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RETAILER'S DECISION SELECTION WITH DUAL SUPPLY UNCERTAINTIES UNDER DIFFERENT RELIABILITY LEVELS OF SERVING THE MARKET

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Abstract. The main reason why supply uncertainty reduces supply chain performance is that it is difficult to estimate whether uncertain supply matches demand. Seldom do papers study retailers' decision-selection problems according to the reliability of uncertain supply in satisfying demand. This paper considers the optimal decision selection of a retailer working with a main supplier facing supply uncertainty and a backup supplier whose yield is infinite or uncertain. The retailer can enforce demand management by adjusting prices, seeking the backup supplier to make up for the lack of products or mixing the two decisions. We provide the definition called the reliability level of serving the market (RLSM) to characterize the reliability of uncertain supply in satisfying market demand. Under different RLSMs, the participants maximize their profits based on a confidence level in three scenarios: benchmark, infinite backup supply and uncertain backup supply. Whether the main supplier determines the wholesale price or not, we find that in the benchmark, the retailer orders from the main supplier if the RLSM is low; otherwise, the retailer gives up purchasing the product. In the latter two scenarios, our results show that the particular order strategy chosen by the retailer depends on the values of the RLSM and that the retailer's order quantity follows threshold rules. It is interesting that for different RLSMs, the retailer chooses either a price adjustment strategy, a backup supply strategy or neither of them but does not choose the mixed one, which is counterintuitive. We also derive the particular scenario that is good for the retailer by comparing the results in the three scenarios. Finally, a proper RLSM is suggested for the retailer to balance the reliability of serving the market and her profit.

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1. Introduction

Supply uncertainty has become a major concern in supply-chain risk management. Many reasons may cause supply uncertainty; for example, weather conditions and diseases often affect the yield of agricultural products. Machine faults or technology limitations may directly result in the yield uncertainty of a product. The mismatch of supply and demand due to the yield uncertainty can cause losses to both upstream and downstream firms

Keywords. Supply uncertainty, uncertain yield, confidence level, profit risk level of retailer.

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in supply chains. Firms may use different strategies to alleviate the risk caused by the supply uncertainty. One strategy is that firms adjust prices to enforce demand management [9]. Another strategy is that firms seek backup suppliers to make up for the lack of products caused by supply disruption [27]. Furthermore, the backup suppliers may supply a sufficient number of products or may face supply uncertainty [30].

However, little or no observed data on supply uncertainty information, such as yield uncertainty caused by sudden bad weather, a new type of machine failure or other factors, that cannot be exactly predicted in advance, may exist in some supply-chain risk management problems. Only subjective judgments about it are provided by people, which results in the inapplicability of this type of uncertain information to be evaluated by probability theory. Moreover, almost all works on supply uncertainty problems use the conventional expected utility maximization rule to characterize the participants' goal. Nevertheless, Ellsberg [5] demonstrates that this expectation cannot always be considered individuals' decision rule.

Therefore, this paper investigates how to apply uncertainty theory [14], which is an essential tool to conduct expert subjective judgments regarding uncertain information, to depict uncertain supply information in the supply-chain risk management environment. Due to supply uncertainty, a downstream firm, such as a retailer, may not provide sufficient products to customers, *i.e.*, the service provided by the retailer to the customers may be interrupted or inadequate. The service interruptions or inadequate service can cause losses to the retailer. For example, a supplier sells high-quality artwork or readily marketable products to a retailer. The retailer is willing to accept the uncertain supply of the supplier and bears the risk caused by supply uncertainty because she could earn considerable profits. Obviously, the belief degree of service interruptions is important to the retailer because its magnitude directly affects the retailer's profit. Specifically, the belief degree that service interruptions will fall below a certain range or the possibility that an uncertain market supply can satisfy market demand will be above a certain level, is beneficial to the retailer. This level is called the reliability level of (the retailer) serving the market (RLSM). The higher the RLSM is, the higher the possibility of uncertain market supply to satisfy market demand (implying that the lower the belief degree of service interruptions). Under different RLSMs, we study an optimal decision-selection problem for a retailer with uncertain backup supply, that is, the retailer enforces demand management by adjusting prices, seeking backup supplies to compensate for the lack of products or mixing the two decisions. Particularly, we want to answer the following questions. How does the RLSM affect the retailer's decision selection if the main supplier's wholesale price is exogenous? How would the impact of the RLSM on the retailer's decision change if the main supplier could determine the wholesale price? How does the retailer select a proper RLSM for balancing the reliability of serving the market and her profit?

To address these questions, we research optimal decision-selection models for one retailer ordering from two suppliers. One supplier is regarded as the main supplier by the retailer since he has a cost advantage or better reputation. However, the main supplier faces uncertain yield, and thus, the retailer needs to decide to adopt price adjustment, seek a backup supplier (another supplier) or mix the two strategies to enforce demand management when the supply cannot satisfy the market demand. When the main supplier's wholesale price is exogenous or endogenous, we build the corresponding models for maximizing the participants' profits based on a confidence level under different RLSMs in three scenarios: benchmark, infinite backup supply and uncertain backup supply. In the benchmark, the retailer only orders from the main supplier. In the infinite (or uncertain) backup supply scenario, the retailer first places an order from the main supplier and then orders from the backup supplier whose yield is infinite (or uncertain). We analyze the effect of different RLSMs on the retailer's decision selection and compare them. Finally, we suggest a proper RLSM for balancing the reliability of serving the market and the retailer's profit when the main supplier determines his wholesale price or not.

When the main supplier's wholesale price is exogenous, in the benchmark, the low RLSM implies that the retailer is risky in judging whether the uncertain supply satisfies the demand and believes that the belief degree of service interruptions is high. For this reason, the retailer will raise the retail price to reduce the market demand that is price-dependent. This results in the decrease of the retailer's profit. When the RLSM increases to a certain value, the retailer has no profit and then forgoes purchasing the product. Thus, in the benchmark, the retailer prefers to enforce demand management by adjusting prices if the RLSM is low; otherwise, she gives up ordering from the main supplier. In the infinite backup supply scenario, the retailer can adopt a flexible price to enforce demand management. In fact, if the RLSM is low, the retailer chooses the same strategy as that in the benchmark due to the cost advantage or better reputation of the main supplier. The backup supplier participating in the competition narrows the range of the RLSM interval on which the retailer only orders from the main supplier. As the RLSM increases, the profit of the retailer from the main supplier decreases. When the RLSM reaches a certain value, the main supplier is completely replaced by the backup supplier. That is, the retailer only orders from the backup supplier if the RLSM is high. In the uncertain backup supply scenario, the retailer has three strategies: price adjustment strategy (only orders from the main supplier), backup supply strategy (only orders from the backup supplier) or neither of them. Determining which strategy the retailer chooses depends on the RLSM.

Comparing the above three scenarios, we find that for different RLSMs, the retailer can adopt a flexible price to enforce the demand management in the best way with the infinite backup supply scenario, can adopt it in a better way with the uncertain backup supply scenario and can adopt it in the worst way with the benchmark. The conclusion is straightforward, because a supply breakdown in the benchmark is partially replaced by the added shipments from the backup supplier due to the uncertainty of the backup supply in the uncertain backup supply scenario, whereas it is completely replaced due to the sufficiency of the backup supply in the infinite backup supply scenario. It is interesting that for different RLSMs, the retailer chooses either the price adjustment strategy, the backup supply strategy or neither of them but does not choose the mixed one in the infinite and uncertain backup supply scenarios, which is counterintuitive. It is reasonable because as long as the retailer does not have a low belief degree of service interruption, she is willing to order from the main supplier and believes that her potential profit has be achieved due to the main supplier's cost advantage or better reputation. If the retailer has a sufficiently low belief degree of service interruption, she only orders from the backup supplier because she believes that the sufficiently high RLSM forces her to be completely unprofitable if she orders from the main supplier. Therefore, the retailer is not willing to choose the mixed strategy.

When the main supplier's wholesale price is endogenous, we show that the optimal decision selection of the retailer is threshold controlled by the changed unit production cost of this main supplier due to the supply uncertainty that depends on the RLSM. Because the retailer's profit decreases with respect to the RLSM, that is, the higher profit of the retailer implies a lower RLSM, this may result in the retailer losing control of the market. We show that a proper RLSM can be suggested by maximizing the order quantity to balance the reliability of the retailer serving the market and her profit.

The rest of the paper is organized as follows. Section 2 reviews the literature relevant to our study. Section 3 describes and builds the fundamental model in the three supply scenarios: benchmark, infinite and uncertain backup supply scenarios. Section 4 extends the fundamental model under the assumption that the main supplier's wholesale price is endogenous. Section 5 suggests a reliability level of serving the market for the retailer. Section 6 concludes the paper.

2. Literature review

The literature of strategy selection of a retailer relevant to this paper comes from three streams. The first one focuses on supply uncertainty; the second stream studies supply chain risk measures; the last one addresses the application of uncertainty theory in operations management and service management.

The existing literature relevant to supply uncertainty can be split into three lines of studies: supply disruption, lead time and random yield. Supply disruption usually happens infrequently and temporarily and can cause a significant change to the system. In this line of research, Hendricks and Singhal [7] provide many examples to explain that supply disruptions often cause devastating economic consequences. Li *et al.* [13] study how the manufacturer may force the supplier to recover its production capability as much as possible according to ex ante penalty terms and ex post financial assistance in a better financial situation. The latter two lines often suppose that the lead time or the supply level is a random variable or a random variable function. For example, Zipkin [34] provides a comprehensive review of literature on lead time. Wang and Gerchak [28] consider a production planning problem in a periodic review environment with variable production capacity, random yields and uncertain demand. Bollapragada *et al.* [2] research stock positioning in a pure assembly system controlled by using installation base-stock policies and characterize the system's inventory dynamics when component suppliers face random capacity and uncertain end-product demand. Li *et al.* [12] study the infinite-horizon decision problem of an inventory system with single-stage, single-item periodic review under uncertain yield and demand. Xu *et al.* [30] study a buyer's configuration of flexibility strategies under supply yield uncertainty. Our study is particularly relevant to the work of Xu *et al.* [30]. However, there are two main differences between this literature and our research. First, they assume that the stochastic yield proportion of the main supplier follows a two-point distribution, whereas we suppose that the supply rates of the suppliers are continuous uncertain variables with general uncertainty distributions. Second, they demonstrate that the configuration flexibility strategies are determined by comparing the marginal cost with several marginal benefit thresholds, while we define the RLSM to characterize the reliability of supply in satisfying demand, and provide the decision selection of the retailer under different RLSMs.

The second stream considers supply chain risk measure methods, such like variance, standard deviation, mean-variance, value-at-risk (VaR) and conditional-value-at-risk (CVaR). The common feature is that all these measure methods are in order to characterize the interaction of uncertainty and the extent of its related harm or benefit. Variance or standard deviation is widely applied in measuring the supply chain risk, and they are to evaluate the wideness of a distribution and consider the positive and negative deviations (i.e, upside and downside risk) from expected revenues [1,4]. Nevertheless, Cox [4] thinks that the mean-variance analysis is more applicable than the standard deviation analysis in the theoretical decision analysis and financial economics. The mean-variance criterion is first proposed by Markowitz [20,21] and is widely used in the asset allocation industry. Its theoretical foundation lies in the classic Arrow-Pratt approximation of certainty equivalent wealth for such an investor. Maccheroni *et al.* [19] extend it to a static framework and introduce ambiguity in the standard portfolio analysis. Markowitz [22] himself comprehensively reviews the relative merits of this criterion. VaR and CVaR are widely applied in portfolio management problems as popular percentile measures of downside risk and characterize different parts of a profit or loss distribution. Their use is determined by the goal of the decision maker as well as by the availability and quality of distribution estimates. A close correspondence between them is that VaR is a lower bound for CVaR with the same confidence level [24]. The application of downside risk measures within the context of supply chain problems is very common as can be deduced from the literature analysis. For example, VaR is applied by Poojari *et al.* [23] and Lockamy and McCormack [18], and CVaR is applied by Sawik [25] and Soleimani and Govindan [26]. Heckmann *et al.* [6] comprehensively review the existing methods to measure supply chain risks by setting the focus on the supply chain risk definition and relevant concepts. VaR and CVaR are defined according to the assumption that the distribution function of a random variable is not less than a given confidence level. Different from this processed method, under the assumption that the belief degree of uncertain supply in satisfying market demand is not less than a given confidence level, we analyze the optimal decision selection of a retailer with dual supply uncertainties under different RLSMs.

Our research is also related to the literature on the application of uncertainty theory in operations management and service management. Uncertainty theory is founded by Liu [14] and refined by Liu [16]. Now uncertainty theory regarded as a branch of axiomatic mathematics is widely applied in the operations management and service management. For instance, Zhou *et al.* [32, 33] consider a dynamic recruitment problem with enterprise performance in the uncertain environment and study a model of principal-agent problem under loss aversion and inequity aversion, respectively. Cheng *et al.* [3] analyze how store-brand introduction affects a supply chain with uncertain information. Ke *et al.* [10] study a competitive pricing and remanufacturing problem in an uncertain closed-loop supply chain with risk-sensitive retailers. Wu *et al.* [29] use a decision rule based on confidence level to measure the participants' potential income to consider a principal-agent problem with multi-dimensional incomplete information between one principal and one agent. Liu *et al.* [17] apply the same rule to research a contract designing problem for two competing heterogeneous suppliers working with one common retailer, and analyze the effect of the confidence levels on the selection of the retailer facing two types of market demand. Different from the works mentioned above [17, 29], this paper adopts confidence level to measure the reliability level of supply in satisfying demand. Furthermore, we analyze the impact of the reliability level on the retailer's strategy selection and suggest a suitable reliability level for each of different strategies.

3. The fundamental model

3.1. Problem description

Consider a supply chain with two suppliers, indexed by $i \in \{1, 2\}$, and one retailer (she). One supplier (supplier 1) has a cost c_1 and provides a wholesale price $w_1 \geqslant c_1$ to the retailer. We assume that the wholesale price w_1 is exogenously determined in this section, while it is endogenous in the next section. Supplier 1 faces uncertain yield for some reasons such as natural disasters, technological limitations, imperfect manufacturing processes, and so on, that is, if the retailer orders q_1 units of the product from supplier 1, she would receive the uncertain quantity $\xi_1 q_1$, where ξ_1 (is called an uncertainty supply rate of supplier 1) is an uncertain variable on the interval $[m_1, n_1] \subseteq [0, 1]$ with uncertainty distribution $\Phi_1(x)$. However, since supplier 1 has a cost advantage or better reputation, the retailer signs a long-term contract with supplier 1, who becomes the retailer's main supply source. For example, a supplier sells high-quality artwork or readily marketable products to a retailer. The retailer is willing to accept the uncertain supply of the supplier and bears the risk caused by supply uncertainty because she could earn considerable profits.

As a temporary or contingent source of supply (a backup supply source), the other supplier (supplier 2) sells the completely substitutable product to the retailer with an exogenous wholesale price w_2. The retailer can obtain enough supply from supplier 2 at a higher purchasing cost (*e.g.*, olive oil in [8]), or cannot find a suitable supplier even at a higher cost (*e.g.*, Ericsson in [11]). We consider two situations to characterize the retailer's searching abilities (or supplier 2's supply abilities). In the first situation, the retailer is powerful and has access to adequate supply at a higher purchasing cost, that is, supplier 2's supply quantity is infinite. In the second situation, supplier 2 also faces uncertain supply, that is, if the retailer orders q_2 units of the product from supplier 2, she would receive the uncertain quantity $\xi_2 q_2$, where ξ_2 (is called an uncertainty supply rate of supplier 2) is an uncertain variable on the interval $[m_2, n_2] \subseteq [0, 1]$ with uncertainty distribution $\Phi_2(x)$. We assume that both the uncertain variables ξ_1 and ξ_2 are independent and that their distributions are known to all the participants.

The retailer is a monopolist in her market and enforces demand management by adjusting prices on the basis of the realized supply. The ultimate demand of the product is price independent and is denoted as

$$D(p) = a - bp,$$

where p is the market price, a is the market size and b is a price-sensitive parameter. Moreover, suppose that $a \geqslant bw_2 \geqslant bc_1$ in order to guarantee that the ultimate demand is nonnegative and supplier 1 has a cost advantage.

In the following three subsections, we consider three supply scenarios. The first supply scenario indicates that the retailer only orders the product from supplier 1. The second or third supply scenario implies that the retailer orders the product from supplier 1 and supplier 2 whose product yield is infinite or uncertain, respectively.

The sequence of events in three stages is as follows.

- In stage I, the retailer orders q_1 units of the product and receives $\xi_1 q_1$ from supplier 1.
- In stage II, the retailer orders q_2 units of the product and receives q_2 or $\xi_2 q_2$ from supplier 2.
- In stage III, the retailer determines the market price p according to the total realized supply quantity.

3.2. The benchmark

In this subsection, we investigate the first supply scenario, *i.e.*, the retailer only places an order from supplier 1, whose product yield is uncertain. For convenience, we suppress supplier 2, and then the problem is degenerated to two stages.

3.2.1. Retailer's price decision in stage II

Due to supply uncertainty, the service provided by the retailer to customers may be interrupted or inadequate. The service interruptions or inadequate service can cause losses to the retailer. Therefore, the retailer needs to reduce the belief degree of service interruptions to diminish losses, that is, she needs to improve the possibility that uncertain market supply can satisfy market demand. Thus, the retailer allows for the belief degree of the realized supply that exceeds the ultimate demand to be not less than the given confidence level denoted by β, *i.e.*,

$$\mathcal{M}\{\xi_1 q_1 \geqslant D(p)\} \geqslant \beta.$$

According to the above inequation, a higher value of β indicates a lower belief degree of service interruptions (or a higher possibility that an uncertain market supply can satisfy market demand). We call the confidence level β the reliability level of serving the market (RLSM). Therefore, the model for maximizing the retailer's ex post profit (*i.e.*, market sales amount) denoted by $v_e(p)$ is

$$\begin{cases} \max\limits_{p} \ v_e(p) = pD(p) \\ \text{subject to:} \\ \qquad \mathcal{M}\{\xi_1 q_1 \geqslant D(p)\} \geqslant \beta. \end{cases} \tag{3.1}$$

Lemma 3.1. *The optimal price and profit of the retailer are as follows.*

(i) *If* $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, *then* $\overline{p}(q_1) = a/(2b)$ *and* $v_e\big(\overline{p}(q_1)\big) = a^2/(4b)$.

(ii) *If* $\Phi_1^{-1}(1-\beta)q_1 < a/2$, *then*

$$\overline{p}(q_1) = \left(a - \Phi_1^{-1}(1-\beta)q_1\right)/b, \ v_e\big(\overline{p}(q_1)\big) = a\Phi_1^{-1}(1-\beta)q_1/b - \left(\Phi_1^{-1}(1-\beta)\right)^2 q_1^2/b.$$

The quantity $\Phi_1^{-1}(1-\beta)q_1$ essentially represents the realized supply of supplier 1 in the market under the RLSM β, which is called the β-supply of supplier 1, and $\Phi_1^{-1}(1-\beta)$ is named the β-supply rate of supplier 1. Therefore, Lemma 3.1 implies that the retailer sets a stable market price and earns a stable ex post profit at the cost of some unsold inventory when the β-supply of supplier 1 is sufficient (*i.e.*, case (i)). Oppositely, when the β-supply of supplier 1 is lacking (*i.e.*, case (ii)), the retailer uses the market clearing price to match the β-supply with demand. In case (i), since the market price is lower than that in case (ii), while the retailer's ex post profit and the β-supply of supplier 1 are higher, the retailer provides a better service to the customers due to a lower price and more market supply in case (i) than in case (ii).

3.2.2. Retailer's order decision in stage I

Denote the retailer's profit as $v(p, q_1; \xi_1) = v_e(p) - w_1\xi_1 q_1$, which is also an uncertain variable because it includes the uncertain variable ξ_1. Let $\alpha \in [0,1]$ be the confidence level of the retailer, which reflects the retailer's attitude to risk and has been used in Wu *et al.* [29] and Yang *et al.* [31]. Particularly, a higher value of α implies a conservative retailer, while a lower value of α means a risky one. Under a given confidence level α, the profit of the retailer can be expressed as v_0 belonging to $\left\{v_0 \mid \mathcal{M}\{v(p,q_1;\xi_1) \geqslant v_0\} \geqslant \alpha\right\}$, which is a set of the retailer's profits. Given the market price p and the order quantity q_1, the retailer's α-profit (the retailer's profit, for short) $v_m(p, q_1; \alpha)$ can be denoted by

$$v_m(p, q_1; \alpha) = \max\left\{v_0 \mid \mathcal{M}\{v(p,q_1;\xi_1) \geqslant v_0\} \geqslant \alpha\right\}.$$

Particularly, the retailer's α-profit is her expected profit when $\alpha = 0.5$. Here, we propose a decision rule based on a confidence level rather than the expected value to measure the retailer's profit, which is similar to Wu *et al.* [29] and Yang *et al.* [31].

According to the optimal price in stage II, the retailer maximizes her α-profit to determine her order quantity from supplier 1, *i.e.*,

$$\max_{q_1 \geqslant 0} \ v_m\big(\overline{p}(q_1), q_1; \alpha\big). \tag{3.2}$$

We call the confidence level α the profit risk level of the retailer (PRLR) to distinguish it from the RLSM β. Furthermore, the higher the confidence level is, the lower the PRLR. For notation convenience, we denote $t_1 = \Phi_1^{-1}(1-\beta)/\Phi_1^{-1}(\alpha)$, which decreases in α and β. For any given PRLR, the parameter t_1 reflects the RLSM, that is, the higher the value of t_1 is, the lower the RLSM β, and *vice versa*.

Lemma 3.2. *In the benchmark, the order strategy and profit of the retailer are as follows.*

(i) *When* $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, *we obtain*

$$\overline{p} = a/(2b),\ \overline{q}_1 = a/\left(2\Phi_1^{-1}(1-\beta)\right),\ v_m(\overline{p},\overline{q}_1;\alpha) = a^2/(4b) - aw_1/(2t_1).$$

(ii) *When* $\Phi_1^{-1}(1-\beta)q_1 < a/2$, *we obtain*
 – *If* $w_1/t_1 \leqslant a/b$, *then*

$$\overline{p} = \frac{at_1 + bw_1}{2bt_1},\ \overline{q}_1 = \frac{at_1 - bw_1}{2t_1\Phi_1^{-1}(1-\beta)}, v_m(\overline{p},\overline{q}_1;\alpha) = \frac{(at_1 - bw_1)^2}{4bt_1^2}.$$

 – *If* $w_1/t_1 > a/b$, *then* $\overline{p} = a/b,\ \overline{q}_1 = 0,\ v_m(\overline{p},\overline{q}_1;\alpha) = 0.$

We call the ratio w_1/t_1 the purchasing belief degree cost of the retailer from supplier 1. In fact, the ratio w_1/t_1, which increases in α and β, can be understood as the purchasing cost per unit believed by the retailer due to the supply uncertainty of supplier 1 under the given RLSM and PRLR. The reasons are as follows, on the one hand, the retail price $\overline{p} = a/(2b)+w_1/(2t_1)$ in case (ii) of Lemma 3.2 is similar to the traditional monopoly price $\overline{p} = a/(2b) + w/2$. This implies that the ratio w_1/t_1 just replaces the unit purchasing cost w in the traditional monopoly price in our settings. On the other hand, this understanding is intuitive, because as the RLSM or PRLR increases, the retailer believes that her purchasing cost w_1/t_1 increases. In other words, the conservative retailer believers that her purchasing cost is high and the risky retailer believers that her purchasing cost is low.

For example, we assume that supplier 1 has an uncertainty supply rate $\xi_1 \sim \mathcal{L}(0.4, 0.7)$, which means experts estimate that the uncertainty supply rate is in the interval $[0.4, 0.7]$, and the exogenous wholesale price $w_1 = 12$. Then for a given $\alpha = 0.5$ (meaning that the risk attitude of the retailer to her profit is risk-neutral), the retailer's purchasing belief-degree cost is estimated as $w_1/t_1 = w_1/[(0.7-0.3\beta)/0.55]$ under the RLSM β. The term $\Phi_1^{-1}(1-\beta) = 0.7-0.3\beta \in [0.4, 0.7]$ is the uncertainty supply rate estimated by the experts, and the item $\Phi_1^{-1}(\alpha) = 0.55 \in [0.4, 0.7]$ can be understood as the noise caused by the risk attitude of the retailer to her profit. This implies that the retailer regards 14.35 as her purchasing belief-degree cost if she is risk-averse and has a higher RLSM of $\beta = 0.8$. The purchasing belief-degree cost is estimated as 10.82 if she is a risk lover with a lower RLSM $\beta = 0.3$ and is 12 if she is risk-neutral (*i.e.*, $\beta = 0.5$). To summarize, For a given PRLR, the retailer believes that increasing the RLSM (*i.e.*, decreasing the belief degree of service interruptions) is at the price of increasing the purchasing cost. This means that the retailer uses the purchasing belief degree cost to quantify the cost of change due to the supply uncertainty of a supplier. This also implies that the dual supply uncertainties in our study are ultimately reflected in the costs of change (*i.e.*, purchasing belief degree costs).

Lemma 3.2 reveals that the retailer chooses the order decision for optimizing her profit and sets a stable market price when the β-supply of supplier 1 is sufficient (*i.e.*, $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$). Otherwise, the retailer adopts a flexible price to match the β-supply of supplier 1 with demand, and her order decision depends on the RLSM. A higher value of the RLSM indicates a lower belief degree of service interruptions. The retailer ignores the order from the supplier if the RLSM is too high because of the sufficiently low belief degree of service interruptions, which satisfies that the purchasing belief degree cost of the retailer from supplier 1 is higher than the purchasing threshold (*i.e.*, $w_1/t_1 > a/b$), and forces the market demand to be zero such that the retailer has no profit. Oppositely, the retailer selects the order decision for optimizing her profit if the RLSM is low (*i.e.*, $w_1/t_1 \leqslant a/b$) because the retailer has a positive profit.

Having compared the two order decisions, the decision corresponding to case (i) is dominated by those corresponding to case (ii) from the perspective of the profit of the retailer because the profit of the retailer in case (i) is lower than each situation in case (ii) (*i.e.*, $a^2/(4b) - aw_1/(2t_1) \leqslant (at_1 - bw_1)^2/(4bt_1^2)$ and $a^2/(4b) - aw_1/(2t_1) \leqslant 0$). However, the order decision corresponding to case (i) provides a better service to customers due to a lower market price and a more β-supply. In this case, the retailer essentially raises her order quantity to stabilize the market price at the cost of the decrease of her profit. Therefore, in the benchmark, the two order decisions corresponding to case (ii) are optimal for the retailer and are described as the following proposition. Moreover, the following proposition characterizes the impact of the RLSM β and PRLR α on the optimal solution and profit in the benchmark.

Proposition 3.3. *In the benchmark, the optimal solution and profit are as follows.*

(i) *If $w_1/t_1 \leqslant a/b$, then*

$$\overline{q}_1 = (at_1 - bw_1)/\left(2t_1\Phi_1^{-1}(1-\beta)\right),\ \overline{p} = (at_1 + bw_1)/(2bt_1), v_m(\overline{p}, \overline{q}_1; \alpha) = (at_1 - bw_1)^2/(4bt_1^2)\cdot$$

Moreover, the market price increases in α and β, whereas the profit of the retailer decreases in α and β. The order quantity of the retailer decreases in α; while it increases in β if $t_1 \geqslant 2bw_1/a$; otherwise it decreases in β.

(ii) *If $w_1/t_1 > a/b$, then $\overline{p} = a/b$, $\overline{q}_1 = 0$ and $v_m(\overline{p}, \overline{q}_1; \alpha) = 0$.*

Proposition 3.3 means that when the belief degree of service interruptions is high (*i.e.*, $w_1/t_1 \leqslant a/b$), the retailer prefers to set a flexibly higher price to adjust market demand and order less products rather than set a stably lower retail price and order more products to better service customers because she can earn more profit by adopting the former price and order decisions. Moreover, when the belief degree of service interruptions decreases (*i.e.*, the RLSM increases), the market price increases, the profit of the retailer decreases and the order quantity increases first and then decreases. This is because the retailer reduces the possibility of service interruptions by raising the retail price to decrease the market demand, which results in the decrease in her profit. In other words, the retailer's price of reducing the belief degree of service interruptions is an increase in the retail price and a decrease in the market demand and the profit. Although the order quantity increases monotonically and then decreases with the RLSM, the realized supply decreases with the RLSM because the increase of the retail price results in the decrease of the market demand. When the PRLR decreases, the order quantity and profit of the retailer increase, while the retail price decreases. The conclusion is intuitive because regarding profit, a risk-loving retailer likes to set a lower retail price to increase the market demand, order more products to meet the market demand and, at the same time, bear a higher potential risk to earn more profit.

However, when the belief degree of service interruptions is low (*i.e.*, $w_1/t_1 > a/b$), the retailer gives up purchasing the product. In this case, the retailer still adopts a flexible price decision since her profit is higher than that in the situation of setting a stable price. Nevertheless, when the purchasing belief degree cost of the retailer from supplier 1 is higher than the purchasing threshold (*i.e.*, the belief degree of service interruptions is low), the market demand is zero, that is, the retailer's profit is negative, and thus, she is not willing to sell the product if she orders from supplier 1.

We numerically analyze the effect of the PRLP α and RLSM β on the optimal order strategy for the benchmark (see Fig. 1, where $\xi_1 \sim \mathcal{L}(m_1, n_1), a = 190, b = 8, m_1 = 0.1, n_1 = 0.9$ and $w_1 = 12$). In Figure 1, the decreasing curve represents the function curve of α varying with β (*i.e.*, $t_1 = \Phi_1^{-1}(1-\beta)/\Phi_1^{-1}(\alpha) = bw_1/a$). On the left side of the curve (*i.e.*, $t_1 \geqslant bw_1/a$), the retailer chooses the order strategy corresponding to case (i) in Proposition 3.3, while on its right side (*i.e.*, $t_1 < bw_1/a$), the retailer's order quantity is zero. In other words, for a given PRLR α, the retailer chooses the former order strategy when the RLSM β is low; otherwise, she chooses the latter one. Particularly, the latter order strategy does not appear if the given PRLR is too small regardless of whether the RLSM is low or high, that is, the value of the RLSM β when $t_1 < bw_1/a$ does not exist. Figure 2 illustrates that the order quantity $\overline{q}_1$ changes with the RLSM β, where $\alpha = 0.5$ and the other parameter values are the same as those in Figure 1. In Figure 2, the order quantity is maximized when the RLSM β is equal to β_1. The value β_2 distinguishes whether the retailer purchases the product from supplier 1. That is, when $\beta < \beta_2$, the retailer purchases the product; otherwise, she does not.

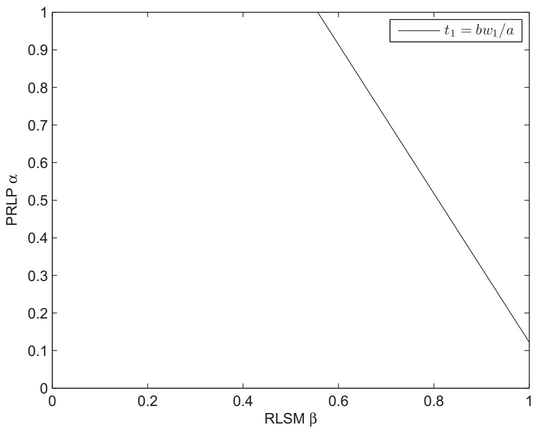


FIGURE 1. Effect of RLSM β and confidence level α on order strategy.

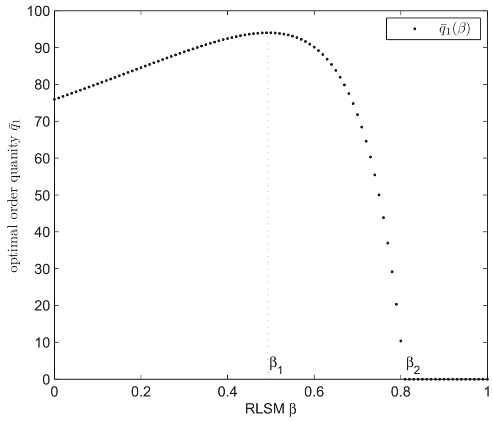


FIGURE 2. Effect of RLSM β on order quantity $\overline{q}_1$.

3.3. The infinite backup supply model

Consider the second supply scenario, *i.e.*, the retailer places an order from supplier 1 and then orders from supply 2 with infinite product volume. For convenience, we use the superscript I to represent this supply scenario.

3.3.1. Retailer's price decision in stage III of the infinite backup supply scenario

Since the retailer needs to place an order from supplier 2, the model for maximizing her ex post profit becomes

$$\begin{cases} \max\limits_{p} \ v_e(p) = pD(p) \\ \text{subject to:} \\ \qquad \mathcal{M}\{\xi_1 q_1 + q_2 \geqslant D(p)\} \geqslant \beta. \end{cases} \tag{3.3}$$

Lemma 3.4. *The optimal price in model (3.3) is as follows.*

(i) *If* $\Phi_1^{-1}(1-\beta)q_1 + q_2 \geqslant a/2$, *then* $\overline{p}^I(q_1, q_2) = a/(2b)$.
(ii) *If* $\Phi_1^{-1}(1-\beta)q_1 + q_2 < a/2$, *then* $\overline{p}^I(q_1, q_2) = [a - \Phi_1^{-1}(1-\beta)q_1 - q_2]/b$.

The analysis for this lemma is similar to that for Lemma 3.1.

3.3.2. Retailer's backup order decision in stage II of the infinite backup supply scenario

In this stage, the retailer has obtained the supply from supplier 1, and the supply uncertainty of supplier 1 has been resolved. The retailer needs to determine the backup order quantity from supplier 2 according to the β-supply of supplier 1 and the price strategy in stage III. In this stage, the profit of the retailer is denoted as

$$v_c^I(p, q_2) = v_e(p) - w_2 q_2.$$

According to the β-supply and the price strategy in stage III, the model for maximizing the profit of the retailer is

$$\max_{q_2 \geqslant 0} \ v_c^I\big(\overline{p}^I(q_1, q_2), q_2\big). \tag{3.4}$$

Lemma 3.5. *The backup order quantity and optimal price in model (3.4) are as follows.*

(i) *If* $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, *then* $\overline{q}_2^I(q_1) = 0$ *and* $\overline{p}^I(q_1) = a/(2b)$.

(ii) *If* $(a-bw_2)/2 < \Phi_1^{-1}(1-\beta)q_1 < a/2$, *then* $\overline{q}_2^I(q_1) = 0$ *and* $\overline{p}^I(q_1) = [a - \Phi_1^{-1}(1-\beta)q_1]/b$.

(iii) *If* $\Phi_1^{-1}(1-\beta)q_1 \leqslant (a-bw_2)/2$, *then* $\overline{q}_2^I(q_1) = (a-bw_2)/2 - \Phi_1^{-1}(1-\beta)q_1$ *and* $\overline{p}^I(q_1) = (a+bw_2)/(2b)$.

Lemma 3.5 implies that when the β-supply of supplier 1 reaches a threshold $(a-bw_2)/2$, the retailer only places an order from supplier 1, that is, she abandons the backup order. Moreover, if the β-supply of supplier 1 is sufficient, the retailer sets a stable market price (*i.e.*, case (i)); otherwise, she adopts a flexible price (*i.e.*, case (ii)). When the β-supply of supplier 1 is lower than the threshold, the retailer's backup order quantity is the difference between the threshold and the β-supply of supplier 1 (*i.e.*, case (iii)). In this case, although the retailer adopts a flexible price strategy, the retail price is not affected by the order quantities due to the market demand equaling the threshold. By summarizing the above analyses, the following proposition is obtained.

Proposition 3.6. *In the infinite backup supply scenario, the retailer's backup order quantity is*

$$[(a-bw_2)/2 - \Phi_1^{-1}(1-\beta)q_1]^+ = \max\{(a-bw_2)/2 - \Phi_1^{-1}(1-\beta)q_1, 0\}.$$

Proposition 3.6 states that the retailer raises the total realized quantity in the market (*i.e.*, the sum of the β-supply of supplier 1 and the order quantity from supplier 2) to the level of a threshold $(a-bw_2)/2$. The reason is that the retailer can earn the highest profit in stage III with the market demand equaling the threshold $(a-bw_2)/2$ at the retail price $\overline{p}^I(q_1) = (a+bw_2)/(2b)$.

3.3.3. Retailer's main order decision in stage I of the infinite backup supply scenario

In the infinite backup supply scenario, we denote the retailer's profit as $v^I(p, q_1, q_2; \xi_1) = v_c^I(p, q_2) - w_1\xi_1 q_1$. By using the method similar to the benchmark and for the given market price p and order quantity q_1, the retailer's α-profit $v_m^I(p, q_1; \alpha)$ can be denoted by

$$v_m^I(p, q_1, q_2; \alpha) = \max\left\{v_0 \;\middle|\; \mathcal{M}\{v^I(p, q_1, q_2; \xi_1) \geqslant v_0\} \geqslant \alpha\right\}.$$

In view of the price and order decisions in the above two stages, the retailer maximizes her α-profit in order to determine her order quantity from supplier 1, *i.e.*,

$$\max_{q_1 \geqslant 0} \; v_m^I\left(\overline{p}^I(q_1), q_1, \overline{q}_2^I(q_1); \alpha\right). \tag{3.5}$$

Lemma 3.7. *The order strategy in the infinite backup supply scenario is as follows.*

(i) *When* $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, *we have* $\overline{q}_1^I = a/\left(2\Phi_1^{-1}(1-\beta)\right)$, $\overline{q}_2^I = 0$.

(ii) *When* $(a-bw_2)/2 \leqslant \Phi_1^{-1}(1-\beta)q_1 < a/2$, *we have*
- *If* $w_1/t_1 \leqslant w_2$, *then* $\overline{q}_1^I = (at_1 - bw_1)/\left(2t_1\Phi_1^{-1}(1-\beta)\right)$, $\overline{q}_2^I = 0$.
- *If* $w_1/t_1 > w_2$, *then* $\overline{q}_1^I = (a-bw_2)/\left(2\Phi_1^{-1}(1-\beta)\right)$, $\overline{q}_2^I = 0$.

(iii) *When* $\Phi_1^{-1}(1-\beta)q_1 \leqslant (a-bw_2)/2$, *we have*
- *If* $w_1/t_1 \leqslant w_2$, *then* $\overline{q}_1^I = (a-bw_2)/\left(2\Phi_1^{-1}(1-\beta)\right)$, $\overline{q}_2^I = 0$.
- *If* $w_1/t_1 > w_2$, *then* $\overline{q}_1^I = 0$, $\overline{q}_2^I = (a-bw_2)/2$.

Lemma 3.7 provides the five order decisions when the β-supply of supplier 1 is sufficient or lacking. When the β-supply of supplier 1 is sufficient, the retailer orders more units of product to stabilize the market price, which is the same as the order decision corresponding to the first case in Lemma 3.2.

When the β-supply of supplier 1 is lacking and higher than the threshold $(a-bw_2)/2$, the retailer abandons the backup supplier and only orders from supplier 1 due to a cost advantage or better reputation of the main supplier. In this case, there are two order decisions that depend on the RLSM. If the RLSM β is low (*i.e.*, $w_1/t_1 \leqslant w_2$), the retailer only orders the product from supplier 1, and her realized order quantity is higher than the threshold (*i.e.*, $\Phi_1^{-1}(1-\beta)\overline{q}_1^I \geqslant (a-bw_2)/2$). This order decision is the same as that in the first subcase of the second case of Lemma 3.2. Otherwise, the retailer only places an order from supplier 1, and her realized order quantity equals the threshold. Under the latter order decision, a sufficiently low belief degree of service interruptions (*i.e.*, sufficiently high RLSM) forces the realized order quantity of the retailer to only equal the threshold. In this subcase, the realized order quantity of the retailer cannot attain the level under the former order decision to earn more profit because increasing the RLSM will decrease the realized supply of supplier 1. Therefore, these two order decisions depend on the RLSM.

When the β-supply of supplier 1 is no higher than the threshold, the retailer places an order from supplier 1 and enforces demand management by adjusting prices, or seeks the backup supplier. Her realized order quantity (market supply) is equal to the threshold regardless of which strategy the retailer chooses, and the chosen strategy depends on the RLSM. If the RLSM is low, the retailer chooses the former strategy; otherwise, she selects the latter strategy. The reason is that when the RLSM is low, the retailer first orders from supplier 1 due to a cost advantage or better reputation of the main supplier. In addition, the supply of supplier 1 can satisfy the retailer's realized order demand that is equal to the threshold. However, sufficiently high RLSM can force the supply of supplier 1 to be unable to satisfy the realized order demand that is equal to the threshold, thus losing the advantage of cost or reputation. In fact, in this subcase, the retailer's optimal order quantity from supplier 1 is zero, and the retailer's profit decreases due to losing the advantage of the main supplier when her order quantity is more than zero. Thus, the retailer abandons the main supplier and seeks the backup supplier if the RLSM is high.

Comparing the above five order decisions from the perspective of the retailer's profit, we obtain two dominated decisions that are characterized by the following proposition.

Proposition 3.8. *In the infinite backup supply scenario, the optimal solution and profit are as follows.*

(i) *If* $w_1/t_1 \leqslant w_2$, *then*

$$\overline{q}_1^I = \frac{a}{2\Phi_1^{-1}(1-\beta)} - \frac{bw_1}{2t_1\Phi_1^{-1}(1-\beta)},\ \overline{q}_2^I = 0,\ \overline{p}^I = \frac{a}{2b} + \frac{w_1}{2t_1}, v_m^I(\overline{p}^I,\ q_1^I, q_2^I; \alpha) = \frac{(at_1 - bw_1)^2}{4bt_1^2}\cdot$$

Moreover, the market price increases in α and β, whereas the profit of the retailer decreases in α and β. The order quantity from supplier 1 *decreases in α, while it increases in β if $t_1 \geqslant 2bw_1/a$; otherwise it decreases in β.*

(ii) *If* $w_1/t_1 > w_2$, *then*

$$\overline{q}_1^I = 0,\ \overline{q}_2^I = (a-bw_2)/2,\ \overline{p}^I = (a+bw_2)/(2b),\ v_m^I(\overline{p}^I, q_1^I, q_2^I; \alpha) = (a-bw_2)^2/(4b).$$

Proposition 3.8 means that in the infinite backup supply scenario, the retailer sets a flexible price to adjust market demand and orders more than the threshold from the main supplier if the RLSM is low (*i.e.*, $w_1/t_1 \leqslant w_2$), and this strategy is the same as that in the benchmark (*i.e.*, case (i) of Prop. 3.3). Otherwise, she seeks the backup supplier, under which strategy the retailer also sets a flexible price to adjust market demand; however, her price decision is not affected by the order quantities because the market demand (or the retailer's order quantity) is equal to the threshold and only depends the exogenous wholesale price of supplier 2 due to her order quantity, which is all from the backup supplier. The reason is as follows. First, the retailer prefers a flexible price strategy rather than a stable price strategy because she can obtain more profit in the former strategy than in the latter strategy. Second, the retailer prefers to order more of the product, such that her realized market demand is more than the threshold rather than just equal to the threshold, to obtain the optimal profit if the RLSM is low. However, sufficiently high RLSM can force the supply of supplier 1 to be unable to satisfy the realized order demand that is equal to the threshold and to lose the advantage of cost or reputation. Moreover, in this subcase, the retailer's order quantity that is more than zero from supplier 1 can decrease her profit due to losing the advantage of the main supplier. Thus, the retailer abandons the main supplier and seeks the backup supplier if the RLSM is high. Third, the retailer chooses either the price adjustment strategy (only orders from the main supplier) or the backup supply strategy but does not choose the mixed one, which is counterintuitive. This is reasonable because as long as the retailer does not have a low belief degree of service interruption, she is willing to order from the main supplier and believes that her potential profit has achieved her request due to the main supplier's cost advantage or better reputation. If the retailer has a sufficiently low belief degree of service interruption, she only orders from the backup supplier because she believes that the sufficiently high RLSM forces her to be completely unprofitable if she orders from the main supplier. Therefore, the retailer is not willing to choose the mixed strategy.

According to the definition of the purchasing belief degree cost of the retailer from supplier 1, the implication of Proposition 3.8 can also be described as follows. In case (i), since the purchasing belief degree cost of the retailer from the main supplier is no more than the purchasing cost from the backup supplier (*i.e.*, $w_1/t_1 \leqslant w_2$), that is, the main supplier has his cost advantage, the retailer only orders from the main supplier; otherwise (*i.e.*, in case (ii)), she only orders from the backup supplier.

Comparing Propositions 3.3 and 3.8, we observe that when the RLSM is low, the retailer's order decisions in the benchmark and infinite backup supply scenarios are the same. When the RLSM is high, the retailer gives up ordering from supplier 1 in the benchmark, whereas in the infinite backup supply scenario, the retailer's order quantity is still zero from supplier 1 and is supplemented to the level of the threshold from supplier 2. Moreover, the value of the RLSM β when $t_1 = bw_1/a$ in the benchmark is higher than that when $t_1 = w_1/w_2$ in the infinite backup supply scenario because $w_1/w_2 \geqslant bw_1/a$ and t_1 decreases in β. Therefore, the following proposition is obtained.

Proposition 3.9. *In the infinite backup supply scenario, the range of the RLSM interval, on which the retailer only places an order from the main supplier and adopts a flexible price strategy, is narrower than that in the benchmark.*

Proposition 3.9 reveals that in the infinite backup supply scenario, the retailer can better adopt a flexible price to enforce the demand management for different RLSMs than in the benchmark because supplier 2 participates in the competition. In fact, compared with the benchmark, the threshold of the upper limit of the RLSM is lower, and the same flexible price and order strategies are adopted in the infinite backup supply scenario when the RLSM is low. When the RLSM is high, the backup order strategy instead of the no-order strategy is used. In other words, the competition between two suppliers is beneficial for the retailer because she can better adopt a flexible price to obtain more profit regardless of whether the RLSM is low or high.

We do a numerical analysis of the effect of the RLSM β (and the PRLR α) on the order quantities (the order strategy) in the infinite backup supply scenario (see Figs. 3 and 4, where $w_2 = 14$ and the other parameter values are the same as those in Figs. 1 and 2, respectively). Comparing Figures 1 and 3, we observe that the range of the left area of the curve in Figure 3, on which only supplier 1 supplies the product to the retailer, is narrower than that in Figure 1. This implies that for any given PRLR α, the range of the RLSM β in the infinite backup supply scenario, on which the product is supplied to the market only from supplier 1, is narrower than that in the benchmark. Comparing Figures 2 and 4, we see that in the infinite backup supply scenario, the order strategy is the same as that in the benchmark when $\beta < \beta_3$, whereas the retailer orders the product from supplier 2 instead of from supplier 1 and her order quantity is stable when $\beta > \beta_3$. The reason of stable order quantity is that the backup supply is infinite.

3.4. The uncertain backup supply model

Now, we study the uncertain backup supply scenario, *i.e.*, the retailer first places an order from supplier 1 and then orders from supplier 2 with uncertain product volume. For convenience, we use the superscript U to represent this supply scenario.

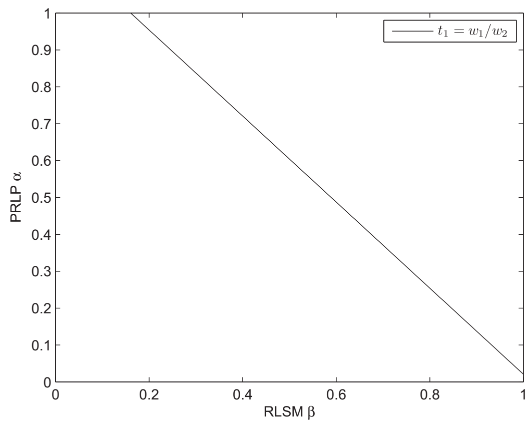


FIGURE 3. Effect of RLSM β and confidence level α on order strategy.

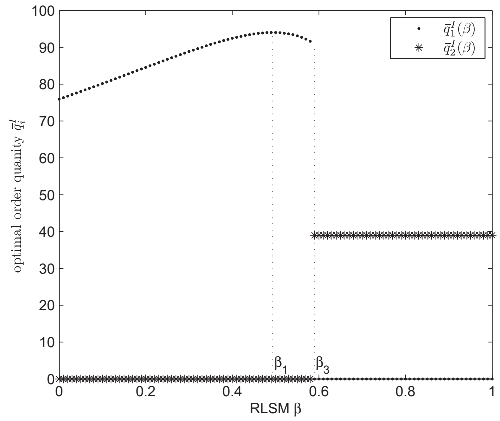


FIGURE 4. Effect of RLSM β on order quantity $\overline{q}_i^I$.

3.4.1. Retailer's price decision in stage III of the uncertain backup supply scenario

Since the retailer needs to place an order from supplier 2 with uncertain product volume, the model for maximizing her ex post profit becomes

$$\begin{cases} \max\limits_{p} \ v_e(p) = pD(p) \\ \text{subject to:} \\ \qquad \mathcal{M}\{\xi_1 q_1 + \xi_2 q_2 \geqslant D(p)\} \geqslant \beta. \end{cases} \tag{3.6}$$

Lemma 3.10. *The optimal price in model (3.6) is as follows.*

(i) *If* $\Phi_1^{-1}(1-\beta)q_1 + \Phi_2^{-1}(1-\beta)q_2 \geqslant a/2$, *then* $\overline{p}^U(q_1, q_2) = a/(2b)$.
(ii) *If* $\Phi_1^{-1}(1-\beta)q_1 + \Phi_2^{-1}(1-\beta)q_2 < a/2$, *then* $\overline{p}^U(q_1, q_2) = \left[a - \left(\Phi_1^{-1}(1-\beta)q_1 + \Phi_2^{-1}(1-\beta)q_2\right)\right]/b$.

The analysis for this lemma is similar to that for Lemma 3.1.

3.4.2. Retailer's backup order decision in stage II of the uncertain backup supply scenario

Similar to the second stage in the infinite backup supply scenario, the retailer has obtained the supply from supplier 1, whose supply uncertainty has been addressed. Then, the retailer needs to determine the backup order quantity from supplier 2 according to the β-supply of supplier 1 and the price strategy in stage III. Since supplier 2's yield is also uncertain, the profit of the retailer becomes

$$v_{cu}^U(p, q_2; \xi) = v_e(p) - w_2 \xi_2 q_2.$$

By using the method similar to the benchmark and for the given market price p and order quantity q_1, the retailer's α-profit $v_c^U(p, q_2; \alpha)$ can be denoted by

$$v_c^U(p, q_2; \alpha) = \max\left\{v_0 \;\middle|\; \mathcal{M}\{v_{cu}^U(p, q_2; \xi_2) \geqslant v_0\} \geqslant \alpha\right\}.$$

In view of the β-supply of supplier 1 and the price strategy in stage III, the model for maximizing the retailer's α-profit is

$$\max_{q_2 \geqslant 0} \ v_c^U\left(\overline{p}^U(q_1, q_2), q_2; \alpha\right). \tag{3.7}$$

Similarly, for notation convenience, we denote $t_2 = \Phi_2^{-1}(1-\beta)/\Phi_2^{-1}(\alpha)$, which decreases in α and β. We call the ratio w_2/t_2 the purchasing belief degree cost of the retailer from supplier 2. This implies that the retailer uses the purchasing belief degree cost to quantify the cost of change due to the supply uncertainty of the supplier. That is, the supply uncertainty is ultimately reflected in the cost of change (*i.e.*, the purchasing belief degree cost).

Lemma 3.11. *The backup order quantity and optimal price in model (3.7) are as follows.*

(i) *When $w_2/t_2 \leqslant a/b$, we obtain*
  (a) *If $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, then $\overline{q}_2^U(q_1) = 0$ and $\overline{p}^U(q_1) = a/(2b)$.*
  (b) *If $(at_2 - bw_2)/(2t_2) < \Phi_1^{-1}(1-\beta)q_1 < a/2$, then $\overline{q}_2^U(q_1) = 0$ and $\overline{p}^U(q_1) = a/b - \Phi_1^{-1}(1-\beta)q_1/b$.*
  (c) *If $\Phi_1^{-1}(1-\beta)q_1 \leqslant (at_2 - bw_2)/(2t_2)$, then*

$$\overline{q}_2^U(q_1) = \frac{at_2 - bw_2}{2t_2\Phi_2^{-1}(1-\beta)} - \frac{\Phi_1^{-1}(1-\beta)q_1}{\Phi_2^{-1}(1-\beta)},\ \overline{p}^U(q_1) = \frac{a}{2b} + \frac{w_2}{2t_2}\cdot$$

(ii) *When $w_2/t_2 > a/b$, we obtain*
  (a) *If $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, then $\overline{q}_2^U(q_1) = 0$ and $\overline{p}^U(q_1) = a/(2b)$.*
  (b) *If $0 \leqslant \Phi_1^{-1}(1-\beta)q_1 < a/2$, then $\overline{q}_2^U(q_1) = 0$ and $\overline{p}^U(q_1) = a/b - \Phi_1^{-1}(1-\beta)q_1/b$.*

Lemma 3.11 provides the five backup order decisions of the retailer. Compared with Lemma 3.5, the threshold of the market supply becomes $(at_2 - bw_2)/(2t_2)$ when $w_2/t_2 \leqslant a/b$, that is, the retailer abandons the backup order and only orders from supplier 1 when the β-supply of supplier 1 is higher than the threshold. Similarly, a stable market price is set by the retailer if supplier 1's β-supply is sufficient (*i.e.*, case (ia)); otherwise, a flexible price is employed (*i.e.*, case (ib)). If supplier 1's β-supply is lower than the threshold, the retailer intends to raise the total realized supply in the market to the level of the threshold. Thus, the retailer places an order from the backup supplier such that the realized supply $\Phi_2^{-1}(1-\beta)q_2$ under the RLSM β (is named the β-supply of supplier 2 and $\Phi_2^{-1}(1-\beta)$ is named the β-supply rate of supplier 2) is equal to the difference between the threshold and the β-supply of supplier 1 (*i.e.*, $\Phi_2^{-1}(1-\beta)\overline{q}_2^U(q_1) = (at_2 - bw_2)/(2t_2) - \Phi_1^{-1}(1-\beta)q_1$). Since suppler 2's supply is uncertain, the retailer's backup order quantity is the ratio of supplier 2's β-supply to his β-supply rate (*i.e.*, case (ic)). When $w_2/t_2 > a/b$, the threshold degenerates to zero. Correspondingly, the three subcases in case (i) degenerate to the two in case (ii). According to the above analyses, we obtain the following proposition.

Proposition 3.12. *In the uncertain backup supply scenario, when $w_2/t_2 \leqslant a/b$, the retailer's backup order quantity is*

$$\left((at_2 - bw_2)/(2t_2) - \Phi_1^{-1}(1-\beta)q_1\right)^+ /\Phi_2^{-1}(1-\beta),$$

where $\left((at_2 - bw_2)/(2t_2) - \Phi_1^{-1}(1-\beta)q_1\right)^+ = \max\{(at_2 - bw_2)/(2t_2) - \Phi_1^{-1}(1-\beta)q_1, 0\}$; otherwise, it is zero.

Proposition 3.12 implies that the retailer raises the total realized supply in the market to the level of a threshold $(at_2 - bw_2)/(2t_2)$. The reason is that the retailer can earn the highest profit in stage III with the market demand equaling the threshold $(at_2 - bw_2)/(2t_2)$ at the retail price $\overline{p}^U(q_1) = a/(2b) + w_2/(2t_2)$. In fact, when the RLSM is low or the belief degree of service interruption is high (*i.e.*, $w_2/t_2 \leqslant a/b$), the retailer determines her backup order quantity according to the β-supply of supplier 1. However, the sufficiently high RLSM (*i.e.*, the sufficiently low belief degree of service interruption) can reduce the β-rate of the backup supplier, which results in the level of the threshold being no more than zero. Thus, the threshold degenerates to zero when the RLSM is high (*i.e.*, $w_2/t_2 > a/b$).

Compared with the infinite backup supply scenario, the threshold of the market supply decreases (because $(at_2 - bw_2)/(2t_2) \leqslant (a - bw_2)/2$) in this scenario. The reason is that the lacking supply is completely complemented by the backup supplier because the supply is infinite in the infinite backup supply scenario. In contrast, in the uncertain backup supply scenario, the lacking supply cannot be completely complemented due to supply uncertainty, leading to the lower threshold.

3.4.3. Retailer's main order decision in stage I of the uncertain backup supply scenario

In this stage, the retailer's profit is denoted as $v^U(p, q_1, q_2; \xi_1, \xi_2) = v_e(p) - w_2\xi_2 q_2 - w_1\xi_1 q_1$. Since the retailer is single, we use the same confidence level as that in stage II, α, to characterize the retailer's profit, which is called the retailer's α-profit and is denoted as

$$v_m^U(p, q_1, q_2; \alpha) = \max\left\{v_0 \mid \mathcal{M}\{v^U(p, q_1, q_2; \xi_1, \xi_2) \geqslant v_0\} \geqslant \alpha\right\}.$$

On the basis of the price and order decisions in stages III and II, the retailer maximizes her α-profit to determine her order quantity from supplier 1, *i.e.*,

$$\max_{q_1 \geqslant 0}\ v_m^U\left(\overline{p}^U(q_1), q_1, \overline{q}_2^U(q_1); \alpha\right). \tag{3.8}$$

Lemma 3.13. *The order strategy in the uncertain backup supply scenario is as follows.*

(i) *When $w_2/t_2 \leqslant a/b$, we obtain*
  (a) *If $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, then $\overline{q}_1^U = a/\left(2\Phi_1^{-1}(1-\beta)\right)$, $\overline{q}_2^U = 0$.*
  (b) *If $(at_2 - bw_2)/(2t_2) \leqslant \Phi_1^{-1}(1-\beta)q_1 < a/2$, then*
    – *If $t_1/t_2 \geqslant w_1/w_2$, then $\overline{q}_1^U = (at_1 - bw_1)/\left(2t_1\Phi_1^{-1}(1-\beta)\right)$, $\overline{q}_2^U = 0$.*
    – *If $t_1/t_2 < w_1/w_2$, then $\overline{q}_1^U = (at_2 - bw_2)/\left(2t_2\Phi_1^{-1}(1-\beta)\right)$, $\overline{q}_2^U = 0$.*
  (c) *If $\Phi_1^{-1}(1-\beta)q_1 < (at_2 - bw_2)/(2t_2)$, then*
    – *If $t_1/t_2 \geqslant w_1/w_2$, then $\overline{q}_1^U = (at_2 - bw_2)/\left(2t_2\Phi_1^{-1}(1-\beta)\right)$, $\overline{q}_2^U = 0$.*
    – *If $t_1/t_2 < w_1/w_2$, then $\overline{q}_1^U = 0$, $\overline{q}_2^U = (at_2 - bw_2)/\left(2t_2\Phi_2^{-1}(1-\beta)\right)$.*

(ii) *When $w_2/t_2 > a/b$, we obtain*
  (a) *If $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, then $\overline{q}_1^U = a/\left(2\Phi_1^{-1}(1-\beta)\right)$, $\overline{q}_2^U = 0$.*
  (b) *If $\Phi_1^{-1}(1-\beta)q_1 < a/2$, then*
    – *If $t_1 \geqslant bw_1/a$, then $\overline{q}_1^U = (at_1 - bw_1)/\left(2t_1\Phi_1^{-1}(1-\beta)\right)$, $\overline{q}_2^U = 0$.*
    – *If $t_1 < bw_1/a$, then $\overline{q}_1^U = 0$, $\overline{q}_2^U = 0$.*

Lemma 3.13 provides the eight order decisions for different RLSMs and different β-supplies of supplier 1. When the RLSM is low (*i.e.*, case (i)), the analysis of this lemma is similar to that of Lemma 3.7. The order decisions corresponding to case (iia) and the first subcase of case (iib) are the same as those of case (ia) and the first subcase of case (ib), respectively. Their differences are that they hold under the different conditions about the RLSM. The last order decision indicates that the retailer does not purchase the product from any supplier due to no profit when the RLSM is too high. By comparing the eight order decisions in accordance with the profit of the retailer, the optimal solution and profit in the uncertain backup supply scenario are described by the following proposition.

Proposition 3.14. *The optimal solution and profit in the uncertain backup supply scenario are as follows.*

(i) *When $w_2/t_2 \leqslant a/b$, we have*
  (a) *If $w_2/t_2 \geqslant w_1/t_1$, then*

$$\overline{q}_1^U = \frac{at_1 - bw_1}{2t_1\Phi_1^{-1}(1-\beta)},\ \overline{q}_2^U = 0,\ \overline{p}^U = \frac{at_1 + bw_1}{2bt_1},\ v_m^U(\overline{p}^U, \overline{q}_1^U, \overline{q}_2^U; \alpha) = \frac{(at_1 - bw_1)^2}{4bt_1^2}.$$

Moreover, the market price increases in α and β, whereas the profit of the retailer decreases in α and β. The order quantity from supplier 1 decreases in α, while it increases in β if $t_1 \geqslant 2bw_1/a$; otherwise it decreases in β.

  (b) *If $w_2/t_2 < w_1/t_1$, then*

$$\overline{q}_1^U = 0,\ \overline{q}_2^U = \frac{at_2 - bw_2}{2t_2\Phi_2^{-1}(1-\beta)},\ \overline{p}^U = \frac{at_2 + bw_2}{2bt_2}, v_m^U(\overline{p}^U, \overline{q}_1^U, \overline{q}_2^U; \alpha) = \frac{(at_2 - bw_2)^2}{4bt_2^2}.$$

Moreover, the market price increases in α and β, whereas the profit of the retailer decreases in α and β. The order quantity from supplier 2 *decreases in α, while it increases in β if $t_2 \geqslant 2bw_2/a$; otherwise it decreases in β.*

(ii) *When $w_2/t_2 > a/b$, we have*
  (a) *If $w_1/t_1 \leqslant a/b$, then the results are the same as those in case (ia).*
  (b) *If $w_1/t_1 > a/b$, then $\overline{q}_1^U = 0$, $\overline{q}_2^U = 0$, $\overline{p}^U = a/b$, $v_m^U(\overline{p}^U, \overline{q}_1^U, \overline{q}_2^U; \alpha) = 0$.*

Proposition 3.14 indicates that the retailer has three strategies: price adjustment strategy (only orders from the main supplier), backup supply strategy (only orders from the backup supplier) or neither of them. When the retailer selects a low RLSM due to the supply uncertainty of supplier 2 (*i.e.*, $w_2/t_2 \leqslant a/b$), she chooses the price adjustment strategy if supplier 1 has a purchasing belief degree cost advantage (*i.e.*, $w_2/t_2 \geqslant w_1/t_1$); otherwise, she chooses the backup supply strategy because the advantage of the purchasing belief degree cost from supplier 2 (*i.e.*, $w_2/t_2 < w_1/t_1$) induces the retailer to obtain more profits. When the retailer selects a high RLSM such that the purchasing belief degree cost from supplier 2 is higher than the purchasing threshold (*i.e.*, $w_2/t_2 > a/b$), she chooses the price adjustment strategy if she selects a low RLSM such that the purchasing belief degree cost from supplier 1 is no higher than the purchasing threshold (*i.e.*, $w_1/t_1 \leqslant a/b$); otherwise, she chooses neither of the above two strategies because the sufficiently high RLSM chosen by the retailer (satisfying $w_1/t_1 > a/b$ and $w_2/t_2 > a/b$) forces the market demand to be zero, which forces the retailer to be unprofitable.

According to Propositions 3.3, 3.8 and 3.14, we observe that the decisions of the retailer in the uncertain backup supply scenario are the synthesis of those in the benchmark and the infinite supply scenario. In fact, the conclusions in cases (ia) and (iia) in Proposition 3.14 are the same as that in case (i) in Proposition 3.8 or 3.3. Their differences are that they hold under the different ranges about the RLSM. The conclusion in case (ib) in Proposition 3.14 is similar to that in case (ii) in Proposition 3.8. The difference is that the threshold $(a - bw_2)/2$ in Proposition 3.8, which is the order quantity of the retailer from supplier 2, becomes $(at_2 - bw_2)/(2t_2)$, which is the β-supply of supplier 2. And the results hold under the different ranges about the RLSM. The conclusion in case (iib) in Proposition 3.14 is the same as that in case (ii) in Proposition 3.3. That is, the retailer's order quantity is zero due to too high RLSM. Comparing Propositions 3.3, 3.8 and 3.14, we have the following proposition for the range of the interval about the RLSM.

Proposition 3.15. *In the uncertain backup supply scenario, the range of the RLSM interval, on which the retailer does not order from either supplier, is narrower than that in the benchmark. In contrast, the range does not exist in the infinite backup supply scenario, that is, the retailer orders from either supplier* 1 *or* 2 *in this scenario.*

Proposition 3.15 reveals that for different RLSMs, the retailer can adopt a flexible price to enforce the demand management in the best way in the infinite backup supply scenario, can adopt it in a better way in the uncertain backup supply scenario and can adopt it in the worst way in the benchmark. The conclusion is straightforward, because a supply breakdown in the benchmark is partially replaced by the added shipments from supplier 2 due to the uncertainty of the backup supply in the uncertain backup supply scenario, whereas it is completely replaced due to the infinite supply of the backup supplier in the infinite backup supply scenario.

In the uncertain backup supply scenario, Figure 5 states the effect of the PRLP and RLSM on the optimal order strategies, and Figure 6 characterizes the impact of the RLSM on the optimal order quantities, where $\xi_2 \sim \mathcal{L}(m_2, n_2), m_2 = 0.2, n_2 = 0.85$ and the other parameter values are the same as those in Figures 3 and 4, respectively. In Figure 5, the four areas correspond to the ranges of the PRLP and RLSM intervals in the four cases in Proposition 3.14, respectively. From the figure, we observe that for any given PRLP, the retailer selects the order decision corresponding to case (ia) or (ib) in Proposition 3.14 when the RLSM is low (*i.e.*, in the first or second area). Otherwise, she chooses the order decision corresponding to case (iia) or (iib) in Proposition 3.14 (*i.e.*, in the third or fourth area). Particularly, the retailer's total order quantity is zero when both the PRLP and RLSM are high (*i.e.*, in the fourth area). The analysis for Figure 6 is similar to that for Figure 4. Comparing the two figures, we see that the range of the RLSM interval in Figure 6 is wider than that in Figure 4 when the

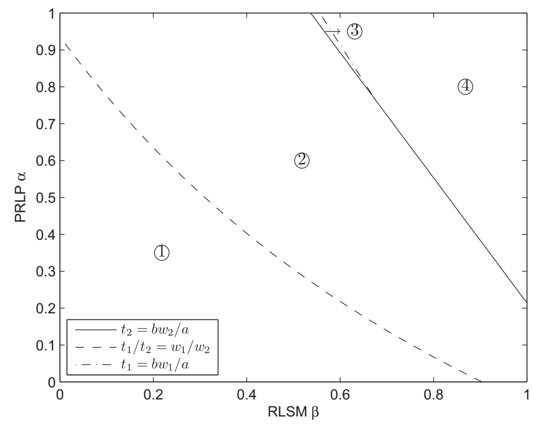


FIGURE 5. Effect of RLSM β and confidence level α on order strategy.

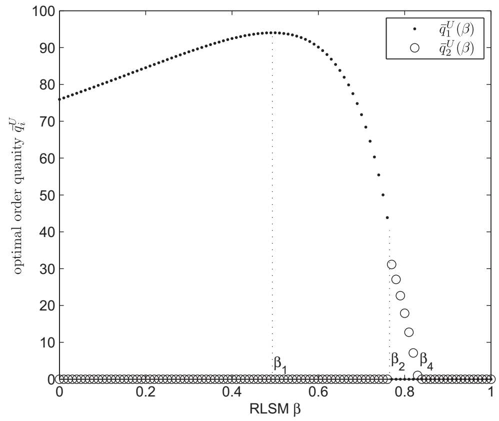


FIGURE 6. Effect of RLSM β on order quantity $\overline{q}_i^U$.

retailer only orders from supplier 1. In Figure 6, when the RLSM increases, the retailer's total order quantity increases monotonically and then decreases to zero. In Figure 4, the front part of the total order quantity curve is the same as that in Figure 6, whereas it is constant when the RLSM is high, that is, it is not affected by the RLSM.

4. Extended model

In this section, we relax the condition that the wholesale price of supplier 1 is exogenously assumed in Section 3 and that of supplier 2 is still exogenous to consider the retailer's order strategy and supplier 1's price decision. Additionally we analyze the impact of the RLSM on the retailer's order strategy and supplier 1's price decision.

4.1. Supplier 1's price decision in the benchmark

Denote supplier 1's profit as $u_1(w_1, q_1; \xi_1) = (w_1 - c_1)\xi_1 q_1$ and use the confidence level γ to describe his profit risk. Under a given confidence level γ, the profit of supplier 1 can be expressed as u_0 belonging to $\left\{u_0 \mid \mathcal{M}\{u_1(w_1, q_1; \xi_1) \geqslant u_0\} \geqslant \gamma\right\}$, which is a set of supplier 1's profits. Given the wholesale price w_1 and the order quantity q_1, supplier 1's γ-profit $u_1(w_1, q_1; \xi_1)$ can be represented as

$$u_{m1}(w_1, q_1; \gamma) = \max\left\{u_0 \mid \mathcal{M}\{u_1(w_1, q_1; \xi_1) \geqslant u_0\} \geqslant \gamma\right\}. \tag{4.1}$$

According to the retailer's order strategy for the first case in Proposition 3.3, the model for maximizing the γ-profit of supplier 1 is

$$\max_{w_1 \geqslant c_1} \; u_{m1}(w_1, \overline{q}_1; \gamma). \tag{4.2}$$

Proposition 4.1. *In the benchmark, when $c_1/t_1 \leqslant a/b$, the optimal wholesale price of supplier 1 and the optimal order quantity are*

$$\overline{w}_1 = (bc_1 + at_1)/(2b), \; \overline{q}_1 = (at_1 - bc_1)/\left(4t_1\Phi_1^{-1}(1-\beta)\right).$$

Otherwise, the profits of the retailer and supplier 1 are zero.

Proposition 4.1 provides the main supplier's price decisions according to the retailer's order decisions for different RLSMs in the benchmark. In fact, when the RLSM is low (*i.e.*, $c_1/t_1 \leqslant a/b$), the optimal wholesale price of supplier 1 decreases with respect to the RLSM. In this case, when the RLSM increases, implying that the retailer believes that the belief degree of supply interruption is low (*i.e.*, the realized supply is low), the retailer will increase its retail price to adjust/decrease the market demand. Since the main supplier determines the wholesale price according to the realized supply, maximizing the main supplier's profit will force the wholesale price to decrease. However, when the RLSM is high (*i.e.*, $c_1/t_1 > a/b$), the retailer and supplier 1 are not willing to purchase or sell the product. The reason is that the ratio c_1/t_1 (named the belief degree cost of supplier 1) being no lower than the purchasing threshold (a/b) forces the demand to be zero, implying that the profits of retailer and supplier 1 are zero.

4.2. Supplier 1's price decision in the infinite backup supply scenario

According to Eq. (4.1) and the retailer's order strategy in Proposition 3.8, the model for maximizing the γ-profit of supplier 1 in the infinite backup supply scenario is

$$\max_{w_1 \geqslant c_1} \; u_{m1}(w_1, \overline{q}_1^I; \gamma). \tag{4.3}$$

Proposition 4.2. *In the infinite backup supply scenario, the optimal wholesale price of supplier* 1 *and the order quantity of the retailer are as follows.*

(i) *When* $w_2 \geqslant c_1/(2t_1) + a/(2b)$*, we obtain*

$$\overline{w}_1^I = (bc_1 + at_1)/(2b), \; \overline{q}_1^I = (at_1 - bc_1)/\left(4t_1\Phi_1^{-1}(1-\beta)\right).$$

(ii) *When* $c_1/t_1 \leqslant w_2 < c_1/(2t_1) + a/(2b)$*, we obtain*

$$\overline{w}_1^I = w_2 t_1, \; \overline{q}_1^I = (a - bw_2)/\left(2\Phi_1^{-1}(1-\beta)\right).$$

(iii) *When* $w_2 < c_1/t_1$*, then the retailer only orders from the backup supplier.*

Proposition 4.2 indicates that in the infinite backup supply scenario, the main supplier has three price decisions when the RLSM increases. From cases (i) to (iii), we observe that when the RLSM increases, the belief degree cost of supplier 1 (*i.e.*, c_1/t_1) increases. Since the wholesale price of supplier 2 is exogenous and constant, the purchasing cost advantages of the retailer from supplier 2 are gradually strengthened. This forces the wholesale price of supplier 1 and the order quantity from supplier 1 to decrease. When the order quantity from supplier 1 reduces to zero. The order from supplier 1 is replaced by that from supplier 2.

4.3. Supplier 1's price decision in the uncertain backup supply scenario

In the uncertain backup supply scenario, the maximization profit model of supplier 1 becomes

$$\max_{w_1 \geqslant c_1} \; u_{m1}(w_1, \overline{q}_1^U; \gamma). \tag{4.4}$$

Proposition 4.3. *In the uncertain backup supply scenario, the optimal wholesale price of supplier* 1 *and the order quantity of the retailer are as follows.*

(i) *When* $w_2/t_2 \leqslant a/b$*, we have*
   (a) *If* $w_2/t_2 \geqslant c_1/(2t_1) + a/(2b)$*, then*

$$\overline{w}_1^U = (bc_1 + at_1)/(2b), \; \overline{q}_1^U = (at_1 - bc_1)/\left(4t_1\Phi_1^{-1}(1-\beta)\right).$$

(b) *If* $c_1/t_1 \leqslant w_2/t_2 < c_1/(2t_1) + a/(2b)$, *then*

$$\overline{w}_1^U = w_2 t_1/t_2,\ \overline{q}_1^U = (at_2 - bw_2)/\left(2t_2\Phi_1^{-1}(1-\beta)\right).$$

(c) *If* $w_2/t_2 < c_1/t_1$, *then the retailer only orders from the backup supplier.*

(ii) *When* $w_2/t_2 > a/b$ *and* $c_1/t_1 \leqslant a/b$, *the results are the same as those in case (ia).*

(iii) *When* $w_2/t_2 > a/b$ *and* $c_1/t_1 > a/b$, *the profits of the retailer and the two suppliers are zero.*

Proposition 4.3 provides the main supplier's price decisions and supply decisions in the uncertain backup supply scenario for different RLSMs. When the purchasing belief degree cost of the retailer from supplier 2 is lower than the purchasing threshold, from cases (ia) to (ic), we observe that when the RLSM changes, the purchasing belief degree cost advantages of the retailer from supplier 2 are gradually strengthened. This forces the wholesale price of supplier 1 and the order quantity from supplier 1 to decrease. When the order quantity from supplier 1 reduces to zero, the order from supplier 1 is replaced by that from supplier 2. When the purchasing belief degree cost of the retailer from supplier 2 is higher than the purchasing threshold, the retailer has no profit if she orders from supplier 2. Thus, the retailer orders either from supplier 1, whose price decision is the same as that in case (ia), or from neither of the two suppliers. The retailer selects the former strategy if the RLSM is low and selects the latter strategy if the RLSM is high. The reason is that the retailer has positive profit under the former decision, while the retailer has no profit under the latter decision, according to the comparison between the purchasing belief degree cost of the retailer from supplier 1 and the purchasing threshold.

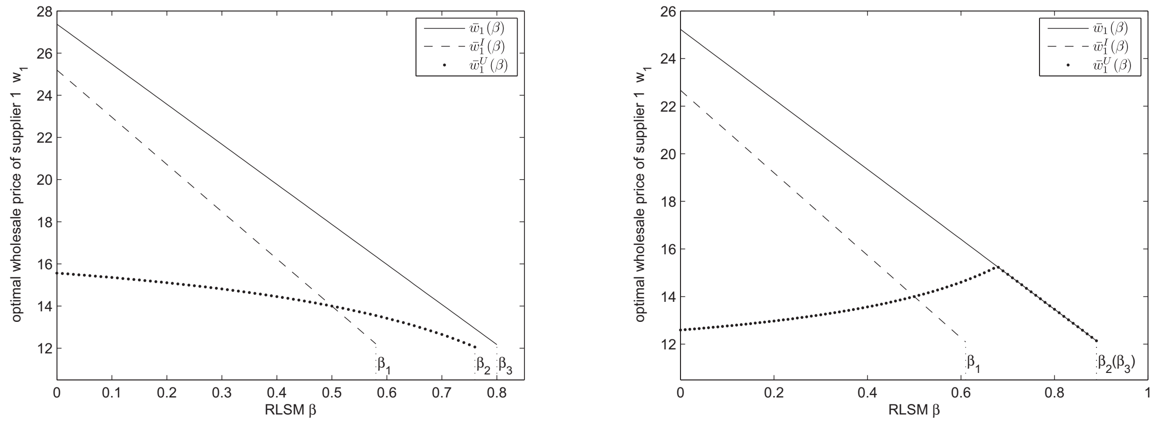
Comparing the results in Propositions 4.1–4.3, we obtain the following proposition.

Proposition 4.4. *In the benchmark, the wholesale price of supplier* 1 *is not lower than those in the infinite and uncertain backup supply scenarios. Moreover, the wholesale price decreases with the RLSM and PRLR in each case of the benchmark or the infinite backup supply scenario.*

Proposition 4.4 implies that in the three supply scenarios, it is best for supplier 1 in the benchmark, whereas the respective advantages exist for supplier 1 in the other two supply scenarios. The conclusion is straightforward, because supplier 1 needs to decrease his wholesale price due to the competition from supplier 2 in the infinite and uncertain backup scenarios. Furthermore, the higher RLSM or PRLR determined by the retailer forces supplier 1 to decrease his wholesale price to maximize his profit, due to the conservative decision of the retailer in the benchmark or the infinite backup supply scenario.

Comparing the results for the fundamental and extended models, we find that the retailer's decision selections in the three scenarios are not essential difference. The difference is that the retailer's order strategies for the extended model are more specific.

Figure 7 illustrates the effect of the RLSM on the price strategies of supplier 1 in the three scenarios, where $w_1 = 12$ is replaced by $c_1 = 12$, $m_1 = 0.1, n_1 = 0.9, m_2 = 0.2, n_2 = 0.85$ in (a), $m_1 = 0.2, n_1 = 0.85, m_2 = 0.1, n_2 = 0.9$ in (b), and the other parameter values are the same as those in Figure 6, respectively. From Figure 7, we observe that for any given RLSM, the wholesale price of supplier 1 in the benchmark is not lower than those in the infinite and uncertain backup supply scenarios. Whereas the wholesale price in the infinite backup supply scenario may be higher or lower than that in the uncertain backup supply scenario. In the uncertain backup supply scenario, the range of the RLSM interval, on which the retailer orders from supplier 1, is narrower than that in the benchmark, whereas is wider than that in the infinite backup supply scenario because $\beta_1 < \beta_2 \leqslant \beta_3$, where β_1, β_2 and β_3 respectively represent the supremum of the RLSM such that the retailer orders from supplier 1 in the three scenarios. Moreover, in each of the three scenarios, the wholesale price of supplier 1 decreases with the RLSM for the given PRLR in Figure 7a. However, from Figure 7b, we know that the wholesale price first increases and then decreases in the uncertain backup supply scenario, whereas it decreases in the other scenarios. This is because the monotonicity of the wholesale price about the RLSM depends on the distribution functions of the two suppliers' uncertain supply rates in the uncertain backup supply



(a) $m_1 = 0.1, n_1 = 0.9, m_2 = 0.2, n_2 = 0.85$ (b) $m_1 = 0.2, n_1 = 0.85, m_2 = 0.1, n_2 = 0.9$

FIGURE 7. Effect of RLSM β on wholesale price w_1 in three scenarios.

scenario, while it only depends on that of supplier 1's uncertain supply rate in the other scenarios. Comparing the two figures, in the three scenarios, we observe that the ranges of the interval about the RLSM in Figure 7b, on which the retailer only orders from supplier 1, are wider than those in Figure 7a because the values of β_1, β_2 and β_3 in Figure 7b are greater than those in Figure 7a, respectively. This means that different related parameter values in the two distribution functions may result in the change of supplier 1's price decisions.

5. Selection of RLSM

According to Propositions 3.3, 3.8 and 3.14, we know that the retailer's profit decreases with respect to the RLSM. In other words, the higher profit of the retailer implies the lower RLSM and then may result in the retailer losing control of the market. Therefore, the retailer needs to select an appropriate RLSM to balance her profit and the reliability of servicing the market. Since the models built in the above two sections are maximizing the retailer's profit, the retailer chooses the appropriate RLSM to serve the market better; for example, she maximizes her order quantity about the RLSM.

5.1. In the fundamental model

Based on the results in Propositions 3.3, 3.8 and 3.14, the following proposition is obtained.

Proposition 5.1. *In order to maximize the order quantity of the retailer about the RLSM, we have*

(i) *In the benchmark, the optimal RLSM is the solution of the equation $t_1 = 2bw_1/a$ when the RLSM is low.*
(ii) *In the infinite backup supply scenario, the optimal RLSM is the solution of the equation*

$$t_1 = \max\{w_1/w_2, 2bw_1/a\}$$

when the RLSM is low. Otherwise, it is equal to one.
(iii) *In the uncertain backup supply scenario, the optimal RLSM is attained at the maximum point or one of the endpoints of the interval when the RLSM is low. When the RLSM is moderate, the RLSM is equal to*

$$\max\{\beta \mid t_1 = 2bw_1/a, t_2 = bw_2/a\}.$$

Proposition 5.1 demonstrates that there exists an appropriate RLSM such that the order quantity of the retailer is maximized in the three scenarios in the fundamental model. For example, when the values of parameters are the same as those in Figures 2, 4 and 6, the optimal RLSM in the benchmark equals 0.4934. In the infinite backup supply scenario, the optimal RLSM equals 0.4934 when the RLSM is low, and equals 1 when it is high. In the uncertain backup supply scenario, the optimal RLSM equals 0.4934 and 0.7652 when $\beta < \beta_2$ and $\beta_2 \leqslant \beta < \beta_4$ in Figure 6, respectively. From Figures 2, 4 and 6, the conclusions are straightforward. These results indicate that the retailer will obtain a considerable profit and not bear a higher risk (the highest risk is $0.5066 = 1 - 0.4934$) regardless of whether she is risk-loving or risk-averse.

5.2. In the extended model

In view of the results in Propositions 4.1–4.3, we obtain the following proposition.

Proposition 5.2. *In order to maximize the order quantity of the retailer about the RLSM, we have*

(i) *In the benchmark, the optimal RLSM is the solution of the equation $t_1 = 2bc_1/a$ when the RLSM is low.*
(ii) *In the infinite backup supply scenario, when the RLSM is low, the optimal RLSM is the solution of the equation $t_1 = \max\{bc_1/(2bw_2 - a), 2bc_1/a\}$. When the RLSM is moderate, the optimal RLSM is the solution of the equation $t_1 = c_1/w_2$. When the RLSM is high, the RLSM is equal to one.*
(iii) *In the uncertain backup supply scenario, the optimal RLSM is attained at the maximum point or one of the endpoints of the interval when the RLSM is low. When the RLSM is moderate, the RLSM is equal to*

$$\max\{\beta \mid t_1 = 2bc_1/a, t_2 = bw_2/a\}.$$

The analysis of Proposition 5.2 is similar to that of Proposition 5.1. The difference between them is that the wholesale price of supplier 1 is replaced by the expression about his cost or the wholesale price of supplier 2 since the wholesale price of supplier 1 is endogenous in the extension model.

6. Conclusion

This paper researches a decision selection problem of a retailer working with a main supplier and a backup supplier. The main supplier faces supply uncertainty and the backup supplier's supply is infinite or uncertain. Different from the relevant literature, the uncertainty supply rates of the two suppliers are characterized by uncertain variables on the interval from zero to one, and the RLSM is defined to measure the reliability of supply in satisfying demand. In this setting, we build the corresponding models for maximizing the participants' profits based on confidence level in the benchmark, the infinite and uncertain backup supply scenarios when the main supplier's wholesale price is exogenous or endogenous.

The RLSM reflects the retailer's risk attitude to the degree of supply in satisfying demand. Specifically, the low RLSM means that the retailer is risk-loving, while the high RLSM implies the retailer's opposite risk preference. Whether the main supplier's wholesale price is exogenous or endogenous, whether in the benchmark, the infinite or uncertain backup supply scenario, when the retailer chooses the low RLSM, she orders only from the main supplier and use a flexible price to enforce demand management. In this case, as the RLSM increases, the retailer increases the retail price to decrease the market demand, which results in the decrease of the retailer's profit. When the RLSM increases to a certain value, the retailer gives up buying from the main supplier due to no profit in the benchmark. The main supplier's supply disruption is supplemented to the threshold level from the backup supplier in the infinite backup supply scenario. This is because as the RLSM increases, the retailer believes that the cost advantage or better reputation of the main supplier is broken by the backup supplier. In the uncertain backup supply scenario, the supply disruption at the main supplier is supplemented to a threshold level by the backup supplier due to the uncertainty of the backup supply when the RLSM increases to a certain value. Different from the infinite backup supply in which the number of supplements is fixed, this quantity of replenishment decreases with the increase of the RLSM. The conclusion is intuitive because the supply is infinite in the former case while is uncertain in the latter case. When the RLSM is too high, the retailer has no profit and gives up purchasing from either of the backup main and suppliers due to the uncertainty of their supplies.

Interestingly, for different RLSMs, the retailer chooses either price adjustment strategy or backup supply strategy or neither of them, but does not choose the mixed one in the infinite and uncertain backup supply scenarios, which is counterintuitive. It is reasonable because as long as the retailer does not have a low belief degree of service interruption, she is willing to order from the main supplier and believes that her potential profit has achieved her request due to the main supplier's cost advantage or better reputation. If the retailer has a sufficiently low belief degree of service interruption, she only orders from the backup supplier because she believes that the sufficiently high RLSM forces her to be completely unprofitable if she orders from the main supplier. Therefore, the retailer is not willing to choose the mixed strategy.

Finally, since the order quantity is concave about the RLSM, maximizing it implies a better service to market or customers. For this reason, a proper RLSM is suggested by maximizing the order quantity to balance the reliability of the retailer's service market with her profit. This shows that the expectation is not always an appropriate decision rule.

In this paper, we only study the decision selection problem of one retailer with a main supplier facing supply uncertainty and a backup supplier whose yield is infinite or uncertain. If the retailer is not a monopolist in her market, that is, there exit other retailers who compete with her, it is interesting to consider the order strategy problems of the retailers under different RLSMs. In addition, we only research the profit maximization problem under different RLSMs. Considering both profit maximization and risk minimization problems under different RLSMs implies abundant research themes that would expand the scope of the current supply chain risk management literature.

Appendix A.

Lemma A.1 ([15]). *Let the function $g(x, x_1, x_2, \ldots, x_n)$ be strictly increasing with respect to $x_1, x_2, \ldots, x_k$ and strictly decreasing with respect to $x_{k+1}, x_{k+2}, \ldots, x_n$. If $\xi_1, \xi_2, \ldots, \xi_n$ are independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively, then the chance constraint*

$$M\{g(x, \xi_1, \xi_2, \ldots, \xi_n) \le 0\} \ge \alpha$$

holds if and only if

$$g\left(x, \Phi_1^{-1}(\alpha), \ldots, \Phi_k^{-1}(\alpha), \Phi_{k+1}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)\right) \le 0,$$

where $0 \leqslant \alpha \leqslant 1$.

Proof of Lemma 3.1. By Lemma A.1, Model (3.1) can be equivalently transformed into the following one,

$$\begin{cases} \max\limits_{p} \ v_e(p) = p(a - bp) \\ \text{subject to:} \\ \Phi_1^{-1}(1-\beta) q_1 \geqslant a - bp. \end{cases}$$

The first-order condition of the objective function $v_e(p)$ is $p = a/(2b)$, then there are two cases.

(i) If $\left(a - \Phi_1^{-1}(1-\beta)q_1\right)/b \leqslant a/(2b)$, *i.e.*, $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, then $\overline{p}(q_1) = a/(2b)$. Therefore, the profit $v_e\big(\overline{p}(q_1)\big) = a^2/(4b)$, implying that the first conclusion of Lemma 3.1 can be obtained.

(ii) If $\left(a - \Phi_1^{-1}(1-\beta)q_1\right)/b > a/(2b)$, *i.e.*, $\Phi_1^{-1}(1-\beta)q_1 < a/2$, then $\overline{p}(q_1) = \left(a - \Phi_1^{-1}(1-\beta)q_1\right)/b$. Therefore, the profit$v_e\big(\overline{p}(q_1)\big) = a\Phi_1^{-1}(1-\beta)q_1/b - \left(\Phi_1^{-1}(1-\beta)\right)^2 q_1^2/b$, meaning that the second conclusion of Lemma 3.1 can be obtained.

Proof of Lemma 3.2. By Lemma A.1, the retailer's α-profit $v_m(p, q_1; \alpha) = v_e(p) - w_1\Phi_1^{-1}(\alpha)q_1$.

According to Lemma 3.1, there are two cases.

(i) If $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, then the optimal solution is attained at the endpoint, and thus we have

$$p = \frac{a}{2b}, q_1 = \frac{a}{2\Phi_1^{-1}(1-\beta)}, v_e(p) = \frac{a^2}{4b}, v_m(p, q_1; \alpha) = \frac{a^2}{4b} - \frac{aw_1}{2t_1}.$$

(ii) Since the objective function in Model (3.2) is concave about the order quantity q_1, the first-order condition is $q_1 = (at_1 - bw_1)/\left(2t_1\Phi_1^{-1}(1-\beta)\right)$. Therefore, if $\Phi_1^{-1}(1-\beta)q_1 < a/2$, then

$$p = \frac{a}{2b} + \frac{w_1}{2t_1}, q_1 = \frac{at_1 - bw_1}{2t_1\Phi_1^{-1}(1-\beta)}, v_e(p) = \frac{a^2}{4b} - \frac{b^2w_1^2}{4bt_1^2}, v_m(p, q_1; \alpha) = \frac{a^2}{4b} - \frac{aw_1}{2t_1} + \frac{bw_1^2}{4t_1^2}.$$

The lemma can be proved because the objective function value in case (ii) is no smaller than that in case (i).

Proof of Proposition 3.3. By comparing the profits of the retailer in the two cases of Lemma 3.2, the strategy corresponding to case (i) is dominated by those corresponding to case (ii) from the perspective of the profit of the retailer because $a^2/(4b) - aw_1/(2t_1) \leqslant (at_1 - bw_1)^2/(4bt_1^2)$ and $a^2/(4b) - aw_1/(2t_1) \leqslant 0$. According to the results, the monotonicity of the market price, order quantity and profit about α and β can be obtained.

Proof of Lemma 3.4. The proof of the lemma is similar to that of Lemma 3.1.

Proof of lemma 3.5. By Lemma 3.4, there are two cases.

Case 1: If $\Phi_1^{-1}(1-\beta)q_1 + q_2 \geqslant a/2$, then

$$v_c^I\left(\overline{p}^I(q_1, q_2), q_2\right) = v_e\left(\overline{p}^I(q_1, q_2)\right) - w_2q_2 = a^2/(4b) - w_2q_2,$$

and thus, there are two subcases.

(a) If $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, then the optimal solution is attained at the endpoint (*i.e.*, $\overline{q}_2^I(q_1) = 0$), and thus the first conclusion in this lemma holds.

(b) If $\Phi_1^{-1}(1-\beta)q_1 < a/2$, then

$$q_2^I(q_1) = a/2 - \Phi_1^{-1}(1-\beta)q_1,\ v_c^I\left(\overline{p}^I(q_1, q_2), q_2\right) = a^2/(4b) - aw_2/2 + w_2\Phi_1^{-1}(1-\beta)q_1.$$

Case 2: If $\Phi_1^{-1}(1-\beta)q_1 + q_2 < a/2$, then

$$v_c^I\left(\overline{p}^I(q_1, q_2), q_2\right) = \frac{a(\Phi_1^{-1}(1-\beta)q_1 + q_2)}{b} - \frac{(\Phi_1^{-1}(1-\beta)q_1 + q_2)^2}{b} - w_2q_2.$$

Its first-order condition about q_2 is $q_2 = (a - bw_2)/2 - \Phi_1^{-1}(1-\beta)q_1$, and then there are two subcases.

(a) If $(a - bw_2)/2 < \Phi_1^{-1}(1-\beta)q_1 < a/2$, then

$$\overline{q}_2^I(q_1) = 0, v_c^I\left(\overline{p}^I(q_1, q_2), q_2\right) = \frac{a\Phi_1^{-1}(1-\beta)q_1}{b} - \frac{\left(\Phi_1^{-1}(1-\beta)\right)^2 q_1^2}{b}.$$

(b) If $\Phi_1^{-1}(1-\beta)q_1 \leqslant (a - bw_2)/2$, then

$$q_2^I(q_1) = (a - bw_2)/2 - \Phi_1^{-1}(1-\beta)q_1,\ v_c^I\left(\overline{p}^I(q_1, q_2), q_2\right) = a^2/(4b) - aw_2/2 + w_2\Phi_1^{-1}(1-\beta)q_1 + bw_2^2/4.$$

Next, we compare case 1(b) with cases 2(a) and 2(b) from the following two scenarios.

– If $(a - bw_2)/2 < \Phi_1^{-1}(1-\beta)q_1 < a/2$, then case 2(a) is better than case 1(b) because

$$\left(\frac{a\Phi_1^{-1}(1-\beta)q_1}{b} - \frac{\left(\Phi_1^{-1}(1-\beta)\right)^2 q_1^2}{b}\right) - \left(\frac{a^2}{4b} - \frac{aw_2}{2} + w_2\Phi_1^{-1}(1-\beta)q_1\right)$$
$$= -\frac{1}{b}\left(\Phi_1^{-1}(1-\beta)q_1 - \frac{a - bw_2}{2}\right)^2 + \frac{bw_2^2}{4} > 0.$$

– If $\Phi_1^{-1}(1-\beta)q_1 \leqslant (a-bw_2)/2$, then case 2(b) is better than case 1(b) because

$$\frac{a^2}{4b} - \frac{aw_2}{2} + w_2\Phi_1^{-1}(1-\beta)q_1 + \frac{bw_2^2}{4} \geqslant \frac{a^2}{4b} - \frac{aw_2}{2} + w_2\Phi_1^{-1}(1-\beta)q_1.$$

According the above comparison, case 1(b) is eliminated. The proof of this lemma is completed.

Proof of Lemma 3.7. By Lemma A.1, the retailer's α-profit in the infinite backup supply scenario

$$v_m^I(p, q_1, q_2; \alpha) = v_c^I(p, q_2) - w_1\Phi_1^{-1}(\alpha)q_1.$$

According to Lemma 3.5, there are three cases.

Case 1: If $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, then

$$v_m^I\left(\overline{p}^I(q_1), q_1, q_2^I(q_1); \alpha\right) = v_c^I\left(\overline{p}^I(q_1), q_2^I(q_1)\right) - w_1\Phi_1^{-1}(\alpha)q_1 = a^2/(4b) - w_1\Phi_1^{-1}(\alpha)q_1.$$

Thus, we have

$$q_1 = \frac{a}{2\Phi_1^{-1}(1-\beta)}, v_m^I = \frac{a^2}{4b} - \frac{aw_1}{2t_1}.$$

Case 2: If $(a-bw_2)/2 \leqslant \Phi_1^{-1}(1-\beta)q_1 < a/2$, then

$$v_m^I\left(\overline{p}^I(q_1), q_1, q_2^I(q_1); \alpha\right) = \frac{a\Phi_1^{-1}(1-\beta)q_1 - \left(\Phi_1^{-1}(1-\beta)\right)^2 q_1^2}{b} - w_1\Phi_1^{-1}(\alpha)q_1.$$

Its first-order condition about q_1 is $q_1 = \dfrac{at_1 - bw_1}{2t_1\Phi_1^{-1}(1-\beta)}$, and then there are three subcases.

– If $at_1 \geqslant bw_1$ and $\dfrac{a}{2\Phi_1^{-1}(1-\beta)} - \dfrac{bw_1}{2t_1\Phi_1^{-1}(1-\beta)} \geqslant \dfrac{a-bw_2}{2\Phi_1^{-1}(1-\beta)}$ (*i.e.*, $t_1 \geqslant w_1/w_2$ because $a \geqslant bw_2$), then

$$q_1 = \frac{at_1 - bw_1}{2t_1\Phi_1^{-1}(1-\beta)}, v_m^I = \frac{(at_1 - bw_1)^2}{4bt_1^2}.$$

– If $at_1 \geqslant bw_1$ and $w_2t_1 < w_1$ (*i.e.*, $bw_1/a \leqslant t_1 < w_1/w_2$), then

$$q_1 = \frac{a-bw_2}{2\Phi_1^{-1}(1-\beta)}, v_m^I = \frac{a^2}{4b} - \frac{bw_2^2}{4} - \frac{(a-bw_2)w_1}{2t_1}.$$

– If $t_1 < bw_1/a$, then

$$q_1 = \frac{a-bw_2}{2\Phi_1^{-1}(1-\beta)}, v_m^I = \frac{a^2}{4b} - \frac{bw_2^2}{4} - \frac{(a-bw_2)w_1}{2t_1}.$$

The latter two subcases can be combined as one scenario.

Case 3: If $\Phi_1^{-1}(1-\beta)q_1 \leqslant (a-bw_2)/2$, then

$$v_m^I\left(\overline{p}^I(q_1), q_1, q_2^I(q_1); \alpha\right) = \frac{a^2}{4b} - \frac{aw_2}{2} + \frac{bw_2^2}{4} + (w_2\Phi_1^{-1}(1-\beta) - w_1\Phi_1^{-1}(\alpha))q_1,$$

and thus there are two subcases.

– If $t_1 \geqslant w_1/w_2$, then

$$q_1 = \frac{a-bw_2}{2\Phi_1^{-1}(1-\beta)}, v_m^I = \frac{a^2}{4b} - \frac{bw_2^2}{4} - \frac{(a-bw_2)w_1}{2t_1}.$$

– If $t_1 < w_1/w_2$, then

$$q_1 = 0, v_m^I = (a-bw_2)^2/(4b).$$

This completes the proof of this lemma.

Proof of Proposition 3.8. Comparing the objective function value in case (i) with those in the first subcases of cases (ii) and (iii) in the proof process of Lemma 3.7, we know the value in the first subcase of case (ii) is greatest. Similarly, the value in the second subcase of case (iii) is greater than those in case (i) and the second subcase of case (ii). Therefore, this proposition can be proved.

Proof of Lemma 3.10. The proof of the lemma is similar to that of Lemma 3.1.

Proof of Lemma 3.11. By Lemma A.1, the retailer's α-profit in the uncertain backup supply scenario

$$v_c^U(p, q_2; \alpha) = v_e(p) - w_2\Phi_2^{-1}(\alpha)q_2.$$

According to Lemma 3.10, if $\Phi_1^{-1}(1-\beta)q_1 + \Phi_2^{-1}(1-\beta)q_2 \geqslant a/2$, then

$$v_c^U\left(\overline{p}^U(q_1,q_2), q_2\right) = v_e\left(\overline{p}^U(q_1,q_2)\right) - w_2\Phi_2^{-1}(\alpha)q_2 = a^2/(4b) - w_2\Phi_2^{-1}(\alpha)q_2,$$

and thus, there are two subcases.

- If $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, then the optimal solution is attained at the endpoint (*i.e.*, $\overline{q}_2^U(q_1) = 0$), and thus the conclusions in cases (ia) and (iia) of this lemma hold.
- If $\Phi_1^{-1}(1-\beta)q_1 < a/2$, then $q_2^U(q_1) = \dfrac{a}{2\Phi_2^{-1}(1-\beta)} - \dfrac{\Phi_1^{-1}(1-\beta)q_1}{\Phi_2^{-1}(1-\beta)}$ and $v_c^U\left(\overline{p}^U(q_1, q_2^U(q_1)), q_2^U(q_1)\right) = \dfrac{a^2}{4b} - \dfrac{aw_2}{2t_2} + \dfrac{w_2\Phi_1^{-1}(1-\beta)q_1}{t_2}$. This subcase is eliminated by the comparison method similar to that in the proof of Lemma 4.

If $\Phi_1^{-1}(1-\beta)q_1 + \Phi_2^{-1}(1-\beta)q_2 < a/2$, then

$$v_c^U\left(\overline{p}^U(q_1,q_2), q_2\right) = \frac{a\left(\Phi_1^{-1}(1-\beta)q_1 + \Phi_2^{-1}(1-\beta)q_2\right)}{b} - \frac{\left(\Phi_1^{-1}(1-\beta)q_1 + \Phi_2^{-1}(1-\beta)q_2\right)^2}{b} - w_2\Phi_2^{-1}(\alpha)q_2.$$

Its first-order condition about q_2 is $q_2 = \dfrac{at_2 - bw_2}{2t_2\Phi_2^{-1}(1-\beta)} - \dfrac{\Phi_1^{-1}(1-\beta)q_1}{\Phi_2^{-1}(1-\beta)}$, and then there are two subcases.

(1) When $t_2 \geqslant bw_2/a$, we obtain
   - If $(at_2 - bw_2)/(2t_2) < \Phi_1^{-1}(1-\beta)q_1 < a/2$, then the optimal solution is attained at the endpoint (*i.e.*, $\overline{q}_2^U(q_1) = 0$), and thus the conclusion in case (ib) of this lemma holds.
   - If $\Phi_1^{-1}(1-\beta)q_1 \leqslant (at_2 - bw_2)/(2t_2)$, then

$$q_2^U(q_1) = \frac{at_2 - bw_2}{2t_2\Phi_2^{-1}(1-\beta)} - \frac{\Phi_1^{-1}(1-\beta)q_1}{\Phi_2^{-1}(1-\beta)},$$

   and thus the conclusion in case (ic) of this lemma holds.

(2) When $t_2 < bw_2/a$ and $0 \leqslant \Phi_1^{-1}(1-\beta)q_1 < a/2$, we obtain $\overline{q}_2^U(q_1) = 0$, and thus the conclusion in case (iib) of this lemma holds. This completes the proof of the lemma.

Proof of Lemma 3.13. By Lemma A.1, the retailer's α-profit in the infinite backup supply scenario

$$v_m^U(p, q_1, q_2; \alpha) = v_e(p) - w_2\Phi_2^{-1}(\alpha)q_2 - w_1\Phi_1^{-1}(\alpha)q_1 = v_c^U(p, q_2) - w_1\Phi_1^{-1}(\alpha)q_1.$$

According to Lemma 3.11, there are five cases.

(i) When $t_2 \geqslant bw_2/a$, we obtain
   (a) If $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, then

$$v_m^U\left(\overline{p}^U(q_1), q_1, \overline{q}_2^U(q_1); \alpha\right) = v_c^U\left(\overline{p}^U(q_1), \overline{q}_2^U(q_1)\right) - w_1\Phi_1^{-1}(\alpha)q_1 = a^2/(4b) - w_1\Phi_1^{-1}(\alpha)q_1.$$

   Thus, we have $\overline{q}_1^U = a/\left(2\Phi_1^{-1}(1-\beta)\right)$, $\overline{q}_2^U = 0$, $v_m^U = a^2/(4b) - aw_1/(2t_1)$.

(b) If $(at_2 - bw_2)/(2t_2) \leqslant \Phi_1^{-1}(1-\beta)q_1 < a/2$, then

$$v_m^U\left(\overline{p}^U(q_1), q_1, \overline{q}_2^U(q_1); \alpha\right) = \frac{a\Phi_1^{-1}(1-\beta)q_1 - \left(\Phi_1^{-1}(1-\beta)\right)^2 q_1^2}{b} - w_1\Phi_1^{-1}(\alpha)q_1.$$

Its first-order condition about q_1 is $q_1 = \dfrac{at_1 - bw_1}{2t_1\Phi_1^{-1}(1-\beta)}$, and then there are three subcases.

– If $\dfrac{a}{2\Phi_1^{-1}(1-\beta)} - \dfrac{bw_1}{2t_1\Phi_1^{-1}(1-\beta)} \geqslant \dfrac{at_2 - bw_2}{2t_2\Phi_1^{-1}(1-\beta)}$, *i.e.*, $t_1 \geqslant \dfrac{w_1t_2}{w_2}$, then $t_1 \geqslant \dfrac{bw_1}{a}$, and thus we have $\overline{q}_1^U = \dfrac{at_1 - bw_1}{2t_1\Phi_1^{-1}(1-\beta)}$, $\overline{q}_2^U = 0$, $v_m^U = \dfrac{(at_1 - bw_1)^2}{4bt_1^2}$.

– If $t_1 < w_1t_2/w_2$ and $t_1 \geqslant bw_1/a$, then

$$\overline{q}_1^U = \frac{at_2 - bw_2}{2t_2\Phi_1^{-1}(1-\beta)},\ \overline{q}_2^U = 0, v_m^U = \frac{a^2}{4b} - \frac{bw_2^2}{4t_2^2} - \frac{aw_1}{2t_1} + \frac{bw_1w_2}{2t_1t_2}.$$

– If $t_1 < bw_1/a$, then $\overline{q}_1^U = \dfrac{a}{2\Phi_1^{-1}(1-\beta)} - \dfrac{bw_2}{2t_2\Phi_1^{-1}(1-\beta)}$, $\overline{q}_2^U = 0$, $v_m^U = \dfrac{a^2}{4b} - \dfrac{bw_2^2}{4t_2^2} - \dfrac{aw_1}{2t_1} + \dfrac{bw_1w_2}{2t_1t_2}$.

The latter two subcases can be combined as one scenario.

(c) If $\Phi_1^{-1}(1-\beta)q_1 \leqslant (at_2 - bw_2)/(2t_2)$, then

$$v_m^U\left(\overline{p}^U(q_1), q_1, \overline{q}_2^U(q_1); \alpha\right) = \frac{a^2}{4b} - \frac{aw_2}{2t_2} + \frac{bw_2^2}{4t_2^2} + \left(w_2\Phi_1^{-1}(1-\beta)/t_2 - w_1\Phi_1^{-1}(\alpha)\right)q_1,$$

and thus there are two subcases.

– If $t_1 \geqslant \dfrac{w_1t_2}{w_2}$, then $\overline{q}_1^U = \dfrac{at_2 - bw_2}{2t_2\Phi_1^{-1}(1-\beta)}$, $\overline{q}_2^U = 0$, $v_m^U = \dfrac{a^2}{4b} - \dfrac{bw_2^2}{4t_2^2} - \dfrac{aw_1}{2t_1} + \dfrac{bw_1w_2}{2t_1t_2}$.

– If $t_1 < \dfrac{w_1t_2}{w_2}$, then $\overline{q}_1^U = 0$, $\overline{q}_2^U = \dfrac{at_2 - bw_2}{2t_2\Phi_2^{-1}(1-\beta)}$, $v_m^U = \dfrac{(at_2 - bw_2)^2}{4bt_2^2}$.

(ii) When $t_2 < bw_2/a$, we obtain

(a) If $\Phi_1^{-1}(1-\beta)q_1 \geqslant a/2$, then $v_m^U\left(\overline{p}^U(q_1), q_1, \overline{q}_2^U(q_1); \alpha\right) = a^2/(4b) - w_1\Phi_1^{-1}(\alpha)q_1$. Thus, we have

$$\overline{q}_1^U = \frac{a}{2\Phi_1^{-1}(1-\beta)},\ \overline{q}_2^U = 0,\ v_m^U = \frac{a^2}{4b} - \frac{aw_1}{2t_1}.$$

(b) If $\Phi_1^{-1}(1-\beta)q_1 < a/2$, then

$$v_m^U\left(\overline{p}^U(q_1), q_1, \overline{q}_2^U(q_1); \alpha\right) = \frac{a\Phi_1^{-1}(1-\beta)q_1 - \left(\Phi_1^{-1}(1-\beta)\right)^2 q_1^2}{b} - w_1\Phi_1^{-1}(\alpha)q_1.$$

Its first-order condition about q_1 is $q_1 = \dfrac{at_1 - bw_1}{2t_1\Phi_1^{-1}(1-\beta)}$, and then there are two subcases.

– If $t_1 \geqslant bw_1/a$, then

$$\overline{q}_1^U = \frac{at_1 - bw_1}{2t_1\Phi_1^{-1}(1-\beta)},\ \overline{q}_2^U = 0,\ v_m^U = \frac{(at_1 - bw_1)^2}{4bt_1^2}.$$

– If $t_1 < bw_1/a$, then $\overline{q}_1^U = 0$, $\overline{q}_2^U = 0$, $v_m^U = 0$.

This completes the proof of this lemma.

Proof of Proposition 3.14. Comparing the objective function value in case (ia) with those in the first subcases of cases (ib) and (ic) in the proof process of Lemma 3.13, we obtain that the value in the first subcase of case (ib) is greatest. Similarly, the value in the second subcase of case (ic) is greater than those in case (ia) and the second subcase of case (ib). The first or second subcase of case (iib) is better than case (iia). Therefore, this proposition can be proved.

Proof of Proposition 3.15. According to case (ii) in Proposition 3.3 and case (iib) in Proposition 3.14, we obtain that the range of the RLSM interval in the benchmark, on which the retailer does not order from either supplier, is wider than that in the uncertain backup supply scenario. In the infinite backup supply scenario, the retailer orders from either supplier 1 or 2. This completes the proof of this proposition.

Proof of Proposition 4.1. According to Lemma A.1 and the order decision for the first case in Proposition 3.3, the retailer's profit

$$u_{m1}(w_1, \overline{q}_1; \gamma) = \Phi_1^{-1}(1-\gamma)(w_1 - c_1)\frac{at_1 - bw_1}{2t_1\Phi_1^{-1}(1-\beta)}.$$

Its first-order condition about w_1 is $w_1 = \dfrac{c_1}{2} + \dfrac{at_1}{2b}$. Since $t_1 \geqslant bw_1/a$ and $w_1 \geqslant c_1$, we obtain $\dfrac{c_1}{2} + \dfrac{at_1}{2b} \leqslant \dfrac{at_1}{b}$, *i.e.*, $t_1 \geqslant bc_1/a$, and then

$$\overline{w}_1 = \frac{c_1}{2} + \frac{at_1}{2b},\ \overline{q}_1 = \frac{a}{4\Phi_1^{-1}(1-\beta)} - \frac{bc_1}{4t_1\Phi_1^{-1}(1-\beta)}.$$

This completes the proof of this proposition.

Proof of Proposition 4.2. According to Lemma A.1 and the order decision for the first case in Proposition 3.8, the retailer's profit

$$u_{m1}(w_1, \overline{q}_1^I; \gamma) = \Phi_1^{-1}(1-\gamma)(w_1 - c_1)\frac{at_1 - bw_1}{2t_1\Phi_1^{-1}(1-\beta)}.$$

Its first-order condition about w_1 is $w_1 = c_1/2 + at_1/(2b)$, and then there are three subcases.

- If $w_2 t_1 \geqslant c_1/2 + at_1/(2b)$, *i.e.*, $w_2 \geqslant c_1/(2t_1) + a/(2b)$, which implies that $at_1 \geqslant bc_1$, then

$$\overline{w}_1^I = c_1/2 + at_1/(2b) \geqslant c_1,\ \overline{q}_1^I = (at_1 - bc_1)/\left(4t_1\Phi_1^{-1}(1-\beta)\right).$$

- If $c_1/t_1 \leqslant w_2 < c_1/(2t_1) + a/(2b)$, then $\overline{w}_1^I = w_2 t_1 \geqslant c_1$, $\overline{q}_1^I = (a - bw_2)/\left(2\Phi_1^{-1}(1-\beta)\right)$.
- If $w_2 < c_1/t_1$, then the the retailer only orders from the backup supplier.

This completes the proof of this proposition.

Proof of Proposition 4.3. According to Lemma 3.1 and the order decision for cases (ia) and (iia) in Proposition 3.14, the retailer's profit

$$u_{m1}(w_1, \overline{q}_1^U; \gamma) = \Phi_1^{-1}(1-\gamma)(w_1 - c_1)\frac{at_1 - bw_1}{2t_1\Phi_1^{-1}(1-\beta)}.$$

Its first-order condition about w_1 is $w_1 = c_1/2 + at_1/(2b)$, and we prove this theorem from the following two situations.

(1) When $t_2 \geqslant bw_2/a$, there are three subcases.
   - If $w_2 t_1/t_2 \geqslant c_1/2 + at_1/(2b)$, then the results in case (ia) of this theorem is obtained.
   - If $c_1 \leqslant w_2 t_1/t_2 < c_1/2 + at_1/(2b)$, then the results in case (ib) of this theorem is obtained.
   - If $w_2 < c_1/t_1$, then the the retailer only orders from the backup supplier.

(2) When $t_2 < bw_2/a$, there are two subcases.
   - If $at_1/b \geqslant c_1/2 + at_1/(2b)$, *i.e.*, $t_1 \geqslant bc_1/a$, then the results in case (ii) of this theorem is obtained.
   - If $t_1 < bc_1/a$, then the results in case (iii) of this theorem is obtained.

   This completes the proof of this proposition.

Proof of Proposition 4.4. According to the results in Propositions 4.1–4.3, this proposition can be proved.
Proof of Proposition 5.1. According to the monotonicity of the order quantity about the RLSM and its establishment conditions in Propositions 3.3, 3.8 and 3.14, this proposition can be proved.
Proof of Proposition 5.2. According to the monotonicity of the order quantity about the RLSM and its establishment conditions in Propositions 4.1–4.3, this proposition can be proved.

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