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# MULTI-CHOICE AND STOCHASTIC PROGRAMMING FOR TRANSPORTATION PROBLEM INVOLVED IN SUPPLY OF FOODS AND MEDICINES TO HOSPITALS WITH CONSIDERATION OF LOGISTIC DISTRIBUTION

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**Abstract.** The objective of the proposed article is to minimize the transportation costs of foods and medicines from different source points to different hospitals by applying stochastic mathematical programming model to a transportation problem in a multi-choice environment containing the parameters in all constraints which follow the Logistic distribution and cost coefficients of objective function are also multiplicative terms of binary variables. Using the stochastic programming approach, the stochastic constraints are converted into an equivalent deterministic one. A transformation technique is introduced to manipulate cost coefficients of objective function involving multi-choice or goals for binary variables with auxiliary constraints. The auxiliary constraints depends upon the consecutive terms of multi-choice type cost coefficient of aspiration levels. A numerical example is presented to illustrate the whole idea.

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## 1. Introduction

Timely supply of pharmaceutical products from different sources to different hospitals is extremely essential for healthcare industry. A good transport service can fulfill the need that essentially requires reliability, efficiency and safety. The problem for a transportation service is that there are multiple sources and demand points, where availability and requirement of different types of medicines are not known exactly because depending upon the patients condition and situation production at sources and requirement at hospitals are changed randomly. The problem further intensifies as the decision makers have multiple number of choices of medicines. In such situation transportation service provider has to approximate units of medicines from different sources to different hospitals in the cost effective way such that total transportation cost is minimized given fulfillment of multiple choices. The problem can be addressed by applying multi-choice stochastic programming for transportation problem.

*Keywords.* Multi-choice programming, stochastic programming, Logistic distribution, transportation problem, transformation technique, mixed-integer programming.

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A stochastic transportation model in which the constraints are stochastic in nature and the cost coefficients are multi choice type is considered. The transportation problem is well known problem of operations research, that can be formulated and solved as a linear programm. The traditional transportation problem can be described as a special case of linear programming problem. The available amounts at the supply points and the amounts required at the demand points are the parameters of the transportation problem. These parameters are not always stable and/or not known exactly. In such a situation,the parameters in all constraints are considered as random variables to follow Logistic distribution.

Transportation model deals with the determination of minimization of the total cost during transportation plan for transporting a commodity from a number of sources to a number of destinations. At the $i$th sources $(i = 1, 2, \ldots, m)$, there are $s_i$ unit of commodity available. The demand at the $j$th destination $(j = 1, 2, \ldots, n)$, is denoted by $d_j$. The sources may be production facilities, warehouses, supply points and the destinations may be consumption facilities, warehouse or demand points. The coefficient $C^k_{ij}$ of the $k$th objective function could represent the unit transportation cost of product from source $i$ to destination $j$. Hitchcock [9] first considered the problem of minimizing the cost of distribution of products from several factories to a number of customers. He developed a procedure to solve the transportation problems, which have close resemblance with the primal simplex transportation method developed by Dantzig [7].

Stochastic programming (SP) is an optimization technique in which constraints and/or objective functions contain random variables, where stochastic element is present in the data. Then these numbers follow stochastic distributions such as Logistic and others distribution. The random variables associated with the sources and destinations may be many in number, depending on the nature and the type of model. More generally, such models are formulated, solved analytically or numerically and analyzed in order to provide useful information to the decision maker. Goicoechea *et al.* [8] presented the deterministic equivalents for some probabilistic programming involving normal and other distributions. Mahapatra *et al.* [16], discussed the solution procedure of multi-objective stochastic transportation problem involving normal distribution with joint constraints. The probability density function of a random variable $x$, a Logistic distribution, is

$$f(x;\alpha,\beta) = \frac{1}{\beta}\frac{e^{\frac{x-\alpha}{\beta}}}{[1+e^{\frac{x-\alpha}{\beta}}]^2};\ \ -\infty \leq x \leq \infty; \text{and}\ \ \alpha > 0,\ \ \beta > 0. \tag{1.1}$$

where $\alpha$ and $\beta$ are the location parameter and scale parameter, respectively. Here,

$$\int_{-\infty}^{+\infty} f(x;\alpha,\beta)\mathrm{d}x = -\left[\frac{1}{1+e^{(x-\alpha)/\beta}}\right]_{-\infty}^{\infty} = 1.$$

The graph of the above pdf is symmetric about the location parameters. The Logistic distribution is a continuous probability distribution. The shape of the logistic distribution is similar to that of normal distribution. It has heavier tails than the normal distribution with negligible error in the respective models. The importance of the Logistic distribution has already been felt in many areas of transportation distribution, warehousing, materials handling, food procedures and companies, postal delivery utilities and public transportation, among others. The Logistic distribution has been used in a variety of fields, for details description of the various properties and applications are referred to the monograph of Balakrishnan [2]. The two-parameter Logistic distribution was originally proposed as a generalization of the Logistic distribution by Ahuja and Nash [1], Olapade [14].

Multi-choice programming is a mathematical programming problem, in which decision maker is allowed to set multiple number of choices for a parameter. In recent years, methods of multi-choice stochastic optimization become increasingly important in solving scientifically decision making problems arising in economics, industry, health care, transportation, agriculture, military, engineering and technology. Biswal and Acharya [3] presented the transformation of a multi-choice linear programming problem in which constraints are associated with multi-choice parameters. A method for modeling the multi-choice programming problem, using the multiple terms of binary variables was presented by Chang [4]. He considered a mathematical model where the multiplicative terms of binary variables were replaced by continuous variable [5]. He also proposed a revised method for multi-choice goal programming model which did not involve multiplicative terms of binary variables to model the multiple aspiration levels [6].

Considerable amount of researches were done addressing how to improve the quality of service to patients. Based on data collection through interviews Mohapatra [11] discussed about the automation of business processes in healthcare system. To minimize the treatment cost and improve quality of treatment in Indian context Mohapatra and Murarka [13] identified different parameters that govern performance of hospitals. Mohapatra [12] explained how hospital information system is used as a comprehensive integrated information system designed to manage the administrative, financial and clinical aspects of a hospital in urban India.

All the aforementioned papers discuss about the improvement of patient care system, manage administrative and financial goals in hospitals. But as indicated, timely availability of clinical materials is very important for smooth running of a hospital. The papers mentioned in this literature did not consider these issues. This paper considers the problem through multi-choice stochastic transportation problem. The multi-choice stochastic transportation problem is presented with two different type of probabilistic constraints. Both the probabilistic constraints involves inequality type associated with random variables which follow Logistic distribution. The cost coefficients of the objective function are multi-choice type, where the transformation technique is used. Binary variable and additional restrictions are introduced to formulate a non-linear mixed integer programming model. A new methodology is proposed to solve multi-choice stochastic transportation problem.

## 2. Mathematical model

In this paper, a mathematical model for multi-choice stochastic transportation problem involving Ligistic distribution is considered as follows:

**Model 1:**

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k\}x_{ij}, \qquad k = 1, 2, \ldots, K \tag{2.1}$$

$$\text{subject to} \quad \Pr\left(\sum_{j=1}^{n} x_{ij} \leq s_i\right) \geq 1 - \gamma_i, \quad i = 1, 2, \ldots, m \tag{2.2}$$

$$\Pr\left(\sum_{i=1}^{m} x_{ij} \geq d_j\right) \geq 1 - \delta_j, \quad j = 1, 2, \ldots, n \tag{2.3}$$

$$x_{ij} \geq 0, \quad \forall \;\; i \text{ and } \;\; j \tag{2.4}$$

where $0 < \gamma_i < 1, \; \forall \;\; i$ and $0 < \delta_j < 1, \; \forall \;\; j$.

Assumed that $s_i \quad (i = 1, 2, \ldots, m)$ and $d_j \quad (j = 1, 2, \ldots, n)$ are Logistic random variables and $C_{ij}^k = \{C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k\}$ is a multi-choice parameter.

The following cases are to be considered

(1) Only $s_i, (i = 1, 2, \ldots, m)$ follows Logistic distribution.
(2) Only $d_j, (j = 1, 2, \ldots, n)$ follows Logistic distribution.
(3) Both $s_i, (i = 1, 2, \ldots, m)$ and $d_j, (j = 1, 2, \ldots, n)$ follow Logistic distribution.

### 2.1. Only $s_i, (i = 1, 2, \ldots, m)$ follows Logistic distribution

Assumed that $s_1, s_2, \ldots, s_m$ are independent random variables and $s_i, (i = 1, 2, \ldots, m)$ follows Logistic distribution with location and scale parameters as $\alpha_i$ and $\beta_i$ respectively, where $\gamma_i$, is the aspiration level, $0 < \gamma_i < 1$.

The constraints (2.2) of Model 1 can be rewritten as follows

$$\Pr\left(\sum_{j=1}^{n} x_{ij} \leq s_i\right) \geq 1-\gamma_i, \quad i=1,2,\ldots,m.$$

The probability density function of $s_i$ $(i=1,2,\ldots,m)$ is given by

$$f(s_i;\alpha_i,\beta_i) = \frac{1}{\beta_i}\frac{e^{\frac{s_i-\alpha_i}{\beta_i}}}{[1+e^{\frac{s_i-\alpha_i}{\beta_i}}]^2}; \quad s_i \geq 0; \text{and } \ \alpha_i>0, \ \ \beta_i>0. \tag{2.5}$$

Hence the cumulative density function can be presented as:

$$\int_{\sum_{j=1}^{n} x_{ij}}^{\infty} f(s_i;\alpha_i,\beta_i)\mathrm{d}(s_i) \geq 1-\gamma_i. \tag{2.6}$$

The above integral can be expressed as:

$$\int_{\sum_{j=1}^{n} x_{ij}}^{\infty} \frac{1}{\beta_i}\frac{e^{\frac{s_i-\alpha_i}{\beta_i}}}{[1+e^{\frac{s_i-\alpha_i}{\beta_i}}]^2}\mathrm{d}(s_i) \geq 1-\gamma_i. \tag{2.7}$$

It can be further simplified as:

$$e^{\frac{\sum_{j=1}^{n} x_{ij}-\alpha_i}{\beta_i}} \leq \frac{\gamma_i}{1-\gamma_i}\cdot \tag{2.8}$$

Taking logarithm in both sides twice,we have

$$\sum_{j=1}^{n} x_{ij} - \alpha_i \leq \ \beta_i \ln\frac{\gamma_i}{1-\gamma_i}\cdot \tag{2.9}$$

Thus finally, the probabilistic constraints (2.3) can be transformed into a deterministic linear constraints as:

$$\sum_{j=1}^{n} x_{ij} \leq \alpha_i + \beta_i \ln\frac{\gamma_i}{1-\gamma_i}\cdot \tag{2.10}$$

A multi-choice deterministic transportation problem has been obtained (see Model 2) instead of multi-choice stochastic transportation model.

**Model 2:**

$$\min: z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2,\ldots,C_{ij}^k\}x_{ij}, \qquad k=1,2,\ldots,K \tag{2.11}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq \alpha_i + \beta_i \ln\frac{\gamma_i}{1-\gamma_i} \tag{2.12}$$

$$\sum_{i=1}^{m} x_{ij} \geq d_j, \quad j=1,2,\ldots,n \tag{2.13}$$

$$x_{ij} \geq 0, \quad \forall \ i \ \text{ and } \ j \tag{2.14}$$

where $\sum_{j=1}^{n} \alpha_i + \beta_i \ln\frac{\gamma_i}{1-\gamma_i} \geq \sum_{j=1}^{n} d_j$ (feasibility condition).

### 2.2. Only $d_j, (j = 1, 2, \ldots, n)$ follows Logistic distribution

It is assumed that $d_1, d_2, \ldots, d_n$ are independent random variables and $d_j (j = 1, 2, ...n)$ follows Logistic distribution with location and scale parameters as $\alpha'_j$ and $\beta'_j$ respectively, where the aspiration levels is $\delta_j$, $0 < \delta_j < 1$. The constraint of Model 1 is rewritten as follows.

$$\Pr\left(\sum_{i=1}^{m} x_{ij} \geq d_j\right) \geq 1 - \delta_j, \quad j = 1, 2, \ldots, n.$$

Rearranging the above constraints, we write

$$\Pr\left(\sum_{i=1}^{m} x_{ij} \leq d_j\right) \leq \delta_j, \quad j = 1, 2, \ldots, n. \tag{2.15}$$

The probability density function of $d_j \;\; (j = 1, 2, \ldots, n)$ is given by

$$f(d_j; \alpha'_j, \beta'_j) = \frac{1}{\beta'_j} \frac{e^{\frac{d_j - \alpha'_j}{\beta'_j}}}{[1 + e^{\frac{d_j - \alpha'_j}{\beta'_j}}]^2}; \quad d_j \geq 0; \text{and} \;\; \alpha'_j > 0, \;\; \beta'_j > 0. \tag{2.16}$$

Hence the cumulative density function can be presented as:

$$\int_{\sum_{i=1}^{m} x_{ij}}^{\infty} f(d_j; \alpha'_j, \beta'_j) \mathrm{d}(d_j) \leq \delta_j. \tag{2.17}$$

The above integral can be expressed as:

$$\int_{\sum_{i=1}^{m} x_{ij}}^{\infty} \frac{1}{\beta'_j} \frac{e^{\frac{d_j - \alpha'_j}{\beta'_j}}}{[1 + e^{\frac{d_j - \alpha'_j}{\beta'_j}}]^2} \mathrm{d}(d_j) \leq \delta_j. \tag{2.18}$$

It can be further simplified as:

$$e^{\frac{\sum_{j=1}^{n} x_{ij} - \alpha'_j}{\beta'_j}} \geq \frac{\delta'_j}{1 - \delta'_j}. \tag{2.19}$$

Taking logarithm in both sides twice,we have

$$\sum_{j=1}^{n} x_{ij} - \alpha'_j \geq \;\beta'_j \ln \frac{\delta'_j}{1 - \delta'_j}. \tag{2.20}$$

Thus finally, the probabilistic constraints (2.3) can be transformed into a deterministic linear constraints as:

$$\sum_{j=1}^{n} x_{ij} \geq \alpha'_j + \beta'_j \ln \frac{\delta'_j}{1 - \delta'_j}. \tag{2.21}$$

Thus, a multi-choice deterministic transportation problem has been obtained (see Model 3) instead of multi-choice stochastic transportation problem.

**Model 3:**

$$\min: z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^{1}, C_{ij}^{2}, \ldots, C_{ij}^{k}\}x_{ij}, \qquad k = 1, 2, \ldots, K \tag{2.22}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq s_i, \quad i = 1, 2, \ldots, m \tag{2.23}$$

$$\sum_{j=1}^{n} x_{ij} \geq \alpha'_j + \ \beta'_j \ln \frac{\delta'_j}{1-\delta'_j} \tag{2.24}$$

$$x_{ij} \geq 0, \quad \forall \ i \ \text{ and } \ j \tag{2.25}$$

where $\sum_{i=1}^{m} s_i \geq \sum_{i=1}^{m} \alpha'_j + \ \beta'_j \ln \frac{\delta'_j}{1-\delta'_j}$ (feasibility condition).

### 2.3. Both $s_i, (i = 1, 2, \ldots, m)$ and $d_j, (j = 1, 2, \ldots, n)$ follow Logistic distributions

The location and scale parameters of the random variables $s_i$ and $d_j$ are known and previously defined. In this case, both $s_i$ and $d_j$ follow Logistic distribution. Model 1 is transferred into an equivalent deterministic model as:

**Model 4:**

$$\min: z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^{1}, C_{ij}^{2}, \ldots, C_{ij}^{k}\}x_{ij}, \quad k = 1, 2, \ldots, K \tag{2.26}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq \alpha_i + \beta_i \ln \frac{\gamma_i}{1-\gamma_i} \tag{2.27}$$

$$\sum_{j=1}^{n} x_{ij} \geq \alpha'_j + \beta'_j \ln \frac{\delta'_j}{1-\delta'_j} \tag{2.28}$$

where $\sum_{j=1}^{n}[\alpha_i + \beta_i \ln \frac{\gamma_i}{1-\gamma_i}] \geq \sum_{j=1}^{n}[\alpha'_j + \beta'_j \ln \frac{\delta'_j}{1-\delta'_j}]$, (feasibility condition).

## 3. Transformation of the multi-choice cost parameter to an equivalent model

The proposed model is derived for maximum of ten choices of any cost coefficients of the objective function. Ten cases are presented below for $k = 2, 3, \ldots, 10$.

**Case 1:** *When* $k = 2$.

The objective function (2.1) is presented as follows:

$$\min: z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^{1}, C_{ij}^{2}\}x_{ij}.$$

The cost coefficients have two choices as $\{C_{ij}^{1}, C_{ij}^{2}\}$, out of these two choices one is to be selected. Since the total number of elements of the set is 2, only one binary variable is required. Taking the binary variable as $z_{ij}^{1}$, the objective function is formulated as:

$$\min: z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^{1} z_{ij}^{1} + C_{ij}^{2}(1 - z_{ij}^{1})\}x_{ij}, \tag{3.1}$$

$$z_{ij}^{p} = 0/1, \quad p = 1, \ \text{for all } i \text{ and } j. \tag{3.2}$$

**Case 2:** *When* $k = 3$.

The objective function (2.1) is presented as follows:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, C_{ij}^3\}x_{ij}.$$

The cost coefficients have three choices as $\{C_{ij}^1, C_{ij}^2, C_{ij}^3\}$, out of which one is to be selected. Since $2^1 < 3 < 2^2$, so the total number of elements of the set is 3. Denoting the binary variables by $z_{ij}^1, z_{ij}^2$ and introducing additional constraints, two models are obtained as:

*Model 2(a)*:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1(1-z_{ij}^1)(1-z_{ij}^2) + C_{ij}^2 z_{ij}^1(1-z_{ij}^2) + C_{ij}^3(1-z_{ij}^1)z_{ij}^2\}x_{ij} \tag{3.3}$$

$$z_{ij}^1 + z_{ij}^2 \leq 1 \tag{3.4}$$

$$z_{ij}^p = 0/1, \;\; p = 1, 2, \;\; \text{for all } i \text{ and } j.$$

*Model 2(b)*:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1 z_{ij}^1 z_{ij}^2 + C_{ij}^2 z_{ij}^1(1-z_{ij}^2) + C_{ij}^3(1-z_{ij}^1)z_{ij}^2\}x_{ij} \tag{3.5}$$

$$z_{ij}^1 + z_{ij}^2 \geq 1 \tag{3.6}$$

$$z_{ij}^p = 0/1, \;\; p = 1, 2, \;\; \text{for all } i \text{ and } j.$$

**Case 3:** *When* $k = 4$.

The objective function (2.1) is presented as follows:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4\}x_{ij}.$$

The cost coefficients of the objective function have four choices as $\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4\}$, out of which one is to be selected. Here the total number of choices is $4 = 2^2$. Denoting the binary variables by $z_{ij}^1, z_{ij}^2$, the following problem is obtained as below:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1 z_{ij}^1 z_{ij}^2 + C_{ij}^2 z_{ij}^1(1-z_{ij}^2) + C_{ij}^3(1-z_{ij}^1)z_{ij}^2 + C_{ij}^4(1-z_{ij}^1)(1-z_{ij}^2)\}x_{ij} \tag{3.7}$$

$$z_{ij}^p = 0/1, \;\; p = 1, 2, \;\; \text{for all } i \text{ and } j.$$

**Case 4:** *When* $k = 5$.

The objective function (2.1) is presented as follows:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5\}x_{ij}.$$

The cost coefficients have five choices as $\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5\}$, out of which one is to be selected. Since $2^2 < 5 < 2^3$, we need three binary variables as: $z_{ij}^1, z_{ij}^2, z_{ij}^3$. The restriction is imposed to remaining three terms $(8 - 5)$ by introducing additional constraints and obtain three different models expressed below:

*Model 4(a)*:

$$\begin{aligned}\min : z = &\sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3) + C_{ij}^2(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3)\\ &+ C_{ij}^3(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3 + C_{ij}^4 z_{ij}^1 z_{ij}^2(1-z_{ij}^3) + C_{ij}^5(1-z_{ij}^1)z_{ij}^2 z_{ij}^3\}x_{ij} \quad (3.8)\\ 1 \le\ & z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 2 \quad (3.9)\\ & z_{ij}^1 + z_{ij}^3 \le 1 \quad (3.10)\\ z_{ij}^p &= 0/1,\ p = 1,2,3,\ \text{for all } i \text{ and } j.\end{aligned}$$

*Model 4(b)*:

$$\begin{aligned}\min : z = &\sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3) + C_{ij}^2(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3)\\ &+ C_{ij}^3(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3 + C_{ij}^4 z_{ij}^1 z_{ij}^2(1-z_{ij}^3) + C_{ij}^5 z_{ij}^1(1-z_{ij}^2)z_{ij}^3\}x_{ij} \quad (3.11)\\ 1 \le\ & z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 2 \quad (3.12)\\ & z_{ij}^2 + z_{ij}^3 \le 1 \quad (3.13)\\ z_{ij}^p &= 0/1,\ p = 1,2,3,\ \text{for all } i \text{ and } j.\end{aligned}$$

*Model 4(c)*:

$$\begin{aligned}\min : z = &\sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3) + C_{ij}^2(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3)\\ &+ C_{ij}^3(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3 + C_{ij}^4(1-z_{ij}^1)z_{ij}^2 z_{ij}^3 + C_{ij}^5 z_{ij}^1(1-z_{ij}^2)z_{ij}^3\}x_{ij} \quad (3.14)\\ 1 \le\ & z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 2 \quad (3.15)\\ & z_{ij}^1 + z_{ij}^2 \le 1 \quad (3.16)\\ z_{ij}^p &= 0/1,\ p = 1,2,3,\ \text{for all } i \text{ and } j.\end{aligned}$$

**Case 5:** *When* $k = 6$.

The objective function (2.1) is presented as follows:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5, C_{ij}^6\}x_{ij}.$$

The cost coefficients of the objective function have six choices as $\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5, C_{ij}^6\}$, out of which one is to be selected. Since $2^2 < 6 < 2^3$, three binary variables are needed as $z_{ij}^1, z_{ij}^2, z_{ij}^3$. Then the restriction is imposed to remaining two terms $(8-6)$ by introducing auxiliary constraints to get the mathematical model as given below:

$$\begin{aligned}\min : z = &\sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3) + C_{ij}^2(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3)\\ &+ C_{ij}^3(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3 + C_{ij}^4 z_{ij}^1 z_{ij}^2(1-z_{ij}^3)\\ &+ C_{ij}^5 z_{ij}^1(1-z_{ij}^2)z_{ij}^3 + C_{ij}^6(1-z_{ij}^1)z_{ij}^2 z_{ij}^3\}x_{ij} \quad (3.17)\\ 1 \le\ & z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 2 \quad (3.18)\\ z_{ij}^p &= 0/1,\ p = 1,2,3,\ \text{for all } i \text{ and } j.\end{aligned}$$

**Case 6:** *When* $k = 7$.

The objective function from (2.1) is presented as follows:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5, C_{ij}^6, C_{ij}^7\}x_{ij}.$$

The cost coefficients of the objective function have seven choices as $\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5, C_{ij}^6, C_{ij}^7\}$, and one of them is to be selected. Since $2^2 < 7 < 2^3$, we need three binary variables as: $z_{ij}^1, z_{ij}^2, z_{ij}^3$. Then the restriction is imposed to remaining one term $(8-7)$ by introducing additional constraint in the mathematical model. Two different models are formulated as given below:

*Model 6(a)*:

$$\begin{aligned}\min : z = &\sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1(1-z_{ij}^1)(1-z_{ij}^2)(1-z_{ij}^3) + C_{ij}^2 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3)\\ &+ C_{ij}^3(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3) + C_{ij}^4(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3 + C_{ij}^5 z_{ij}^1 z_{ij}^2(1-z_{ij}^3)\\ &+ C_{ij}^6 z_{ij}^1(1-z_{ij}^2)z_{ij}^3 + C_{ij}^7(1-z_{ij}^1)z_{ij}^2 z_{ij}^3\}x_{ij} \qquad (3.19)\\ &z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 2 \qquad (3.20)\\ z_{ij}^p = 0/1,\ &p = 1,2,3,\ \text{for all } i \text{ and } j.\end{aligned}$$

*Model 6(b)*:

$$\begin{aligned}\min : z = &\sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3) + C_{ij}^2(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3)\\ &+ C_{ij}^3(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3 + C_{ij}^4 z_{ij}^1 z_{ij}^2(1-z_{ij}^3) + C_{ij}^5 z_{ij}^1(1-z_{ij}^2)z_{ij}^3\\ &+ C_{ij}^6(1-z_{ij}^1)z_{ij}^2 z_{ij}^3 + C_{ij}^7 z_{ij}^1 z_{ij}^2 z_{ij}^3\}x_{ij} \qquad (3.21)\\ &z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \ge 1 \qquad (3.22)\\ z_{ij}^p = 0/1,\ &p = 1,2,3,\ \text{for all } i \text{ and } j.\end{aligned}$$

**Case 7:** *When* $k = 8$.

The objective function from (2.1) is presented as follows:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5, C_{ij}^6, C_{ij}^7, C_{ij}^8\}x_{ij}.$$

The cost coefficients of the objective function have eight choices as $\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5, C_{ij}^6, C_{ij}^7, C_{ij}^8\}$, out of which one is to be selected. Since the total number of elements of the set is $8 = 2^3$. Taking the help of three binary variables as: $z_{ij}^1, z_{ij}^2, z_{ij}^3$, then only one model is formulated as given below:

$$\begin{aligned}\min : z = &\sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1 z_{ij}^1 z_{ij}^2 z_{ij}^3 + C_{ij}^2(1-z_{ij}^1)z_{ij}^2 z_{ij}^3 + C_{ij}^3 z_{ij}^1(1-z_{ij}^2)z_{ij}^3\\ &+ C_{ij}^4 z_{ij}^1 z_{ij}^2(1-z_{ij}^3) + C_{ij}^5(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3 + C_{ij}^6 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3)\\ &+ C_{ij}^7(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3) + C_{ij}^8(1-z_{ij}^1)(1-z_{ij}^2)(1-z_{ij}^3)\}x_{ij} \qquad (3.23)\\ &z_{ij}^p = 0/1,\ p = 1,2,3,\ \text{for all } i \text{ and } j.\end{aligned}$$

**Case 8:** *When* $k = 9$.

The objective function from equation (2.1) is presented as follows:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5, C_{ij}^6, C_{ij}^7, C_{ij}^8, C_{ij}^9\}x_{ij}.$$

The cost coefficients of the objective function have nine choices as $\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5, C_{ij}^6, C_{ij}^7, C_{ij}^8, C_{ij}^9\}$, out of which one of them is to be selected. Since $2^3 < 9 < 2^4$, so four binary variables $z_{ij}^1, z_{ij}^2, z_{ij}^3, z_{ij}^4$ are required. Then the restriction is imposed to the introducing of auxiliary and additional constraints in two different mathematical models. Then the two models are expressed in respect to the consecutive terms of binomial coefficients as:$\{C_{ij}^1, C_{ij}^2\}$ and $\{C_{ij}^2, C_{ij}^3\}$, whose sum is 10. The auxiliary constraints are also depend upon the range of coefficients in each sets. But the additional constraints are expressed as the difference between sum of above with aspiration levels. In each model will be performed six ype of similar models. In this case, only one model has stated as given below:

*Model 9(a)*:

$$\begin{aligned}\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{&C_{ij}^1(1 - z_{ij}^1(1 - z_{ij}^2)z_{ij}^3 z_{ij}^4) + C_{ij}^2(1 - z_{ij}^1)z_{ij}^2(1 - z_{ij}^3)\\ &z_{ij}^4 + C_{ij}^3(1 - z_{ij}^1)z_{ij}^2 z_{ij}^3(1 - z_{ij}^4) + C_{ij}^4 z_{ij}^1(1 - z_{ij}^2)(1 - z_{ij}^3)z_{ij}^4 + C_{ij}^5 z_{ij}^1 \qquad (3.24)\\ &(1 - z_{ij}^2)z_{ij}^3(1 - z_{ij}^4) + C_{ij}^6 z_{ij}^1 z_{ij}^2(1 - z_{ij}^3)(1 - z_{ij}^4) + C_{ij}^7(1 - z_{ij}^1)z_{ij}^2\\ &z_{ij}^3 z_{ij}^4 + C_{ij}^8 z_{ij}^1(1 - z_{ij}^2)z_{ij}^3 z_{ij}^4 x_{ij} + C_{ij}^9 z_{ij}^1 z_{ij}^2(1 - z_{ij}^3)z_{ij}^4\} \qquad (3.25)\\ &2 \le z_{ij}^1 + z_{ij}^2 + z_{ij}^3 + z_{ij}^4 \le 3 \qquad (3.26)\\ &z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 2 \qquad (3.27)\\ &x_{ij} \ge 0,\ \ \forall\ \ i,\ \ \forall\ \ j,\ \ \forall\ \ k,\ \text{and}\ \ z_{ij}^p = 0/1,\ p = 1, 2, 3, 4. \qquad (3.28)\end{aligned}$$

*Model 9(b)*:

$$\begin{aligned}\min : z = \ &\sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1(1 - z_{ij}^1)(1 - z_{ij}^2)(1 - z_{ij}^3)z_{ij}^4) + C_{ij}^2(1 - z_{ij}^1)(1 - z_{ij}^2)z_{ij}^3(1 - z_{ij}^4)\\ &z_{ij}^4 + C_{ij}^3(1 - z_{ij}^1)z_{ij}^2(1 - z_{ij}^3)(1 - z_{ij}^4) + C_{ij}^4 z_{ij}^1(1 - z_{ij}^2)(1 - z_{ij}^3)(1 - z_{ij}^4)\\ &+ C_{ij}^5(1 - z_{ij}^1)(1 - z_{ij}^2)z_{ij}^3 z_{ij}^4 + C_{ij}^6(1 - z_{ij}^1)z_{ij}^2(1 - z_{ij}^3)z_{ij}^4 + C_{ij}^7(1 - z_{ij}^1)z_{ij}^2\\ &z_{ij}^3(1 - z_{ij}^4) + C_{ij}^8 z_{ij}^1(1 - z_{ij}^2)(1 - z_{ij}^3)z_{ij}^4 x_{ij} + C_{ij}^9 z_{ij}^1(1 - z_{ij}^2)z_{ij}^3(1 - z_{ij}^4\}) \qquad (3.29)\\ 1 \le\ &z_{ij}^1 + z_{ij}^2 + z_{ij}^3 + z_{ij}^4 \le 2 \qquad (3.30)\\ &z_{ij}^1 + z_{ij}^2 \le 1 \qquad (3.31)\\ x_{ij} \ge 0,\ &\ \forall\ \ i,\ \ \forall\ \ j,\ \ \forall\ \ k,\ \text{and}\ \ z_{ij}^p = 0/1,\ p = 1, 2, 3, 4. \qquad (3.32)\end{aligned}$$

**Case 9 (c):** *When $k = 10$.*

The objective function from ( 2.2 ) is presented as follows:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^{10}\}x_{ij}.$$

The cost coefficients of the objective function have ten choices as $\{C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^{10}\}$, out of which one of them is to be selected in the multi-choice stochastic transportation problem. Since $2^3 < 10 < 2^4$, so four binary variables as: $z_{ij}^1, z_{ij}^2, z_{ij}^3, z_{ij}^4$ are established. The set of binomial coefficients $\{C_1^4, C_2^4\}$ and $\{C_2^4, C_3^4\}$ are expressed of whose sum is equal to 10 which is the same of goals as $k_i = 10$ in respect to the different models. So there does not arise any additional restriction. Two different models are formulated as given below:

*Model 10(a)*:

$$\begin{aligned}\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{&C_{ij}^1(1-z_{ij}^1)(1-z_{ij}^2)(1-z_{ij}^3z_{ij}^4 + C_{ij}^2(1-z_{ij}^1)\\ &(1-z_{ij}^2)z_{ij}^3(1-z_{ij}^4) + C_{ij}^3(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3)(1-z_{ij}^4) + C_{ij}^4 z_{ij}^1(1-z_{ij}^2)\\ &(1-z_{ij}^3)(1-z_{ij}^4) + C_{ij}^5(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3z_{ij}^4 + C_{ij}^6(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3)z_{ij}^4 \quad (3.33)\\ &+C_{ij}^7(1-z_{ij}^1)z_{ij}^2z_{ij}^3(1-z_{ij}^4) + C_{ij}^8 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3)z_{ij}^4 + C_{ij}^9 z_{ij}^1(1-z_{ij}^2)\\ &z_{ij}^3(1-z_{ij}^4) + C_{ij}^{10} z_{ij}^1 z_{ij}^2(1-z_{ij}^3)(1-z_{ij}^4)\}x_{ij} \quad (3.34)\end{aligned}$$

$$1 \le z_{ij}^1 + z_{ij}^2 + z_{ij}^3 + z_{ij}^4 \le 2 \tag{3.35}$$

$$x_{ij} \ge 0, \ \ \forall \ \ i, \ \ \forall \ \ j \text{ and } \ z_{ij}^p = 0/1, \ p = 1,2,3,4. \tag{3.36}$$

*Model 10(b)*:

$$\begin{aligned}\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{&C_{ij}^1(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3z_{ij}^4 + C_{ij}^2(1-z_{ij}^1)z_{ij}^2\\ &(1-z_{ij}^3)z_{ij}^4 + C_{ij}^3(1-z_{ij}^1)z_{ij}^2z_{ij}^3(1-z_{ij}^4) + C_{ij}^4 z_{ij}^1(1-z_{ij}^2)\\ &(1-z_{ij}^3)z_{ij}^4 + C_{ij}^5 z_{ij}^1(1-z_{ij}^2)z_{ij}^3(1-z_{ij}^4) + C_{ij}^6 z_{ij}^1 z_{ij}^2(1-z_{ij}^3(1-z_{ij}^4)\\ &+C_{ij}^7(1-z_{ij}^1)z_{ij}^2z_{ij}^3(1-z_{ij}^4) + C_{ij}^8 z_{ij}^1(1-z_{ij}^2)z_{ij}^3(1-z_{ij}^4) + C_{ij}^9 z_{ij}^1\\ &z_{ij}^2(1-z_{ij}^3)z_{ij}^4 + C_{ij}^{10} z_{ij}^1 z_{ij}^2 z_{ij}^3(1-z_{ij}^4)\}x_{ij}\end{aligned} \tag{3.37}$$

$$2 \le z_{ij}^1 + z_{ij}^2 + z_{ij}^3 + z_{ij}^4 \le 3 \tag{3.38}$$

$$x_{ij} \ge 0, \ \ \forall \ \ i, \ \ \forall \ \ j \text{ and } \ z_{ij}^p = 0/1, \ p = 1,2,3,4. \tag{3.39}$$

## 4. Case study

A reputed medicine supplier Agency transports a variety of medicines from two sources points by lorry or train to the four hospitals center through the six routes. The main objective is minimization of the transportation cost and maximization of the profit against the market price at different market. The production of materials at sources points and delivery of goods at the demands points are randomly changed in situation as term in Logistic distribution. The transportation costs to carry medicines from sources to destinations are multi-choice parameters and are related with fluctuation in road condition, distance of destination point from source point, consumption of fuel, and durability or lifetime of particular brand of tires used etc. The problem can not be solved without using the multi-choice programming methodology. They are appended below in Table 1.

A multi-choice stochastic transportation problem where the objective function and the constraints are formulated as:

$$\min : z = \sum_{i=1}^{2}\sum_{j=1}^{3}\{C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k\}x_{ij}, \qquad k = 1,2,\ldots,6 \tag{4.1}$$

$$\text{subject to} \quad \Pr\left(\sum_{j=1}^{4} x_{ij} \le s_i\right) \ge 1-\gamma_i, \quad i = 1,2 \tag{4.2}$$

$$\Pr\left(\sum_{i=1}^{3} x_{ij} \ge d_j\right) \ge 1-\delta_j, \quad j = 1,2,3 \tag{4.3}$$

Table 1. Unit transportation cost in different routes.

| Sl. No. | Route: $x_{ij}$ | Transportation cost (in Rupees) $C^k_{ij}$: per unit (1 unit= 20 Kg) |
|---|---|---|
| 1 | $(1,1)$: $x_{11}$ | 10 or 11 or 12 |
| 2 | $(1,2)$: $x_{12}$ | 15 or 16 |
| 3 | $(1,3)$: $x_{13}$ | 17 or 18 or 19 or 20 |
| 4 | $(2,1)$: $x_{21}$ | 30 or 33 or 35 or 37 or 40 or 45 |
| 5 | $(2,2)$: $x_{22}$ | 20 or 21 or 22 or 23 or 24 or 25 or 26 |
| 6 | $(2,3)$: $x_{23}$ | 25 or 26 or 27 or 28 or 29 or 30 or 31 or 32 or 33 |

Table 2. Probability levels, location and scale parameter values of supplies.

| Parameters $s_i$ | Location parameters | Scale parameters | Specified prob.levels |
|---|---|---|---|
| $s_1$ | $\alpha_1 = 40$ | $\beta_1 = 5.0$ | $\gamma_1=0.09$ |
| $s_2$ | $\alpha_2=32$ | $\beta_2 = 3$ | $\gamma_2=0.08$ |

Table 3. Probability levels, location and scale parameter values of demands.

| Parameters $b_j$ | Shape parameters | Scale parameters | Specified prob.levels |
|---|---|---|---|
| $b_1$ | $\alpha'_1 = 10$ | $\beta'_1 = 2$ | $\delta_1=0.05$ |
| $b_2$ | $\alpha'_2 = 8$ | $\beta'_2 = 1$ | $\delta_2 = 0.04$ |
| $b_3$ | $\alpha'_3 = 5$ | $\beta'_3 = 1$ | $\delta_3 = 0.03$ |

where, $x_{ij} \geq 0,\ i = 1,2;\ j = 1,2,3$ and $0 < \gamma_i < 1,\ 0 < \delta_j < 1$.

Assume that two parameters $s_1, s_2$ with Logistic distribution. The specified probability levels and the location and the scale parameters of supplies $s_1, s_2$ are given in the Table 2.

Further, the specified probability levels and the location and the scale parameters of demands parameters $d_1, d_2, d_3$ which follow Logistic distribution are given in Table 3.

Using the data provided in Tables 1–3 the following deterministic multi-choice transportation problem is formulated as:

$$\begin{aligned} \min : z = {} & \{10,11,12\}x_{11} + \{15,16\}x_{12} + \{17,18,19,20\}x_{13} \\ & + \{30,33,35,37,40,45\}x_{21} + \{20,21,22,23,24,25,26\}x_{22} \\ & + \{25,26,27,28,29,30,31,32,33\}x_{23} \end{aligned} \tag{4.4}$$

$$\text{subject to} \quad \sum_{j=1}^{3} x_{1j} \leq 28.43182535 \tag{4.5}$$

$$\sum_{j=1}^{3} x_{2j} \leq 20.23061186 \tag{4.6}$$

$$\sum_{j=1}^{2} x_{i1} \geq 15.88887796 \tag{4.7}$$

$$\sum_{i=1}^{2} x_{i2} \geq 11.17805383 \tag{4.8}$$

$$\sum_{i=1}^{2} x_{i3} \geq 8.47609869 \tag{4.9}$$

$$x_{ij} \geq 0, \quad i = 1,2 \text{ and } j = 1,2,3.$$

Now using a new transformation technique, a multi-choice deterministic transportation problem is obtained as below:

$$\min: z = t_{11}x_{11} + t_{12}x_{12} + t_{13}x_{13} + t_{21}x_{21} + t_{22}x_{22} + t_{23}x_{23} \tag{4.10}$$

subject to (4.74) − (4.78)

where, $t_{11} = 10z_{11}^1 z_{11}^2 + 11z_{11}^1(1 - z_{11}^2) + 12(1 - z_{11}^1)z_{11}^2$ (4.11)

$$t_{12} = 15z_{12}^1 + 16(1 - z_{12}^1) \tag{4.12}$$

$$t_{13} = 17z_{13}^1 z_{13}^2 + 18z_{13}^1(1 - z_{13}^2) + 19(1 - z_{13}^1)z_{13}^2 + 20(1 - z_{13}^1)(1 - z_{13}^2) \tag{4.13}$$

$$t_{21} = 30z_{21}^1(1 - z_{21}^2)(1 - z_{21}^3) + 33(1 - z_{21}^1)z_{21}^2(1 - z_{21}^3) + 35(1 - z_{21}^1)$$
$$(1 - z_{21}^2)z_{21}^3 + 37z_{21}^1 z_{21}^2(1 - z_{21}^3) + 40z_{21}^1(1 - z_{21}^2)z_{21}^3 + 45(1 - z_{21}^1)(1 - z_{21}^1)z_{21}^2 z_{21}^3 \tag{4.14}$$

$$t_{22} = 20(1 - z_{22}^1)(1 - z_{22}^2)(1 - z_{22}^3) + 21z_{22}^1(1 - z_{22}^2)(1 - z_{22}^3) + 22(1 - z_{22}^1)$$
$$z_{22}^2(1 - z_{22}^3) + 23(1 - z_{22}^1)(1 - z_{22}^2)z_{22}^3 + 24z_{22}^1 z_{22}^2(1 - z_{22}^3)$$
$$+ 25z_{22}^1(1 - z_{22}^2)z_{22}^3 + 26z_{22}^1(1 - z_{22}^2)z_{22}^3 \tag{4.15}$$

$$t_{23} = 25(1 - z_{23}^1)(1 - z_{23}^2 z_{23}^3 z_{23}^4 + 26z_{23}^1(1 - z_{23}^2)z_{23}^3(1 - z_{23}^4)$$
$$+ 27(1 - z_{23}^1)z_{23}^2 z_{23}^3(1 - z_{23}^4) + 28z_{23}^1(1 - z_{23}^2)(1 - z_{23}^3)z_{23}^4 + 29z_{23}^1(1 - z_{23}^2)$$
$$z_{23}^3(1 - z_{23}^4) + 30z_{23}^1 z_{23}^2(1 - z_{23}^3)(1 - z_{23}^4) + 31(1 - z_{23}^1)z_{23}^2 z_{23}^3 z_{23}^4$$
$$+ 32z_{23}^1(1 - z_{23}^2)z_{23}^3 z_{23}^4 + 33z_{23}^1 z_{23}^2(1 - z_{23}^3)z_{23}^4 \tag{4.16}$$

$$1 \leq z_{11}^1 + z_{11}^2 \leq 2 \tag{4.17}$$

$$1 \leq z_{21}^1 + z_{21}^2 + z_{21}^3 \leq 2 \tag{4.18}$$

$$z_{22}^1 + z_{22}^2 + z_{22}^3 \leq 2 \tag{4.19}$$

$$2 \leq z_{23}^1 + z_{23}^2 + z_{23}^3 \leq 3 \tag{4.20}$$

$$z_{23}^1 + z_{23}^2 + z_{23}^3 \leq 2 \tag{4.21}$$

where, $x_{ij} \geq 0, \quad i = 1,2$ and $j = 1,2,3$.

## 5. Result and discussion

To find the optimal solutions the problem is solved by Lingo package developed by Schrage [17] The optimal solutions are $x_{11} = 15.88888$, $x_{12} = 4.066849$, $x_{13} = 8.476099$, $x_{22} = 7.111205$, and rest of the decision variables are zero. The minimum cost of the objective function is 506.2093. The optimal value for multi-choice cost coefficients $x_{ij}$ to the objective function are $10, 15, 17, 20, 27, 30$ per hundred rupees respectively.

The introduction of binary variables is an important concept in multi-choice programming for selection of one choice from the set of multi-choice. To formulate the proposed model in this paper, the additional/ auxiliary constraints involve the binary variables. The number of binary variables for each choice or goal are dependent on the relation $\frac{\ln(p)}{\ln(2)}$, where $p$ is the number of choice or goals. Depending on the number of choice or goals, different models can be formulated for the proposed problem.

It is quite natural that requirement of a variety of medicines in hospitals is completely random and their timely availability to apply on patients is a big challenge. Generally medicines are available in different units at different sources and the requirements are also different in different hospitals. The primary objective of a transporter is to transport medicine timely from different sources to different hospitals such that total transportation cost is minimized. The problem for the transporter is complicated to some extent because there is a menu of choices of routes based on transportation cost. The model presented here provides an idea about how a transporter supplies medicines timely by minimizing total transportation cost by suitably choosing best routes based on transportation cost.

## 6. Conclusion

This paper develops an optimization model of transportation problem having uncertainties as multi-choice type and Logistic distribution. There are two uncertainties in the problem. Firstly, the cost coefficients of the objective function are multi-choice rather than of single choice. The second uncertainty involves in supply and demand which are all followed by Logistic distribution. Initially, the stochastic constraints are transformed into equivalent deterministic constraints by applying the stochastic programming. Then, a transformation technique selects only one choice from a set of multi-choice for each coefficient of the objective function and provides an optimal solution. This article explores three types of mathematical models of multi-choice transportation problem involving Logistic distribution in the demand, in the source and in both.

Although the model provides idea about how the random requirements of medicines in hospitals can be fulfilled by a transporter by suitably choosing parameters from sets of parameters, still it has some limitations. For example, during the entire transportation process if the requirement of a medicine arises suddenly then how can it be supplied? In emergency system of healthcare, time is the most precious component for better service. The model does not take it into consideration. For better applicability of the model, in future an extended model can be developed by considering the time while a transporter will take to supply the medicine. Obviously, the time a transporter will take will depend on various factors such as type of the carrier, choice of route, distances between sources and hospitals, etc. Certainly, in such case, the model will be more realistic and robust with higher grade of complicity. Furthermore, degree of emergency of different medicines are different. Thus choice of routes aiming at minimizing transportation cost is not acceptable always. Consequently, it is a big challenge that how urgently required medicines can be transported without cost minimization independently and the rest can be transported by fulfilling the transportation cost minimization objective.

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