STORE-BRAND INTRODUCTION AND PRODUCTION ARRANGEMENT IN THE PRESENCE OF MULTIPLE RETAILERS

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Abstract. Store brands play an increasingly important role in retailing business, leading more and more retailers to introduce store brands. Abundant research focuses on competition between store brands and national brands and counterstrategies that national-brand manufacturers can take to counter store-brand introduction. A little research studies the store-brand production issue, however, all under single-retailer scenarios. To approach the real world, we employ game theory to model inter-action between a national-brand manufacturer and multiple locally monopolist retailers, one of whom has capability and motivation to introduce a store brand. Five Stackelberg games are build and solved to investigate: how the presence of the non-store-brand retailers affects the store-brand retailer's decision on and profitability in the store-brand introduction; how the store-brand retailer should arrange store-brand production; whether there is a win–win situation where both the store-brand retailer and the national-brand manufacturerare better off with the latter producing the store brand. Accordingly, our study offers a novel rationale for why so many, especially leading, national-brand manufacturers are involved in the store-brand production. Some useful managerial suggestions are proposed on the store-brand introduction and production arrangement.

1. Introduction

Store brands (SBs), also called private label products, are merchandise controlled and sold by a retailer and have witnessed an increasing growth and success in many countries and product categories in the past two decades, especially in Europe and food industry. According to the report from the 2016 International Private Label Yearbook, SB volume share was nearly 25% in the US [28], 46% in the UK, and increased in 13 of the 20 Europe countries [27]. Due to the successful expansion of SBs and continuing economic downturns, more and more retailers are eager to initiate their SBs to attract customers, leading to an unavoidable brands competition between SB and national brand (NB). Abundant research has paid attention to the price competition between two brands and the counterstrategies that NB manufacturers can use to moderate the impacts of SBs. However, limited attention has been paid to the SB production issue [33]. According to the classifications of the Private Label Manufacturers Association, the manufacturers that produce SB products can fall into the following three categories: the NB manufacturers that produce both their own NBs and SBs; the specialist manufacturers who

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almost exclusively concentrate on producing SBs; the major retailers and wholesalers who operate their own manufacturing plants and provide SBs for their own stores [27].

Empirically, it is shown that NB manufacturers producing their counterpart retailers' SBs is prevalent in the real world. More than half of US NB manufacturers of consumer packaged goods produce SBs as well and provide more than 60% of SBs [18, 29]. Then, the question is why NB manufacturers are willing to supply SBs for their retailers, although SBs compete with their own NBs head-to-head, and why the retailer would accept it on the other side. This important question has also been highlighted by Sethuraman and Raju [33] and Sethuraman [32] recently. In the literature, the typical motives for NB manufacturers' involvement in this practice include utilizing excess production capacity [29, 38], strengthening bargaining power on NB [3], buffering competition from follower brands [10], and cultivating a better relation with SB retailers [20, 39]. In addition, Wang and Wu [40] analytically show that all supply chain members are better off with SB being produced by one of the two competing NB manufacturers, which can mitigate promotional competition between them. When the two competing NB manufacturers are differentiated in product quality, Lewin *et al.* [19] reveal that the high-quality NB manufacturer supplying the premium SB and the low-quality NB manufacturer supplying the copycat SB can make all game players better off. Bergès and Bouamra-Mechemache [2] show that when the NB manufacturers have production efficiency advantage and excess capacity, they would be chosen to provide SB and benefit from a more powerful position on NB. Chambolle *et al.* [5] demonstrate that there exists a mutually beneficial arrangement with the SB supplied by the NB manufacturers, which can facilitate category innovation and hence improve quality of both brands. More recently, Hara and Matsubayashi [16] reveal that when a typical SB of the retailer faces intense competition from NBs of two competing NB manufacturers, a premium store brand introduced by the retailer and produced by one of NB manufacturers would make all channel members better off. However, all these studies are conducted under single-retailer scenarios. In contrast, we try to get a better understanding of this issue under a multiple-retailer scenario which more corresponds to the real business practice.

This article is also related to the studies on how to coordinate NB manufacturers and SB retailers. Gabrielsen and Sørgard [12] reveal the NB manufacturer can prevent the retailer to introduce its SB by offering an exclusivity contract on the NB wholesale price. However, Groznik and Heese [14] find that the commitment ability to deter the retailer' SB introduction can be owned only by a long-term contract on the NB wholesale price. By contrast, Fang *et al.* [11] show that the NB manufacturer may not be able to stop the retailer's SB entry when the cost per unit quality of NB is larger than that of SB. Therefore, a minimum order quantity contract is needed to coordinate the supply chain with uncertain demands. Amrouche and Yan [1] show that revenue sharing strategy can foster collaboration between the NB manufacturer and the SB retailer and all channel member are better off with a profit sharing mechanism. Distinguishing from previous research, our paper investigates whether the NB manufacturer and SB retailer have incentive to collaborate on the SB production issue.

Our logic is formulated as follows. One main insight of the existing research is that the NB manufacturer should lower the NB wholesale price strategically as a reaction to the SB entry, and the retailer, on the other hand, always benefits from the SB entry by improving its profitability on the NB as well as increasing total demands due to consumer discrimination [4,8,17,22–25,30,31]. However, this common wisdom is obtained under single-retailer scenarios. When a manufacturer distributes its NB through multiple local monopolist retailers and should comply with the Robinson–Patman Act, the NB manufacturer may not behave the same as in the single-retailer setting. According to the the Robinson–Patman Act, the NB manufacturer should charge a uniform wholesale price to all its retailers (unless there is a good and fair business reason for differentiation), and thus the NB manufacturer might have to adjust its wholesale price globally in reaction to the retailers' SB entry [15]. Therefore, the NB manufacturer may not reduce its NB wholesale price as much as that under single-retailer scenarios. Consequently, SB entry may not be so beneficial to the retailer and an incentive to withdraw NB from its shelves may arise [22]. In order to avoid this situation, the NB manufacturer should find some ways, other than lowering wholesale price, to compensate the retailer [25]. In this article, it is found that the production of SB is an implicit and effective way for the NB manufacturer to do so.

To the best of our knowledge, our work takes the first step to investigate the NB manufacturer's reaction to the SB introduction and motive for supplying SB in a more realistic setting with multiple retailers distributing

NB simultaneously. Most analytical literature regarding the impacts of the SB introduction has been conducted in single-retailer scenarios. In contrast, our findings show that the presence of the NB-manufacturer's non-SB retailers does affect the NB manufacturer's decision and hence affect the profitability of the retailer's SB introduction, which predicts that the impacts of the SB introduction may be quite different in the context of multiple retailers. Moreover, there exists a win–win situation where both the NB manufacturer and the SB retailer are better off when the former produces SB with a contract. Our findings reveal an implicit collusion between SB retailers and NB manufacturers in form of SB production arrangement, and provide another rationale for why more and more, especially the leading, NB manufacturers produce SBs for their retailers. Some empirical evidences can partially confirm our main results. For instance, Ter Braak *et al.* [39] empirically find that the leading NB manufacturers that are involved in SB production have a higher likelihood of procuring shelf presence for their brands. Steiner [36,37] show us a new variation in the category management structure wherein the leading NB manufactures act as a "Category Captain", not only to supply SB but also to price and manage the category instead of the retailers, which can be regarded as an implicit collusion between the two brands. Chen *et al.* [6] use structural model estimates for fluid milk in a major metropolitan area and verify that supplying SBs can facilitate joint pricing of two brands and benefit both NB manufacturers and retailers.

The remainder of this paper is structured as follows. In Section 2, we make some assumptions and formulate our game. In Section 3, by solving the models, we obtain the equilibrium outcomes and the main insights. In Section 4, we make some discussions and conclude our studies and offer some avenues for future research.

2. Model

Consider a setting where an NB manufacturer (he) distributes his NB product of a particular category through k systematic and local monopolist retailers, one of which (she) has motivation and capability to introduce her SB within the same product category (we will discuss the case with more than one retailer having options of SB introduction in the final section). Initially, we investigate the case with only NB as a benchmark. We assume that the NB production cost is constant and equal to c_n. The NB manufacturer complies with the Robinson–Patman Act, and thus wholesales it to his retailers at a uniform wholesale price w_n. In turn, the retailer in question and the other $k-1$ non-SB retailers resell NB to the end consumers with retail price p_n, p_n^{k-1}, respectively. We denote the case with superscript 0. For simplicity, the NB production cost c_n can be assumed to be zero without loss of generality. Then, the retailer in question is allowed to introduce her SB within the same category. If the SB is introduced, the retailer will sell SB at retail price p_s alongside with NB.

As a result, the retailer has to decide how to produce her SB product. There are two alternative choices available to her: a fringe manufacturer (hereafter called "the competitive fringe" for short) that specializes in producing SBs from a perfectly competitive market, and the NB manufacturer in question. In line with many existing studies [2–4], as for the same production quality, we assume that the NB manufacturer has a comparative advantage in production efficiency and hence has a lower SB production cost, compared with the competitive fringe. It is believable that relative to competitive fringe manufacturers, experienced large NB manufacturers can possess a technology or services difference in producing a SB within some certain categories [9, 13]. Therefore, we assume the SB production cost of the competitive fringe equals w_s. The competitive fringe is assumed to act as a dumb player and make no margin since it is from a perfectly competitive market. Therefore, the competitive fringe supplies SB at its production cost w_s if it is entrusted to produce SB for the retailer. Alternatively, the NB manufacturer can produce SB at production cost $c_n = 0$ identical with that of NB due to the same production process and product quality. To exclude the widely accepted wisdom that NB manufacturers supplying SBs is to share SB market by making use of excess capacity, we assume that the NB manufacturer supplies SB at its production cost, to explore whether NB manufacturers have other incentives to supply SBs.

We assume k locally monopolistic markets are symmetric and there is no direct competition among them, except an SB is introduced in one of local markets. We focus on the interesting market with an SB. Within this particular product category, consumers make their purchasing decisions by maximizing their utility through

comparing the prices and quality of the products. Consumers are assumed to be heterogeneous in the valuation of every unit perceived quality. We denote this value by v and assume v is uniformly distributed within the consumer population in $[0, 1]$ with unit density. Each consumer buys at most one unit product, either NB, or SB product, or nothing. Due to the heavy NB advertisements [7, 21, 34, 35], consumers would deem SB as an inferior substitute for NB, although NB and SB are produced in the same production process and product quality. In other words, consumers perceive a lower quality of SB than NB. For simplicity, the perceived quality of NB is normalized and thus the SB perceived quality is γ ($0 < \gamma < 1$). As a consequence, the consumers' valuation of NB is v and SB is γv. Given retail prices p_n, p_s and the consumers' valuation $v, \gamma v$ of NB and SB respectively, the consumer derives a utility of $v - p_n$ by buying NB, and $\gamma v - p_s$ by buying SB. Accordingly, consumers with the valuation $v \geq p_n$ would consider purchasing NB. Likewise, consumers with the valuation $v \geq \frac{p_s}{\gamma}$ would consider purchasing SB. If $v - p_n > \gamma v - p_s$, then NB would be preferred; if $v - p_n < \gamma v - p_s$ and $\gamma v - p_s > 0$, then SB would be preferred; if $v - p_n < \gamma v - p_s$ and $\gamma v - p_s < 0$, then consumers would buy nothing. It is straightforward to derive the valuation of the two kinds of marginal consumers: $\frac{p_n - p_s}{1 - \gamma}$ for those indifferent between buying NB and SB; $\frac{p_s}{\gamma}$ for those indifferent between buying SB and nothing. Explicitly, when $p_s > \gamma p_n$, there is no sale of SB at all. Hence, $p_s > \gamma p_n$ is a necessary condition for the SB introduction. So the demands for NB and SB, respectively, are

$$
\begin{aligned}
q_n &= \begin{cases} 1 - \dfrac{P_n - P_s}{1 - \gamma} & \text{if } p_s < \gamma p_n, \\ 1 - p_n & \text{otherwise}, \end{cases} \\
q_s &= \begin{cases} \dfrac{\gamma P_n - P_s}{\gamma(1 - \gamma)} & \text{if } p_s < \gamma p_n, \\ 0 & \text{otherwise}. \end{cases}
\end{aligned}
\tag{2.1}
$$

In this study, we assume that the NB manufacturer is power and dominates its k retailers, thus acting as a Stakerberg-game leader. As we have mentioned above, the competitive fringe acts as a nonstrategic player. Therefore, w_s is an exogenous variable and taken as a parameter throughout this paper. Thus the interaction between the NB manufacturer and its retailers can be formulated as a two-stages Stakerberg game.

– Stage 1, the NB manufacturer, as a Stakerberg leader, firstly announces a uniform NB wholesale price w_n and leaves take-it-or-leave-it contracts to all his retailers, together with an additional SB supply contract to the retailer in question.
– Stage 2, given the NB wholesale price w_n, the other $k-1$ non-SB retailers, as followers, make their decisions on the NB retail price p_n^{k-1}. At the same time, given the NB wholesale price w_n, the SB supply cost w_s of the competitive fringe, and the SB supply contract of the NB manufacturer, the SB retailer, as a follower, decides whether to introduce her SB, and if so, simultaneously decides who produces SB for her and price both brands.

TABLE 1. List of notations.

Notation	Definition
γ	Store-brand perceived quality
w_n	National-brand wholesale price
w_s	Store-brand supply price by the competitive fringe
p_n, p_n^{k-1}, p_s	The store-brand-retailer's and non-store-brand-retailer's national brand retail prices and the store brand retail price
q_n, q_s	Demands for the national brand and the store brand
π_m	Manufacturer's profits
π_r, π_r^{k-1}	Profits of the store-brand retailer and the non-store-brand retailer

Our game is different from [2,3]. They both study SB production issue under the bargaining-model framework, and assume that the leading NB manufacturer can obtain the equal bargaining power on both NB and SB by supplying SB. However, we use a Stackelberg game to investigate this issue. Our model corresponds to the business practice where product profits are divided in the form of wholesale prices and retail markups, which is very common in the real word. All notations used in our model are summarized in Table 1.

3. EQUILIBRIUM MARKET OUTCOMES

3.1. NB only

In this section, as a benchmark model, we investigate the case that the NB manufacturer distributes his NB through k systematic retailers who are local monopolists. We focus our analysis on the interaction between the NB manufacturer and the retailer in question (hereinafter called "the retailer" for short) with and without SB option. In this setting, the retailer only sells the NB and has no option of her SB. Therefore, all k retailers are systematic and make the consistent decisions, $i.e.$, $p_n = p_n^{k-1}$. Consumers with valuation no less than p_n will buy NB, and so the demands for NB are $q_n = 1 - p_n$. As a consequence, the interaction between the dominant NB manufacturer and the dominated retailer can be modelled as

$$\begin{cases} \max_{w_n} \ \pi_m = kw_n(1 - p_n^*) \\ \text{where } p_n^* \text{ derive from problem:} \\ \quad \max_{p_n} \ \pi_r = (p_n - w_n)(1 - p_n). \end{cases} \tag{3.1}$$

The NB manufacturer first decides the wholesale price w_n, and then the retailer determines the retail price p_n. We can solve this game with backward induction. Given wholesale price w_n, the best response of the retailer is $p_n = (w_n + 1)/2$. Anticipating the retailer's reaction, the NB manufacturer determines his wholesale price w_n by maximizing his profits $\pi_m = kw_n(1 - w_n)/2$, yielding the optimal wholesale price $w_n^0 = 1/2$. The equilibrium profits of the NB manufacturer and the retailer respectively are $\pi_m^0 = k/8$ and $\pi_r^0 = 1/16$, and the profits of each of all other $k - 1$ retailers are also $1/16$.

3.2. With SB supplied by the competitive fringe

Now, we allow the retailer has the option of introducing SB and can choose a fringe supplier from a perfectly competitive market to produce her SB at unit supply price w_s equal to its production cost. To explore how the presence of other $k - 1$ retailers and the SB introduction of the retailer affect each other, we firstly suppose there were no other $k - 1$ retailers. In this case, the NB manufacturer firstly sets a wholesale price w_n, and given w_s and w_n, the retailer decides whether or not to introduce her SB and sequently prices NB and SB if introduced. Therefore, the pricing and SB introduction decision problems can be modeled as follows

$$\begin{cases} \max_{w_n} \ \pi_m = w_n(1 - \dfrac{p_n - p_s}{1 - \gamma}) \\ \text{where } (p_n^*, p_s^*) \text{ derive from problem:} \\ \quad \begin{cases} \max_{p_n, p_s} \ \pi_r = (p_n - w_n)(1 - \dfrac{p_n - p_s}{1 - \gamma}) + (p_s - w_s)\dfrac{\gamma p_n - p_s}{\gamma(1 - \gamma)} \\ \text{subject to:} \\ \quad p_s < \gamma p_n. \end{cases} \end{cases} \tag{3.2}$$

Likewise, we solve this problem backwards. By comparing the retailer's profits with SB and without SB in Section 3.1 and ensuring nonnegative demands for SB, we can obtain the condition for the retailer's final decision on the SB introduction. Table 2 summarizes all equilibrium outcomes. We characterize decisions of the retailer and the NB manufacturer without the presence of the other $k - 1$ retailers as follows.

TABLE 2. Equilibrium outcomes with SB supplied by competitive fringe and without the present of other $k-1$ retailers.

w_s	$\left(0, \dfrac{\gamma(1-\gamma)}{2-\gamma}\right)$	$\left[\dfrac{\gamma(1-\gamma)}{2-\gamma}, 1\right)$
w_n^*	$\dfrac{1-\gamma+w_s}{2}$	$\dfrac{1}{2}$
p_n^*	$\dfrac{3-\gamma+w_s}{4}$	$\dfrac{3}{4}$
p_s^*	$\dfrac{\gamma+w_s}{2}$	N/A
q_n^*	$\dfrac{1-\gamma+w_s}{4(1-\gamma)}$	$\dfrac{1}{4}$
q_s^*	$\dfrac{\gamma-\gamma^2+\gamma w_s-2w_s}{4\gamma(1-\gamma)}$	N/A
π_r^*	$\dfrac{\gamma+2\gamma^2-3\gamma^3+6\gamma^2 w_s-3\gamma w_s^2-6\gamma w_s+4w_s^2}{16\gamma(1-\gamma)}$	$\dfrac{1}{16}$
π_m^*	$\dfrac{(1-\gamma+w_s)^2}{8(1-\gamma)}$	$\dfrac{1}{8}$

Proposition 3.1. *Without the presence of the other $k-1$ retailers and if $w_s < \frac{\gamma-\gamma^2}{2-\gamma}$, anticipating the retailer will always introduce her SB, the NB manufacturer will lower the NB wholesale price as $w_n = \frac{1-\gamma+w_s}{2}$, and the resulting profits of the retailer and the NB manufacturer respectively are $\pi_r^* = \frac{\gamma+2\gamma^2-3\gamma^3+6\gamma^2 w_s-3\gamma w_s^2-6\gamma w_s+4w_s^2}{16\gamma(1-\gamma)}$, $\pi_m^* = \frac{(1-\gamma+w_s)^2}{8(1-\gamma)}$. Otherwise, the retailer never introduces SB.*

Proposition 3.1 shows that the SB introduction would lead a decrease in the NB wholesale price and hence hurt the NB manufacturer while benefit the retailer, which echoes with many other studies [22–25, 30, 31]. Next, we consider a more realistic case with the presence of other $k-1$ retailers. Now that the retailer decides to introduce her SB, according to the Robinson–Patman Act, the NB manufacturer should charge an identical NB wholesale price to all retailers (unless there are exceptional reasons for differentiation). As a result, the manufacturer must take other retailers' reaction into account when he makes final decision. Then, the pricing and SB introduction decision problems with the presence of k retailers can be modeled as follows

$$
\begin{cases}
\max\limits_{w_n}\ \pi_m = w_n(1-\dfrac{p_n^*-p_s^*}{1-\gamma}) + (k-1)w_n(1-p_n^{k-1*}) \\
\text{where } (p_n^*, p_s^*, p_n^{k-1*}) \text{ derive from problems:} \\
\quad\begin{cases}
\max\limits_{p_n^{k-1}}\ \pi_r^{k-1} = (p_n^{k-1}-w_n)(1-p_n^{k-1}) \\
\max\limits_{p_n,p_s}\ \pi_r = (p_n-w_n)(1-\dfrac{p_n-p_s}{1-\gamma}) + (p_s-w_s)\dfrac{\gamma p_n-p_s}{\gamma(1-\gamma)} \\
\text{subject to:} \\
\quad p_s < \gamma p_n.
\end{cases}
\end{cases}
\tag{3.3}
$$

Similarly, backward induction is used to derive the equilibria of the game. The corresponding equilibrium outcomes are summarized in Table 3. The decisions of the retailer and the NB manufacturer with the presence of the other $k-1$ retailers are illustrated in the Proposition 3.2.

Proposition 3.2. *With the presence of the other $k-1$ retailers and if $w_s < \frac{k\gamma-k\gamma^2}{2k+\gamma-2k\gamma}$, anticipating the retailer will always introduce her SB and the other $k-1$ retailers' reaction, the NB manufacturer will reduce the NB wholesale price as $w_n = \frac{k+w_s-k\gamma}{2(k+\gamma-k\gamma)}$, and the resulting profits of the retailer, every single non-SB retailer and the*

TABLE 3. Equilibrium outcomes with SB and with the present of other $k-1$ retailers.

w_s	$\left(0, \dfrac{k\gamma - k\gamma^2}{2k + \gamma - 2k\gamma}\right)$	$\left[\dfrac{k\gamma - k\gamma^2}{2k + \gamma - 2k\gamma}, 1\right)$
w_n^*	$\dfrac{k + w_s - k\gamma}{2(k + \gamma - k\gamma)}$	$\dfrac{1}{2}$
p_n^*, p_n^{k-1*}	$\dfrac{3k + 2\gamma - 3k\gamma + w_s}{4(k + \gamma - k\gamma)}$	$\dfrac{3}{4}$
p_s^*	$\dfrac{w_s + \gamma}{2}$	N/A
q_n^*	$\dfrac{k + 2\gamma - w_s + 2kw_s + 2\gamma w_s - 2\gamma^2 - 3k\gamma - 2k\gamma w_s + 2k\gamma^2}{4(k + \gamma - k\gamma)(1 - \gamma)}$	$\dfrac{1}{4}$
q_s^*	$\dfrac{k\gamma + 2k\gamma w_s - 2kw_s - \gamma w_s - k\gamma^2}{4\gamma(k + \gamma - k\gamma)(1 - \gamma)}$	N/A
π_r^*	$\dfrac{X^1}{16\gamma(1 - \gamma)(k + \gamma - k\gamma)^2}$	$\dfrac{1}{16}$
π_r^{k-1*}	$\dfrac{(k + 2\gamma - w_s - k\gamma)^2}{16(k + \gamma - k\gamma)^2}$	$\dfrac{1}{16}$
π_m^*	$\dfrac{(k + w_s - k\gamma)^2}{8(k + \gamma - k\gamma)(1 - \gamma)}$	$\dfrac{k}{8}$

Notes. $^1 X = \gamma(k + 2\gamma - k\gamma - w_s)(k + 2\gamma - w_s + 2kw_s + 2\gamma w_s - 2\gamma^2 - 3k\gamma - 2k\gamma w_s + 2k\gamma^2) + 2(\gamma - w_s)(k + \gamma - k\gamma)(k\gamma + 2k\gamma w_s - 2kw_s - \gamma w_s - k\gamma^2)$.

manufacturer respectively are $\pi_r^* = \frac{X^1}{16\gamma(1-\gamma)(k+\gamma-k\gamma)^2}, \pi_r^{k-1*} = \frac{(k+2\gamma-w_s-k\gamma)^2}{16(k+\gamma-k\gamma)^2}, \pi_m^* = \frac{(k+w_s-k\gamma)^2}{8(k+\gamma-k\gamma)(1-\gamma)}.$ *Otherwise, the retailer never introduces SB.*

We illustrate the threshold condition w_s for the SB introduction in Figure 1. From the figure, we can straightforward see that the threshold condition w_s for the SB introduction is increasing with the number of retailers k. In other words, when the number of retailers k, through which the NB manufacturer distributes his NB products, the higher likelihood of that the retailer will introduce her SB. Intuitively, when there are more retailers and the NB manufacturer should set a uniform NB wholesale price, the NB manufacturer will not lower the uniform NB wholesale price as that much as when there are less retailers to correspond to the SB introduction. As a result, the retail price of NB will also become higher correspondingly and less competitive for SB, and the retailer will more likely introduce SB despite a higher SB supply price.

By comparing the equilibrium results under the two cases with and without the presence of the other $k-1$ retailers, we can reveal how the presence of the other $k-1$ retailers affects the performance of the retailer with SB introduction, and in contrast how the SB introduction affects the profits of the other $k-1$ retailers. The Corollary 3.3 illustrates the impacts of the presence of the other $k-1$ retailers.

Corollary 3.3. (i) *By comparing the condition for the SB introduction on* w_s, *since* $\frac{\gamma(1-\gamma)}{2-\gamma} < \frac{k\gamma-k\gamma^2}{2k+\gamma-2k\gamma}$, *it shows that the presence of the other $k-1$ retailers leads to a higher likelihood of SB being introduced by the retailer;*

(ii) *By comparing the NB wholesale price, since* $\frac{1-\gamma+w_s}{2} < \frac{k+w_s-k\gamma}{2(k+\gamma-k\gamma)}$, *it shows that the NB wholesale price is higher when the other $k-1$ retailers exist;*

(iii) *By comparing the retailer's profits, when the related parameters satisfy the following conditions:* $\frac{2}{3} < \gamma \le \frac{4}{5}, \frac{3\gamma}{3\gamma-2} < k, 0 < w_s < \frac{(2k+3\gamma-3k\gamma)(1-\gamma)}{1-3k-3\gamma+3k\gamma}$; *or* $\frac{4}{5} < \gamma \le 1, \frac{3\gamma}{3\gamma-2} < k < \frac{5\gamma}{5\gamma-4}, 0 < w_s < \frac{(2k+3\gamma-3k\gamma)(1-\gamma)}{1-3k-3\gamma+3k\gamma}$; *or* $\frac{4}{5} < \gamma \le 1, \frac{5\gamma}{5\gamma-4} < k, 0 < w_s < \frac{\gamma(1-\gamma)}{2-\gamma}$, *then the retailer earns more profits by introducing her SB with the presence of the other $k-1$ retailers than without. Otherwise, the situation reverses.*

To generalize, although there is no direct competition between the retailer in question and the other $k-1$ retailers, the presence of the other $k-1$ retailers does affect the decision and performance of the retailer. More
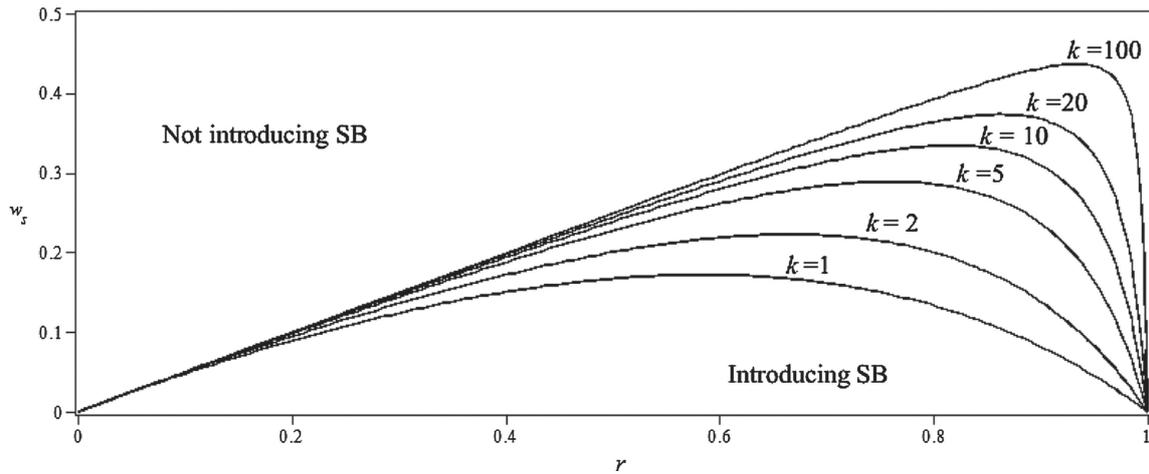
FIGURE 1. Retailer's SB introduciton strategy with SB supplied by the competitive fringe.

interestingly, the presence of the other $k-1$ retailers leads the NB manufacturer to set a higher NB wholesale price, which, however, may affect the retailer in two totally opposite ways. The intuition behind this goes as follows: a higher NB wholesale price can result in a higher NB retail price, and as such SB can cannibalize more demand from NB when the SB perceived quality is high enough. Meanwhile when the SB supply price by the competitive fringe is relatively low, the SB retail margin may surpass NB retail margin. With an increase in SB sales and a higher retail margin from SB than NB, It turns out that the retailer may be better off with the more gain in SB than the loss in NB. However, the presence of the other $k-1$ retailers may also hurt the retailer. When the SB perceived quality is low while the SB supply price of the competitive fringe is high, the SB retail margin becomes very low relative to NB. As a consequence, when the increased NB wholesale price results in more demands eroded by SB, the retailer would be worse off with the more loss in NB than the gain in SB. In contrast, we explore how the other $k-1$ retailers are affected by the retailer's SB introduction by comparing their profits with and without SB entry.

Corollary 3.4. *Since every single non-SB retailer's profits in the case without SB introduced by the retailer are lower than those with SB, i.e.,* $\frac{1}{16} < \frac{(k+2\gamma-w_s-k\gamma)^2}{16(k+\gamma-k\gamma)^2}$, *it shows that the other $k-1$ retailers always benefit from the retailer's SB introduction.*

Corollary 3.4 echoes the main conclusion made by Groznik and Heese [15], who demonstrate that when the retailer introduces SB, she may share the benefit with its competing retailer in the form of a lowered NB wholesale price when the NB manufacturer is constrained by the Robinson–Patman Act. Notwithstanding the similar conclusion, we make this under the scenario with the multiple retailers without direct competition, which is different from that with two competing railers in Groznik and Heese's [15] study.

To sum up, with the presence of other $k-1$ retailers, when the retailer determines to introduce her SB and make the competitive fringe produce for her, the NB manufacturer, constrained by the Robinson–Patman Act, always suffers from the SB entry with a lowered NB wholesale price which, however, benefits the retailer and the other $k-1$ retailers. Nevertheless, compared with the case without the presence of other $k-1$ retailers, how much that the retailer benefits from its SB introduction is quite differentiated. The presence of other $k-1$ retailers may make the retailer more or less profit from its SB introduction, and it all depends on the related parameters. Therefore, it is believable that the retailer may have incentive to withdraw NB from her shelves when she is less profitable by introducing her SB, due to the presence of the NB-manufacturer's $k-1$ non-SB retailers. Empirical findings predict that NB manufacturers supplying SBs can cultivate a better relation with

SB retailers and thus lead a higher likelihood of procuring shelf presence for their NBs [20, 39]. Accordingly, we step further to investigate whether both the retailer and the NB manufacturer have economic incentives to make SB supplied by the latter in the next subsection.

3.3. With SB supplied by the NB manufacturer

In this section, we assume the NB manufacturer has excess production capacity to produce SB at the same production cost with NB. At the beginning, we examine whether both the retailer and the NB manufacturer have the motive to make SB supplied by the NB manufacturer without any contract. To exclude the common wisdom that it is to make use of the idle production capacity that the NB manufacturer supplies SB, we assume that the NB manufacturer supplies SB at production cost without any contract. Consequently, the pricing and SB introduction decision problems with a dominant NB manufacturer can be modeled as follows

$$\begin{cases} \max_{w_n} \ \pi_m = w_n(1 - \dfrac{p_n^* - p_s^*}{1 - \gamma}) + (k-1)w_n(1 - p_n^{k-1*}) \\ \text{where } (p_n^*, p_s^*, p_n^{k-1*}) \text{ derive from problems:} \\ \quad \begin{cases} \max_{p_n^{k-1}} \ \pi_r^{k-1} = (p_n^{k-1} - w_n)(1 - p_n^{k-1}) \\ \max_{p_n, p_s} \ \pi_r = (p_n - w_n)(1 - \dfrac{p_n - p_s}{1 - \gamma}) + p_s \dfrac{\gamma p_n - p_s}{\gamma(1 - \gamma)} \\ \text{subject to:} \\ \quad p_s < \gamma p_n. \end{cases} \end{cases} \tag{3.4}$$

To solve this problem with backward induction, we obtain the equilibrium solution and characterize strategies in Proposition 3.5. All equilibrium outcomes are summarized in Table 4

Proposition 3.5. *When the retailer has the option of SB introduction and the NB manufacturer supplies SB at production cost equal to zero, the retailer will always introduce her SB, and the NB manufacturer will set the NB wholesale price as $w_n = \frac{k(1-\gamma)}{2(k+\gamma-k\gamma)}$. The resulting profits of the retailer, every single non-SB retailer and the manufacturer respectively are $\pi_r^* = \frac{4\gamma^2 + 4k\gamma + k^2 - 4k\gamma^2 - k^2\gamma}{16(k+\gamma-k\gamma)^2}, \pi_r^{k-1*} = \frac{(k+2\gamma-k\gamma)^2}{16(k+\gamma-k\gamma)^2}, \pi_m^* = \frac{k^2(1-\gamma)}{8(k+\gamma-k\gamma)}$.*

By comparing the NB-manufacturer's profits in this case and those when SB is supplied by the competitive fringe in Section 3.2, we can obtain the following Corollary on the NB manufacturer's final decision on the SB production.

Corollary 3.6. *Since the NB manufacturer suffers more from the SB introduction when himself supplies SB than when the competitive fringe supplies SB, i.e., $\frac{k^2(1-\gamma)}{8(k+\gamma-k\gamma)} < \frac{(k+w_s-k\gamma)^2}{8(k+\gamma-k\gamma)(1-\gamma)}$, if supplying SB at production cost without any contract on NB, the NB manufacturer never chooses to produce SB for the retailer.*

From Table 4, we note that the NB wholesale price is much lower than that when SB is supplied by the competitive fringe. As a result, the NB manufacturer would let the competitive fringe to supply SB rather than himself, since he will lose more by doing so. On the other hand, the retailer would always more benefit from the SB entry when the NB manufacturer supplies SB. As a consequence, the retailer would have strong incentive to induce the NB manufacturer to supply SB for her. To this end, we go further to investigate whether there is a possibility for the NB manufacturer to do so by signing a contract.

In the literature, it is shown that the NB manufacturer supplying the retailer's SB can coordinate NB wholesale price to minimize the SB effect on the NB manufacturer's profits [19]. Furthermore, a stable price gap between NB and SB which profits the whole channel has been widely suggested by existing research [30, 31]. In practice, Steiner [36, 37] find that leading NB manufacturers often act "Category Captain", not only to supply SB but also to price and manage the category instead of the retailers. In line with theses observations, we assume the retailer agrees to assign a fixed wholesale price $w_n = \frac{1}{2}$ for NB, and maintain the price gap between

TABLE 4. Equilibrium outcomes with SB supplied by NB manufacturer without contract.

w_n^*	p_n^*, p_n^{k-1*}	p_s^*	q_n^*	q_s^*	π_r^*	π_r^{k-1*}	π_m^*
$\dfrac{k(1-\gamma)}{2(k+\gamma-k\gamma)}$	$\dfrac{3k+2\gamma-3k\gamma}{4(k+\gamma-k\gamma)}$ $\dfrac{\gamma}{2}$	$\dfrac{k+2\gamma-2k\gamma}{4(k+\gamma-k\gamma)}$	$\dfrac{k}{4(k+\gamma-k\gamma)}$	$\dfrac{4\gamma^2+4k\gamma+k^2-4k\gamma^2-k^2\gamma}{16(k+\gamma-k\gamma)^2}$	$\dfrac{(k+2\gamma-k\gamma)^2}{16(k+\gamma-k\gamma)^2}$	$\dfrac{k^2(1-\gamma)}{8(k+\gamma-k\gamma)}$

the brands that meets $\frac{p_n-p_s}{1-\gamma}=\frac{3}{4}$, to induce the NB manufacturer to supply SB for her by avoiding any losses from SB introduction. Hence, the corresponding model is

$$
\begin{cases}
\max_{w_n}\ \pi_m = w_n(1-\dfrac{p_n^*-p_s^*}{1-\gamma})+(k-1)w_n(1-p_n^{k-1*})\\
\text{where } (p_n^*,p_s^*,p_n^{k-1*}) \text{ derive from problems:}\\
\begin{cases}
\max_{p_n^{k-1}}\ \pi_r^{k-1}=(p_n^{k-1}-w_n)(1-p_n^{k-1})\\
\max_{p_n,p_s}\ \pi_r=(p_n-w_n)(1-\dfrac{p_n-p_s}{1-\gamma})+p_s\dfrac{\gamma p_n-p_s}{\gamma(1-\gamma)}\\
\text{subject to:}\\
\quad p_s<\gamma p_n\\
\quad w_n=\dfrac{1}{2}\\
\quad \dfrac{p_n-p_s}{1-\gamma}=\dfrac{3}{4}.
\end{cases}
\end{cases}
\tag{3.5}
$$

Under this scenario, we can derive the equilibria straightforward. Proposition 3.7 characterizes the main equilibrium outcomes of the manufacturer and all retailers. All the equilibrium outcomes are summarized in Table 5

Proposition 3.7. *If the NB manufacturer supplies SB with the production cost $c_n=0$ with a contract specifying $w_n=\frac{1}{2}$ and $\frac{p_n-p_s}{1-\gamma}=\frac{3}{4}$, then the manufacturer keeps the total profits $\pi_m=\frac{k}{8}$ without any suffering from the SB entry, and the retailer makes profits $\pi_r=\frac{1+\gamma}{16}$, and the other individual non-SB retailer earns $\pi_r^{k-1}=\frac{1}{8}$, also without any impact from the SB entry.*

By comparing the profits of the NB manufacturer and the retailer with SB supplied by the former and those by the competitive fringe, it is shown that the NB manufacturer is always better off with SB supplied by the NB manufacturer with this contract. However, it is maybe not the case for the retailer. By comparing the retailer's profits with SB supplied by the competitive fringe and by the NB manufacturer, we can obtain the win–win situation where both the NB manufacturer and the retailer are better off with SB supplied by the NB manufacturer.

Corollary 3.8. *There exists a win–win situation that is $k>\frac{1+\sqrt{1+\gamma}}{2(1-\gamma)}$ and*
$\frac{\gamma(Y+2k+3\gamma-6k\gamma-4\gamma^2+4k\gamma^2-2(k+\gamma-k\gamma)\sqrt{(1-\gamma)(Y-4\gamma-4k+4k\gamma+1)})}{2Y} < w_s < \frac{k\gamma-k\gamma^2}{2k+\gamma-2k\gamma}$*, where $Y = 4k^2 + \gamma + 4k\gamma - 8k^2\gamma - 4k\gamma^2 + 4k^2\gamma^2$, wherein both the retailer and the NB manufacturer are better off with SB supplied by the latter with a contract specifying $w_n=\frac{1}{2}$ and $\frac{p_n-p_s}{1-\gamma}=\frac{3}{4}$; otherwise, the competitive fringe would be left to supply SB.*

Interestingly, Corollary 3.8 reveals that both the NB manufacturer and the retailer do have incentive to make SB supplied by the NB manufacturer, even though the NB manufacturer supplies SB at its production cost and thus earns nothing directly from SB sales at all. Accordingly, we suggest that if the NB manager could not stop his retailer introducing SB, one of the most wise choices would be to supply SB for the retailer in spite of no direct profits from doing this. On the other hand, when the SB supply price from the competitive market

TABLE 5. Equilibrium outcomes with SB supplied by NB manufacturer with contract.

w_n^*	p_n^*	p_n^{k-1*}	p_s^*	q_n^*	q_s^*	π_r^*	π_r^{k-1*}	π_m^*
$\dfrac{1}{2}$	$\dfrac{3-\gamma}{4}$	$\dfrac{3}{4}$	$\dfrac{\gamma}{2}$	$\dfrac{1}{4}$	$\dfrac{1}{4}$	$\dfrac{1+\gamma}{16}$	$\dfrac{1}{16}$	$\dfrac{k}{8}$

is relatively high and the number of the NB-manufacturer's non-SB retailers is sufficiently big, the retailer is better off making the NB manufacturer produce SB, otherwise a competitive fringe would be a better choice.

Someone may question that whether this win–win situation exists without the presence of the other $k-1$ retailers. The answer is no. Noting that with the contract specifying the NB wholesale price, the presence of the other $k-1$ retailers has no impact on the retailer. As a result, by comparing the retailer's profits in this case and those in the case with SB supplied by the competitive fringe without the presence of the other $k-1$ retailers, we find that the former is always lower than the latter, *i.e.*, $\frac{1+\gamma}{16} < \frac{\gamma+2\gamma^2-3\gamma^3+6\gamma^2 w_s-3\gamma w_s^2-6\gamma w_s+4w_s^2}{16\gamma(1-\gamma)}$. In other words, the retailer has no incentive to make the NB manufacturer supply SB by signing this contract, and hence an analogous win–win situation would never show up without the presence of the other $k-1$ retailers, ceteris paribus. One may also question that how about collaborating between the NB manufacturer and the retailer by increasing (or lowering) the NB wholesale price, it also cannot reach a win–win situation. If increasing the NB wholesale price by a contract, the NB manufacturer will suffer more with less profits from other $k-1$ retailers due to a more serious double-marginalization problem. By contrast, a lowered NB wholesale price can benefit the whole supply chain by lessening the double-marginalization problem. However, it needs that the NB manufacturer negotiates and signs a revenue-sharing contract with every single retailer. This is a traditional supply chain coordination problem. Novelly, we provide an easy-applicable collaborating method to reach a win–win situation. More importantly, our results also coincide with some empirical findings [6, 36, 37]. In our study, when the leading NB manufacturer supplies SB with the contract, we show that the retail prices of both NB and SB are lower than those when he dose not, *i.e.*, $\frac{3-\gamma}{4} < \frac{3k+2\gamma-3k\gamma+w_s}{4(k+\gamma-k\gamma)}, \frac{\gamma}{2} < \frac{\gamma+w_s}{2}$. Similarly, Chen *et al.* [6] find that introducing SB can benefit both NB manufacturers and retailers and a vertically integrated supply of SB can result in lower prices for end consumers. Further more, since the more retailers through whom the NB manufacturer distributes his NB (*i.e.*, k is bigger), the higher market share he might have, then the more likely that he is a leading manufacturer. In turn, the more inclined he will be to supply SB since the more losses if SB is introduced and supplied by the competitive fringe. Thus, this result provides another rationale for why more and more, especially the leading, NB manufacturers engage in the SB production [36, 37].

4. DISCUSSION AND CONCLUDING REMARKS

In practice, NB manufacturers produce SBs for their counterpart retailers are not uncommon. However, the rationale for why NB manufacturers, especially the leading NB manufacturers, are willing to involve in SB production is far from clear. A few extant studies shed light on this issue and propose some possible explanations, however, under single-retailer scenarios. Thus, these explanations may be invalid under multiple-retailer scenarios. With this in mind, we use game-theory-based framework to model the interaction between an NB manufacturer and its k local monopolist retailers, one of whom has capability and motivation to introduce its SB. In the meanwhile, the NB manufacturer is constrained by the Robinson–Patman Act, which defines a uniform wholesale price for all retailers unless there is a fair business reason for differentiation. Furthermore, to exclude the reason for NB manufacturers supplying SBs is to share SB market by utilizing excess production capacity, the NB manufacturer is assumed to supply SB at its production cost. According to received theory, the NB manufacturer should reduce NB wholesale price to response to the retailer's SB introduction. However, when the NB manufacturer deals with multiple retailers and complies with the Robinson–Patman Act, he would not lower NB wholesale price globally as much as that under the single-retailer scenarios. As a result, surprisingly

we find that the impacts of the other non-SB-retailers' presence on the SB retailer are twofold, *i.e.*, benefiting or hurting the SB retailer under different conditions, although there is no direct competition among the retailers. Consequently, the NB manufacturer suffers from the retailer's SB introduction, while the retailer may less profits from introducing its SB due to the presence of the NB-manufacturer's other non-SB retailers. Accordingly, we design a contract whereby a win–win situation can be created for both the NB manufacturer and the retailer.

Our study offers a novel explanation for why the leading NB manufacturers involve in SB production in practice. Some important managerial implications also can be put forward. Firstly, when a retailer plans to introduce its SB, it had better catch as much information as possible about its NB manufacturer, since how many other retailers the NB manufacturer deal with will affect how much it will profit from and whether introducing its SB. More importantly, when the SB perceived quality and the SB supply price of the competitive fringe are both relatively high, the retailer had better choose the NB manufacturer to suppy its SB with a contract, otherwise the competitive fringe would be a better choice. As for the leading NB manufacturer, it does not have to take its retailer's SB as a threat for granted. Supplying SB can be a very natural and implicit collusion with its SB retailer. Therefore, the NB managers can collaborate with their retailers by producing SBs with the contract designed in this study, to counter the impacts of their retailers' SBs. In deed, taking an "if you can't beat them, join them" stand to engage in the SB production would be a better choice for NB manufacturers [10, 29], whereby category leader NB manufacturers may remain relatively safe despite SBs growth, according to the report of the A.C. Nielsen company in 2014 [26].

One may question what if there were more than one retailer have option of their SBs. First of all, not all retailers in the real business practice have capability and enough resources to initiate a SB business. On the other hand, just as our finding in this study, the other $k-1$ retailers can indirectly benefit from the SB introduction of the retailer in question, and therefore the motivation for introducing SB of the other $k-1$ retailers diminishes. Consistently, Groznik and Heese [15] also conclude that retailers would play "chicken" and prefer others to introduce SB rather than themselves, since introducing SB incurs the associated costs and risks, while sharing the benefit with other retailers. Nevertheless, we argue that our results are robust with the case where more than one retailer can and are willing to introduce SB. Since the k local monopolist retailers are systematic, the more the SB retailers, then the more NB wholesale price would be lowered by the NB manufacture, and in turn the more non-SB retailers are need for the retailer is better off letting the NB manufacture producing SB. For the NB manufacturer, it is always better off supplying SB with the contract. Therefore, a win–win situation still exists.

Now, we discuss some limitations of this study. Firstly, we assume the NB-manufacturer's k retailers are local monopolists and thus there is no competition among them. However, in the real word, there is more or lese competition among the retailers selling the same products even though in different areas. As a consequence, one can extend this study into multiple competing retailers setting. In addition, incorporating competition from other NB manufacturers also would be an interesting research avenue. Furthermore, the NB manufacturer is assumed to enjoy an edge in SB production efficiency, ending up with a lower supplying cost than that of the competitive fringe. In practice, it is possible that some specialist manufacturers that concentrate SBs may possess production efficiency advantage in some certain categories, and hence can supply SBs at lower prices than NB manufacturers instead. In this case, the win–win situation created in this study would disappear. Someone can examine other aspects which can give NB manufacturers an edge in producing SBs under multiple-retailers scenarios, such as promoting cooperative advertisement, improving SB perceived quality by NB manufacturers labeled, spurring innovation of product category and so on.

Appendix

Proof of Proposition 3.1. Firstly, we suppose the retailer decided to introduce her SB and purchased it from the competitive fringe at cost w_s, then her profits by distributing both SB and NB are

$$\pi_r = (p_n - w_n)\left(1 - \frac{p_n - p_s}{1 - \gamma}\right) + (p_n - w_s)\frac{\gamma p_n - p_s}{\gamma(1 - \gamma)}.$$

For given w_n, the retailer maximizes her profits with respect to p_n and p_s respectively, yielding the optimal prices

$$p_n = \frac{1 + w_n}{2}, \qquad p_s = \frac{\gamma + w_s}{2}.$$

Anticipating the retailer's response functions, the NB manufacturer chooses his wholesale price to maximize his profits

$$\pi_m = w_n \left(1 - \frac{p_n - p_s}{1 - \gamma} \right).$$

Solving this problem we can get the equilibrium prices and profits of this subgame for both the retailer and the manufacturer

$$w_n^* = \frac{1 - \gamma + w_s}{2}, \qquad p_n^* = \frac{3 - \gamma + w_s}{4}, \qquad p_s^* = \frac{\gamma + w_s}{2}$$

$$\pi_r^* = \frac{\gamma + 2\gamma^2 - 3\gamma^3 + 6\gamma^2 w_s - 3\gamma w_s^2 - 6\gamma w_s + 4w_s^2}{16\gamma(1 - \gamma)}, \qquad \pi_m^* = \frac{(1 - \gamma + w_s)^2}{8(1 - \gamma)}.$$

To ensure the nonnegative demand for SB, the constraint $p_s < \gamma p_n$ must hold, then we obtain

$$w_s < \frac{\gamma - \gamma^2}{2 - \gamma}.$$

We find that when $w_s < \frac{\gamma - \gamma^2}{2 - \gamma}$, the retailer's profits with SB are always higher than without, that is

$$\frac{\gamma + 2\gamma^2 - 3\gamma^3 + 6\gamma^2 w_s - 3\gamma w_s^2 - 6\gamma w_s + 4w_s^2}{16\gamma(1 - \gamma)} > \frac{1}{16}.$$

Therefore, when $w_s < \frac{\gamma - \gamma^2}{2 - \gamma}$, then the retailer will always introduce her store brand. Otherwise, she will not. $\square$

Proof of Proposition 3.2. In this scenario, suppose the retailer decided to introduce her SB and purchased it from the competitive fringe at cost w_s, then the profit functions of supply chain members respectively are

$$\pi_m = w_n(1 - \frac{p_n^* - p_s^*}{1 - \gamma}) + (k - 1)w_n(1 - p_n^{k-1*})$$

$$\pi_r = (p_n - w_n) \left(1 - \frac{p_n - p_s}{1 - \gamma} \right) + (p_s - w_s)\frac{\gamma p_n - p_s}{\gamma(1 - \gamma)}$$

$$\pi_r^{k-1} = (p_n^{k-1} - w_n)(1 - p_n^{k-1}).$$

For given w_n, all retailers maximizes their profits, yielding response functions

$$p_n = p_n^{k-1} = \frac{1 + w_n}{2}, \qquad p_s = \frac{\gamma + w_s}{2}.$$

Substituting all retailers' response functions into the NB-manufacturer's profits function, the NB manufacturer chooses his wholesale price to maximize his profits

$$\pi_m = \frac{w_n(k\gamma w_n - k\gamma - kw_n - \gamma w_n + k + w_s)}{2(1 - \gamma)}.$$

Solving this problem we can get the equilibrium prices and profits of this subgame

$$w_n^* = \frac{k + w_s - k\gamma}{2(k + \gamma - k\gamma)}, \qquad p_n^* = p_n^{k-1*} = \frac{3k + 2\gamma - 3k\gamma + w_s}{4(k + \gamma - k\gamma)}, \qquad p_s^* = \frac{\gamma + w_s}{2}$$

$$\pi_r^* = \frac{X}{16\gamma(1-\gamma)(k + \gamma - k\gamma)^2}, \qquad \pi_r^{k-1*} = \frac{(k + 2\gamma - w_s - k\gamma)^2}{16(k + \gamma - k\gamma)^2},$$

$$\pi_m^* = \frac{(k + w_s - k\gamma)^2}{8(k + \gamma - k\gamma)(1-\gamma)}$$

where $X = \gamma(k + 2\gamma - k\gamma - w_s)(k + 2\gamma - w_s + 2kw_s + 2\gamma w_s - 2\gamma^2 - 3k\gamma - 2k\gamma w_s + 2k\gamma^2) + 2(\gamma - w_s)(k + \gamma - k\gamma)(k\gamma + 2k\gamma w_s - 2kw_s - \gamma w_s - k\gamma^2)$. To ensure the nonnegative demand for SB, the constraint $p_s < \gamma p_n$ must hold, then we obtain

$$w_s < \frac{k\gamma - k\gamma^2}{2k + \gamma - 2k\gamma}.$$

Under this condition, the retailer's profits with SB are always higher than without. Therefore, when $w_s < \frac{k\gamma - k\gamma^2}{2k + \gamma - 2k\gamma}$, then the retailer will always introduce her store brand. Otherwise, she will not. $\qquad\square$

Proof of Corollary 3.3. The proof is very straightforward and left out for simplicity. $\qquad\square$

Proof of Corollary 3.4. The proof is very straightforward and left out for simplicity. $\qquad\square$

Proof of Proposition 3.5. If the retailer introduces her SB and the NB manufacturer supplies SB at production cost equal to zero, then the corresponding profits functions of all supply chain members are

$$\pi_m = w_n(1 - \frac{p_n^* - p_s^*}{1 - \gamma}) + (k - 1)w_n(1 - p_n^{k-1*})$$

$$\pi_r = (p_n - w_n)\left(1 - \frac{p_n - p_s}{1 - \gamma}\right) + p_s\frac{\gamma p_n - p_s}{\gamma(1 - \gamma)}$$

$$\pi_r^{k-1} = (p_n^{k-1} - w_n)(1 - p_n^{k-1}).$$

For given w_n, the retailer maximizes her profits with respect to p_n and p_s respectively, and the other $k - 1$ retailers maximize their profits with respect to p_n^{k-1}, yielding the optimal prices

$$p_n = p_n^{k-1} = \frac{1 + w_n}{2}, \qquad p_s = \frac{\gamma}{2}.$$

Substituting above retailers' reaction functions into the NB-manufacturer's profits function, there is

$$\pi_m = \frac{w_n(k\gamma w_n - k\gamma - kw_n - \gamma w_n + k)}{2(1 - \gamma)}.$$

Maximizing the NB-manufacturer's profits function with respect to w_n, we can obtain all equilibrium prices and profits as follows

$$w_n^* = \frac{k(1 - \gamma)}{2(k + \gamma - k\gamma)}, \qquad p_n^* = p_n^{k-1*} = \frac{3k + 2\gamma - 3k\gamma}{4(k + \gamma - k\gamma)}, \qquad p_s^* = \frac{\gamma}{2}$$

$$\pi_r^* = \frac{4\gamma^2 + 4k\gamma + k^2 - 4k\gamma^2 - k^2\gamma}{16(k + \gamma - k\gamma)^2}, \qquad \pi_r^{k-1*} = \frac{(k + 2\gamma - k\gamma)^2}{16(k + \gamma - k\gamma)^2},$$

$$\pi_m^* = \frac{k^2(1 - \gamma)}{8(k + \gamma - k\gamma)}.$$

Since $k > 1$ and $0 < \gamma < 1$, then the retailer's profits with SB are always higher than without. As a result, in this scenario, the retailer will always be willing to introduce her SB. $\qquad\square$

Proof of Corollary 3.6. The proof is very straightforward and left out for simplicity. □

Proof of Proposition 3.7. If the NB manufacturer supplies SB with the production cost $c_n = 0$ with a contract specifying $w_n = \frac{1}{2}$ and $\frac{p_n - p_s}{1 - \gamma} = \frac{3}{4}$, then the profits of all chain members are

$$\pi_m = \frac{k}{8}$$

$$\pi_r = \left(p_n - \frac{1}{2}\right)\left(1 - \frac{3}{4}\right) + p_s \frac{\gamma p_n - p_s}{\gamma(1 - \gamma)}$$

$$\pi_r^{k-1} = \left(p_n^{k-1} - \frac{1}{2}\right)(1 - p_n^{k-1}).$$

Since $\frac{p_n - p_s}{1 - \gamma} = \frac{3}{4}$, then $p_s = p_n - \frac{3(1-\gamma)}{4}$. Consequently, we can maximize the retailer's profits with respect to p_n, obtaining

$$p_n^* = \frac{3 - \gamma}{4}.$$

Meanwhile, maximizing all other $k - 1$ retailers' profits with respect to p_n^{k-1} yields

$$p_n^{k-1*} = \frac{3}{4}.$$

Therefore, the equilibrium profits of all retailers are, respectively

$$\pi_r^* = \frac{1 + \gamma}{16}, \qquad \pi_r^{k-1*} = \frac{1}{16}.$$

□

Proof of Corollary 3.8. Under the condition $w_s < \frac{k\gamma - k\gamma^2}{2k + \gamma - 2k\gamma}$, if the retailer's profits with SB, which is purchased from the competitive fringe at cost w_s, are lower than those with SB, which is supplied by the NB manufacturer with a contract, then the retailer will choose the NB manufacturer as its supplier, that is

$$\frac{X}{16\gamma(1 - \gamma)(k + \gamma - k\gamma)^2} < \frac{1 + \gamma}{16}.$$

where $X = \gamma(k + 2\gamma - k\gamma - w_s)(k + 2\gamma - w_s + 2kw_s + 2\gamma w_s - 2\gamma^2 - 3k\gamma - 2k\gamma w_s + 2k\gamma^2) + 2(\gamma - w_s)(k + \gamma - k\gamma)(k\gamma + 2k\gamma w_s - 2kw_s - \gamma w_s - k\gamma^2)$.

Meanwhile, only when the NB-manufacturer's profits when supplying SB with a contract are higher than those when let the competitive fringe supplies SB, the NB manufacturer will be willing to supply SB, that is

$$\frac{(k + w_s - k\gamma)^2}{8(k + \gamma - k\gamma)(1 - \gamma)} < \frac{k}{8}.$$

Solving these two inequations under $w_s < \frac{k\gamma - k\gamma^2}{2k + \gamma - 2k\gamma}$ and $1 < k, 0 < \gamma < 1$, we can obtain the following conditions

$$k > \frac{1 + \sqrt{1 + \gamma}}{2(1 - \gamma)}$$

and

$$\frac{\gamma(Y + 2k + 3\gamma - 6k\gamma - 4\gamma^2 + 4k\gamma^2 - 2(k + \gamma - k\gamma)\sqrt{(1 - \gamma)(Y - 4\gamma - 4k + 4k\gamma + 1)})}{2Y}$$

$$< w_s < \frac{k\gamma - k\gamma^2}{2k + \gamma - 2k\gamma}$$

where $Y = 4k^2 + \gamma + 4k\gamma - 8k^2\gamma - 4k\gamma^2 + 4k^2\gamma^2$, In other words, when the above two conditions hold, both the retailer and the the NB manufacturer are better off with SB supplied by the latter with a contract. $\square$

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