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INVENTORY MANAGEMENT STRATEGIES FOR THE CALABRIAN HOSPITALS SYSTEM

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Abstract. This paper addresses some inventory management problems arising in the Italian health care system. We aim at defining inventory management strategies for the hospitals located in Calabria, a Region in the South of Italy. We present some classical inventory management formulations together with the description of a new model, devised to take into account some peculiarities of the case under study. An extensive testing phase is carried out on real data. Numerical results show that the use of the presented mathematical models can be valuable in the decision making process.

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1. Introduction

The development of inventory management strategies in health care keeps the same motivations of the other industries: the availability of all the resources necessary for ensuring an appropriate level of assistance; the reduction of the costs generated from high inventory and the prevention of goods shortage.

The literature in health care inventory management is generally concentrated either on real case studies like in [15–17, 19] or on supply chain and general management issues, like in [2, 5, 8, 18].

General inventory management and control have attracted the attention of many researchers over the years and there is a huge body of related literature, here we just cite the seminal works of [12, 13] for inventory management motivations, main optimization models and solution approaches.

This paper addresses inventory management problems arising in the Italian health care system. In particular, we focus on the specific situation of Calabria, a region located in the South of Italy. Starting from the current situation, where no inventory strategies are used and implemented, we apply some well known inventory policies to our case study and introduce a new optimization model designed to take into account the features of the Calabrian inventory health care management system which is described in detail in [10].

We consider a network composed by 13 public hospitals located in Calabria (see Fig. 1) and coordinated by two governmental organizations called ASP ("Azienda Sanitaria Provinciale") and AO ("Azienda Ospedaliera") of Cosenza. Each hospital is constituted by wards as next shown in Table 2 of Section 4.

The main aim of our work is to support decision makers in efficiently managing the system constituted by the ward and the hospital pharmacies. The hospital pharmacy (called also central) acts as the supplier of drugs and

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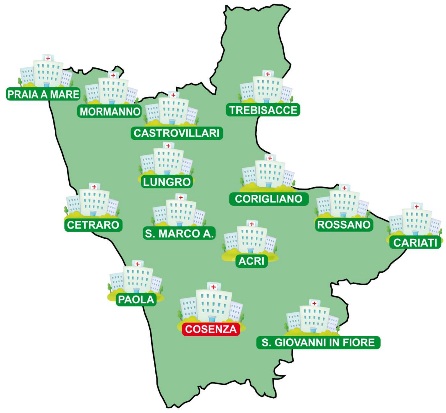

FIGURE 1. Healthcare network: province of Cosenza.

other medical materials for the whole hospital. The ward pharmacies, after estimating the necessary quantity of drugs (and of the other health care materials), on a given time horizon, make a request to the central pharmacy. The ward pharmacies also control the received medicines and place them in a dedicated storage space, taking into account both the expiration date and the inventory level. The central pharmacy makes the necessary orders to satisfy the requests coming from all the ward pharmacies. In this scenario, critical issues can arise either between ward and central pharmacies or among the latter and its suppliers: insufficient storage space for both central and ward pharmacies; over-estimation of the demand resulting in large stock levels for drugs and health care materials with consequent higher probability of managing expired medicines; presence of broken drugs packages; possible troubles in positioning and managing the delicate drugs and health care materials. At present the drugs supply process is of the order-delivery type: the requests are made day-by-day and are fulfilled without any possible strategy to contain the costs deriving from the frequency of the orders. Moreover, the absence of any schedule causes an excess of stock with high inventory costs. With this premises, we tried to rationalize and reduce the order and inventory costs by applying appropriate inventory control strategies.

The rest of the paper is organized as follows: in Section 2, we remind and adapt to the case under study some well known inventory mathematical models; is Section 3 a new optimization model aimed at rationalizing the logistic system constituted by the ward and the hospital pharmacies is described; in Section 4, the presented mathematical models are applied to the Calabrian case, while in Section 5 we draw some conclusions.

2. Optimization models for hospital inventory management

This section describes some of the models that can be useful to support the decision makers in designing the most common inventory management strategies (see [14] for a interesting survey).

It is important to underline that the models, we describe below, can be used to manage orders and inventory for two systems, the first composed by the ward and the hospital pharmacies, and the latter made up of the hospital pharmacy and the (health care) material suppliers. We consider the specific scenario where the medical products are not perishable.

The parameters and variables we need to formulate our first model are reported in the following.

- Parameters
  - $i = 1, \ldots, n$ the drug index;
  - $h = 1, \ldots, H$ the drug-typology (phials, pills, bottles, etc.) index;
  - $t = 1, \ldots, T$ the period index;

- R_h the capacity of the pharmacy related to the drug-typology h, *i.e.* the maximum number of drugs of type h, $h = 1, \dots, H$ that can be stocked in the pharmacy;
- S_{it} the cost for ordering drug i at the period t;
- a_{ih} equal to 1 if drug i is of type h, and 0 otherwise;
- h_{it} the unit inventory cost of drug i at the end of period t;
- d_{it} the demand for drug i in period t;
- $M_t = \sum_{i=1}^{n} d_{it}$ an upper bound on the quantity of drug i to be ordered in period t;
- $I^i_{\min} \geq 0$ the safety stock of drug i.

– Variables
- x_{it} the quantity of drug i to be ordered in period t;
- I_{it} the inventory of drug i at the end of period t;
- y_{it} a binary variable equal to 1 if drug i is ordered in period t and 0 otherwise.

The Multi-product Capacitated Inventory Problem (MCIP) can be formulated as:

$$\min \sum_{i=1}^{n} \sum_{t=1}^{T} (S_{it} y_{it} + h_{it} I_{it}) \tag{2.1}$$

s.v.

$$\sum_{i=1}^{n} a_{ih} x_{it} \leq R_h - \sum_{i=1}^{n} a_{ih} I_{i(t-1)} \quad \forall h = 1, \dots, H, \quad t = 1, \dots, T \tag{2.2}$$

$$x_{it} + I_{i(t-1)} - I_{it} = d_{it} \quad \forall i = 1, \dots, n, \quad t = 1, \dots, T \tag{2.3}$$

$$x_{it} \leq M_t y_{it} \quad \forall i = 1, \dots, n, \quad t = 1, \dots, T \tag{2.4}$$

$$y_{it} \leq x_{it} \quad \forall i = 1, \dots, n, \quad t = 1, \dots, T \tag{2.5}$$

$$I_{it} \geq I^i_{\min}, \quad \forall i = 1, \dots, n, \quad t = 1, \dots, T \tag{2.6}$$

$$x_{it} \geq 0, \quad \forall i = 1, \dots, n, \quad t = 1, \dots, T \tag{2.7}$$

$$y_{it} \in (0, 1), \quad \forall i = 1, \dots, n, \quad t = 1, \dots, T \tag{2.8}$$

The objective function (2.1) minimizes the total ordering and inventory costs. Constraints (2.2) impose that, in each time period, the space filled by all drugs of typology h, does not exceed the available space R_h. Constraints (2.3) state that, at each time period, the ordered quantity plus the inventory of drug i must be equal to the total demand d_{it}. Constraints (2.4) and (2.5) account for the order cost only if drug i is requested in period t. Constraints (2.6) require the inventory to be at least equal to the safety stock level, while constraints (2.7) and (2.8) impose the non negativity of variables x_{it} and the binariety of variables y_{it}, respectively.

We now present a version of the problem aimed at minimizing the number of orders in the considered planning horizon. We need to introduce the following additional variables:

– y_t equal to 1 if an order is made in period t, 0 otherwise.

The new formulation is obtained from (2.1)–(2.8), by reformulating constraints (2.4) as follows:

$$\sum_{i=1}^{n} x_{it} \leq M_t y_t, \quad \forall t = 1, \dots, T, \tag{2.9}$$

and by replacing the objective function (2.1) with:

$$\min \sum_{t=1}^{T} y_t \tag{2.10}$$

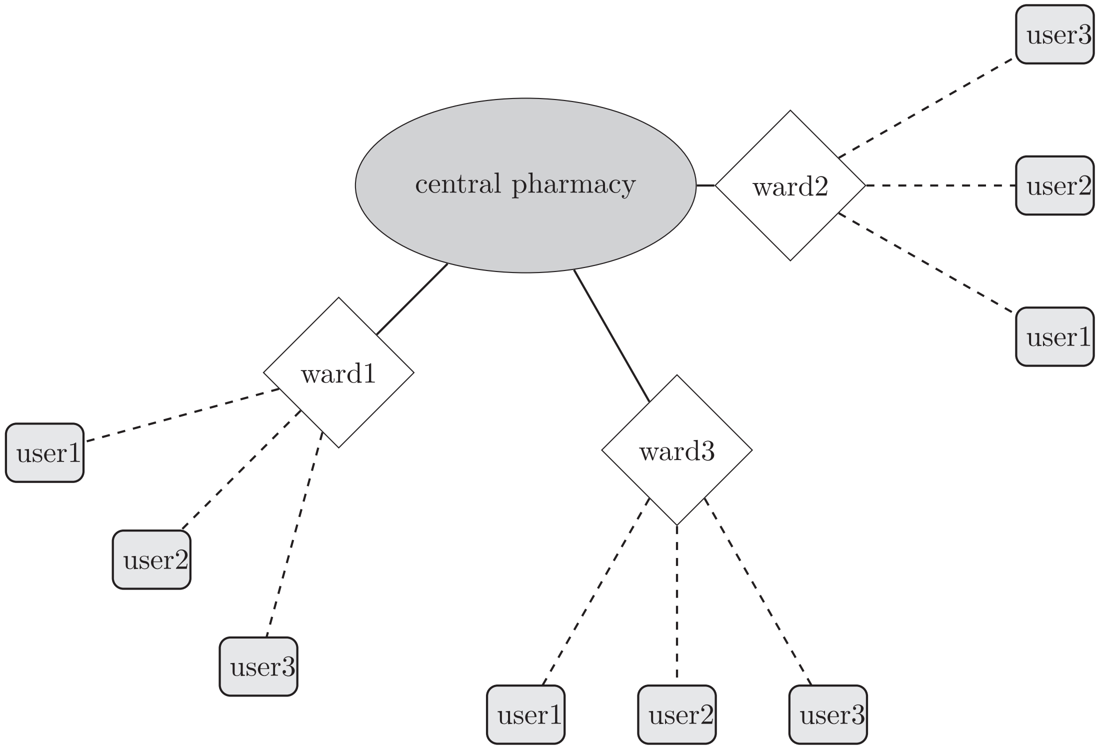

Figure 2. A two echelon scheme for the hospital pharmacy system.

So far, we have considered ward and hospital pharmacies as separate parts of the same system: they decide autonomously the drug quantities to be ordered and when the orders have to be placed. On one side, the suppliers for the hospital pharmacies are the drug producers, while the consumers are the ward pharmacies; from the other, the suppliers for ward pharmacies are the hospital pharmacies and the consumers the ward patients. In the next section, we introduce a new mathematical model aimed at taking into account possible synergies between the two types of pharmacies (ward and hospital).

3. The Integrated Inventory Problem (IIP)

We introduce a new mathematical programming model where the decision maker is the hospital pharmacy, which has to decide its own inventory level together with the quantity of drugs to be sent to the ward pharmacies and their inventory levels.

The objective is to minimize both the total inventory cost of all pharmacies and the dispatching cost from the hospital to the ward pharmacies. The hospital pharmacy behaves as the primary depot and holds the quantity of drugs to satisfy the demand from all the ward pharmacies. A pictorial representation of the system under consideration is reported in Figure 2. Thus, in the proposed model, the hospital and the ward pharmacies operate under a vendor management inventory (VMI) system ([1,3,7]).

In particular, the hospital pharmacy is the manufacturer while the ward pharmacies are the retailers . We also assume that: (a) there is a single supplier and r retailers; (b) there are n different products; (c) all the pharmacies (including the supplier) are characterized by a limited storage capacity; (d) the demands of all products are deterministic.

To state the Integrated Inventory Problem (IIP) model, we need to introduce the following parameters and variables:

- $k = 0, \ldots, r$ the pharmacy indexes. In particular $k = 0$ refers to the hospital pharmacy;
- h_k^{it} the unit inventory cost of drug i in pharmacy k, $k = 0, \ldots, r$, at end of period t;
- c_{0k} the fix dispatching (cost) from the hospital pharmacy to the ward pharmacy k, $k \neq 0$;
- d_k^{it} the demand of pharmacy k, $k \neq 0$, for drug i at time t;
- R_h^k the capacity of the pharmacy k, $k = 0, \ldots, r$, related to the drug-typology h, *i.e.* the maximum number of drugs of type h, $h = 1, \ldots, H$ that can be stocked in pharmacy k;
- I_k^{i0} the initial level of inventory for drug i in pharmacy k $k = 0, \ldots, r$;
- $I_{\min}^i \geq 0$ the safety stock of drug i.

The decision variables are:

- I_k^{it} the inventory levels for drug i at the end of period t for pharmacy k , $k = 0, \ldots, r$;
- r^{it} the quantity of drug i purchased by the hospital pharmacy at time t;
- x_k^{it}, $k \neq 0$, the quantity of drug i sent from the hospital pharmacy to ward pharmacy k at time t;
- y_k^t, $k \neq 0$, a binary variable equal to 1 if ward pharmacy k is served by the hospital pharmacy at time t, 0 otherwise.

The IPP model can be stated as follows:

$$\min \sum_{t=1}^{T} \sum_{i=1}^{n} \sum_{k=0}^{r} h_k^{it} I_k^{it} + \sum_{k=1}^{r} \sum_{t=1}^{T} c_{0k} y_k^t \tag{3.1}$$

s.v.

$$I_0^{it} = I_0^{i(t-1)} + r^{it} - \sum_{k=1}^{r} x_k^{it} \quad \forall\, i,\ t \tag{3.2}$$

$$I_k^{it} = I_k^{i(t-1)} + x_k^{it} - d_k^{it} \quad \forall\, i,\ t,\ k \neq 0 \tag{3.3}$$

$$\sum_{i=1}^{n} a_{ih} I_k^{it} \leq R_h^k \quad \forall\, t,\ k,\ h \tag{3.4}$$

$$\sum_{i=1}^{n} a_{ih} x_k^{it} \leq R_h^k - \sum_{i=1}^{n} a_{ih} I_k^{i(t-1)} \quad \forall\, t,\ k,\ h \tag{3.5}$$

$$x_k^{it} \leq M_t y_k^t \quad \forall\, i,\ t,\ k : k \neq 0 \tag{3.6}$$

$$y_k^t \leq x_k^{it} \quad \forall\, i,\ t,\ k : k \neq 0 \tag{3.7}$$

$$I_k^{it} \geq I_{\min}^i \quad \forall\, i,\ t,\ k \tag{3.8}$$

$$x_k^{it} \geq 0 \quad \forall\, i,\ t,\ k : k \neq 0 \tag{3.9}$$

$$r^{it} \geq 0 \quad \forall\, i,\ t \tag{3.10}$$

$$y_k^t \in (0, 1), \quad \forall\, k : k \neq 0,\ t \tag{3.11}$$

The objective function (3.1) minimizes the inventory costs for ward and hospital pharmacies and the cost of handling medicines from the hospital to the ward pharmacies.

Constraints (3.2) and (3.3) define the inventory level for pharmacies by considering the initial inventory, the drugs delivered from the hospital pharmacy and the arrival at (3.2) or the supply from (3.3) ward pharmacies.

Constraints (3.4) impose that, in each time period, the level of inventory of each drug typology at each ward pharmacy does not exceed the available capacity, whereas constraints (3.5) control that the inventory level, after the arrival of the drugs from the hospital pharmacy, does not exceed the ward capacity. Constraints (3.6) and (3.7) define logical links between the variables x and y, constraints (3.8) state that the inventory must be at least equal to the safety stock while constraints (3.9)–(3.11) impose the non-negativity of x and r and the binariety of y variables.

The IIP model can be modified by introducing the possibility of moving drugs, from ward pharmacies, to other wards where the drugs are currently unavailable. In the related model, we need to introduce the following variables:

– $\delta^{it}_{k\bar{k}},\ k, \bar{k} = 1, \dots, r,\ i = 1, \dots, n,\ t = 1, \dots, T$: the quantity of drug i sent from ward pharmacy k to ward pharmacy $\bar{k}$ at time t.

Once introduced the δ variables, a transfer costs should be also considered. In particular, letting $c^i_{k\bar{k}}$ the cost for sending an unit of drug i from ward pharmacy k to ward pharmacy $\bar{k}$, the total inter-ward pharmacy shipment cost is equal to $\sum_{t=1}^{T} \sum_{i=1}^{n} \sum_{k=1}^{r} \sum_{\bar{k} \neq k} c^i_{k\bar{k}} \delta^{it}_{k\bar{k}}$ and should de added to the objective function. Nevertheless, in the case under study, the transfer costs between wards are not considered, since they are assumed to be negligible.

We also need to reformulate constraints (3.3) as follows:

$$I^{it}_k = I^{i(t-1)}_k + x^{it}_k + \sum_{\bar{k} \neq k} \delta^{it}_{\bar{k}k} - \sum_{\bar{k} \neq k} \delta^{it}_{k\bar{k}} - d^{it}_k \quad \forall\ i = 1, \dots, n, \quad t = 1, \dots, T,\ k = 1, \dots, r \tag{3.12}$$

A further formulation can be introduced with the objective of minimizing the number of purchases made by the hospital pharmacy in a given planning horizon. We need to introduce the following variables:

– γ_t a binary variable equal to 1 if a purchase is made at time t, 0 otherwise.

Constraints (3.6) and (3.7) are substituted by:

$$\sum_{i=1}^{n} r_{it} \leq M_t \gamma^t, \quad \forall\ t = 1, \dots, T \tag{3.13}$$

while the objective (3.1) is replaced by:

$$\min \sum_{t=1}^{T} \gamma^t \tag{3.14}$$

4. Computational study

The data and information used to test the models of Sections 2 and 3, are provided by the ASP and the AO of Cosenza, a city in the south of Italy.

The ASP, together with the AO, coordinates the public provincial health care activities, related to the management of hospitals and other qualified structures, such as care homes, clinical laboratories and homes for disabled and aged.

The ASP includes thirteen hospitals, reported in Table 1, where the name of the city hosting each hospital is given. The list of the hospital wards is reported in Table 2. The AO includes the Annunziata and the Mariano Santo hospitals, located in Cosenza, and the Santa Barbara hospital, in Rogliano, a small city situated in the Calabria region.

TABLE 1. Cosenza's ASP hospitals.

	Hospitals
1	Acri
2	Castrovillari
3	Cetraro
4	Corigliano
5	Lungro
6	Mormanno
7	Paola
8	Praia a mare
9	Rossano
10	Trebisacce
11	San Giovanni in Fiore
12	San Marco Argentano
13	Cariati

TABLE 2. Wards.

	Wards
1	Cardiology
2	General surgery
3	Geriatric ward
4	General medicine
5	Nephrology
6	Nursery
7	Neurology
8	Ophthalmology
9	Orthopedics
10	Obstetrics and Gynecology
11	Otorhinolaryngology
12	Pediatrics and neonatology
13	Psychiatry
14	Urology
15	Intensive Care
16	Cardiological Intensive Care
17	Intensive Rehabilitation
18	Long-Term Care
19	Oncology
20	Pneumology
21	Multidisciplinary
22	Rheumatology

To test our models we have considered the following data:

- the daily and monthly consumption of drugs in all the wards of all the hospitals active in 2012;
- the drug movements between ward and hospital pharmacies of all the hospitals active in 2012;
- the inventory of the ward pharmacies of all the hospitals available on December, 31 for the years 2009-2010-2011-2012;
- with reference to the hospital pharmacies: drug names, ATC/AIC code, description, measure unit, unit price, ordered quantity and its value (quantity $\times$ unit price).

It was not possible to have information on the drug demands at the ward pharmacies, since only hard copies of these information are available. However the hospital pharmacies track the drug flows towards the wards by an informative system, so that, to test the models, we could use the ward consumptions (*i.e.* the quantities of drugs provided by hospital pharmacy to the wards) as ward pharmacies demand. We individuated the subset of drugs to use in our experiments by taking into account the total quantity requested in the 2012. In particular, the 13 hospitals were ordered in a decreasing way on the basis of the quantity of drugs requested to the central pharmacy. The highest number of requests was made by the Corigliano hospital. Thus, we operated an ABC analysis, a well known and practical classification based on the Pareto principle, on the drugs requested by the Corigliano hospital. After selecting, among the wards active in Corigliano, those with an annual drug request greater than 10 000 units (general surgery, general medicine, orthopedics, obstetrics and gynecology, pediatrics, psychiatry), we performed the ABC analysis, individuating the set of 20 drugs with the highest values (*i.e.* the product between the ordered drug quantity and the unit price). Thus, we carried out the computational experiments by considering the 7 typologies of each of the 20 selected drugs (*i.e.* phials, bottles, cavities, pipes, vaginal suppository, syringe, pills).

We also assumed: a monthly planning horizon, with the time unit being one day; an initial inventory, for each drug, equal to the total inventory of the hospital pharmacy available on December 31, 2011 divided by 12 (months); a demand, for each drug, equal to the sum of the ward consumptions on each day of the considered months; a pharmacy capacity, for each drug, equal to the maximum quantity available in each ward (calculated as the highest possible demand value plus a safety stock); a safety stock calculated so that, with a probability of α, there is no shortage during the lead time $L = 1$ unit of time. Since it was not possible to quantify the ordering and inventory costs, we restated, as follows, the objectives of the MCIP model while maintaining the same constraints:

– MCIP_1: $\min \sum_{t=1}^{T} y_t$.
– MCIP_2: $\min \sum_{i=1}^{n}\sum_{t=1}^{T} y_{it}$. MCIP with $S_{it} = 1$ and $h_{it} = 0$, $\forall i, t$.
– MCIP_3: $\min \sum_{i=1}^{n}\sum_{t=1}^{T} I_{it}$. MCIP with $S_{it} = 0$ and $h_{it} = 1$, $\forall i, t$.

The first model MCIP_1 is focused on the minimization of the number of orders made on the whole time horizon, independently from the type of drug while the MCIP_2 model encourages an order distribution where the number of orders of each drug i is minimized over T. The MCIP_3 model minimizes the total inventory level on T. It is worth noting that the first two models against the third one, present conflicting objectives. Nevertheless, they represent very important tools to support the decision maker, who can select the most suitable one, depending on the specific features to take into account. In addition, they can be used to analyse the behaviour of the inventory system under different operative conditions.

The models were tested under the scenarios described in Table 3 that differ for the initial inventory, the security stock and the service level values. We remark that ensuring a service level of $\alpha\%$ means that the safety stock has been dimensioned so that with a probability of $\alpha\%$ the system will be not under stock during the fixed lead time L. The initial inventory values, not equal to zero, are reported in Table 4. The two considered values for the safety stock are reported in the Table 5, the first guaranteeing a service level of 90% while the second of 60%.

In all our computational experiments, the model were solved in few seconds, so that we do not present the computational times.

Table 6 reports the $\sum_{t=1}^{T} y_t$ values, corresponding to the optimal solution of each models, under the considered scenarios.

TABLE 3. Scenarios.

Scenarios	I_{i0}	$I^i_{\min}$	Service levels
1	=0	=0	–
2	=0	>0	90%
3	=0	>0	60%
4	>0	=0	–
5	>0	>0	90%
6	>0	>0	60%

TABLE 4. Initial inventory.

Drugs	I_{i0}
1	11
2	160
3	106
4	30
5	214
6	95
7	5
8	427
9	2
10	139
11	3
12	395
13	2
14	42
15	35
16	60
17	8
18	3
19	60
20	23

Due to the objective function definitions, model MCIP_1 exhibits the smallest total number of orders per month, followed by MCIP_2 and MCIP_3. The behaviour of the models is well represented in Figure 3 where we focused on scenario 5 of Table 3.

In Table 7, we report the $\sum_{i=1}^{n}\sum_{t=1}^{T} y_{it}$ values for each model under each scenario. Also in this case, the highest values of $\sum_{i=1}^{n}\sum_{t=1}^{T} y_{it}$ is obtained with the model MCIP_3, which minimizes the total inventory. The model behaviour is highlighted in Figure 4, focusing, in particular, on scenario 5 of Table 3.

Table 8 reports the inventory values for each considered models and scenarios and, obviously, the smallest values are those corresponding to the optimal solution of model MCIP_3. Also in this case, we give a graphical view of the model behaviour in Figure 5, focussing the attention on scenario 5 of Table 3. Looking at $\sum_{i=1}^{n}\sum_{t=1}^{T} I_{it}$ values in Table 8, it is also evident that the inventory levels, as expected, are smaller if a lower safety stock level is imposed.

TABLE 5. Safety stock $I^i_{\min}$ with service levels of 90% and 60%.

	$I^i_{\min}$	
Drugs	LS = 90%	LS = 60%
1	27	5
2	467	94
3	77	16
4	54	11
5	60	12
6	61	12
7	7	1
8	232	47
9	8	2
10	82	17
11	5	1
12	105	21
13	28	6
14	26	5
15	44	9
16	56	11
17	28	6
18	14	3
19	35	7
20	15	3

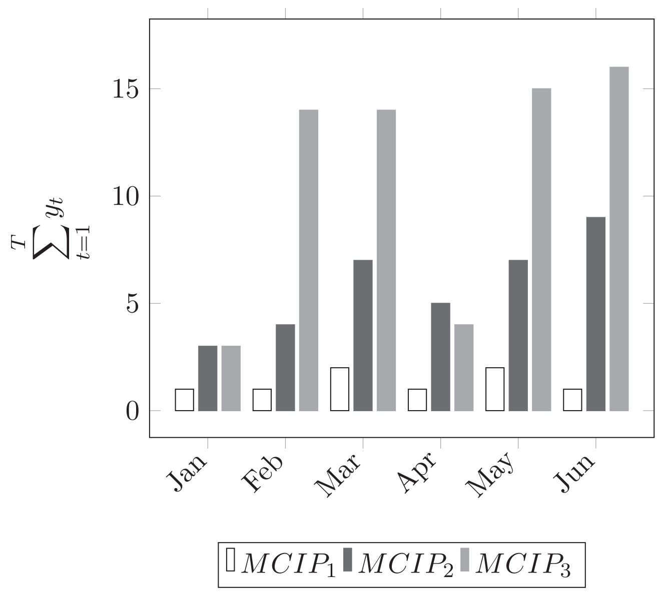

FIGURE 3. Trend of $\sum_{t=1}^{T} y_t$ values for MCIP models in the case of scenario 5.

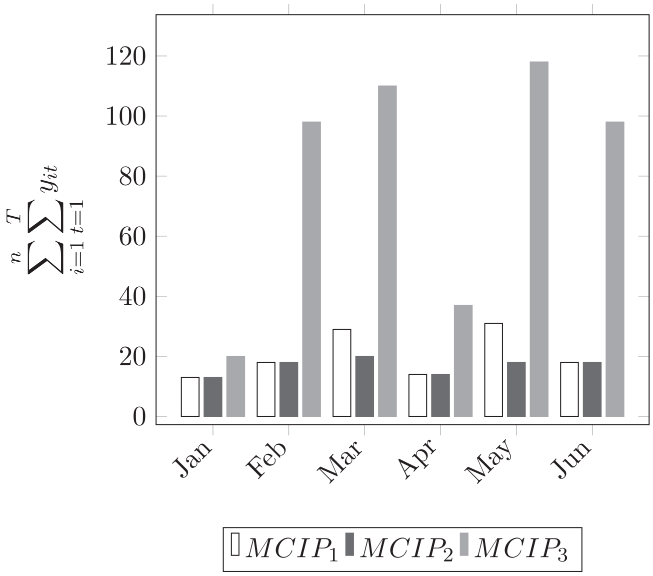

FIGURE 4. Trend of $\sum_{i=1}^{n}\sum_{t=1}^{T} y_{it}$ values for MCIP models in the case of scenario 5.

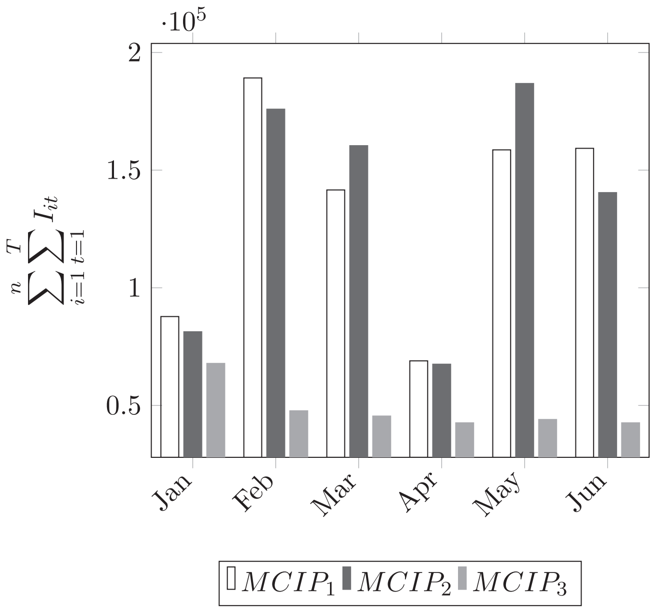

FIGURE 5. Trend of $\sum_{i=1}^{n}\sum_{t=1}^{T} I_{it}$ values for MCIP models in the case of scenario 5.

TABLE 6. $\sum_{t=1}^{T} y_t$ values for the 3 models and the 6 scenarios.

$\sum_{t=1}^{T} y_t$	January	February	March	April	May	June
$MCIP_1.1$	1	2	1	1	2	1
$MCIP_1.2$	1	2	1	1	2	1
$MCIP_1.3$	1	2	1	1	2	1
$MCIP_1.4$	1	1	2	1	2	1
$MCIP_1.5$	1	1	2	1	2	1
$MCIP_1.6$	1	1	2	1	2	1
$MCIP_2.1$	4	4	6	5	6	9
$MCIP_2.2$	1	4	6	5	6	9
$MCIP_2.3$	1	6	6	5	7	7
$MCIP_2.4$	3	6	5	7	7	9
$MCIP_2.5$	3	4	7	5	6	9
$MCIP_2.6$	3	6	7	5	7	10
$MCIP_3.1$	2	13	14	4	15	16
$MCIP_3.2$	3	14	14	4	15	16
$MCIP_3.3$	3	13	14	4	15	16
$MCIP_3.4$	2	14	14	4	15	16
$MCIP_3.5$	3	14	14	4	15	16
$MCIP_3.6$	3	14	14	4	15	16

TABLE 7. $\sum_{i=1}^{n}\sum_{t=1}^{T} y_{it}$ values for the 3 models and the 6 scenarios.

$\sum_{i=1}^{n}\sum_{t=1}^{T} y_{it}$	January	February	March	April	May	June
$MCIP_1.1$	9	31	19	14	34	18
$MCIP_1.2$	19	33	19	14	32	18
$MCIP_1.3$	19	31	19	14	34	18
$MCIP_1.4$	3	18	26	14	19	18
$MCIP_1.5$	13	18	29	14	31	18
$MCIP_1.6$	6	18	29	14	28	18
$MCIP_2.1$	9	18	19	14	18	18
$MCIP_2.2$	19	18	19	14	18	18
$MCIP_2.3$	19	18	19	14	18	18
$MCIP_2.4$	3	18	20	14	18	18
$MCIP_2.5$	13	18	20	14	18	18
$MCIP_2.6$	6	18	20	14	18	18
$MCIP_3.1$	13	102	113	37	118	98
$MCIP_3.2$	32	102	113	37	118	98
$MCIP_3.3$	32	102	113	37	118	98
$MCIP_3.4$	4	94	109	37	118	98
$MCIP_3.5$	20	98	110	37	118	98
$MCIP_3.6$	8	96	109	37	118	98

TABLE 8. $\sum_{i=1}^{n}\sum_{t=1}^{T} I_{it}$ value for the 3 models and the 6 scenarios.

$\sum_{i=1}^{n}\sum_{t=1}^{T} I_{it}$	January	February	March	April	May	June
$MCIP_1$.1	1452	64 745	127 171	26 336	97 349	116 663
$MCIP_1$.2	70 075	122 932	171 098	68 846	153 247	159 173
$MCIP_1$.3	35 014	62 019	136 037	34 916	71 782	125 243
$MCIP_1$.4	58 313	141 963	113 802	26 426	146 877	116 733
$MCIP_1$.5	87 776	189 183	141 579	68 936	158 613	159 263
$MCIP_1$.6	65 735	149 505	106 854	35 006	114 801	125 333
$MCIP_2$.1	7019	136 350	119 976	24928	139205	88934
$MCIP_2$.2	70 075	178 856	163 903	67 438	186 804	140 355
$MCIP_2$.3	35 014	146 132	128 842	33 508	142 957	98 798
$MCIP_2$.4	52 959	134 198	115 969	25 018	138 170	88 974
$MCIP_2$.5	81 305	175 905	160 391	67 528	186 817	140 445
$MCIP_2$.6	53 824	148 989	124 394	33 598	143 050	98 888
$MCIP_3$.1	0	0	0	0	0	0
$MCIP_3$.2	43 927	41 093	43 927	42 510	43 927	42 510
$MCIP_3$.3	8866	8294	8866	8580	8866	8580
$MCIP_3$.4	52 729	11 738	3548	90	93	90
$MCIP_3$.5	67 852	47 710	45 480	42 600	44 020	42 600
$MCIP_3$.6	53 538	18 755	11 973	8670	8959	8670

We have also considered three variants of the IIP model by introducing the following objective functions:

– IIP_1 Model: $\min \sum_{i=1}^{n}\sum_{k=0}^{r}\sum_{t=1}^{T} I_k^{it}$. IPP with $c_{0k} = 0$ and $h_k^{it} = 1$, $\forall i, k, t$.

– IIP_2 model: $\min \sum_{k=1}^{r}\sum_{t=1}^{T} y_k^t$. IPP with $c_{0k} = 1$ and $h_k^{it} = 0$, $\forall i, k, t$.

– IIP_3 model: $\min \sum_{t=1}^{T} \gamma^t$

We refer to the models that consider the interactions among ward pharmacies, by adding the symbol δ to their name.

In IIP_1 model, the objective is to minimize the inventory of the overall system constituted by hospital and ward pharmacies, while the objective of the IIP_2 model is to minimize the number of times that the hospital pharmacy supplies the wards. All the models were tested on scenario 5 of Table 3, with an initial inventory level and safety stock not equal to zero and a level service of 90% (Tab. 5).

In Table 10, we report the $\sum_{i=1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{T} I_k^{it}$ values for all the considered models. The inventory level values obtained by solving the IIP_1, IIP_2 and IIP_3 models are greater, of 0.082%, 2.25% and 0.06%, than those achieved with $IIP_{1\delta}$, $IIP_{2\delta}$ e $IIP_{3\delta}$ models. The possibility of moving drugs among wards reduce the stock values. Furthermore, $IIP_{2\delta}$ and $IIP_{2\delta}$ models exhibit the highest values of inventory, since minimizing the number of times when ward pharmacies are furnished by the hospital pharmacy, leads to higher inventory values for the ward pharmacies.

It is worth noting that for the $IIP_{1\delta}$, $IIP_{2\delta}$ and $IIP_{3\delta}$ models, the drugs sent from hospital to ward pharmacies are concentrated, in all the months, in one or two wards from which the drugs will be successively sent to all the

TABLE 9. $\sum_{i=1}^{n}\sum_{t=1}^{T} x_k^{it}$ values.

Wards	IIP_1	$IIP_{1\delta}$
1	3432	8600
2	2052	
3	1895	
4	831	
5	243	
6	415	

TABLE 10. $\sum_{i=1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{T} I_k^{it}$ values for the 3 models. Basic and δ versions.

$\sum_{i=1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{T} I_k^{it}$	January	February	March	April	May	June
IIP_1.5	264 313	255 538	282 439	279 098	294 938	302 082
IIP_2.5	289 834	1 097 995	1 142 256	1 375 807	1 574 200	1 489 945
IIP_3.5	290 020	336 737	398 909	309 559	323 079	396 023
$IIP_{1\delta}$.5	263 843	247 798	264 297	255 630	264 151	255 630
$IIP_{2\delta}$.5	289 834	393 462	390 387	299 328	403 587	370 823
$IIP_{3\delta}$.5	289 927	272 429	378 491	265 205	372 840	359 916

others. The situation is different for IIP_1, IIP_2 and IIP_3 models, where the drugs from the hospital pharmacy are distributed among the wards. To give an example, Table 9 reports the $\sum_{i=1}^{n}\sum_{t=1}^{T} x_k^{it}$ values for IIP_1 and $IIP_{1\delta}$ models in March. For the IIP_1 model and for all the wards, the flow between hospital and wards pharmacies is not equal to zero, thus showing that there is a movement of drugs from the hospital pharmacy to the ward pharmacies for almost all the drugs and in almost all the periods. On the other hand, for the $IIP_{1\delta}$ model there is a drug flow from the hospital pharmacy to only ward 1, and from this ward, the drugs will be sent to the other wards.

From Table 12, where the $\sum_{k=1}^{r}\sum_{t=1}^{T} y_k^t$ values are reported, it can be noticed that, as expected, the lowest number of movements, from hospital to ward pharmacies, is obtained by IIP_2 and $IIP_{2\delta}$ models.

In particular, for the $IIP_{2\delta}$ model the average number of movements is equal to 1.3 whereas in the case there is no flow between ward and central pharmacies the average number increase to 6.

Table 11 reports the $\sum_{t=1}^{T}\gamma^t$ values, *i.e.* the number of purchases that the hospital pharmacy makes every month to satisfy the ward requests. As expected, the smallest values are those obtained with IIP_2 e IIP_3 models, whereas the number of purchases is quite high in the cases the inventory is minimized. For models embedding variables $\delta_{k\bar{k}}^{it}$, the lowest value is obtained by $IIP_{2\delta}$, in February, when only one purchase is made.

4.1. Practical hints

To give some insights into the application of our models to the current situation, we compared the actual values of $\sum_{t=1}^{T}\gamma^t$ and $\sum_{k=1}^{r}\sum_{t=1}^{T} y_k^t$ for the ASP of Cosenza, reported in Table 13 with reference to Corigliano

TABLE 11. $\sum_{t=1}^{T} \gamma^t$ value for the 3 models. Basic and δ versions.

$\sum_{t=1}^{T} \gamma^t$	January	February	March	April	May	June
$IIP_1.5$	3	13	15	4	15	14
$IIP_2.5$	1	1	3	3	3	3
$IIP_3.5$	1	2	1	1	2	1
$IIP_{1\delta}.5$	3	12	15	4	15	16
$IIP_{2\delta}.5$	1	1	1	1	2	1
$IIP_{3\delta}.5$	1	2	1	1	2	1

TABLE 12. $\sum_{k=1}^{r} \sum_{t=1}^{T} y_k^t$ values for the 3 models. Basic and δ versions.

$\sum_{k=1}^{r} \sum_{t=1}^{T} y_k^t$	January	February	March	April	May	June
$IIP_1.5$	11	34	27	13	34	23
$IIP_2.5$	6	6	6	6	6	6
$IIP_3.5$	6	30	6	7	43	6
$IIP_{1\delta}.5$	3	12	15	4	16	18
$IIP_{2\delta}.5$	1	2	1	1	2	1
$IIP_{3\delta}.5$	3	19	4	17	7	5

hospital, with respect to those obtained by applying our models. In particular, we report in Figures 6 and 7 the values of the number of purchases made by the hospital pharmacy and the number of times the ward pharmacies are served, in the case of scenario 5 of Table 3.

From Figure 6, it is evident that $IIP_2.5$ and $IIP_3.5$ models always give a better solution in terms of number of monthly purchases made by the hospital pharmacy, while $IIP_1.5$ and $IIP_{1\delta}.5$ models give a solution quite similar to the one currently adopted.

Hence the use and the implementation of ad-hoc optimization models can lead to a consistent reduction in terms of number of purchases from the hospital pharmacy, thus obtaining an important reduction of inventory and ordering costs with respect to the current situation.

The results plotted in Figure 7 (that is the $\sum_{k=1}^{r} \sum_{t=1}^{T} y_k^t$ values) underline clearly that $IIP_{1\delta}.5$, $IIP_{2\delta}.5$, $IIP_{3\delta}.5$ models give a better solution and, in particular, the best performances are obtained by the $IIP_{2\delta}.5$ model, minimizing the number of movements between hospital pharmacy and wards. It is worth noting that in the case of Corigliano hospital, the actual average number of movements between hospital pharmacy and wards is 29.5 while in the case of $IIP_1.5$ model, the worst performing model, the average value of handling is 23.7. The application of our models leads to a substantial improvement with respect to the current situation also in terms of number of times a ward pharmacy is served by the hospital one. These improvements can lead to a significant reduction of the operational costs related to the smaller drugs flows between the ward and the hospital pharmacies.

We have also compared the strategy actually adopted in practice with those obtained by applying the MCIP models. The $\sum_{t=1}^{T} y_t$ and $\sum_{i=1}^{n} \sum_{t=1}^{T} y_{it}$ values for Corigliano hospital are reported in Table 14.

TABLE 13. $\sum_{t=1}^{T}\gamma^t$ and $\sum_{k=1}^{r}\sum_{t=1}^{T}y_k^t$ values for Corigliano hospital.

ASP orders	January	February	March	April	May	June
$\sum_{t=1}^{T}\gamma^t$	2	14	14	4	15	16
$\sum_{k=1}^{r}\sum_{t=1}^{T}y_k^t$	5	43	35	14	48	32

TABLE 14. $\sum_{t=1}^{T}y_t$ and $\sum_{i=1}^{n}\sum_{t=1}^{T}y_{it}$ values for Corigliano hospital.

ASP orders	January	February	March	April	May	June
$\sum_{t=1}^{T}y_t$	2	14	14	4	15	16
$\sum_{i=1}^{n}\sum_{t=1}^{T}y_{it}$	14	102	113	37	118	98

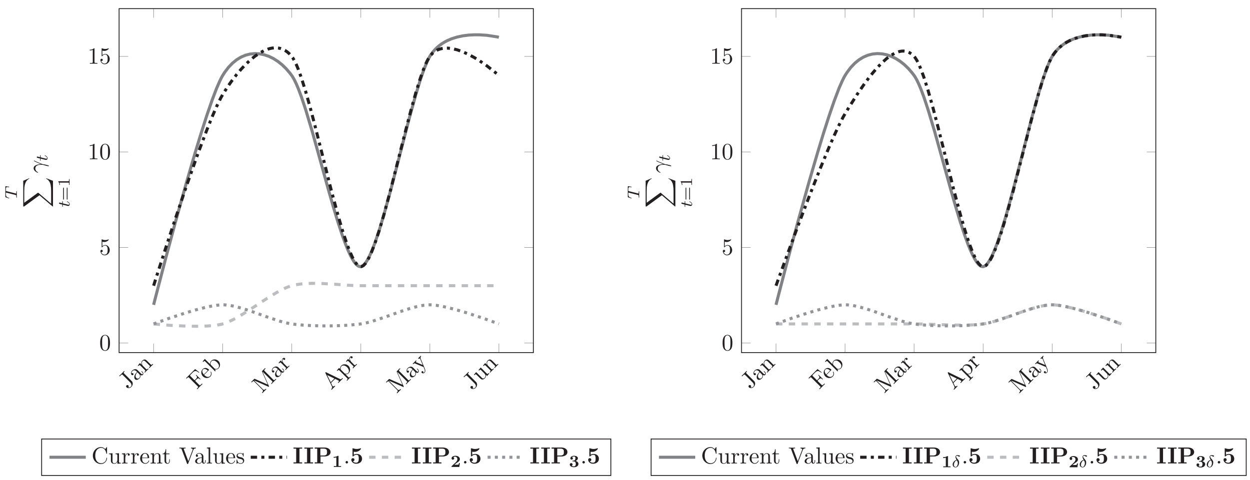

FIGURE 6. $\sum_{t=1}^{T}\gamma_t$ current and IIP model values in the case of scenario 5.

It is worth noting that the $\sum_{t=1}^{T}y_t$ values in Table 14 are equal to $\sum_{t=1}^{T}\gamma^t$ values in Table 13, since they are both the number of orders that the pharmacy makes every month by considering the MCIP and IPP models, respectively.

The lowest value for $\sum_{t=1}^{T}y_t$ (see Tab. 6) is obtained by the MCIP$_1$ model, providing in all the scenarios an average monthly value for the number of orders equal to 1.3, significantly lower than the real value of 10.33.

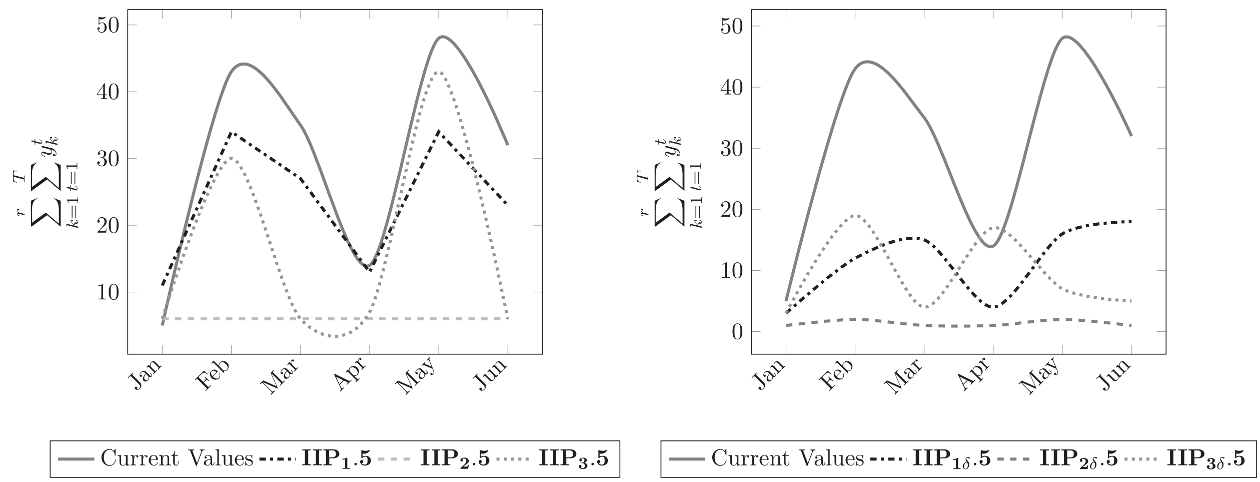

FIGURE 7. $\sum_{k=1}^{r}\sum_{t=1}^{T} y_k^t$ current and IIP model values in the case of scenario 5.

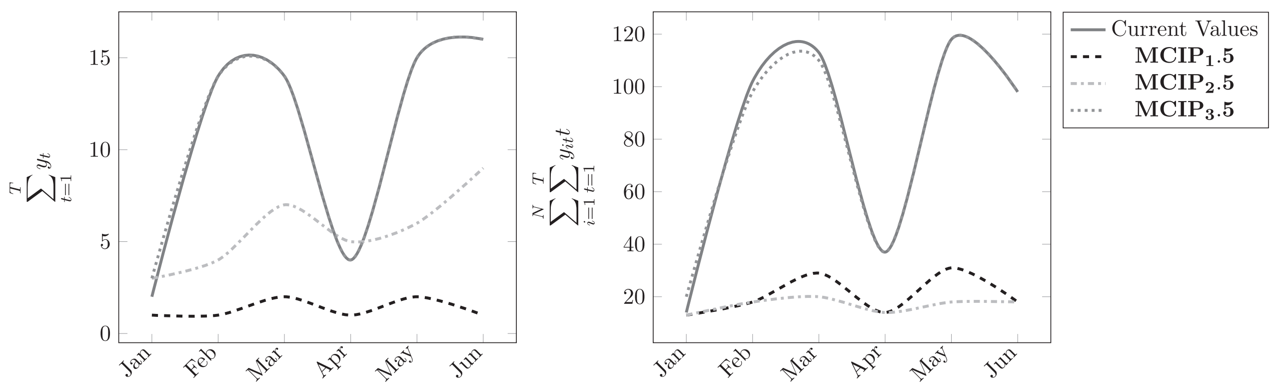

FIGURE 8. $\sum_{t=1}^{T} y_t$ and $\sum_{i=1}^{N}\sum_{t=1}^{T} y_{it}$ current and MCIP model values in the case of scenario 5.

Also for the $MCIP_2$ model, under all the scenarios, the obtained solution is better than the current one, whereas in the case of the $MCIP_3$ model, the minimization of the inventory increases the number of times the orders are made, generating a solution quite similar to that currently adopted for Corigliano hospital for all the considered scenarios.

With reference to $\sum_{i=1}^{n}\sum_{t=1}^{T} y_{it}$ values (see Tab. 7), we can observe a significant improvement for the $MCIP_1$ and $MCIP_2$ models (with an average number of orders, in the best scenario, of 6.33 and 15.16 respectively) with respect to the current situation where, on average, we have 80.33 orders for month. The worst situation is observed for the $MCIP_3$ model, which, due to its objective, gives a minimum improvement in case of scenarios $1, 4, 5$ and 6. Furthermore, in case of scenarios 2 and 3 the obtained solution is even worse than the current one with an average monthly value of 83.33 orders. The described models trend is well highlighted in Figure 8, where the case of scenario 5 of Table 3 is considered. The collected results confirm, also in this case, that the application of the proposed mathematical programming models leads to an important reduction of the orders made by the pharmacies that can be translated in an immediate reduction of the related costs.

Summing up, by comparing the actual situation with those obtained by applying our formulations, it is evident that a significant improvement in terms of cost can be obtained by embedding the optimization models in a decision support system. Moreover, the proposed formulations can be used to analyse various scenarios by considering different objectives and constraints. It is also important to underline that the obtained cost reduction and higher efficiency are crucial in the context under study where the inventory is managed with public resources and in a sector of primary importance.

5. Concluding remarks

The models discussed in the paper represent an important tool for implementing inventory control and management strategies in all the organizations. In the Italian settings, considered here, where the hospitals are public, the availability of tools supporting the decision makers, in a perspective of cost reduction, plays a fundamental role. Optimizing the flow of each drug within the wards of a given hospital is a crucial task especially when the products to manage are of vital importance. The models we proposed can be used to help hospital and ward pharmacies decide their inventory strategies. Indeed decision makers, operating in both hospital and ward pharmacies, can be supported in choosing the quantities of each drugs to be ordered in each period; the inventory of each drug at the end of each period and the best quantity of each drug to be transferred from the hospital to the ward pharmacies at each time period.

The information obtained by solving the models, with different parameters, can be also used at strategic level to evaluate the impact of possible variation in the current inventory strategies. The extensive computational study we have presented, on a set of real data provided by the ASP of Cosenza, has underlined the correctness of the proposed formulations and shown that the application of mathematical programming models can be fundamental to reduce health care inventory costs.

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