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GENERAL LOT-SIZING AND SCHEDULING FOR PERISHABLE FOOD PRODUCTS

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Abstract. General lot-sizing and scheduling is a well-studied problem in the literature, but for perishable or time-sensitive products is less investigated. Also, most of studies on perishable product supply chains focus on strategic and tactical decision levels rather than operational decision level and integrated operational and tactical decision levels. We focus on a general lot-sizing and scheduling problem faced by perishable food products. The lifespan and shelf life are two important key features of perishable products that are considered in the problem. This problem can be described as a multi-product, multi-parallel line, multi-period general lot-sizing and scheduling problem with sequence dependent change over time. The objective function is sum of production costs, inventory holding costs, waste costs, and lifespan related cost function. We apply two mixed-integer programming based heuristics to solve generated instances. The heuristics are compared in terms of solution quality and computational time. Also, the sensitivity analysis is presented to analyze the effects of parameters' changes.

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1. Introduction

Inspite of existence of many studies on general lot-sizing and scheduling problem in the literature, perishable or time-sensitive products is less investigated. Also, the strategic and tactical decision levels are more concentrated than the operational decision level in recent perishable products studies [29]. Some of perishable products that are investigated for production scheduling problems in the literature are food, typically yogurt, newspapers, blood bank, ready-mix concrete, chemical adhesive materials, so on. In light of customers' tendency to consume fresh and more healthily food, perishable food industry has received more attention in recent years. Fresh agricultural products, fresh meat, fish, and some dairy products are examples of perishable food industries with one day up to a month shelf life. There exists limited research on perishable food supply chains compared to other nonperishable supply chains. Perishable food industry can be distinguished from other industries with its characteristics such as, flow shop technology with sequence dependent set-up time (cost), and perishability with limited fixed or random shelf life of raw materials, intermediate and final products [34]. The fixed shelf life means

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product has predefined lifespan, instead, when shelf life of product is strongly dependent on holding situations such as temperature or humidity, the shelf life is considered random. Additionally, there is a predefined sequence of products, based on their cup size, taste or fat [15].

Perishability is one of the most crucial factors in modeling of perishable food supply chains. Amorim *et al.* [5] presented a new perishability classification with three dimensions, physical product deterioration (yes or no), authority limits (fixed or loose) and customer value (constant or decreasing). Based on their framework, perishable food physically deteriorates and we can classify perishable products in two dimensions: fixed shelf life or random shelf life, constant customer value (willingness) or decreasing customer value. The customer value means the value of product in each point of product lifespan in customer stand point. In this situation, to encourage customer to buy a product, pricing and discount strategies are applied based on product lifespan [40]. For example, yogurt as an important perishable food product, has fixed shelf life with decreasing customer value based on its lifespan.

The limited shelf life and high sequence dependent change over time (cost) in the production of perishable food, result in integrated lot-sizing and scheduling problem (LSP) [27]. The model changed to capacitated lot-sizing and scheduling (CLSP) with considering capacity constraints. Also existence of multiple product families, produced on single line or multiple parallel lines, led to general lot-sizing and scheduling problem (GLSP). In GLSP, the finite planning horizon is divided into T macro periods and each macro period is divided into set of micro periods with variable length. The number of micro periods in each macro periods is predefined. We refer the interested reader to the work of Copil *et al.* [10] for more information and notice of lot-sizing and scheduling problems' classification.

In this paper, we focus on a general lot-sizing and scheduling problem faced by perishable food products. The lifespan and shelf life are two key features of perishable products that are considered in the problem. This problem can be described as a multi-product, multi-parallel line, multi-period general lot-sizing and scheduling problem with sequence dependent change over time. The objective function is the sum of the production costs, inventory holding costs, waste costs, and lifespan related cost function. Our model is an extension of Amorim *et al.* [4] model, with considering quality control cost and storage possibility in the production plant. We apply two mixed-integer programming based heuristics to solve randomly generated instances. The heuristics are compared in terms of the solution quality and the computational time. Also, the sensitivity analysis is presented to analyze the effects of parameters' changes.

So, the main contributions of this paper are:

- An GLSP model considering lifespan related function in the objective.
- Considering quality control time in the model.
- Applying heuristics for proposed GLSP.

The paper is organized as follows: Section 2 presents a literature review on managing perishability as well as production planning and scheduling in perishable food supply chains. In Section 3, an MILP model for GLSP is proposed. Section 4 represents the solution method. Section 5 is dedicated to computational analysis. Finally, Section 6 presents conclusions and suggestions for future research.

2. Literature review

According to the relevant literature, in Section 2.1, managing perishability in perishable food supply chains is investigated, also in Section 2.2, we focus on production planning and scheduling in perishable food supply chain with fixed shelf life. Finally, Section 2.3 presents MIP-based decomposition heuristics.

2.1. Managing perishability

Perishable food has limited shelf life, from one day to a month. Also, the possibility of quality reduction exists through product lifespan. Quality reduction is strongly related to holding situation such as temperature and humidity as well as storage time. Then, lifespan and quality have important role on perishable food supply chain planning. There are different viewpoints in classifying perishable products. Nahmias [28] classified perishable products into two main categories based on the possibility of predetermining their shelf life; fixed shelf life and uncertain shelf life. The shelf life of products with fixed lifespan is predetermined and after passing that time the products will not have any value, such as some dairy products and yogurt. On the other hand, products with uncertain shelf life, for instance, fruit, vegetables and fresh meat, don't have any predefined shelf life and their shelf life is dependent on storage conditions such as temperature and humidity. Because of public health and waste reduction, quality and safety have an essential role on perishable food supply chain management [23]. Rong *et al.* [30] presented an MILP model for production and distribution planning of perishable food supply chain with considering quality degradation in tactical level. In the aforementioned paper quality degradation is computed based on a function proposed by Labuza and Man *et al.* [22, 25]. Tsiros and Heilman [37] presented another classification, from the customer viewpoint, of perishable products categorized to constant and decreasing customer value. They investigated purchasing behavior of customer and presented the willingness to pay (WTP) functions for high or moderate quality risk products. Amorim *et al.* [5] presented a new perishability classification with three dimensions, physical product deterioration (yes or no), authority limits (fixed or loose) and customer value (constant or decreasing). Based on their framework, perishable food physically deteriorates and we can classify perishable products in two dimensions: fixed shelf life or random shelf life, constant customer value (willingness) or decreasing customer value. Consequently, Amorim *et al.* [4] presented three concave, linear (for yogurt) and convex (for fresh meat) WTP functions for products with different quality risks (low, moderate, high quality risks) based on their lifespan and remained shelf life.

2.2. Production planning and scheduling in perishable food supply chain with fixed shelf life

Entrup *et al.* [24] introduced three models for production planning and scheduling for perishable food with integrating shelf life in the objective function. They developed an MILP model with block planning approach. Marinelli *et al.* [26] modeled parallel packaging machines with buffers as an CLSP with sequence independent set up time and cost, then, applied two-stage optimization decomposition approach. Doganis and Sarimveis [11] proposed an MILP scheduling model for a single machine packaging line with sequence dependent set up time and cost, consequently, they extended their model in another paper [12] for multiple paralleled machines, also they supplemented their model by considering shelf life related cost in the objective function and presented a new model in [13]. Kopanos *et al.* [19] modeled fermentation and packaging line together with considering families of products and sequence dependent set up time and cost. In the other paper, Kopanos *et al.* [20] presented a deterministic model for production planning and scheduling of yogurt packaging lines with capacity constraints in which the objective function is to minimize inventory holding costs, operating costs, changeover costs between families, plus possibility of outsourcing cost as a penalty cost. Also because of the less manpower use policy, the overlapping of production of families is minimized. Amorim *et al.* [3] proposed two multi-objective models for make to order (MTO) and hybrid make to order- make to stock (MTO-MTS) strategy for LSP with considering lifespan of products and block planning approach. They used Non-dominated Sorting Genetic Algorithm II (NSGA II) as a solution method. In consequence, Amorim *et al.* [4] integrated production planning together with perishability, and they defined demand function based on consumer behavior function which is related to price and lifespan of perishable products. Then stochastic programming is applied to tackle with demand uncertainty. Consequently, Amorim *et al.* [2], perceived the possibility of risk management in perishable food planning. They presented two risk-neutral and risk-averse scenarios based on stochastic models with considering demand, decay rate, consumer purchasing behavior uncertainties. In recent work, they made a review to all lot-sizing and scheduling models presented for yogurt production. Bilgen and Celebi [8] addressed an MILP model for production scheduling and distribution planning of yogurt production line based on the model presented by Doganis and Sarimveis [11] and shelf life function presented by Entrup *et al.* [24]. They considered operation times as dynamic factor and applied hybrid simulation- optimization approach. Finally, Sel *et al.* [33] presented a complete review of yogurt planning papers, and presented an MILP model for integrated production and distribution planning and scheduling of yogurt packaging and incubation line. They used big bucket and small bucket decomposition approach, and solved each sub model separately.

2.3. MIP-based decomposition heuristics

Finding optimal solution for real world LSP is a challenge for commercial solvers in computing time viewpoint [6]. Many heuristics and meta-heuristics are presented in the literature, but MIP-based heuristics such as relax-and-fix as well as fix-and-optimize are suitable approaches for LSP.

2.3.1. Relax-and-fix

Relax-and-fix heuristic "RF" is a constructive algorithm that can be used to obtain good initial solutions. In RF variables are divided into three groups. First group is considered fixed, the second group is optimized and the third one is relaxed. In this algorithm, at each iteration only the binary variables in optimized group should be optimized and considered fixed at the next iteration. The remaining variables are considered relaxed. Araujo *et al.* [6] presented RF to an LSP model. James and Almada-Lobo [18] presented a hybrid heuristic to solve single and parallel-machine CLSP which is NP-hard problem. They applied RF to construct an initial solution, and they performed a local search heuristic to improve the obtained initial solution.

2.3.2. Fix-and-optimize

The fix-and-optimize heuristic "FO" is an improvement heuristic and needs an initial solution. The variables are divided into two groups. First group is considered fix, the second group is optimized. In this algorithm, at each iteration only the binary variables in optimized group should be optimized and considered fixed at the next iteration. The remaining variables are considered fixed. There are three decomposition approaches to divide variables; product oriented decomposition, resource oriented decomposition, and time decomposition approach. Also, heuristics and meta-heuristics are used to construct initial solutions. Additionally, some heuristics are combined with FO to improve solution quality. Helber and Sahling [16] proposed several decomposition approaches to solve multi-level CLSP. They extended this model in their next paper [17] and proposed its stochastic version with considering demand uncertainty. They applied FO again. Lang and Shen [21] applied hybrid RF&FO to solve CLSP with sequence dependent setups with time decomposition approach. Goren *et al.* [14] combined Genetic Algorithm (GA) with FO to solve CLSP. Seeanner *et al.* [31] combined variable neighborhood search (VNS) with FO to solve GLSP. Toledo *et al.* [35] combined multi-population GA with FO to solve multi-item LSP with backlogging. Xiao *et al.* [39] proposed different hybrid heuristics based on RF&FO to solve CLSP. Sel and Bilgen [32] applied hybrid RF&FO to solve production and distribution planning problem in the soft drink industry. Chen [9] proposed FO to solve multi-level CLSP. Also, VNS is applied to improve the solution. Belo-Filho *et al.* [7] presented a lot-sizing and scheduling as well as vehicle routing model for perishable food supply chain. They applied the branch-and-bound tree to find an initial solution and use FO to improve the initial solution. They also, combined adaptive large neighborhood search with FO to improve solution quality. Wei *et al.* [38] proposed a tactical production and distribution planning model and used hybrid RF&FO joined with VNS to solve the problem.

3. Model formulation

The formulation of our GLSP model is presented in this section.

3.1. General lot-sizing and scheduling model formulation

The main assumptions of the model are as follows:

- Perishable food industry.
- Parallel packaging lines.
- Multiple family of products, based on the recipe.
- Each family contains a predefined sequence of products.

– Sequence dependent change over time (cost) between families.
– Sequence independent set up time (cost) for products.
– There is a quality control time after production to maintain product at the quality control room.
– There is a daily shut down time for the quality assurance
– Storage possibility in production plant.
– Products in inventory whose shelf life came to an end and cannot be sold are considered as wasted amount.
– Sequencing and scheduling is determined for families.

The notations are used in the model:

Indices and sets	
t	Time index for macro periods (days).
s	Time index for micro periods, $s \in \{1, \ldots, S\}$ (h).
a	Product lifespan (days).
f, $\acute{f}$	Product families index.
z	Products index.
l	Parallel production lines.
Z_f	Set of products belonging to family f.
S_{tl}	Set of micro periods in macro period t for production line l.
Costs and times	
$\mathrm{cc}_{f\acute{f}l}(\mathrm{ct}_{f\acute{f}l})$	Sequence dependent change over cost (time) from family f to family $\acute{f}$ on line l (\$ or h).
$\mathrm{sc}_{zl}(\mathrm{st}_{zl})$	Sequence independent set up cost (time) to product z on line l (\$ or h).
ic_z	Inventory holding cost of product z (\$/unit).
$\mathrm{ic}_z^{\mathrm{QC}}$	Inventory holding cost of product z in quality control room (\$/unit).
wc_z	Waste cost of product z (\$/unit).
$\mathrm{pc}_{zl}(p_{zl})$	Production cost (time) of product z on line l (\$/unit or h).
Other parameters	
m_{fl}	Minimum lot-size of family f on line l (kg).
d_{zt}	Demand of product z on macro period t (unit).
M	Maximum inventory storage capacity (kg).
pr_z	Price of product z (\$).
at_{lt}	Available time of line l on macro period t (h).
sht_{lt}	Daily shut down time in line l on macro period t (quality assurance time) (h).
Qt	Quality control time after production (days).
we_z	Weight of product z (kg/unit).
sl_z	Shelf life of product z (days).
lf_z	Loss factor of product z.
U_l	Line (Machine) utility.
λ	Customer sensitivity to product lifespan.
Binary variables	
δ_{fls}	Takes value 1, if line l is used for production of family f in micro period s, 0 otherwise.
$\varphi_{f\acute{f}ls}$	Takes value 1, if family f is immediately followed by family $\acute{f}$ on line l in the beginning of micro period s, 0 otherwise.
ϑ_{zls}	Takes value 1, if line l is used for production of product z in micro period s, 0 otherwise.
Decision variables	
q_{zls}	Production quantity of product z on line l in micro period s.
I_{zt}^a	Inventory of product z at macro period t with age a.
x_{zt}^a	Quantity of inventory I_{zt}^a used to satisfy in macro period t.
w_{zt}	Wasted amount of product z in macro period t.

The objective functions and constraints are as follows:

$$\text{Minimize } Z = \sum_f \sum_{\acute f} \sum_l \sum_s \text{cc}_{f\acute f l} \varphi_{f\acute f ls} + \sum_z \sum_l \sum_s (\text{sc}_{zl} \vartheta_{zls} + \text{pc}_{zl} q_{zls}) + \sum_z \sum_t \sum_{a=0}^{\min(t-1,\text{Qt}-1)} \text{ic}_z^{\text{QC}} I_{zt}^a$$
$$+ \sum_z \sum_{t>\text{Qt}} \sum_{a=\text{Qt}}^{\min(t-1,\text{sl}_z)} \text{ic}_z I_{zt}^a + \sum_z \sum_{t>\text{Qt}} \text{wc}_z w_{zt} + \sum_z \sum_{t>\text{Qt}} \sum_{a=\text{Qt}}^{\min(t-1,\text{sl}_z)} \text{pr}_z x_{zt}^a \frac{\lambda a}{\text{sl}_z - 1}. \tag{3.1}$$

Subject to
The time and production capacity constraints:

$$q_{zls} \le \frac{(\text{at}_{lt} - \text{sht}_{lt})}{p_{zl}} \vartheta_{zls} \qquad \forall z,l,s,t \tag{3.2}$$

$$\sum_{z \in Z_f} q_{zls} \ge m_{lf} \left(\delta_{fls} - \delta_{fl,s-1}\right) \qquad \forall l,s,f \tag{3.3}$$

$$\sum_f \sum_{\acute f} \sum_{s \in S_{tl}} \text{ct}_{f\acute f l} \varphi_{f\acute f ls} + \sum_z \sum_{s \in S_{tl}} (p_{zl} q_{zls} + \text{st}_{zl} \vartheta_{zls}) \le \text{at}_{lt} - \text{sht}_{lt} \qquad \forall l,t. \tag{3.4}$$

$$\tag{3.5}$$

The set up and change over constraints:

$$\sum_{z \in Z_f} \vartheta_{zls} \le \delta_{fls} \left|Z_f\right| \qquad \forall f,l,s \tag{3.6}$$

$$\sum_f \delta_{fls} = 1 \qquad \forall l,s \tag{3.7}$$

$$\varphi_{f\acute f ls} \ge \delta_{fl,s-1} + \delta_{\acute f ls} - 1 \qquad \forall l,f,\acute f,s. \tag{3.8}$$

Inventory balance constraints:

$$\sum_l \sum_{s \in S_{tl}} q_{zls} = I_{z,t}^0 \qquad \forall z,t \tag{3.9}$$

$$I_{z,t+1}^{a+1} = I_{z,t}^0 \qquad \forall a = 0,\ldots,\text{Qt}-1, z, t \tag{3.10}$$

$$I_{z,t+1}^{a+1} = (1 - \text{lf}_z)\, I_{zt}^a - x_{zt}^a \qquad \forall z, t > \text{Qt}, a = \text{Qt},\ldots,\min(t-1,\text{sl}_z) \tag{3.11}$$

$$w_{zt} = I_{zt}^{\text{sl}_z+1} \qquad \forall z, t \ge \max\{\text{sl}_z + 2, \text{Qt}+1\}. \tag{3.12}$$

Demand satisfaction constraints:

$$\sum_{a=\text{Qt}}^{\min(t-1,\text{sl}_z)} x_{zt}^a \ge d_{zt} \qquad \forall z, t > \text{Qt}. \tag{3.13}$$

Binary and non-negative variables related constraints:

$$q_{zls} \ge 0, \qquad \forall z,l,s \tag{3.14}$$

$$x_{zt}^a \ge 0, \qquad \forall z, t > \text{Qt},\ a = \text{Qt},\ldots,\ \min(t-1,\text{sl}_z) \tag{3.15}$$

$$I_{zt}^a \ge 0, \qquad \forall z,t,a \tag{3.16}$$

$$w_{zt} \ge 0, \qquad \forall z, t > \text{Qt} \tag{3.17}$$

$$\vartheta_{zls}, \delta_{fls}, \varphi_{f\acute f ls} \in \{0,1\} \qquad \forall z,l,f,\acute f,s. \tag{3.18}$$

The objective function in equation (3.1) is the sum of sequence dependent change over cost, sequence independent set up cost, production cost, inventory holding costs and waste cost, and the lifespan related cost. Our model takes into account fixed shelf life with decreasing linear function for customer value based on [4,37]. They presented equation (3.19) to calculate price or value of a product based on its age in customer viewpoint, so we will have:

$$\mathrm{pr}_{za} = \mathrm{pr}_z - \frac{\mathrm{pr}_z \lambda a}{\mathrm{sl}_z - 1}. \tag{3.19}$$

In equation (3.19) pr_{za} is the value of a product based on its lifespan a. Since profit of product z with lifespan a, (B_{za}), is equal to the price multiplied by the transmitted amount, we will have equation (3.20):

$$B_{za} = \mathrm{pr}_{za} x^a_{zt} = \mathrm{pr}_z x^a_{zt} - \frac{\mathrm{pr}_z \lambda a}{\mathrm{sl}_z - 1} x^a_{zt}. \tag{3.20}$$

And the lost profit, ΔB_{za}, is commutated by equation (3.21):

$$\Delta B_{za} = \left(\mathrm{pr}_{za} - \mathrm{pr}_z\right) x^a_{zt} = -\mathrm{pr}_z x^a_{zt} \frac{\lambda a}{\mathrm{sl}_z - 1}. \tag{3.21}$$

Based on equation (3.21), the lost profit should be reduced for all z with all a in each time period t. This equation is considered as lifespan related cost function in the objective function.

Constraints (3.2) ensure that production quantity is less than available capacity of line l at period s. Constraints (3.3) satisfy minimum production quantity (minimum lot size) of family f. Additionally, constraints (3.5) guarantee that total time consumed to set up, change over and production is less than total available time minus daily shut down time for quality assurance, on each line in each macro period t. Constraints (3.6) show that the sum of products types produced in line l in micro period s must not exceed the total number of product types of this particular family. Moreover, using the line in period s for production of the product types is possible only if the line l is used for production of this family f in micro period s. Constraints (3.7) ensure that each family is assigned only to one line at each micro period s. Constraints (3.8) guarantee that the change over from family f to family $\acute{f}$ on line l in micro period s is taken place, if family f has been assigned to micro period $s-1$, besides family f and $\acute{f}$ have been set up on the same line l. According to constraints (3.9), the total amount of production quantity of product z on each line l and in each micro period s in macro period t, is considered as inventory of product z with age zero. Constraints (3.10) show the amount of inventory after quality control time. Constraints (3.11) ensure that inventory of product z in each period is equal to previous period inventory minus quantity transmitted at period t for different ages. Also, Constraints (3.12) determine wasted amount of products. Constraints (3.13) satisfy demands. Finally, constraints (3.14)–(3.18) show binary and non-negative variables related constraints.

4. Solution method

Because of the commercial solvers inability to find the optimal solution for real world cases within a reasonable computational time, we apply two MIP-based heuristics to solve the presented model. First, we apply RF as a constructive heuristic to find an initial feasible solution. In the second approach, a hybrid heuristic presented, we use FO to improve the obtained initial solution with RF. Then the quality of solutions as well as performance of heuristics are analyzed. Also, the sensitivity analysis is implemented to show heuristics parameters' changes.

4.1. RF heuristic

In this method, the main problem is decomposed to sub-problems. The time horizon divided into three sub time windows. In the first time window, known as fixed window, the binary variables are fixed. In the second time window, the optimized window, the binary variables should be optimized and in the latest time window, relaxed window, the binary variables are relaxed. For instance, the time decomposition approach is shown in

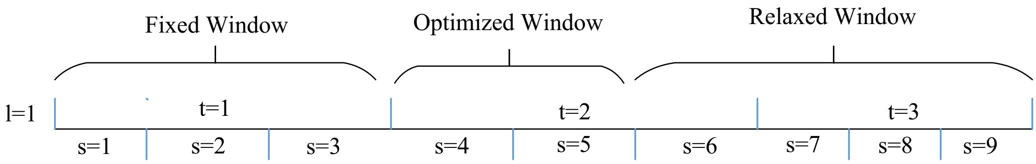

FIGURE 1. The time decomposition approach in RF heuristic.

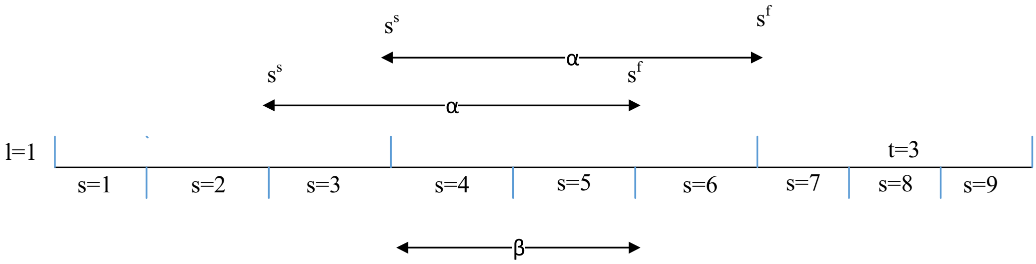

FIGURE 2. Two consecutive optimized windows.

Figure 1, for a hypothetical GLSP problem with a line l, 3 macro periods t, and 9 micro periods s. Each macro period is divided into three micro periods. The fixed window includes micro periods 1–3, the optimized window includes micro periods 4,5 and finally, the relaxed window includes micro periods 6–9.

Both micro and macro periods can be considered as time window unit. For the presented model in this paper we consider micro periods as time unit based on Seeanner *et al.* [31]'s paper. We show the starting point and the end point of the optimized window respectively with s^s and s^f, then the optimized window period is (s^s, s^f). In the same way, the fixed window period will be $(1, s^s - 1)$, and the relaxed window period is $(s^f + 1, S)$. Where the planning horizon spans from 1 to S. To determine each window, two parameters should be defined; the overlapped periods, β, and the interval length, α. The parameter α shows the number of time units in the optimized window and the parameter β indicates the number of intervals that are re-optimized. In fact, it shows the amount of overlap between two optimized windows in two consecutive iterations. Figure 2 presents a planning horizon with 9 micro periods for line l. Two consecutive optimized windows are shown with time windows (s^s, s^f). The parameter α is 3 and the parameter β is 2.

The RF starts with first micro period, then defines the optimized window and relaxed window based on the parameter α (in the first iteration there is no fixed window). The binary variables in the relaxed window are considered relaxed. The obtained sub problem is solved. The computed binary variables' measures for the optimized window in the feasible solution are considered fixed in the next iteration. The new fixed window, optimized window and relaxed window are defined based on parameters α and β. This algorithm is repeated until the end of the planning horizon.

In our model, there are three types of binary variables. The binary variable ϑ_{zls}, takes value 1 if product z *is produced* on line l in micro period s. The binary variable δ_{fls}, takes value 1 if family f *is produced* on line l in micro period s and finally, the binary variable $\varphi_{f\acute{f}ls}$, indicates the production change over from family f to $\acute{f}$ from micro period $s-1$ to s. Because of this binary variable, $\varphi_{f\acute{f}ls}$, considering overlap between two consecutive optimized windows is necessary.

In each iteration the obtained values for three binary variables $\vartheta_{zls}, \delta_{fls}, \varphi_{f\acute{f}ls}$, in optimized window will be named with $\bar{\vartheta}_{zls}$, $\bar{\delta}_{fls}, \bar{\varphi}_{f\acute{f}ls}$ and will be fixed in the fixed window for the next iteration. So, $\bar{\vartheta}_{zls}$, $\bar{\delta}_{fls}, \bar{\varphi}_{f\acute{f}ls}$ are the optimized value of binary variables $\vartheta_{zls}, \delta_{fls}, \varphi_{f\acute{f}ls}$. In each iteration of the relax-and-fix algorithm,

TABLE 1. The new constraints added to the main GLSP problem.

$\vartheta_{zls} = \bar{\vartheta}_{zls},\ \delta_{fls} = \bar{\delta}_{fls},\ \varphi_{ffls} = \varphi_{ffls}$	$s \in \{1, s^s - 1\}, \forall z, l, f$	For the fixed window
$\vartheta_{zls},\ \delta_{fls},\ \varphi_{ffls} \in \{0, 1\}$	$s \in \{s^s, s^f\}, \forall z, l, f$	For the optimized window
$\vartheta_{zls},\ \delta_{fls},\ \varphi_{ffls} \in [0, 1]$	$s \in \{s^f + 1, S\}, \forall z, l, f$	For the relaxed window

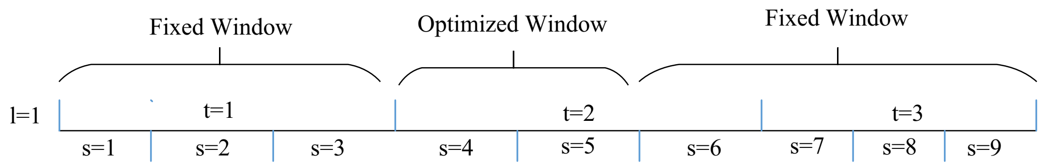

FIGURE 3. The time decomposition approach in FO heuristic.

some new constraints are appended to the main GLSP problem to construct a new mixed integer sub problem, $Sub\,\text{MIP}^{RF}$. These constraints are shown in Table 1.

The RF is presented in Algorithm 1.

Algorithm 1. Relax-and-Fix heuristic ($\boldsymbol{\alpha, \beta}$).

$s^s \leftarrow 1, s^f \leftarrow \alpha$
while $s^f \leq S$ *do*
  Solve Sub MIP^{RF}
  $\vartheta = \bar{\vartheta}, \delta = \bar{\delta}, \varphi = \bar{\varphi}$
  $s^s \leftarrow s^f - \beta, \quad s^f \leftarrow s^f + \alpha - \beta$
  if $s^f > S$ then
    $s^f \leftarrow S$
  end if
end while

To improve the initial feasible solution obtained by RF, we can apply some improvement heuristics. In the next section the FO is presented as improvement heuristic.

4.2. Hybrid RF& FO

In the FO two time windows, fixed window and optimized window, are defined. Figure 3 represents a planning horizon with 9 micro periods, the optimized window's length is assumed 2, and all of the remaining micro periods are considered as the fixed window. As it is known to implement this algorithm, we first need an initial feasible solution. In each iteration an optimized window is chosen and the remained ones are considered as fixed windows. In this paper we use time decomposition approach to decompose the binary variables.

There are two parameters in the time decomposition approach; time interval α and time overlapping β. The FO starts with first micro period, then defines the optimized window and fixed window based on the parameter α. The binary variables in the fixed window are considered fixed based on the initial solution, "$Ini\ Sol^{RF}$", obtained by RF. Firstly, the initial solution considered as the best solution, "BestSol". The obtained sub problem is solved. If the current solution in the iteration, "IterSol", is better than "BestSol", then "BestSol" will be replaced. The computed values for binary variables the "BestSol" are considered fixed in the next iteration for the optimized window. The new fixed window and optimized window are defined based on parameters α and β. This algorithm is repeated until the end of the planning horizon. For three binary variables $\vartheta_{zls}, \delta_{fls}, \varphi_{ffls}$, consider the obtained values in "BestSol" with $\bar{\vartheta}_{zls},\ \bar{\delta}_{fls}, \bar{\varphi}_{ff\acute{l}s}$. In each iteration of FO, some new constraints

TABLE 2. The new constraints added to construct $Sub\ MIP^{FO}$.

$\vartheta_{zls} = \bar{\vartheta}_{zls},\ \delta_{fls} = \bar{\delta}_{fls},\ \varphi_{ffls} = \bar{\varphi}_{ffls}$	$\forall s \in \{S\} \setminus \{s^s, s^f\}, \forall z, l, f$	For the fixed window
$\vartheta_{zls},\ \delta_{fls},\ \varphi_{ffls} \in \{0, 1\}$	$\forall s \in \{s^s, s^f\}, \forall z, l, f$	For the optimized window

are appended to the main GLSP problem to construct a new mixed integer sub problem, *Sub* MIP^{FO}. These constraints are shown at Table 2.

The hybrid RF&FO is presented in Algorithm 2.

Algorithm 2. hybrid relax-and-fix & fix-and-optimize heuristic ($\boldsymbol{Ini\ Sol^{RF}, \alpha, \beta}$).

```
BestSol=Ini Sol^RF
s^s ← 1, s^f ← α
while      s^f ≤ S   do
           Solve Sub MIP^FO
           Obtain IterSol
           If      IterSol<BestSol then
                   ϑ = ϑ̄, δ = δ̄, φ = φ̄
                   BestSol=IterSol
           end if
           s^s ← s^f − β,   s^f ← s^f + α − β
           if      s^f > S then
                   s^f ← S
           end if
end while
```

5. Computational analysis

The problem is an GLSP with L parallel lines, to produce F families and Z products. The problem parameters are presented in Table 3 that are extended based on the literature [1].

The values of heuristics parameters (α and β) are considered at each next sections separately. To investigate the heuristics performance three approaches are presented in three subsections. Subsection 5.1 reviews the presented heuristic solution quality for problems with different sizes; small-size, medium-size and large-size problem. Subsection 5.2 investigates the heuristic performance based on changes in key input parameters, for instance, demand variation, setup costs and so on. Finally, subsection 5.3 presents sensitivity analysis based on changes in heuristic parameters α and β.

5.1. The heuristic solution quality

The heuristic solution quality is investigated with three indices, objective value, computation time, and GAP, for generated instances. To evaluate the performance of presented heuristics, the problems are solved by FR, FR&FO and the CPLEX solver directly. The time limit for CPLEX solver is 3600 s. Goren *et al.* [14] introduce the index GAP to compare solution quality between different heuristics. GAP is the relative difference between the heuristic solution and the best feasible solution by the CPLEX solver within the CPU time limitation. The equation (5.1) shows computation method of GAP:

$$\text{GAP} = \frac{heuristic\ obj. - best\ obj.}{best\ obj.}. \tag{5.1}$$

TABLE 3. The values of parameters.

Description	Parameter	Values
Change over time from family f to family $\acute{f}$ on line l (h)	$\mathrm{ct}_{f\acute{f}l}$	$\sim U[2,5]$
Change over cost from family f to family $\acute{f}$ on line l (\$)	$\mathrm{cc}_{f\acute{f}l}$	$\mathrm{cc}_{f\acute{f}l} = 50 * \mathrm{ct}_{f\acute{f}l}$ $\mathrm{cc}_{f\acute{f}l} = 0$ for $f = \acute{f}$
Set up time for products (h)	st_{zl}	$\sim U[0.5,1]$
Set up cost for products (\$)	sc_{zl}	$50 * \mathrm{st}_{zl}$
Product price (value) (\$)	pr_z	2
Inventory holding cost (\$/unit)	ic_z	$0.1 * \mathrm{pr}_z$
Inventory holding cost in quality control room (\$/unit)	$\mathrm{ic}_z^{\mathrm{QC}}$	$0.15 * \mathrm{pr}_z$
Waste cost (\$/unit)	wc_z	pr_z
Production time (h)	p_{zl}	1
Production cost (\$/unit)	pc_{zl}	$0.5 * p_{zl}$
Minimum production capacity (unit)	m_{fl}	1
Demand (unit)	d_{zt}	$\sim U[40,60]$.
Line (machine) utility	U_l	0.7
Available time (h)	at_{lt}	$\mathrm{at}_{lt} = \frac{\sum_z d_{zt} p_{zl}}{U_l}$
Shut down time (h)	sht_{lt}	1
Quality control time (day)	Qt	1
Product shelf life (day)	sl_z	$\sim U[1,T]$
Loss factor of product z	lf_z	0.1
Customer sensitivity to product lifespan	λ	0.5
Number of micro periods s in macro period t for production line l	$\lvert S_{tl}\rvert$	F
Total number of micro periods	S	$T \times \lvert S_{tl}\rvert$
FR heuristics parameters		
The fixed window	s	$s \in \{1, s^s - 1\}$
The optimized window	s	$s \in \{s^s, s^f\}$
The relaxed window	s	$s \in \{s^f + 1, S\}$
FO heuristics parameters		
The fixed window	s	$s \in \{1, \cdots, S\} \setminus \{s^s, s^f\}$
The optimized window	s	$s \in \{s^s, s^f\}$

Also, the GAP cumulative distribution function can be applied to compare different heuristic solutions quality based on Belo-Filho *et al.* [7] paper. The equation (5.2) represents the formula:

$$F_a^{\mathrm{GAP}}(\lambda) = \frac{\left|\{i \in I : \mathrm{GAP}^{a,i} \leq \lambda\}\right|}{|I|} \forall\ a \in A,\ \lambda \in [0,1] \tag{5.2}$$

where, A is set of applied heuristics, I is a set of instances with cardinality $|I|$, a is the studied heuristic, λ is a scalar in [0,1], $\mathrm{GAP}^{a,i}$ is the GAP of heuristic a for instance$i$, and $F_a^{\mathrm{GAP}}(\lambda)$ is the GAP cumulative distribution function value for heuristic a and λ.

One way to evaluate the performance of the presented solution approaches is to generate problems of different sizes and to analyze the GAPs and the computational time. By giving different values to the effective parameters, iterations with different size can be generated. In this problem the number of families (F), the total number of products (Z), the number of lines (L), and the planning horizon (T) have major impact on the problem size as well as computational time needed to verify the optimal solutions. The given measures to these parameters are shown in Table 4.

Table 4. The effective parameters values on problem size.

Parameter's name	# of families (F)	Total # of products (Z)	# of lines (L)	Planning horizon (T)
Parameter's value	5,10	10,15,20	5,10	7,15

Table 5. Summary of the results for all instances ($\alpha = 10, \beta = 5$).

Instances (i)	# of bin. V	RF			FR&FO			CPLEX	
$F * Z * L * T$		Obj.	GAP	Time(s)	Obj.	GAP	Time(s)	Obj.	Time(s)
$2 * 10 * 2 * 7$	392	1852.71	0	0.328	1852.71	0	0.171	1852.71	1.50
$5 * 10 * 5 * 7$	6125	1642.08	0.353	3.869	1636.8	0.030	3.229	1562.212	2.11
$5 * 15 * 5 * 7$	7000	2644.01	0.390	4.056	2633.73	0	10.904	2633.845	5.79
$5 * 20 * 5 * 7$	7875	3283.45	0.246	6.816	3275.51	0.004	8.221	3275.383	10.86
$5 * 10 * 5 * 15$	13 125	4457.04	0.269	8.345	4445.07	0	8.905	4445.069	14.37
$5 * 15 * 5 * 15$	15 000	6685.5	0.400	11.31	6659.33	0.007	14.585	6658.844	21.09
$5 * 20 * 5 * 15$	16 875	8197.45	0.330	14.367	8170.59	0.002	23.385	8170.44	73.11
$10 * 10 * 10 * 7$	77 000	1526.56	0.970	51.716	1511.91	0.001	24.286	1511.891	81.49
$10 * 20 * 10 * 7$	84 000	3246.36	0.690	51.776	3224.11	0	75.552	3224.11	286.699
$10 * 10 * 10 * 15$	165 000	4084.68	0.521	258.666	4066.92	0.084	581.04	4063.5	>3600

In this section, the number of micro periods in each macro period is considered equal to the number of families, so the number of micro periods will be $S = F * T$. Each problem runs 10 times and the average value are shown in Table 5.

Table 5 shows, the RF heuristic in spite of having more GAP than the FR&FO heuristic, could find near optimal solution in less computational time. This means that both of heuristics are applicable and competitive for large-size problems to obtain the near optimal solutions in reasonable time and with acceptable GAP.

Figure 4 illustrates the cumulative distribution functions for the computed GAPs. It shows that the hybrid relax-and-fix & fix-and-optimize heuristic (RF-FO) has less gaps in comparison with the relax-and-fix [36] heuristic.

5.2. The heuristic performance based on scenarios

Another way to evaluate the heuristics performances is to generate scenarios by changing the key parameters values for a typical problem. For this typical problem, based on the literature and the expert views, the main parameters are listed in Table 6. Two levels for each parameter are determined, L for low level and H for high level.

In this subsection, we consider problem $5 * 10 * 5 * 7$ from the previous subsection. The number of generated scenarios based on five parameters' levels is equal to $2^5 = 32$. Each scenario runs 10 times and the obtained averages are considered in Table 7.

The importance of parameters and their impact need to be analyzed. For this purpose, the average GAPs for different parameters levels are computed based on data from Table 7. Table 8 shows these average GAPs for scenarios.

According to the Table 8, the FR heuristics again has more average GAP than FR&FO heuristic in different scenarios for a specific problem. The maximum GAP is for line utility at low level and the minimum GAP belongs to inventory holding cost at its high level for FR heuristic. Also, for the next heuristic the maximum GAP is for demand variation at high level and the minimum GAP belongs to line utility at high level. These

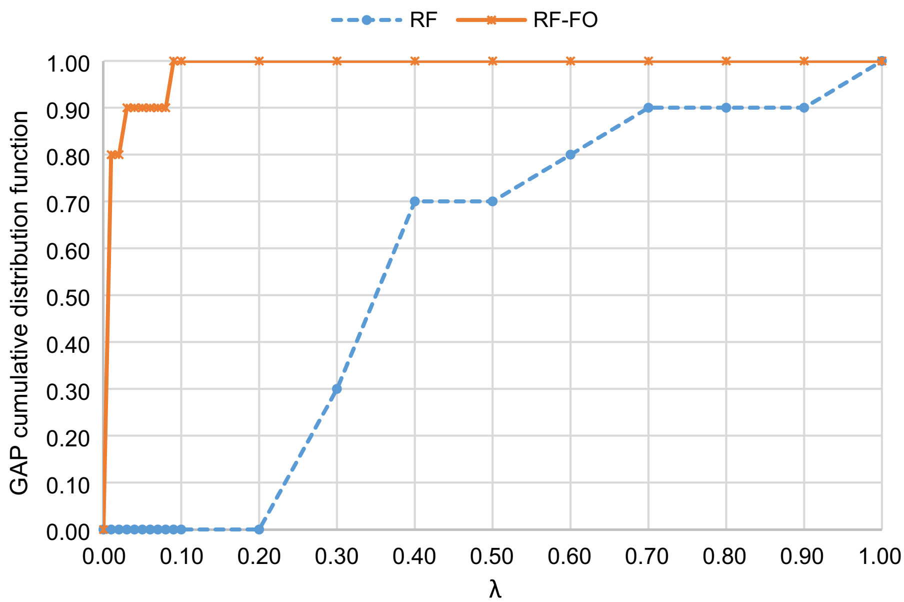

Figure 4. The performance of heuristics.

Table 6. Notations and settings of key parameters.

Notations	Settings
Families change over time (ct_{ffl})	L:$\sim U[2,5]$, H: $\sim U[10,25]$
Inventory holding cost (ic_z)	L: $0.1 * \mathrm{pr}_z$, H: $1 * \mathrm{pr}_z$
Lose factor (lf_z)	L: 0.1, H: 0.5
Demand variation (d_{zt})	L:$\sim U[40,60]$, H: $\sim U[0,100]$
Line utility (U_l)	L: 0.7, H:0.9

results show that the performance of the presented heuristics are related to key parameters changes and in different scenarios they could have variant performance.

To analyze Table 8 accurately, a new index called the relative increase in GAPs, $Rel.Inc^{GAP}$, is defined. It shows the effect of parameter changes on the average GAPs changes. The relative deviation of average GAP for parameter's high level, $ave.GAP^{H}$, from average GAP for parameter's low level, $ave.GAP^{L}$, is defined as relative increase in GAPs, $Rel.Inc^{GAP}$, and is shown in equation (5.3).

$$Rel.Inc^{GAP} = \frac{ave.GAP^{H} - ave.GAP^{L}}{ave.GAP^{L}} * 100. \tag{5.3}$$

The relative increase in GAPs, $Rel.Inc^{GAP}$, for five key parameters are summarized in Table 9. Based on Table 9, the change over time between families, loss factor and demand variations have more relative increase in GAPs in comparison with the other parameters. Based on this analysis, the changes of these parameters might effect on the heuristics performance more than the other parameters.

5.3. Sensitivity analysis of heuristic's parameters

The third way to evaluate the performance of the heuristics is sensitivity analysis of heuristics parameters. The sensitivity analysis is applied to evaluate the effect of heuristic's parameters α and β on computation time,

Table 7. The parameters effect and scenarios.

Scenarios	Obj. in 3600 s			Mean GAP (%) in 3600 s		Time		
ct-ic-lf-d-U	FR	FR&FO	Cplex	FR	FR&FO	FR	FR&FO	Cplex
H-H-H-H-H	9770.67	9770.67	9770.67	0.00000%	0.00000%	2.309	2.434	1.888
H-H-H-H-L	8947.92	8943.25	8942.72	0.05222%	0.00593%	3.151	2.091	1.435
H-H-H-L-H	9941.6	9937.49	9937.49	0.04136%	0.00000%	1.903	2.589	7.005
H-H-H-L-L	9290.47	9285.71	9285.71	0.05126%	0.00000%	33.134	9.705	45.568
H-H-L-H-H	5555.81	5554.71	5554.19	0.02917%	0.00936%	3.604	2.451	1.638
H-H-L-H-L	5373.8	5372.06	5371.73	0.03239%	0.00614%	3.261	4.945	1.934
H-H-L-L-H	5531.53	5529.75	5529.75	0.03219%	0.00000%	2.138	3.619	1.482
H-H-L-L-L	5202.61	5199.43	5199.43	0.06116%	0.00000%	2.543	12.839	1.488
H-L-H-H-H	2788.75	2786.96	2786.92	0.06566%	0.00144%	2.497	5.226	16.364
H-L-H-H-L	2797.39	2796.43	2795.45	0.06940%	0.03506%	62.51	10.109	393.528
H-L-H-L-H	2541.8	2539.72	2539.65	0.08190%	0.00276%	3.229	3.508	1.482
H-L-H-L-L	2675.36	2670.17	2670.17	0.19437%	0.00000%	3.885	14.383	767.432
H-L-L-H-H	1596.08	1592.42	1592.42	0.22984%	0.00000%	1.824	4.96	2.606
H-L-L-H-L	1723.96	1715.78	1715.75	0.47851%	0.00175%	3.448	3.231	3.51
H-L-L-L-H	1714.29	1711.9	1711.82	0.13961%	0.00467%	3.119	2.981	1.778
H-L-L-L-L	1672.86	1663.82	1663.82	0.54333%	0.00000%	3.728	5.678	3.229
L-H-H-H-H	8983.13	8981.12	8980.73	0.02672%	0.00434%	3.259	2.854	2.074
L-H-H-H-L	8783.52	8781.59	8781.59	0.02198%	0.00000%	12.87	4.273	4.774
L-H-H-L-H	9865.77	9862.8	9862.8	0.03011%	0.00000%	2.448	6.054	17.394
L-H-H-L-L	8633.13	8628.44	8627.94	0.06015%	0.00580%	2.45	5.711	9.282
L-H-L-H-H	5411.49	5400.24	5400.09	0.20832%	0.00278%	3.79	2.075	1.326
L-H-L-H-L	5149.95	5143.76	5143.76	0.12034%	0.00000%	1.996	2.886	2.184
L-H-L-L-H	5220.84	5215.53	5215.53	0.10181%	0.00000%	3.541	6.115	239.134
L-H-L-L-L	5346.81	5343.08	5343.08	0.06981%	0.00000%	1.918	2.091	1.919
L-L-H-H-H	2638.48	2637.37	2637.37	0.04209%	0.00000%	3.37	4.258	6.989
L-L-H-H-L	2656.68	2651.27	2651.27	0.20405%	0.00000%	2.528	8.361	77.408
L-L-H-L-H	2691.85	2688.84	2688.84	0.11194%	0.00000%	3.602	6.007	2.106
L-L-H-L-L	2714.76	2712.14	2712.14	0.09660%	0.00000%	3.416	3.12	1.591
L-L-L-H-H	1683.54	1678.43	1678.43	0.30445%	0.00000%	3.385	4.243	59.452
L-L-L-H-L	1787.25	1781.59	1781.59	0.31769%	0.00000%	2.667	2.652	1.748
L-L-L-L-H	1744.05	1736.29	1736.29	0.44693%	0.00000%	2.823	4.541	8.736
L-L-L-L-L	1629.26	1624.46	1624.46	0.29548%	0.00000%	2.45	2.963	2.153
Mean				0.14232%	0.00090%			

solution quality as well as GAP index for two different size typical problems In RF, we defined time interval α and window overlapping β based on the problem parameters. They are computed based on equation (5.4).

$$\alpha = \left[\frac{T \times |S_{tl}| \times L}{4}\right], \beta = [0.3 \times \alpha]\,. \tag{5.4}$$

In contrast, for FO the time interval α and time overlapping β have different values based on equation (5.5).

$$\alpha = 10, 20, 30, 40, \ldots, T \times |S_{tl}| \qquad \beta = 0, \ldots, \alpha - 1. \tag{5.5}$$

For problem instance $5*10*5*7$ the initial optimal objective value from RF algorithm is 1709.24. This value is improved by FO to 1699.13. By changing the parameters α, β, the objective function did not change, but the computational time changed. The computation time changes based on different values of parameters are shown in Table 10.

TABLE 8. The average gaps based on parameters levels.

Parameter	Level	Average gap% FR	Average gap% FR&FO
Families change over time ($\text{ct}_{f\acute{f}l}$)	H	0.132617	0.002975
	L	0.15383	0.000634
Inventory holding cost (ic_z)	H	0.059616	0.001219
	L	0.226831	0.00239
Lose factor (lf_z)	H	0.072407	0.002914
	L	0.21404	0.000694
Demand variation (d_{zt})	H	0.138606	0.003247
	L	0.147841	0.000362
Line utility (U_l)	H	0.118896	0.000946
	L	0.167551	0.002663
Mean		0.143224	0.001804

TABLE 9. The relative increase in gaps.

Parameter's change	$Rel.Inc^{gap}$ FR	$Rel.Inc^{gap}$ FR&FO
ct**(L→H)**	−13.7898	**369.5619**
ic(L→H)	−73.7179	−49.0071
lf**(L→H)**	−66.1713	**319.6815**
d(L→H)	−6.24696	**796.3685**
U(L→H)	−29.0391	−64.4601

TABLE 10. Sensitivity analysis for problem $(5 * 10 * 5 * 7)$.

Parameters' value α	Parameters' value β	Computation time for hybrid heuristic
10	5	2.714
15	5	4.119
	10	5.538
20	5	2.73
	10	3.542
	15	4.928
25	15	7.144
	20	8.3

As Table 10 shows, by increasing the time interval value (α), which means the larger optimized window, the consumption time is increased. Also, for a specific value of α, by increasing the overlap time (β), which leads to smaller optimized window, the consumption time would be decreased. The relationship between two paramers α and β with computation time is shown in Figure 5.

This analysis would be helpful to find the best values for α and β with considering computational time. For instance, for this specific problem the best values for time interval (α) is 10 and for time overlap (β) is 5 and the worst values are $\alpha = 25$ and $\beta = 20$. For the next problem instance $5 * 10 * 5 * 15$ the initial optimal objective value obtained from RF is 4364.81. The optimal solution obtained by CPLEX is 4354.87. The improved solutions

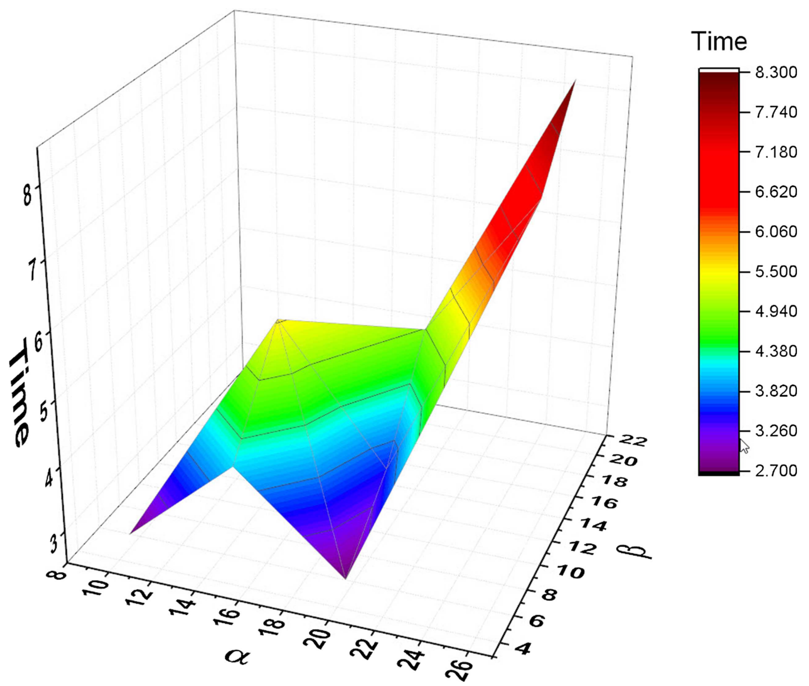

FIGURE 5. Sensitivity analysis of parameters for problem $(5 * 10 * 5 * 7)$.

TABLE 11. Sensitivity analysis for instance (5*10*5*15).

α	β	FR&FO		α	β	FR&FO		α	β	FR&FO		α	β	FR&FO	
		Obj.	Time			Obj.	Time			Obj.	Time			Obj.	Time
10	5	4355.42	14.57	30	5	4354.87	7.442	40	5	4354.87	11.95	50	25	4354.98	11.638
15	5	4355.1	10.795		10	4354.87	8.003		10	4354.87	13.385		30	4354.87	13.478
	10	4354.87	17.253		15	4354.87	14.57		15	4354.87	20.686		35	4354.87	17.706
20	5	4354.87	8.626		20	4354.87	15.489		20	4354.97	13.166		40	4354.87	18.86
	10	4355.65	28.532		25	4354.87	27.346		25	4354.87	17.566		45	4354.87	33.946
	15	4354.87	21.03	35	5	4354.87	7.379		30	4354.87	20.873	55	35	4355.07	18.393
25	5	4355.03	6.911		10	4354.97	8.16		35	4354.87	29.716		40	4354.9	24.071
	10	4354.87	8.704		15	4355.22	8.581	45	15	4354.87	11.217		45	4354.87	24.632
	15	4354.87	11.637		20	4354.87	11.387		20	4354.87	13.992		50	4354.87	41.34
	20	4354.87	23.024		25	4354.87	14.445		25	4355.18	13.088	60	45	4354.97	14.665
					30	4358.06	13.151		30	4354.87	16.677		50	4354.87	22.886
									35	4354.87	20.779		55	4354.87	33.37
									40	4354.87	63.365	65	55	4354.87	18.86
													60	4354.87	36.349
												70	65	4354.87	18.986

by FO for different values of α and β are shown at Table 11. By changing the parameters α and β, the objective function and computation time are changed.

Relationship between different values of α and β with computation time are shown in Figure 6 and with objective GAP in Figure 7.

As mentioned above, by considering computational time, for this specific problem the best values for time interval (α) is 35 and for time overlap (β) is 5 with computation time 7.379 s and the worst values are $\alpha = 45$ and $\beta = 40$ with computation time 63.365s.

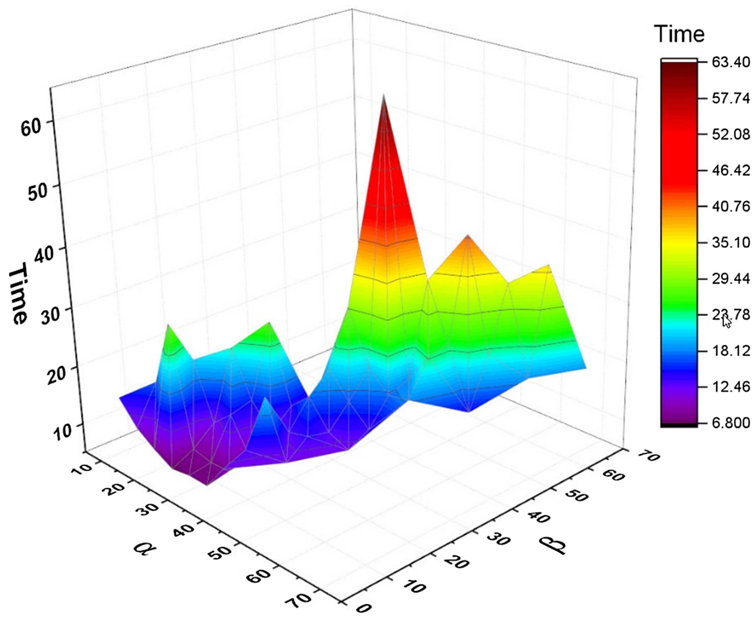

FIGURE 6. Relationship between different values of α and β with computation time.

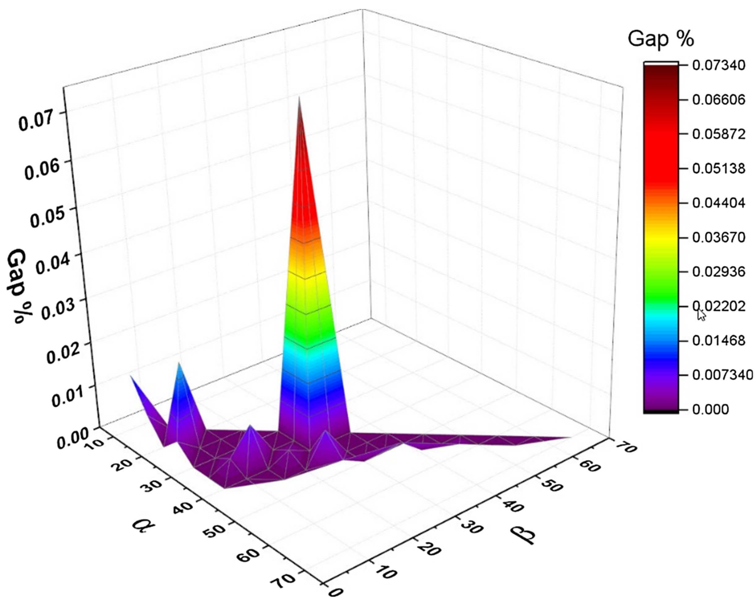

FIGURE 7. Relationship between different values of α and β with objective gap.

The objective GAP, $Obj.GAP$, is defined with equation (5.6).

$$Obj.GAP = \frac{heuristic\ obj. - best\ obj.}{best\ obj.} * 100. \tag{5.6}$$

According to the Figures 6 and 7, there are meaningful relationships between heuristics parameters values and computation time and parameter determination has major effect on heuristics performance. It should be noted here that the MILP formulations are modeled in ILOG's OPL Studio as a modelling environment and are solved by CPLEX as the standard optimization software, on a PC, 64-bit windows 8, with CPU Intel (R) Core^{TM} i7-2600 K 3.40 GHz and 4.00 GB RAM.

6. Conclusions and future research

In this paper firstly, we developed an MILP model for multi- family, multi-product, multi-parallel line GLSP for perishable food industry with limited lifespan and fixed shelf life. The main contribution of this paper is presenting a new lifespan cost function as an objective with extra assumptions such as possibility of storage finished products at plant and quality control time.

Because of the commercial solvers inability to find the optimal solution for real world cases within a reasonable time, we apply two MIP-based heuristics to solve the presented model. Then the quality of solutions as well as performance of heuristics are analyzed. Also, the sensitivity analysis is implemented to show heuristics parameters' changes.

Our model can be extended and applied to the other types of perishable products such blood bank, chemical adhesive materials and newspapers, and food. Further research could be on presenting multi objective models with quality, safety, or sustainability related objective functions. Furthermore, the uncertainty of parameters could be taken into account and the model would be more realistic. Moreover, because of limited shelf life and quality degradation, the model could be extended and distribution planning and scheduling could be integrated with the presented model.

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