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PRICE STRATEGIES AND SALESFORCE COMPENSATION DESIGN WITH OVERCONFIDENT SALES AGENT

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Abstract. Salesforce compensation and pricing decisions have invoked the interest of several academicians and practitioners for a long period of time. However, dilemma of whether the pricing decisions should be made by the firm or delegated to the sales agent, especially the overconfident agent, is still unexplored. This study tries to investigate the problems associated with this dilemma by conducting a thorough study of the scenario, it studies a supply chain that the rational manufacturer hiring an overconfident sales agent to sell its products, the agent might overestimate the demand, or underestimate the variability of the demand. These behaviors are characterized as ability-based and precision-based overconfidence respectively. The models are designed for centralized pricing and delegated pricing settings, and the sensitivity analysis are conducted. Moreover, comparative studies have also been conducted to highlight the impacts of the two types of overconfidence on the compensation decisions under different pricing strategies. It was found out that, the manufacturer favors centralized pricing, while the sales agent prefers delegated pricing. The final decisions of both sides deviate considerably from the rational scenario, overconfidence prompts the agent to exert more efforts, which ultimately enhances manufacturer's profits that the manufacturer should hire a more overconfident agent, while not guaranteeing a higher commission rate. Overconfidence leads to the decline of the agent's actual utility, and the loss amount increases with the overconfidence level. The influences of the both types of overconfidence are substitutable. Managerial insights are also provided for various scenarios and propositions along with numerical illustration of the finding.

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1. Introduction

Nowadays, majority of firms heavily rely on sales agents to sell their products to the customers. A vital dilemma faced by sales managers today is precisely how much compensation should be offered to the salesforce. Reportedly, for companies involved in B2B operations the compensation cost is about 40% of their total sales spending. In the U.S. pharmaceutical industry, manufacturers spend about 1.5 billion dollars a year in promoting pharmaceutical products, 45% of which is used to pay sales representatives for introducing products to customers (see [17]). As discussed by Zoltners *et al.* [48], in 2006, salesforce compensation by US firms alone amounted

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to a robust 800 billion dollars. Since this figure was three times of the incumbent advertisement costs hence, an extensively focused research had been dedicated to salesforce compensation design and its sub-disciplines, *e.g.* marketing, economics and operations. Additionally, price is also an important factor which affects manufacturers, dealers, customer and market future. Therefore, a rational pricing policy is imperative to maintain manufacturer's interest, mobilizing dealers' enthusiasm, customer attraction, winning against the competitors, and market development consolidation. An important issue which has attracted the interest of several scholars in both marketing and operations is, whether the pricing decisions should be made by the firm or delegated to the salesforce? There appears to be no unconditional answer to this question, because examples of both pricing policies can be found in marketing practice and theoretical recommendations (see [4, 15, 23, 31, 32]). A question and the main challenge to the manufacturer is dealing with the problem of designing salesforce compensation and setting sensible sales pricing strategies.

Recently, increased attention had been given to phenomena of salesforce compensation and pricing strategies. On the contrary, the previous models were based on complete rationality, where decision taken by the decision makers were aimed explicitly at utility maximization. But in reality, pride or arrogance could lead to decision makers making irrational decisions, resulting in overestimating one's competence or excessive precision in one's beliefs. Overconfidence is quiet common in our daily life, for example, the Royal Dutch Shell Group noticed that the newly hired geologists often make mistakes due to overconfidence in detecting oil, and hence designed a training program to improve the accuracy of the geologist's predictions. Eastman Kodak Company overestimated its technical ability and ignored future planning which lead to a decline in sales and eventual bankruptcy (see [39]). Decision-makers are liable to fall into their own trap of overconfidence (see [3, 28, 35, 38, 39]).

This paper addresses two aspects of overconfidence behavior, the ability-biased overconfidence and the precision-biased overconfidence. The ability-biased overconfidence is defined as the sales agent's overestimation of his selling ability, while the precision-biased overconfidence means underestimating the variability (standard deviation) of the stochastic market demand (overprecision). Such overconfidence behavior could seriously induce negative affect to the manufacturer's profit and ultimately the entire supply chain. Therefore it is imperative to account for the sales agent's overconfidence while making compensation and pricing decisions. This article investigates the impact of sales agent's overconfidence on the manufacturer as well as his personal self, when the market demand is stochastic and effort-dependent. Therefore, in this paper, salesforce compensation models were developed for two pricing strategies, *i.e.*, centralized pricing and delegated pricing settings respectively, while considering the degree of overconfidence based on both the overestimation on his selling ability and overprecision on the stochastic market demand's accuracy. Specifically, this study aims to address the following issues: (1) Which of the the two pricing strategies (centralized pricing or delegated pricing) is better from the perspective of the rational manufacturer and which one is beneficial to the overconfident agents? (2) How does each type of the sales agent's overconfidence influences the supply chain players' decisions? Should the manufacturer ignore the sales agent's overconfident behavior? (3) Could the sales agent's overconfidence generate more profit for manufacturer, and be beneficial or harmful to the sales agent himself? To address these questions, firstly, this study investigates the salesforce contract under centralized pricing and delegated pricing strategy respectively, followed by further analysis of the influence of two overconfidence behavioral aspects on the decision making and actual income of both sides followed by results comparison under centralized pricing and delegated pricing strategies.

The rest of the paper has been organized as follows. In the next section, an extensive review of literature has been done followed by problem formulation in Section 3. Section 4 analyzes the contract design under two pricing strategies, and how does the sales agent's overconfidence affects the manufacturer's compensation and pricing decision? In Section 5, the overall results have been compared under centralized pricing and delegated pricing strategies. In Section 6, a numerical study has been conducted along with discussion on managerial insights. Finally, this study concludes with general remarks and managerial implications in Section 7.

2. Literature review

There are two important dimensions that relate to this study: the literature on the salesforce compensation and pricing decisions, and the literature on overconfidence behavior.

The salesforce compensation and pricing problem has been extensively studied in the analytical marketing literature (see [2, 10, 18]), and the modeling framework had been used extensively in marketing for exploring the salesforce-firm relationship. Here it should be noted that these two factors have been jointly considered in the model. For the salesforce compensation models with effort-dependent demand, it usually assumes a stochastic sales response function of selling effort. This, together with the fact that the firm cannot observe the salesperson's selling effort, leads to the moral hazard problem. The basic model of the moral hazard problem is built on agency theory in economics. The classical references in agency theory include Holmstrom and Milgrom [19], Gonik [18] and Basu *et al.* [2], which have been investigated extensively in salesforce compensation (see [6–8, 13, 24, 25, 40]). Most of the research is associated with salesforce compensation in the area of marketing-operations interface. For example, Chen [7] studied how a firm can induce its salespeople to exert effort in a way that actually streamlines the demand process, such that demand uncertainty during lead time is reduced, which facilitates a firm's production planning. Chen [8] further compared Gonik's scheme (see [18]) with menus of linear contracts and studied how the market information possessed by the salesforce influences the firm's production planning decisions. Saghafian and Chao [40] studied the dependency of the operational decisions of production/inventory management and the design of salesforce incentives, and considered the problem of joint salesforce incentive design and inventory/production control under asymmetric market information. Dai and Chao [14] assumed that the sales agent's risk attitude is private, and they demonstrated that the firm should offer a higher commission rate contract to a salesperson with less risk adverse attitude. Lee and Yang [24, 25] employed a screening model to examine the compensation scheme problem involving competing players under asymmetric demand information. Here it is note worthy that although these studies only consider the pricing decision, and not the delegating pricing decision. Conversely, this paper makes a major contribution by studying compensation, and delegating pricing decisions simultaneously.

Different from the above earlier salesforce research, Mishra and Prasad [31] extra investigated the issue of delegating pricing to the salesforce versus centralized pricing for salesforce compensation, and found out that when the salesperson's private information can be revealed to the firm through contracting, centralized pricing performs at least as well as price delegation. Dai and Chao [15] examined how the asymmetric information on salesperson's risk aversion coefficient affects the firm's pricing decision and analyzed the impact of delegation on prices and incentives in presence of information asymmetry. The empirical results stated that both the firm and the agents prefer centralized pricing over delegated pricing when the agents are very risk-averse and place high value on their efforts. For salesforce management, should pricing decisions be made by the firm or delegated to the salesforce is also a sensitive issue, delegating pricing responsibility to the salesforce has been investigated extensively in marketing (see [4, 15, 23, 31]). Lal [23] showed that price delegation is appropriate subject to existence of information asymmetry. Bhardwaj [4] and Mishra and Prasad [32] examined the price delegation issue with competitive settings. The current study is different from these papers since it focuses on the decision of delegating pricing. Moreover, this study consideres the salesforce contract design for irrational salesforce with two types of overconfidence behavior.

The above mentioned studies have hardly addressed the issue of the decision on pricing and salesforce under incomplete rational, and ignoring the behavioral characteristics. Conversely, this study analyzes the sales agent's overconfidence, a phenomenon which is very prevalent among decision-makers. Overconfidence has been empirically shown to be persistent in one's decision-making processes under uncertain environment. Meanwhile, many people are studied as overconfident decision makers in behavioral economics (see [1, 9, 26, 37, 44, 46]). This overconfidence behavior definitely affects one's decisions. Russo and Schoemaker [39], Steen [42], and Moore and Healy [35] theorized the existence of overconfidence. There has been limited work in operation literature considering overconfidence effect, and it is not studied in the salesforce compensation.

For the overconfidence behavior in supply chains, Ren and Croson [38] performed two experiments assuming that underestimating the variance of demand causes orders to deviate from optimal in predictable ways, *i.e.*, the overconfidence on the belief of the demand. Moore and Healy [35] proposed three different ways of modeling overconfidence: (i) overestimating in one's actual performance, (ii) overplacement in one's performance relative to others, and (iii) excessive precision in one's beliefs. The first two stress on the overestimate of ability in different perspective, and the third kind means the overestimating of their forecast accuracy. Different kinds of overconfidence phenomenon have been extensively studied in the literature. Van den Steen [45] studied the mechanism of Bayesian-rational individual's overconfidence, which includes overestimation of the estimation precision. Lin *et al.* [27] claimed that the greater the forecast error, the higher would be the manager's overconfidence, and the higher the overestimation in firms' earnings. The study of behavior of the supply chain and operational context considering overconfidence is rare, Lu *et al.* [29] studied how green efforts impact supply chain performances in a market with an overconfident supplier and a rational retailer, and developed models under two types of overconfidence, *i.e.*, the overconfident supplier might overestimate product demand due to carbon-reduction green efforts or underestimate the variability of the stochastic demand. While researching the overconfidence behavior under the asymmetric information, the majority is the effects of overconfidence on incentive contract in a principal-agent framework. Moreover, there is a increasing volume of literature that takes the overconfidence into account. Russo and Schoemaker [39] and Busenitz and Barney [5] confirmed that overconfidence exists in principal-agent scenario, Keiber [21], Ludwig *et al.* [30], and De la Rosa [16] studied the wage contract design that provides the principal and the agent with the same level of overconfidence and different levels of overconfidence in the moral hazard framework respectively. Sandroni and Squintani [41] indicated that overconfidence may overturn fundamental relations between observable variables in perfect-competition asymmetric information insurance markets. In this paper, the source of asymmetric information comes from the sales agent's selling effort based on traditional principal-agent theory. From different papers mentioned above, this study considered a supply chain with an overconfident sales agent who keeps two aspects of overconfidence behavior *i.e.* the ability-biased overconfidence and the precision-biased overconfidence simultaneously. This study contributes to this line of research by investigating a salesforce contract design problem under two pricing strategies (centralized pricing and delegated pricing settings).

The main contributions of this paper are three-fold. Firstly, we investigated the compensation plan while considering two pricing strategies in this model: centralized pricing and delegated pricing followed by cross comparison. Secondly, the sales agent's two types of overconfidence behavior as the ability-biased overconfidence and the precision-biased overconfidence simultaneously are addressed, and further analyzed to gauge their influence decisions of supply chain players respectively. Finally, study model indicates that the manufacturer should not ignore the sales agent's overconfidence behavior, which is actually harmful to the sales agent himself, and it states that the manufacturer favors centralized pricing over delegated pricing with certain overconfidence level.

3. Problem formulation and model assumption

This study considers a supply chain in which a rational manufacturer (she) relies on an overconfident sales agent (he) to sell her products. Suppose that the sales quantity, or demand is stochastic and depends on the sales effort exerted by the sales agent and the selling price.

Table 1 lists the notation used in this paper.

In the base case, we adopt the following linear demand function for an unbiased cognitive sales agent (see [33, 34, 36, 47]), to model the sales quantity for the product in the following additive form:

$$X = a - bp + e + \theta, \tag{3.1}$$

where the sales parameter $a > 0$ is the market potential, $b > 0$ represents the price elasticity of demand. Here θ is a normally distributed random noise with mean 0 and variance σ^2. Rational individuals can correctly recognize

TABLE 1. Notations.

Parameters	
X	Market demand
a	Market potential
b	Sensitivity coefficient of demand on retail price (the price elasticity of demand)
e	The level of sales effort devoted by the sales agent
θ	Random variable of the market demand
σ^2	Standard deviation of the random variable θ
p	Product retail price
λ	The sales agent's ability-biased overconfidence factor (Factor of overestimation behavior of the sales agent)
η	The sales agent's precision-biased overconfidence factor (Factor of overprecision behavior of the sales agent)
X_0	The overconfident sales agent's subjective demand
b'	The effort cost parameter
α	The fixed salary
β	The commission rate
π_M	Profit function of the manufacturer
ρ	The risk aversion level
$-U_0$	The sales agent's reservation utility

the distribution of θ. Moreover, we assume that θ is sufficiently large such that the probability of being negative is negligible.

The sales agent has overconfident tendency, as it has superior information due to close contact to the customers and direct marketing through selling of products. Since the sales agent is an overconfident player, his overconfidence behavior is modeled in two different ways as ability-biased overconfidence and precision-biased overconfidence, *i.e.*, the overconfident sales agent may overestimate their selling abilities (λ) and overconfident about the stochastic market demand accurate (η). The demand function in the overconfident sales agent's mind (the overestimated demand) can be expressed as:

$$X_0 = a - bp + \mu(\lambda)e + \omega(\eta)\theta, \tag{3.2}$$

where $\mu(\lambda) \geq 1$, $\mu'(\lambda) \geq 0$, $\mu(0) = 1$, and $0 < \omega(\eta) \leq 1$, $\omega'(\eta) \leq 0$, $\omega(0) = 1$. Here $\omega(\eta)\theta$ is normally distributed with mean 0 and variance $\omega^2(\eta)\sigma^2$, $\omega^2(\eta)\sigma^2 \leq \sigma^2$ holds, λ and η represent the sales agent's ability-biased and precision-biased overconfidence level respectively, larger λ implies more overconfidence, and large η also implies more overconfidence, *i.e.*, the sales agent is too confident about his precision, thus $\lambda = \eta = 0$ meaning that the sales agent is completely rational.

The cost incurred from exerting the effort, which is assumed to be $C(e) = b'e^2/2$, which is increasing convex in e, and b' is the effort cost parameter.

The rational and risk-neutral manufacturer aims to maximize her expected profit by designing the compensation contract for the overconfident sales agent. Since the manufacturer cannot directly observe the agent's effort level, the compensation to the sales agent must be based on the realized sales value. We consider a linear compensation contract (α, β) where the agent is compensated on his subjective total sales pX_0 using base salary α and commission rate β, while the demand in the overconfident sales agent's mind is X_0. We restrict our attention to the class of linear contract as $s(X_0) = \alpha + \beta pX_0$.

Although linear contracts are suboptimal, they are widely accepted to be a good measure to tackle the incentive provision problems and normally adopted by practitioners as well as in the academic literatures (see [8, 19, 22, 23]).

The manufacturer is risk-neutral and tends to maximize the expected profit. Therefore, her expected profit function can be expressed as follows:

$$\pi_M = E[pX - s(X_0)]. \tag{3.3}$$

The overconfident sales agent's net profit is given by

$$\pi_S = s(X_0) - C(e) = \alpha + \beta p[a - bp + \mu(\lambda)e + \omega(\eta)\theta] - b'e^2/2, \tag{3.4}$$

The sales agent is risk-averse and tends to maximize expected utility. Without losing generality, we adopt the widely used negative exponential utility function in the agency literature, *i.e.*,

$$U = -\exp(-\rho\pi_S),$$

where π_S is the net income of the agent, and $\rho > 0$ is a constant measuring risk tolerance with larger ρ representing more risk aversion. Here $-U_0$ denotes the sales agent's reservation utility representing the best outside opportunity for the sales agent, and the corresponding certainty equivalence is $\underline{\pi} = -\ln U_0/\rho$. Thus, for the sales agent to accept a contract, the contract has to maximize his expected utility among his choices and the expected utility value has to be at least $-U_0$, *i.e.*, $E[-\exp(-\rho\pi_S)] \geq -U_0$, Hence the expected utility of the sales agent modified by the certainty equivalence (CE) principle is given by

$$\text{(IR)} \quad \text{CE} = \alpha + \beta p[a - bp + \mu(\lambda)e] - \rho\sigma^2\beta^2p^2\omega^2(\eta)/2 - b'e^2/2, \tag{3.5}$$

the Individual Rationality (IR) constraint (3.5) ensures the participation of the sales agent.

The above assumptions, including the linear payment structure, the negative exponential utility, and the normally distributed randomness, together referred to as the LEN (Linear-Exponential-Normal) assumption, are commonly adopted in the agency literature for tractability (see [14, 22, 40]). The above assumptions are common knowledge to all the parties concerned.

We assume $2bb' > \max(1, [\mu^2(\lambda) - b'\rho\sigma^2\omega^2(\eta)]\beta)$ holds, the assumption ensures that when price sensitivity b is low, the cost of sales effort should be high enough (high b') to prevent the manufacturer from setting an infinite effort level e and making infinite profit (see [20, 36]).

The manufacturer's problem is to design the compensation contract for the overconfident sales agent, when the prices are determined by herself or delegated to the overconfident sales agent, that maximize her expected total profit. She is also interested in comparing her expected maximum profits under the two pricing strategies. We first analyzed the centralized pricing strategy and salesforce contract with overconfident sales agent, and then investigated delegated pricing scenario, further we analyzed the impact of two aspects of overconfidence behavior on the decision and income of both sides. Finally, we compared the results under centralized pricing and delegated pricing strategies. Throughout the paper, we use increasing and decreasing in non-strict sense, *i.e.*, they represent non-decreasing and non-increasing respectively. For the sake of simplicity, the traditional factor $A = b'\rho\sigma^2$ is defined as the product of the effort cost parameter b', the risk aversion level ρ and the actual variability of the stochastic demand σ^2. We use the following notation: We use $E[\cdot]$ to represent the mathematical expectation. The subscripts "R", "C", "D", "M" and "S", respectively, denote the results of corresponding to Rational case, Centralized pricing, Delegation pricing, the Manufacturer and the Sales agent respectively, and the superscript "*" denotes the optimal cases.

4. The Contract for different pricing strategies

4.1. Centralized pricing

This subsection analyzes the contract design for centralized pricing. The sequence of events for centralized pricing is as follows: (1) the manufacturer offers the contract for the overconfident sales agent; (2) the sales agent decides whether or not to accept the contract; (3) if the sales agent accepts, it then exerts a level of unobservable selling effort, (4) finally, the manufacturer determines the price and the agent exerts the selling effort under the signed contract.

Recall that the overconfident sales agent's problem is to maximize his expected utility function

$$E[-\exp(-\rho[\alpha+\beta p(a-bp+\mu(\lambda)e+\omega(\eta)\theta)-b'e^2/2])]. \tag{4.1}$$

By the certainty equivalence principle, the agent's Certainty Equivalence corresponding to the expected utility is given by

$$\mathrm{CE}_C=\alpha+\beta p[a-bp+\mu(\lambda)e]-b'e^2/2-\rho\sigma^2\beta^2p^2\omega^2(\eta)/2. \tag{4.2}$$

Similar to a principal-agent framework, the sales agent acts as the follower, and the manufacturer as a leader designs the incentive contract and sets the product price to maximize her expected utility while satisfying the sales agent's IR and Incentive Compatibility (IC) constraints. The manufacturer's decision problem can be expressed as follows:

$$\max_{\alpha,\beta,p} E[pX-s(X_0)], \tag{4.3}$$

$$\text{subject to(IR)}\mathrm{CE}_C=\alpha+\beta p[a-bp+\mu(\lambda)e]-b'e^2/2-\rho\sigma^2\beta^2p^2\omega^2(\eta)/2\geq\underline{\pi},$$

$$\text{(IC) } e^*=\arg\max_{e\geq 0}\mathrm{CE}_C. \tag{4.4}$$

The IR constraint ensures the participation of the sales agent, because of exceeding the reservation profit. Equation (4.4) is IC constraint, assuring that the sales agent does not pretend to choose the other effort level.

The sales agent aims to maximize his expected utility, and his IC constraint can be replaced by first-best effort level as:

$$e^*=\frac{\mu(\lambda)p\beta}{b'}\cdot \tag{4.5}$$

Substituting this first-best effort level into equation (4.2), we can obtain

$$\mathrm{CE}_C=\alpha+\beta p(a-bp)+\frac{\mu^2(\lambda)-A\omega^2(\eta)}{2b'}p^2\beta^2. \tag{4.6}$$

Then, we have the following theorem, which characterizes the optimal contract for centralized pricing strategy.

Theorem 4.1. *For centralized pricing strategy, facing the overconfident sales agent, the optimal contract* (α_C^*,β_C^*) *and the optimal central selling price* p_C^* *are given by*

$$\beta_C^*=\frac{\mu(\lambda)}{A\omega^2(\eta)+\mu^2(\lambda)}, \tag{4.7}$$

$$\alpha_C^*=\underline{\pi}+\frac{a^2b'[b\mu(\lambda)(A\omega^2(\eta)-\mu^2(\lambda))-2bb'(A\omega^2(\eta)+\mu^2(\lambda))+2\mu^2(\lambda)]}{[2bb'(A\omega^2(\eta)+\mu^2(\lambda))-\mu^2(\lambda)]^2}, \tag{4.8}$$

$$p_C^*=\frac{a}{2b}\frac{2bb'[A\omega^2(\eta)+\mu^2(\lambda)]}{2bb'[A\omega^2(\eta)+\mu^2(\lambda)]-\mu^2(\lambda)}, \tag{4.9}$$

and the optimal sales effort strategy e_C^* *can be easily derived by*

$$e_C^*=\frac{\mu(\lambda)p_C^*\beta_C^*}{b'}=\frac{a\mu^2(\lambda)}{2bb'[A\omega^2(\eta)+\mu^2(\lambda)]-\mu^2(\lambda)}. \tag{4.10}$$

The manufacturer's optimal expected profit is given by

$$E_C^*(\pi_M)=\frac{a^2}{4b}\left(1+\frac{\mu^2(\lambda)}{2bb'[A\omega^2(\eta)+\mu^2(\lambda)]-\mu^2(\lambda)}\right)-\underline{\pi}, \tag{4.11}$$

where $A=b'\rho\sigma^2$ *and* $2bb'-1>0$.

The proof of Theorem 4.1 and the other propositions are given in the Appendix.

Theorem 4.1 states that, the optimal contract (α_C^*, β_C^*), the optimal central selling price p_C^*, the optimal sales effort strategy e_C^* and the manufacturer's optimal expected profit $E_C^*(\pi_M)$ are all dependent on both λ and η.

From Theorem 4.1, we deduce the decisions of centralized pricing strategy under completely rational case by considering the special case when $\lambda = \eta = 0$ as follows.

Observation 4.2. *If the sales agent is completely rational ($\lambda = \eta = 0$), the optimal central selling price p_{CR}^*, the commission rate β_{CR}^* and the effort e_{CR}^* constitute the unique Bayesian Nash equilibrium (The subscripts "R" denotes Rational):*

$$\beta_{\mathrm{CR}}^* = \frac{1}{1+A}, \quad p_{\mathrm{CR}}^* = \frac{ab'(1+A)}{2bb'(1+A)-1}, \quad e_{\mathrm{CR}}^* = \frac{a}{2bb'(1+A)-1}, \tag{4.12}$$

correspondingly, the optimal base salary is

$$\alpha_{\mathrm{CR}}^* = \underline{\pi} + \frac{a^2 b'[b(A-1) - 2bb'(A+1) + 1]}{2[2bb'(A+1)-1]^2}, \tag{4.13}$$

the manufacturer's optimal expected profit is

$$E_{\mathrm{CR}}^*(\pi_M) = \frac{a^2}{4b}\left(1 + \frac{1}{2bb'(A+1)-1}\right) - \underline{\pi}, \tag{4.14}$$

where $A = b'\rho\sigma^2$ and $2bb' - 1 > 0$.

Conducting sensitivity analysis of the ability-biased overconfidence and precision-biased overconfidence under centralized pricing strategy, we have the following results.

Proposition 4.3. *For centralized pricing strategy, when $2bb' - 1 > 0$,*

(i) The commission rate β_C^* is increasing in η, and if $A > \frac{\mu^2(\lambda)}{\omega^2(\eta)}$, β_C^* is increasing in λ, otherwise, β_C^* is decreasing in λ; and
(ii) $E_C^*(\pi_M)$, e_C^* and p_C^* are all jointly increasing in λ and η; and
(iii) $E_C^*(\pi_M)$, β_C^*, e_C^* and p_C^* are all decreasing in A.

Proposition 4.3 states that, the sales effort e_C^*, the manufacturer's optimal expected profit $E_C^*(\pi_M)$, the commission rate β_C^*, and the central selling price p_C^* are all decreasing in A, *i.e.*, they are all jointly decreasing in the cost parameter b', risk aversion level ρ and the sales quantity variance σ^2. It implies that strong risk aversion, high sales quantity variance and high effort valuation hinder the sales agent from making right pricing and effort decisions to maximize his profit, for centralized pricing strategy. It also hinders the manufacturer from increasing her profitability. Meanwhile, the risk-averse coefficient, sales quantity variance and the effort valuation could be interpreted as three substitutable factors for the manufacturer and sales agent's preference.

Proposition 4.3 also indicates that for centralized pricing strategy, the sales agent's effort, the central selling price and the manufacturer's optimal expected profit are all jointly increasing in the two types of overconfidence level, *i.e.*, the decisions of both sides are highly deviate from the rational scenario, and the influence of the two types of overconfidence on the decisions of both sides are substitutable. The commission rate is increasing in the sales agent's precision-based overconfidence level, while it is not always increasing in the sales agent's ability-biased overconfidence level, and its monotonicity with respect to the ability-biased overconfidence level depends on the degree of the two aspects of overconfidence.

The manufacturer's optimal expected profit is jointly increasing in the two types of overconfidence level, it also implies that the manufacturer should hire a more overconfident sales agent, while a higher commission rate is not guaranteed.

From Proposition 4.3 and Observation 4.2, we have the following Observation 4.4.

Observation 4.4. *$e_C^* \geq e_{\mathrm{CR}}^*$, $p_C^* \geq p_{\mathrm{CR}}^*$ and $E_C^*(\pi_M) \geq E_{\mathrm{CR}}^*(\pi_M)$ hold, if $A = b'\rho\sigma^2$ is sufficiently large, $\beta_C^* \geq \beta_{\mathrm{CR}}^*$, otherwise, $\beta_C^* < \beta_{\mathrm{CR}}^*$ holds.*

From Observation 4.4, we know that the agent's sales effort, the manufacturer's expected profit, and the centralized pricing decision are all more than the rational scenario, while the relationship between the commission rates is dependent on $A = b'\rho\sigma^2$, or two types of the sales agent's overconfidence level. Observation 4.4 also implies that the manufacturer prefers to hire the overconfident sales agent under centralized pricing strategy.

We now derive, discuss and contrast the impacts of the ability-biased overconfidence and precision-biased overconfidence on the players' actual decisions and performance.

Proposition 4.5. *For centralized pricing, if the manufacturer ignores the sales agent's overconfidence behavior, she will get less profit; when A is sufficiently large, the overconfident sales agent's actual profit will decrease with his degree of two types of overconfidence.*

Proposition 4.5 states that, for centralized pricing, the manufacturer should not ignore the sales agent's overconfidence behavior while pricing and designing the salesforce compensation. Overconfidence behavior leads to the loss of actual expected profit of the sales agent, and the loss amount increases with the increasing overconfidence level, which implies the overconfidence behavior is actually harmful to the sales agent himself.

4.2. Price delegation

This subsection proceed to investigate the case when the price setting responsibility is delegated to the sales agent, the sequence of events for delegating pricing is as follows: (1) the manufacturer offers the compensation contract for the sales agent; (2) the sales agent decides whether or not to participate and if so, which contract to sign based on his overconfidence; (3) finally, the agent determines the price and then exerts the selling effort. The manufacturer's decision problem can be expressed as follows:

$$\max_{\alpha,\beta} E[pX - s(X_0)], \tag{4.15}$$

$$\text{subject to (IR) } \mathrm{CE}_D = \alpha + \beta p[a - bp + \mu(\lambda)e] - b'e^2/2 - \rho\sigma^2\beta^2p^2\omega^2(\eta)/2 \geq \underline{\pi},$$

$$\text{(IC) } \{e^*, p^*\} = \arg\max_{e,p\geq 0} \mathrm{CE}_D. \tag{4.16}$$

By solving the above model, we have the following theorem, which characterizes the optimal contract for delegated pricing strategy.

Theorem 4.6. *For delegated pricing strategy, when $\max(1, [\mu^2(\lambda) - b'\rho\sigma^2\omega^2(\eta)]\beta) < 2bb'$, the optimal contract (α_D^*, β_D^*) and the delegated selling price p_D^* are given by*

$$\beta_D^* = \frac{2bb'\mu(\lambda)}{2bb'[A\omega^2(\eta) + \mu^2(\lambda)] + [A\omega^2(\eta) - \mu^2(\lambda)]^2 + \mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]}, \tag{4.17}$$

$$\alpha_D^* = \underline{\pi} - \frac{a^2b'}{2}\frac{\mu(\lambda)}{2bb'[A\omega^2(\eta) + \mu^2(\lambda)] + [A\omega^2(\eta) - \mu^2(\lambda)]^2 + 2\mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]}, \tag{4.18}$$

$$p_D^* = \frac{a}{2b}\frac{2bb'[A\omega^2(\eta) + \mu^2(\lambda)] + [A\omega^2(\eta) - \mu^2(\lambda)]^2 + \mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]}{2bb'[A\omega^2(\eta) + \mu^2(\lambda)] + [A\omega^2(\eta) - \mu^2(\lambda)]^2 + 2\mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]}. \tag{4.19}$$

And other results can be easily derived

$$e_D^* = \frac{a\mu^2(\lambda)}{2bb'[A\omega^2(\eta) + \mu^2(\lambda)] + [A\omega^2(\eta) - \mu^2(\lambda)]^2 + 2\mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]}, \tag{4.20}$$

$$E_D^*(\pi_M) = -\underline{\pi} + \frac{a^2}{4b}\left(1 + \frac{\mu^2(\lambda)}{2bb'[A\omega^2(\eta) + \mu^2(\lambda)] + [A\omega^2(\eta) - \mu^2(\lambda)]^2 + \mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]}\right). \tag{4.21}$$

where $2bb' - 1 > 0$ and $A = b'\rho\sigma^2$.

We deduce the decisions of delegated pricing strategy under completely rational case by considering the special case when $\lambda = \eta = 0$ as follows.

Observation 4.7. *If the sales agent is rational ($\lambda = \eta = 0$), the delegated selling price p^*_{DR} and the commission rate β^*_{DR} as well as the sales effort e^*_{DR} constitute the unique Bayesian Nash equilibrium (The subscripts "R" denotes Rational):*

$$\beta^*_{\mathrm{DR}} = \frac{2bb'}{2bb'(A+1) + A(A-1)}, \tag{4.22}$$

$$p^*_{\mathrm{DR}} = \frac{a}{2b}\frac{2bb'(A+1) + A(A-1)}{2bb'(A+1) + A^2 - 1}, \tag{4.23}$$

$$e^*_{\mathrm{DR}} = \frac{a}{2bb'(A+1) + A^2 - 1}, \tag{4.24}$$

the manufacturer's optimal expected profit is

$$E^*_{\mathrm{DR}}(\pi_M) = \frac{a^2}{4b}\left(1 + \frac{1}{2bb'(A+1) + A^2 - 1}\right) - \underline{\pi}. \tag{4.25}$$

where $A = b'\rho\sigma^2$ and $2bb' - 1 > 0$.

Conducting sensitivity analysis of the ability-biased overconfidence and precision-biased overconfidence under delegated pricing strategy, we have the following proposition.

Proposition 4.8. *For delegated pricing, when $2bb' - 1 > 0$, we have*

(i) If A is sufficiently large, the commission rate β^*_D is jointly increasing in λ and η, while β^*_D is decreasing in A, and vice versa.
(ii) If A is sufficiently large, both e^*_D and $E^*_D(\pi_M)$ are jointly increasing in λ and η, while are decreasing in A, and vice versa.
(iii) If A is sufficiently large or sufficiently small, the delegated selling price p^*_D is increasing in η, while decreasing in A; if A is sufficiently small, p^*_D is increasing in λ, and vice versa.

Proposition 4.8 indicates that, for delegated pricing strategy, if A is sufficiently large, the sales effort e^*_D, the manufacturer's optimal expected profit $E^*_D(\pi_M)$, the commission rate β^*_D, and the central selling price p^*_D are all decreasing in A, *i.e.*, they are all jointly decreasing in the cost parameter b', risk aversion level ρ and the sales quantity variance σ^2, and *vice versa*.

If A is sufficiently large or sufficiently small, the delegated selling price is increasing in the sales agent's precision-based overconfidence level, while its monotonicity with respect to the ability-biased overconfidence level is not guaranteed.

When A is sufficiently large, the manufacturer's optimal expected profit is jointly increasing in the two types of overconfidence level, which implies that the manufacturer should hire a more overconfident sales agent, for delegated pricing strategy.

From Proposition 4.8 and Observation 4.7, we have the following Observation 4.9.

Observation 4.9. *We have the following inequalities hold: if A is sufficiently large, $\beta^*_D \geq \beta^*_{\mathrm{DR}}$, $e^*_D \geq e^*_{\mathrm{DR}}$ and $E^*_D(\pi_M) \geq E^*_{\mathrm{DR}}(\pi_M)$ hold.*

The proofs are obvious and are omitted.

We now derive, discuss and contrast the impacts of the ability-biased overconfidence and precision-biased overconfidence on the players' actual decisions and performance under delegated pricing strategy.

Proposition 4.10. *For delegated pricing, if the manufacturer ignores the sales agent's overconfidence behavior, she will get less expected profit; if A is sufficiently large, the overconfident sales agent's actual profit will decrease with his degree of overconfidence.*

Proposition 4.10 implies that for delegated pricing, the manufacturer designs the salesforce compensation should consider the sales agent's two types of overconfidence behavior. Overconfidence behaviors are actually harmful to the sales agent.

5. Comparison

An important issue faced by the rational manufacturer is, should pricing decisions be made by herself or delegated to the overconfidence sales agent? Both pricing strategies are valid when $2bb' > 1$ holds. In this section, we proceed to compare the optimal results derived in Sections 4.1 and 4.2. We mainly compare the equilibrium values among the two pricing strategies with respect to the optimal commission rate, sales agent's optimal effort, optimal pricing strategies and manufacturer's expected profit. Based on the equilibrium results in Section 4, the following propositions can be derived.

We start with comparing the commission rate under centralized pricing and delegated pricing, we obtain the following proposition.

Proposition 5.1. *Comparing the commission rates under centralized pricing and delegated pricing strategies, when $2bb' > 1$, we have, if the sales agent's two aspects of overconfidence satisfies $A > \frac{\mu^2(\lambda)}{\omega^2(\eta)}$ or $0 < A < \frac{\mu^2(\lambda)-\mu(\lambda)}{\omega^2(\eta)}$, $\beta_C^* > \beta_D^*$; otherwise, $\beta_C^* \leq \beta_D^*$.*

Since

$$\frac{1}{\beta_D^*} - \frac{1}{\beta_C^*} = \frac{[A\omega^2(\eta) - \mu^2(\lambda) + \mu(\lambda)][A\omega^2(\eta) - \mu^2(\lambda)]}{2bb'\mu(\lambda)},$$

when $A\omega^2(\eta) - \mu^2(\lambda) > 0$ or $A\omega^2(\eta) - \mu^2(\lambda) + \mu(\lambda) < 0$, $\frac{1}{\beta_D^*} > \frac{1}{\beta_C^*}$, *i.e.*, $\beta_C^* > \beta_D^*$ holds, and *vice versa*.

From the proof of Proposition 5.1, $A\omega^2(\eta) - \mu^2(\lambda) > 0$ implies $0 \leq \lambda \leq \mu^{-1}(\sqrt{A}\omega(\eta))$ or $0 \leq \eta \leq \omega^{-1}(\mu(\lambda)/\sqrt{A})$ holds, which implies that the sales agent's two aspects of overconfidence have joint impacts on the commission rates.

Proposition 5.1 states that, given the traditional factor $A = b'\rho\sigma^2$ (the product of the effort valuation b', the risk aversion level ρ and the actual variability of the stochastic demand σ^2), which pricing strategy leads to a higher commission rate depends on the extent of the agent's overconfidence, and two types of overconfidence have joint impacts. When the sales agent is overconfidence, *i.e.*, he overestimates his selling ability, or underestimates the variability of the stochastic market demand, price delegation leads to a lower commission rate than centralized pricing.

Proposition 5.2. *Comparing the selling prices under centralized and delegated pricing, given A, when $2bb' > 1$, if the sales agent's two aspects of overconfidence satisfies $0 \leq \eta \leq \omega^{-1}\left(\sqrt{\frac{\mu^2(\lambda)-\mu(\lambda)}{A}}\right)$ or $0 \leq \lambda \leq \mu^{-1}\left(\frac{1+\sqrt{1+4A\omega^2(\eta)}}{2}\right)$, the centralized price is larger than the delegated price, i.e., $p_C^* \geq p_D^*$, otherwise, $p_C^* < p_D^*$.*

Since

$$p_C^* - p_D^* * = \frac{a}{2b}\frac{\mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda) + \mu(\lambda)]}{2bb'[A\omega^2(\eta) + \mu^2(\lambda)] - \mu^2(\lambda)} * \frac{(2bb'+1)A\omega^2(\eta) + (2bb'-1)\mu^2(\lambda)}{2bb'[A\omega^2(\eta) + \mu^2(\lambda)] + [A\omega^2(\eta) - \mu^2(\lambda)]^2 + 2\mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]}.$$

From the proof of Proposition 5.1, we can obtain the results.

Proposition 5.2 states that, if A is sufficiently large, the manufacturer tends to price higher than the agent, and this might partially due to the two aspects of overconfidence behavior.

Proposition 5.3. *Comparing the sales effort under centralized and delegated pricing, at the same level of overconfidence, when* $2bb' > 1$*, we have* $e^*_C > e^*_D$ *holds.*

By equations (4.10) and (4.20), we have

$$\frac{1}{e^*_D} - \frac{1}{e^*_C} = \frac{[A\omega^2(\eta) - \mu^2(\lambda) + \mu(\lambda)]^2}{a\mu^2(\lambda)},$$

which implies that the sales agent exerts more effort under centralized pricing.

Proposition 5.3 states that, it is beneficial for the sales agent to adopt delegated pricing as he can exert less effort.

Proposition 5.4. *Comparing the sales agent's actual expected utility under centralized and delegated pricing, at the same level of overconfidence, we have* $\mathrm{CE}_D(actual) \geq \mathrm{CE}_C(actual)$*.*

From the proof of Theorems 4.1 and 4.6, we have $\beta^*_C p^*_C = b' e^*_C/\mu(\lambda)$, and $\beta^*_D p^*_D = b' e^*_D/\mu(\lambda)$ hold, by Propositions 5.2 and 5.4, we can obtain

$$\triangle_C = \frac{b'^2 e^{*2}_C}{\mu^2(\lambda)} \left(\frac{\mu(\lambda)[\mu(\lambda) - 1]}{b'} + \frac{1 - \omega^2(\eta)}{2b'} A \right),$$

$$\triangle_D = \frac{b'^2 e^{*2}_D}{\mu^2(\lambda)} \left(\frac{\mu(\lambda)[\mu(\lambda) - 1]}{b'} + \frac{1 - \omega^2(\eta)}{2b'} A \right).$$

From Proposition 5.3, we know $e^*_C \geq e^*_D$, which implies $\triangle_C \geq \triangle_D$ holds. As the expected profit in overconfident sales agent's mind is $\underline{\pi}$ under centralized and delegated pricing, we can obtain $\mathrm{CE}_D(actual) \geq \mathrm{CE}_C(actual)$ holds.

Proposition 5.4 states that, it is actually beneficial for the sales agent to apply delegated pricing, his actual expected utility is higher, while exerting less effort (see Prop. 5.3).

Proposition 5.5. *Comparing the manufacturer's expected profit under centralized and delegated pricing, at the same level of overconfidence, we have* $E^*_C(\pi_M) \geq E^*_D(\pi_M)$*.*

By equations (4.11) and (4.21), we have

$$\frac{1}{[E^*_D(\pi_M) + \underline{\pi}]\frac{4b}{a^2} - 1} - \frac{1}{[E^*_C(\pi_M) + \underline{\pi}]\frac{4b}{a^2} - 1} = \frac{[A\omega^2(\eta) - \mu^2(\lambda) + \mu(\lambda)]^2}{\mu^2(\lambda)} > 0,$$

which implies that the manufacturer 's expected profit is higher under centralized pricing.

Proposition 5.5 states that, at the same level of overconfidence, it is beneficial for the manufacturer to adopt centralized pricing.

From Propositions 5.3–5.5, we can conclude that at the same level of overconfidence, the manufacturer preferences centralized pricing for higher expected profit, while the sales agent favors delegated selling strategies, this might because that the agent who exerts less sales effort, on the contrary, he receives higher actual expected utility relative to centralized pricing.

6. Numerical examples

In this section, we conduct numerical studies to complement our analytical findings in the previous sections. We implement the following base parameter set: $a = 7$, $b = 1$, $\underline{\pi} = 0$, $b' = 1$, $\rho = 1$, $\sigma^2 = 2$, thus $A = 2$. In addition, $\mu(\lambda) = 1 - \lambda$, $\omega(\eta) = 1 - \eta$. Based on our analytical results in previous sections, the optimal decisions under two pricing scenarios can be calculated out.

From Theorem 4.1, for centralized pricing strategy, from equations (4.7)–(4.11), we can obtain the optimal contract as

$$\alpha_C^* = \frac{49(1+\lambda)}{2}\frac{2(1+\lambda)(1-\eta)^2-(1+\lambda)^3-4(1-\eta)^2}{[4(1-\eta)^2+(1+\lambda)^2]^2}, \quad \beta_C^* = \frac{1+\lambda}{2(1-\eta)^2+(1+\lambda)^2}.$$

The optimal price and effort are given by

$$p_C^* = \frac{14(1-\eta)^2+7(1+\lambda)^2}{4(1-\eta)^2+(1+\lambda)^2]}, \quad e_C^* = \frac{7(1+\lambda)^2}{4(1-\eta)^2+(1+\lambda)^2},$$

and the manufacturer's optimal expected profit is

$$E_C^*(\pi_M) = \frac{49}{4}\left(1+\frac{(1+\lambda)^2}{4(1-\eta)^2+(1+\lambda)^2}\right).$$

From equations (4.12)–(4.14), when the sales agent is rational, we obtain $e_{\mathrm{CR}}^* = 1.4$, the optimal contract is $\alpha_{\mathrm{CR}}^* = -2.94$, $\beta_{\mathrm{CR}}^* = 0.33$, the optimal price is $p_{\mathrm{CR}}^* = 4.2$, the manufacturer's corresponding expected profit is $E_{\mathrm{CR}}^*(\pi_M) = 14.7$.

For delegated pricing strategy, from Theorem 4.6, we obtain the optimal effort as

$$e_D^* = \frac{7(1+\lambda)^2}{2H+[2(1-\eta)^2-(1+\lambda)^2]^2+2(1+\lambda)[2(1-\eta)^2-(1+\lambda)^2]},$$

the optimal of contract are given by

$$\alpha_D^* = -\frac{49}{2}\frac{(1+\lambda)}{2H+[2(1-\eta)^2-(1+\lambda)^2]^2+2(1+\lambda)[2(1-\eta)^2-(1+\lambda)^2]},$$
$$\beta_D^* = \frac{2(1+\lambda)}{2H+[2(1-\eta)^2-(1+\lambda)^2]^2+(1+\lambda)[2(1-\eta)^2-(1+\lambda)^2]},$$

and the optimal price is given by

$$p_D^* = \frac{7}{2}\frac{2H+[2(1-\eta)^2-(1+\lambda)^2]^2+(1+\lambda)[2(1-\eta)^2-(1+\lambda)^2]}{2H+[2(1-\eta)^2-(1+\lambda)^2]^2+2(1+\lambda)[2(1-\eta)^2-(1+\lambda)^2]},$$

and the manufacturer's optimal expected profit is

$$E_D^*(\pi_M) = \frac{49}{2}\left(1+\frac{(1+\lambda)^2}{2H+[2(1-\eta)^2-(1+\lambda)^2]^2+2(1+\lambda)[2(1-\eta)^2-(1+\lambda)^2]}\right).$$

where $H = 2(1-\eta)^2+(1+\lambda)^2$

From Observation 4.7, when the sales agent is rational, we have

$$\beta_{\mathrm{DR}}^* = 0.25, \quad p_{\mathrm{DR}}^* = 3.11, \quad e_{\mathrm{DR}}^* = 0.78, \quad E_{\mathrm{DR}}^*(\pi_M) = 13.6.$$

To examine the impact of sales agent's ability-biased overconfidence level (λ) and the precision-biased overconfidence level (η) on the optimal commission rate, the sales agent's optimal effort, optimal pricing strategies and manufacturer's expected profit. Other parameters are kept constant, Figures 1–4 illustrates they change in centralized pricing and delegated pricing respectively.

The commission rate under centralized pricing and delegated pricing strategies are shown in Figure 1, we verify that the optimal commission rate under centralized pricing and delegated pricing strategies both grow with the sales agent's degree of ability-biased and precision-biased overconfidence. The commission rate under centralized pricing maybe less than the delegated scenario.

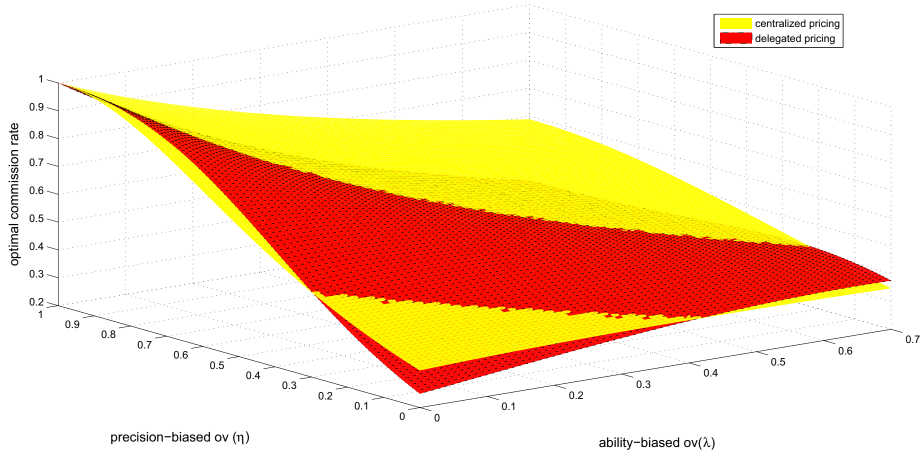

FIGURE 1. The impact of the sales agent's degree of ability-biased and precision-biased overconfidence on the commission rate.

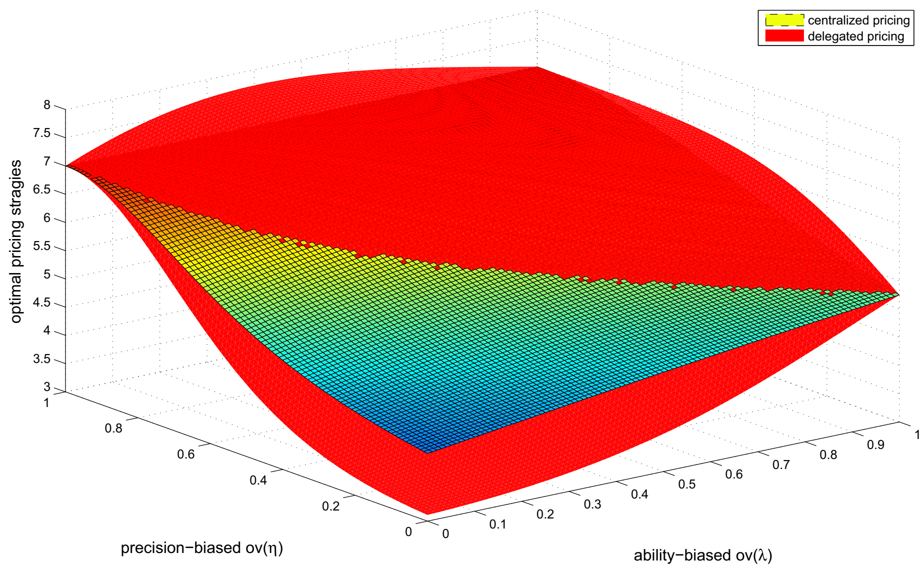

FIGURE 2. The impact of the sales agent's degree of ability-biased and precision-biased overconfidence on the price.

The pricing strategies are shown in Figure 2, we verify that the central selling price and delegated selling price both grow with the sales agent's degree of ability-biased and precision-biased overconfidence. Moreover, the central selling price is not always larger than delegated selling price regardless of the type of the overconfidence.

The sales agent's optimal effort level under centralized pricing and delegated pricing strategies are shown in Figure 3, we verify that the manufacturer's expected profits under centralized pricing and delegated pricing strategies both grow with the sales agent's degree of ability-biased and precision-biased overconfidence. Moreover, the effort level under centralized pricing strategy is always larger than delegated scenario.

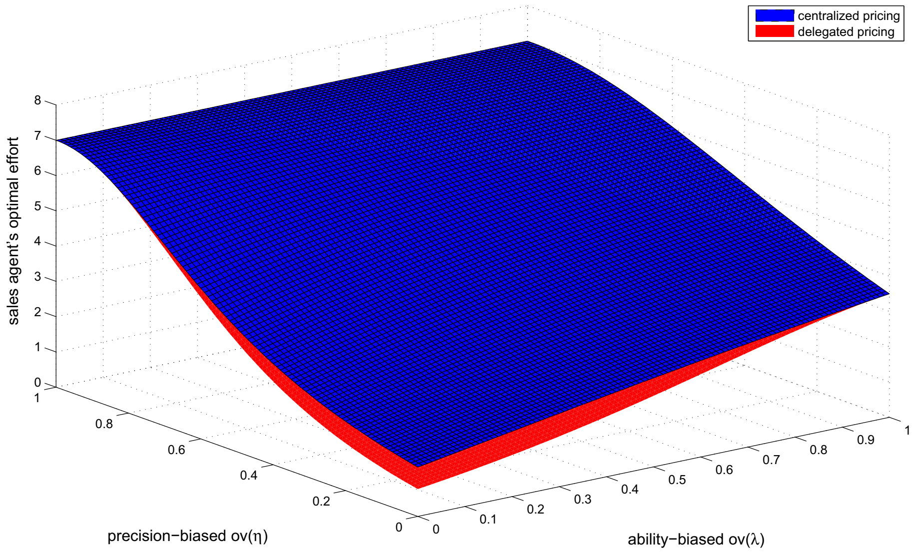

FIGURE 3. The impact of the sales agent's degree of ability-biased and precision-biased overconfidence on his optimal effort.

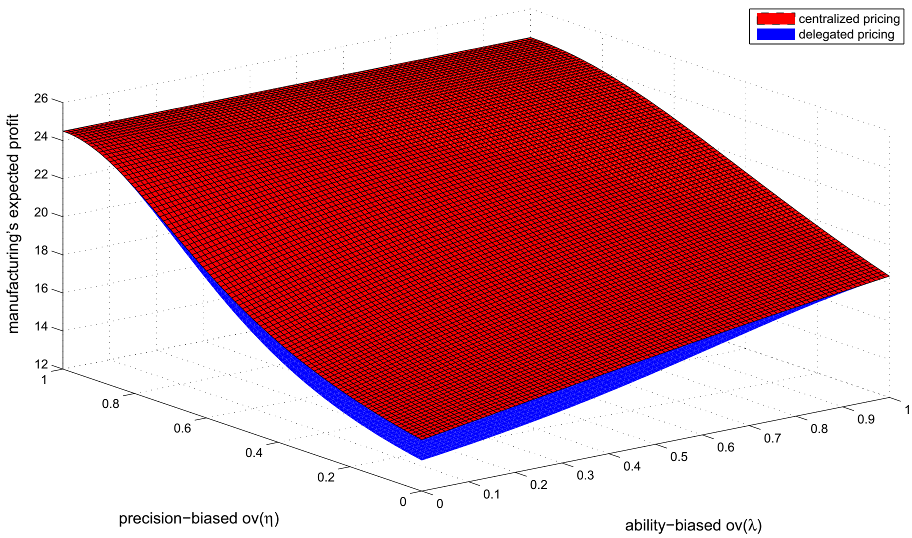

FIGURE 4. The impact of the sales agent's degree of ability-biased and precision-biased overconfidence on the manufacturer's expected profit.

The manufacturer's expected profits under centralized pricing and delegated pricing strategies are displayed in Figure 4. We verify that they both grow with the two types of overconfidence, and the expected profit under centralized pricing strategy is larger.

Through our quantitative analysis, we demonstrate that both the ability-biased and precision-biased overconfidence have significant effects on salesfore incentive strategies, pricing decisions, and resulted profit.

7. Conclusions

Significantly large investment is observed by many firms, as reported in a Harvard Business Review study which is also discussed by Steenburgh and Ahearne [43], where US companies were shown spending a huge sum of 800 billion dollars annually on salesforce compensation. In this paper, we studied a supply chain with a rational manufacturer who sells products through an overconfident sales agent. By developing models under two pricing scenarios (centralized pricing and delegated pricing strategies) with consideration of two aspects of overconfidence behaviors as ability-biased and precision-biased cognition simultaneously. The purpose of this paper is to investigate how can a manufacturer make the pricing and compensation decisions considering the agent's two aspects of overconfidence behaviors. For both pricing policies, the compensation contract and pricing are designed to address sales agent's overconfidence issue, and the sensitivity analyses are conducted to examine how they affect the decisions and revenue of both sides. Numerical studies are conducted to understand the proposed models and validate our propositions.

Some interesting results and management insights are likewise found. For both centralized and delegated pricing strategies, the sales agent's effort level, the commission rate and the manufacturer's expected profit are all jointly increasing in the two types of overconfidence level with certain overconfidence, *i.e.*, the decisions of both sides are highly deviate from the rational scenario. The increasing of the sales agent's overconfidence means the sales agent is optimistic about the market and his selling ability, thus he is willing to exert more sales effort to gain more returns, the increasing effort resulted in the increasing in the overall income and the manufacturer's profit. It also indicates that the ability-biased and precision-biased overconfidence behaviors have substitutable effects on the pricing policy preference of the manufacturer and the sales agent. An important finding in the paper is that the manufacturer should not ignore the sales agent's overconfidence behavior for the two pricing strategies, as the sales agent's overconfidence behavior may be beneficial for the manufacturer. On the contrary, overconfidence behavior leads to the loss of actual expected profits of the sales agent, and the loss amount increases with the increasing overconfidence level, *i.e.*, the sales agent's overconfidence behavior is actually harmful to himself. At the same level of overconfidence, compared with centralized pricing strategy, the sales agent gets higher expected utility while exerting less effort under delegated pricing strategy, we further show that, it is beneficial for the manufacturer to utilize centralized pricing, while the sales agent prefers delegated pricing.

Certainly, there are some limitations in this paper. First, we only considered the decision setting in which the sales agent has overconfident behaviors, and ignored the manufacturer's overconfident behaviors. As such, in future research, we will investigate similar scenarios and simultaneously consider the manufacturer's overconfident behaviors. Second, in our paper, we only considered the moral hazard problem, the sales agent's two aspects of overconfidence information are assumed to be common knowledge. In reality, the overconfidence information available to each other may be imperfect, and thus adverse selection problem or screening could occur, which may yield interesting insights. Finally, only the traditional linear compensation contract and additive demand function are considered, the quota-based compensation plan (see [24]) and the multiplicative demand model (see [12]) should likewise be investigated. It still remains for future research to cite the more reasonable overconfidence measure. Future work also includes considering salvage value and lost sales penalty (see [11]), and extending the primal model to other behavioral economics context, behavioral operation research especially the fairness concern behavior supported strongly by behavior experiments.

Appendix

Proof of Theorem 4.1. Substituting equation (4.6) and the IR constraint (replace α) into the optimal problem (4.3), we can get the manufacturer's equivalence problem as

$$\begin{aligned}\max_{\beta,p} E(\pi_M) &= p(a-bp+e) - \alpha - \beta p(a-bp+\mu(\lambda)e)\\ &= p(a-bp) + \frac{\mu(\lambda)\beta p^2}{b'} - \underline{\pi} - \frac{A\omega^2(\eta)+\mu^2(\lambda)}{2b}\beta^2.\end{aligned}$$

Taking the second-order partial derivatives of $E(\pi_M)$ with respect to p and β respectively, we have

$$\begin{aligned}\frac{\partial^2 E(\pi_M)}{\partial p^2} &= -2b + \frac{2\mu(\lambda)\beta}{b'} - \frac{A\omega^2(\eta)+\mu^2(\lambda)}{b'}\beta^2 < 0,\\ \frac{\partial^2 E(\pi_M)}{\partial p\partial\beta} &= \frac{\partial^2 E(\pi_M)}{\partial\beta\partial p} = \frac{2\mu(\lambda)p}{b'} - \frac{2[A\omega^2(\eta)+\mu^2(\lambda)]}{b'}\beta p,\\ \frac{\partial^2 E(\pi_M)}{\partial\beta^2} &= -\frac{A\omega^2(\eta)+\mu^2(\lambda)}{2b'}p^2 < 0.\end{aligned}$$

We obtain the Hessian matrix H_1 by second-order derivatives. It is easy to verify that, when $2bb'-1>0$, H_1 is negative definite concavity. Therefore, $E(\pi_M)$ is a strictly jointly concave function of β and p, that is, the first-order derivatives are therefore sufficient. Based on the first-order necessary condition,

$$\begin{cases}\frac{\partial E(\pi_M)}{\partial p} = a - 2bp + \frac{2\mu(\lambda)\beta p}{b'} - \frac{A\omega^2(\eta)+\mu^2(\lambda)}{b'}\beta^2 p = 0,\\ \frac{\partial E(\pi_M)}{\partial\beta} = \frac{\mu(\lambda)p^2}{b'} - \frac{A\omega^2(\eta)+\mu^2(\lambda)}{b'}p^2\beta = 0,\end{cases}$$

we have

$$\begin{aligned}\beta_C^* &= \frac{\mu(\lambda)}{A\omega^2(\eta)+\mu^2(\lambda)},\\ p_C^* &= \frac{a}{2b}\frac{2bb'[A\omega^2(\eta)+\mu^2(\lambda)]}{2bb'[A\omega^2(\eta)+\mu^2(\lambda)]-\mu^2(\lambda)},\end{aligned}$$

and the optimal effort strategy is given by

$$e_C^* = \frac{\mu(\lambda)p_C^*\beta_C^*}{b'} = \frac{a\mu^2(\lambda)}{2bb'[A\omega^2(\eta)+\mu^2(\lambda)]-\mu^2(\lambda)}.$$

From the binding IR constraint (3.5), the base salary α_C^* can be easily derived, to substitute the above e_C^*, α_C^*, β_C^* and p_C^* into the optimal problem (4.3), the maximum objective value $E_C^*(\pi_M)$ can be calculated. Therefore, this Theorem is proved.

Proof of Proposition 4.3. Owing that $\frac{1}{\beta_C^*} = \frac{A\omega^2(\eta)}{\mu(\lambda)} + \mu(\lambda)$, we have $\frac{\partial(1/\beta_C^*)}{\partial(A\omega^2(\eta))} = \frac{1}{\mu(\lambda)} > 0$, so $\frac{1}{\beta_C^*}$ is increasing in $A\omega^2(\eta)$, *i.e.*, β_C^* is jointly decreasing in A and $\omega(\eta)$, and β_C^* is increasing in η ($\omega'(\eta) \le 0$);

Similarly, $\frac{\partial(1/\beta_C^*)}{\partial\lambda} = \frac{\mu^2(\lambda)-A\omega^2(\eta)}{\mu^2(\lambda)}\mu'(\lambda)$, we have: if $A > \frac{\mu^2(\lambda)}{\omega^2(\eta)}$, $\frac{\partial(1/\beta_C^*)}{\partial\lambda} > 0$, $\frac{1}{\beta_C^*}$ is decreasing in λ ($\mu'(\lambda) \ge 0$), thus β_C^* is increasing in λ, and *vice versa*.

For simplification, we denote

$$R = \frac{\mu^2(\lambda)}{2bb'[A\omega^2(\eta)+\mu^2(\lambda)]-\mu^2(\lambda)} = \frac{1}{2bb'[A\omega^2(\eta)/\mu^2(\lambda)+1]-1},$$

obviously, $p_C^* = \frac{a(1+R)}{2b}$, $e_C^* = aR$, and $E_C^*(\pi_M) = \frac{a^2(1+R)}{4b} - \underline{\pi}$ have the same monotonicity with R respect to A, λ and η.

We know that R is decreasing in $\frac{A\omega^2(\eta)}{\mu^2(\lambda)}$ ($2bb' > 1$), *i.e.*, R is jointly decreasing in A and $\omega(\eta)$, R is increasing in η ($\omega'(\eta) \le 0$), and R is increasing in $\mu(\lambda)$ and λ.

So $E_C^*(\pi_M)$, e_C^* and p_C^* are are all decreasing in A, while are all jointly increasing in λ and η. Therefore, this Proposition is proved.

Proof of Observation 4.4. From the proof of Proposition 4.3, $E_C^*(\pi_M)$, e_C^* and p_C^* are all jointly increasing in λ and η, if $\lambda = \eta = 0$, they are reduced to $E_{\mathrm{CR}}^*(\pi_M)$, e_{CR}^* and p_{CR}^* respectively, thus $e_C^* \ge e_{\mathrm{CR}}^*$, $p_C^* \ge p_{\mathrm{CR}}^*$ and $E_C^*(\pi_M) \ge E_{\mathrm{CR}}^*(\pi_M)$ hold. Then, we compare β_C^* and β_{CR}^*, due to

$$\beta_C^* - \beta_{\mathrm{CR}}^* = \frac{A[\mu(\lambda)-\omega^2(\eta)] - [\mu^2(\lambda)-\mu(\lambda)]}{[A\omega^2(\eta)+\mu^2(\lambda)](1+A)},$$

so if $A \geq \frac{\mu^2(\lambda)-\mu(\lambda)}{\mu(\lambda)-\omega^2(\eta)}$, $\beta_C^* \geq \beta_{\mathrm{CR}}^*$, otherwise if $0 < A < \frac{\mu^2(\lambda)-\mu(\lambda)}{\mu(\lambda)-\omega^2(\eta)}$, $\beta_C^* < \beta_{\mathrm{CR}}^*$. Therefore, this observation is proved.

Proof of Proposition 4.5. For centralized pricing, if the manufacturer ignores the sales agent's overconfidence behavior, her actual expected net profit is given by

$$E^*[\pi_C(\alpha_{\mathrm{CR}}^*, \beta_{\mathrm{CR}}^*, p_{\mathrm{CR}}^*)] = p_{\mathrm{CR}}^*(a - bp_{\mathrm{CR}}^* + e_C^*) - \alpha_{\mathrm{CR}}^* - \beta_{\mathrm{CR}}^* p_{\mathrm{CR}}^*(a - bp_{\mathrm{CR}}^* + e_C^*),$$

while her expected net profit with the sales agent's overconfidence consideration is given by

$$E^*[\pi_C(\alpha_C^*, \beta_C^*, p_C^*)] = p_C^*(a - bp_C^* + e_C^*) - \alpha_C^* - \beta_C^* p_C^*(a - bp_C^* + e_C^*),$$

since $(\alpha_C^*, \beta_C^*, p_C^*)$ satisfies $\max_{\alpha,\beta,p} E[\pi_C(\alpha, \beta, p)]$, thus $E^*[\pi_C(\alpha_{\mathrm{CR}}^*, \beta_{\mathrm{CR}}^*, p_{\mathrm{CR}}^*)] \leq E^*[\pi_C(\alpha_C^*, \beta_C^*, p_C^*)]$ holds.

Additionally, from the proof of Proposition 4.3, $E_C^*(\pi_M)$ is jointly increasing in λ and η, when $\lambda = \eta = 0$, $E^*[\pi_C(\alpha_C^*, \beta_C^*, p_C^*)]$ degrades into $E^*[\pi_C(\alpha_{\mathrm{CR}}^*, \beta_{\mathrm{CR}}^*, p_{\mathrm{CR}}^*)]$, so it is easy to verify that $E^*[\pi_C(\alpha_{\mathrm{CR}}^*, \beta_{\mathrm{CR}}^*, p_{\mathrm{CR}}^*)] \leq E^*[\pi_C(\alpha_C^*, \beta_C^*, p_C^*)]$.

The CE in overconfident sales agent's mind is given by

$$\mathrm{CE}_C(\alpha_C^*, \beta_C^*, p_C^*) = \alpha_C^* + \beta_C^* p_C^*[a - bp_C^* + \mu(\lambda)e_C^*] - b' e_C^{*2}/2 - \rho\sigma^2 \beta_C^{*2} p_C^{*2} \omega^2(\eta)/2,$$

while the overconfident sales agent's actual CE is given by

$$\mathrm{CE}_C(actual) = \alpha_C^* + \beta_C^* p_C^*(a - bp_C^* + e_C^*) - b' e_C^{*2}/2 - \rho\sigma^2 \beta_C^{*2} p_C^{*2}/2,$$

we obtain

$$\begin{aligned}
\triangle_C &= \mathrm{CE}_C(\alpha_C^*, \beta_C^*, p_C^*) - \mathrm{CE}_C(actual) = \beta_C^* p_C^* e_C^*[\mu(\lambda) - 1] + \frac{1-\omega^2(\eta)}{2b'} A\beta_C^{*2} p_C^{*2} \\
&= \beta_C^* p_C^*[\mu(\lambda) - 1]\frac{\mu(\lambda)\beta_C^* p_C^*}{b'} + \frac{1-\omega^2(\eta)}{2b'} A\beta_C^{*2} p_C^{*2} \\
&= \beta_C^{*2} p_C^{*2} \left(\frac{\mu(\lambda)[\mu(\lambda)-1]}{b'} + \frac{1-\omega^2(\eta)}{2b'} A \right) \geq 0,
\end{aligned}$$

$\triangle_C \geq 0$ holds, which implies that the sales agent's overconfidence behavior seriously hurt the profit of himself, which means overconfidence behavior is harmful to himself for centralized pricing.

To substitute the optimal β_C^* and p_C^* into the above $\triangle_C$, we have

$$\triangle_C = \left(\frac{ab'}{2bb'[A\omega^2(\eta)/\mu(\lambda) + \mu(\lambda)] - \mu(\lambda)} \right)^2 \left(\frac{\mu(\lambda)[\mu(\lambda)-1]}{b'} + \frac{1-\omega^2(\eta)}{2b'} A \right).$$

It is easy to verify that, $\triangle_C$ is jointly increasing in A and η; if A is sufficiently large, *i.e.*, $A \geq \frac{\mu^2(\lambda)}{\omega^2(\eta)} \frac{2bb'-1}{2bb'}$, $\triangle_C$ is jointly increasing in λ and η. Therefore, this Proposition is proved.

Proof of Theorem 4.6. By equation (4.16), we have the second-order partial derivatives of CE_D with respect to e and p respectively as

$$\frac{\partial^2 \mathrm{CE}_D}{\partial e^2} = -b' < 0, \quad \frac{\partial^2 \mathrm{CE}_D}{\partial e \partial p} = \frac{\partial^2 \mathrm{CE}_D}{\partial p \partial e} = \mu(\lambda)\beta, \quad \frac{\partial^2 \mathrm{CE}_D}{\partial p^2} = -2\beta b - \rho\sigma^2 \beta^2 \omega^2(\eta) < 0.$$

The Hessian matrix of CE_D is given by

$$\begin{vmatrix} \frac{\partial^2 \mathrm{CE}_D}{\partial e^2} & \frac{\partial^2 \mathrm{CE}_D}{\partial e \partial p} \\ \frac{\partial^2 \mathrm{CE}_D}{\partial p \partial e} & \frac{\partial^2 \mathrm{CE}_D}{\partial p^2} \end{vmatrix} = 2bb'\beta + \beta^2[A\omega^2(\eta) - \mu^2(\lambda)].$$

We obtain the Hessian matrix H by second-order derivatives. It is easy to verify that, when $2bb' > [\mu^2(\lambda) - A\omega^2(\eta)]\beta$, H is negative definite concavity. Hence, CE_D is strictly jointly concave in e and p, that is, the first-order derivatives are therefore sufficient. Using the first-order necessary conditions,

$$\begin{cases} \frac{\partial \mathrm{CE}_D}{\partial e} = \mu(\lambda)\beta p - b'e = 0, \\ \frac{\partial \mathrm{CE}_D}{\partial \beta} = \beta a - 2\beta bp + \mu(\lambda)\beta p - \rho p\beta^2\sigma^2\omega^2(\eta) = 0, \end{cases}$$

we can derive that

$$\begin{cases} e = \frac{\mu(\lambda)}{b'}\frac{a\beta}{2b-L\beta}, \\ p = \frac{a}{2b-L\beta}, \end{cases} \quad \text{where } L = \frac{\mu^2(\lambda) - A\omega^2(\eta)}{b'}.$$

To substitute the above e and p into the sales agent's optimal problem (21), we have

$$\begin{aligned} \mathrm{CE}_D = \alpha + \beta p[a - bp + \mu(\lambda)e] - b'e^2/2 - \rho\sigma^2\beta^2p^2\omega^2(\eta)/2 = -b'\left(\frac{a}{2b - L\beta}\right)^2/2 \\ + \alpha + \beta\frac{\mu(\lambda)}{b'}\frac{a\beta}{2b-L\beta}\left(a - b\frac{\mu(\lambda)}{b'}\frac{a\beta}{2b-L\beta} + \frac{a\mu(\lambda)}{2b-L\beta}\right) \\ - \rho\sigma^2\beta^2\omega^2(\eta)\left(\frac{\mu(\lambda)}{b'}\frac{a\beta}{2b-L\beta}\right)^2/2 = \alpha + \frac{a^2\beta}{2(2b-L\beta)}. \end{aligned}$$

Then, we can obtain

$$\alpha = \underline{\pi} - \frac{a^2\beta}{2(2b-L\beta)},$$

to substitute the above α, e and p into the optimal problem (4.15), we can get the equivalence problem as

$$\begin{aligned} E_D(\pi_M) &= p(a - bp + e) - \alpha - \beta p[a - bp + \mu(\lambda)e] \\ &= (1-\beta)\frac{a^2(2b-L\beta)}{(2b-L\beta)^2} + \frac{a^2\beta}{2(2b-L\beta)} + \frac{\mu(\lambda)\beta - \mu^2(\lambda)\beta^2}{b'}\frac{a^2}{(2b-L\beta)^2} - \underline{\pi} \\ &= a^2\frac{[L/2 + \mu(\lambda)/b']\beta^2 + [\mu(\lambda)/b' - L]\beta + b}{(2b-L\beta)^2} - \underline{\pi}. \end{aligned}$$

Taking first derivative of $E_D(\pi_M)$ with respect to β, we have

$$\frac{\mathrm{d}E_D(\pi_M)}{\mathrm{d}\beta} = a^2\frac{[2bL + L\mu(\lambda)/b' - L^2 - 4b\mu^2(\lambda)/b']\beta + 2\mu(\lambda)b/b'}{(2b-L\beta)^3}.$$

Since $(2b - L\beta)^3$ is positive and

$$\begin{aligned} &2bL + L\mu(\lambda)/b' - L^2 - 4b\mu^2(\lambda)/b' \\ &= -\frac{2bb'[A\omega^2(\eta) + \mu^2(\lambda)] + [A\omega^2(\eta) - \mu^2(\lambda)]^2 + \mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]}{b'^2} < 0, \end{aligned}$$

which means $2bL + L\mu(\lambda)/b' - L^2 - 4b\mu^2(\lambda)/b'$ is decreasing in β when $2bb' > [\mu^2(\lambda) - b'\rho\sigma^2\omega^2(\eta)]\beta$ holds, we know that $E_D(\pi_M)$ is quasi-concave in β, and therefore the optimal commission rate can be obtained by first-order condition. And other results can be easily derived.

It is easy to verify that, when $2bb' - 1 > 0$, $2bb' > [\mu^2(\lambda) - b'\rho\sigma^2\omega^2(\eta)]\beta_D^*$ holds, so we can omit the condition $2bb' > [\mu^2(\lambda) - b'\rho\sigma^2\omega^2(\eta)]\beta_D^*$.

Proof of Proposition 4.8. Owing that

$$\frac{1}{\beta_D^*} = \frac{A\omega^2(\eta) + \mu^2(\lambda)}{\mu(\lambda)} + \frac{[A\omega^2(\eta) - \mu^2(\lambda)]^2}{2bb'\mu(\lambda)} + \frac{A\omega^2(\eta) - \mu^2(\lambda)}{2bb'},$$

we have $\frac{\partial(1/\beta_D^*)}{\partial(A\omega^2(\eta))} = \frac{bb'+A\omega^2(\eta)-\mu^2(\lambda)+\mu(\lambda)/2}{bb'\mu(\lambda)}$, if $A > \frac{[\mu^2(\lambda)-bb'-\mu(\lambda)/2]^+}{\omega^2(\eta)}$, $\frac{\partial(1/\beta_D^*)}{\partial(A\omega^2(\eta))} > 0$, $\frac{1}{\beta_D^*}$ is increasing in $A\omega^2(\eta)$, *i.e.*, β_D^* is jointly decreasing in A and $\omega(\eta)$, and β_D^* is increasing in η ($\omega'(\eta) \leq 0$), and *vice versa*.

Since

$$\frac{\partial(1/\beta_D^*)}{\partial(\mu(\lambda))} = \frac{3\mu^4(\lambda) - 2\mu^3(\lambda) + 2bb'\mu^2(\lambda) - 2bb'A\omega^2(\eta) - 2A\omega^2(\eta)\mu^2(\lambda) - A^2\omega^4(\eta)}{2bb'\mu^2(\lambda)},$$

if

$$A > \frac{\sqrt{[bb' + \mu(\lambda)]^2 + 2bb'\mu^2(\lambda) + 3\mu^4(\lambda) - 2\mu^3(\lambda)} - [bb' + \mu(\lambda)]}{\omega^2(\eta)},$$

$\frac{1}{\beta_D^*}$ is decreasing in λ ($\mu'(\lambda) \geq 0$), thus β_D^* is increasing in λ, and *vice versa*.

For simplification, we denote

$$G = \frac{\mu^2(\lambda)}{2bb'[A\omega^2(\eta) + \mu^2(\lambda)] + [A\omega^2(\eta) - \mu^2(\lambda)]^2 + 2\mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]}.$$

Obviously, G, $e_D^* = aG$ and $E_D^*(\pi_M) = \frac{a^2(1+R)}{4G} - \underline{\pi}$ have the same monotonicity with respect to A, λ and η, we have

$$\frac{\partial(1/G)}{\partial(A\omega^2(\eta))} = \frac{2[bb' + A\omega^2(\eta) + \mu(\lambda) - \mu^2(\lambda)]}{\mu^2(\lambda)},$$

if $A > \frac{[\mu^2(\lambda)-\mu(\lambda)-bb']^+}{\omega^2(\eta)}$, $\frac{\partial(1/G)}{\partial(A\omega^2(\eta))} > 0$, $\frac{1}{G}$ is increasing in $A\omega^2(\eta)$, *i.e.*, G is jointly decreasing in A and $\omega(\eta)$, G is increasing in η ($\omega'(\eta) \leq 0$). both e_D^* and $E_D^*(\pi_M)$ are decreasing in A, while increasing in η, and *vice versa*.

As

$$\frac{\partial(1/G)}{\partial(\mu^2(\lambda))} = 1 - \frac{2bb'A\omega^2(\eta) + A^2\omega^4(\eta) + \mu^3(\lambda) + A\omega^2(\eta)\mu(\lambda)}{\mu^4(\lambda)},$$

if $A > \frac{\sqrt{[bb'+\mu(\lambda)/2]^2+\mu^4(\lambda)-\mu^3(\lambda)}-[bb'+\mu(\lambda)/2]}{\omega^2(\eta)}$, $\frac{\partial(1/G)}{\partial(\mu^2(\lambda))} < 0$, which implies that $\frac{1}{G}$ is decreasing in $\mu^2(\lambda)$, G is increasing in λ ($\mu'(\lambda) > 0$), both e_D^* and $E_D^*(\pi_M)$ are increasing in λ, and *vice versa*.

Obviously,

$$p_D^* = \frac{a}{2b}\left(1 - \frac{\mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]}{2bb'[A\omega^2(\eta) + \mu^2(\lambda)] + [A\omega^2(\eta) - \mu^2(\lambda)]^2 + 2\mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]}\right).$$

For simplification, we denote

$$N = \frac{2bb'[A\omega^2(\eta) + \mu^2(\lambda)] + [A\omega^2(\eta) - \mu^2(\lambda)]^2 + 2\mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]}{\mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]}.$$

Thus, both p_D^* and N have the same monotonicity with respect to A, λ and η, we have

$$\frac{\partial N}{\partial(A\omega^2(\eta))} = \frac{[A\omega^2(\eta) - \mu^2(\lambda)]^2 - 4bb'\mu^2(\lambda)}{\mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]^2},$$

if $A > \frac{\mu^2(\lambda)+2\mu(\lambda)\sqrt{bb'}}{\omega^2(\eta)}$ or $0 < A < \frac{[\mu^2(\lambda)-2\mu(\lambda)\sqrt{bb'}]^+}{\omega^2(\eta)}$, $\frac{\partial N}{\partial(A\omega^2(\eta))} < 0$, both p_D^* and N are decreasing in A, while increasing in η. When $\frac{[\mu^2(\lambda)-2\mu(\lambda)\sqrt{bb'}]^+}{\omega^2(\eta)} < A < \frac{\mu^2(\lambda)+2\mu(\lambda)\sqrt{bb'}}{\omega^2(\eta)}$, p_D^* is increasing in A, while decreasing in η.

We can obtain

$$\frac{\partial N}{\partial(\mu(\lambda))} = \frac{4bb'\mu^2(\lambda)[A\omega^2(\eta) + \mu^2(\lambda)] - [2bb' + A\omega^2(\eta) + \mu^2(\lambda)][A\omega^2(\eta) - \mu^2(\lambda)]^2}{\mu(\lambda)[A\omega^2(\eta) - \mu^2(\lambda)]^2},$$

when $A\omega^2(\eta) = \mu^2(\lambda) + t$, and t satisfies $t^3 + 2[bb' + \mu^2(\lambda)]t^2 - 4bb'\mu^2(\lambda)t - 8bb'\mu^4(\lambda) < 0$, $\frac{\partial N}{\partial(\mu(\lambda))} > 0$. Therefore, this proposition is proved.

Proof of Proposition 5.4. For delegated pricing, if the manufacturer ignores the sales agent's overconfidence behavior, her actual expected net profit is given by

$$E^*[\pi_D(\alpha^*_{\rm DR},\beta^*_{\rm DR})] = p^*_D(a-bp^*_D+e^*_D) - \alpha^*_{\rm DR} - \beta^*_{\rm DR}p^*_D(a-bp^*_D+e^*_D),$$

while her expected net profit with the sales agent's overconfidence consideration is given by

$$E^*[\pi_D(\alpha^*_D,\beta^*_D)] = p^*_D(a-bp^*_D+e^*_D) - \alpha^*_D - \beta^*_Dp^*_D(a-bp^*_D+e^*_D),$$

since (α^*_D,β^*_D) satisfies $\max_{\alpha,\beta} E[\pi_D(\alpha,\beta)]$, thus $E^*[\pi_D(\alpha^*_{\rm DR},\beta^*_{\rm DR})] \leq E^*[\pi_D(\alpha^*_D,\beta^*_D)]$ holds.

From the proof of Proposition 5.3, if A is sufficiently large, $E^*_D(\pi_M)$ is jointly increasing in λ and η, we can also get $E^*[\pi_D(\alpha^*_{\rm DR},\beta^*_{\rm DR})] \leq E^*[\pi_D(\alpha^*_D,\beta^*_D)]$.

For delegated pricing, the CE in overconfident sales agent's mind is given by

$${\rm CE}_D(\alpha^*_D,\beta^*_D) = \alpha^*_D + \beta^*_Dp^*_D[a-bp^*_D+\mu(\lambda)e^*_D] - b'e^{*2}_D/2 - \rho\sigma^2\beta^{*2}_Dp^{*2}_D\omega^2(\eta)/2,$$

while the overconfident sales agent's actual CE is given by

$${\rm CE}_D(actual) = \alpha^*_D + \beta^*_Dp^*_D(a-bp^*_D+e^*_D) - b'e^{*2}_D/2 - \rho\sigma^2\beta^{*2}_Dp^{*2}_D/2.$$

We obtain

$$\begin{aligned}\triangle_D &= {\rm CE}_D(\alpha^*_D,\beta^*_D) - {\rm CE}_D(actual) = \beta^*_Dp^*_De^*_D[\mu(\lambda)-1] + \frac{1-\omega^2(\eta)}{2b'}A\beta^{*2}_Dp^{*2}_D\\ &= \beta^*_Dp^*_D[\mu(\lambda)-1]\frac{\mu(\lambda)\beta^*_Dp^*_D}{b'} + \frac{1-\omega^2(\eta)}{2b'}A\beta^{*2}_Dp^{*2}_D\\ &= \beta^{*2}_Dp^{*2}_D\left(\frac{\mu(\lambda)[\mu(\lambda)-1]}{b'} + \frac{1-\omega^2(\eta)}{2b'}A\right) \geq 0,\end{aligned}$$

so $\triangle_D \geq 0$ holds, which implies that overconfident sales agent's actual profit is lower, the sales agent's overconfidence behaviors actually hurt his profit.

It can be easily verified that, when A is sufficiently large, $\triangle_D$ is jointly increasing in λ and η. Therefore, this proposition is proved.

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