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ROBUST BI-LEVEL RISK-BASED OPTIMAL SCHEDULING OF MICROGRID OPERATION AGAINST UNCERTAINTY

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Abstract. The model introduced in this paper is the first to propose a decentralized robust optimal scheduling of MG operation under uncertainty and risk. The power trading of the MG with the main grid is the first stage variable and power generation of DGs and power charging/discharging of the battery are the second stage variables. The uncertain term of the initial objective function is transformed into a constraint using robust optimization approach. Addressing the Decision Maker's (DMs) risk aversion level through Conditional Value at Risk (CVaR) leads to a bi-level programming problem using a data-driven approach. The model is then transformed into a robust single-level using Karush–Kahn–Tucker (KKT) conditions. To investigate the effectiveness of the model and its solution methodology, it is applied on a MG. The results clearly demonstrate the robustness of the model and indicate a strong almost linear relationship between cost and the DMs various levels of risk aversion. The analysis also outlines original characterization of the cost and the MGs behavior using three well-known goodness-of-fit tests on various Probability Distribution Functions (PDFs), Beta, Gumbel Max, Normal, Weibull, and Cauchy. The Gumbel Max and Normal PDFs, respectively, exhibit the most promising goodness-of-fit for the cost, while the power purchased from the grid are well fitted by Weibull, Beta, and Normal PDFs, respectively. At the same time, the power sold to the grid is well fitted by the Cauchy PDF.

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Nomenclature

Indices:

t	Index of time
i	Index of distributed generator
j	Index of battery
s	Index of scenario

Parameters:

$C_t^{\text{grid,in}}$	Price of purchased power from main grid (\$/kWh)
$C_t^{\text{grid,out}}$	Price of power sold to main grid (\$/kWh)
$E_j^{\text{batt,ini}}$	Initial energy stored in battery (kWh)
$E_j^{\text{batt,max}}$	Maximum energy stored in battery (kWh)
$E_j^{\text{batt,min}}$	Minimum energy stored in battery (kWh)
$P_j^{\text{batt,max}}$	Maximum power charging/discharging of battery (kW)
P_t^{Demand}	Power demand (kW)
$P_{i,t}^{\text{DG,max}}$	Maximum DG capacity limit (kW)
$P_{i,t}^{\text{DG,min}}$	Minimum DG capacity limit (kW)
$P^{\text{grid,max}}$	Maximum limit for trading power with main grid (kW)
η_j^{charge}	Charging efficiency of battery
$\eta_j^{\text{discharge}}$	Discharging efficiency of battery
η^{grid}	Efficiency of grid's transformer
α	Decision makers risk aversion level
β	Significance level for the goodness of fit tests

Variables:

$E_{j,t}^{\text{batt}}$	Energy stored in battery (kWh)
$P_{i,t}^{\text{DG}}$	Power generation of DG (kW)
$P_{j,t}^{\text{charge}}$	Power charging of battery (kW)
$P_{j,t}^{\text{discharge}}$	Power discharging of battery (kW)
$P_t^{\text{grid,in}}$	Power purchased from main grid (kW)
$P_t^{\text{grid,out}}$	Power sold to main grid (kW)
$U_{i,t}$	Binary variables used for on/off of DGs
$X_{j,t}^{\text{batt}}$	Binary variables used for power charging/discharging status of battery
X_t^{grid}	Binary variables used for purchased/sold power from/to main grid

Uncertain parameters:

$\tilde{P}_t^{\text{PHEV}}$	Power consumed by PHEV (kWh)
$\tilde{C}_{j,t}^{\text{batt}}$	Charging/discharging cost of battery (\$/kWh)
$\tilde{C}_{i,t}^{\text{DG}}$	Generation cost of DG (\$/kWh)

Abbreviations:

ADG	Active Distribution Grid
BLP	Bi-Level Programming
CRM	Coherent Risk Measure
CVaR	Conditional Value at Risk
DG	Distributed Generator

DISCO	Distribution Company
DM	Decision Maker
IGDT	Information Gap Decision Theory
KKT	Karush–Kuhn–Tucker
MG	Microgrid
MILP	Mixed-Integer Linear Problem
PHEV	Plug-in Hybrid Electric Vehicle
RO	Robust Optimization
MILP	Mixed Integer Linear Programming
MINLP	Mixed Integer Nonlinear Programming
DNO	Distributed Network Operator
KS	Kolmogrove–Smirnov
AD	Anderson–Darling
CS	Chi-Square
CDF	Cumulative Distribution Function
PDF	Probability Distribution Function
CHP	Combined Heat and Power

1. Introduction

Today, traditional power systems are facing the problems of reduced fossil fuel resources, poor energy efficiency, and environmental pollutions. Electric power consumption, on the other hand, is increasing rapidly in the last few decades. Therefore, Distributed Generations (DGs) are connected into the system to meet the load locally [4]. However there are several issues concerning such an integration. It changes the system from passive to active networks that affects the reliability and operation of the main power system [2]. In such an Active Distribution Grid (ADG), the Distribution Company (Disco) and the Microgrids (MGs) may play the roles of the leader and the followers of a non-cooperative game, called Stackelberg game theory. In this dynamic game player 1, *i.e.* DISCO, chooses a strategy first and then player 2, *i.e.* MG, makes its decision accordingly. The mathematical model designed to deal with this type of game is called Bi-Level Programming (BLP). The theory is applicable, especially when a number of the problem inputs are subject to uncertainty [31], which is the case, in this paper, in the presence of Plug-in Hybrid Electric Vehicles (PHEVs).

Nowadays, PHEVs become key elements within the MGs that enhance the reliability of the resulting integrated system [29]. However, the uncertainty of drivers' behaviors toward vehicle charging significantly affects the optimality of the system operation [4]. The costs arising from such uncertain demand, *e.g.* the generation cost of the DG and charging/discharging cost of battery, are also uncertain. Such uncertainties resulting in operational risks of a system. Robust Optimization (RO) approach is therefore needed to deal with vulnerability of the system in presence of such undeniable uncertainties and risk considerations. Although, the idea of RO is widely used in other engineering desciplines, there exists a very limited literature about using it in the field of the MG operation. Constructing a robust counterpart based on an initial uncertain problem using ellipsoidal uncertainty sets is first introduced by Ben-Tal and Nemirovski [5]. Bertsimas and Sim [6] imployed polyhedral uncertainty sets to keep the linearity of the initial problem. Bertsimas and Brown [7] constructed uncertainty sets for robust linear optimization problems. Their approach relies directly on Decision Maker's (DMs) risk preferences using Conditional Value at Risk (CVaR) as a controllable means of multiple chance constraints. The key idea behind the approach is that minimizing the CVaR is a strong alternative to maximizing the expectation of any risk-averse utility function.

This paper formulates a grid-connected MG operation scheduling problem using BLP under the uncertainty of not only the PHEVs' electricity demand, but also the costs arising from such uncertainty. The risk arising from such decentralized system leads to obtain robustness against uncertainties. The resulting model is formulated as a bi-level optimization problem using CVaR considering the DMs level of risk aversion. Finally, an exact solution approach is addopted to solve the model based on the Karush–Kuhn–Tucker (KKT) conditions. The CVaR appraoch is used in many research to handle the risk of the DM by adding its relative equations to the objective function and constraints of the initial problem. However, in this paper, an original appraoch is develped for modeling the risk of the DM through the use of the CVaR by fully transforming the initial problem into its robust bi-level counterpart. (BLP) resulting in a Stackelberg equilibrium solution, in which the MG's resources, as the follower, cost function is minimized with respect to the main grid's, as the leader, potential choices on the amount of power purchased/sold from/to the MGs. From this, a virtual cost reduction incentive response function is constructed. The grid, according to its leadership, surely anticipates this response. Obtaining an equilibrium solution, in the approach introduced in this paper, Stachelberg game concept proceeds with minimization of the grid cost, while incorporating the MG's resources response into it, based on any realizations of the scenarios, which is more like real life conditions. To the best of authors knowledge this is the first to use such a decentralized apprach in the field of the MG operation scheduling and optimization. The key contributions of the paper are summarized as follows:

(a) Modeling an original decentralized grid-connected MG operation scheduling problem considering PHEVs' demand uncertainty using BLP approach with respect to DMs risk aversion level is the main contribution of the paper. Uncertainty handling in a bi-level framework is still a key point for the operation of MGs.
(b) Literature reveals that it is still a challenging issue to cover uncertainties caused by random loads while optimizing bi-level problems in economy. It is noticable that, in contrast to the relative literature, in this paper the final (BLP) problem is directly formulated through the CVaR approach and, therefore, rely on a stronger mathematical theory.
(c) Addressing the DMs attitude toward risk using CVaR approach through a non-cooperative game in a robust manner is the other key contribution of the paper. Accordingly, the model is fully interactive while providing full control of the system operation elements with respect to different viewpoints of DMs.
(d) The model leads to a strong almost linear relationship between the system total cost and the DMs risk aversion level.
(e) Using the Kolmogrove–Smirnov (KS), Chi-Square (CS), and Anderson–Darling (AD) tests, extensive goodness-of-fit analysis is carried out to test which probability distribution fits, satisfactorily, the empirical distribution of the economic costs and power exchange between the MG and upstream network as the key variables of the problem.

The rest of the paper is organized as follows. A literature review and the motivation of the study are given in Section 2. The problem description and modeling framework are described in Section 3. Numerical results, discussion, and robust solution analysis are presented in Section 4. Finally, conclusions and some future directions are presented in Section 5.

2. Literature review

It is acknowledged from the literature that the MG operation scheduling problem is an optimization problem with the aim of determining optimal trading of MG with the DISCO in a grid-connected mode. There exist some recent studies about investigating the operation of energy systems, especially under uncertainty. Nosratabadi *et al.* [30] comprehensively studied the scheduling problem of MGs and virtual power plant concepts from various aspects such as modeling techniques, solving methods, reliability, emission, uncertainty, stability, demand response, and multi-objective standpoint. Accordingly, this section reports some more recent, more relevant research papers to avoid duplication of effort. Considering multiple interconnected MGs in a certain environment, Lee *et al.* [23] studied the economic benefits of a hierarchical energy trading mechanism as a multi-leader multi-follower Stachelberg game. Based on a Stachelberg equiliberium and a heuristic solution approach, Ma *et al.* [25] analyzed a dynamic pricing and formulated a deterministic energy management optimization model for the joint operation of Combined Heat and Power (CHP) system and prosumers in a MG. Lu *et al.* [24] introduced a multi-objective optimal model for the MG under grid-connected mode considering the operation cost and the environmental protection cost of the system in the presence of electric vehicles. The uncertain nature of the vehicles charge-discharge behavior is fully neglected in the model. Umeozor and Trifkovic [34] presented models of energy devices in the MG and pose the energy management goal as an optimization problem. They exploited the bounded uncertainties in the renewable power through parameterizations approach to transform the problem from a Mixed-Integer Non-Linear Programming (MINLP) to a bi-level parametric Mixed-Integer Linear Programming (MILP). Fang *et al.* [12] fomulated a bi-level strategic scheduling model to maximize the profit of the load serving entities as the primary objective and minimize the cost of independent system operator's generation as the objective of the sub-problem. They addressed the uncertainty of renewable power such as wind in their model formulation. Although the optimization approaches are not considered to be robust, the models successfully consider the uncertain nature of the renewable power. The existance of the electric vehicles is also neglected in the formulation models. Laying stress on the inadequacy of the deterministic methods to provide a precise analysis of the MG operation, Nikmehr *et al.* [29] introduced a stochastic algorithm to deal with daily optimal scheduling problem of MGs. Gazijahani *et al.* [16] presented a new risk-based multi-objective energy exchange optimization for MGs from economic and reliability standpoints under uncertainty using CVaR approach. Characterizing the interactions between Distributed Network Operator (DNO) and clusters of MGs, Wang *et al.* [35] introduced a bi-level stochastic formulation to model the energy management problem taking into consideration the strategic behaviors of all entities and the intermittent outputs of renewable power. Developing a multiple-leader multiple-follower stochastic Stachelberg-based game-theoritic model, Mondal *et al.* [27] characterized the effect of storage in a home energy management system. Comparing stochastic optimization and RO approaches to energy management for residential appliances, Chen *et al.* [10] outlined that; although, both the approaches are fairly suitable, stochastic approache involves higher computational burden. Thus, an approach which has gained substantial attention in recent years is RO [20]. Considering uncertainties caused by intermittent renewable power and random loads, Wang *et al.* [36] proposed an integrated scheduling approach for MGs based on robust multi-objective optimization. Gazijahani and Salehi [13] suggested a novel RO approach to optimally design MGs considering reconfigurable topology. They addressed uncertainties in technical and economical information including a lack of full information on the nature of uncertainty. Bahramara and Golpîra [4] optimally formulated the MG operation problem in a robust manner. The resulting decision-making model is the first to cover the MG operation in the presence of PHEVs that addresses not only the solution robustness, but also model robustness. Using a robust two stage optimization model, Gazijahani and Salehi [14] examined an integrated MGs planning problem and time-based demand response program aiming to minimize the investment costs and maintaining the reliability of system in an acceptable level. Although the models are designed to be robust, the risk aversion level of the system operator (system DM) is neglected in their formulation. However, the attitude of the DM toward risk is a crucial issue in the process of decision-making under uncertainty and risk [19]. In compliance with this concept Morales *et al.* [28] reported that the DMs risk-aversion level has a remarkable impact on the amount of market-integrable wind resources in energy management systems. In this way, based on Stackelberg equilibria, Gazijahani and Salehi [15] proposed a cooperative game theory-based model to determine optimal participation of MGs aggregators in retail market to increase its own profit. They used Information Gap Decision Theory (IGDT) to protect the flexible decision variables of MGs power provision against unknown risk, allowing for the ambiguity and variability of uncertain parameters. Mehdizadeh *et al.* [26] proposed the IGDT to obtain the bidding strategy of the MG. The approach covers MGs attitude toward risk for upstream grid price uncertainty modeling. Rezaei *et al.* [32] presented a MILP framework to manage frequency excursions produced from load and renewable generation fluctuations. They firstly formulate the frequency-dependent behavior of the distributed energy resources and, then, utilize the IGDT to handle MGs uncertainties in a robust way. Although the foregoing research papers simultaniously reflect the uncertainty and the risk factors, the IGDT allows them considering only optimistic and pessimistic strategies and they are, therefore, limitted to consider the other levels of risk aversion of the DM.

Reviewing the aforementioned literature reveals that there exists a comprehensive literature about decentralizing power generation using MGs. However, there is a few effort to obtain decentralization in the system modeling and scheduling approach, especially in the presence of uncertainties. In this paper, a novel approach is introduced to formulate the grid-connected MG scheduling problem using BLP as a non-cooperative game formulation framework. The main motivation of the study is to integrate the BLP and the CVaR approach to obtain a decentralized framework in which the attitude of the DM toward risk is mathematicaly considered in a robust manner. In the proposed model, the following characteristics are successfully addressed: (1) Uncertain PHEVs electricity demand, (2) uncertain charge/discharge cost of the battery, (3) uncertain generation cost of DG, and (4) the DMs risk aversion level. It is remarkable that most, if not all, models in the literature considered part of the above characteristics only. The resulting bi-level MILP model incorporates BLP concept and a MG scheduling problem in a robust manner. The last aim is to build an optimal energy exchange regime between MG and the DISCO considering both technical and economical aspects in a robust and more reliable manner. The initial objective function contains two overall terms: (1) certain cost of any energy exchange between the MG and the DISCO, (2) uncertain costs of DG power generation and charging/discharging the batterry. Incorporating CVaR approach into the BLP to reformulate the second term enhances the reliability of the system through the idea of risk-based robustness against uncertainty. The leader of the hierarchical model is the DISCO, and the follower is the MG. Finally, the model is transformed into a single level MILP problem via the KKT conditions to obtain an exact analytical solution.

3. Model formulation

This section begins with a sub-section devoted to the model formulation and will be continued by reformulation of the model in a robust manner.

3.1. Mathematical model presentation

The resulting uncertain MILP formulation of the mentioned model is described through equations (3.1)–(3.10).

$$\text{Minimize}\,\xi = \sum_{t=1}^{T} C_t^{\text{grid,in}} P_t^{\text{grid,in}} - C_t^{\text{grid,out}} P_t^{\text{grid,out}} + \sum_{t=1}^{T}\left[\sum_{i=1}^{I} \tilde{C}_{i,t}^{\text{DG}} P_{i,t}^{\text{DG}} + \sum_{j=1}^{J} \tilde{C}_{j,t}^{\text{batt}} \left(P_{j,t}^{\text{charge}} + P_{j,t}^{\text{discharge}}\right)\right] \tag{3.1}$$

Subject to:

$$\sum_{i=1}^{I} P_{i,t}^{\text{DG}} + \sum_{j=1}^{J}\left(P_{j,t}^{\text{discharge}} \eta_j^{\text{discharge}} - \frac{P_{j,t}^{\text{charge}}}{\eta_j^{\text{charge}}}\right) + P_t^{\text{grid,in}} \eta^{\text{grid}} - \frac{P_t^{\text{grid,out}}}{\eta^{\text{grid}}} - P_t^{\text{Demand}} \geq \tilde{P}_t^{\text{PHEV}} \quad \forall t \tag{3.2}$$

$$P_{i,t}^{\text{DG,min}} U_{i,t} \leq P_{i,t}^{\text{DG}} \leq P_{i,t}^{\text{DG,max}} U_{i,t} \quad \forall i,t \tag{3.3}$$

$$0 \leq P_t^{\text{grid,in}} \leq P^{\text{grid,max}} X_t^{\text{grid}} \quad \forall t \tag{3.4}$$

$$0 \leq P_t^{\text{grid,out}} \leq P^{\text{grid,max}} (1 - X_t^{\text{grid}}) \quad \forall t \tag{3.5}$$

$$0 \leq P_{j,t}^{\text{charge}} \leq P_j^{\text{batt,max}} (1 - X_{j,t}^{\text{batt}}) \quad \forall j,t \tag{3.6}$$

$$0 \leq P_{j,t}^{\text{discharge}} \leq P_j^{\text{batt,max}} X_{j,t,}^{\text{batt}} \quad \forall j,t \tag{3.7}$$

$$E_j^{\text{batt,min}} \leq E_{j,t}^{\text{batt}} \leq E_j^{\text{batt,max}} \quad \forall j,t \tag{3.8}$$

$$E_{j,t,}^{\text{batt}} = E_{j,t-1}^{\text{batt}} + P_{j,t}^{\text{charge}} - P_{j,t}^{\text{discharge}} \quad \forall j, t > 1 \tag{3.9}$$

$$E_{j,t,}^{\text{batt}} = E_j^{\text{batt,ini}} + P_{j,t}^{\text{charge}} - P_{j,t}^{\text{discharge}} \quad \forall j, t = 1. \tag{3.10}$$

The objective function of the model is defined by equation (3.1) to minimize the operation cost of the MG in its operation time. The first term in equation (3.1) represents the cost of electricity exchange between the MG and the DISCO. The second term exhibits the uncertain costs of the power generation of DGs and charging/discharging of battery. Constraints (3.2) ensure that the power supply is within operational range. Since the problem is formulated from the view point of the MG operator, the power flow equations of the distribution network is not modeled in the problem and merely the power balance on the MG's bus is considered as equation (3.2). The minimum and maximum boundary of DGs power generation are defined by constraint set (3.3). The power exchange between the MG and the DISCO is limited using constraints (3.4) and (3.5). Technical constraints of battery are represented by constraints (3.6)–(3.10). As mentioned in constraints (3.3)–(3.8), the decision variables of the problem are denoted by $P_{i,t}^{\mathrm{DG}}$ as the amount of power generated by DG i at time t; $P_{j,t}^{\mathrm{discharge}}/P_{j,t}^{\mathrm{charge}}$ as the amount of power discharging/charging of jth battery at time t; $P_{t}^{\mathrm{grid,in}}/P_{t}^{\mathrm{grid,out}}$ as the amount of power purchased/sold from/to the main grid at time t; $E_{j,t}^{\mathrm{batt}}$ as the amount of power stored in jth battery at time t; $U_{i,t}$ as the binary variable used for on/off of DG i at time t; $X_{j,t}^{\mathrm{batt}}$ as a binary variable used for power charging/discharging status of jth battery at time t, and X_{t}^{grid} as a binary variable used for power purchased/sold from/to the main grid at time t.

3.2. Robust mathematical model

The model formulated in Section 3.1 aims at minimizing the total cost of the energy supply system, which is varying from scenario to scenario, regarding the uncertain parameters. An effective approach to deal with this bias is RO. Bertsimas and Brown [7] introduced an approach to construct uncertainty sets and, consequently, a robust counterpart for an uncertain linear optimization problem based on Coherent Risk Measure (CRM). CRM is a particular risk measure that satisfies the axioms of Convexity, Positive Homogeneity, Monotonicity, and Translation in Variance [21]. They focused on discrete probability space motivated by sampling considerations. This paper adopts a same approach to deal with the uncertainty of the system. Accordingly, equation (3.1) can be firstly transformed into equations (3.11) and (3.12). It is noticeable that capturing the uncertainty of the right hand side value ω needs to add an additional variable ς, and including ω as its coefficient in agreement with $\varsigma = 1$ as an additional constraint, proposed in Bertsimas and Brown [7].

$$\text{Minimize}\,\xi = \sum_{t=1}^{T} P_t^{\mathrm{grid,out}} C_t^{\mathrm{grid,out}} - P_t^{\mathrm{grid,in}} C_t^{\mathrm{grid,in}} + \omega\varsigma, \quad \varsigma = 1 \tag{3.11}$$

Subject to:

$$\sum_{t=1}^{T}\left[\sum_{i=1}^{I}\tilde{C}_{i,t}^{\mathrm{DG}}P_{i,t}^{\mathrm{DG}} + \sum_{j=1}^{J}\tilde{C}_{j,t}^{\mathrm{batt}}\left(P_{j,t}^{\mathrm{charge}} + P_{j,t}^{\mathrm{discharge}}\right)\right] \geq \omega \tag{3.12}$$

where $\tilde{\mathbf{C}}^{\mathbf{DG}} \in \mathrm{R}^{1\times I}, \tilde{\mathbf{C}}^{\mathbf{batt}} \in \mathrm{R}^{1\times J}$ are the uncertain matrices. The idea behind the concept of RO is that the design under this approach will be fairly good, *i.e.* robust, regardless of which scenario, *i.e.* $\mathbf{C}^{\mathbf{DG}}, \mathbf{C}^{\mathbf{batt}}$, is realized. To do so, this paper adopts the concept of CVaR based on the DMs, *i.e.* the MG Operator (MGO), attitude toward risk. To avoid technical details, the basics of the approach are not explained, but the necessary interpretations will be mentioned. Given $\boldsymbol{\mu}$ as a CRM, and uncertain vectors $\tilde{\mathbf{c}}^{\mathbf{DG}}, \tilde{\mathbf{c}}^{\mathbf{batt}}$ as random vectors in $\mathrm{R}^{1\times I}$, $\mathrm{R}^{1\times J}$, respectively on finite probability spaces. It is clear that $\mathbf{c}_s^{\mathbf{DG}} \triangleq \tilde{\mathbf{c}}_s^{\mathbf{DG}}, \mathbf{c}_s^{\mathbf{batt}} \triangleq \tilde{\mathbf{c}}_s^{\mathbf{batt}}$ and the support of $\tilde{\mathbf{c}}^{\mathbf{DG}}, \tilde{\mathbf{c}}^{\mathbf{batt}}$ can be defined as $\Psi^{\mathbf{DG}} = \left\{\mathbf{c}_1^{\mathbf{DG}}, \mathbf{c}_2^{\mathbf{DG}}, \ldots, \mathbf{c}_S^{\mathbf{DG}}\right\}$ and $\Psi^{\mathbf{batt}} = \left\{\mathbf{c}_1^{\mathbf{batt}}, \mathbf{c}_2^{\mathbf{batt}}, \ldots, \mathbf{c}_S^{\mathbf{batt}}\right\}$, respectively. $\Psi^{\mathbf{DG}}$ and $\Psi^{\mathbf{batt}}$ can be alternatively defined by $\Xi^{\mathbf{DG}} = \left[\mathbf{c}_1^{\mathbf{DG}}, \mathbf{c}_2^{\mathbf{DG}}, \ldots, \mathbf{c}_S^{\mathbf{DG}}\right]$ and $\Xi^{\mathbf{batt}} = \left[\mathbf{c}_1^{\mathbf{batt}}, \mathbf{c}_2^{\mathbf{batt}}, \ldots, \mathbf{c}_S^{\mathbf{batt}}\right]$ as their corresponding matrix form, respectively. These are the only information available about $\tilde{\mathbf{c}}^{\mathbf{DG}}, \tilde{\mathbf{c}}^{\mathbf{batt}}$. Based on constraint (3.12) a risk-based constraint of the form $\mu\left(\tilde{\mathbf{c}}^{\mathbf{DG}}\mathbf{x} + \tilde{\mathbf{c}}^{\mathbf{batt}}\mathbf{y} - \omega\right) \leq 0$ is formulated in which $\mathbf{x} = P_{i,t}^{\mathrm{DG}}$ and $\mathbf{y} = \left(P_{j,t}^{\mathrm{charge}}, P_{j,t}^{\mathrm{discharge}}\right), i = 1, 2, \ldots, I, j = 1, 2, \ldots, J$. In other words, the MGO, as a DM, wants to assign some levels of confidence to satisfy the constraint that gives rise to $p\left\{\tilde{\mathbf{c}}^{\mathbf{DG}}\mathbf{x} + \tilde{\mathbf{c}}^{\mathbf{batt}}\mathbf{y} - \omega\right\} \geq 1 - \alpha$, which affects the convexity of the problem at all. However, it can be transformed using CVaR approach that leads to $\mathrm{CVaR}_\alpha\left\{\tilde{\mathbf{c}}^{\mathbf{DG}}\mathbf{x} + \tilde{\mathbf{c}}^{\mathbf{batt}}\mathbf{y} - \omega\right\} \leq 0$ as its convex counterpart. As a result, equation (3.13) can be formed according to CRMs axioms.

$$\mathrm{CVaR}_\alpha\left\{\tilde{\mathbf{c}}^{\mathbf{DG}}\mathbf{x} + \tilde{\mathbf{c}}^{\mathbf{batt}}\mathbf{y} - \omega\right\} = -\inf_{\{V\in\Delta^s : v_s \leq 1/(S_\alpha)\}} E_V\left[\mathbf{c}^{\mathbf{DG}}\mathbf{x} + \mathbf{c}^{\mathbf{batt}}\mathbf{y}\right] + \omega, \Delta^s \triangleq \left\{\mathbf{p} \in \mathrm{R}_+^s : \mathbf{e}^t\mathbf{p} = 1\right\}, \tag{3.13}$$

where $\mathbf{e}$ is the vector of ones and S_α is the number of scenarios remaining after trimming to the level α (*i.e.* $S_\alpha = \lfloor S.(1-\alpha)+\alpha \rfloor \approx S.(1-\alpha)$). $\chi_{(S_\alpha)}$ is the S_α^{th} best component regarding the objective of the problem. Accordingly, CRM can be formulated as $\sum_{s=1}^{S} \theta_s \text{CVaR}_\alpha \left\{ \tilde{\mathbf{c}}^{\mathbf{DG}}\mathbf{x} + \tilde{\mathbf{c}}^{\mathbf{batt}}\mathbf{y} - \omega \right\}$, which resulting in equation (3.14).

$$\sum_{s=1}^{S} \theta_s \min_{\mathbf{c}_s^{\mathbf{DG}} \in \prod_{\mathbf{p}_{\mathbf{DG}}^{\mathbf{s}}} (\Psi^{\mathbf{DG}}), \mathbf{c}_s^{\mathbf{batt}} \in \prod_{\mathbf{p}_{\mathbf{batt}}^{\mathbf{s}}} (\Psi^{\mathbf{batt}})} \text{CVaR}_\alpha \left\{ \tilde{\mathbf{c}}^{\mathbf{DG}}\mathbf{x} + \tilde{\mathbf{c}}^{\mathbf{batt}}\mathbf{y} \right\} + \omega. \tag{3.14}$$

In equation (3.14), for example $\prod_{\mathbf{p}_{\mathbf{batt}}^{\mathbf{s}}} \left(\Psi^{\mathbf{batt}}\right) = \text{conv}\left(\left\{ \sum_{s=1}^{S} p_{\sigma(s)} \mathbf{c}_s^{\mathbf{DG}} : \sigma \in \rho(s) \right\} \right)$is the $\mathbf{P} - permuthall$ of $\Psi^{\mathbf{batt}}$ and $\rho(s)$ is introduced to define all permutations of S elements. θ_s is defined by equation (3.15), where $\mathbf{P} = \sum_{s=1}^{S} \theta_s \mathbf{P}_s$, in which $\mathbf{P} \in \Delta^s$ and $\mathbf{P}_s \leq \mathbf{P}_{s+1}$.

$$\theta_s = \begin{cases} s.p_s & s = S \\ (S-s).(p_{S-s} - p_{S-s+1}) & s = 1, 2, \ldots, S-1 \end{cases}, \sum_{s=1}^{S} \theta_s = 1. \tag{3.15}$$

Accordingly, the new coefficients, *i.e.* $C_{i,t,s}^{\text{DG}\prime}$, $C_{j,t,s}^{\text{batt}\prime}$ are calculated to be substituted by the corresponding initial coefficients and, therefore, constraint (3.12) is transformed into constraint (3.16), regarding constraint (3.14).

$$\text{Min}\xi' = \left\{ \sum_{t=1}^{T} \left(\sum_{i=1}^{I} \sum_{s=1}^{S} p_{\sigma(s)} \left(C_{i,t,s}^{\text{DG}\prime} P_{i,t}^{\text{DG}} \right) + \sum_{j=1}^{J} \sum_{s=1}^{S} p_{\sigma(s)} \left(C_{j,t,s}^{\text{batt}\prime} \left(P_{j,t}^{\text{charge}} + P_{j,t}^{\text{discharge}} \right) \right) \right) \right\} + \omega. \tag{3.16}$$

The power demand of PHEVs in constraint set (3.2), on the other hand, is uncertain. The "data-driven" approach, used in this paper, is originally developed to avoid of considering complicated distributional assumptions and specific pattern for the uncertain input data. The approach is well suited to a practical setting, in which the realizations of the uncertain parameters are the only information available. Leveraging the data-driven approach, more like real life situations, this paper assumes that the only information on the uncertain vector $\tilde{\mathbf{P}}_t^{\text{PHEV}}$ is the finite set of sampled vector ${P_t^{\text{PHEV}}}_s$ with no specific pattern for PHEVs drivers' behavior. Thus, the paper relies on a discrete space for the PHEVs demand possibilities over the time horizon that is sub-divided into 24 time slots each representing one hour. So, all available realizations of PHEVs demand per time slot are defined over scenarios, as ${P_t^{\text{PHEV}}}_s \in \left\{ {P_t^{\text{PHEV}}}_1, {P_t^{\text{PHEV}}}_2, \ldots, {P_t^{\text{PHEV}}}_S \right\}$, where S is the total number of realizations available. In line with this approach, equation (3.17) is employed to transform the uncertain demand to the appropriate certain demand where, S_α is the number of cases remaining after trimming to the level α $\quad (S_\alpha = \lfloor S \times (1-\alpha) + \alpha \rfloor \approx S \times (1-\alpha))$ and ${P_t^{\text{PHEV}}}_{(s)}$ is the s^{th} smallest component of ${P_t^{\text{PHEV}}}_1, {P_t^{\text{PHEV}}}_2, \ldots, {P_t^{\text{PHEV}}}_S$, yielding ${P_t^{\text{PHEV}}}_1 \leq {P_t^{\text{PHEV}}}_2 \leq \ldots \leq {P_t^{\text{PHEV}}}_S$.

$$\tilde{P}_t^{\text{PHEV}} = \frac{1}{s_\alpha} \sum_{s=1}^{\lfloor s_\alpha \rfloor} {P_t^{\text{PHEV}}}_{(s)} - \left(\frac{s_\alpha - \lfloor s_\alpha \rfloor}{\lfloor s_\alpha \rfloor} \right) {P_t^{\text{PHEV}}}_{(\lceil s_\alpha \rceil)} \forall t. \tag{3.17}$$

Taking into account equations (3.11), (3.16), and (3.17) makes the model a bi-level model.

3.3. Bi-level programming formulation

BLP was originally developed as Stackelberg game and was later generalized by several researchers in various disciplines such as supply chain network design [21] and cooperative transmission [3]. This game is based on the hierarchical relationship between two autonomous and possibly conflicting DMs, called leader and follower. In this game, a leader makes the first move, knowing that the follower tracks its action, and the follower reacts optimally. In this paper, the DISCO as the leader determines the prices of power exchange with MG regarding which the power trading between the DISCO and the MG is determined. Then, the MG as the follower DM, decides on optimal scheduling of the DGs and charging/discharging of battery as described in Figure 1.

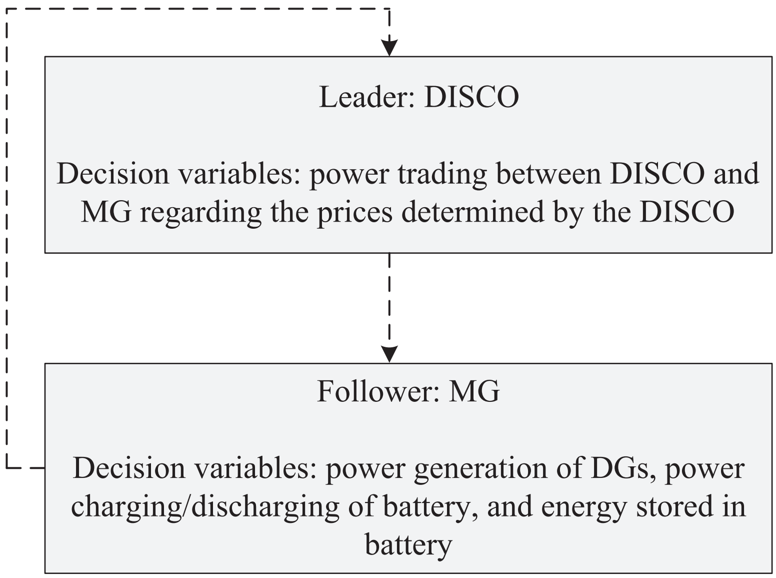

FIGURE 1. The Resulting bi-level appraoch to model the uncertainty in the MG problem.

For a more complete view of this hierarchical theory, it would be appropriate to at least refer the reader to see [11]. Several methods are also suggested by researchers to transform the initial BLP into its single level counterpart. In this regard, KKT conditions are more and more taken into consideration. Using the KKT conditions approach, the model's lower level (Eqs. (3.18)–(3.26)) is completely replaced by its KKT conditions. Constraints (3.4) and (3.5) are embedded in the model's upper level and the others are remaining for the model's lower level. The resulting BLP problem is then ready to solve adopting KKT conditions approach. The KKT conditions for model's lower level are represented through equations (3.27)–(3.35).

$$\text{Minimize}\, \xi = \sum_{t=1}^{T} P_t^{\text{grid,out}} C_t^{\text{grid,out}} - P_t^{\text{grid,in}} C_t^{\text{grid,in}} + \omega \tag{3.11}$$

$$0 \le P_t^{\text{grid,in}} \le P^{\text{grid,max}} X_t^{\text{grid}} \quad \forall t \tag{3.4}$$

$$0 \le P_t^{\text{grid,out}} \le P^{\text{grid,max}} (1 - X_t^{\text{grid}}) \quad \forall t \tag{3.5}$$

$$\text{Minimize}\, \xi' = \left\{ \sum_{t=1}^{T} \left(\sum_{i=1}^{I} C_{i,t}^{\text{DG}} P_{i,t}^{\text{DG}} + \sum_{j=1}^{J} C_{j,t}^{\text{batt}} \left(P_{j,t}^{\text{charge}} + P_{j,t}^{\text{discharge}} \right) \right) \right\} + \omega \tag{3.18}$$

$$\sum_{i=1}^{I} P_{i,t}^{\text{DG}} + \sum_{j=1}^{J} \left(P_{j,t}^{\text{discharge}} \eta_j^{\text{discharge}} - \frac{P_{j,t}^{\text{charge}}}{\eta_j^{\text{charge}}} \right) + P_t^{\text{grid,in}} \eta^{\text{grid}} - \frac{P_t^{\text{grid,out}}}{\eta^{\text{grid}}} - P_t^{\text{Demand}} \ge \tilde{P}_t^{\text{PHEV}} \ \forall t : \lambda_t^1 \tag{3.19}$$

$$\tilde{P}_t^{\text{PHEV}} = \frac{1}{s_\alpha} \sum_{s=1}^{\lfloor s_\alpha \rfloor} P_t^{\text{PHEV}}{}_{(s)} - \left(\frac{s_\alpha - \lfloor s_\alpha \rfloor}{\lfloor s_\alpha \rfloor} \right) P_t^{\text{PHEV}}{}_{(\lceil s_\alpha \rceil)} \ \forall t, \tag{3.20}$$

$$E_{j,t}^{\text{batt}} = E_j^{\text{batt,ini}} + P_{j,t}^{\text{charge}} - P_{j,t}^{\text{discharge}} \ \forall j, t = 1, \lambda_t^2 \tag{3.21}$$

$$E_{j,t}^{\text{batt}} = E_{j,t-1}^{\text{batt}} + P_{j,t}^{\text{charge}} - P_{j,t}^{\text{discharge}} \ \forall j, t > 1, \lambda_t^3 \tag{3.22}$$

$$E_j^{\text{batt,min}} \le E_{j,t}^{\text{batt}} \le E_j^{\text{batt,max}} \ \forall j, t, \mu_{j,t}^{\text{energy}}, \bar{\mu}_{j,t}^{\text{energy}} \tag{3.23}$$

$$0 \le P_{j,t}^{\text{discharge}} \le P_j^{\text{batt,max}} \ \forall j, t, \mu_{j,t}^{\text{dis}}, \bar{\mu}_{j,t}^{\text{dis}} \tag{3.24}$$

$$0 \le P_{j,t}^{\text{charge}} \le P_j^{\text{batt,max}} \ \forall j, t, \mu_{j,t}^{\text{ch}}, \bar{\mu}_{j,t}^{\text{ch}} \tag{3.25}$$

$$P_{i,t}^{\text{DG,min}} \le P_{i,t}^{\text{DG}} \le P_{i,t}^{\text{DG,max}} \ \forall i, t, \mu_{j,t}^{\text{DG}}, \bar{\mu}_{j,t}^{\text{DG}}. \tag{3.26}$$

The final model is described as follows:

$$\text{Minimize}\, \xi = \sum_{t=1}^{T} P_t^{\text{grid,out}} C_t^{\text{grid,out}} - P_t^{\text{grid,in}} C_t^{\text{grid,in}} + \omega\varsigma, \varsigma = 1 \tag{3.11}$$

Subject to:

(3.4), (3.5)

$$C_{i,t}^{\text{DG}'} + \lambda_t^1 - \mu_{j,t}^{\text{DG}} + \bar{\mu}_{j,t}^{\text{DG}} = 0 \tag{3.27}$$

$$C_{j,t}^{\text{batt}'} + \lambda_t^1 \eta_j^{\text{discharge}} + \lambda_t^2\big|_{t=1} + \lambda_t^3\big|_{t>1} - \mu_{j,t}^{\text{dis}} + \bar{\mu}_{j,t}^{\text{dis}} = 0 \tag{3.28}$$

$$C_{j,t}^{\text{batt}'} - \frac{\lambda_t^1}{\eta_j^{\text{charge}}} - \lambda_t^2\big|_{t=1} - \lambda_t^3\big|_{t>1} - \mu_{j,t}^{\text{ch}} + \bar{\mu}_{j,t}^{\text{ch}} = 0 \tag{3.29}$$

$$\lambda_t^2\big|_{t=1} + \lambda_t^3\big|_{t>1} - \lambda_{t+1}^3 - \mu_{j,t}^{\text{energy}} + \bar{\mu}_{j,t}^{\text{energy}} = 0 \tag{3.30}$$

(3.19)–(3.26)

$$\mu_{j,t}^{\text{energy}} \ge 0, \bar{\mu}_{j,t}^{\text{energy}} \ge 0 \tag{3.31}$$

$$\mu_{j,t}^{\text{dis}} \ge 0, \bar{\mu}_{j,t}^{\text{dis}} \ge 0 \tag{3.32}$$

$$\mu_{j,t}^{\text{ch}} \ge 0, \bar{\mu}_{j,t}^{\text{ch}} \ge 0 \tag{3.33}$$

$$\mu_{i,t}^{\text{DG}} \ge 0, \bar{\mu}_{i,t}^{\text{DG}} \ge 0 \tag{3.34}$$

$$\begin{aligned}\sum_{t=1}^{T} \left(\sum_{i=1}^{I} C_{i,t}^{\text{DG}'} P_{i,t}^{\text{DG}} + \sum_{j=1}^{J} C_{j,t}^{\text{batt}'} \left(P_{j,t}^{\text{charge}} + P_{j,t}^{\text{discharge}} \right) \right) - \omega = \sum_{t=1}^{T} (P_t^{\text{Demand}} + \tilde{P}_t^{\text{PHEV}})\lambda_t^1 \\ + \sum_{t=1}^{T} \sum_{j=1}^{J} \left[E_j^{\text{batt,ini}} \lambda_t^2 + E_j^{\text{batt,min}} \mu_{j,t}^{\text{energy}} - \bar{\mu}_{j,t}^{\text{energy}} E_j^{\text{batt,max}} - \bar{\mu}_{j,t}^{\text{dis}} P_j^{\text{batt,max}} - \bar{\mu}_{j,t}^{\text{ch}} P_j^{\text{batt,max}} \right] \\ \times \sum_{t=1}^{T} \sum_{i=1}^{I} \left[\mu_{i,t}^{\text{DG}} P_{i,t}^{\text{DG,min}} - \bar{\mu}_{i,t}^{\text{DG}} P_{i,t}^{\text{DG,max}} \right].\end{aligned} \tag{3.35}$$

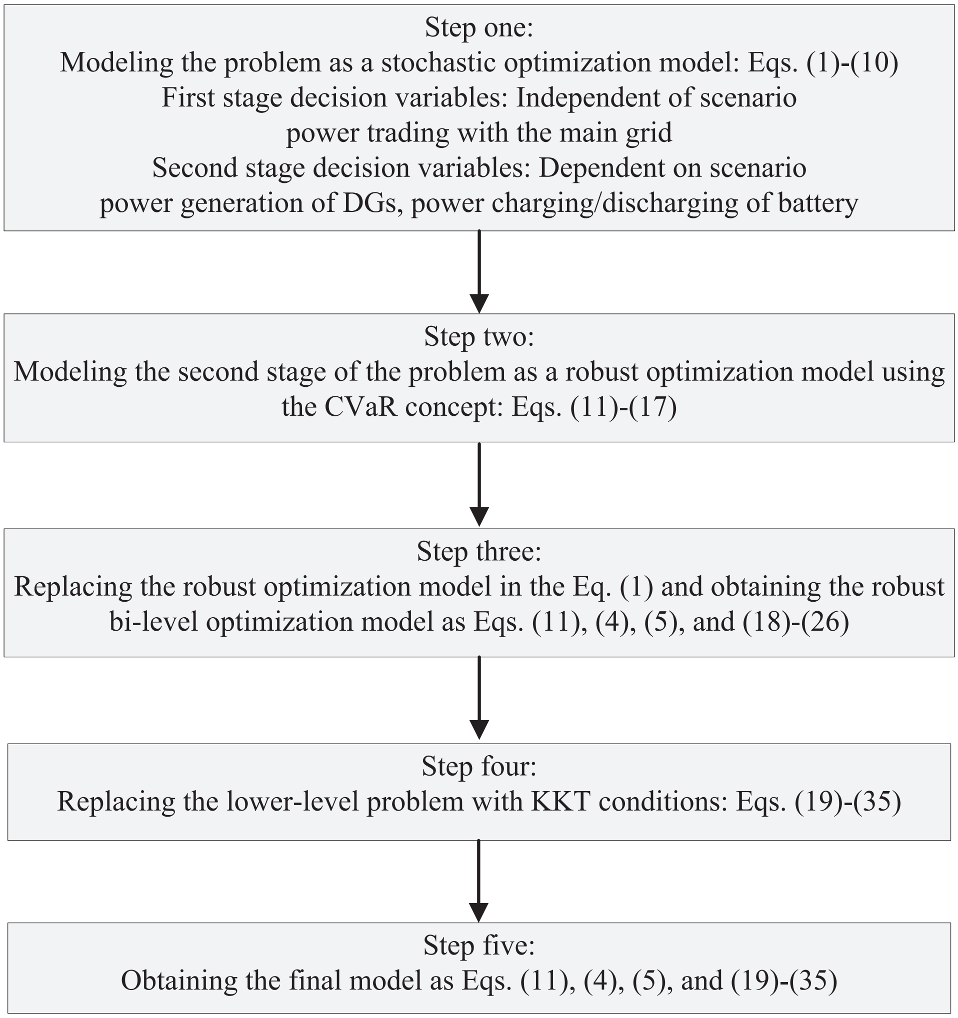

Figure 2. The proposed flowchart to model the operation problem of the MG as a robust single-level optimization one.

3.4. The proposed approach

In this paper, the operation problem of a MG consisting of DGs, PHEVs and energy storages is modeled as a robust bi-level optimization problem as shown in Figure 2. Since the cost of DGs, the cost of battery, and the energy consumptions of the PHEVs are considered as uncertain parameters, the operation problem of the MG is modeled as a two-stage stochastic optimization problem. The decision variables consist of purchased/sold power of the MG from/to the DISCO, which do not depend on realization of the stochastic process are considered as the first-stage or here-and-now decisions. The power generation of the DGs and power charging/discharging of the battery are considered as the second stage or wait-and-see decision variables that depend on the actual realization of the stochastic process. Since the cost of DGs, the cost of battery, and the energy consumptions of the PHEVs are considered as stochastic parameters, the operation problem of the MG is modeled as an uncertain optimization problem. The first stage decision variables are purchased/sold power of the MG from/to the DISCO, which do not depend on realization of the stochastic process. The power generation of the DGs and

Table 1. Characterictics of MGs eqiupment.

Parameter	Value
PHEVs electricity consumption (kWh)	Unif (35,45)
Generation cost of DG1 (€ct/kWh)	Unif (2.5,3.5)
Generation cost of DG2 (€ct/kWh)	Unif (1.5,2.5)
Charging/discharging cost of battery (€ct/kWh)	Unif (0.7,1)
Min, Max DG1 capacity limit (kW)	6, 60
Min, Max DG2 capacity limit (kW)	3, 60
Maximum limit of power exchange with main grid (kW)	200
Min, Max energy stored in battery (kWh)	12, 30
Initial energy stored in battery (kWh)	12
Charging/discharging efficiency of battery	0.98
Efficiency of grid's transformer	0.98

Table 2. Time dependent input parameters.

Time (h)	Demand (kW)	Price of power exchange (€ct/kWh)
1	26.33	3.949
2	25.37	3.305
3	23.84	3
4	20.97	2.782
5	19.44	2.791
6	21.91	2.744
7	30.31	3.147
8	40.42	3.752
9	42.52	4.033
10	45	6.601
11	49.77	8.26
12	50.34	7.54
13	55.3	7.95
14	59.88	8
15	61.02	7
16	54.91	6.5
17	40.38	7.49
18	65.6	10
19	85.1	11.5
20	99.62	9.365
21	90.06	7.498
22	75.34	6.5
23	55.26	4.797
24	30.6	3.406

power charging/discharging of the battery are considered as the second stage decision variables that depend on the actual realization of the stochastic process. Therefore, the objective function of the problem described in equation (3.1) has the two terms consisting of the certain and uncertain terms.

To deal with the uncertain parameters, the second term of equation (3.1) is replaced with variable ω and a constraint is formulated as equation (3.12) using the RO concept. Therefore, equation (3.1) is replaced with equations (3.11) and (3.17). The resulting model from the previous step is a robust bi-level optimization, which can be transformed into a robust single-level problem using KKT conditions. The final model is described as equations (3.11), (3.4), (3.5), (3.19)–(3.35).

TABLE 3. Results of optimization for different values of α.

#	α	Total power generation of DG1	Total power generation of DG2	Total power charging of battery	Total power discharging of battery	Purchased power from grid	Power sold to grid	Total cost
1	0.01	1440	1440	38	36	40.53	655.70	3130.193
2	0.02	1440	1440	38	36	40.47	656.52	3134.05
3	0.03	1440	1440	38	36	40.37	657.95	3137.42
4	0.04	1440	1440	38	36	40.14	658.15	3155.30
5	0.05	1440	1440	38	36	39.42	660.21	3159.11
6	0.06	1440	1440	38	36	39.66	661.24	3167.03
7	0.07	1440	1440	38	36	40.24	661.64	3179.40
8	0.08	1440	1440	38	36	39.69	662.53	3181.12
9	0.09	1440	1440	38	36	39.95	660.70	3190.16
10	0.10	1440	1440	20	18	39.04	666.96	3201.10
11	0.90	1278	1440	20	18	36.21	596.31	3740.81
12	0.91	1278	1440	20	18	35.21	597.53	3746.41
13	0.92	1278	1440	20	18	35.31	597.15	3749.77
14	0.93	1224	1440	20	18	34.61	548.06	3751.59
15	0.94	1224	1440	20	18	34.70	548.68	3761.18
16	0.95	1224	1440	20	18	34.10	546.04	3767.22
17	0.96	1224	1440	20	18	33.75	552.38	3772.22
18	0.97	1224	1440	20	18	33.37	550.91	3778.66
19	0.98	1224	1440	20	18	33.43	549.22	3801.50
20	0.99	1224	1440	20	18	33.28	553.13	3802.60

4. Computational results, discussion and robust solution analysis

The MG under study in this paper consists of two DGs, one energy storage, and PHEVs. The PHEVs electricity consumption, characteristics of MG's equipment, and cost of DGs and energy storage are given in Table 1. Total demand of the MG is sum of the electricity demand, which is given in Table 2, and electricity consumption of PHEVs, which is generated randomly in each scenario. The price of power exchange between the MG and the main grid is also given in Table 2. As aforementioned, the final proposed framework is a MILP problem. The model is solved in GAMS v.24.7.1 using CPLEX 11.1.1. Simulations are run on an Intel(R) Core (TM) i7-6700HQ CPU @ 2.60 GHz with 16 GB memory. The problem consists of 675 single variables, 48 discrete variables and 2140 non-zero coefficients. Simulation of the model using 100 scenarios takes approximately 100 seconds that is allowing the current framework to be configured for real time applications with much shorter time intervals. Before proceeding, it is needed to clarify why the number of scenarios is set to 100. Developing a theorem named Reduced Robust Problem, Thiele [33] discussed that if the DM keeps the S_α worst cases among the realizations, he/she has observed so far, and has observed a number $S \geq S_\alpha$, which can be very large, he/she only needs to consider $S_\alpha + 1$ scenarios. Accordingly; although, any number of scenarios is allowed, the literature suggests that it is better to set the number of scenarios to $S_\alpha + 1$ [17, 18, 20, 21]. Golpîra, Khan and Zhang [22] further outlined that, although, the greater number of scenarios has no effect on the results, it has negative effect on the simulation run time. Thus, it is reasonable to consider only 100 scenarios with respect to the available values of the parameterα, *i.e.* $\alpha \in (0, 1)$.

Obtaining more comprehensive insight, Table 3 reflects the results in a wide range of the DMs risk aversion level. According to the wide range of the DMs risk aversion level, *i.e.* $0 < \alpha < 1$, the similarity of the results

Table 4. Optimal scheduling of MGs equipment and power exchange with grid in $\alpha = 0.01$.

Time (h)	Power generation of DG1 (kW)	Power generation of DG2 (kW)	Power charging of battery (kW)	Power discharging of battery (kW)	Power purchased from grid	Power sold to the grid (kW)	Power demand (kW)
1	60	60	2	0	0	45.75	71.28
2	60	60	0	0	0	48.67	70.34
3	60	60	0	0	0	50.15	68.83
4	60	60	0	0	0	52.98	65.94
5	60	60	0	0	0	54.51	64.38
6	60	60	18	0	2.92	34.25	66.68
7	60	60	0	0	0	43.9	75.2
8	60	60	0	0	0	34.08	85.23
9	60	60	0	0	0	31.93	87.43
10	60	60	0	0	0	29.43	89.98
11	60	60	0	18	0	42.22	90.55
12	60	60	0	0	0	24.12	95.29
13	60	60	0	0	0	19.33	100.27
14	60	60	0	0	0	19.73	104.85
15	60	60	0	0	0	13.9	105.82
16	60	60	18	0	0	1.75	99.85
17	60	60	0	0	0	34.03	85.28
18	60	60	0	0	0	9.25	110.56
19	60	60	0	18	0	7.44	130.05
20	60	60	0	0	25.06	0	144.55
21	60	60	0	0	15.28	0	134.97
22	60	60	0	0	0.2	0	120.2
23	60	60	0	0	0	19.5	100.11
24	60	60	0	0	0	43.58	75.54
sum	1440	1440	38	36	40.53	655.7	2247.2

reported in Table 3, and due to time and computer resource limitations, the detailed results are preferably illustrated at only two common risk aversion levels, *i.e.* $\alpha = 0.01, \alpha = 0.99$. These levels of α can better reflect the effect of DMs levels of risk aversion on the results and, therefore, better illustrate the validity and superiority of the proposed framework. In this way, the operation results for $\alpha = 0.01$ and $\alpha = 0.99$ are given in Tables 4 and 5, respectively.

As shown in Table 4, the MGO purchases energy from the DISCO in hour 6 due to low energy price of the market and in hours 20–22 due to high-energy demand of the MG. In other hours, the MGO dispatches DGs in their maximum capacity, meets its demand from these resources, and sells its extra energy to the DISCO. Moreover, the MGO charges its battery in hours 2, 6, and 16 due to low market energy prices and discharge it in hours 11 and 19 due to high market energy prices to meet its demand.

As shown in Table 5, on the other hand, the MGO purchases energy from the DISCO in hour 6 due to low energy price of the market and in hours 20 and 21 due to high demand consumption of the MG. In other hours, the MGO dispatches DGs to meet its demand from these resources and sells its extra energy to the DISCO. Moreover, the MGO charges its battery in hours 2 and 4–6 due to low market energy prices and discharge it in hour 19 due to high market energy prices to meet its demand.

As shown in Tables 4 and 5, in $\alpha = 0.01$ where the MGO accepts high risk in its decision-making, its power sold to the main grid is greater than $\alpha = 0.99$ where the MGO dispatches low risk. Moreover, in $\alpha = 0.01$ the

TABLE 5. Optimal scheduling of MGs equipment and power exchange with grid in $\alpha = 0.99$.

Time (h)	Power generation of DG1 (kW)	Power generation of DG2 (kW)	Power charging of battery (kW)	Power discharging of battery (kW)	Power purchased from grid (kW)	Power sold to the grid (kW)	Power demand (kW)
1	60	60	2	0	0	50.555	66.38
2	60	60	0	0	0	53.74	65.16
3	6	60	0	0	0	1.83	64.14
4	6	60	5	0	0	0	60.9
5	6	60	6.289	0	0	0	59.59
6	6	60	6.71	0	2.92	0	62
7	60	60	0	0	0	48	71
8	60	60	0	0	0	38.58	80.63
9	60	60	0	0	0	36.82	82.44
10	60	60	0	0	0	33.79	85.52
11	60	60	0	0	0	28.96	90.45
12	60	60	0	0	0	29.26	90.14
13	60	60	0	0	0	23.93	95.59
14	60	60	0	0	0	19.73	99.87
15	60	60	0	0	0	18.65	100.97
16	60	60	0	0	0	24.43	95.08
17	60	60	0	0	0	38.5	80.72
18	60	60	0	0	0	13.5	106.22
19	60	60	0	18	0	12.1	125.3
20	60	60	0	0	20.16	0	139.76
21	60	60	0	0	10.35	0	130.14
22	60	60	0	0	0	4.16	115.75
23	60	60	0	0	0	24.18	95.32
24	60	60	0	0	0	48.54	70.47
sum	1224	1440	20	18	33.44	549.23	2133.6

power generation of DGs and power charging/discharging of the battery are greater than $\alpha = 0.99$ due to the different risk aversion level of the MGO.

To clearly check the sensitivity of the reported results to various levels of DMs risk aversion, Figures 2 and 3 are presented containing the regression line for $0.01 \leq \alpha \leq 0.1$ and $0.9 \leq \alpha \leq 0.99$. From the mathematical viewpoint, the greater the value of the α, the higher the risk aversion level of the DM. The DM with highest risk aversion level makes the decision under the worst-case. The worst-case is the situation in which the amount of the uncertain parameter, *i.e.* $\tilde{P}_t^{\text{PHEV}}$ in the proposed model, takes the worst value, *i.e.* greater value, regarding the constraint, *i.e.* equation (3.19), satisfaction. This leads to greater power generation, which causes the greater amount of cost regarding the objective function of the model, *i.e.* equation (3.11).

On the other hand, when $\alpha = 0.01$ the MGO accepts high risk in decisions, which means that the MGO schedules its resources and trading power with the main grid without considering the occurrence of the worst scenarios. Therefore, the MGO has the lowest operation cost in its period of the operation. With increasing the value of α, the risk of the MGO decreases and the MGO sells lower energy to the main grid and utilization of its resources decreases to consider the effect of the worst scenarios. So, the operation cost of the MGO increases and in $\alpha = 0.99$ the MGO has the most operation cost.

Not only does Figures 3 and 4 illustrate a logical relationship between the system costs and the DMs risk aversion level, but it also obtains some more details about it. It clearly indicates a strong linear relationship, *i.e.* $R^2 = 0.9841$ for $0.01 \leq \alpha \leq 0.1$ and $R^2 = 0.9399$ for $0.9 \leq \alpha \leq 0.99$, between the cost and the various values of α.

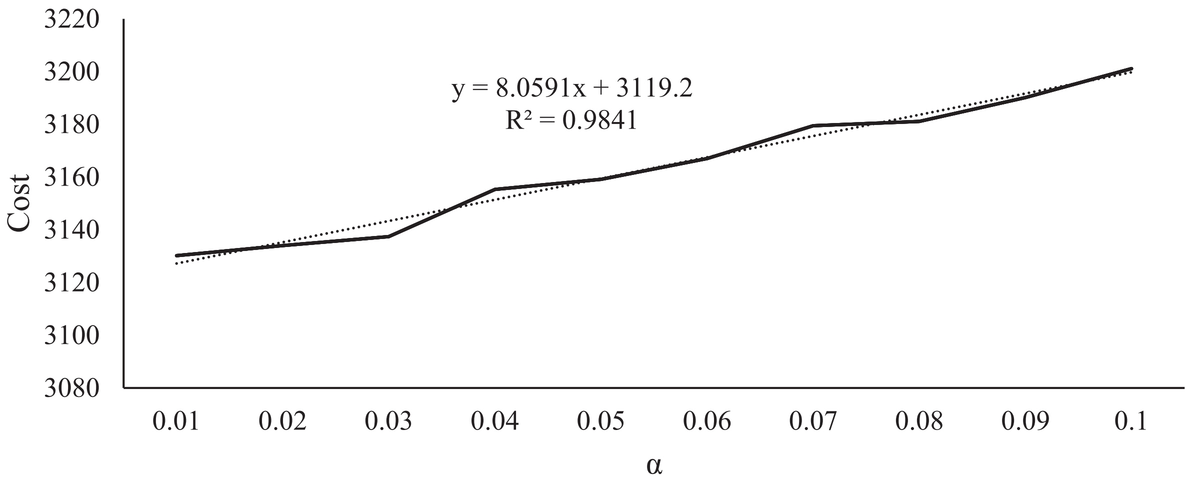

FIGURE 3. Sensitivity of operation cost with respect to$0.01 \leq \alpha \leq 0.1$.

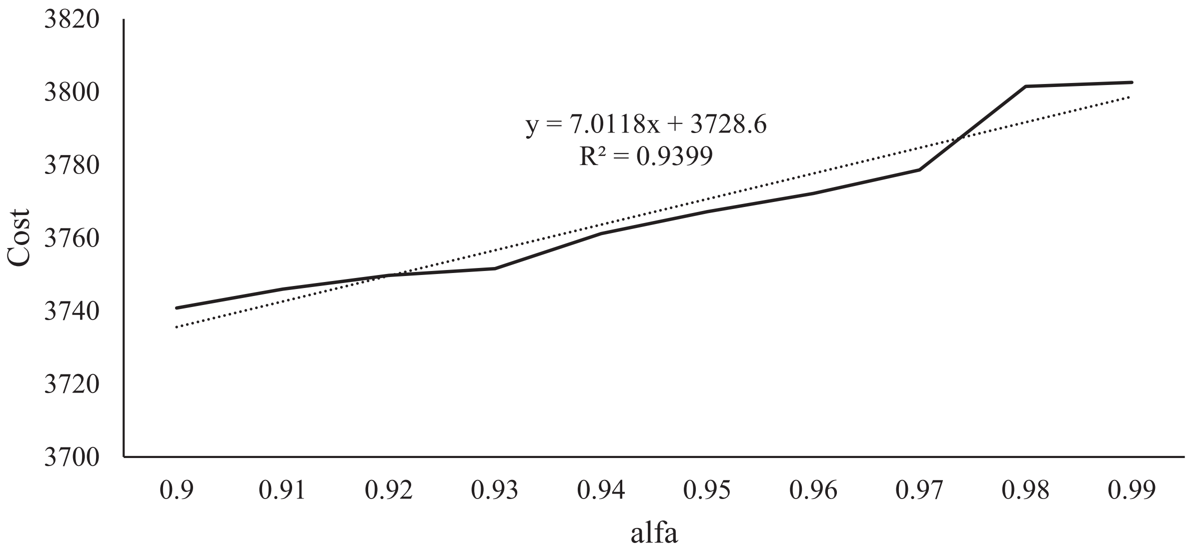

FIGURE 4. Sensitivity of operation cost with respect to $0.9 \leq \alpha \leq 0.99$.

On the other hand, in order to justify the robustness of the presented model, α should define an upper bound for cost variation. As one can see, the cost variability for the ten lower level of risk aversion and for the ten upper level of risk aversion is approximately 70 and 60. The mean cost variability is then equals to very small negligible value, *i.e.* 0.19%, per unit of the parameter α, *i.e.* 0.01. This behavior of the MGO clearly shows the solution robustness of the proposed model.

According to the results, an extensive goodness-of-fit analysis can also be carried out to test which probability distribution fits, satisfactorily, the empirical distribution of the economic costs and power exchange between the MG and the DISCO. Following the concepts introduced in [1,8,9], the KS, CS, and AD tests are done for various probability distributions, based on a set of results obtained from the simulations for a hundred times, graphically outlined in Figure 5.

TABLE 6. The results obtained for the KS, AD, and CS tests with respect to $\beta = 0.05$.

PDF		ξ			$P^{\text{grid,in}}$			$P^{\text{grid,out}*}$		
Beta	Critical value	KS	AD	CS	KS	AD	CS	KS	AD	CS
		0.13403	2.5018	12.592	0.13403	2.5018	12.592	0.13403	2.5018	12.592
	Statistic value	KS	AD	CS	KS	AD	CS	KS	AD	CS
		0.12443	3.395	13.12	0.08759	0.72809	11.891	0.07007	1.0627	8.5952
Hypothesis final result		Accept	Reject	Reject	Accept	Accept	Accept	Accept	Accept	Accept
Gumbel Max	Critical value	KS	AD	CS	KS	AD	CS	• The only distribution can be fitted to the results obtained from the $P^{\text{grid,out}}$ is Cauchy. Almost all of other distributions are not fitted to the results for this decision variable in almost all of the αvalues.		
		0.13403	2.5018	12.592	0.13403	2.5018	12.592			
	Statistic value	KS	AD	CS	KS	AD	CS			
		0.10123	2.0246	4.1456	0.13253	3.1927	14.038			
Hypothesis final result		Accept	Accept	Accept	Accept	Reject	Reject			
Normal	Critical value	KS	AD	CS	KS	AD	CS			
		0.13403	2.5018	12.592	0.13403	2.5018	12.592			
	Statistic value	KS	AD	CS	KS	AD	CS			
		0.10354	1.1491	10.609	0.08829	0.74522	11.895			
Hypothesis final result		Accept	Accept	Accept	Accept	Accept	Accept			
Weibull	Critical value	KS	AD	CS	KS	AD	CS			
		0.13403	2.5018	11.07	0.13403	2.5018	12.592			
	Statistic value	KS	AD	CS	KS	AD	CS			
		0.1449	4.6379	21.726	0.07208	1.5701	1.8609			
Hypothesis final result		Reject	Reject	Reject	Accept	Accept	Accept			

Given the null and the alternative hypotheses as: H_0 : the data follow the specified distribution, and H_1 : the data do not follow the specified distribution. Based on the empirical Cumulative Distribution Function (CDF), the KS test is used to decide if a sample comes from a hypothesized continuous distribution. The KS statistic is defined based on the largest vertical difference between the theoretical and the empirical CDF. The CS test is also used to determine if a sample comes from a population with a specific distribution. This test is applied to binned data, so the value of the test statistic depends on how the data is binned. For both the tests, the hypothesis regarding the distributional form is rejected at a significance level β if the test statistic is greater than the critical value. The fixed values of χ, which is typically sets to 0.05, are also used to evaluate the null hypothesis at various significance levels. The P-value, in contrast to fixed β values, is calculated based on the test statistic, and denotes the threshold value of the significance level in the sense that the null hypothesis will be accepted for all values of β less than the P-value. The AD gives more weight to the tails than the KS test. In this test, the hypothesis regarding the distributional form is also rejected at β if the test statistic is greater than the obtained critical value. Using some well-known probability distributions, the results of the tests for $\beta = 0.05$ are summarized in Table 6. It is noteworthy that $P_{i,t}^{\text{DG}}$, $P_{j,t}^{\text{charge}}$, and $P_{j,t}^{\text{discharge}}$are take the constant values of 2664, 20, and 18, all over the day. Therefore, no distribution is fitted to these variables.

According to the results obtained for the cost, the Gumbel Max and Normal distributions, respectively, exhibit the most promising goodness-of-fit, whereas the AD and CS tests are not successful for the Beta probability distribution. And, all the three tests are not successful for the Weibull distribution. The Weibull, Betta, and Normal probability distributions are the most promising goodness-of-fit for the results achieved for $P^{\text{grid,in}}$, while the AD and CS tests are not successful for the Gumbel Max probability distribution. Further, the only distribution can be fitted to the results obtained for $P^{\text{grid,out}}$ is Cauchy. Almost all of the other probability distributions are not fitted to the results obtained for the amount of power soled to the grid.

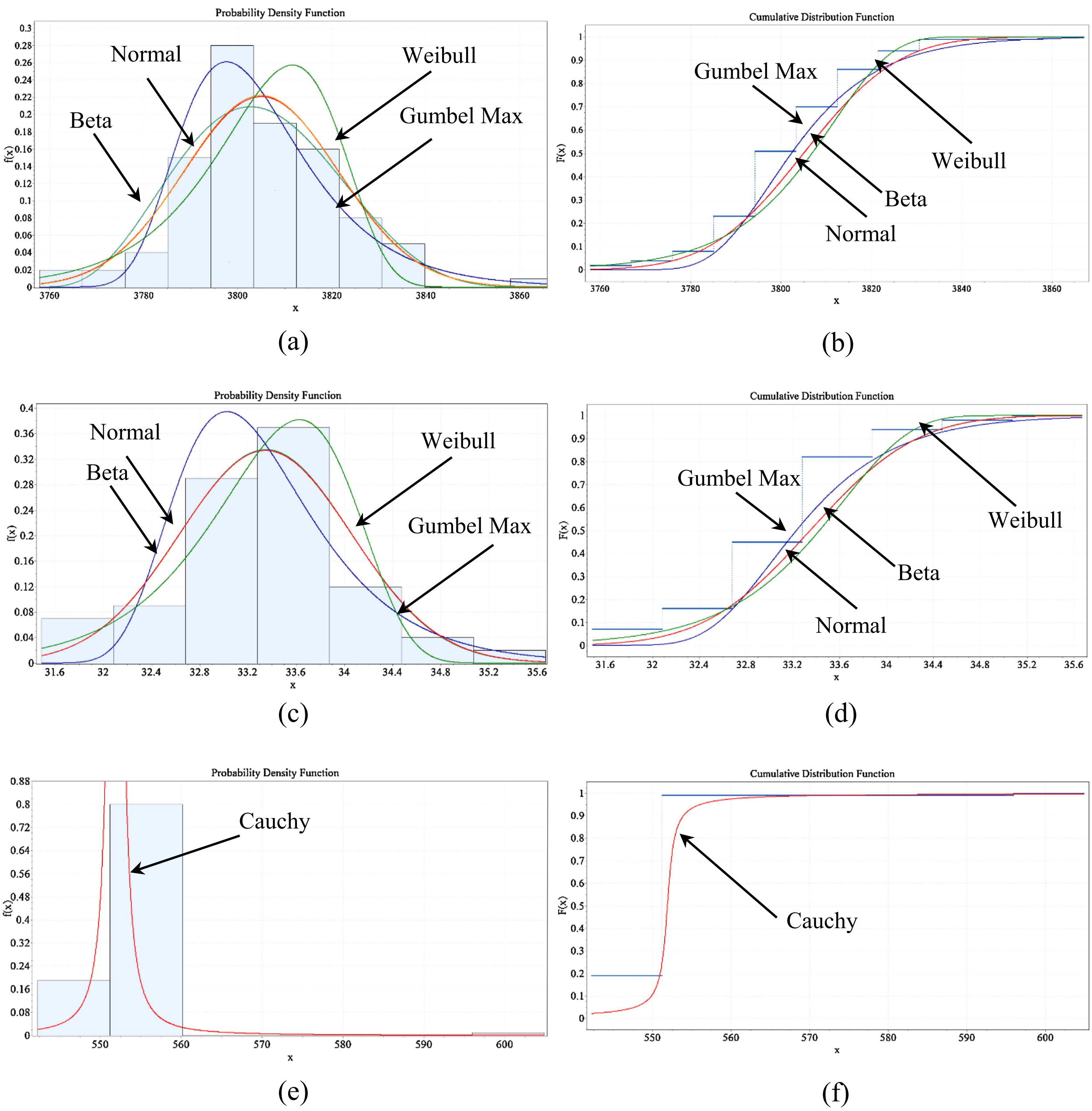

FIGURE 5. The Probability Distribution Functions (PDFs) and the CDFs of variouse probability distributions fitted to the results obtained for ξ (Figs. 5a and 5d), $P^{\text{grid,in}}$ (Figs. 5b and 5e), and $P^{\text{grid,out}}$ (Figs. 5c and 5f).

5. Conclusion and future works

In this paper, the operation problem of MGs in the presence of PHEVs is modeled as a robust bi-level optimization model. The MGO decisions are determined in the two-stage consists of trading power with the main grid and optimal scheduling of MG's resources as the first- and the second-stage decision making. The CVaR approach is used to model the risk aversion level of the MGO, *i.e.* the DM of the system. The main conclusions from the proposed model, which are obtained from numerical studies are as follows:

(a) When the risk of the MGO increases (where the value of α decreases), the operation cost of the MGO decreases since the power sold to the main grid increases. In this manner, the sum of the power generation of DGs and power charging/discharging of the battery increases.
(b) When the risk of the MGO decreases (where the value of α increases), the operation cost of the MGO increases since the power sold to the main grid decreases. In this manner, the sum of the power generation of DGs and the power charging/discharging of the battery decreases.
(c) The robustness of the model is demonstrated through the sensitivity of the MGOs decisions to the risk aversion level.
(d) The interactive model proposed in this paper can be used by the MGO with different risk aversion level in the real world situations.
(e) Applying CVaR approach to deal with the uncertainties of input data through a BLP approach leads to a strong almost linear relationship between the MGOs risk aversion level and the cost of the operation cost of the system. This attractive result enables the MGO to forecast the cost of the system with respect to the MGOs risk aversion level. This may enable the MGO to establish a good trade-off between the overall cost and the DMs risk aversion level.
f) The Gumbel Max and Normal distributions, respectively, exhibit the most promising goodness-of-fit. The Weibull, Betta, and Normal probability distributions are the most promising goodness-of-fit for the results achieved for the amount of power purchased from the grid. Further, the only distribution can be fitted to the results obtained for the amount of power soled to the grid is Cauchy.

The proposed model in this paper can be extended as the future works to model the other problems of MGs as follows:

(a) Modeling the planning problem of the MGs to determine the optimal size of their resources.
(b) Modeling the operation problem of the MGs in the presence of different demand response programs.
(c) Modeling the operation problem of the MGs when it cooperates with other DMs in the distribution network.

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