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INTEGRATED VENDOR–BUYER STRATEGIES FOR IMPERFECT PRODUCTION SYSTEMS WITH MAINTENANCE AND WARRANTY POLICY

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Abstract. Collaboration has evolved as a key component of many modern supply chains, supporting the competitive advantages of companies in a range of operations from manufacturing to sales. With this viewpoint, the present paper develops an integrated inventory model in which manufacturing is carried out at the vendor's end so as to fulfill demand at the buyer's doorway. As the production process is presumed to be imperfect it shifts from an "in-control" state to "out-of-control" state at any random time and yields non-conforming items. The vendor uses regular preventive maintenance actions for the efficient operation of the production system and offers free minimal repair warranty on the products sold to the buyer. Along with preventive maintenance actions the vendor also uses the rework process and restoration process as effective steps towards minimizing the imperfections of the production system. The proposed model solves the non-linear cost minimization problem through a generalized reduced gradient method by using Lingo 15.0. The aim is to jointly optimize the order size, backorder size and the number of shipments in order to minimize the integrated cost of the vendor and the buyer. Numerical analysis and sensitivity analysis is performed on key parameters that render some important supervisory insights.

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1. Introduction and literature overview

In past, a lot of research in the field of inventory management focused on imperfect production systems, in which at some point of time the manufacturing process shifts from an in-control state to the out-of-control state. In such a scenario, to maintain the quality in production, a restoration must be done before initializing the production process for the subsequent cycle. Further, during the non-production time, it is imperative to keep a check on the production system so as to avoid any kind of malfunctioning or breakdown for the next cycle. Such a check can be sustained by taking proper preventive maintenance actions. In today's globally active markets, supply chain players work together to plan and execute various supply chain operations viz. manufacturing, transportation, sales etc. Such alliance among the supply chain players has substantial benefits to the overall

Keywords. Inventory, imperfect production systems, preventive maintenance, warranty, shortages.

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business and is widely implemented. Prompted to this, an integrated inventory model has been presented by considering rework of non-conforming items, restoration of the production system and use of regular preventive maintenance actions during the non-production time. Also, the vendor offers free minimal repair warranty to the buyer as a promotional tool. An integrated model is developed with the motive to serve the decision makers with some important inferences.

Almost all the organizations these days consider quality and reliability as a genuine technique to achieve improved profitability and greater benefits for the organization. In the manufacturing sector occurrence of imperfect quality items due to the faulty production process is unavoidable. Some of the interesting work in this area is given by [1–18]. In practical production systems, imperfect items can be reworked, which significantly reduces the overall costs of production and inventory [19–26].

Since the production process is imperfect so it is essential to take proper maintenance actions to deliver breakdown-free operations. Preventive maintenance comes with a motto to ensure optimum life for the equipment and it also increases productivity by reducing the downtime of a production unit. The primary goal of maintenance actions is to safeguard the system against any breakdown and malfunction during run-time. Reference [29] investigated the effects of pull systems and level material shipping to derive formulas for the buffer in limited demand variability. Reference [30] developed a flow approximation of a production system with finite buffers and unreliable machines. Later, [29–34] etc. investigated the area further.

Further, the organizations believe that in order to gain loyal customers and to elevate sales one needs to provide better post-sale services and other prevailing offers at the forefront. Warranty is known to be a renowned promotional strategy to boost sales and it can also be interpreted as an assurance given by the vendor to the buyer to ensure the reliability of their purchases. Reference [35] developed a framework for the production policy so as to achieve an optimal safety stock, production rate and production lot size. Reference [36] proposed a model for imperfect production system considering preventive maintenance and optimal buffer inventory with warranty. Reference [37–39] explored the imperfect production systems with maintenance and warranty.

Moreover, in the era of the supply chain, the crucial aspects of coordination and collaboration with the channel partners have got a lot of attention worldwide. The various perks that come up with integrated modelling include financial savings, sharing new risks, competitive advantage, expanding the overall profits and other mutual benefits. In real-world inventory situations, the manufacturing and sales operations are performed by different players of the supply chain. Numerous researchers have put efforts to establish coordination between vendor and buyer by using integrated optimization in their modelling. Some of the pioneering work in the area of integrated modelling was proposed by [40–49]. Later, Chung [50] presented an updated study of [47] by providing the complete proofs and justifications for the algorithm proposed by Wu and Ouyang [47]. Reference [51] proposed an inventory model to showcase coordination between vendor and buyer under an imperfect production environment, the model assumes that the inspection is carried out simultaneously with production process at vendor's end and items of good quality are delivered to the buyer in small sizes via compound supplies. Reference [52–56] have contributed recently in the area of integrated inventory modelling.

This paper extends the existing inventory literature by developing a model for the two important players of supply chain viz. the vendor and the buyer. The production process is governed by the vendor and it is assumed to be imperfect *i.e.* the system might deviate from an in-control state to out-of-control state and it ends up in the production of defectives in both the states. In order to maintain the smooth flow of the production system the model investigates the use of regular preventive maintenance actions. Also, the vendor should respond to the buyer needs efficiently, thus, to enhance the post-sales experience, a free minimal repair warranty policy is considered. The model further takes into account the rework process as an effective step towards managing the defectives. The process restoration is employed so as to keep the system under an in-control state for the subsequent cycle which is quite a necessary step towards maintaining the efficiency of the production system. Shortages are assumed to occur at the buyer's place which are fully backordered. The study contributes in overcoming the traditional vendor–buyer models by constructing an integrated framework under the influence of the above-discussed scenarios which indeed helps in increasing the operational efficiency of the supply chain. A solution procedure is proposed in order to obtain the optimal solution. The model is validated with the help

Table 1. Key features of integrated inventory models with imperfect-production systems.

Article	Imperfect quality/ production	Rework	Allow for shortages	Maintenance actions	Warranty policy	Integrated
Huang (2002)	Yes	No	No	No	No	Yes
Goyal *et al.* (2003)	Yes	No	No	No	No	Yes
Huang (2004)	Yes	No	No	No	No	Yes
Ouyang *et al.* (2006)	Yes	No	No	No	No	Yes
Chen and Lo (2006)	Yes	No	Yes	No	Yes	No
Lin (2009)	Yes	No	No	No	No	Yes
Sana (2011)	Yes	No	No	No	No	Yes
Hsu and Hsu (2012)	Yes	No	No	No	No	Yes
Sana (2012)	Yes	Yes	Yes	Yes	Yes	No
Yedes and Rezg (2012)	Yes	No	Yes	Yes	No	Yes
Hsu and Hsu (2013)	Yes	No	Yes	No	No	Yes
Treviño-Garza *et al.*(2015)	Yes	Yes	No	No	No	Yes
Cheng *et al.* (2018)	Yes	No	No	Yes	No	Yes
Nobil *et al.* (2018)	Yes	No	Yes	No	No	Yes
Present Study	Yes	Yes	Yes	Yes	Yes	Yes

of numerical examples. Sensitivity analysis is presented so as to provide important managerial inferences. In the end, the paper is summarized with a conclusion and future research directions.

The present study is compared with the related literature and the summary is listed in Table 1.

2. Assumptions

1. A single vendor and a single buyer are considered.
2. Demand rate is known and constant over time at the buyer.
3. At any time-point, the production process can be in "in-control state" or "out-of-control" state. In each state, a constant percentage of non-conforming items are produced.
4. The "out-of-control" state cannot be spotted till the production process terminates.
5. The out-of-control state is restored to "in-control" state at a restoration cost for the subsequent cycle.
6. Non-conforming items are reworked straightway at a parallel manufacturing setup.
7. Production rate is higher than the demand rate.
8. Shortages at buyer are completely backordered.
9. Preventive maintenance actions are carried out during non-production time and it safeguards the system against any breakdown during run-time for the subsequent cycle.
10. In the imperfect production process, products are sold with a free minimal repair warranty (FRW) policy.

3. Notations

Parameters	
λ	The annual demand at the buyer
ψ	Production rate; $\psi > \lambda$
K_V	The setup cost per production run for the vendor
K_B	The ordering cost per order for the buyer
T_B	The freight cost per consignment
V_V	The unit variable cost for order handling and receiving
B_B	The backordering cost per unit per year at the buyer
I_V	The holding cost per unit per year for the vendor
I_B	The holding cost per unit per year for the buyer
M_V	Cost of material, energy and labor to produce an item
M_B	Cost of purchasing per unit
PM_V	Cost of preventive maintenance
T	Time interval between successive shipments of y units.
T_1	Production time, period during which the vendor produces
T_2	The non-production and preventive maintenance time, period during which the vendor supplies to the buyer, which is a random variable.
T_{cyc}	Cycle time ($mT = T_1 + T_2$)
RW_V	Rework cost per unit non-conforming item
RS_V	Restoration cost per cycle
χ	A random variable that is the time elapsed after which the production process shifts to the out-of-control state
u	The parameter for exponential distribution $f(\chi)$
π	Fraction of total products that are non-conforming items
w	Warranty period
C_W	Warranty cost
γ_1	Probability of non-conforming items in the "in-control" state
γ_2	Probability of non-conforming items in the "out-of-control" state; and $0 < \gamma_1 < \gamma_2 < 1$
NC	Number of non-conforming items
Y_ψ	Size of the production batch
Decision variables	
m	Number of consignments, $Y_\psi = my$
y	The size of the shipment from the vendor to the buyer
S	Backordering quantity
Functions	
$\kappa_1(x)$	Failure rate function of conforming items
$\kappa_2(x)$	Failure rate function of non-conforming items
$f(T_2)$	Probability density function of T_2
$f(\chi)$	Probability density function of χ and it is assumed to be exponentially distributed with parameter u
$\mathrm{TC}_V(m, y)$	The vendor's cost per cycle
$\mathrm{TC}_B(m, y, S)$	The buyer's cost per cycle
$\mathrm{TC}_{\mathrm{cyc}}(m, y, S)$	Total integrated vendor–buyer cost per cycle
$\mathrm{TC}(m, y, S)$	The total integrated annual cost of the vendor and buyer.
Optimal values	
m^*	Optimal number of shipments from vendor to the buyer
$y^*(m)$	optimal lot size (integrated model)
$S^*(m)$	Optimal backorder level (integrated model)

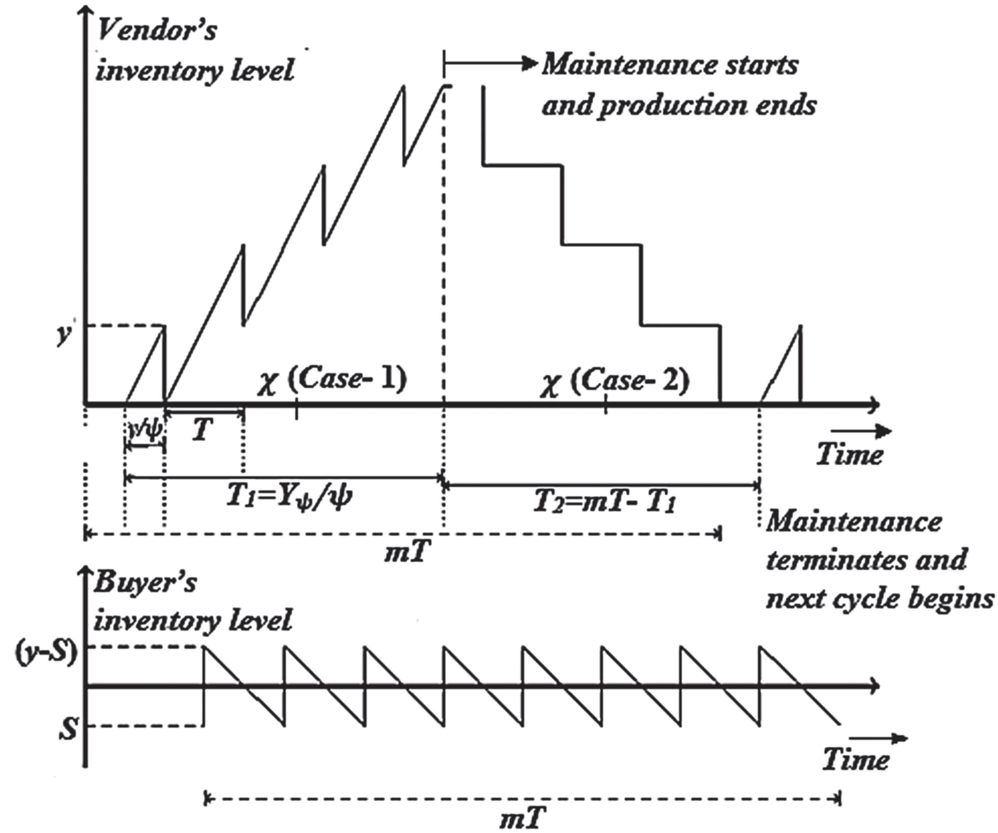

FIGURE 1. Behaviour of inventory over time for vendor and buyer.

4. Model formulation and analysis

The key to achieving an efficient supply chain system lies behind the effective collaborative management of the participating players. The present scenario explores the situation of a single vendor and a single buyer. The annual demand rate of the buyer is λ units for the given product, for which the buyer places consistent orders of fixed size Y_ψ to the vendor. The vendor caters to the needs of the buyer by manufacturing the products in the batches of sizes Y_ψ with a plan to ship the respective batch in m shipments with a lot of y units each, which are of good quality. It is assumed that all the customers are eager to wait for a later delivery at some known cost, shortages are considered with complete backlogging. The Figure 1 below depicts the inventory representation for the vendor and the buyer. For the case of the vendor, the total time length mT comprises of the production time T_1 and non-production time T_2. The non-production time is considered to be the preventive maintenance time for the production system. For the case of the buyer, during the time length mT, equal sized shipments of Q units takes place from the vendor to the buyer. The expressions for the vendor's and the buyer's cost per cycle along with the total annual integrated cost are then derived and the different scenarios are presented as follows:

4.1. Vendor's perspective

The production process is imperfect and at any random time shifts from in-control state to out-of-control state and it produces non-conforming items in both states with the probability of non-conforming items in the

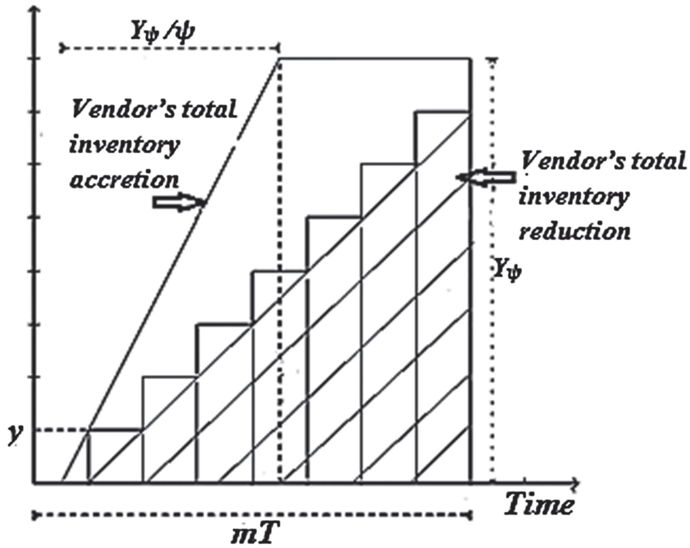

FIGURE 2. Total accumulation and depletion of vendor's inventory per cycle.

former state being less than the latter state. The non-conforming items are reworked straightway at a parallel manufacturing system at some known cost. If the production process is found in the out-of-control state at the end of any production cycle then it is restored back to its original condition at the beginning of subsequent production cycle. The vendor uses regular preventive maintenance actions during the non-production time so as to prevent any possible malfunctioning or breakdown of the production process for the succeeding cycle. Further, the vendor offers free minimal repair warranty policy on the sold products for a given period of time. Hence, the vendor's annual total cost consists of the setup cost, variable cost, holding cost, the cost of restoring the system to the functional state, the cost of reworking the non-conforming items, the preventive maintenance cost and the minimal free repair cost for warranty. The various individual cost components are evaluated as follows:

(1)
$$\text{Setup cost} = K_V. \tag{4.1}$$

(2)
$$\text{Variable Cost} = M_V \psi T_1. \tag{4.2}$$

(3) The expected holding cost of vendor is represented by Figure 2

$$\begin{aligned} &= [\text{Bold area} - \text{Shaded area}] \\ &= I_V \left\{ \left[my \left(\frac{y}{\psi} + (m-1)T \right) - \frac{my\,(my/\psi)}{2} \right] - T\left[y + 2y + ... + (m-1)\,y\right] \right\} \\ &= I_V \left\{ \frac{my^2}{\psi} - \frac{m^2y^2}{2\psi} + \frac{m(m-1)y^2}{2\lambda} \right\} \end{aligned} \tag{4.3}$$

where, bold area represents the total inventory accumulation and shaded area represents the total inventory depletion.

The production unit at vendor runs for a period T_1 after which a preventive maintenance interruption occurs and continues till the next cycle starts. The time T_2 (an exponentially distributed random variable that denotes the non-production time in which preventive maintenance takes place) follows probability density function $f(T_2)$. The out-of-control state may occur at a time χ (an exponentially distributed random variable symbolizing elapsed time to shift to "out-of-control" state) which follows probability density function $f(\chi)$.

The random variable χ may occur within the period T_1 or may occur after T_1(see Fig. 1, Case 1 and 2). In this situation, the number of non-conforming items, NC is given as

$$\mathrm{NC} = \begin{cases} \gamma_1 \psi T_1, & \text{if } \chi \geq T_1, \\ \gamma_1 \psi \chi + \gamma_2 \psi (T_1 - \chi) & \text{if } \chi < T_1 \end{cases}. \tag{4.4}$$

The expected value of NC is given by

$$\begin{aligned} E[\mathrm{NC}] &= \gamma_1 \psi T_1 \int_{T_1}^{\infty} f(\chi)\,\mathrm{d}\chi + \gamma_1 \psi \int_0^{T_1} \chi f(\chi)\,\mathrm{d}\chi + \gamma_2 \psi \int_0^{T_1} (T_1 - \chi) f(\chi)\,\mathrm{d}\chi \\ &= \gamma_1 \psi T_1 + (\gamma_2 - \gamma_1)\,\psi \int_0^{T_1} (T_1 - \chi) f(\chi)\,\mathrm{d}\chi. \end{aligned} \tag{4.5}$$

Let us define the fraction of non-conforming items of the total manufactured as

$$\begin{aligned} \pi &= \frac{E[\mathrm{NC}]}{\psi T_1} \\ &= \gamma_1 + \frac{(\gamma_2 - \gamma_1)}{T_1} \int_0^{T_1} (T_1 - \chi)\, f(\chi)\,\mathrm{d}\chi. \end{aligned} \tag{4.6}$$

Hence, the expected cost of rework is given as:

(4)

$$\text{Rework Cost} = \mathrm{RW}_V \pi \psi T_1. \tag{4.7}$$

(5) The restoration cost per cycle can be obtained by multiplying r, the per unit restoration-cost, by $\mathrm{Prob}(\chi < T_1)$ the probability that the imperfect manufacturing system is in "out-of-control" state at the completion of the manufacturing cycle. The restoration cost is $= \mathrm{RS}_V * \mathrm{Prob}(\chi < T_1)$

$$= \mathrm{RS}_V * (1 - \mathrm{e}^{-uT_1}). \tag{4.8}$$

(6)

$$\text{The expected cost of maintenance is} = \mathrm{PM}_V \int_0^{\infty} T_2 f(T_2)\,\mathrm{d}T_2 = \mathrm{PM}_V E[T_2]. \tag{4.9}$$

The vendor offers free minimal repair warranty policy on the products sold to the buyer for a specified period called warranty period. The probability of a product failing within the warranty period $[0, w]$ is $= (1-\pi)\int_0^w \kappa_1(x)\,\mathrm{d}x + \pi \int_0^w \kappa_2(x)\,\mathrm{d}x = (1-\pi) A_1 + \pi A_2$, where $A_1 = \int_0^w \kappa_1(x)\,\mathrm{d}x$ and $A_2 = \int_0^w \kappa_2(x)\,\mathrm{d}x$. Hence, the warranty cost is given as:

(7)

$$\text{Warranty Cost} = C_W \psi T_1 \left[(1-\pi) A_1 + \pi A_2\right]. \tag{4.10}$$

By adding equations (4.1)–(4.4) and (4.8)–(4.10), the vendor's cost per cycle can be obtained as:

$$\begin{aligned} \mathrm{TC}_V(m, y) &= K_V + M_V \psi T_1 + I_V \left\{ \frac{my^2}{\psi} - \frac{m^2 y^2}{2\psi} + \frac{m(m-1)y^2}{2\lambda} \right\} + \mathrm{RS}_V (1 - \mathrm{e}^{-uT_1}) + \mathrm{RW}_V \pi \psi T_1 \\ &\quad + \mathrm{PM}_V \int_0^{\infty} T_2 \phi(T_2) \mathrm{d}T_2 + C_W \psi T_1 \left[A_1 + (A_2 - A_1)\pi\right]. \end{aligned} \tag{4.11}$$

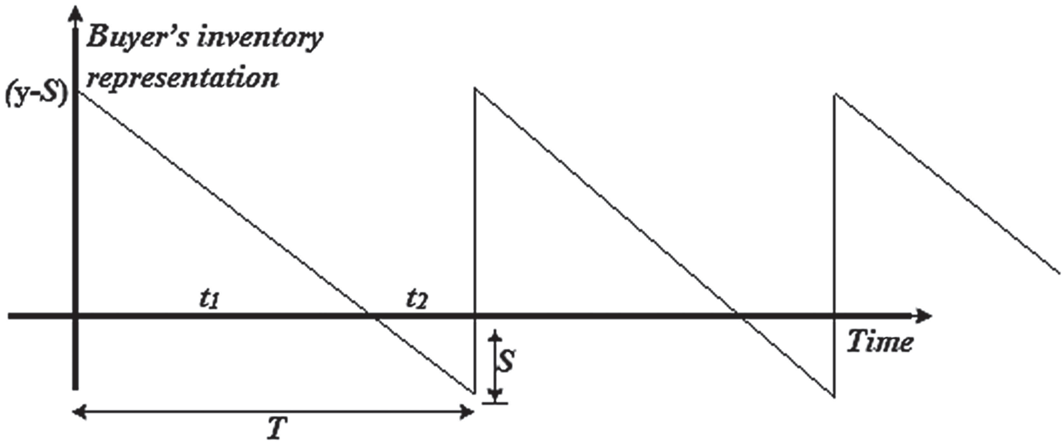

Figure 3. Inventory representation of the buyer.

4.2. Buyer's perspective

Figure 3 depicts the behaviour of inventory level over time T (time interval between successive shipments of y units) for the buyer, where the time T is divided into the inventory time t_1 and shortage time t_2. During t_1 the inventory depletes due to demand and during t_2 shortages are accumulated which are fully backlogged. The buyer's total inventory cost per cycle comprises of the following cost components:

(8)
$$\text{Cost due to placing an order} = K_B, \tag{4.12}$$

(9)
$$\text{Purchasing cost} = M_B my, \tag{4.13}$$

(10)
$$\text{Shipment Cost}(mT_B + V_V my), \text{which is the sum of transportation and handling cost}, \tag{4.14}$$

(11)
$$\text{Backordering cost} = B_B m \frac{1}{2} \frac{S^2}{\lambda}\cdot \tag{4.15}$$

The maximum inventory level is $(y - S)$ and the minimum inventory level is 0, so the average inventory level is $(1/2)\,(y - S)$. Since here $t_1 = (y - S)\,/\lambda$ and $T = y/\lambda$, the buyer's holding cost per cycle is:

(12)
$$\text{Holding cost} = I_B m \left\{ \frac{1}{2} \frac{(y - S)^2}{\lambda} \right\}\cdot \tag{4.16}$$

Hence, the buyer's cost per cycle can be obtained by adding equations (4.12)–(4.16):

$$\text{TC}_B\,(m, y, S) = K_B + M_B my + (mT_B + V_V my) + I_B m \left\{ \frac{1}{2} \frac{(y - S)^2}{\lambda} \right\} + B_B m \frac{1}{2} \frac{S^2}{\lambda}\cdot \tag{4.17}$$

4.3. The supply chain integrated inventory cost model

Adding both the vendor and buyer costs we obtain the total integrated vendor–buyer cost per cycle as given by

$$\begin{aligned}\mathrm{TC_{cyc}}\,(m,y,S) &= K_V + K_B + mT_B + V_V my + M_V\psi T_1 + M_B my + B_B m\frac{1}{2}\frac{S^2}{\lambda} + I_B m\left\{\frac{1}{2}\frac{(y-S)^2}{\lambda}\right\}\\ &\quad + I_V\left\{\frac{my^2}{\psi} - \frac{m^2y^2}{2\psi} + \frac{m(m-1)y^2}{2\lambda}\right\} + \mathrm{RS}_V(1-\mathrm{e}^{-uT_1}) + \mathrm{RW}_V\pi\psi T_1\\ &\quad + \mathrm{PM}_V\int_0^\infty T_2\phi(T_2)\mathrm{d}T_2 + C_W\psi T_1\left[A_1 + (A_2 - A_1)\pi\right]. \end{aligned} \tag{4.18}$$

If the vendor and buyer work in alliance to minimize their integrated annual cost which means that the two players agree on the terms and conditions where the buyer places order of size Y_ψ and the vendor delivers the items to the buyer in m number of shipments each of y units, the total annual integrated cost of the vendor and buyer is given as:

$$\begin{aligned}\mathrm{TC}(m,y,S) &= \frac{(K_V + K_B)\,\lambda}{my} + \frac{(mT_B + V_V my + M_V\psi T_1 + M_B my)\,\lambda}{my} + B_B\frac{S^2}{2y} + I_B\left\{\frac{1}{2}\frac{(y-S)^2}{y}\right\}\\ &\quad + I_V\lambda\left\{\frac{y}{\psi} - \frac{my}{2\psi} + \frac{(m-1)y}{2\psi}\right\} + \frac{\mathrm{RS}_V(1-\mathrm{e}^{-uT_1})\lambda}{my} + \frac{\mathrm{RW}_V\pi\psi T_1\lambda}{my}\\ &\quad + \frac{\lambda}{my}\left(\mathrm{PM}_V\int_0^\infty T_2\phi(T_2)\mathrm{d}T_2 + C_W\psi T_1\left[A_1 + (A_2 - A_1)\pi\right]\right), \end{aligned} \tag{4.19}$$

where $T_{\mathrm{cyc}} = \frac{my}{\lambda}$ is the replenishment cycle length.

5. Optimality and solution procedure

5.1. Optimality

The purpose is to find the optimal values of m, y and S for which the total annual integrated cost is minimum. So, for a particular value of m the necessary conditions for $\mathrm{TC}(m,y,S)$ to be optimum are

$$\frac{\partial \mathrm{TC}(m,y,S)}{\partial y} = 0, \text{ and} \tag{5.1}$$

$$\frac{\partial \mathrm{TC}(m,y,S)}{\partial S} = 0. \tag{5.2}$$

The sufficient conditions to establish optimality are: $\partial^2\mathrm{TC}(m,y,S)/\partial y^2 > 0, \partial^2\mathrm{TC}(m,y,S)/\partial S^2 > 0$, and $(\partial^2\mathrm{TC}(m,y,S)/\partial y^2)(\partial^2\mathrm{TC}(m,y,S)/\partial S^2) - ((\partial^2\mathrm{TC}(m,y,S)/\partial y\partial S)^2 > 0$, which infers at a specific m value, the $\mathrm{TC}(m,y,S)$ is a convex function and there exists a unique value of y and S that minimize (4.19), from (5.1) and (5.2), we have

$$y^*\,(m) = \sqrt{\frac{\left[2\,(K_V+K_B)\,\lambda + 2\,(mT_B + M_V\psi T_1)\,\lambda + B_B S^2 m + I_B S^2 m + 2\mathrm{RS}_V\left(1-\mathrm{e}^{-uT_1}\right)\lambda + 2\mathrm{RW}_V\pi\psi T_1\lambda + 2\left(\mathrm{PM}_V\int_0^\infty T_2\phi(T_2)\mathrm{d}T_2 + C_W\psi T_1\left[A_1 + (A_2 - A_1)\pi\right]\right)\lambda\right]}{m\left[I_B + I_V\left(\frac{\lambda}{\psi}\right)\right]}}, \tag{5.3}$$

$$S^*\,(m) = \frac{I_B y^*\,(m)}{(I_B + B_B)}\cdot \tag{5.4}$$

All the first order derivatives are obtained in Appendix A.

The second derivative with respect to m shows the total integrated annual cost to be a convex function of m. All the second order derivatives are calculated in Appendix B.

TABLE 2. Optimal solutions of number of shipments.

m	y	S	TC$(m,\ y,\ S)$
1	4929.29	1297.17	879 825.50
2	2955.15	777.66	878 336.60
3	2150.90	566.02	877 880.90
4	1708.01	449.47	877 715.20
5*	**1426.26**	**375.33**	**877 671.00**
6	1230.75	323.87	877 688.00
7	1086.90	286.02	877 741.10

Notes. *Denotes the optimal value.

5.2. Solution procedure

The number of deliveries, m, from the vendor to the buyer is a discrete variable, so in order to obtain the optimal value of m which minimizes the total integrated cost $\mathrm{TC}(m, y, S)$, following steps are followed:

Step 1: For a given range of m values, determine the corresponding $y^*(m)$ and $S^*(m)$ using (5.3) and (5.4) and compute $\mathrm{TC}(m, y^*(m), S^*(m))$by substituting $y^*(m)$ and $S^*(m)$ into (4.19).

Step 2: Derive the optimal value of m, denoted by m^*, such that

$$\mathrm{TC}(m, y^*(m), S^*(m)) \leq \mathrm{TC}(m-1, y^*(m-1), S^*(m-1)),$$
$$\mathrm{TC}(m, y^*(m), S^*(m)) \leq \mathrm{TC}(m+1, y^*(m+1), S^*(m+1)).$$

Step 3: Once we obtain the m^* value, the optimal production batch Y_ψ^* is given by$Y_\psi^* = m^* y^*(m^*)$.

6. Numerical analysis

Example 1. The following parameters values are to be taken in suitable units: $\lambda = 50\,000$ units, $\psi = 160\,000$ units, $K_V = \$300$, $I_V = 1.5$, $M_V = \$6$, $M_B = 10$, $\mathrm{RW}_V = \$3$, $\mathrm{RS}_V = \$150$, $u = 0.1$, $\mathrm{PM}_V = \$100$, $K_B = \$100$, $I_B = \$2.5$, $T_B = \$25$, $\gamma_1 = 0.05$, $\gamma_2 = 0.09$, $C_W = \$50$, $w = 2$, $\sigma_1 = 1/36$, $\sigma_2 = 1/12$, $\rho_1 = 2$, $\rho_2 = 2$, $\kappa_1(x) = \sigma_1^{\rho_1}\rho_1 x^{\rho_1-1}$, $\kappa_2(x) = \sigma_2^{\rho_2}\rho_2 x^{\rho_2-1}$, $f(\chi) = 0.5\mathrm{e}^{-0.5\chi}$, $f(T_2) = 0.7\mathrm{e}^{-0.7T_2}$, $B_B = \$7$, $V_V = \$1$ (Apart from additional data for this paper the remaining parameter values are devised from Sana [36]).

Table 2 presents the solution for the number of shipments. The optimal result is $m^* = 5$, $y^* = 1426.26$ units, $S^* = 375.33$ units and the integrated total cost per year is \$877 671.00 (Lingo 15.0 as a tool is used to obtain the results).

Example 2. The values of the following parameters are to be taken in appropriate units: $\lambda = 40\,000$ units, $\psi = 160\,000$ units, $K_V = \$300$, $I_V = 1.5$, $M_V = \$6$, $M_B = \$10$, $\mathrm{RW}_V = \$3$, $\mathrm{RS}_V = \$150$, $u = 0.1$, $\mathrm{PM}_V = \$100$, $K_B = \$100$, $I_B = \$2.5$, $T_B = \$25$, $\gamma_1 = 0.05$, $\gamma_2 = 0.09$, $C_W = \$50$, $w = 2$, $\sigma_1 = 1/36$, $\sigma_2 = 1/12$, $\rho_1 = 2$, $\rho_2 = 2$, $\kappa_1(x) = \sigma_1^{\rho_1}\rho_1 x^{\rho_1-1}$, $\kappa_2(x) = \sigma_2^{\rho_2}\rho_2 x^{\rho_2-1}$, $f(\chi) = 0.5\mathrm{e}^{-0.5\chi}$, $f(T_2) = 0.7\mathrm{e}^{-0.7T_2}$, $B_B = \$7$, $V_V = \$1$.

The optimal result is $m^* = 5$, $y^* = 1251.58$ units $S^* = 329.36$ units and the integrated total cost per year is \$703182.30.

7. Sensitivity analysis

The current section presents sensitivity analysis of optimal order quantity, backordering quantity, number of shipments and total integrated cost with respect to key model parameters viz. buyer's holding cost, vendor's holding cost, backordering cost, freight cost, variable cost, warranty period, rework cost, restoration cost and maintenance cost the other parameters are held fixed at their original values. The aim is to illustrate some important aspects of developed model.

7.1. Managerial insights

In reference to the modern organizations which are being operated in a continuously changing environment, the decision-makers has to have the knowledge of various scenarios starting from manufacturing to sales and then post-sales. The present paper incorporates policies viz. rework, preventive maintenance, restoration and warranty under an integrated vendor–buyer environment. The integrated modelling gives a centralized control to the decision makers so as to take care of varying degrees of complexities. The dynamics of the parameter changes and their effects as discussed below presents the business enterprises with useful insights.

From the computational results presented in Table 3 following implications are obtained:

– With an increase in buyer's holding cost, the optimal order quantity decreases, the backordering quantity increases, and the total integrated cost tends to increase. It is therefore advisable to stock less in the inventory and backorder more of the demand so as to prevent higher holding cost of the buyer.
– The rise in the vendor's holding cost increases the shipments size and the total integrated cost but the number of shipments tends to decrease. In such a case it is beneficial for the vendor to store less in the inventory and deliver more quantity to the buyer so as to avoid larger holding cost of the produced lot.
– An increase in the backordering cost results into smaller optimal order quantity and backordering quantity with an increase in the total integrated cost. For such a scenario it is suggested that the buyer should avoid backorders up to the desired possible limit to prevent the high backordering cost.
– When the freight cost increases, the number of shipments reduces however both the optimal order quantity and backordering quantity increases. So, in this case, it is advisable to decrease the frequency of shipments from vendor to the buyer, but the size of shipments should increase so as to avoid high wages in transportation and handling. Obviously, the total integrated cost tends to increase with an increase in freight cost.
– The incurrence of higher warranty cost implies a longer warranty period, due to this, the buyer's cost remains unaffected but the vendor's cost increases, and this ultimately increases the total integrated cost. For such a case it is suggested to control the production of non-conforming items so as to avoid larger warranty costs.
– With an increase in the rework cost the total integrated cost increases because the rework process is carried out in a parallel manufacturing system. For such a case it is recommended that one should opt for various preventive measures to lessen the number of non-conforming items which will ultimately assist in reducing the rework costs.
– The vendor's cost tends to increase with an increase in the restoration cost, which ultimately contributes to increasing the total integrated cost. However, in the present scenario there is a slight increase in the total cost, so one should take advantage of this and cater the system with frequent restorations in order to increase the efficiency of the production system. Instead for the case where restoration costs are larger, it is mandatory to prevent frequent restorations which can be achieved by prolonging the production run time.
– The preventive maintenance actions are necessary for smooth operations and to gear-up the production system for the succeeding cycle. The cost-benefit analysis of the preventive maintenance actions always suggests its importance in terms of maintaining breakdown free operation of any production system. As the preventive maintenance cost increases, the vendor's cost increases and the buyer's cost remains unaffected since maintenance actions are performed at the vendor's end only. Moreover, the optimal order quantity, backordering quantity, and the total integrated cost tend to increase with an increase in the preventive maintenance cost.

The aforementioned findings illustrate the significance of various parameters on the integrated vendor–buyer system. No doubt the emergence of various policies viz. rework, preventive maintenance, restoration and warranty increases the total integrated cost but in the long run it aids in improvising the overall system. For instance, the rework policy helps in managing defectives efficiently. The use of preventive maintenance actions has been

TABLE 3. Sensitivity analysis on the key parameters.

	m^*	y^*	S^*	TC(m, y, S)
I_B				
1.250	3	2368.89	358.92	876 999.90
1.875	4	1767.28	373.36	877 399.70
2.500	5	1426.26	375.33	877 671.00
3.125	6	1205.82	372.16	877 882.40
3.750	6	1185.01	413.37	878 050.60
I_V				
0.750	8	1255.25	330.33	875 704.00
1.125	6	1362.87	358.65	876 779.00
1.500	5	1426.26	375.33	877 671.00
1.875	4	1584.70	417.02	878 447.00
2.250	4	1484.74	390.72	879 130.00
B_B				
3.50	4	1770.82	737.82	877 381.60
5.25	5	1442.67	465.38	877 565.50
7.00	5	1426.26	375.33	877 671.00
8.75	5	1415.27	314.50	877 743.70
10.50	5	1407.40	270.65	877 796.50
T_B				
12.50	7	1018.50	268.02	877 147.40
18.75	6	1196.97	314.99	877 431.10
25.00	5	1426.26	375.33	877 671.00
31.25	5	1459.25	384.01	877 887.60
37.50	4	1773.20	466.63	878 074.30
C_W				
25.00	5	1428.32	375.87	872 257.10
37.50	5	1427.29	375.60	874 964.10
50.00	5	1426.26	375.33	877 671.00
62.50	5	1425.23	375.05	880 378.00
75.00	5	1424.20	374.78	883 084.90
RW_V				
1.50	5	1431.30	376.65	873 887.80
2.25	5	1428.77	375.99	875 779.40
3.00	5	1426.26	375.33	877 671.00
3.75	5	1423.76	374.67	879 562.60
4.50	5	1421.27	374.01	881 454.10
RS_V				
75.00	5	1426.26	375.33	877 668.70
112.50	5	1426.26	375.33	877 669.80
150.00	5	1426.26	375.33	877 671.00
187.50	5	1426.26	375.33	877 672.20
225.00	5	1426.26	375.33	877 673.40
PM_V				
25	5	1306.84	343.90	876 887.00
50	5	1347.82	354.69	877 156.00
100	5	1426.26	375.33	877 671.00
125	5	1463.90	385.23	877 918.20
150	5	1500.60	394.89	878 159.10

signified as it increases the order size. The restoration helps in avoiding the system breakdowns and thus its absence can turn up in a huge disaster for the production managers. Further, the warranty policy enhances the post sales experience.

8. Concluding remarks

8.1. Conclusion

The present paper highlights the significance of preventive maintenance actions, warranty, and process restoration for an imperfect manufacturing system. The model is developed in an integrated environment by considering a single vendor and a single buyer. The imperfect production system may shift to the out-of-control state at any random time and non-conforming items are produced in both the states. The non-conforming items are reworked on a secondary machine and incur rework cost to the vendor. Further, the production process is restored back to its original conditions if found in "out-of-control" state at the end of every production cycle with a cost of restoration. The model also incorporates preventive maintenance actions in order to ensure the smooth functioning of the production system. Further, free minimal repair warranty policy is offered to the buyer on sold products. The buyer places regular orders of fixed size to the vendor, and shortages are allowed and completely backlogged at the buyer's end. The aim is to minimize the total annual integrated cost by optimizing the number of shipments, the size of shipments and backordering quantity. An algorithm has been given to obtain the optimal solution efficiently. Also, numerical analysis and sensitivity analysis is performed to exemplify the key benefits of the model.

The findings suggest the following managerial insights to the decision makers under varying parameters: (i) the vendor may increase the shipment size with an increase in its holding cost so as to avoid high wages in holding; (ii) whereas with an increase in the buyer's holding cost, it is advisable for the buyer to stock less in the inventory and backorder more of the demand; (iii) as the freight cost increases, a large shipment size with less frequency of the number of shipments from the vendor to the buyer is recommended; (iv) although higher preventive maintenance cost increases the total integrated cost but at the same time it helps in improving the system performance by increasing the production quantity; (v) for the case of the rework, it is beneficial for the vendor to reduce the rework cost as it will assist in decreasing the total cost with an increase in the order size; (vi) there is a slight increase in the total cost with increase in the restoration cost, so one should take advantage of this and cater the system with frequent restorations in order to increase the efficiency of the production system.

8.2. Limitations and future research directions

Every form of research hold some limitations, the present study has a limitation in the view of static nature of demand, *i.e.* the demand of the product has been considered to be constant, however, practically the demand is influenced by many factors depending upon the product. One of the most basic nature of demand is being price-sensitive. In future, such a limitation can be addressed by considering price-dependent demand function. Further, the impact of carbon-emissions is not incorporated in this study, thus, in future this can be overcome through a green supply chain model by considering emissions while transportation of goods from the vendor to the buyer and during the production run.

Future extension of the proposed model can be made in different ways. The model can be extended to the case of multiple buyers, different warranty policy, inspection errors, quantity discounts on material costs etc. The production and demand rate can also be taken as a function of different parameters. Taking into account the effect of inflation and trade credit policy would be another contribution to the existing literature.

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Appendix A.

$$\begin{aligned}\frac{\partial \mathrm{TC}(m,y,S)}{\partial y} = &-\frac{(K_V+K_B)\,\lambda}{my^2}-\left\{\frac{\lambda}{y^2}+\frac{M_V\psi T_1\lambda}{my^2}\right\}-\frac{B_BS^2}{2y^2}-\frac{I_B}{2}\left(\frac{S^2}{y^2}-1\right)\\ &+I_V\left[\frac{\lambda}{\psi}-\frac{m\lambda}{2\psi}+\frac{(m-1)}{2}\right]-\frac{\mathrm{RS}_V\lambda\left(1-\mathrm{e}^{-uT_1}\right)}{my^2}\\ &-\frac{\mathrm{RW}_V\pi\psi T_1\lambda}{my^2}-\frac{\lambda}{my^2}\left(\mathrm{PM}_V\int_0^\infty T_2\phi(T_2)\mathrm{d}T_2+C_W\psi T_1\left[A_1+(A_2-A_1)\pi\right]\right)\end{aligned} \tag{A.1}$$

$$\frac{\partial \mathrm{TC}(m,y,S)}{\partial S}=I_B\left[\frac{S}{y}-1\right]+\frac{B_BS}{y}\cdot \tag{A.2}$$

Appendix B.

$$\begin{aligned}\frac{\partial^2 \mathrm{TC}(m,y,S)}{\partial y^2} = &\frac{2\,(K_V+K_B)\,\lambda}{my^3}+\left\{\frac{2\lambda}{y^3}+\frac{2M_V\psi T_1\lambda}{my^3}\right\}+\frac{B_BS^2}{y^3}+\frac{I_BS^2}{y^3}+\frac{2\mathrm{RS}_V\lambda\left(1-\mathrm{e}^{-uT_1}\right)}{my^3}\\ &+\frac{2\mathrm{RW}_V\pi\psi T_1\lambda}{my^3}+\frac{2\lambda}{my^3}\left(\mathrm{PM}_V\int_0^\infty T_2\phi(T_2)\mathrm{d}T_2+C_W\psi T_1\left[A_1+(A_2-A_1)\pi\right]\right)\end{aligned} \tag{B.1}$$

$$\begin{aligned}\frac{\partial^2 \mathrm{TC}(m,y,S)}{\partial m^2} = &\frac{2\,(K_V+K_B)\,D}{m^3y}+\frac{2M_V\psi T_1\lambda}{m^3y}+\frac{2\mathrm{RS}_V\lambda\left(1-\mathrm{e}^{-uT_1}\right)}{m^3y}+\frac{2\mathrm{RW}_V\pi\psi T_1\lambda}{m^3y}\\ &+\frac{2\lambda}{m^3y}\left(\mathrm{PM}_V\int_0^\infty T_2\phi(T_2)\mathrm{d}T_2+C_W\psi T_1\left[A_1+(A_2-A_1)\pi\right]\right)>0\end{aligned} \tag{B.2}$$

$$\frac{\partial^2 \mathrm{TC}(m,y,S)}{\partial S^2}=\frac{1}{y}\left(I_B+B_B\right), \tag{B.3}$$

$$\frac{\partial^2 \mathrm{TC}(m,y,S)}{\partial y\partial S}=-\frac{(I_B+B_B)\,S}{y^2} \tag{B.4}$$

$$\begin{aligned}&\left(\frac{\partial^2 \mathrm{TC}(m,y,S)}{\partial y^2}\right)\left(\frac{\partial^2 \mathrm{TC}(m,y,S)}{\partial S^2}\right)-\left(\frac{\partial^2 \mathrm{TC}(m,y,S)}{\partial y\partial S}\right)^2\\ &=\left(\begin{array}{l}(I_B+B_B)(S^2my^3B_B+S^2my^3I_B-2\mathrm{RS}_V(1-\mathrm{e}^{-uT_1})\lambda y^3-2\lambda y^3K_B-2\lambda y^3K_V-B_BS^2my^3\\ -I_BS^2my^3-2M_V\psi T_1\lambda y^3-2\lambda my^3-2\mathrm{RW}_V\pi\psi T_1\lambda y^3-y^3\\ \left(\mathrm{PM}_V\int_0^\infty T_2\phi(T_2)\mathrm{d}T_2+C_W\psi T_1\left[A_1+(A_2-A_1)\pi\right]\right)\end{array}\right)/(y^4my^3).\end{aligned} \tag{B.5}$$

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