RAIRO-Oper. Res. 54 (2020) 1103–1117
https://doi.org/10.1051/ro/2019015

RAIRO Operations Research
www.rairo-ro.org

IMPACT OF CARBON EMISSION ON IMPERFECT PRODUCTION INVENTORY SYSTEM WITH ADVANCE PAYMENT BASE FREE TRANSPORTATION

Amalesh Kumar Manna[1,*], Barun Das[2] and Sunil Tiwari[3]

Abstract. Most of the works in EPQ model considered that retailer's purchasing cost of an order should be paid to the manufacturer at the time of its delivering. In the real world situations, few manufacturers may be expect to receive full or a fraction of the purchasing cost in advance and sometimes allow to prepayment into several equal instalments. Also, in classical inventory models it is assumed that no defective items are produced, but real production process may shift from in-control state to out-of-control state due to occurrence of some assignable causes, which results in the quality loss of items. To make the study more realistic, we develop an imperfect production inventory model that incorporates random carbon emissions under consecutive prepayments. Also, production processes are assumed to be imperfect, so they can produce some defective items and some portion of them are reworked in the same cycle. In addition, it is assumed that manufacturer offers an advanced payment based free transportation to the retailer. Carbon emissions are associated with the decisions of production and transportation from manufacturer to retailer. A carbon emission tax is need to pay by the manufacturer due to the environmental regulations. Numerical examples illustrate the proposed models and sensitivity analysis provides some managerial insights for managers.

Mathematics Subject Classification. 90B05, 90B15, 90B25, 90B30, 90B50.

Received September 26, 2018. Accepted January 23, 2019.

1. Introduction

A large number of EOQ model established based on the assumption that the buyer must pay the purchasing cost of the items to the seller at the time of order quantity received. But, in practice there are three different possible situations arise for the payment of purchasing cost: (i) payment is made at the time of order quantity delivery such as in Harris [8], (ii) permissible partial delay in payment (*i.e.*, payment is done after the time of order quantity delivery) such as Tiwari *et al.* [35], and (iii) partial advance payment (before the time of order quantity delivery) such as Taleizadeh *et al.* [32]. Advance payment is one of the most secure and risk-less methods of the manufacturers for controlling the cash flow. We can expect that advance payment strategy is

Keywords. Inventory, stochastic, inspection, production models, management decision making.

[1] Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore 721102, WB, India.

[2] Department of Mathematics, Sidho-Kanho-Birsha University, Purulia, WB, India.

[3] The Logistics Institute – Asia Pacific, National University of Singapore, 21 Heng Mui Keng Terrace, #04-01, 119613, Singapore.

*Corresponding author: akmanna1987@gmail.com

suggested by manufacturers in exclusive business markets. Sometimes the manufacturers offer to the buyers a price discount, free transportation *etc.* for full or partial advance payment (pay a fraction of the purchasing cost as prepayment with the remaining purchasing cost to be paid at the time of delivery is called partial advance payment). Also many researchers studied and discussed on advanced payment inventory control model since the advance of quantity ensure the production management for sure exhaust of quantity. According to Zhang [37], prepayment scheme is used when the purchasing cost is larger than the billed amount. Zhang [37] extended a model to determine the optimal amount of cash deposit when there are fix prepayment costs. Taleizadeh [33] developed an EOQ model for deteriorating items in a purchasing system with multiple prepayments. Teng *et al.* [34] proposed an inventory lot-size policies for deteriorating items with expiration dates and advance payments. Recently, Li *et al.* [16] established the pricing and lot-sizing policies for perishable products with advance-cash-credit payments by a discounted cash-flow analysis. In all the existing literature, the partial advance payment is made for the economic lot size model, in which system, it is quite absurd since for these cases guarantee of advance payment does not felt any effect on the amount of manufacturer. Here to felt an effect it is consider in a manufacturing system. More-over, that here for the first time in research a facility of transportation is given for the advance payment.

Production systems prone to malfunctions have gained a lot of attention from researchers at some time or the other. When we talk about manufacturing systems it is important to take into account the malfunctions as they lead to defective items which directly result in an economic loss for the organization. Moreover, it is necessary to manage the defective so as to extract the monetary value of the products as much as possible. In a real manufacture world, the production system is not failures free, in other words all produced items are not perfect. So in any production system, production of defective items is inevitable because of many reasons such as defects of machine, and other related factors. In 1986, Rosenblatt and Lee [25] considered an imperfect production process at which the process shifts from the in-control state to the out-of-control state. Salameh and Jaber [26] developed a modified EPQ model which accounted for imperfect quality items. Goyal and Cárdenas-Barrón [6] developed an economic production quantity (EPQ) model to determine both the optimal lot size and manufacturing processing cost in an imperfect production system. Sana [27] developed an imperfect production model to determine the optimal product reliability and production rate to obtain maximum profit. Sana [28] discussed an imperfect production model where a certain percent of total product is defective occurred in out-of-control state, which depends on production rate and production run time. Sarkar [29] developed two inventory models under the presence of imperfect items during out-of-control state. Manna *et al.* [19] proposed an imperfect production inventory model in which the defective items are sold at reduced price. Nobil *et al.* [23] proposed economic production quantity model in a cleaner production environment. Manna *et al.* [21] developed a supply chain model for an imperfect production system at which the process shifts from the in-control state to the out-of-control state at any random time. Most of the manufacturing organizations due to government environmental tax rules, are worried about the carbon emission. And most the EPQ model till now does not considered with these phenomena. The research gap of the existing literature are shown in the following Table 1.

Every production management system has been based on the any one of the economic criteria, either cost minimization or profit maximization for a long period regard less of the negative impacts that these activities may have on environment, basically in terms of carbon emissions. In the last decade, both developed and developing Governments are under growing pressure to enact legislation to curb the amount of these emissions. Most of these works focus on the environmental damages (mainly, carbon emissions) associated with the decisions of facility location technology selection, and supplier selection in either forward or reverse supply chain. In developed countries, beyond the limitation of carbon cap, a carbon regulated tax is charged to the manufacturing system. Penkuhn *et al.* [24] present a joint production planning problems by integrating emission taxes using nonlinear programming *etc.* Wu and Chang [36] considered an environmental costs like pollution charges and water resources fees in a production-planning optimization problem in an uncertain environment for the textile industry. Hua *et al.* [10] derived the sub scenarios of an EOQ model with carbon-trade and carbon tax. Harris *et al.* [9] assessing the impact of carbon emission on infra structural modeling. Benjaffar *et al.* [1] take an initiatives in their research to reduce the carbon footprint on reducing emissions due to the manufacturing

TABLE 1. Summary of related literature survey of inventory models based on different assumptions.

Author(s) and year	Imperfect production process	Rework process	Advance payments	Carbon emission treatment	Random environment
Benjaffar *et al.* [1]	×	×	×	√	×
Cárdenas-Barrón [2]	√	√	×	×	×
Flappera and Teunterb [3]	√	√	×	×	×
Goyal and Cárdenas-Barrón [5]	√	×	×	×	√
Hammami *et al.* [7]	√	×	×	√	×
Harris *et al.* [9]	×	×	×	√	×
Hayek and Salameh [11]	√	√	×	×	√
Hua *et al.* [10]	×	×	×	√	×
Inderfurth *et al.* [12]	√	√	×	×	×
Li *et al.* [16]	√		√	×	×
Manna *et al.* [18]	√	√	×	×	√
Manna *et al.* [19]	√	×	×	×	×
Manna *et al.* [20]	√	√	×	√	√
Manna *et al.* [21]	√	√	×	×	√
Manna *et al.* [22]	√	√	×	√	×
Rosenblatt and Lee [25]	√	×	×	×	√
Salameh and Jaber [26]	√	×	×	×	√
Sana [28]	√	√	×	×	√
Sarkar [29]	√	×	×	×	√
Taleizadeh *et al.* [33]	×	×	√	×	×
Teng *et al.* [34]	×	×	√	×	×
Zhang [37]	×	×	√	×	√
Present paper	√	√	√	√	√

processes involved. Hammami *et al.* [7] considered a multi echelon supply chain model, where the main objective of the problem is reduce the emission of carbon. Manna *et al.* [20] construct a two layer green supply chain model that measures carbon emission during the production run time. Recently, Manna *et al.* [22] controlled green house gas emission in a production inventory model by simultaneous consideration of minimization of carbon emission and maximization of total profit of the system. Here, a random carbon tax is charged on the imperfect production inventory system and its transportation system.

In an imperfect production system, the rework policy is an important role for eliminating waste and decreasing manufacturing cost. Hayek and Salameh [11] studied the determination of optimal production lot size with reworking of defective items. Flappera and Teunterb [3] showed how reworking plans could both reduce costs and be environmentally friendly. Inderfurth *et al.* [12] analyzed cost minimizing-scheduling of work and rework processes on a single facility under deterioration of reworkable items. Cárdenas-Barrón [2] proposed an inventory model on optimal batch sizing in a multi-stage production system with rework process. Manna *et al.* [18] studied that the screening process is essentially required to sort out the imperfect items which are to be reworked. Recently, Ghosh *et al.* [4] analyzed the deteriorating manufacturing system with rework process of imperfect items including machine breakdown. After that, Jain *et al.* [13] discussed an imperfect production and repair inventory model with time dependent demand in fuzzy environment. Later, Mallick *et al.* [17] derived a supply chain model for imperfect items with stochastic lead time demand.

The remainder of this paper is organized as follows: In Section 2, we define the problem with its notations and assumptions. The mathematical formulation of our proposed model are described in Section 3. In Section 4, optimal solutions are derives analytically and provide an algorithm for numerically solution. Numerical solutions and sensitivity analysis are performed in Section 5. The practical implication of the model presented in Section 6. Finally, we give some conclusions and recommendations for future researche in Section 7.

2. Problem definition

This paper treats a manufacturing management system of the items with fixed defective rate. Figure 1 shows the inventory level of the perfect items in a complete cycle. The defective items are rework and transformed

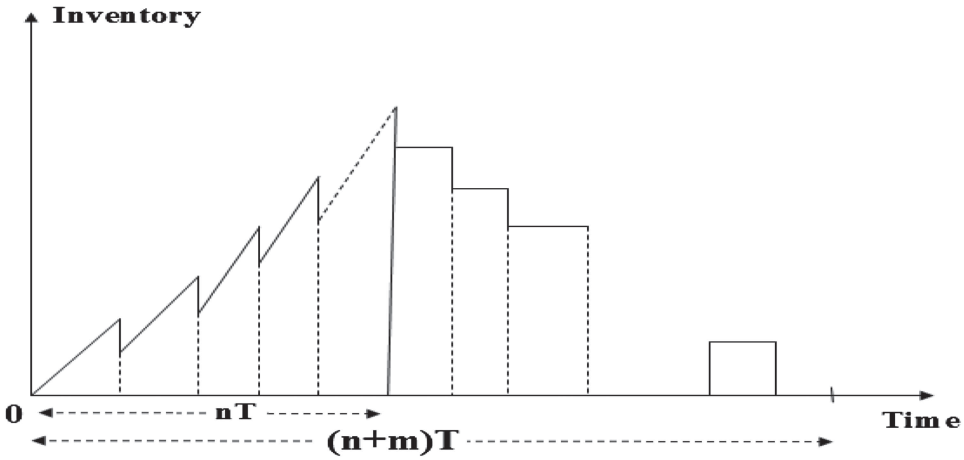

FIGURE 1. Inventory level of perfect quality item in the manufacture.

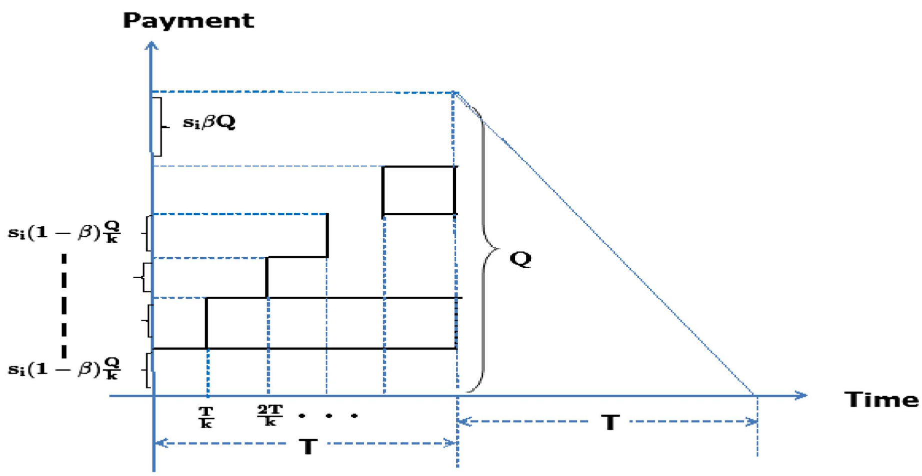

FIGURE 2. Pictorial representation of advance payment policy.

into perfect items. At the time of production and reproduction (rework), the environment is polluted due to the emission of carbon. As per reality the emission of carbon is depend on the production as well as rework. The retailer receive the perfect items through partial advance payment, as shown in Figure 2. Such type of systems are found for any highly demandable item. The amount of advance payment as ensures the production of manufacturer and in this regard, the manufacturer offer a free transportation facility to the retailer.

In this paper, we consider an imperfect production inventory model in which the manufacturer claimed to the retailer a fraction of purchasing cost be prepaid for the ensuring of retailer's ordering and the remaining balance will paid at the time of delivery. The retailer is to decide the prepaid portion given in multiple equal instalments. In addition, it is assumed that manufacture offered to the retailer advanced payment based free transportation. Also, in our proposed model we incorporate random carbon emissions as well as carbon emissions tax for governments' environmental regulations due to production and transportation of produced perfect item to the retailer. We explained our proposed model through the following decisions of manufacturing system:

– how much to production rate in each period of the manufacture?
– how long time to produce the product in each period?
– how much quantity (perfect quality item) to delivery (retailer's order) in each period?

2.1. Notation

To develop the model, the following notation have been used.

P	:	Manufacturer's production rate.
θ	:	Percentage of produced defective items per unit time.
δ	:	Fraction of rework of defective items per unit time.
Q	:	Manufacturer's batch size that delivered to the retailer.
T	:	Time interval at which the manufacturer delivered the order quantity.
n	:	Number of production cycles of manufacturer.
A	:	Manufacturer set-up cost per cycle.
c_{pi}	:	Per unit production cost of ith cycle.
$s_{\rm c}$	:	Per unit screening cost.
$r_{\rm cm}$	:	Reworking cost per unit for manufacturer.
h_m	:	Per unit holding cost per unit time for perfect quality item in manufacturer.
$\hat{\eta}^{\varepsilon}$	:	Random carbon emissions rate associated for per unit production.
$\hat{\eta}^{\gamma}$	:	Random carbon emissions rate associated for transportation of each batch size.
$g(\eta)$	:	Probability density function of $\hat{\eta}$, $\eta \in [0,1)$.
$e_{\rm c}$	:	Carbon emission tax due to per unit production.
$e'_{\rm c}$	:	Carbon emission tax due to per unit transportation.
s_i	:	Selling price per unit of perfect quality item for manufacturer.
$c_{\rm t}$	:	Unit transportation cost for transfer the quantity from manufacturer to retailer.

2.2. Assumptions

The proposed model have been develop based on the following assumptions.

1. In real production system, it is seen that 100% produced item may not be perfect quality due to different factors involved in the system such as machine, un-expertise labour *etc.* (see Salameh and Jaber [26], Sarkar [29], Liao [15], Manna *et al.* [19]). In this point of view, we assume a manufacturer produces a mixture of perfect and imperfect quality items. Also we assume that some portion of imperfect items are reworked and transformed into a perfect quality items.
2. Due to the existence of defective production, there exist some imperfect quality items. Hence the manufacturer decides to sale the perfect quality item after inspect each item to check whether the item is perfect or not (see Ghosh *et al.* [4], Sett *et al.* [31] Mallick *et al.* [17]).
3. A carbon emissions tax is pay by the manufacture for production and transportation of goods due to governments' environmental regulations (see Hua *et al.* [10], Harris *et al.* [9], Hammami *et al.* [7]). For this reason, we incorporate random carbon emission tax in our model due to production and transportation of produced perfect item to the retailer. The carbon emission rate during the production and transportation are multiple of $\hat{\eta}^{\varepsilon}$ and $\hat{\eta}^{\gamma}$ respectively. Also it assume that carbon emission tax due to per unit production, $e_{\rm c} = \alpha_0 + \alpha_1 \theta P$, ($\alpha_0 = 0$ when $P = 0$) where the unit tax α_0 is charged for the production and second part ($\alpha_1 \theta P$) is charged for defective production amount (θP). On the otherhand, the unit carbon emission tax due to transportation of perfect item from manufacture to the retailer is $e'_{\rm c} = \alpha'_0 + \alpha'_1 Q$ ($\alpha'_0 = 0$ when $Q = 0$).
4. Manufacturer sales the produced perfect quality item to the the retailers in the fixed time interval (T). *i.e.*, retailer's received the order quantity with fixed time interval (T).
5. Retailer pays an advance payment to the manufacturer for $(1-\beta)$ portion of purchase cost in multiple (k) installments. The price of remaining portion (β) will be paid by the retailer at the time of receiving of item. For this advance part payment the manufacturer gain an interest (see Taleizadeh *et al.* [33], Teng *et al.* [34]).

6. The unit production cost of the item is not fixed through out the period but increases in exponential rate which is of the form $c_{pi} = a_0 \mathrm{e}^{(i-1)a_1}$, $i = 1, 2, \ldots, n$. The selling price of the item revised in each cycle and following the form $s_i = s_0 \mathrm{e}^{(i-1)s_1}$, $i = 1, 2, \ldots, (m+n)$.
7. In this proposed model, we assume that an advantage of advanced payment will be given on transportation cost by the manufacturer to the retailer. If the retailer made payment in advance for the total received amount then the manufacturer transport the quantity in his/her own risk. Other wise, the manufacturer reduce his/her responsibility of transportation on the basis of advance payment. In this assumption, the transportation cost in each batch can be considered as $c_t(1-\beta)^{\lambda} Q$, where $0 \le \lambda \le 1$.

3. Mathematical formulation of the model

We consider a manufacturing system which produces both perfect and imperfect item in each production run at the rate $(1-\theta)P$ and θP respectively. Among the imperfect item few items are repaired at a rate $\delta\theta P$. We consider the manufacturer delivered a lot size Q of perfect items to the retailer in each cycle and the retailer sold the perfect item to the customer. During the production and transportation unavoidable carbons are emission at the rate $\hat{\eta}^{\epsilon}$ and $\hat{\eta}^{\gamma}$ respectively. Such supply chain inventory model is derived to formulate different cost expression.

$$\begin{aligned}
\text{Manufacturer's production cost during } [0,\, nT] &= \sum_{i=1}^{n} a_0 \mathrm{e}^{(i-1)a_1} PT = \frac{\mathrm{e}^{na_1}-1}{\mathrm{e}^{a_1}-1} a_0 PT \\
\text{Manufacturer's inspection cost during } [0,\, nT] &= n s_c PT \\
\text{Manufacturer's reworking cost during } [0,\, nT] &= n r_{\mathrm{cm}} \delta\theta PT \\
\text{Manufacturer's disposal cost during } [0,\, nT] &= n c_d (1-\delta)\theta PT.
\end{aligned}$$

$$\begin{aligned}
&\text{Manufacturer's holding cost during } [0,\, (m+n)T] \\
&= h_{\mathrm{c}}\Big[\frac{1}{2}\{1-(1-\delta)\theta\} n^2 PT^2 - \Big\{QT + 2QT + \cdots + (n-1)QT\Big\} \\
&+ \Big\{QT + 2QT + \cdots + (m-1)QT\Big\}\Big] \\
&= h_{\mathrm{c}}\Big[\frac{1}{2}\{1-(1-\delta)\theta\} n^2 PT^2 - \frac{n(n-1)}{2} QT + \frac{m(m-1)}{2} QT\Big] \\
&= \frac{h_{\mathrm{c}}}{2}\Big[\{1-(1-\delta)\theta\} n^2 PT^2 - n(n-1)QT + m(m-1)QT\Big].
\end{aligned}$$

The carbon emission tax during the production and transportation is given by

$$\begin{aligned}
\mathrm{CE}(P,T;\hat{\eta}) &= (\alpha_0 + \alpha_1\theta P)\hat{\eta}^{\varepsilon} \int_0^{nT} P \,\mathrm{d}t + (m+n)(\alpha_0' + \alpha_1' \hat{\eta}^{\gamma} Q) \\
&= (\alpha_0 + \alpha_1\theta P)\hat{\eta}^{\varepsilon} nPT + (m+n)(\alpha_0' + \alpha_1' \hat{\eta}^{\gamma} Q).
\end{aligned}$$

The expected carbon emission tax during the production and transportation is given by

$$\begin{aligned}
E[\mathrm{CE}(P,T;\hat{\eta})] &= (\alpha_0 + \alpha_1\theta P) E[\hat{\eta}^{\varepsilon}] nPT + (m+n)(\alpha_0' + \alpha_1' E[\hat{\eta}^{\gamma}] Q) \\
&= (m+n)\alpha_0' + (m+n)\alpha_1' E[\hat{\eta}^{\gamma}] Q + n\alpha_0 E[\hat{\eta}^{\varepsilon}] PT + n\alpha_1\theta E[\hat{\eta}^{\varepsilon}] P^2 T.
\end{aligned}$$

Manufacturer receive advance part payment in $(k-1)$ equal shipment for each delivery Q and the kth installment for the remaining part is received by manufacturer at the time of delivery. So an interest will be gain by the manufacturer for $(k-1)$ times for the advance part payment. Therefore, Manufacturer's interest earn for each shipment due to advanced payment

$$
\begin{aligned}
&= I_e\Big\{s_i(1-\beta)\frac{Q}{k}\times\frac{T}{k}+s_i(1-\beta)\frac{Q}{k}\times\frac{2T}{k}+\cdots+s_i(1-\beta)\frac{Q}{k}\times\frac{(k-1)T}{k}\Big\}\\
&= s_iI_e(1-\beta)\frac{Q}{k}\Big\{1+2+\cdots+(k-1)\Big\}\frac{T}{k}\\
&= s_iI_e(1-\beta)\frac{Q}{k}\times\frac{(k-1)k}{2}\times\frac{T}{k}\\
&= \frac{s_iI_e}{2k}(1-\beta)(k-1)QT,
\end{aligned}
$$

Manufacturer's total interest earn during whole business period due to advanced payment

$$
\begin{aligned}
&= \sum_{i=1}^{(m+n)}\frac{s_iI_e}{2k}(1-\beta)(k-1)QT\\
&= \frac{s_0I_e\{\mathrm{e}^{(m+n)s_1}-1\}}{2k(\mathrm{e}^{s_1}-1)}(1-\beta)(k-1)QT,
\end{aligned}
$$

Manufacturer's selling revenue during $[0,\,(m+n)T] = \sum_{i=1}^{(m+n)} s_0\mathrm{e}^{(i-1)s_1}Q = \frac{\mathrm{e}^{(m+n)s_1}-1}{\mathrm{e}^{s_1}-1}s_0Q.$

Expected total profit of manufacturer during $[0,(m+n)T]$ is given by

$$
\begin{aligned}
E[\Pi_m(P,T)] = {} & \frac{\mathrm{e}^{(m+n)s_1}-1}{\mathrm{e}^{s_1}-1}\Big\{s_0Q+\frac{s_0I_e}{2k}(1-\beta)(k-1)QT\Big\}-(m+n)c_t(1-\beta)^{\lambda}Q-A\\
& -\frac{\mathrm{e}^{na_1}-1}{\mathrm{e}^{a_1}-1}a_0PT-n\{s_{\mathrm{c}}+(\alpha_0+\alpha_1\theta P)E[\hat{\eta}^{\varepsilon}]\}PT-nr_{cm}\delta\theta PT-nc_d(1-\delta)\theta PT\\
& -\frac{h_{\mathrm{c}}}{2}\Big[\{1-(1-\delta)\theta\}n^2PT^2-n(n-1)QT+m(m-1)QT\Big]-(m+n)(\alpha_0'+\alpha_1'E[\hat{\eta}^{\gamma}]Q).
\end{aligned}
$$

Therefore, expected average profit of the manufacturer is given by

$$
\begin{aligned}
\mathrm{EAP}(P,T) = {} & \frac{\Pi_m(P,T)}{(m+n)T}=\Big\{\frac{s_0\{\mathrm{e}^{(m+n)s_1}-1\}}{(m+n)(\mathrm{e}^{s_1}-1)}-c_t(1-\beta)^{\lambda}-\alpha_1'E[\hat{\eta}^{\gamma}]\Big\}\frac{Q}{T}+\frac{s_0I_e\{\mathrm{e}^{(m+n)s_1}-1\}}{2k(m+n)(\mathrm{e}^{s_1}-1)}(1-\beta)(k-1)Q\\
& -\Big[\frac{\mathrm{e}^{na_1}-1}{n(\mathrm{e}^{a_1}-1)}a_0+s_{\mathrm{c}}+\alpha_0E[\hat{\eta}^{\varepsilon}]+r_{cm}+c_d(1-\delta)\theta\Big]\frac{nP}{m+n}-\frac{n\alpha_1\theta}{m+n}E[\hat{\eta}^{\varepsilon}]P^2\\
& -\frac{h_{\mathrm{c}}}{2(m+n)}\{1-(1-\delta)\theta\}n^2PT-\frac{h_{\mathrm{c}}}{2(m+n)}(m-n)(n+m-1)Q-\frac{A+(m+n)\alpha_0'}{(m+n)T}.
\end{aligned}
$$

Lemma 3.1. *In the manufacturing system, despatch quantity (Q) must satisfy the following relation in terms of the production rate (P) and business cycle period (T)*

$$
Q=\frac{n}{m+n}\{1-(1-\delta)\theta\}PT.
$$

Proof. Rate of producing perfect item and imperfect item of manufacturer are $(1-\theta)P$ and θP respectively. Among the imperfect item few items are repaired at a rate $\delta\theta P$. On the otherhand, the manufacture sale lot size Q in each cycle. Therefore,

$$
\begin{aligned}
& n(1-\theta)PT+n\delta\theta PT=(m+n)Q\\
& i.e.,\ Q=\frac{n}{m+n}\{1-(1-\delta)\theta\}PT.
\end{aligned}
$$

Hence the proof.

Using Lemma 3.1 in the expression of expected average profit of the manufacturer, we have

$$\mathrm{EAP}(P,T)=\frac{n}{m+n}\Big\{\frac{s_0\{\mathrm{e}^{(m+n)s_1}-1\}}{(m+n)(\mathrm{e}^{s_1}-1)}-c_t(1-\beta)^\lambda-\alpha_1' E[\hat\eta^\gamma]\Big\}\{1-(1-\delta)\theta\}P-\frac{n\alpha_1\theta}{m+n}E[\hat\eta^\varepsilon]P^2$$
$$-\frac{n}{m+n}\Big[\frac{\mathrm{e}^{na_1}-1}{n(\mathrm{e}^{a_1}-1)}a_0+s_\mathrm{c}+\alpha_0E[\hat\eta^\varepsilon]+r_{cm}+c_d(1-\delta)\theta\Big]P-\frac{A+(m+n)\alpha_0'}{(m+n)T}$$
$$+\frac{n\{1-(1-\delta)\theta\}}{2(m+n)^2}\Big\{\frac{\{\mathrm{e}^{(m+n)s_1}-1\}}{k(\mathrm{e}^{s_1}-1)}(1-\beta)(k-1)s_0I_e-h_\mathrm{c}\{m(m+n-1)+n\}\Big\}PT.$$

□

4. The optimal solution of the model

Taking the first order partial derivatives of $\mathrm{EAP}(P,T)$ with respect to P and T, we obtain

$$\frac{\partial}{\partial P}\{\mathrm{EAP}(P,T)\}=\frac{n}{m+n}\Big\{\frac{s_0\{\mathrm{e}^{(m+n)s_1}-1\}}{(m+n)(\mathrm{e}^{s_1}-1)}-c_t(1-\beta)^\lambda-\alpha_1' E[\hat\eta^\gamma]\Big\}\{1-(1-\delta)\theta\}$$
$$-\frac{n}{m+n}\Big[\frac{\mathrm{e}^{na_1}-1}{n(\mathrm{e}^{a_1}-1)}a_0+s_\mathrm{c}+\alpha_0E[\hat\eta^\varepsilon]+r_{cm}+c_d(1-\delta)\theta\Big]-\frac{2n\alpha_1\theta}{m+n}E[\hat\eta^\varepsilon]P$$
$$+\frac{n\{1-(1-\delta)\theta\}}{2(m+n)^2}\Big\{\frac{\{\mathrm{e}^{(m+n)s_1}-1\}}{k(\mathrm{e}^{s_1}-1)}(1-\beta)(k-1)s_0I_e-h_\mathrm{c}\{m(m+n-1)+n\}\Big\}T, \tag{4.1}$$

and

$$\frac{\partial}{\partial T}\{\mathrm{EAP}(P,T)\}=\frac{n\{1-(1-\delta)\theta\}}{2(m+n)^2}\Big\{\frac{\{\mathrm{e}^{(m+n)s_1}-1\}}{k(\mathrm{e}^{s_1}-1)}(1-\beta)(k-1)s_0I_e$$
$$-h_\mathrm{c}\{m(m+n-1)+n\}\Big\}P+\frac{A+(m+n)\alpha_0'}{(m+n)T^2}\cdot \tag{4.2}$$

Lemma 4.1. *For fixed values of n, expected average profit* $\mathrm{EAP}(P,T)$ *function is concave of* P *and* T *(required for existence of* $P=P^*$ *and* $T=T^*$ *for which* $\mathrm{EAP}(P,T)$ *is global maximum) provided*

$$\frac{4n\alpha_1\theta\{A+(m+n)\alpha_0'\}E[\hat\eta^\varepsilon]}{(m+n)^2T^3}-\frac{n^2\{1-(1-\delta)\theta\}^2}{4(m+n)^4}\Big[\frac{\{\mathrm{e}^{(m+n)s_1}-1\}}{k(\mathrm{e}^{s_1}-1)}(1-\beta)(k-1)s_0I_e-h_\mathrm{c}\{m(m+n-1)+n\}\Big]^2>0.$$

Proof. Further, taking the second order partial derivatives of $\mathrm{EAP}(P,T)$ with respect to P and T, we get

$$\frac{\partial^2}{\partial P^2}\{\mathrm{EAP}(P,T)\}=-\frac{2n\alpha_1\theta}{m+n}E[\hat\eta^\varepsilon]<0 \tag{4.3}$$
$$\frac{\partial^2}{\partial T^2}\{\mathrm{EAP}(n,P,T)\}=-\frac{2\{A+(m+n)\alpha_0'\}}{(m+n)T^3}<0, \tag{4.4}$$

and

$$\frac{\partial^2}{\partial T\partial P}\{\mathrm{EAP}(P,T)\}=\frac{n\{1-(1-\delta)\theta\}}{2(m+n)^2}\Big[\frac{\{\mathrm{e}^{(m+n)s_1}-1\}}{k(\mathrm{e}^{s_1}-1)}(1-\beta)(k-1)s_0I_e-h_\mathrm{c}\{m(m+n-1)+n\}\Big]. \tag{4.5}$$

Therefore, the Hessian's matrix H is given by

$$H = \begin{bmatrix} \frac{\partial^2}{\partial P^2}\{\mathrm{EAP}(P,T)\} & \frac{\partial^2}{\partial P \partial T}\{\mathrm{EAP}(P,T) \\ \frac{\partial^2}{\partial T \partial P}\{\mathrm{EAP}(P,T) & \frac{\partial^2}{\partial P^2}\{\mathrm{EAP}(P,T)\} \end{bmatrix}$$

$$= \frac{4n\alpha_1\theta\{A+(m+n)\alpha_0'\}E[\hat{\eta}^\varepsilon]}{(m+n)^2T^3} - \frac{n^2\{1-(1-\delta)\theta\}^2}{4(m+n)^4}\Big[\frac{\{\mathrm{e}^{(m+n)s_1}-1\}}{k(\mathrm{e}^{s_1}-1)}(1-\beta)(k-1)s_0I_e$$
$$-h_\mathrm{c}\{m(m+n-1)+n\}\Big]^2.$$

For extremum values of $\mathrm{EAP}(P,T)$, the necessary condition is the Hessian matrix should be positive, and, the principle minors of H satisfy

$$\frac{\partial^2}{\partial P^2}\{\mathrm{EAP}(P,T)\} < 0 \text{ and } \frac{\partial^2}{\partial T^2}\{\mathrm{EAP}(P,T)\} < 0,$$

which is trivial. Hence, $\mathrm{EAP}(P,T)$ is a concave function in P and T if

$$i.e., \frac{4n\alpha_1\theta\{A+(m+n)\alpha_0'\}E[\hat{\eta}^\varepsilon]}{(m+n)^2T^3} - \frac{n^2\{1-(1-\delta)\theta\}^2}{4(m+n)^4}\Big[\frac{\{\mathrm{e}^{(m+n)s_1}-1\}}{k(\mathrm{e}^{s_1}-1)}(1-\beta)(k-1)s_0I_e - h_\mathrm{c}\{m(m+n-1)+n\}\Big]^2 > 0.$$

□

In that case $\mathrm{EAP}(P,T)$ has a global maximum at $P = P^*$ and $T = T^*$.

Now, the maximum value of $\mathrm{EAP}(P,T)$ will occur at the point (P,T) which satisfies simultaneously

$$\frac{\partial}{\partial P}\{\mathrm{EAP}(P,T)\} = 0 \text{ and } \frac{\partial}{\partial T}\{\mathrm{EAP}(P,T)\} = 0. \tag{4.6}$$

The above equations gives

$$\xi_{11}P - \xi_{12}T = \xi_{01}, \tag{4.7}$$

and

$$\xi_{12}PT^2 = \xi_{21}, \tag{4.8}$$

where

$$\xi_{01} = \frac{n}{m+n}\Big\{\frac{s_0\{\mathrm{e}^{(m+n)s_1}-1\}}{(m+n)(\mathrm{e}^{s_1}-1)} - c_t(1-\beta)^\lambda - \alpha_1'E[\hat{\eta}^\gamma]\Big\}\{1-(1-\delta)\theta\}$$
$$- \frac{n}{m+n}\Big[\frac{\mathrm{e}^{na_1}-1}{n(\mathrm{e}^{a_1}-1)}a_0 + s_\mathrm{c} + \alpha_0E[\hat{\eta}^\varepsilon] + r_{cm} + c_d(1-\delta)\theta\Big]$$
$$\xi_{12} = \frac{n\{1-(1-\delta)\theta\}}{2(m+n)^2}\Big[\frac{\{\mathrm{e}^{(m+n)s_1}-1\}}{k(\mathrm{e}^{s_1}-1)}(1-\beta)(k-1)s_0I_e - h_\mathrm{c}\{m(m+n-1)+n\}\Big]$$
$$\xi_{11} = \frac{2n\alpha_1\theta}{m+n}E[\hat{\eta}^\varepsilon], \;\; \xi_{21} = \frac{A+(m+n)\alpha_0'}{(m+n)}.$$

After simplification of equations (4.7) and (4.8) by using LINGO software, we get

$$P = P^* \text{ and } T = T^*.$$

Algorithm 1

Step 1: Initialize the set of input data n, m, k, θ, β, λ, δ, c_p, s_c, r_{cm}, c_d, h_c, A, s_0, s^1, c_t, a_0, a_1, α_0, α_1, α'_0, α'_1, I_e, v, w.

Step 2: The expected total profit $\mathrm{EAP}(P,T)$ is a function of P and T which is optimized analytically as follows. Now compute the values of P and T from the equations $\frac{\partial}{\partial P}\{\mathrm{EAP}(P,T)\} = 0$ and $\frac{\partial}{\partial T}\{\mathrm{EAP}(P,T)\} = 0$ using the given input values in step 1.

Step 3: After compute the values of P and T in step 2 cheek the condition

$$\frac{\partial^2}{\partial P^2}\{\mathrm{EAP}(P,T)\} \times \frac{\partial^2}{\partial T^2}\{\mathrm{EAP}(P,T)\} - \left[\frac{\partial^2}{\partial P \partial T}\{\mathrm{EAP}(P,T)\}\right]^2 > 0,$$

as well as either $\frac{\partial^2}{\partial P^2}\{\mathrm{EAP}(P,T)\} < 0$ or $\frac{\partial^2}{\partial T^2}\{\mathrm{EAP}(P,T)\} < 0$. If the conditions are satisfied then $P = P^*$, $T = T^*$ optimal solutions are obtained and follow step 4. Otherwise go to step 1 for set another values of input data and followed step 2.

Step 4: Compute Q^* using the obtained values of P and T from step 3 and the equation

$$Q = \frac{n}{m+n}\{1-(1-\delta)\theta\}PT.$$

Step 5: Compute the value of $\mathrm{EAP}(P^*,T^*)$ which the optimal expected average profit.

5. Numerical example

In this section, we present a numerical study to illustrate the feasibility of the proposed model.

Example 5.1. We consider the standard parameters of production inventory model with the following characteristics: $n = 7$, $m = 5$, $k = 9$, $\theta = 0.08$, $\beta = 0.65$, $\lambda = 0.40$, $\delta = 0.80$, $c_p = \$32$ per unit, $s_c = \$2$ per unit, $r_{cm} = \$4$ per unit, $c_d = \$3$ per unit, $h_c = \$.5$ per unit per unit time, $A = \$420$, $s_0 = \$195$, $s^1 = 0.03$, $c_t = \$1.5$, $a_0 = \$50$, $a_1 = 0.01$, $\alpha_0 = 8$, $\alpha_1 = 4$, $\alpha'_0 = 5$, $\alpha'_1 = 0.10$, $I_e = 0.08$.

The respective carbon-emission rates $\hat{\eta}^\epsilon$ and $\hat{\eta}^\gamma$ due to per unit production and per unit transportation followed a Beta distribution $g(\eta)$ with parameters v, w (v and w are positive integer) *i.e.*., the p.d.f. of $\hat{\eta}$ is

$$g(\eta) = \begin{cases} \frac{\eta^{v-1}(1-\eta)^{w-1}}{\beta(v,w)}, & 0 \le \eta \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

In our model, we take $\epsilon = \frac{1}{2}$, $\gamma = \frac{3}{2}$. So the expected values of $\hat{\eta}^\epsilon$ and $\hat{\eta}^\gamma$ are respectively
$E[\hat{\eta}^\epsilon] = \frac{\Gamma(v+\frac{1}{2})\Gamma(v+w)}{\Gamma(v+w+\frac{1}{2})\Gamma(v)}$, and $E[\hat{\eta}^\gamma] = \frac{\Gamma(v+\frac{3}{2})\Gamma(v+w)}{\Gamma(v+w+\frac{3}{2})\Gamma(v)}$

Again, for in anticipation if we consider $v = 6$, $w = 4$ then $E[\hat{\eta}^\epsilon] = 0.477$ and $E[\hat{\eta}^\gamma] = 0.768$.

Now utilizing the above parametric values $\frac{\partial}{\partial P}\{\mathrm{EAP}(P,T)\} = 0$ and $\frac{\partial}{\partial T}\{\mathrm{EAP}(P,T)\} = 0$ are solve by LINGO and MATHEMATICA softwares, we get the solution $P = 25.59327$ and $T = 1.31039$.

In this model, use the above parametric values, $P = 25.59327$ and $T = 1.31039$ we calculate the Hessian matrix numerically for confirmation of optimality of the above obtained result by LINGO as follows $\frac{\partial^2}{\partial P^2}\{\mathrm{EAP}(P,T)\} \times \frac{\partial^2}{\partial T^2}\{\mathrm{EAP}(P,T)\} - \left[\frac{\partial^2}{\partial P \partial T}\{\mathrm{EAP}(P,T)\}\right]^2 = 9.365576 > 0$. Therefore the obtained solution by LINGO is confirmed to be optimal. Again since $\frac{\partial^2}{\partial P^2}\{\mathrm{EAP}(P,T)\} < 0$ and $\frac{\partial^2}{\partial T^2}\{\mathrm{EAP}(P,T)\} < 0$, hence the obtained solution of P and T are optimal and $\text{EAP}(P,T)$ is maximum for $P = 25.59327$ and $T = 1.31039$. The details of the optimal solution are given in following Table 2.

TABLE 2. Optimal result of the illustrated model.

Production rate (P^*)(unit)	Cycle period (T^*)(unit)	Order quantity (Q^*)(unit)	Expected average profit ($\text{EAP}(P^*, T^*)$)($)
25.59327	1.31039	19.25034	1204.313

The obtained values of P and T are not unique, since the normal equations produce cubic equation for P and T respectively. The other two values $\{(P = 26.6003, T = -0.982256), (P = 0.00708878, T = 60.1703)\}$ are non-feasible in nature. So these values are not discussed here.

5.1. Sensitivity analyses

We have presented the sensitivity analyses on our proposed model. We examined the effects of changes of the parameters β, θ and k. on the decision variables and the results summarized are in the following tables.

– From Table 3, the expected average profit and transportation cost are sensitive to the advance-payment parameter and increases with an increase in the advance-payment parameter, since as the low amount of advanced payment $(1 - \beta)$ motivates the retailers for more order quantity (Q). On the resulting of more order quantity increases the transportation cost as well as expected average profit of the system. On the other hand for increasing the order quantity, as expected here both production rate, cycle period increases.

TABLE 3. Sensitivity analyses w.r.t. advance payment parameter.

Parameter β	Production rate (P^*) (unit)	Cycle period (T^*) (unit)	Order quantity (Q^*) (unit)	Interest amount of advance payment ($)	Cost of free transportation ($)	$\text{EAP}(P^*, T^*)$ ($)
0.55	25.48842	1.06559	15.59003	98.87116	15.85842	1201.728
0.60	25.53836	1.16900	17.13643	96.60301	16.62924	1202.973
0.65	25.59327	1.31039	19.25034	94.95475	17.70898	1204.313
0.70	25.65508	1.52067	22.39334	94.67830	19.36848	1205.768
0.75	25.72753	1.88340	27.81336	97.99497	22.36442	1207.373

– From Table 4, the expected average profit is sensitive to the defective rate parameter and which decreases with an increase in the defective rate parameter. Also, production rate and order quantity are decreases with an increase in the the defective parameter. On the otherhand, cycle period increases with an decrease in the the defective parameter.

TABLE 4. Sensitivity analyses w.r.t. produced defective item parameter

Parameter (θ)	Production rate (P^*)(unit)	Cycle period (T^*)(unit)	Order quantity (Q^*)(unit)	Cost of carbon emission ($)	$\text{EAP}(P^*, T^*)$ ($)
0.04	25.88887	1.29762	19.43974	145.3959	1231.884
0.06	25.74107	1.30398	19.34505	168.0411	1218.060
0.08	25.59327	1.31039	19.25034	190.1457	1204.313
0.10	25.44546	1.31687	19.15562	211.7141	1190.644
0.12	25.29765	1.32342	19.06088	232.7513	1177.053

– From Table 5, the average expected profit is sensitive to the advance-payment times and decreases or increases with an increase or decrease in the advance-payment times, respectively. On the other hand, production rate, cycle period and order quantity are decreases with an increase in the advance-payment times.

TABLE 5. Sensitivity analyses w.r.t advance payment instalment parameter

Parameters k	Production rate (P^*) (unit)	Cycle period (T^*) (unit)	Order quantity (Q^*) (unit)	Interest amount of advance payment ($)	Free transportation cost amount ($)	EAP(P^*, T^*) ($)
3	25.68049	1.76979	26.08789	96.51138	23.99906	1204.403
6	25.61282	1.39115	20.45234	94.57854	18.81474	1204.335
9	25.59327	1.31039	19.25034	94.95475	17.70898	1204.313
12	25.58390	1.27498	18.72328	95.24106	17.22413	1204.301
15	25.57840	1.25507	18.42697	95.43803	16.95154	1204.295

– From Figure 3, we can conclude that, as the rate of defective produced items increases, due to its effect the amount of carbon emission also increase and consequently the cost of carbon emission is increased. But, the relation of total profit with the rate of defective is inversely proposed as expected.

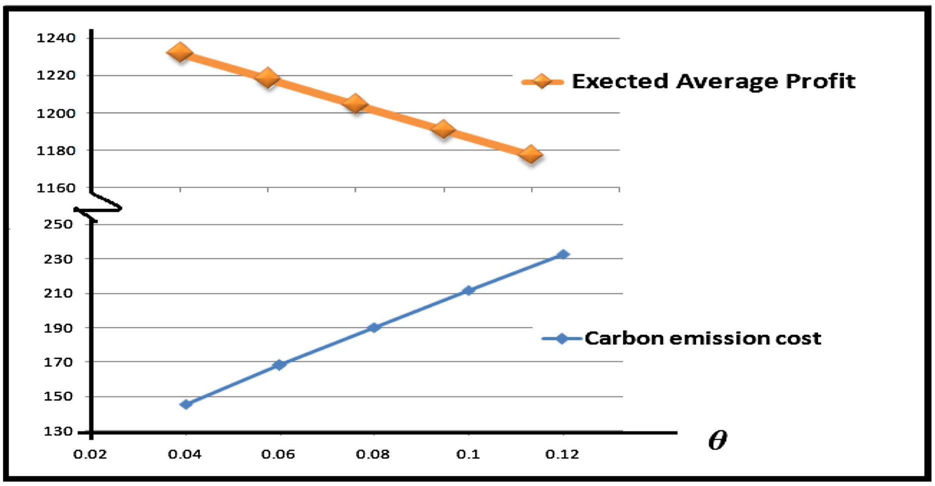

FIGURE 3. Effect of expected average profit and carbon emission cost w.r.t. defective rate parameter.

– From Figure 4, it reveals that, if the amount of advance payment is higher then as expected the order quantity is greater and as order quantity is high then corresponding expected profit become high.

6. Practical implication

In present business era, our proposed model have many practical implications. As for example, it is very usable in the manufacturing system for mobile phones. At the time of production few defective units (like, scratching, disorder shape, *etc.*) are produced and then some of them be repaired to sell at the market. The decision manager of the company always decides the maximization of the profit, considering the advance payment policy to the

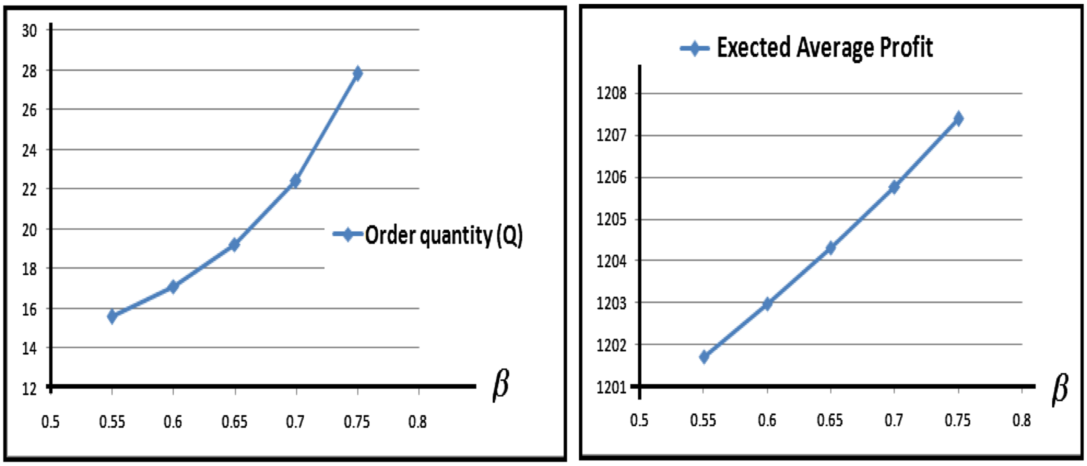

FIGURE 4. Effect of order quantity and expected average profit w.r.t. advance payment parameter.

retailers for ensures the manufacturing to continue the production and earn interest due to advance payment. Moreover, due to governments' environmental regulations, the tax due to carbon emissions is play a vital role of every manufacturing system for minimize the cost. For such a real life problem, the present model can be implemented. From this study some managerial insights have been drawn which are very useful for the decision maker of any newly established mobile company.

7. Conclusions and future research

Nowadays, each manufacturing firms are giving more importance to social and environmental issues, instead of just economic criteria because of various rules and regulations imposed by government authorities on carbon emission. Since efficient management of production and inventory decisions is the backbone for maintaining smooth flow in the supply chain system. The fundamental of any inventory modelling revolves around the products. Therefore, it becomes essential to cogitate about products practicality in the supply chain structure. Thus, it is quite unrealistic to consider the products to be of perfect quality, as due to numerous reasons (faulty production process, mishandling during transportation etc.,) the end product may not withstand with the quality standards set by the manufacturer. Consequently, the firm has to deal with imperfect quality items. So at the same time each manufacturing firm has to consider both quality issue as well as environment issue. This paper deals with both concerns, here we developed an economic ordering quantity model where the production process is imperfect and at the same time we consider the impact of carbon emission which is the normal phenomena of any production system in our model. Here, we consider the random rate of carbon emission, which is the key feature. Again a free transportation is given here on the basis of advanced payment. The model is solved analytically *i.e.*, a closed form solution is obtained which guarantee that the system has global extremum points of the system. The results and findings of present model will help decision makers in manufacturing firms/organisation, while considering the above-mentioned scenarios..

The model can be extended in a number of ways by adopting varying demand patterns. Also, it would be interesting to develop an integrated vendor-buyer model in this direction (like, Mallick *et al.* [17]). It will be interesting to consider integrated vendor-buyer model with partial trade credits (like, Tiwari *et al.* [35]). One can also extend the current model by considering warranty policy, inspection errors (like, Sarkar *et al.* [30]), quantity discounts on material costs *etc.*

Acknowledgements. The authors express heartfelt gratitude and boundless regards to the editor and referees for their several useful comments and valuable suggestions which have helped immensely to improving the paper.

References

[1] S. Benjaafar, Y. Li and M. Daskin, Carbon foot print and the management of supply chains: Insights from simple models. *IEEE Trans. Auto. Sci. Eng.* **10** (2013) 99–116.

[2] L.E. Cárdenas-Barrón, On optimal batch sizing in a multistage production system with rework consideration. *Eur. J. Oper. Res.* **196** (2009) 1238–1244.

[3] S.D.P. Flappera and R.H. Teunterb, Logistic planning of rework with deteriorating work-in-process. *Int. J. Prod. Econ.* **88** (2004) 51–59.

[4] P.K. Ghosh, A.K. Manna and J.K. Dey, Deteriorating manufacturing system with selling price discount under random machine breakdown. *Int. J. Comput. Eng. Manage.* **20** (2017) 8–17.

[5] S.K. Goyal and L.E. Cárdenas-Barrón, Note on: Economic production quantity model for items with imperfect quality- a practical approach. *Int. J. Prod. Econ.* **77** (2002) 85–87.

[6] S.K. Goyal and L.E. Cárdenas-Barrón, Economic production quantity with imperfect production system. *Ind. Eng. J.* **34** (2005) 33–36.

[7] R. Hammami, I. Nouira and Y. Frein, Carbon emissions in a multi-echelon production-inventory model with lead time constraints. *Int. J. Prod. Econ.* **164** (2014) 292–307.

[8] F.W. Harris, How many parts to make at once factory. *Mag. Manage.* **10** (1913) 135–136.

[9] I. Harris, M. Naim, A. Palmer, A. Potter and C. Mumford, Assessing the impact of cost optimization based on infrastructure modelling on CO_2 emissions. *Int. J. Prod. Econ.* **131** (2011) 313–321.

[10] G. Hua, T.C.E. Cheng and S. Wang, Managing carbon foot prints in inventory management. *Int. J. Prod. Econ.* **132** (2011) 178–185.

[11] P.A. Hayek and M.K. Salameh, Production lot sizing with the reworking of imperfect quality items produced. *Prod. Plann. Control* **12** (2001) 584–590.

[12] K. Inderfurth, M.Y. Kovalyov, C.T. Ng and F. Werner, Cost minimizing scheduling of work and rework processes on a single facility under deterioration of reworkables. *Int. J. Prod. Econ.* **105** (2007) 345–356.

[13] S. Jain, S. Tiwari, L.E. Cárdenas-Barrón, A.A. Shaikh and S.R. Singh, A fuzzy imperfect production and repair inventory model with time dependent demand, production and repair rates under inflationary conditions. *RAIRO: OR* **52** (2018) 217–239.

[14] M. Khan, M. Hussain and L.E. Cardenas-Barron, Learning and screening errors in an EPQ inventory model for supply chains with stochastic lead time demands. *Int. J. Prod. Res.* **55** (2017) 4816–4832.

[15] G.L. Liao, Optimal economic production quantity policy for a parallel system with repair, rework, free-repair warranty and maintenance. *Int. J. Prod. Res.* **54** (2016) 6265–6280.

[16] R. Li, Y.-L. Chan, C.-T. Chang and L.E. Cardenas-Barron, Pricing and lot-sizing policies for perishable products with advance-cash-credit payments by a discounted cash-flow analysis. *Int. J. Prod. Econ.* **193** (2017) 578–589.

[17] R. Mallick, A.K. Manna and S.K. Mondal, A supply chain model for imperfect production system with stochastic lead time demand. *J. Manage. Anal.* **5** (2018) 309–333.

[18] A. K. Manna, B. Das, J.K. Dey and S.K. Mondal, An EPQ model with promotional demand in random planning horizon: population varying genetic algorithm approach. *J. Intell. Manuf.* **27** (2016) 1–19.

[19] A.K. Manna, J.K. Dey and S.K. Mondal, Imperfect production inventory model with production rate dependent defective rate and advertisement dependent demand. *Comput. Ind. Eng.* **104** (2017) 9–22.

[20] A.K. Manna, B. Das, J.K. Dey and S.K. Mondal, Two layers green supply chain imperfect productioninventory model under bi-level credit period. *TÉKHNE – Rev. Appl. Manage. Stud.* **15** (2017) 124–142.

[21] A.K. Manna, J.K. Dey and S.K. Mondal, Two layers supply chain in an imperfect production inventory model with two storage facilities under reliability consideration. *J. Ind. Prod. Eng.* **35** (2018) 57–73.

[22] A.K. Manna, J.K. Dey and S.K. Mondal, Controlling GHG emission from industrial waste perusal of production inventory model with fuzzy pollution parameters. *Int. J. Syst. Sci. Oper. Logist.* **6** (2019) 368–393.

[23] A.H. Nobil, S. Tiwari and F. Tajik, Economic production quantity model considering warm-up period in a cleaner production environment. *Int. J. Prod. Res.* **57** (2016) 4547–4560.

[24] T. Penkuhn, T. Spengler, H. Puchert and O. Rentz, Environmental integrated production planning for ammonia synthesis. *Eur. J. Oper. Res.* **97** (1997) 327–336.

[25] M.J. Rosenblatt and H.L. Lee, Economic production cycles with imperfect production process. *IIE Trans.* **18** (1986) 48–55.

[26] M.K. Salameh and M.Y. Jaber, Economic production quantity model for items with imperfect quality. *Int. J. Prod. Econ.* **64** (2000) 59–64.

[27] S.S. Sana, A production-inventory model in an imperfect production process. *Eur. J. Oper. Res.* **200** (2010) 451–464.

[28] S.S. Sana, An economic production lot size model in an imperfect production system. *Eur. J. Oper. Res.* **201** (2010) 158–170.

[29] B. Sarkar, An inventory model with reliability in an imperfect production process. *Appl. Math. Comput.* **218** (2012) 4881–4891.

[30] B. Sarkar, B.K. Sett, S. Sarkar, Optimal production run time and inspection errors in an imperfect production system with warranty. *J. Ind. Manage. Opt.* **14** (2018) 267–282.

[31] B.K. Sett, S. Sarkar, B. Sarkar, Optimal buffer inventory and inspection errors in an imperfect production system with preventive maintenance. *Int. J. Adv. Manuf. Technol.* **90** (2017) 1–16.

[32] A.A. Taleizadeh, D.W. Pentico, M.S. Jabalameli and M. Aryanezhad, An economic order quantity model with multiple partial prepayments and partial backordering. *Math. Comput. Model.* **57** (2013) 311–323.

[33] A.A. Taleizadeh, An economic order quantity model for deteriorating items in a purchasing system with multiple prepayments. *Appl. Math. Model.* 38(23) (2014) 5357-5366.

[34] J.T. Teng, L.E. Cárdenas-Barrón, H.J. Chang, J. Wu and Y. Hu, Inventory lot-size policies for deteriorating items with expiration dates and advance payments. *Appl. Math. Model.* **40** (2016) 8605–8616.

[35] S. Tiwari, L.E. Cárdenas-Barrón, M. Goh and A.A. Shaikh, Joint pricing and inventory model for deteriorating items with expiration dates and partial backlogging under two-level partial trade credits in supply chain. *Int. J. Prod. Econ.* **200** (2018) 16–36.

[36] C.C. Wu and N.B. Chang, Corporate optimal production planning with varying environmental costs: A grey compromise programming approach. *Eur. J. Oper. Res.* **155** (2004) 68–95.

[37] A.X. Zhang, Optimal advance payment scheme involving fixed per-payment costs. *Omega* **24** (1996) 577–582.