrt{L}}{\gamma q(D-kq)}\right]\right.\\ &\quad\left.\times\left(1-e^{-iq/D}\right)+\frac{i\sigma_D\sqrt{L}}{D\gamma}\ln\left(\frac{D}{kq}-1\right)e^{-iq/D}\right\}\times\frac{1}{\left(1-e^{-inq/D}\right)}\\ &\quad+\frac{h_V}{2i}\left(n-1-\frac{D}{P}(n-2)\right),\end{aligned} \tag{17}$$

and

$$\begin{aligned}\frac{\partial^2 \mathrm{JTC}}{\partial q^2} &= \left\{\left(\frac{qi}{D}\right)^2\frac{e^{inq/D}\left(e^{inq/D}+1\right)}{\left(e^{inq/D}-1\right)^3}(C_1+nC_2)+\frac{nh_B}{i}\left\{\frac{ie^{-iq/D}}{D}\right.\right.\\ &\quad+\frac{\sigma_D\sqrt{L}D(D-2kq)}{\gamma q^2(D-kq)^2}(1-e^{iq/D})-\frac{2\sigma_D\sqrt{L}i}{\gamma q(D-kq)}e^{iq/D}\\ &\quad\left.-\frac{i^2\sigma_D\sqrt{L}}{D^2\gamma}\ln\left(\frac{D}{kq}-1\right)e^{-iq/D}\right\}\times\frac{1}{\left(1-e^{-inq/D}\right)}\\ &\quad+\frac{2n}{\left(1-e^{-inq/D}\right)}\times\frac{h_B}{i}\left\{\left[1-\frac{D\sigma_D\sqrt{L}}{\gamma q(D-kq)}\right]\left(1-e^{-iq/D}\right)\right.\\ &\quad\left.\left.+\frac{i\sigma_D\sqrt{L}}{D\gamma}\ln\left(\frac{D}{kq}-1\right)e^{-iq/D}\right\}\right\}>0.\end{aligned} \tag{18}$$

By examining the second order sufficient conditions (SOSC), it can be verified that the expected joint total cost $\mathrm{JTC}(q, n, L)$ is strictly convex in n for (n, L) fixed.

Thus, for fixed (n, L) the minimum value of $\text{JTC}(q, n, L)$ occurs at the point q which satisfies $\frac{\partial \text{JTC}}{\partial q} = 0$. Solving this equation, we yield the value of q as

$$q = \frac{D\left(1-e^{-inq/D}\right)}{ine^{-inq/D}}\left\{\left[1-\frac{D\sigma_D\sqrt{L}}{\gamma q(D-kq)}\right]\left(1-e^{-iq/D}\right) + \frac{i\sigma_D\sqrt{L}}{D\gamma}\ln\left(\frac{D}{kq}-1\right)e^{-iq/D}\right\} + \frac{h_V D\left(1-e^{-inq/D}\right)^2}{2ih_B n^2 e^{-inq/D}} \times \left(n-1-\frac{D}{P}(n-2)\right) - \frac{i}{nh_B}\left[A_B + A_V + n\left[F + V + R(L) + \frac{nh_B}{i}\left\{\left[\frac{\sigma_D\sqrt{L}}{\gamma}\ln\left(\frac{D}{kq}-1\right)\right].-\frac{D}{i}\right\}\left(1-e^{-iq/D}\right)\right]\right]. \quad (19)$$

The equations (17) and (18) suggest that the expected joint total cost is strictly convex in q for (n, L) fixed.

6. Lead time reduction

6.1. Linear function case

In this section, we assume that lead time and ordering cost reductions act dependently with the following relationship

$$\frac{L_0 - L}{L_0} = \alpha\left(\frac{A_{B0} - A_B}{A_{B0}}\right) \quad (20)$$

where $(\alpha > 0)$ is a constant scaling parameter which describes the linear relationship between percentages of reductions in lead time and ordering cost.

By considering the relationship (20), the ordering cost A_B can be formulated as a linear function of L, that is,

$$A_B(L) = u + vL \quad (21)$$

where $u = \left(1 - \frac{1}{\alpha}A_{B0}\right)$ and $v = \frac{A_{B0}}{\alpha L_0}$. Using (21) in (13) we get the problem as

$$\text{JTC}_1(q, n, L) = \left(1-e^{-inq/D}\right)^{-1}\left\{u + vL + nF + \frac{nh_B}{i}\left\{\left[q + \frac{\sigma_D\sqrt{L}}{\gamma}\ln\left(\frac{D}{kq}-1\right)\right] \times \left(1-e^{-iq/D}\right) + qe^{-iq/D} - \frac{D}{i}\left(1-e^{-iq/D}\right)\right\} + nV + nR(L) + A_V + \frac{h_v}{i}\frac{q}{2}\left[(n-1)-(n-2)\frac{D}{P}\right]\left(1-e^{-inq/D}\right)\right\}. \quad (22)$$

To minimize $\text{JTC}_1(q, n, L)$ under the constraints $q \in (0, D/2k]$, $n \in \mathbb{N}$ and $L \in [L_m, L_0]$, we need to find the partial derivatives of $\text{JTC}_1(q, n, L)$ with respect to $q,\ n,\ L$.

$$\frac{\partial}{\partial L}\text{JTC}_1(q, n, L) = v - nc_i + \frac{nh_B}{2i}\frac{\sigma_D}{\gamma\sqrt{L}}\ln\left(\frac{D}{kq}-1\right)\frac{\left(1-e^{-iq/D}\right)}{\left(1-e^{-inq/D}\right)} \quad \text{and}$$

$$\frac{\partial^2}{\partial L^2}\text{JTC}_1(q, n, L) = -\frac{nh_B}{4i}\frac{\sigma_D}{\gamma\sqrt{L}}\ln\left(\frac{D}{kq}-1\right)\frac{\left(1-e^{-iq/D}\right)}{\left(1-e^{-inq/D}\right)} < 0.$$

Since the first and second partial derivatives of $\text{JTC}_1(q, n, L)$ with respect to $q,\ n$ are as same as the partial derivatives of $\text{JTC}(q, n, L)$ with respect to $q,\ n$ respectively. There is no oscillation in the formulation of q.

6.2. Logarithmic function case

In this section, we assume that lead time and ordering cost reductions act dependently with the following relationship

$$\frac{A_{B0}-A_B}{A_{B0}} = \tau\left(\frac{L}{L_0}\right) \tag{23}$$

where ($\tau < 0$) is a constant scaling parameter which describes the logarithmic relationship between percentages of reductions in lead time and ordering cost.

By considering the relationship (23), the ordering cost A_B can be formulated as a logarithmic function of L, that is,

$$A_B(L) = d + e\ln L \tag{24}$$

where $d = A_{B0} + \tau A_{B0}\ln L_0$ and $e = -\tau A_{B0}$. Using (24) in (13) we get the problem as

$$\begin{aligned}\mathrm{JTC}_2(q,n,L) = {}& \left(1-e^{-inq/D}\right)^{-1}\Bigg\{d + e\ln L + nF + \frac{nh_B}{i}\Bigg\{\left[q + \frac{\sigma_D\sqrt{L}}{\gamma}\ln\left(\frac{D}{kq}-1\right)\right]\\ &\times\left(1-e^{-iq/D}\right) + qe^{-iq/D} - \frac{D}{i}\left(1-e^{-iq/D}\right)\Bigg\} + nV + nR(L)\\ &+ A_V + \frac{h_v}{i}\frac{q}{2}\left[(n-1)-(n-2)\frac{D}{P}\right]\left(1-e^{-inq/D}\right)\Bigg\}.\end{aligned} \tag{25}$$

To minimize $\mathrm{JTC}_2(q,n,L)$ under the constraints $q \in (0, D/2k]$, $n \in \mathbb{N}$ and $L \in [L_m, L_0]$, we need to find the second partial derivatives of $\mathrm{JTC}_2(q,n,L)$ with respect to $q,\ n,\ L$.

$$\begin{aligned}\frac{\partial}{\partial L}\mathrm{JTC}_2(q,n,L) &= \tfrac{e}{L} - nc_i + \tfrac{nh_B}{2i}\tfrac{\sigma_D}{\gamma\sqrt{L}}\ln\left(\tfrac{D}{kq}-1\right)\tfrac{\left(1-e^{-iq/D}\right)}{\left(1-e^{-inq/D}\right)} \quad \text{and}\\ \frac{\partial^2}{\partial L^2}\mathrm{JTC}_2(q,n,L) &= -\tfrac{e}{L^2} - \tfrac{nh_B}{4i}\tfrac{\sigma_D}{\gamma\sqrt{L}}\ln\left(\tfrac{D}{kq}-1\right)\tfrac{\left(1-e^{-iq/D}\right)}{\left(1-e^{-inq/D}\right)} < 0.\end{aligned}$$

Since the first and second order partial derivatives of $\mathrm{JTC}_2(q,n,L)$ with respect to $q,\ n$ are as same as the partial derivatives of $\mathrm{JTC}(q,n,L)$ with respect to $q,\ n$ respectively, there is no variation in the formulation of q.

Algorithm

Step 1: Set $n = 1$ and $q = 100$.
Step 2: Determine q from equation (5). Using this q find the values of SL and z.
Step 3: Determine the value of q from equation (19) by substituting the value of z.
Step 4: Using this q find the values of SL and z. Also repeat the step 3 until the value of q remains unchanged.
Step 5: Then calculate the expected joint total cost $\mathrm{JTC}(q,n,L)$.
Step 6: Increase the value of n to $n+1$ and repeat the steps from (1) to (5).
Step 6: Repeat the step 6 until the convexity of the expected joint total cost $\mathrm{JTC}(q,n,L)$ is achieved. The place where the convexity is achieved will give us the optimal values for n and q.

The above algorithm is same for all the three cases constructed.

7. Numerical application

In this section the model has been practiced to a large number of consumable items like metallic and plastic small parts, personal protective equipment, packaging components, etc. All these items are circumscribed, by

TABLE 2. Lead time data.

Component – j	Duration (days) Normal – b_j	Minimum – a_j	Unit Crashing Cost ($/day)
1	12	7	0.3
2	12	7	2.1
3	10	8	5.3

TABLE 3. Values of Parameters considered in the numerical study.

Parameters	Values		
	Normal case	Linear function case	Logarithmic function case
D (units/unit time)	500	500	500
P (units/unit time)	1000	1000	1000
A_V ($/setup)	200	–	–
A_B ($/order)	100	–	–
A_{V0} ($/setup)	–	200	200
A_{B0} ($/order)	–	100	100
V ($/order)	3	3	3
F($/order)	20	20	20
h_B ($/unit/unit time)	30	30	30
h_V ($/unit/unit time)	10	10	10
ρ	0.15	0.15	0.15
η	0.07	0.07	0.07
k	0.5	0.5	0.5
σ_D (unit/week)	5	5	5
γ	1.702	1.702	1.702

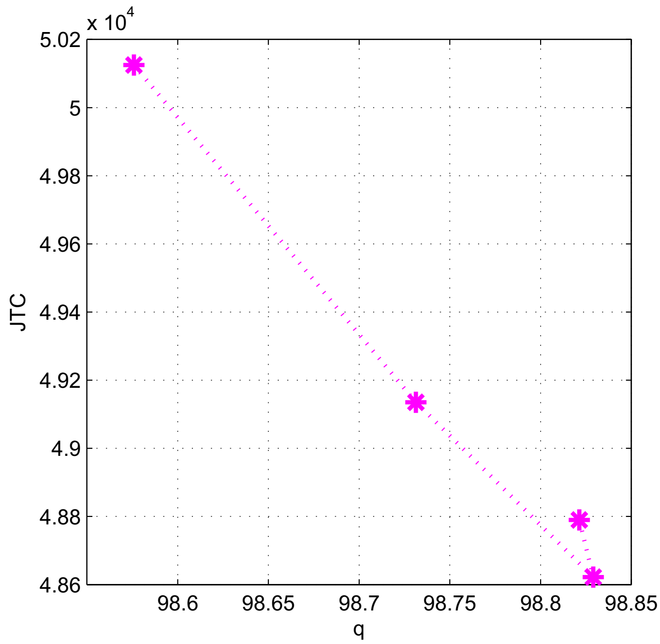


FIGURE 1. The convexity of the expected joint total cost when the ordering cost is fixed to be a constant.

TABLE 4. The change in the value of JTC when the reduction parameters are represented by the values as $\alpha = 0.75$ and $\tau = -1$.

		Normal case		Linear case				Logarithmic case			
L	n	q	JTC	α	A_{B0}	q	JTC_1	τ	A_{B0}	q	JTC_2
34	2	98.5759	50125	0.75	100	98.5759	50125	–1	100	98.5759	50125
29	2	98.7313	49135	0.75	80.3922	98.8645	48514	–1	84.0935	98.8393	48631
24	2	98.8290	48622	0.75	60.7843	99.0954	47381	–1	65.1693	99.0657	47520
22	2	98.8213	48790	0.75	52.9412	99.1412	47301	–1	56.4682	99.1172	47413

TABLE 5. The change in the value of JTC when the reduction parameters are represented by the values as $\alpha = 1$ and $\tau = -0.8$.

		Linear case				Logarithmic case			
L_0	n	α	A_{B0}	q	JTC_1	τ	A_{B0}	q	JTC_2
34	2	0.75	100	98.5759	50125	–0.2	100	98.5759	50125
29	2	0.75	80.3922	98.8645	48514	–0.2	96.8187	98.7529	49034
24	2	0.75	60.7843	99.0954	47381	–0.2	93.0339	98.8762	48401
22	2	0.75	52.9412	99.1412	47301	–0.2	91.2936	98.8805	48514

TABLE 6. The change in the value of JTC when the reduction parameters are represented by the values as $\alpha = 1.25$ and $\tau = -0.5$.

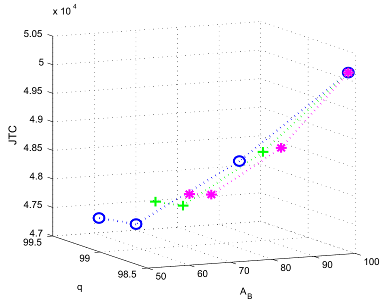
		Linear case				Logarithmic case			
L_0	n	α	A_{B0}	q	JTC_1	τ	A_{B0}	q	JTC_2
34	2	1.25	100	98.5759	50125	–0.5	100	98.5759	50125
29	2	1.25	88.2353	98.7683	48759	–0.5	92.0468	98.7853	48883
24	2	1.25	76.4706	98.9888	47877	–0.5	82.5847	98.9473	48070
22	2	1.25	71.7647	99.0132	47896	–0.5	78.2341	98.9692	48101

TABLE 7. The change in the value of JTC when the reduction parameters are represented by the values as $\alpha = 2.5$ and $\tau = -0.2$.

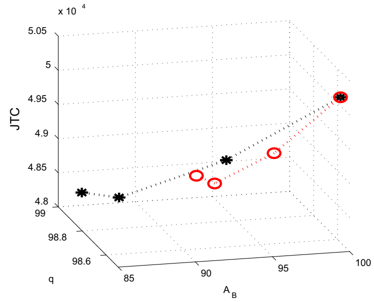
		Linear case				Logarithmic case			
L_0	n	α	A_{B0}	q	JTC_1	τ	A_{B0}	q	JTC_2
34	2	2.5	100	98.5759	50125	–0.2	100	98.5759	50125
29	2	2.5	94.1176	98.7713	48949	–0.2	96.8187	98.7529	49034
24	2	2.5	88.2353	98.9089	48249	–0.2	93.0339	98.8762	48401
22	2	2.5	85.8824	99.9172	48343	–0.2	91.2936	98.8805	48514

TABLE 8. The change in the value of JTC when the reduction parameters are represented by the values as $\alpha = 5$ and $\tau = 0$.

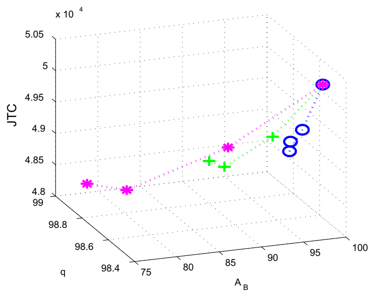
		Linear case				Logarithmic case			
L_0	n	α	A_{B0}	q	JTC_1	τ	A_{B0}	q	JTC_2
34	2	5	100	98.5759	50125	0	100	98.5759	50125
29	2	5	97.0588	98.7513	49042	0	100	98.7313	49135
24	2	5	94.1176	98.8690	48435	0	100	98.8290	48622
22	2	5	92.9412	98.8693	48566	0	100	98.8213	48790



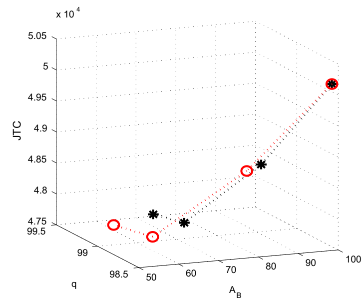
(a) $\alpha = 1.25,\ 5,\ 2.5.$



(b) $\alpha = 1,\ 0.75.$

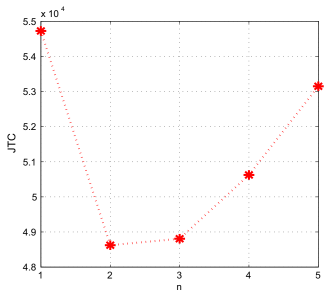


(c) $\tau = -0.5,\ 0,\ -0.2.$

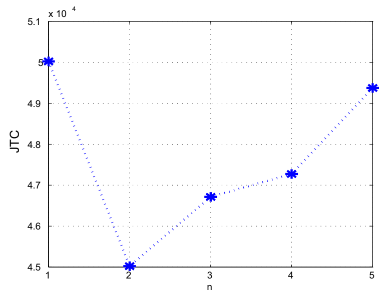


(d) $\tau = -0.8,\ -1.$

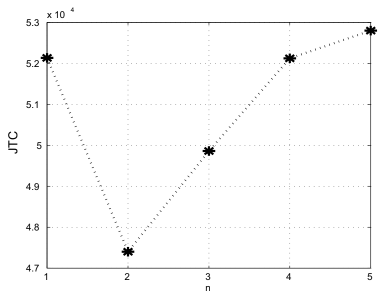
FIGURE 2. Convexity of JTC with the variable q for both the linear and logarithmic cases respectively.



(a) $\alpha = 1.25,\ 5,\ 2.5.$

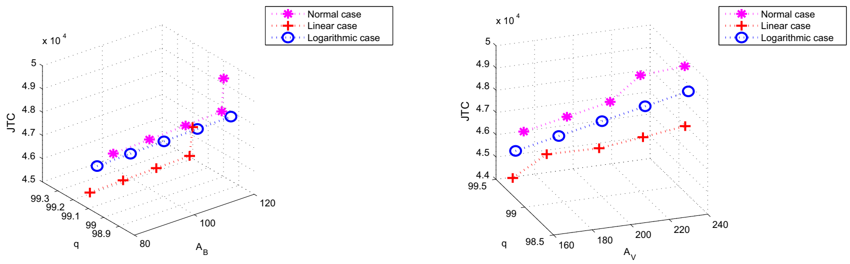


(b) $\alpha = 1,\ 0.75.$



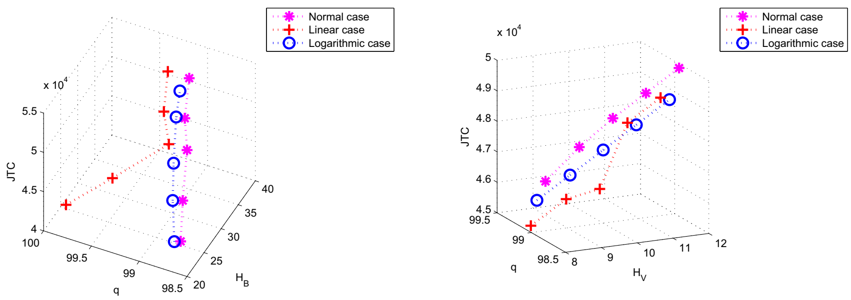
(c) $\tau = -0.5,\ 0,\ -0.2.$

FIGURE 3. Convexity of JTC with the variable n for all the three cases.



(a) Buyer. **(b)** Vendor.

FIGURE 4. Sensitivity on Ordering cost of buyer and vendor respectively.



(a) Buyer. **(b)** Vendor.

FIGURE 5. Sensitivity on holding cost of buyer and vendor.

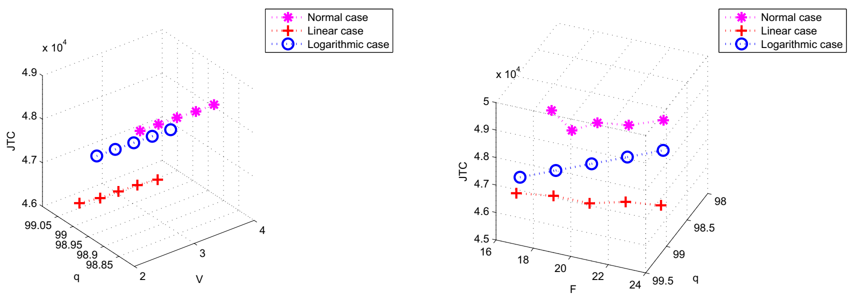


FIGURE 6. Sensitivity on Buyer's variable cost and Transportation cost respectively.

low price, high annual consumption, small dimensions and ease of storage, but, they achieve a considerable annual monetary volume when considered cummulatively.

In order to illustrate the proposed model, we follow the data used in Braglia *et al.* [11] which is shown in Table 2 and some newly introduced parameters are included in Table 3.

The variation of the expected joint total cost and the ordering cost of the buyer can be observed through through the following figures when there is an oscillation in the values of α and τ:

The sensitivity analysis has been conducted based on the variation of the following parameters:

(i) Standard deviation in market demand (in % of the average monthly demand).
(ii) Average demand rate.

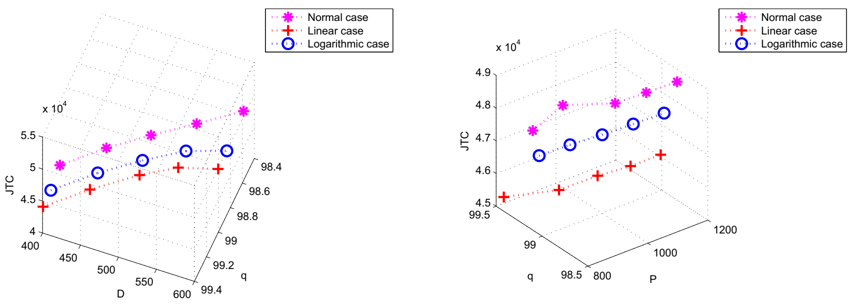


Figure 7. Sensitivity on Demand rate and Production rate respectively.

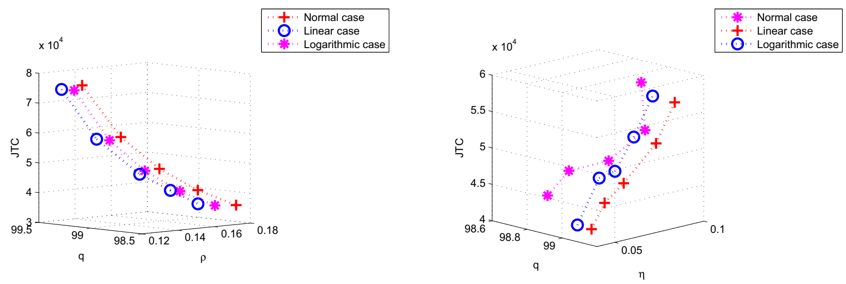


Figure 8. Sensitivity on Dicount rate and Inflation rate.

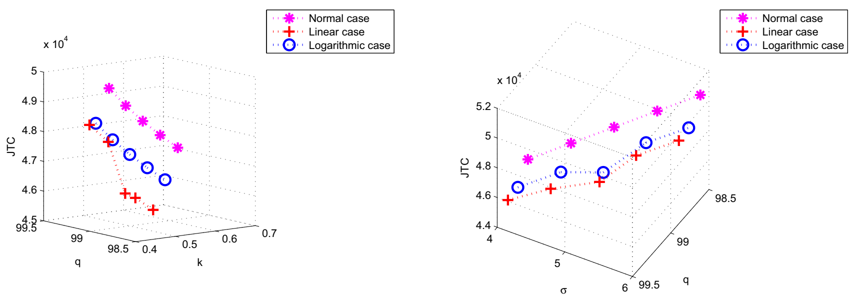


Figure 9. Sensitivity on number of admissible stockouts and standard deviation of demand rate.

(iii) Various cost used by both vendor and buyer.
(iv) Inflation rate and discount rate of present value.

Based on our numerical results, we achieve the following managerial phenomena:

(1) From Figure 1 the convexity of the expected joint total cost JTC can be observed clearly with reference to the values tablulated in the Table 4.
(2) From Tables 4–8, we observe the results of sensitivity analysis of the change in the parameters α and τ and the optimized values are figured out in Figure 2.
(3) We can find that that the demand D and P have not a significant impact on the expected joint total cost per unit time JTC in both the cases.
(4) For both the cases, we could see a moderate significance for the role of inflation rate η, number of admissible stockouts per unit time.

Table 9. Sensitivity of various cost used by both vendor and buyer when $\alpha = 0.75$, $L_0 = 22$ and $\tau = -1$, $L_0 = 22$.

		Normal case		Linear case		Logarithmic case	
Parameters	Values	q	JTC	q	JTC_1	q	JTC_2
	80	98.9373	47986	99.0902	45722	99.0431	47043
	90	98.8969	48305	99.0688	45893	99.0203	47223
A_{B0}	100	98.8564	48624	99.0474	46059	98.9974	47402
	110	98.8160	48943	99.0260	46228	98.9746	47583
	120	98.9990	49229	99.2018	46357	98.9517	47762
	160	99.0182	47349	99.2091	44787	99.1593	46130
	180	98.9373	47986	99.2885	45423	99.0784	46766
A_V	200	98.8564	48624	99.0474	46059	98.9974	47402
	220	98.9990	49229	98.9546	46555	98.9165	48040
	240	98.8987	49650	98.8913	46976	98.8356	48678
	8	98.7940	46945	99.0074	45214	98.9184	46163
	9	98.8233	47876	99.0145	45921	98.9579	46783
H_V	10	98.8564	48624	99.0474	46059	98.9974	47402
	11	98.8959	49243	99.1663	47921	99.0370	48022
	12	98.9354	49862	99.2059	48542	99.0765	48642
	24	98.7083	41235	99.9083	41235	98.7753	40941
	27	98.7944	44267	99.5297	44267	98.8988	43871
H_B	30	98.8564	48624	99.0474	46059	98.9974	46402
	33	98.9871	50334	99.2059	50334	99.0781	50148
	36	99.0487	53368	99.2720	53368	99.1453	51363
	16	98.1545	47125	99.0084	45754	98.9184	46163
	18	98.5684	47493	99.0148	45971	98.9579	46783
K	20	98.8564	48624	99.0474	46059	98.9974	47402
	22	99.0057	49124	99.0784	46471	99.0370	48022
	24	99.0681	49721	99.1294	46745	99.0765	48642
	2.4	98.8596	48624	99.0574	46059	99.0007	47403
	2.7	98.8580	48624	99.0483	46059	98.9990	47403
V	3	98.8564	48624	99.0474	46059	98.9974	47402
	3.3	98.8532	48624	99.0444	46059	98.9958	47402
	3.6	98.8528	48624	99.0368	46059	98.9942	47402

(5) For both the cases, there is a high impact in the discount rate representing the time value of money ρ when the values have some changes.

(6) In both the cases, we could be able to observe a moderate impact in the expected joint total cost with the change in various cost used by vendor and buyer (Figs. 3–9).

8. Conclusion

In this paper we presented a novel approach to safety stock management and investigated the impact of lead time reduction within an integrated vendor–buyer supply chain framework using present value where lead time and ordering cost reductions act dependently. The cost of the safety stock is determined by adopting a logistic approximation to the standard normal cumulative distribution and the service level is formulated in relation

Table 10. Sensitivity of Average demand rate D and Production rate P when $\alpha = 0.75, \quad L_0 = 22$ and $\tau = -1, \quad L_0 = 22$.

Parameters	Values	Normal case		Linear case		Logarithmic case	
		q	JTC	q	JTC_1	q	JTC_2
D	400	99.1103	45037	99.3913	43940	99.2611	44018
	450	98.9670	46869	99.2421	45657	99.1124	45747
	500	98.8564	48624	99.0474	46059	98.9974	47402
	550	98.7281	49850	99.0351	48886	98.9062	49000
	600	98.5789	50847	99.0020	49925	98.8582	49999
P	800	99.1010	48041	99.4157	45434	99.0310	47405
	900	99.1001	48467	99.1416	45801	99.0213	47404
	1000	98.8564	48624	99.0474	46059	98.9974	47402
	1100	98.8442	48623	99.0151	46062	98.9852	47401
	1200	98.8341	48622	99.0128	46062	98.9751	47401

Table 11. Sensitivity of various factors used for inflation and admissible stockouts when $\alpha = 0.75, \quad L_0 = 22$ and $\tau = -1, \quad L_0 = 22$.

Parameters	Values	Normal case		Linear case		Logarithmic case	
		q	JTC	q	JTC_1	q	JTC_2
ρ	0.1200	99.0793	77539	99.2787	75429	99.1562	75592
	0.1350	98.9672	59745	99.2020	58119	99.0761	58245
	0.15	98.8564	48624	99.0474	46059	98.9974	47402
	0.1650	98.7469	41014	99.0128	39898	98.9202	39984
	0.1800	98.6388	35480	99.0091	34514	98.8444	34589
η	0.056	98.7542	43446	99.0057	40318	98.9253	40405
	0.063	98.8052	46746	99.0092	43528	98.9784	46766
	0.07	98.9612	48624	99.0474	46059	98.9974	47402
	0.077	99.0980	53245	99.1616	51796	99.0340	51908
	0.084	99.0054	58846	99.1969	57245	99.0708	57368
k	0.4	98.7894	49953	99.0023	48573	98.9332	48675
	0.45	98.8249	49255	99.0145	47905	98.9672	48007
	0.5	98.8564	48624	99.0474	46059	98.9974	47402
	0.55	98.8846	48046	99.1537	45747	99.0245	46850
	0.6	98.9101	47513	99.1781	45237	99.0489	46339
σ_D	4	99.1000	46231	99.3601	45013	99.2307	45113
	4.5	98.9782	47427	99.2434	45914	99.1104	46258
	5	98.8564	48624	99.0474	46059	98.9974	46402
	5.5	98.7347	49821	99.0100	48445	98.8809	48548
	6	98.6131	51020	98.8934	49590	98.7644	49694

to the dimension of the single shipment, to the average demand of the buyer and to the number of admissible stockouts (Tabs. 9–11).

We then developed an exact algorithm that permits the optimization of inventory replenishment and lead time. Numerical application conferrred that this optimization approach achieves a high level of efficiency, which may offer promising application in practice. One of the repercussions of this work is that if the ordering cost per order could be reduced effectively, then the expected joint total cost per unit time could be automatically minimized.

Further one can extend this work by using various other reduction factors, service level constraints and variable transportation cost. Also one can deal with multi item, trade credit policy, various dependent demand rates, and shortages with partial backorder, etc. A plausible future work deals with the multi-echelon supply chains such as: single vendor-multi buyer, multi vendor-single buyer and multi vendor-multi buyer framework adopting the CS perpective. In this case, the model has either fuzzy or stochastic nature.

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