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A HYBRID FIX-AND-OPTIMIZE HEURISTIC FOR INTEGRATED INVENTORY-TRANSPORTATION PROBLEM IN A MULTI-REGION MULTI-FACILITY SUPPLY CHAIN

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Abstract. In this work, we study an integrated inventory-transportation problem in a supply chain consisting of region-bound warehouses located in different regions. The supply chain deals with multiple items that compete for storage space and transportation capacity with multi-modal transportation considering regional capacity constraint for each mode of transportation. The objective is to determine an optimal storage and transportation plan to satisfy the demand of all regions without shortages for known procurement plan for all items. The problem is formulated as a mixed integer programming (MIP) model for minimizing the total costs over a finite planning horizon. An MIP-based fix-and-optimize (F&O) heuristic with several decomposition schemes is proposed to solve the problem efficiently. The performance of the decomposition schemes is investigated against the structure of the sub-problems obtained. To enhance the performance, F&O is crossbred with two metaheuristics – genetic algorithm (GA) and iterated local search (ILS) separately, which lead to hybrid heuristic approach. Extensive numerical experiments are carried out to analyze the performance of the proposed solution methodology by randomly generating several problem instances built using data collected from the Indian Public Distribution System. The proposed solution approach is found to be computationally efficient and effective, and outperforming state of the art MIP solver Cplex for practical size problem instances. Also, the hybridization of F&O heuristic with GA and ILS boosts its performance although with a justified increase in the computational time.

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1. Introduction

In today's raging and competitive environment, supply chain structure is becoming more complex and challenging for decision making. An effective supply chain management system is the key for retaining the competitive edge for any organization. To achieve the same, management of inventory and transportation, the two major aspects representing tactical level decisions, in a supply chain need to be captivated. Many times, these two

Keywords. inventory-transportation problem, fix-and-optimize, genetic algorithm, iterated local search, public distribution system.

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aspects are practiced independently which may result in suboptimal performance of a supply chain. Therefore, the need for close coordination between inventory control and transportation planning is highlighted in the literature [35].

Along with the traditional supply chain, the challenge of competitive performance through efficient management of inventory and transportation is equally applicable to a non-profit supply chain such as public distribution system (PDS). PDS is practiced in most of the countries for distribution of essential commodities, particularly foodgrains, at subsidized prices to the needy people of society to ensure the food security. In the Indian PDS, foodgrains, mainly wheat and rice, are procured from farmers and distributed to the beneficiaries at subsidized prices to ensure availability and affordability of foodgrains. Serving to more than 813.5 million beneficiaries, the Indian PDS is the largest in the world. The supply chain of the Indian PDS has some special characteristics. It consists of multiple storage facilities (warehouses) located across the country to store the foodgrains to meet the demand of different regions. Each region has a procurement and demand strategy that is aggregated over all the warehouses located in that region. Consequently, it is necessary to decide which warehouse to store the procured items and when to withdraw the items to meet the demand of each region. Because of the limited available storage capacity, the major challenge includes the storage of a large amount of foodgrains in different warehouses. Also, due to the mismatch in procurement, demand, and the storage capacity across different regions, the other challenge is the transportation of foodgrains from regions with a surplus to regions with a deficit over a notably large geographic area by selecting an optimal mode of transportation. At present, these aspects are practiced in a decoupled fashion, which results in a higher cost. Therefore, in the present study, we propose an integrated inventory-transportation problem to jointly optimize the storage and transportation related costs over a finite planning horizon. This problem of storage and inter-state transportation of foodgrains is first studied by Tanksale and Jha [56] for the Indian public distribution system. Later, Tanksale and Jha [57] envisioned a multi-region multi-facility supply chain structure and analyzed an integrated inventory-transportation problem over a finite planning horizon. However, they have not considered the detailed characteristics of transportation of items. Therefore, in this research, we aim to generalize and extend the problem presented in [58] using multiple modes of transportation under regional capacity constraint for each mode of transportation. A capacitated multi-item inventory-transportation problem in a multi-period setting is formulated as mixed-integer programming (MIP) model to determine an optimal storage and transportation plan for all items by minimizing the total transportation and inventory related costs. Further, to deal with the increased complexity of the problem in the present study, we implement MIP-based fix-and-optimize (F&O) heuristic originally proposed by Helber and Sahling [24]. The idea behind F&O heuristic is to decompose the original problem into sub-problems such that each sub-problem has a fewer number of free binary variables to be optimized while keeping other binary variables fixed, and iteratively solving the series of sub-problems that will provide the solution to the original problem. This approach has been found efficient and effective in case of many complex production planning, lot-sizing, and scheduling problems. In this work, we demonstrate the application of F&O heuristic to the problem under consideration. We propose several decomposition schemes obtained from the structure of the problem and an efficient greedy heuristic to construct an initial feasible solution. We demonstrated that the performance of a decomposition scheme depends on the sub-problem size as well as the number of sub-problems formed. We further enhance F&O heuristic by hybridizing it with two metaheuristics – genetic algorithm (GA) and iterated local search (ILS) separately, as diversification tools.

The contributions of the present study to the body of research are multi-fold, as illustrated below.

- We have presented a capacitated inventory-transportation problem in a multi-region multi-facility supply chain over a finite planning horizon, which is stemmed from the Indian PDS by fully characterizing the transportation of items in the form of multi-modal transportation and region-wise transportation capacity constraints. The problem can be seen as capacitated multi-period inventory transportation problem. In this way, we extend and generalize the previous works of Tanksale and Jha [56, 57].

– An application of MIP-based F&O heuristic to solve the capacitated multi-period inventory transportation problem is presented in this paper.
– An analysis of several decomposition schemes used in the F&O heuristic is demonstrated and their performance is evaluated with respect to the decomposition structure. To the best of our knowledge, this is the first time such an analysis is being carried out for F&O heuristic.
– Two hybrid heuristics combining GA and ILS independently with the F&O heuristic are proposed, where F&O is used as a local search heuristic in the architecture of GA and ILS. The novel approach has been used in the amalgamation of the matheuristic and metaheuristics.

The organization of the rest of the paper is as follows. Review of the relevant literature is presented in Section 2. Section 3 includes the problem description and mathematical model. The proposed solution approach is demonstrated in Section 4. The results of the computational experiments are presented in Section 5. Finally, Section 6 concludes the paper.

2. Literature review

In this section, the literature related to multi-period inventory transportation problems and F&O heuristic is presented in the following sub-sections.

2.1. Multi-period inventory-transportation problem

Inventory-transportation problem (ITP) arises in a situation when transportation cost is significant and proportional to the distance covered/load carried, and inventory is carried for a relatively longer period [35]. The literature dealing with the integration of inventory and transportation decisions in supply chain is replete. A recent comprehensive taxonomic review in this area can be found in Sancak and Salman [52] and Ali and O'Connor [1]. Also, the consideration of freight cost in the recent literature along with inventory aspect can be found in Bravo and Vidal [7]. Here, we do not delve into the literature dealing with the inventory transportation problem in an infinite planning horizon, which calls for an EOQ-based approach to address the trade-off between inventory and transportation costs [6]. Considering the relevance of this study, we confine the discussion on the literature dealing with multi-period ITP. Further, the contributions related to multi-period ITP can be classified into two broad categories: dynamic lot-sizing based ITP, and ITP dealing with joint minimization of inventory and transportation costs.

The well-known dynamic lot-sizing problem has been studied extensively in production and inventory management literature since the seminal work of Wagner and Whitin [64]. Integration of transportation aspect in the classical dynamic lot-sizing problem for shipment of multiple items has attracted significant interest among researchers for optimization of supply chain operations. Several studies have extended the classical dynamic lot-sizing problem considering transportation lot size dependent fixed ordering cost.

An early study of Lippman [43] considered a single-item lot-sizing model with container-based transportation cost and presented properties for optimality, which became a cornerstone for several posterior studies in this domain. Lee *et al.* [38], and Jin and Muriel [33] extended the work of [43] for a two-echelon, and a multi-echelon supply chain, respectively. In an early study, Hwang and Sohn [27] addressed a dynamic lot-sizing problem for deteriorating item considering the order lot-sizing and multi-modal transportation simultaneously. Anily and Tzur [2, 3] studied an integrated multi-item lot-sizing problem considering shipment using a fleet of vehicles. Several researchers have focused on inbound and outbound logistics in lot-sizing problems. For example, Lee *et al.* [38] studied the problem of determining the lot size and outbound shipment policy in a two-stage supply chain consisting of a warehouse and a distribution center. The problem was further enhanced by Jaruphongsa *et al.* [30,31] to consider the economy of scale in transportation and multiple modes of transportation. Ertogral [15] studied a single-stage supply chain for multiple items to determine the optimal order size and inbound shipment policy and proposed a Lagrangean-based approach. Recently, Venkatachalam and Narayanan [63] proposed a metaheuristic approach to efficiently solve the same problem as of Ertogral [15]. Sancak and Salman [52] considered a supplier–manufacturer supply chain to determine the optimal ordering and inbound shipment policy.

The case of a company which has a transportation capacity reservation contract with a 3PL company can be found in van Norden and van de Velde [62]. They analyzed the transportation lot-sizing under piecewise linearly increasing freight rate. Several other authors [15, 49, 63] also developed dynamic lot-sizing models assuming piecewise transportation cost. Li *et al.* [42] studied a transportation lot-sizing problem considering the freight cost proportional to a full truckload. Lee *et al.* [39] analyzed a multi-item dynamic lot-sizing problem considering the ordering cost proportional to the number of containers. They proposed a heuristic procedure based on the properties of the optimal solution of the problem. Later, Kim and Lee [36] proposed several metaheuristics approaches to the problem similar to [39]. Li and Chen [41] followed a similar approach to the problem where the transportation cost is proportional to the number of carriers/containers used. In general, dynamic lot-sizing based inventory-transportation problems in supply chains have been explored for different transportation cost structures. Several authors succeeded in developing exact solution approaches [33, 38, 43]. A common approach followed by these authors is to explore structural properties for optimality to devise solution approaches. Some researchers [2, 30] also used dynamic programming to obtain an exact solution. Owing to the complexity of the problem, several authors proposed heuristic [39, 52] and metaheuristics [36, 63] to solve the problems. In all these studies, the limitation on storage capacity has not been considered. Moreover, a little attention is paid on multi-modal transportation.

Apart from dynamic lot-sizing problems, there are some other studies which also deal with joint minimization of inventory and transportation costs in multi-period supply chains. The problem of multi-period inventory transportation planning is studied by Kim and Kim [35], and Kang and Kim [34]. In a similar context, Zhao *et al.* [68] presented the case of a company delivering coal to its four subsidiaries through a central warehouse using different modes of transportation. Lee at al. [40] presented an inventory-transportation problem in a generic multi-echelon supply chain. Jawahar and Balaji [32] studied a fixed-charge transportation problem with inventory decisions in a multi-supplier multi-retailer supply chain with backorders at the retailers. A trade-off between the fixed transportation and inventory carrying costs in a multi-period logistics system is analyzed by Ali and O'conor [1], and they emphasized the necessity of a heuristic solution approach. In the context of foodgrain supply chain, Asgari *et al.* [4] presented a MIP model to determine the storage and inter-province transportation of wheat to minimize the total cost and proposed a genetic algorithm (GA) based solution approach. Pourghannnad *et al.* [48] analyzed the trade-off between inventory and transportation cost considering the time value of money in a vendor–buyer two-stage divergent supply chain. In this domain, there are some case studies that deal with a large pulp producer [8], and a soft-drink distribution company [22]. In general, a linear transportation cost is considered by most of the researchers and solution methodologies ranging from Lagrangean-based heuristics to metaheuristics have been attempted as a solution approach. However, capacity constraints on storage facility and multi-modal transportation of multiple items have not been sufficiently addressed in the literature.

Most of the studies in the literature dealing with multi-period ITP consider an arborescent supply chain network (suppliers – plants – warehouses – distribution centers). However, Tanksale and Jha [56, 57] studied a multi-region multi-facility supply chain recently. Further, the existing studies dealing with the multi-period ITP do not sufficiently address the complicacies arising due to multiple items, storage and transportation capacity constraints and multi-modal transportation simultaneously. The present work take into account all these factors together to extend the work of Tanksale and Jha [56, 57].

2.2. Literature on Fix-and-optimize heuristic

In general, several important supply chain planning problems are modelled as MIP problem. However, the state of art solvers fail to produce optimal or high-quality solutions for large-sized problem instances within an acceptable computational time. To overcome this limitation, MIP-based heuristics have emerged as promising solution methodologies [54]. The recent past has witnessed an ever-growing interest in MIP-based fix-and-optimize (F&O) heuristic. The root of F&O can be traced back due to the work of Pochet and Wolsey [47] who described an improvement heuristic named as "exchange heuristic" to solve a multi-level capacitated lot-sizing problem (CLSP). Later, F&O heuristic was formally introduced by Helber and Sahling [24] to solve a multi-level CLSP. The idea behind F&O heuristic is to decompose original problem into sub-problems such that each sub-problem has a fewer number of free binary variables to be optimized while keeping other binary variables fixed, and iteratively solving the series of sub-problems that will provide the solution to the original problem. F&O has a simple and transparent architecture and it has been found efficient and effective for many complex production planning, lot-sizing, and scheduling problems in the literature. An overview of the relevant literature in the domain of fix-and-optimize according to application area and solution approach is presented in Table 1.

The pioneering study of Helber and Sahling [24] was to solve a multi-level CLSP with positive lead times using F&O approach. Inspired from this work, different problems such as stochastic CLSP [25], cooperative lot-sizing problem [14], and CLSP with setup carryover [9, 19, 20] have been solved using F&O heuristic. Some other applications of F&O heuristic to solve lot-sizing problems include CLSP with backlogging [59, 60], CLSP with sequence dependent setup [37, 58], and multi-level CLSP [17]. F&O heuristic has also been established as a competitive solution technique for integrated lot-sizing and scheduling problem [5, 21, 29, 53, 55, 67].

Apart from lot-sizing, production planning, and scheduling problems, there are other areas in which the application of F&O heuristic can be found. For example, Ghaderi and Jabalameli [18] proposed a hybrid F&O and simulated annealing heuristic to solve a facility location-network design problem. Sel and Bilgen [54] addressed a production-distribution problem for a European soft drink manufacturer. A similar application can also be found in the work of Wie *et al.* [65]. Dorneles *et al.* [13] undertook a high school time-tabling problem in Brazilian schools and proposed F&O heuristic enriched with variable neighborhood search. Wolter and Helber [66] applied F&O heuristic to a dynamic production and maintenance planning problem. In recent years, F&O heuristic is becoming popular to solve the problems of variety of domains such as vendor selection [50], facility location-transportation problem [46], patient admission scheduling problem [61], network function virtualization [45], unit commitment problem [16] and gantry crane scheduling problem for intermodal transportation [23]. These studies have enriched the body of knowledge for F&O heuristic with distinct methods for obtaining an initial feasible solution, parameter tuning and integrating F&O with other heuristic or metaheuristic. However, to the best of our knowledge, there is no study in the literature that applies F&O heuristic to an integrated inventory-transportation problem in a supply chain.

One can note that F&O is an improvement heuristic and needs a starting solution. Table 1 indicates that the use of a greedy heuristic, other matheuristics (*e.g.* relax-and-fix) or a metaheuristics (*e.g.* genetic algorithm) to obtain an initial solution is a popular approach among researchers. Further, it is observed that in many cases the solution obtained by F&O heuristic has the possibility of getting trapped into a local optima [9]. Unfortunately, the basic F&O search procedure lacks a search space diversification mechanism. Therefore, with the zest of exploring more promising regions in the search space, F&O heuristic is usually blended with other metaheuristics. For the purpose, variable neighborhood search-based strategy is commonly adopted by many researchers [9, 13, 53]. This leads to a hybrid solution approach.

In the present work, we presented a novel approach for hybridization of F&O with two metaheuristics – genetic algorithm (GA) and iterated local search (ILS), separately. In the hybrid F&O- GA approach, we obtain initial population for GA by iterating F&O algorithm, reproduction of these solutions, and a random F&O routine where a few binary variables are randomly selected for re-optimization. Following the reproduction, crossover and mutation operations, GA then hopefully produces a better solution. In this way, we embed GA into the F&O routine as a diversification tool. We also propose a hybrid ILS-F&O heuristic. For the purpose, we first obtain a solution using F&O heuristic with single iteration and apply ILS with a random F&O routine and perturbation of the local optima in each iteration. To the best of our knowledge, this is the first attempt to hybridize the ILS and F&O in the literature. Further, in this work we have proposed and analyzed several decomposition schemes and their effects on the heuristic performance. Thus, with the novel application area of F&O, performance analysis of decomposition schemes, and two hybrid heuristics, we contribute to fill the research gap.

TABLE 1. Relevant literature on F&O heuristic.

Application area	Author(s)	Solution methodology
Production planning and dynamic lot sizing	Sahling *et al.* [51]	F&O heuristic
	Helber and Sahling [24]	F&O heuristic
	Drechsel and Kimms [14]	F&O heuristic to obtain upper bound
	Goren *et al.* [20]	F&O embedded into GA procedure
	Lang and Shen [37]	Relax & Fix (R&F) with F&O heuristic
	Helber *et al.* [25]	F&O heuristic
	Toledo *et al.* [59]	F&O embedded into multi-population GA
	Chen [9]	F&O heuristic with Variable neighborhood search (VNS) strategy
	Goren and Tunali [19]	GA to obtain initial solution followed by F&O heuristic
	Tempelmeier and Copil [58]	F&O and Fix & Relax (F&R) heuristic
	Toledo *et al.* [60]	Integrated R&F and F&O heuristic
	Furlan and Santos [17]	Hybrid bee and fix-and-optimize algorithm
Lot-sizing and scheduling	James and Almada-Lobo [29]	R&F for initial solution, F&O heuristic along with iterative neighborhood search procedure
	Guimarães *et al.* [21]	Relax-price-fix and Fix-price-optimize iterative heuristics
	Seeanner *et al.* [53]	F&O hybridized with meta-heuristic Variable Neighborhood Decomposition Search (VNDS)
	Stadtler and Sahling [55]	F&O heuristic
	Xiao *et al.* [67]	R&F for initial solution, F&O heuristic along with iterative neighborhood search procedure
	Baldo *et al.* [5]	R&F as a construction heuristic followed by F&O as an improvement heuristic
Health-care facility location problem	Ghaderi and Jabalameli [18]	Greedy heuristic, F&O heuristic and Simulated annealing
Maintenance planning	Wolter and Helber [66]	F&O heuristic
High school time-tabling problem	Dorneles *et al.* [13]	F&O with VNDS strategy
Production and distribution planning	Sel and Bilgen [54], Wei *et al.* [65]	F&O heuristic
Facility layout planning	Helber *et al.* [26]	F&O heuristic with improvement procedure
Vendor selection	Sahling *et al.* [50]	Integrated R&F and F&O heuristic
Network function virtualization	Luizelli *et al.* [45]	F&O heuristic with VNS
Facility location-transportation	Morenoa *et al.* [46]	Integrated R&F and F&O heuristic
Patient Admission Scheduling	Turhan and Bilgen [61]	Integrated R&F and F&O heuristic
Unit commitment problem	Franz *et al.* [16]	F&O heuristic
Gantry crane scheduling	Guo *et al.* [23]	F&O heuristic

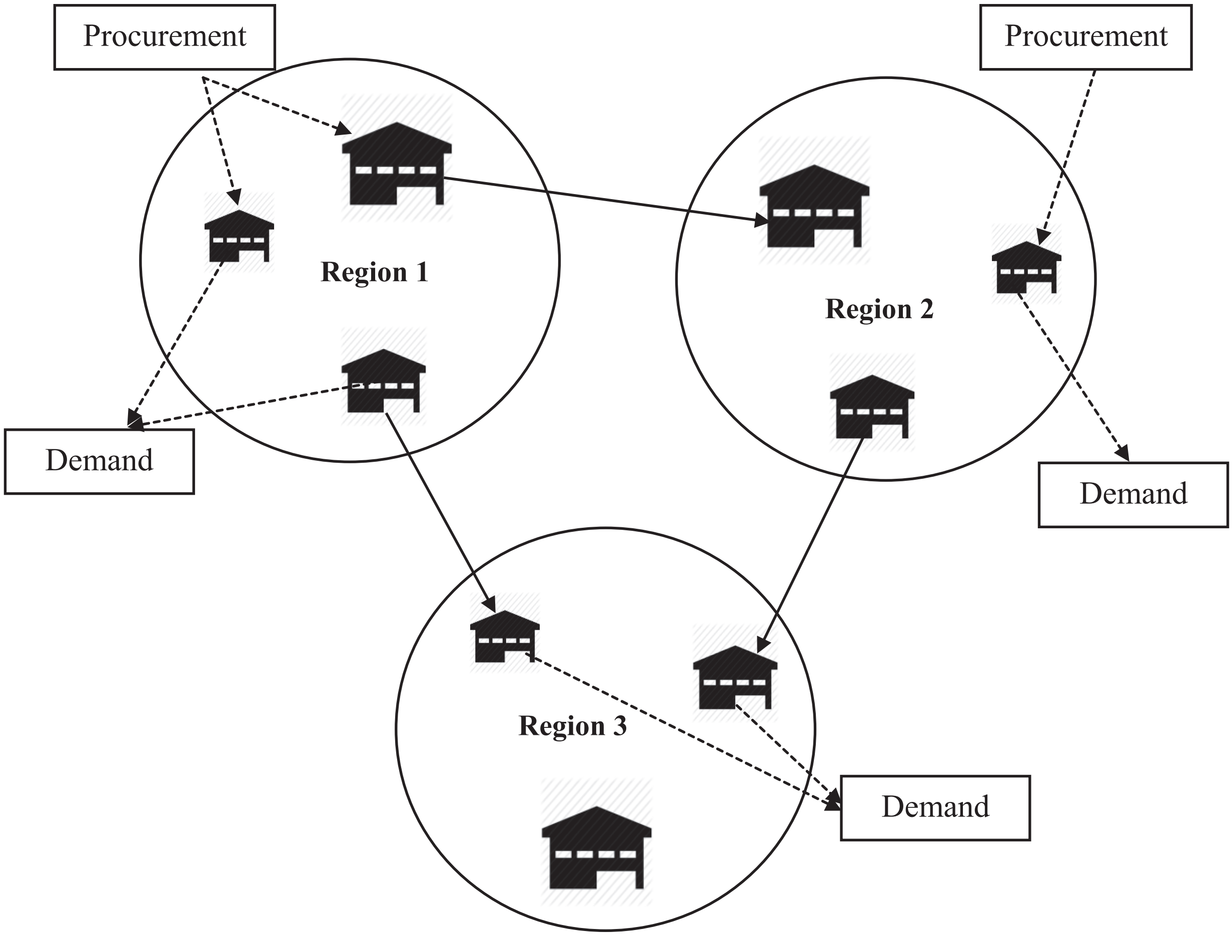

FIGURE 1. Schematic representation of a multi-region multi-facility supply chain.

3. Model formulation

The mathematical model presented here is based on the work of Tanksale and Jha [57] who proposed a multi-period inventory-transportation problem in a multi-region multi-facility supply chain. In this work, we have extended their model considering multiple modes of transportation and region wise transportation capacity constraints, as given below.

The supply chain under consideration consists of multiple warehouses located over a large geographic area which is divided into multiple regions, as shown in Figure 1. The warehouses in each region serve as a common storage facility for multiple items and are distinct in terms of storage capacity and holding cost. Each region has its own demand and procurement strategy that is aggregated over all the warehouses located in that region. An item procured in a region during a period is to be allocated for storage during the same period in the warehouses of that region. Also, the demand for each item in all regions is met completely without shortage during each period from the stock of the item available in the warehouses of the respective regions. In case of insufficient stock of an item in a region during a period, the item is transported from the warehouses of the regions with a surplus. The shipment of the items can be realized through different modes of transportation. Also, each mode of transportation is having a limited capacity that is assumed to be aggregated over all warehouses in a region.

The objective is to determine an optimal plan for storage of each item in the warehouses of each region and plan for movement of the items between the warehouses to minimize the total inventory holding and transportation costs over a finite planning horizon.

The problem is formulated as an MIP model, for which most of the notation is adapted from Tanksale and Jha [57]. The other assumptions required to formulate the model are listed below.

Assumptions

(i) Procurement and demand of all items are deterministic and known.
(ii) Total procurement of each item is sufficient to meet the demand of all regions in each period.
(iii) The storage capacity in all procuring regions is assumed to be sufficient to stock the procured items, *i.e.* $\sum_{i\in W_u} \text{cap}_i \geq \sum_{f\in F} P_u^{ft}, \forall u \in U, t \in T$. However, the holding cost at the warehouses in procuring regions is assumed to be larger than those in the consuming regions.
(iv) Inventory holding cost is charged on the inventory of the item carried at the end of each period.
(v) The cost of allocation of the procured item in a region to a warehouse of that region along with other handling charges contributes towards a fixed cost, which has been considered as a setup cost due to procurement for a warehouse.
(vi) The transportation cost between any two warehouses consists of a fixed cost and a variable cost incurred proportional to the quantity of the item to be transported. The fixed cost of transportation consists of the components such as handling charges, loading charges, and unloading charges, which has been referred as a setup cost due to transportation for a warehouse.
(vii) The item transported from a warehouse to any other warehouse using any mode of transportation is delivered in the same time period.

Mathematical model

Sets and indices

F	Set of items, indexed by f
M	Set of transportation mode, indexed by m
T	Set of time periods, indexed by t
U	Set of regions, indexed by u
W	Set of warehouses, indexed by i
W_u	Set of warehouses located in region $u, W_u \subseteq W$,

Parameters

A_m	Fixed cost of using transportation mode m
cap_i	Storage capacity of warehouse i
C_{ijm}	Transportation cost per unit between warehouses i and j using transportation mode m
D_u^{ft}	Demand of item f in region $u$during period t
H_i^f	Holding cost per unit of item f per period at warehouse i
M_1,M_2	Large positive numbers
N_{um}^t	Total capacity of transportation mode m available at region u during period t
P_u^{ft}	Procurement quantity of item f in region u during period t
S_i	Setup cost incurred if the procured item is allocated for storage in warehouse i,

Decision variables

I_i^{ft}	Inventory of item f in warehouse i at the end of period t
q_{ijm}^{ft}	Quantity of item f transported from warehouses i to j by transportation mode m during period t
λ_{iu}^{ft}	Quantity of item f to be withdrawn from warehouse $i \in W_u$ to fulfill the demand of region u for item f during period t, $\lambda_{ju}^{ft} = 0,$ for $j \notin W_u$
π_{iu}^{ft}	Quantity of procured item f in region u to be allocated for storage to warehouse $i \in W_u$ during period $t, \pi_{ju}^{ft} = 0,$ for $j \notin W_u$

$$x_{iu}^{ft} = \begin{cases} 1, \text{ If} \pi_{\text{iu}}^{\text{ft}} > 0, \\ 0, \text{ Otherwise.} \end{cases}$$

$$y_{ijm}^{ft} = \begin{cases} 1, \text{ If} \text{Q}_{\text{ijm}}^{\text{ft}} > 0, \\ 0, \text{ Otherwise.} \end{cases}$$

Problem: **[P]**
Minimize

$$\sum_{t\in T}\sum_{f\in F}\left[\sum_{i\in W} H_i^f I_i^{ft} + \sum_{u\in U}\sum_{i\in W_u} S_i x_{iu}^{ft} + \sum_{i\in W}\sum_{j\in W}\sum_{m\in M}\left(A_m y_{ijm}^{ft} + C_{ijm} q_{ijm}^{ft}\right)\right] \tag{3.1}$$

subject to

$$I_i^{ft} = I_i^{f,(t-1)} + \pi_{iu}^{ft} + \sum_{j\in W\setminus\{i\}}\sum_{m\in M} q_{jim}^{ft} - \sum_{j\in W\setminus\{i\}}\sum_{m\in M} q_{ijm}^{ft} - \lambda_{iu}^{ft}, \quad \forall f\in F, u\in U, i\in W_u, t\in T \tag{3.2}$$

$$\sum_{f\in F} I_i^{f,(t-1)} + \sum_{f\in F}\pi_{iu}^{ft} + \sum_{f\in F}\sum_{j\in W\setminus\{i\}}\sum_{m\in M} q_{jim}^{ft} \le \text{cap}_i, \quad \forall u\in U, i\in W_u, t\in T \tag{3.3}$$

$$\sum_{f\in F}\sum_{i\in W_u}\sum_{j\in W\setminus\{i\}} q_{ijm}^{ft} \le N_{um}^t, \quad \forall m\in M, u\in U, t\in T \tag{3.4}$$

$$\sum_{i\in W_u}\pi_{iu}^{ft} = P_u^{ft}, \quad \forall f\in F, u\in U, t\in T \tag{3.5}$$

$$\sum_{i\in W_u}\lambda_{iu}^{ft} = D_u^{ft}, \quad \forall f\in F, u\in U, t\in T \tag{3.6}$$

$$\pi_{iu}^{ft} \le \text{M}_1 x_{iu}^{ft}, \quad \forall f\in F, u\in U, i\in W_u, t\in T \tag{3.7}$$

$$q_{ijm}^{ft} \le \text{M}_2 y_{ijm}^{ft}, \quad \forall f\in F, m\in M, u\in U, i\in W_u, j\in W\setminus\{i\}, t\in T \tag{3.8}$$

$$x_{iu}^{ft} \in \{0,1\}, \quad \forall f\in F, u\in U, i\in W_u, t\in T \tag{3.9}$$

$$y_{ijm}^{ft} \in \{0,1\}, \quad \forall f\in F, m\in M, i\in W, j\in W\setminus\{i\}, t\in T \tag{3.10}$$

$$I_i^{ft}, q_{ijm}^{ft}, \lambda_{iu}^{ft}, \pi_{iu}^{ft} \ge 0, \quad \forall f\in F, u\in U, i\in W_u, j\in W\setminus\{i\}, m\in M, t\in T. \tag{3.11}$$

The first term in objective function (3.1) is the inventory holding cost, the second term represents setup cost incurred due to allocation of a procured item for storage in a warehouse, and the third term is the transportation cost (fixed and variable) incurred if the item is moved to a warehouse from any other warehouse. Constraint (3.2) is the inventory balance equation, where the inventory of each item at the end of each period in a warehouse is found based on the allocation of the procured item for storage, withdrawal of the item to satisfy the demand, inflow from the other warehouses and outflow to other warehouses in that period. Constraint (3.3) is due to the finite storage capacity of each warehouse, which ensures that the total quantity of all the items moved in from other warehouses and the quantity of all the procured items to be stored should not exceed the available storage capacity of the warehouses in any point of time during each period. Constraint (3.4) imposes a limit on the aggregate quantity of all items transported from all the warehouses in a region by each mode of transportation during a period. Constraint (3.5) enforces that the procured quantity of each item in each region must be stored in the warehouses located in that region during the same time period. Constraint (3.6) ensures that the demand of all regions for each item during each period must be satisfied by withdrawing the item from the warehouses of the respective regions. Constraint (3.7) enforces that the quantity of each procured item to be stored in a warehouse is zero if there is no setup cost. Constraint (3.8) imposes a similar requirement on setup cost if an item is transported between warehouses by any mode of transportation. In constraints (3.7) and (3.8), M1 and M2 are large positive numbers. To have tighter constraints, we set the value of M1 and M2 as the maximum quantity of the item available for procurement in a region and mode-wise transportation capacity available in the region, respectively. Finally, constraints (3.9)–(3.11) define the nature of decision variables.

4. Solution methodology

The multi-period ITP has been considered to be NP-hard, and several authors have emphasized the need of heuristic solution approach [1, 36, 39, 52, 63]. Practically, problem [**P**] can be solved by any state of the art MIP solvers. However, these solvers cannot prove the optimality or take prohibitively large computational time for large size problem instances. Therefore, to tackle the computational intricacy of the model, a specialized solution approach is necessary. It can be seen that the proposed model is characterized by two sets of binary variables, x_{iu}^{ft} and y_{ijm}^{ft}, which stack most of the computational burden. Hence, if the value of these binary variables is obtained by some means, problem [P] can be solved efficiently. This is the motivation to apply F&O heuristic for the considered problem. In what follows, we demonstrate the architecture of F&O heuristic and two hybrid heuristics with the fusion of GA-F&O heuristic and ILS-F&O heuristic.

4.1. Fix-and-optimize heuristic

The concept of F&O heuristic is to decompose the original problem into sub-problems by controlling the number of binary variables to be optimized in each sub-problem. In each sub-problem, a fewer number of free binary variables along with all continuous decision variables is considered for optimization. Then, sub-problems are solved in an iterative fashion using any MIP solver. Because the value of most of the binary variable is fixed in each sub-problem to those obtained in the previous iteration, solving the series of sub-problems is much easier than the original problem [24]. While implementing F&O heuristic, the key decision is to select a decomposition scheme, *i.e.* selection pattern of the binary variables to be kept free or fixed in each sub-problem, which plays a pivotal role for the quality of the solution. For the problem at hand, we tried several aspects of the problem for obtaining the decomposition schemes. Among all, period, region, and item decompositions are found practically useful. Out of these, the period decomposition is illustrated below in details.

In the period decomposition, in each iteration, the binary variables related to a certain number of time periods are kept free to be optimized while the remaining binary variables are kept fixed. The idea is to optimize the setup decisions in the stipulated time periods to determine the resulting inventory and transportation plan. Starting with period 1 of the planning horizon we proceed in a fashion of rolling horizon. Let $\bar{T}$ be the set of time periods in the planning horizon for which the binary variables x_{iu}^{t} and y_{ijm}^{t} are kept fixed in the current sub-problem. Also, let $\bar{X}$ and $\bar{Y}$ represent set of binary variables x_{iu}^{t} and y_{ijm}^{t} which are kept fixed at $\bar{x}_{iu}^{t}$ and $\bar{y}_{ijm}^{t}$ in the current sub-problem, respectively. Then, the sub-problem can be written as,

Problem [**P**] along with the following additional constraints

$$x_{iu}^{ft} = \bar{x}_{iu}^{ft}, \quad \forall f \in F, u \in U, i \in W_u, t \in \bar{T} \tag{4.1}$$

$$y_{ijm}^{ft} = \bar{y}_{ijm}^{ft}, \quad \forall f \in F, m \in M, i \in W, j \in W, i \neq j, t \in \bar{T} \tag{4.2}$$

Constraints (4.1) and (4.2) represent the sets of binary variables x_{iu}^{ft} and y_{ijm}^{ft} which are kept fixed in the current sub-problem at $\bar{x}_{iu}^{ft}$ and $\bar{y}_{ijm}^{ft}$, respectively, obtained from the previous iteration. Due to these constraints, if $\bar{T} = T \backslash \{t\}, \forall t = 1, 2, \ldots, |T|$, each sub-problem has $|W_u| \times |U| \times |F|$ number of binary variables x_{iu}^{ft} and $|M| \times |W| \times |W| \times |F|$ number of binary variables y_{ijm}^{ft}, which are kept free to be optimized, and the period decomposition scheme results into $|T|$ number of such sub-problems. The working principle of F&O heuristic with period decomposition scheme is presented in Figure 2.

Similarly, the region and the item decompositions can be obtained by keeping the binary variables related to a certain number of regions and a certain number of items, respectively, free to be optimized. Further, a few

TABLE 2. Additional notation related to various decomposition schemes.

Decomposition sequence*	Constraints to be included in problem [P] to obtain a sub-problem	Number of free binary variables x_{iu}^{ft}	Number of free binary variables y_{ijm}^{ft}	Number of sub-problems
T	$x_{iu}^{ft} = \bar{x}_{iu}^{ft}$, $\forall f \in F, u \in U, i \in W_u, t \in \bar{T}$ $y_{ijm}^{ft} = \bar{y}_{ijm}^{ft}$, $\forall f \in F, m \in M, i \in W, j \in W, i \neq j, t \in \bar{T}$	$\|W_u\| \times \|U\| \times \|F\|$	$\|M\| \times \|W\| \times \|W\| \times \|F\|$	$\|T\|$
R	$x_{iu}^{ft} = \bar{x}_{iu}^{ft}$, $\forall f \in F, u \in \bar{U}, i \in W_u, t \in T$ $y_{ijm}^{ft} = \bar{y}_{ijm}^{ft}$, $\forall f \in F, m \in M, u \in \bar{U}, v \in U, i \in W_u, j \in W_v, i \neq j, t \in T$	$\|F\| \times \|W_u\| \times \|T\|$	$\|F\| \times \|M\| \times \|W_u\| \times \|W\| \times \|T\|$	$\|U\|$
I	$x_{iu}^{ft} = \bar{x}_{iu}^{ft}$, $\forall f \in \bar{F}, u \in U, i \in W_u, t \in T$ $y_{ijm}^{ft} = \bar{y}_{ijm}^{ft}$, $\forall f \in \bar{F}, m \in M, u \in U, v \in U, i \in W_u, j \in W_v, i \neq j, t \in T$	$\|W_u\| \times \|U\| \times \|T\|$	$\|M\| \times \|W\| \times \|W\| \times \|T\|$	$\|F\|$
T-R R-T	$x_{iu}^{ft} = \bar{x}_{iu}^{ft}$, $\forall f \in F, u \in \bar{U}, i \in W_u, t \in \bar{T}$ $y_{ijm}^{ft} = \bar{y}_{ijm}^{ft}$, $\forall f \in F, m \in M, u \in \bar{U}, i \in W_u, j \in W, i \neq j, t \in \bar{T}$	$\|W_u\| \times \|F\|$	$\|F\| \times \|M\| \times \|W_u\| \times \|W\|$	$\|T\| \times \|U\|$
T-I I-T	$x_{iu}^{ft} = \bar{x}_{iu}^{ft}$, $\forall f \in \bar{F}, i \in W_u, u \in U, t \in \bar{T}$ $y_{ijm}^{ft} = \bar{y}_{ijm}^{ft}$, $\forall f \in \bar{F}, m \in M, i \in W, j \in W, i \neq j, t \in \bar{T}$	$\|W_u\| \times \|U\|$	$\|M\| \times \|W\| \times \|W\|$	$\|T\| \times \|F\|$
R-I I-R	$x_{iu}^{ft} = \bar{x}_{iu}^{ft}$, $\forall f \in \bar{F}, u \in \bar{U}, i \in W_u, t \in T$ $y_{ijm}^{ft} = \bar{y}_{ijm}^{ft}$, $\forall f \in \bar{F}, m \in M, u \in \bar{U}, i \in W_u, j \in W, i \neq j, t \in T$	$\|W\|_u \times \|T\|$	$\|M\| \times \|W_u\| \times \|W\| \times \|T\|$	$\|F\| \times \|U\|$
T-R-I T- I-R R-T-I R-I-T I-T-R I-R-T	$x_{iu}^{ft} = \bar{x}_{iu}^{ft}$, $\forall f \in \bar{F}, u \in \bar{U}, i \in W_u, t \in \bar{T}$ $y_{ijm}^{ft} = \bar{y}_{ijm}^{ft}$, $\forall f \in \bar{F}, m \in M, u \in \bar{U}, i \in W_u, j \in W, i \neq j, t \in \bar{T}$	$\|W_u\|$	$\|M\| \times \|W_u\| \times \|W\|$	$\|T\| \times \|U\| \times \|F\|$

Notes. *T – Period decomposition, R- Region decomposition, I – Item decomposition.

Iteration	Period	Status of binary variables $\mathbf{X}_{iu}^{ft}$ and $\mathbf{Y}_{ijm}^{ft}$
Iteration 1	$t = 1$	Free to be optimized
	$t = 2$	Fixed
	$t = 3$	
	$t = 4$	
Iteration 2	$t = 1$	Fixed
	$t = 2$	Free to be optimized
	$t = 3$	Fixed
	$t = 4$	
Iteration 3	$t = 1$	Fixed
	$t = 2$	
	$t = 3$	Free to be optimized
	$t = 4$	Fixed

FIGURE 2. Working principle of F&O heuristic with period decomposition for $|T| = 4$.

more other decomposition schemes can be derived from the combination of time, region and item decomposition schemes. For example, in period-region decomposition, we have the period decomposition first followed by the region decomposition. By reversing the sequence, we get region-period decomposition scheme. Like this, 12 decomposition schemes can be obtained based on all possible combinations of period, region and item decompositions considering two decompositions at a time and then all three decompositions simultaneously with different sequences. The structure and characteristics of all the decomposition schemes are illustrated in Table 2.

It can be noted that as the level of decomposition increases (*i.e.* number of decompositions considered simultaneously), the number of free binary variables in each sub-problem decreases with proportionate increase in the number of sub-problems. Also, it can be noted that many decomposition schemes at the same level exhibit similar structure in terms of additional constraints and sub-problems formed. However, they differ in binary variables which are released for optimization, and this may bring a significant difference in the performance of decomposition schemes.

The pseudo-code for the F&O heuristic is presented in Algorithm 1. The F&O heuristic procedure starts with the decomposition scheme, overall time limit (TL), sub-problem time limit (STL) and the desired improvement limit as input. The initial feasible solution for the F&O heuristic can be obtained trivially or using any suitable heuristic. In this work, we follow a greedy heuristic as discussed in Tanksale and Jha [57]. The core concept of the heuristic is that the warehouse with the smallest sum of holding and setup costs is preferred for storage of the procured/transported items and the warehouse with the largest holding cost is preferred for withdrawal of items. Based on this, the decision of procurement allocation, transportation, and withdrawal of items from warehouses in each region are made for each period of the planning horizon sequentially. Using the solution of the greedy heuristic, the objective function value (Z) and the value of binary variables for procurement allocation (x_{iu}^{ft}) and transportation (y_{ijm}^{ft}) are retrieved.

Algorithm 1. F&O (decomposition_scheme, TL, STL, improvement_limit).

$k = 0$
Obtain initial feasible solution.
Update $Z^k \leftarrow Z, \bar{x}_{iu}^{ft} \leftarrow x_{iu}^{ft}$, and $\bar{y}_{ijm}^{ft} \leftarrow y_{ijm}^{ft}$ from initial feasible solution.
Determine nbSubproblem based on decomposition_scheme.
no_improvement = = False;
repeat
    $k \leftarrow k + 1$;
    for each nbSubproblem **do**
        Obtain $\bar{X}, \bar{Y}$ based on decomposition_scheme
        solve$(\bar{X}, \bar{Y}, \mathrm{STL})$;
            if $Z \leq Z^*$ **then**
            update $Z^k \leftarrow Z, Z^* \leftarrow Z, \bar{x}_{iu}^t \leftarrow x_{iu}^t$,and $\bar{y}_{ijm}^t \leftarrow y_{ijm}^t$;
            end if
        if TL reached **then**
            break
        end if
    end for
    improvement = $\left(Z^{k-1} - Z^k / Z^{k-1}\right) \times 100$
        if improvement $\leq$ improvement_limit or TL reached
            no_improvement = = True;
            return Z^{k-1}
        end if
until (no_improvement = = False);
return Z^k

Legend:

TL	Overall time limit
STL	Sub-problem time limit
Z	Objective function value
K	Iteration counter
$\bar{X}, \bar{Y}$	Sets in which binary variables x_{iu}^t,y_{ijm}^t are kept fixed to $\bar{x}_{iu}^{ft}$ and $\bar{y}_{ijm}^{ft}$, respectively

The number of sub-problems (nbSubproblem) is obtained based on the underlying decomposition scheme. For example, if the period decomposition scheme is followed and binary variables related to one time period are kept free to be optimized in each iteration out of $|T|$ number of periods in the planning horizon, then there would be $|T|$ number of sub-problems. The same logic can be extended for other decomposition schemes as well. Initially no improvement condition is set as FALSE, and the algorithm is continued until this condition is fulfilled. The F&O routine starts by incrementing the value of iteration counter (k) by 1. For each sub-problem, sets $\bar{X}$ and $\bar{Y}$in which binary variables x_{iu}^{ft} and y_{ijm}^{ft} are to be kept fixed at $\bar{x}_{iu}^{ft}$ and $\bar{y}_{ijm}^{ft}$, respectively, which are determined based on the specified decomposition scheme. The resulting sub-problem is then solved using an MIP solver to find the value of the free binary variables x_{iu}^{ft} and y_{ijm}^{ft} along with all continuous variables. If the objective function value (Z) of the present sub-problem is less than the incumbent solution value (Z^*), the objective function and binary variable values are updated accordingly. These steps are repeated for each of the sub-problem obtained using the given decomposition scheme, which represents a single iteration. The improvement realized in two consecutive iterations is calculated as the percentage difference in the objective function value obtained in the respective iterations. The algorithm thus iterates once or multiple times till there

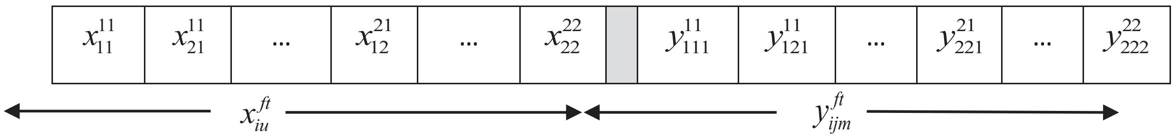

FIGURE 3. A typical chromosome representation for the proposed GA.

is no desired improvement in the objective function or the overall TL is reached. Finally, when terminated, the algorithm returns the value of the best-known objective function and decisions variables.

4.2. Hybrid GA embedded F&O heuristic

In the present study, we propose a novel GA-based diversification strategy to boost the performance of the proposed F&O heuristic. GA is a random search procedure inspired from the natural evolution process, which observes the principle of natural selection and survival of fittest over generations. To start the search, GA requires an initial population of the solutions which are encoded as chromosomes. After the evaluation of the initial population, the chromosomes are selected for reproduction based on some strategy. Then, offsprings are produced through a process of crossover from the parent chromosomes, and a new population is obtained. With the hope of obtaining an extremely desirable characteristic in the offspring, a mutation operator is applied. The process of selection, crossover, and mutation is iterated over many generations till termination criterion is met. The details about GA can be found in Deb [10]. Randomness and diversification of search space are the characteristics of GA that motivates us to embed its framework into F&O heuristic. The procedure for adaptation of GA for the present problem is explained subsequently.

4.2.1. Chromosome representation

In this study, each chromosome is represented as a string of binary variables x_{iu}^{ft} and y_{ijm}^{ft} with the corresponding allele value as zero or one. Therefore, the length of each chromosome is equal to $|T| \times |F| \times |U| \times |W_u|$ (for all x_{iu}^{ft} variables) plus $|T| \times |M| \times |W| \times |W| \times |F|$ (for all y_{ijm}^{ft} variables). Figure 3 shows a typical chromosome for the problem instance having two time periods, two regions, two warehouses in each region, two items, and two modes of transportation.

4.2.2. Initial population

For implementing GA, a common practice is to generate an initial population randomly or based on some decision rules. As we are running GA on the top of the F&O heuristic, we propose a novel initialization scheme which uses the solutions obtained using the F&O heuristic. We apply the principles of GA while generating an initial population also. For the purpose, we identify the size of the population (pop_size) required for GA and run the F&O heuristic for one-fourth times of the population size. The feasible solution found at each iteration of the F&O heuristic is recorded and used as a chromosome of the initial population. Using these chromosomes as parents, we can generate child chromosomes (offsprings) using a crossover function. Naturally, offsprings generated are double the size of parent chromosomes. Therefore, another one half of the population is added to the pool of the initial population. The remaining one-fourth of the population is generated using a random F&O routine. In the random F&O routine, we first identify the best-known solution from the F&O heuristic. Considering this solution as input solution every time, we run the F&O routine by randomly selecting a few variables which are kept free to be re-optimized. As the large of number of variables are already kept fixed, this routine will take a very small computational time. This procedure is also helpful in diversifying the search space. The overall procedure for generating an initial population is explained in Figure 4.

4.2.3. Fitness function

Following the generation of a population, chromosomes are evaluated for their fitness. Since the allele values and the value of binary variables in each chromosome are known, substituting these values in the proposed model

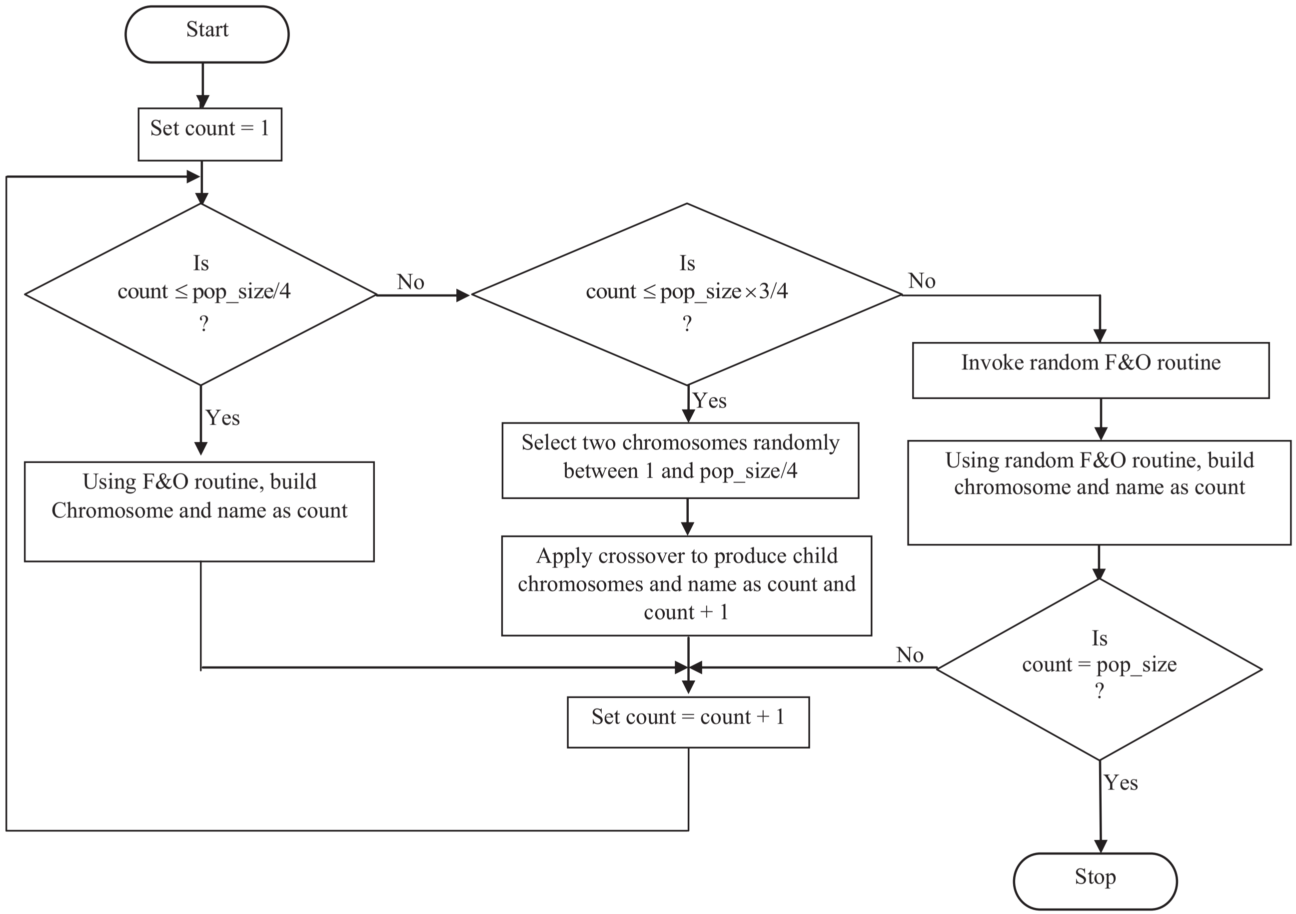

FIGURE 4. Scheme for generating initial population for GA.

provides a simple LP model, which can be easily solved using a solver to find the value of the objective function (total cost). The total cost obtained for each chromosome is regarded as a fitness value of the respective chromosomes.

4.2.4. Genetic operators

After evaluating the fitness of the chromosomes, the next step is to select parent chromosomes for reproduction. In this study, we follow tournament selection method in which two chromosomes are selected randomly from the population, and the chromosome with the better fitness (here, lower total cost) is selected as a candidate for inclusion in the mating pool. In this process, good solutions are preserved, and other solutions are eliminated from the population. Next, a new generation is produced from the selected chromosomes in the mating pool using a single point crossover and single bit flip mutation operator. A crossover point is selected randomly, and the two parts of the parent chromosomes are exchanged. Further, to induce diversity in the population, a random number is generated. Depending upon the predefined probability of mutation, the allele value in the chromosome corresponding to the random number is flipped between zero and one.

4.2.5. Repair operators

After the crossover and the mutation operations, resulting child chromosome may be infeasible. To repair the infeasibility, the following two operators are used. We inspect the case for which procurement of item f takes place in region u during period t and the corresponding setup variables x_{iu}^{ft} for all the warehouses $i \in W_u$ in region u are zero. In such case, starting with warehouse $i = 1$, the repair operator assigns $x_{iu}^{ft} = 1$ for the warehouses in region u in which the procured item is virtually allocated. Similarly, if there is a shortage of item f in region u during period t and $y_{jim}^{ft} = 0, \forall i \in W_u, j \in W \backslash \{i\}, m \in M$, the repair operator assigns $y_{jim}^{ft} = 1$ for the combination of j,i and m for which the transportation cost is lowest. Thus, if there is infeasibility in a chromosome, it is repaired and replaced in the population pool.

GA is iterated till the difference in average fitness of the population in two consecutive generations is less than a pre-specified value or the maximum number of generations is reached. The output of GA is again fed as an initial solution into the F&O routine and the procedure is continued till the stopping criterion is satisfied. The framework of the hybrid GA-F&O approach is illustrated in Figure 5.

4.3. Hybrid F&O heuristic with ILS

ILS has an exceptionally simple framework but has proven highly effective for many complex combinatorial optimization problems. The idea behind ILS is to demeanor the search not on the exhaustive search space consisting of all candidate solutions but on the solutions obtained by some underlying algorithm, typically a local search heuristic. ILS is motivated from the observation that the improvement heuristics that are iterative in nature often trapped in local optima. Therefore, instead of repeating heuristic search iteratively and starting from randomly obtained initial solutions, ILS begins with an initial solution S_0 and applies local search to obtain local optima $\bar{S}$. The solution $\bar{S}$ is perturbed to obtain an intermediate solution S^*. After that, the local search is executed on S^* with the quest of obtaining a new local optima $\bar{S}^*$, which may be better than the previously obtained local optima $\bar{S}$. The newly obtained local optimum is accepted based on some acceptance criteria. The steps for local search, perturbation and acceptance evaluation are repeated iteratively until a termination condition is fulfilled [44]. The architecture of ILS in its simplified form is presented in Algorithm 2.

Algorithm 2. Iterated Local Search.

$S_0 \leftarrow$ Initial Solution();
$\bar{S} \leftarrow$ Local Search();
repeat
    $S^* \leftarrow$ Perturb $(\bar{S})$;
    $\bar{S}^* \leftarrow$ Local Search();
    $\bar{S} \leftarrow$ Acceptance Criteria $(\bar{S}^*, \bar{S})$
Until termination criteria satisfied

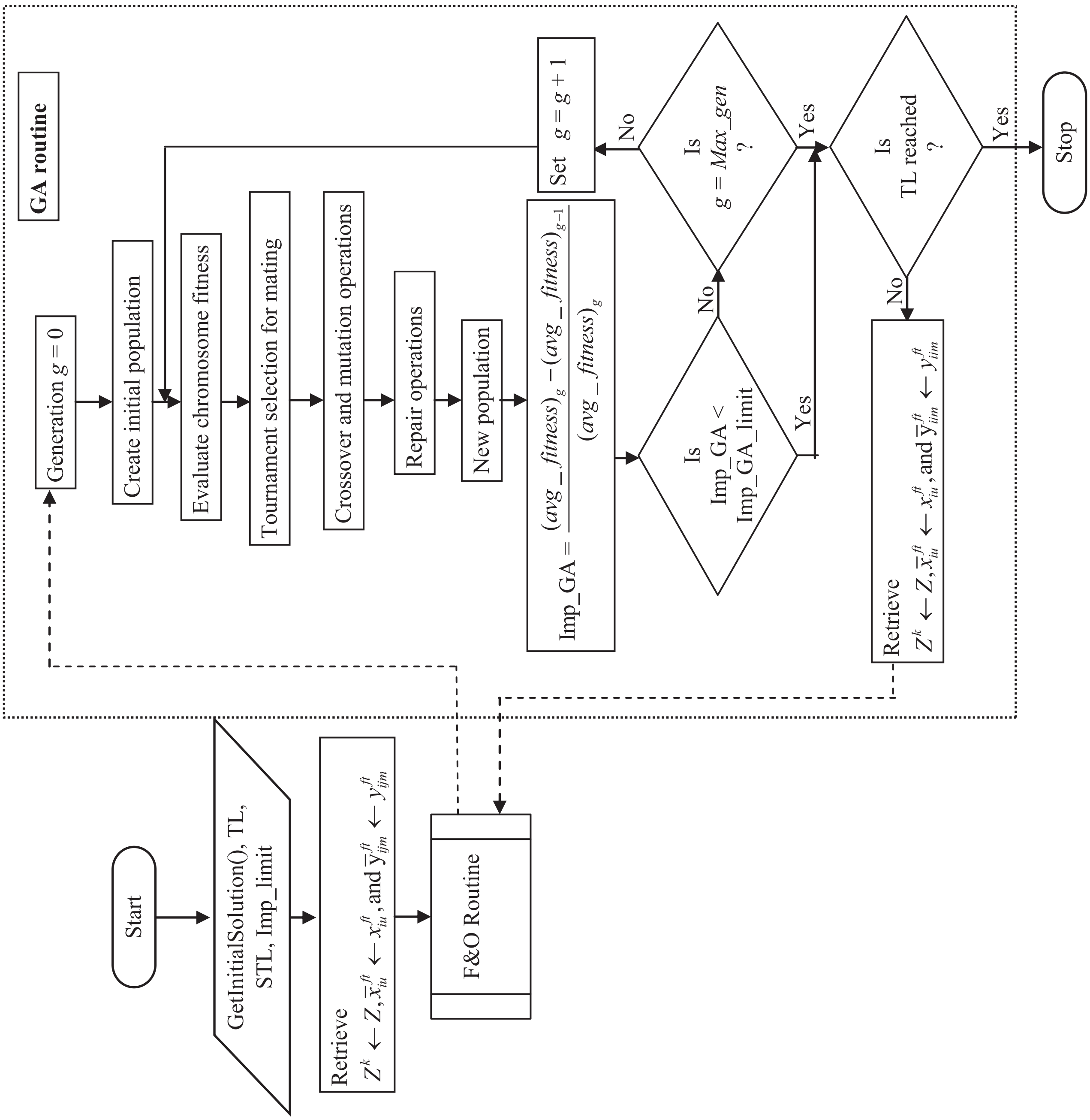

FIGURE 5. Framework of the hybrid GA-F&O heuristic.

In the proposed hybrid approach, instead of executing F&O heuristic in an iterative fashion, we amalgamate the F&O heuristic into the ILS framework. The procedure adopted for the hybrid approach is illustrated below.

4.3.1. Initial solution

In the proposed approach, we use the solution obtained from the F&O heuristic as an initial solution S_0 for the ILS. In the hybrid approach, the only exception is that the F&O routine is executed for one complete iteration and the corresponding solution is retrieved for use in the ILS procedure.

4.3.2. Local search

For carrying out the local search in the ILS routine, we devise a novel mechanism referred as random F&O routine. In the random F&O routine, we select some of the variables (equivalent to the number of free binary variables in a sub-problem) from the initial solution S_0 randomly, which are to be re-optimized. By keeping the value of remaining binary variables fixed, we then solve the resulting sub-problem using a solver similar to the F&O routine. Since most of the binary variables in the sub-problem are fixed, the random F&O routine requires very less computational time. The proposed random F&O is helpful in exploring the solution space in the neighborhoods of the present solution obtained from the F&O routine. The local search phase is continued till a new local optimal solution $\bar{S}$ is found in the neighborhood of the initial solution S_0; otherwise S_0 is treated as the best local optimal solution.

4.3.3. Perturbation

To avoid the basin of attraction, when a local optimal solution $\bar{S}$ is established, it is perturbed to some intermediate solution S^* using multi-bit flip mutation process. For the purpose, some binary variables of solution $\bar{S}$ are randomly selected and their values are flipped (mutated) between zero and one. The choice of the number of binary variables selected for perturbation is critical. A very small number of variables selected for the mutation may not affect the local optima and a very high number of mutated variables may have a random restart effect. It can also be noted that the perturbation may lead to an infeasible solution which needs to be repaired. Hence, we check for feasibility and modify the value of corresponding binary variables in case of infeasibility by following a repair procedure similar to the one discussed in Section 4.2.5. The perturbation phase is again followed by the local search phase, *i.e.* random F&O routine, to find a new local solution $\bar{S}^*$.

4.3.4. Acceptance criterion

The new solution $\bar{S}^*$ is accepted if and only if it exhibits lower objective function value (total cost) as compared to the previous local optimal solution $\bar{S}$. Otherwise, the search is continued with the best-known solution $\bar{S}$.

Algorithm 3. Hybrid ILS-F&O heuristic.

```
k ← 0
S_0 ← F&O heuristic ();
S̄ ← random F&O ();
repeat
    k ← k + 1
    S* ← Perturbation (S̄);
      if S* infeasible then
        repair S*
      end if
     S̄* ← random F&O();
      if S̄* ≤ S̄then
        S̄ ← S̄*
      end if
 Until k    < max_iterations or TL reached
 return S̄
```

Thus, after the initial solution is obtained using the F&O heuristic, the ILS is repeated with the local search, perturbation and acceptance phases until termination criteria (maximum number of iterations or time limit) is satisfied. This procedure is presented in Algorithm 3.

5. Computational experiments

The objectives of the computational analysis are:

- To evaluate the performance of the F&O heuristic
- To investigate the relationship between the decomposition structure and the performance of the F&O heuristic
- To analyze the effectiveness of the hybridization of F&O heuristic with each GA and ILS.

Extensive numerical experimentation is carried out by randomly generating several problem instances based on the data collected from the Indian public distribution system (PDS). All programs are coded in C++ with IBM Concert Technology using Microsoft Visual Studio 2010 ultimate IDE platform on a personal computer equipped with Intel i-5 2.90 GHz processor, 8 GB RAM, and Microsoft Windows 7 professional 64-bit operating system.

5.1. Dataset

For the experimentation purpose, we collected the secondary data for other parameters such as procurement, demand, and storage capacity in each state, from various sources [11,12] for the fiscal year 2014–2015. The data for the holding and the setup costs are not readily available, and so they are estimated based on the prevailing scenario in the Indian PDS. The transportation capacity for each mode of transportation is considered based on the past usage data. The freight charges (transportation costs) are calculated taking into account 47 distance slabs between 1 km and 3500 km, as per the directives of the Indian Railways [28]. The considered dataset is summarized in Table 3.

5.2. Generating problem instances

The testbed consists of the following variations:

- Number of regions (states): 6, 12, 18 and 24
- Number of warehouses in each region (assumed equal for all regions): 3, 5 and 10
- Number of periods (each period equal to one month): 4, 8 and 12
- Number of items (foodgrains): 2 (wheat and rice)
- Mode of transportation: 2 (rail and road).

Table 3. Dataset for problem experimentation.

Parameter	Value	
Annual procurement (MMT)	Wheat	0–11.641
	Rice	0–8.575
Annual demand (MMT)	Wheat	0.00264–4.5475
	Rice	0.01–3.588
Total storage capacity (MMT)	–	0.023–18.266
Total transportation capacity (MMT)	Rail	1–10
	Road	1–25
Transportation cost (INR)	Rail	212.7–3656.7 per ton
	Road	110.8–6044.3 per ton
Holding cost (INR)	–	220–650 per ton per month
Setup cost (INR)	–	3200–8000 per setup

Notes. MMT – million metric tons, INR – Indian rupee.

Total $4 \times 3 \times 3 = 36$ problem configurations are generated from all possible combinations of the number of warehouses in each state, number of states and the number of periods, respectively, while keeping the number of items and the mode of transportation fixed in each configuration. Further, for each problem configuration, five problem instances are generated randomly by selecting the regions from the pool of 24 regions. However, the problem configurations with 24 regions will have only one problem instance for each configuration, and there are nine such configurations. Thus, we have generated a total of 144 problem instances for all 36 problem configurations.

5.3. Results and discussion

In this section, the results of the computational experiments for Cplex and the proposed solution approach are presented, followed by the performance evaluation of the proposed heuristics.

5.3.1. Cplex performance

In order to compare the performance of the proposed solution approach, the problem instances are solved using Cplex solver in concert technology by imposing a maximum time limit of 3600 s. The results for all problem configurations obtained are summarized on an average basis in Table 4. It is observed that except the first problem configuration, Cplex failed to solve the problem instances to optimality within 3600 s of the time limit. For all such configurations, the best-known objective function value along with percentage optimality gap is presented in Table 4.

5.3.2. Analysis of decomposition schemes

While implementing the F&O heuristic, sub-problems are obtained considering one time period, one region, and one item for the variants of the period, region, and the item decompositions, respectively. Each sub-problem obtained corresponding to the problem instances with 3, 5 and 10 warehouses are allowed to run for 10, 20 and 30 s, respectively. Similarly, an overall TL of 600, 900 and 1200 s is imposed for the problem instances with 3, 5 and 10 warehouses, respectively. Multiple iterations are allowed under a strict convergence criterion, *i.e.* the F&O heuristic is terminated if the solution obtained in two consecutive iterations is found same. For each problem instance, the performance of the F&O heuristic is measured in terms of computational time and the solution quality. The deviation of the solution obtained using the F&O heuristic from that of Cplex is calculated as solution gap which is calculated as solution gap (%) = $\left[\left(\mathrm{TC}_{\mathrm{FO}} - \mathrm{TC}_{\mathrm{Cplex}}\right)/\mathrm{TC}_{\mathrm{Cplex}}\right] \times 100$ where, $\mathrm{TC}_{\mathrm{FO}}$ and $\mathrm{TC}_{\mathrm{Cplex}}$ is the total cost obtained using the F&O heuristic and Cplex, respectively. The summary of results for the F&O heuristic with various decomposition schemes is presented in Table 5. It is evident that with an average solution gap of 2.06%, the period decomposition is the most effective decomposition scheme, in which 38.9% of the problem instances of various configurations are having the solution gap of less than 1%. Also, with the lowest average CPU time of 492.2 s, the item decomposition is proved to be the most efficient decomposition scheme. Relatively, T-R-I and I-R-T decomposition schemes have the worst performance in terms of the solution quality and the computational time, respectively. The observed reason behind the weak performance of these strategies is due to the formation of a large number of sub-problems, and the algorithm could not solve all sub-problems within the imposed overall TL for most of the large size instances. Further, considering the sequence in both, two- and three-level decompositions, the item decomposition followed by the region decomposition and the period decomposition has produced the best result. However, in spite of being the best decomposition scheme, when the period decomposition is followed by the region and the item decompositions, it has the weakest performance. In general, with the increase in the level of decomposition, the average computational time increases, but this is not true for the solution quality.

Further analysis of the decomposition structure and the performance of the decomposition scheme have brought some interesting insights. In Figure 6, we presented an analysis of the decomposition scheme considered in this study on an average basis.

Figure 6 has two parts. In part 1, a plot of sub-problem size (the number of free binary variables in each sub-problem) and the number of sub-problems for the various decomposition schemes is presented. Part 2 of

TABLE 4. Performance of Cplex for various problem configurations.

Problem configuration number	Problem configuration*	Average CPU time (s)	Average total cost Average (INR)	Average optimality gap (%)
1	3w-6r-4m	66.51	488 852	0.00
2	3w-6r-8m	3601.61	1 194 079	1.56
3	3w-6r-12m	3605.93	2 043 433	1.07
4	3w-12r-4m	3601.50	626 586	1.88
5	3w-12r-8m	3603.97	1 209 105	3.95
6	3w-12r-12m	3628.53	2 206 363	2.38
7	3w-18r-4m	3605.36	715 998	4.30
8	3w-18r-8m	3601.78	1 540 210	5.17
9	3w-18r-12m	3602.56	2 342 869	5.63
10	3w-24r-4m	3630.96	999 537	3.35
11	3w-24r-8m	3623.48	1 803 995	6.22
12	3w-24r-12m	3609.17	2 629 587	6.02
13	5w-6r-4m	3492.66	326 100	4.03
14	5w-6r-8m	3601.63	987 378	1.91
15	5w-6r-12m	3614.41	3 292 153	1.69
16	5w-12r-4m	3611.47	846 580	2.04
17	5w-12r-8m	3608.10	1 300 849	3.59
18	5w-12r-12m	3602.27	2 100 237	3.78
19	5w-18r-4m	3635.31	850 017	4.20
20	5w-18r-8m	3605.26	1 757 766	3.10
21	5w-18r-12m	3606.32	2 585 050	8.74
22	5w-24r-4m	3602.31	1 042 210	4.77
23	5w-24r-8m	3615.74	1 884 349	7.67
24	5w-24r-12m	3605.82	2 806 019	11.18
25	10w-6r-4m	3605.86	545 631	2.41
26	10w-6r-8m	3630.88	1 940 239	3.19
27	10w-6r-12m	3640.64	3 438 109	4.67
28	10w-12r-4m	3622.29	872 628	3.96
29	10w-12r-8m	3607.19	1 763 201	7.50
30	10w-12r-12m	3611.00	2 711 000	10.97
31	10w-18r-4m	3641.75	919 736	8.29
32	10w-18r-8m	3620.22	1 968 717	11.41
33	10w-18r-12m	3840.87	2 711 112	18.07
34	10w-24r-4m	3618.38	1 239 863	9.77
35	10w-24r-8m	3876.33	2 235 342	12.84
36	10w-24r-12m	3910.48	3 437 425	16.40

Notes. *3w-6r-4m: 3w – 3 warehouses in each region; 6r – 6 regions; 4m – length of planning horizon 4 months.

Figure 6 depicts the performance of the decomposition schemes in terms of CPU time and solution gap. It is observed that lower the sub-problem size, higher is the number of sub-problems and *vice versa*. Thus, there is a trade-off between the number of sub-problems and the size of each sub-problem in a decomposition scheme, which is visible in part 1 of Figure 6. From the second part of Figure 6, we can correlate the performance of a decomposition scheme with the number of sub-problems formed. In general, we observe on the average basis that more the number of sub-problems in a decomposition scheme, higher is the computational time. Also, this reduces the effectiveness of a scheme (*e.g.* T-R-I decomposition). On the other hand, if the number of sub-problems due to a decomposition scheme is less, *i.e.* a large number of free binary variables in each sub-problem, this will also have a detrimental effect on the efficacy of a decomposition scheme (*e.g.* item decomposition). A

TABLE 5. Summary of performance of F&O heuristic for various decomposition schemes.

Decomposition scheme*	CPU time (s) Avg	Min	Max	Average number iterations	Solution gap (%) Avg	Min	Max	% of instances with less than 1% solution gap
T	538.87	3.92	1556.84	4	2.06	−1.95	35.00	38.9
R	591.53	3.93	3629.86	5	2.31	−1.93	22.81	29.2
I	492.92	12.09	1247.35	6	4.40	−0.09	24.03	22.2
T-R	691.32	10.47	3787.94	5	5.58	−1.92	34.40	9.7
R-T	84.96	12.65	3768.29	4	5.43	−1.57	30.50	19.4
T-I	600.83	3.30	3646.38	4	3.33	−8.90	18.94	15.3
I-T	671.47	3.41	3698.60	3	3.62	−9.00	62.17	20.1
R-I	603.55	4.85	3639.26	3	4.06	−7.46	30.92	20.8
I-R	691.74	4.31	3729.08	4	3.09	−7.51	23.27	37.5
T-R-I	959.25	11.51	3843.16	3	6.07	−7.25	66.44	16.7
T-I-R	1055.45	20.13	3830.12	4	4.55	−7.54	44.42	22.2
R-T-I	1041.63	15.15	3867.53	2	3.90	−4.76	29.18	38.9
R-I-T	1012.88	14.82	3852.17	2	3.81	−5.05	31.22	35.4
I-T-R	1098.24	14.46	3716.19	3	2.96	−11.16	33.00	31.9
I-R-T	1157.24	13.59	3871.74	3	2.79	−8.51	38.32	35.4

Notes. *T – Period decomposition, R – Region decomposition, I – Item decomposition. # Avg, Min and Max are calculated based on results of all 144 individual problem instances.

better performance of a decomposition scheme is observed in the central region of the plot, that is, for those decomposition schemes which have a moderate number of sub-problems. Therefore, we can generalize the fact that when several decomposition schemes are possible, a trade-off between the number of sub-problems and the size of each sub-problem can be plotted, and the schemes with the least difference can be selected for further investigation.

5.3.3. Results of GA-F&O hybrid heuristic

In the hybrid approach, the period decomposition scheme is followed for the F&O heuristic keeping rest of the experimental settings same, as explained at the beginning of Section 5.3.2. Further, the population size and the maximum number of generations used in GA are set to 20 and 50, respectively. Similarly, crossover and mutation rates are taken as 0.9 and 0.01, respectively. For each problem instance, the performance of the pure F&O heuristic and the hybrid approach is compared with the solution obtained using Cplex. The summary of the results is presented in Table 6.

The average performance of the pure F&O heuristic in terms of computational time and the solution quality is summarized in Columns 3–9. For the pure F&O heuristic, it is observed that the average CPU time increases with an increase in problem size. However, this increase is polynomial, not exponential. The minimum and maximum CPU time among all instances for the pure F&O heuristic is 3.9 and 2092.5 s, respectively, and the grand average of the CPU time is 683.67 s, about one-sixth of the average CPU time taken by Cplex. This implies that the F&O heuristic is computationally efficient.

Further, the deviation of the solution obtained using the proposed F&O heuristic from that of Cplex is reported as solution gap for each problem instance. The solution gap calculated for all problem configurations is reported in Columns 7–9 on the average basis along with the minimum and maximum value for all problem instances under each problem configuration. The minimum and the maximum solution gaps considering all problem instances are observed as −1.95% and 35.0%, respectively, while the grand average solution gap is 2.01%. For 18 problem instances among all, the F&O gives the better solution than the best-known solution obtained by Cplex. Also, for 90% of the problem instances, solution gap is less than 3%. Next, the performance

TABLE 6. Summary of results for pure F&O and hybrid GA-F&O heuristics.

Problem configuration number	Problem configuration*	Pure F&O heuristic: CPU Time (s) Avg	Min	Max	Number of iterations	Solution gap (%) Avg	Min	Max	Hybrid GA-F&O heuristic: CPU Time (s) Avg	Min	Max	Number of cycles	Solution gap (%) Avg	Min	Max
1	3w-6r-4m	13.0	3.9	26.1	3	0.81	0.39	1.17	548.0	506.0	622.7	6	0.20	0.00	0.44
2	3w-6r-8m	88.8	14.3	303.8	9	2.15	0.50	3.92	594.6	552.4	669.8	5	0.91	0.47	1.58
3	3w-6r-12m	42.0	33.2	50.3	4	1.15	0.24	3.24	640.6	601.8	697.8	5	0.93	−0.41	2.86
4	3w-12r-4m	145.1	90.0	303.5	8	2.55	0.95	4.26	781.9	741.1	864.6	4	1.53	0.50	3.48
5	3w-12r-8m	184.5	67.1	326.0	5	4.32	1.33	13.99	740.3	673.7	807.0	2	1.51	0.03	2.48
6	3w-12r-12m	262.6	201.0	301.6	3	2.38	0.30	4.71	620.9	521.2	688.3	1	1.70	0.49	2.49
7	3w-18r-4m	200.7	82.0	307.4	4	9.76	1.53	35.00	772.5	568.5	920.5	2	3.40	1.14	8.36
8	3w-18r-8m	572.1	215.5	673.7	4	3.34	1.02	7.36	1087.5	1033.3	1172.6	1	2.06	0.75	2.73
9	3w-18r-12m	767.2	751.7	773.6	3	1.75	0.11	3.13	1628.3	1617.3	1643.7	1	0.72	0.35	0.93
10	3w-24r-4m	190.4	–	–	3	2.90	–	–	1060.4	–	–	2	1.48	–	–
11	3w-24r-8m	633.9	–	–	3	1.77	–	–	1440.5	–	–	1	0.34	–	–
12	3w-24r-12m	711.1	–	–	3	2.12	–	–	1719.7	–	–	1	0.99	–	–
13	5w-6r-4m	26.3	13.9	42.1	3	3.33	0.25	9.39	1373.9	1276.7	1545.7	4	−0.25	−2.21	0.88
14	5w-6r-8m	113.4	19.8	180.8	5	2.05	0.11	4.27	1288.6	1220.8	1382.0	4	1.06	−1.41	2.56
15	5w-6r-12m	232.8	107.0	496.4	7	6.35	0.22	27.42	1429.1	1230.3	1572.5	3	0.43	−1.04	2.65
16	5w-12r-4m	416.6	103.4	654.5	6	2.11	1.22	3.78	1810.8	1618.6	1896.3	3	1.33	0.64	1.84
17	5w-12r-8m	633.0	438.8	744.3	4	1.01	0.10	1.78	1809.9	1596.3	2082.0	2	0.90	0.11	1.99
18	5w-12r-12m	680.9	611.6	752.1	3	1.08	−0.16	3.14	1672.2	1516.0	1898.9	1	1.22	0.28	2.71
19	5w-18r-4m	630.9	607.2	682.1	5	2.29	1.81	2.46	1771.8	1654.7	1850.3	2	2.18	1.62	2.87
20	5w-18r-8m	1002.9	708.2	1276.3	4	1.16	0.36	2.24	1483.0	1361.6	1552.8	1	0.74	0.10	1.29
21	5w-18r-12m	958.8	594.1	1227.0	4	0.43	-0.26	1.09	1436.5	1016.4	1761.6	1	−0.15	−2.19	1.31
22	5w-24r-4m	335.4	–	–	4	2.49	–	–	1116.9	–	–	2	2.01	–	–
23	5w-24r-8m	823.9	–	–	3	0.73	–	–	1407.3	–	–	1	0.03	–	–
24	5w-24r-12m	1316.2	–	–	3	2.72	–	–	2316.0	–	–	1	−0.14	–	–
25	10w-6r-4m	239.8	198.4	334.9	5	2.31	0.73	4.14	1942.8	1769.8	2313.2	4	1.12	0.60	1.52
26	10w-6r-8m	663.0	199.7	1035.9	5	1.13	0.20	2.14	2555.3	2167.4	3192.7	3	0.93	−0.38	2.67
27	10w-6r-12m	938.5	611.1	1259.0	3	1.49	0.41	5.03	2397.6	1964.8	2988.7	1	1.11	0.05	2.74
28	10w-12r-4m	927.5	777.1	1017.2	6	1.67	−0.07	3.30	2766.9	2387.1	3293.1	2	0.92	−0.91	1.56
29	10w-12r-8m	1161.4	1064.0	1372.2	4	0.64	−1.59	3.46	2008.0	1713.3	2314.9	1	0.32	−1.25	1.91
30	10w-12r-12m	1491.3	1127.8	1832.0	3	0.12	−1.35	2.15	2498.3	2434.1	2636.0	1	−0.29	−3.14	1.59
31	10w-18r-4m	934.7	485.9	1083.5	5	1.23	−0.07	2.27	2451.9	2103.0	2788.8	2	1.49	−0.12	4.87
32	10w-18r-8m	1540.6	998.6	1956.9	3	0.08	−0.84	2.03	2706.8	2558.4	2832.7	1	−0.25	−0.93	0.58
33	10w-18r-12m	1684.4	1206.5	2092.5	2	−1.04	−1.95	−0.23	3652.5	3607.2	3697.5	1	−2.18	−3.00	−1.00
34	10w-24r-4m	979.8	–	–	5	1.65	–	–	1566.6	–	–	1	−0.31	–	–
35	10w-24r-8m	1012.1	–	–	3	2.45	–	–	2942.8	–	–	1	2.08	–	–
36	10w-24r-12m	2056.8	–	–	2	−0.26	–	–	3609.3	–	–	1	−1.24	–	–
	Grand average	683.7			4	2.01			1712.5			2	0.80		

Notes. *3w-6r-4m: 3w – 3 warehouses in each region; 6r – 6 regions; 4m – length of planning horizon 4 months.

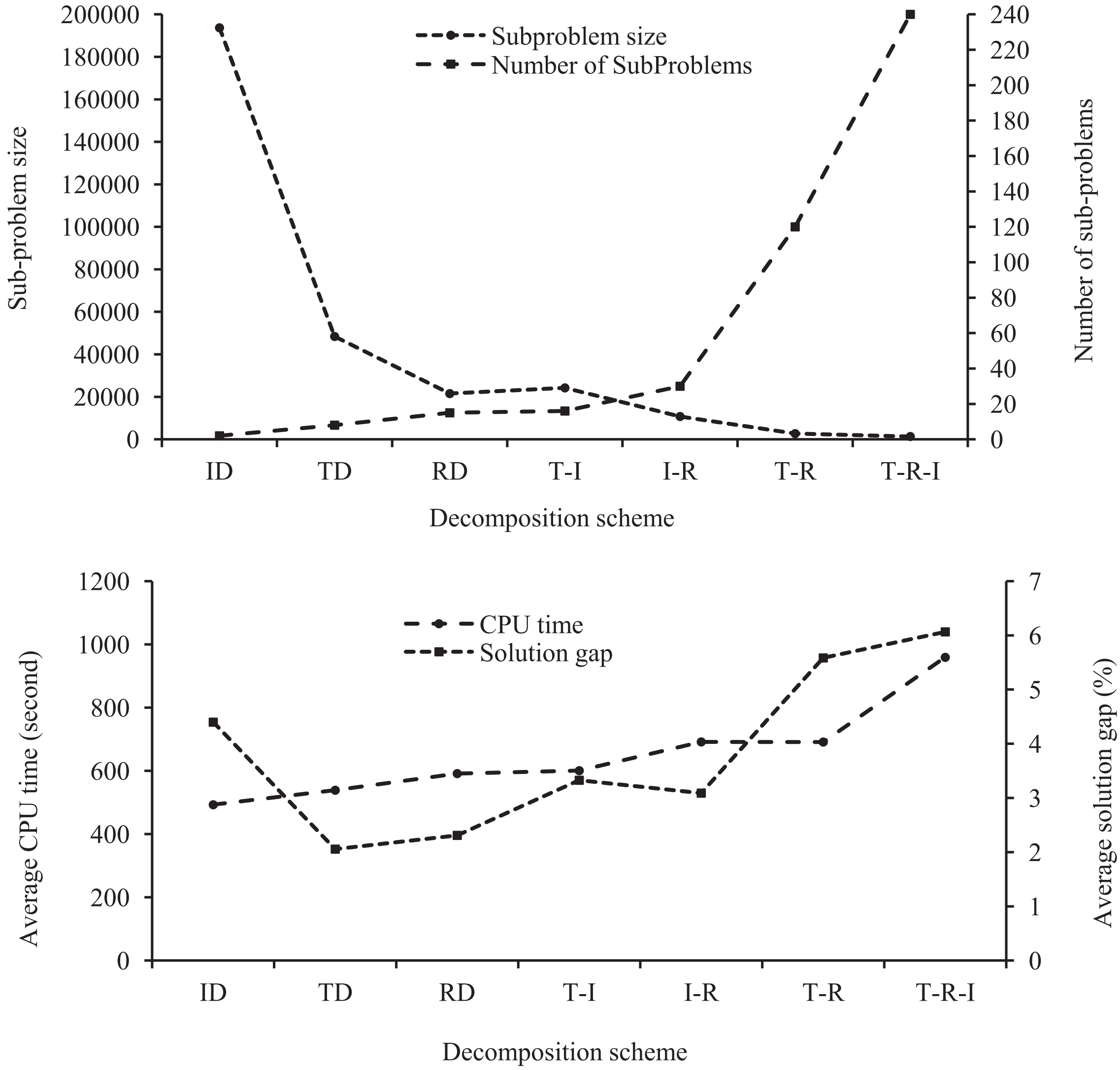

Figure 6. Analysis of decomposition schemes.

of the hybrid GA-F&O heuristic is summarized in Columns 10–16 of Table 6. The average, minimum and the maximum CPU times for each problem configuration are reported in Columns 10–12, along with the number of cycles completed by the hybrid GA-F&O during the allowed run-time. It can be noted that one complete cycle of the hybrid GA-F&O involves iterating the pure F&O and the random F&O routines for one-fourth times of the population size of GA, and then running GA for the maximum number of generations or till the convergence. Table 6 reveals the fact that with the aggregate average CPU time of 1712.5 s, the hybridization of F&O heuristic results in about three-fold increase in the computation time. However, the hybrid GA-F&O heuristic outperforms the pure F&O heuristic in terms of solution quality.

When compared with Cplex, the solution obtained using the hybrid GA-F&O for about one-fifth of the problem instances is better than the best-known solution of Cplex. Also, the solution obtained using the hybrid GA-F&O heuristic for 54.86% of the problem instances are having solution gap of less than 1% from the best known Cplex solution. Using the hybrid GA-F&O heuristic, the minimum and the maximum solution gaps are observed as −3.14% and 8.36%, respectively, while the grand average solution gap is 0.80%. These statistics reveal that the hybridization of F&O heuristic with GA boosts the performance of the pure F&O heuristic and produces solutions in the proximity of the best-known solution given by Cplex.

As GA is used as a diversification tool, which is embedded into the F&O heuristic, we want to evaluate the comparative performance of GA when implemented impartially. To run the pure GA, the population size, maximum generations, and crossover and mutation rates are set to 20, 100, 0.9, and 0.01, respectively. A time limit of 3600 s is imposed along with a strict convergence criterion. Half of the initial population is generated randomly and the other half using a smart approach mentioned in Goren *et al.* [19]. In the smart approach, LP relaxation of the model is solved, and a random number between 0 and 1 is generated. If the value of a binary variable in the relaxed model is less than the random number, then the value of the corresponding variable is set to zero, otherwise to one. This approach is useful in producing a good quality initial solution thereby increasing the chance of finding a better solution in GA. The relative performance of pure GA is summarized in Table 7. It is observed that with the pool of randomly generated initial solutions, GA is able to produce only approximate solutions. The grand average of the average solution gaps in this case is 19.61%. Also, GA hardly converges, or it fails to complete 100 iterations during the imposed time limit. Conversely, when good quality solutions are available as an initial population in the hybrid approach, it is observed that GA has converged well within 10 to 15 generations in most of the cases of the hybrid F&O approach. Therefore, we conclude that the application of standalone GA with randomly generated initial solution pool for the given problem is not so prudent approach.

Next, to answer the question "does the proposed solution approach outperform any state of art MIP solver?", Cplex is allowed to run for the same amount of time as taken by the pure F&O heuristic with the period decomposition to solve the same problem instance, and the corresponding solution value is recorded. Similarly, this procedure is also repeated for all problem instances taking the computational time reported by the hybrid GA-F&O heuristic for running Cplex.

In Figures 7 and 8, the results obtained for all problem instances are presented into three categories according to the number of warehouses in each region. In each category, 48 problem instances are shown representing various problem configurations. Each problem instance is denoted according to the number of regions and the number of time periods in the respective problem configurations. For example, the problem instance 6R-4T-1 denotes the problem configuration with 6 regions and 4 periods and the last digit represents the instance number (here, instance number 1). On the vertical axis, the percentage difference in the solution value of Cplex and the F&O is shown which is calculated as Gap (%) = $\left[\left((\overline{\mathrm{TC}})_{\mathrm{Cplex}} - (\mathrm{TC})_{\mathrm{FO}}\right)/(\mathrm{TC})_{\mathrm{FO}}\right] \times 100$, where $(\overline{\mathrm{TC}})_{\mathrm{Cplex}}$ is the solution offered by Cplex when run for the computational time equivalent to that of the F&O heuristic. Therefore, a positive percentage gap indicates that within the same computational budget Cplex has found a better solution than the F&O heuristic, otherwise the F&O heuristic outclassing Cplex.

For both the cases of the pure and the hybrid GA-F&O heuristics, it is observed that for most of the small size problem instances, Cplex produces a better solution with the same computational effort. It is observed that for these instances Cplex reaches a good quality solution within a short computational time and after that, it takes a long time to converge to an optimal solution. This illustrates the reason why Cplex could not reach the optimality within one hour of execution, for most of the instances. However, with an increase in problem size (right portion of Figs. 7 and 8 in the downward direction), it is evident that the performance of Cplex has deteriorated in terms of solution quality. For medium size problem instances, there is no much difference in the solutions obtained by Cplex and the F&O heuristic. Further, for large size problem instances, it is observed that the proposed F&O heuristic provides better solutions than Cplex. This clearly indicates that within the same computational budget, the proposed F&O heuristic approach outperforms Cplex for the practical size problem instances. This justifies the necessity and superiority of the solution approach undertaken.

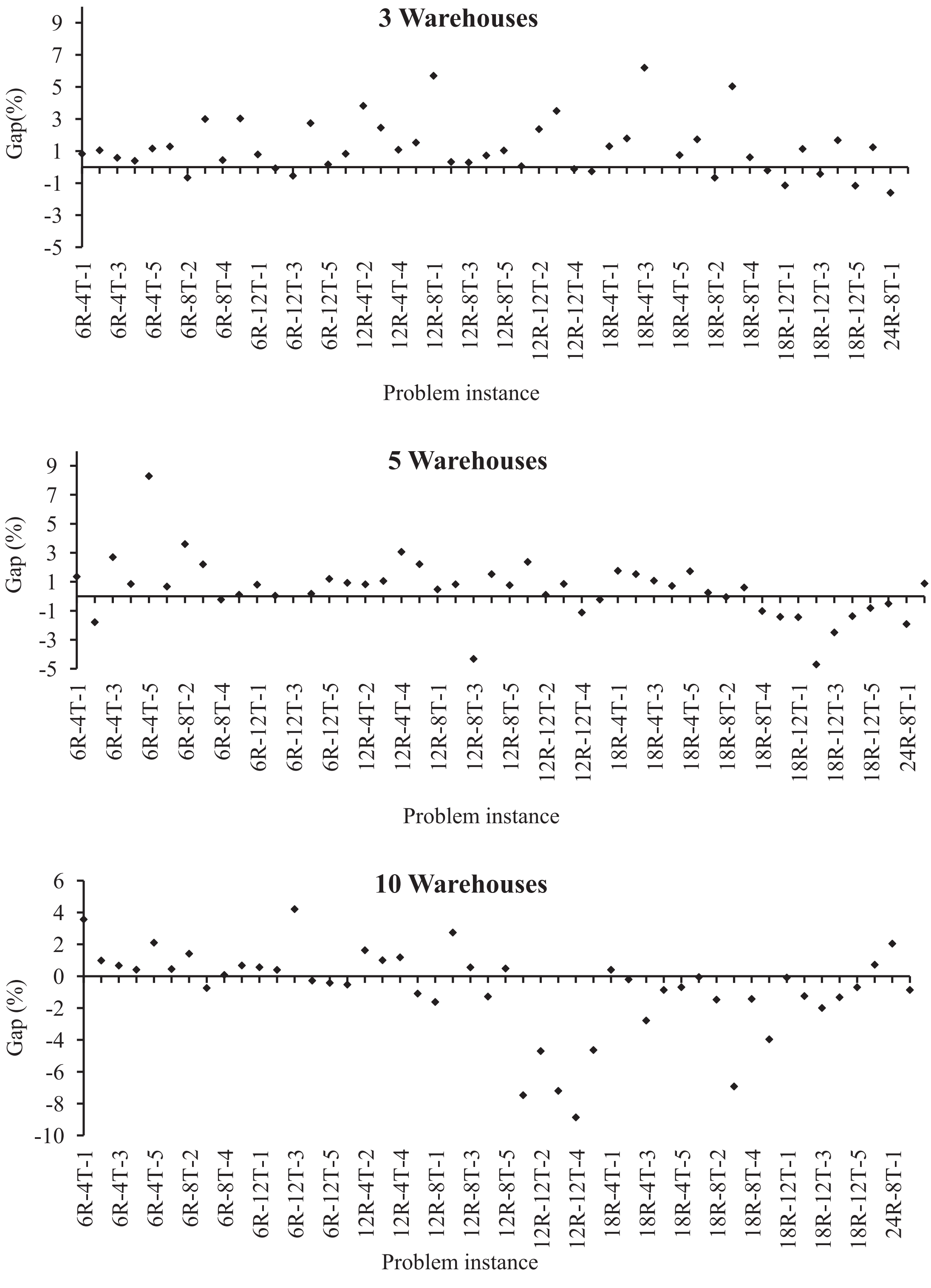

FIGURE 7. Comparison of pure F&O heuristic and Cplex solution for the same CPU time.

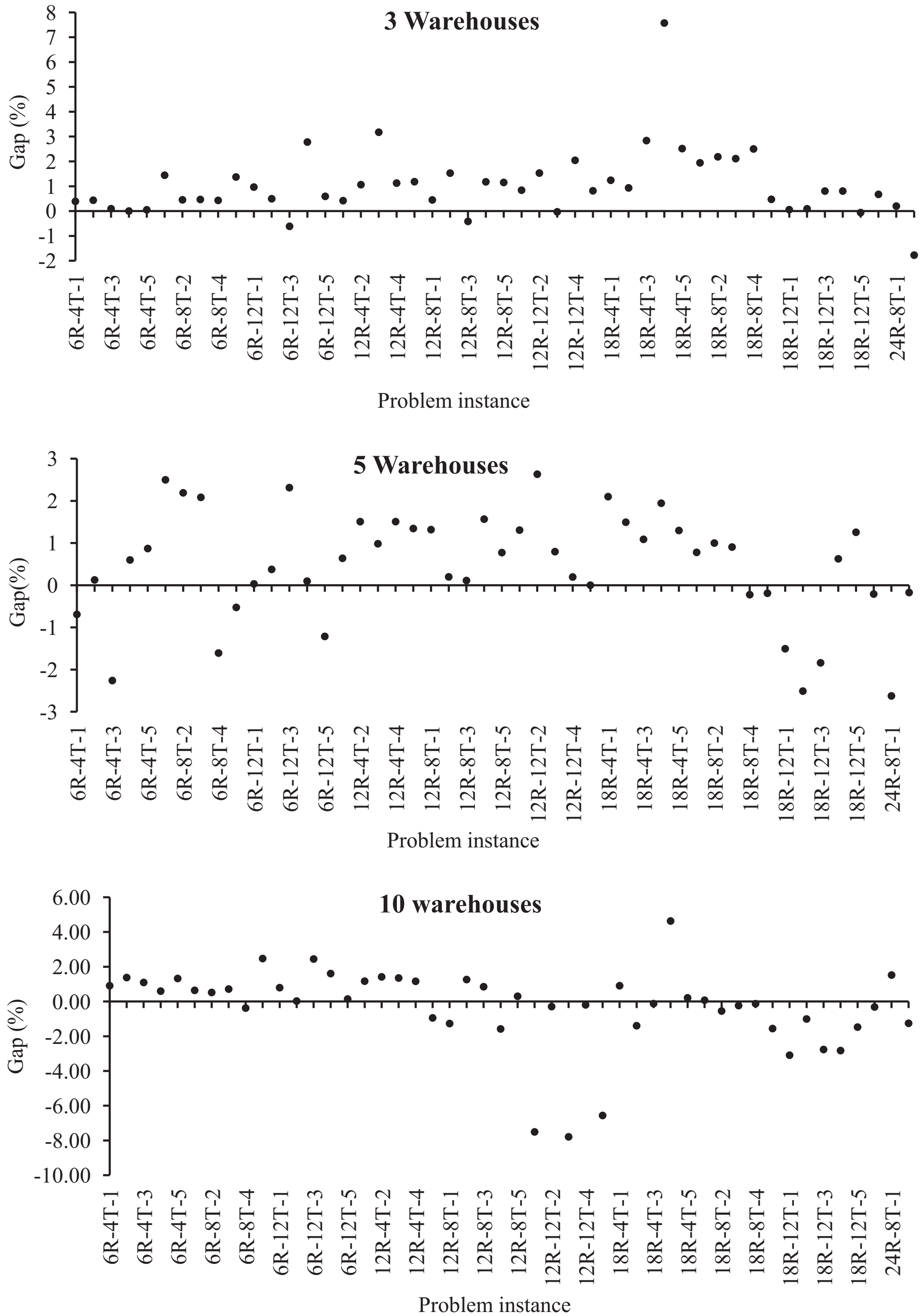

FIGURE 8. Comparison of hybrid GA-F&O heuristic and Cplex solution for the same CPU time.

TABLE 7. Performance of pure GA.

Problem configuration number	Problem configuration*	Pure GA CPU Time (s) Avg	Min	Max	Number of iterations	Solution gap (%) Avg	Min	Max
1	3w-6r-4m	203.23	174.06	223.85	100	11.66	0.70	27.25
2	3w-6r-8m	232.56	231.11	235.88	100	15.84	2.54	24.61
3	3w-6r-12m	342.11	326.37	364.65	100	13.72	5.74	20.86
4	3w-12r-4m	511.58	453.12	554.97	100	20.15	12.42	29.20
5	3w-12r-8m	835.69	800.48	874.94	100	21.03	15.77	25.07
6	3w-12r-12m	1332.22	1247.51	1361.80	100	20.69	15.21	24.70
7	3w-18r-4m	986.02	947.51	1028.47	100	21.04	15.23	32.80
8	3w-18r-8m	1871.42	1730.21	2131.26	100	25.12	21.17	27.40
9	3w-18r-12m	2996.93	2802.13	3145.34	100	23.00	20.84	25.05
10	3w-24r-4m	1628.33	–	–	100	18.55	–	–
11	3w-24r-8m	3339.85	–	–	100	23.97	–	–
12	3w-24r-12m	3645.11	–	–	94	24.71	–	–
13	5w-6r-4m	326.80	289.03	356.08	100	12.96	6.56	20.49
14	5w-6r-8m	587.57	544.34	673.80	100	18.75	7.31	30.07
15	5w-6r-12m	856.16	800.22	1049.71	100	17.87	10.19	30.27
16	5w-12r-4m	1168.75	1064.49	1311.32	100	13.07	10.07	17.38
17	5w-12r-8m	2137.88	2111.01	2201.80	100	19.33	12.67	36.43
18	5w-12r-12m	3330.10	3253.31	3423.92	100	19.65	14.98	28.02
19	5w-18r-4m	2539.33	2409.28	2653.67	100	15.13	11.43	18.89
20	5w-18r-8m	3664.15	3609.25	3761.73	71	16.64	13.41	20.49
21	5w-18r-12m	3661.57	3603.85	3701.26	48	21.62	19.33	26.74
22	5w-24r-4m	3692.89	–	–	84	15.33	–	–
23	5w-24r-8m	3645.52	–	–	43	18.55	–	–
24	5w-24r-12m	3615.57	–	–	28	25.07	–	–
25	10w-6r-4m	1185.87	1041.35	1328.69	100	19.73	11.79	38.28
26	10w-6r-8m	2328.48	2116.86	2550.46	100	19.13	10.38	29.09
27	10w-6r-12m	3202.60	3187.06	3212.82	100	14.06	5.80	27.22
28	10w-12r-4m	3662.31	3622.41	3711.31	78	24.80	19.55	30.08
29	10w-12r-8m	3656.07	3613.79	3726.73	40	31.53	11.83	50.02
30	10w-12r-12m	3689.49	3607.05	3854.19	26	22.12	9.15	42.04
31	10w-18r-4m	3679.92	3608.21	3741.46	31	21.57	2.87	42.44
32	10w-18r-8m	3635.05	3629.76	3762.33	15	15.10	1.87	32.55
33	10w-18r-12m	3618.71	3543.91	3694.02	10	17.80	5.34	33.41
34	10w-24r-4m	3674.65	–	–	20	31.60	–	–
35	10w-24r-8m	3661.02	–	–	14	18.34	–	–
36	10w-24r-12m	3694.42	–	–	10	16.60	–	–
	Grand average	2475.33			19.61			

Notes. *3w-6r-4m: 3w – 3 warehouses in each region; 6r – 6 regions; 4m – length of planning horizon 4 months.

5.3.4. Results of hybrid F&O with ILS

The performance of the proposed hybrid approach where the F&O heuristic is blended with ILS is analyzed. For the hybrid ILS-F&O heuristic, period decomposition scheme is selected for experimentation due to its best performance among all decomposition schemes. For the random F&O routine applied in the local search phase, $|W_u| \times |U| \times |F|$ number of x_{iu}^{ft} variables and $|M| \times |W| \times |W| \times |F|$ number of y_{ijm}^{ft} variables are kept free to be optimized, and 1% of the total binary variables are selected for mutation during the perturbation. The

TABLE 8. Summary of hybrid ILS-F&O heuristic performance.

Problem configuration number	Problem configuration*	Hybrid ILS-F&O heuristic						
		CPU Time (s)			Number of iterations	Solution gap (%)		
		Avg	Min	Max		Avg	Min	Max
1	3w-6r-4m	25	15	37	5	0.42	0.20	0.68
2	3w-6r-8m	78	56	115	10	1.64	0.34	2.31
3	3w-6r-12m	84	53	115	10	0.93	0.02	2.21
4	3w-12r-4m	368	296	439	10	1.13	0.44	2.70
5	3w-12r-8m	319	194	386	8	1.47	0.20	4.14
6	3w-12r-12m	375	342	487	4	1.10	0.20	1.57
7	3w-18r-4m	356	317	394	4	1.36	1.12	1.99
8	3w-18r-8m	422	338	577	3	2.03	1.26	3.07
9	3w-18r-12m	442	369	542	3	2.29	0.16	4.62
10	3w-24r-4m	918	–	–	7	1.74	–	–
11	3w-24r-8m	606	–	–	3	1.17	–	–
12	3w-24r-12m	611	–	–	2	2.44	–	–
13	5w-6r-4m	190	116	273	10	1.77	0.12	4.58
14	5w-6r-8m	103	60	129	10	1.49	0.08	2.89
15	5w-6r-12m	318	100	500	10	0.41	0.04	1.23
16	5w-12r-4m	611	525	674	8	1.03	0.43	1.75
17	5w-12r-8m	556	226	829	7	2.03	0.08	5.34
18	5w-12r-12m	626	607	647	3	0.75	0.11	1.66
19	5w-18r-4m	662	608	687	5	1.76	1.58	1.86
20	5w-18r-8m	766	651	873	3	0.81	0.08	1.59
21	5w-18r-12m	815	793	828	2	0.32	–0.98	1.25
22	5w-24r-4m	1204	–	–	6	1.78	–	–
23	5w-24r-8m	957	–	–	3	0.59	–	–
24	5w-24r-12m	891	–	–	2	2.71	–	–
25	10w-6r-4m	1155	1002	1309	10	1.15	0.60	2.50
26	10w-6r-8m	1140	661	1289	8	0.63	–0.61	1.66
27	10w-6r-12m	1039	602	1290	5	0.97	0.06	1.72
28	10w-12r-4m	1186	946	1863	6	1.28	0.75	1.85
29	10w-12r-8m	1017	969	1098	5	0.17	–0.85	2.03
30	10w-12r-12m	1105	926	1305	2	–0.31	–2.59	2.00
31	10w-18r-4m	1077	1043	1156	4	0.71	–0.51	1.68
32	10w-18r-8m	1336	1146	1507	4	–0.50	–1.02	0.18
33	10w-18r-12m	1248	1106	1318	2	–1.77	–4.23	–0.31
34	10w-24r-4m	1067	–	–	4	0.11	–	–
35	10w-24r-8m	992	–	–	2	–0.05	–	–
36	10w-24r-12m	1106	–	–	1	–0.28	–	–

Notes. *3w-6r-4m: 3w – 3 warehouses in each region; 6r – 6 regions; 4m – length of planning horizon 4 months.

convergence criterion for the hybrid approach is kept the same as that of the pure F&O heuristic. The results of the computational experiments for hybrid ILS-F&O heuristic are summarized in Table 8.

The comparative analysis of the pure F&O and the hybrid ILS-F&O heuristics is depicted in Figures 9 and 10. It is observed that the proposed hybrid approach takes slightly longer computational time for the small size problem instances. However, the hybrid ILS-F&O outperforms the pure F&O heuristic in terms of solution quality. For several medium to large size problem instances, the hybrid approach has slashed down the solution gap drastically. It is also observed that about 88% of the individual problem instances exhibit the solution gap of less than 2% in case of the hybrid ILS-F&O, while this figure is 66% for the pure F&O heuristic. Due

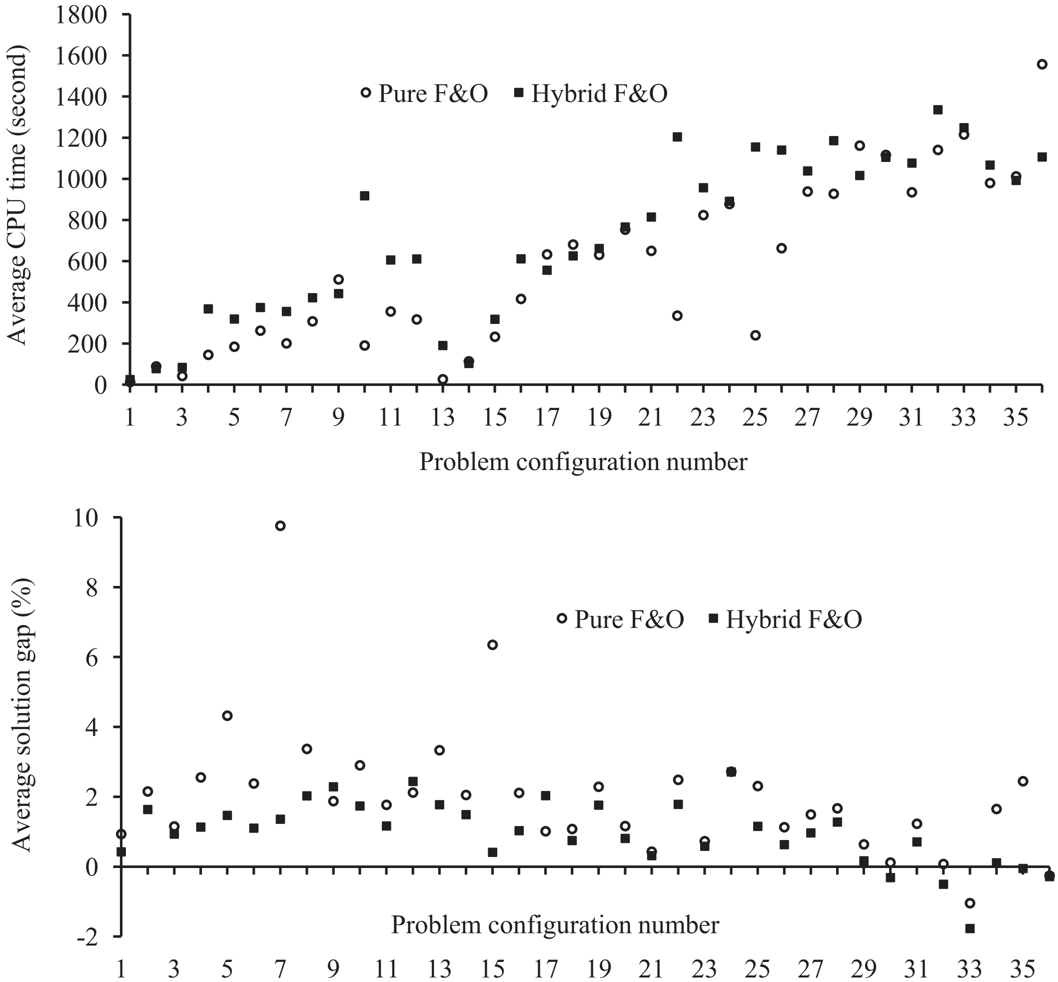

FIGURE 9. Comparison of performance of proposed pure F&O and hybrid ILS-F&O heuristics on average basis.

to this, increased CPU time for the proposed approach is justifiable. It is also observed that the hybrid ILS-F&O heuristic produces solutions which are better than the best-known solutions offered by Cplex in case of several large size problem instances. This indicates the necessity and superiority of the proposed hybrid solution approach.

6. Conclusions

In this work, a generic MIP model is formulated for the capacitated multi-item inventory-transportation problem in a multi-region, multi-facility supply chain and solved using F&O matheuristic. Several decomposition schemes considering period, region, and item and their combinations based decomposition schemes have been proposed, and their performance is evaluated on the basis of decomposition structure. Among the proposed decomposition schemes, the period decomposition outperforms the others. Also, it is observed that the performance of the decomposition schemes in terms of computational time and solution quality deteriorate with an increase in the number of sub-problems formed. A trade-off between the number and the size of sub-problems

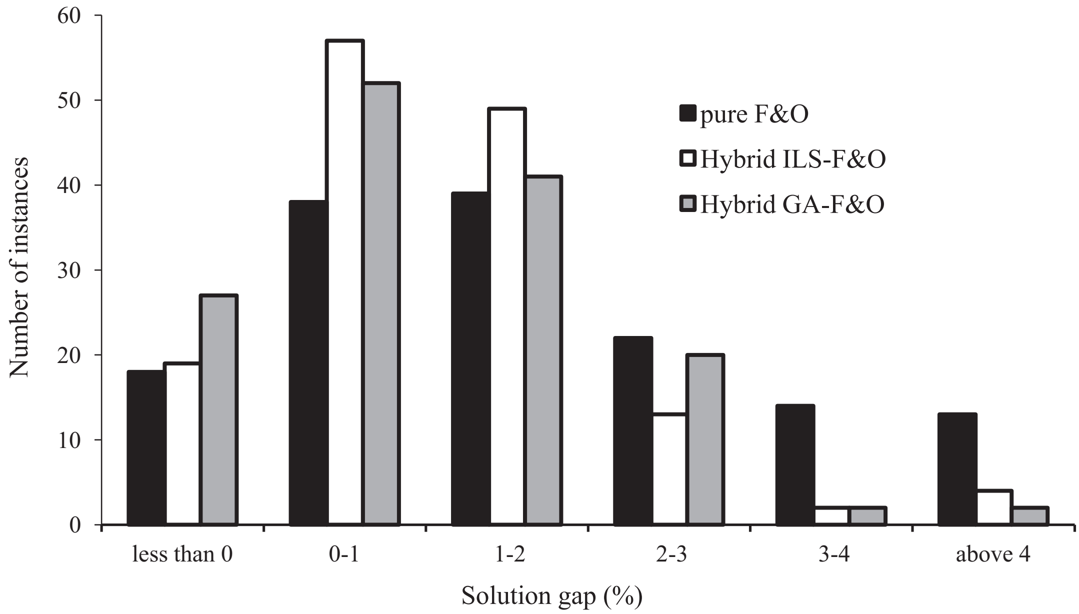

FIGURE 10. Comparison of performance of proposed pure and hybrid F&O on individual instance basis.

formed by decomposition schemes is also established in this work. The results of the computational experiments reveal that more than 90% of the problem instances have less than 3% of the solution gap, and the average CPU time taken is just one-sixth of that of Cplex. Thus, the F&O heuristic found to be computationally efficient and effective. Further, embedding GA into F&O routine as a diversification mechanism is found to be favorable in enhancing the quality of solution obtained from pure F&O heuristic. With an average solution gap of 0.80% from the best-known value, the hybrid approach is proved to be highly effective, but with an additional computational time requirement. In the proposed hybrid GA-F&O heuristic, due to the application of F&O heuristic to generate good quality solutions in the initial population, GA converges quickly. This approach performs better than the classical approach of generating a random initial population for GA. In the case of hybrid ILS-F&O heuristic, the average solution gap is observed as 0.98%, and it took one-fifth of the average CPU time of Cplex. Thus, we conclude that the hybrid GA-F&O is more effective, but hybrid ILS-F&O is more efficient. However, both hybrid heuristics found to be outperforming Cplex for most of the large size problem instances when compared for the same computational budget.

From the perspective of solution procedure, use of another mathheuristic such as relax and fix can be another alternative for future work. Also, the results obtained from the F&O heuristic and the hybrid heuristics can be compared with other metaheuristics such as iterative neighborhood search. Also, a single point crossover is used in GA. It would be interesting to see the results with a two point or multi-point crossover in pure GA and hybrid GA-F&O heuristic. Further, an extensive numerical study can be performed to analyse different impact of the proposed heuristics on solution quality. From the modelling perspective also the work can be extended in future in several other dimensions. The present work focuses on multi-region and multi-facility supply chain. It is possible to extend the model to a multi-level supply chain by incorporating the details of procurement points and distribution centers. Clubbing the distribution level decisions into the tactical level inventory decision would lead to an inventory-routing problem, which could be interesting to explore. Further, in order to plan the storage and transportation capacity optimally, a network design approach, and consideration of uncertainty in supply and demand would make the problem more realistic.

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