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PICTURE FUZZY MULTI-CRITERIA GROUP DECISION-MAKING METHOD TO HOTEL BUILDING ENERGY EFFICIENCY RETROFIT PROJECT SELECTION

Le Wang[1], Hong-yu Zhang[1], Jian-qiang Wang[1,*] and Guo-fang Wu[1]

Abstract. Building energy consumption accounts for a considerable proportion on energy consumption. To reduce building energy consumption, building energy efficiency retrofitting (BEER) based on Energy Performance Contracting mechanism is the most feasible and cost-effective method. With the increase number of BEER projects, BEER project selection has become an essential problem for energy service companies. In this paper, a multi-criteria group decision-making (MCGDM) method is proposed to deal with BEER project selection problem. First, picture fuzzy sets are employed to describe the evaluation information under the complex and uncertain environment. Subsequently, picture fuzzy weighted average operator and Laplace distribution-picture fuzzy order weighted average operator are proposed based on convex combination to aggregate individual evaluations into the overall evaluations. Furthermore, picture fuzzy TOPSIS-based QUALIFLEX method is developed to identify the optimal ranking of alternatives. Moreover, the practicality, effectiveness and advantages of the proposed MCGDM method are illustrated using a case study of hotel BEER project selection and comparative analysis. Finally, conclusions about primary contributions, and future discussions of the proposed method are demonstrated.

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1. Introduction

At present, the energy crisis and environmental problems have got more and more attention. Among various kinds of energy consumption, building energy consumption accounts for a significant proportion. Worldwide, 40% of primary energy consumption is attributed to buildings [4]. In Kuwait, the energy consumption of buildings accounts for about 70% of the total energy consumption [15]. In addition, the amount of building energy consumption is rising year by year. For example, in China, the proportion of building energy consumption in the total energy consumption has grown to 27.5% from 10% in the late 1970s. Thus, to reduce building energy consumption, sustainable development of building energy efficiency becomes a global consensus for a long-term development. One way to improve building energy efficiency is through building energy efficiency retrofit (BEER) [13]. BEER is regarded as a process to reduce the use of building operation (such as building

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[1] School of Business, Central South University, 410083 Changsha, PR China.
*Corresponding author: jqwang@csu.edu.cn

envelope, heating systems, lighting, hot water supply facilities) energy by certain approaches on the premise of maintaining a comfortable indoor environment of building [54]. Currently, a lot of attention has been paid to BEER projects. Wu *et al.* [51] studied the concept, model and control of large-scale BEER. Solmaz *et al.* [31] presented an optimization-based decision support approach to determine the optimal BEER options. Hou *et al.* [12] compared the policies of commercial BEER in four pilot cities in China.

Generally, building owners prefer to select Energy Performance Contracting (EPC) mechanism to improve energy efficiency for ensuring the effective implementation of BEER project [54]. EPC, as an important and effective energy conservation mechanism, has been introduced in China since 1990s [5]. Under EPC mechanism, an energy service company (ESCO) will provide a turnkey service of energy saving to building owners and share the energy cost savings [59]. In China, 95% of existing buildings are "high-energy-consumers". A growing number of building owners are opting to retrofit building energy efficiency in cooperation with ESCO [53]. Therefore, the number of BEER projects has been increasing, and the BEER project selection becomes more and more essential for ESCOs. Similar to other selection problems (such as the supplier selection, personnel selection, outsourcing vendor selection), BEER project selection also can be regarded as multi-criteria decision-making (MCDM) or a multi-criteria group decision-making (MCGDM) problem.

Due to the complexity, fuzziness and the uncertainty of the external environment, utilizing the fuzzy numbers to describe the evaluation information is more appropriate and superior than crisp numbers for BEER project selection. To copy with fuzzy information, Zadeh [57] proposed fuzzy sets (FSs), which were widely used to solve selection problems. Since FSs were introduced, it has been extensively extended, such as interval valued fuzzy sets [3], intuitionistic fuzzy sets (IFS) [1], neutrosophic sets [29], interval neutrosophic probability [40], hesitant fuzzy sets [46], and multi-hesitant fuzzy linguistic [35]. Recently, Cuong and Kreinovich [6] introduced picture fuzzy sets (PFSs), which were characterized by four functions, namely, the degree of positive membership, the degree of neutral membership, the degree of negative membership, and the degree of refusal membership. Compared with FSs and IFSs, PFSs can be applied to the situation where human opinions include more types of answers, such as yes, abstain, no and refusal. Furthermore, many researchers have devoted themselves to the study of PFSs [24,48,58]. Wei [43] proposed picture fuzzy cross-entropy and established a MCDM method based on the picture fuzzy cross-entropy. Le [16] defined generalized picture distance and applied it to picture fuzzy clustering problems. Wang *et al.* [37,38] proposed picture fuzzy normalized projection-based VIKOR method and a prospect theory-based MABAC method for risk evaluation. Afterwards, Wei [44,45,47] defined some operations of picture fuzzy numbers (PFNs) and developed several picture fuzzy aggregation operators and picture 2-tuple linguistic aggregation operators. However, the defined operations of PFNs may have some drawbacks, which will be described in Section 2.2.

Aggregation operators are useful tools to fuse information [20,42,49]. The simplest operators are the weighted arithmetic operator and weighted geometric operator, which take into account the importance of values themselves. Subsequently, Yager [55] proposed order weighted average (OWA) operator, which is to weight the ordered positions of the values instead of weighting the values themselves. Subsequently, many researchers built various fuzzy generalizations of the OWA operator under various fuzzy environment. Wu [50] introduced induced intuitionistic trapezoidal fuzzy OWA (IITFOWA) operator and similarity degree IITFOWA operator under intuitionistic trapezoidal fuzzy environment. Liu and Wang [19] proposed interval neutrosophic prioritized OWA operator with respect to interval neutrosophic numbers. Xian *et al.* [52] developed fuzzy linguistic induced generalized OWA operator with fuzzy linguistic values. Furthermore, Reimann *et al.* [28] studied how well does the OWA operator represent real preferences by an empirical study. In addition, a survey about the development of OWA operators from 1988 to 2014 was presented by Emrouznejad and Marratrace [9].

In recent years, in order to effectively solve MCDM problems from different angles, there is an increasing number of studies concerning the MCDM methods, such as Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) method [11], Vlsekriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method [38,39], an acronym in Portuguese for interactive and multi-criteria decision-making (TODIM) method [56], preference ranking organization method for enrichment evaluations (PROMETHEE) method, elimination and choice expressing reality (ELECTRE) [25,26], and qualitative flexible multiple criteria method

(QUALIFLEX) [14, 32]. QUALIFLEX method aims to obtain the permutation whose ranking best reflects the preorders according to the pair-wise comparisons of all alternatives [36]. Compared with other MCDM methods, the main advantage of QUALIFLEX method is that it can be utilized to effectively deal with complex or large MCDM problems involving numerous criteria and a limited number of alternatives. In addition, QUALIFLEX method treats the cardinal and ordinal information correctly and it is easy to understand and calculate. Recently, QUALIFLEX method has been widely utilized in selection problems under various fuzzy environment. Tian *et al.* [33] utilized extended QUALIFLEX method to copy with green product design selection by life cycle assessment technique with simplified neutrosophic linguistic information. Peng *et al.* [27] studied the selection problem of logistics provider through an extended QUALIFLEX method under probability multi-valued neutrosophic environment. Li *et al.* [18] proposed an extended QUALIFLEX method for selecting green suppliers under probability hesitant fuzzy environment. In previous study, the concordance/discordance index of QUALIFLEX method was obtained based on the signed distance-based comparison approach. However, the final rankings were occasionally incorrect, especially when only the distance closest to the positive ideal solution (PIS) or furthest from the negative ideal solution (NIS) was taken into account [33]. To overcome this defect, a closeness coefficient of TOPSIS considering both PIS and NIS will be introduced to calculate the concordance/discordance index of QUALIFLEX method.

There are two kinds of biases that may affect the final decision results in decision-making process [10, 23]. The first one is cognitive bias. For example, decision-makers (DMs) may produce a prejudice and deviation on judging the same thing because of the different knowledge backgrounds. To overcome the first kind of bias, group decision-making method would be adopted. Moreover, the second one is motivational bias. For example, a DM may deliberately attempt to assign higher weights to a criterion which favors a preferred alternative. Taking into account the stability and robust of Laplace distribution in handing outliers, it will be introduced to minimize the second bias. Because, Laplace distribution assigns the highest weights to the middle values and the smallest weights to the highest and lowest values [21].

This research has four main purposes. The first is to develop a comprehensive MCGDM method considering the cognitive bias and motivational bias for helping ESCO to select an optimum BEER project under complex and uncertain environment. The second lies in exploring several picture fuzzy aggregation operators based on a convex combination of PFNs. The third is to establish an integrated picture fuzzy outranking method considering both PIS and NIS. The final purpose is to demonstrate the application, practicality and effectiveness of the proposed MCGDM method using a case study about hotel BEER project selection.

For the sake of clarity, the rest of this research is organized as follows. In Section 2, concepts regarding PFSs, operations of PFNs, generalized picture distance measure and a convex combination of PFNs are briefly reviewed. In Section 3, picture fuzzy weighted average (PFWA) operator and Laplace distribution picture fuzzy order weighted average LD-PFOWA operator are proposed. In Section 4, the TOPSIS method and a TOPSIS-based QULIFLEX method with picture fuzzy information are developed. In Section 5, a MCGDM method based on LD-PFOWA operator and TOPSIS-based QUALIFLEX method is built. In Section 6, a case study of a hotel BEER project selection problem is demonstrated to illustrate the applicability, effectiveness and advantages of the proposed method. In addition, comparative analyses are conducted. Finally, in Section 7, the conclusions of this research are presented.

2. Preliminaries

This section reviews some basic concepts of PFSs, operations of PFNs, generalized picture distance and a convex combination of PFNs. These concepts will be useful in the reminder of this research.

2.1. Picture fuzzy sets

Since Atanassov [1] introduced IFS, it has been successfully applied in many areas. However IFS can't satisfy the situation when we face human opinions involving four types of answers: yes, abstain, no and refusal. Thus, Coung and Kreinovich [6] proposed the PFSs which are more accurate in describing that situation. Then, the concept of PFS is described as follows.

Definition 2.1 ([6])**.** Let PFS A on a universe X is an object in the form of

$$A\left(x\right)=\left\{\left\langle x,\mu_A\left(x\right),\eta_A\left(x\right),\nu_A\left(x\right)\right\rangle|x\in X\right\},$$

where $\mu_A\left(x\right)\in[0,1]$, $\eta_A\left(x\right)\in[0,1]$ and $\nu_A\left(x\right)\in[0,1]$, with $0\leq\mu_A\left(x\right)+\eta_A\left(x\right)+\nu_A\left(x\right)\leq 1$ for $\forall x\in X$. And, $\mu_A\left(x\right)$, $\eta_A\left(x\right)$, and $\nu_A\left(x\right)$ denote the degrees of positive, neutral, and negative, respectively. Moreover, $\pi_A\left(x\right)=1-\mu_A\left(x\right)-\eta_A\left(x\right)-\nu_A\left(x\right)$ is the degree of refusal membership of x in A. In particular, if X has only one element, then $A\left(x\right)=\left\{\left\langle x,\mu_A\left(x\right),\eta_A\left(x\right),\nu_A\left(x\right)\right\rangle|x\in X\right\}$ is called a PFN. For convenience, a PFN is denoted by $A=\langle\mu_A,\eta_A,\nu_A\rangle$.

2.2. Operations of PFNs

Wei *et al.* [44, 45, 47] defined the operations of PFNs, and proposed several picture fuzzy aggregation operators and picture 2-tuple linguistic aggregation operators. However, the defined operations cannot satisfy the constraint that the sum of the three degrees does not exceed 1. Thereafter, the operations of PFNs and their drawbacks are presented in the following.

Definition 2.2 ([44])**.** Let $a=\langle\mu_a,\eta_a,\nu_a\rangle$ and $b=\langle\mu_b,\eta_b,\nu_b\rangle$ be the two PFNs, then, the operations of them are defined as follows:

(1) $a\oplus b=\langle\mu_a+\mu_b-\mu_a\mu_b,\eta_a\eta_b,\nu_a\nu_b\rangle$;
(2) $a\otimes b=\langle\mu_a\mu_b,\eta_a+\eta_b-\eta_a\eta_b,\nu_a+\nu_b-\nu_a\nu_b\rangle$;
(3) $\lambda a=\left\langle 1-\left(1-\mu_a\right)^\lambda,\eta_a^\lambda,\nu_a^\lambda\right\rangle,(\lambda>0)$;
(4) $a^\lambda=\left\langle\mu_a^\lambda,1-\left(1-\eta_a\right)^\lambda,1-\left(1-\nu_a\right)^\lambda\right\rangle,(\lambda>0)$.

However, the following examples can prove that the proposed operations of PFNs are unreasonable.

Example 2.3. (1) If $a=\langle 0.2,0.3,0.5\rangle$ and $b=\langle 0.3,0.5,0.2\rangle$, then

$$c=a\otimes b=\langle\mu_a\mu_b,\eta_a+\eta_b-\eta_a\eta_b,\nu_a+\nu_b-\nu_a\nu_b\rangle=\langle\mu_c,\eta_c,\nu_c\rangle=\langle 0.06,0.65,0.6\rangle.$$

(2) If $a=\langle 0.2,0.5,0.2\rangle$ and $\lambda=0.2$, then

$$d=\langle\mu_d,\eta_d,\nu_d\rangle=\lambda a=\left\langle 1-\left(1-\mu_a\right)^\lambda,\eta_a^\lambda,\nu_a^\lambda\right\rangle=\langle 0.04,0.87,0.72\rangle.$$

(3) If $a=\langle 0.8,0.1,0.1\rangle$ and $\lambda=2$, then

$$e=\langle\mu_e,\eta_e,\nu_e\rangle=a^\lambda=\left\langle\mu_a^\lambda,1-\left(1-\eta_a\right)^\lambda,1-\left(1-\nu_a\right)^\lambda\right\rangle=\langle 0.64,0.19,0.19\rangle.$$

Thus, the results of $\mu_c+\eta_c+\nu_c=1.31>1$, $\mu_d+\eta_d+\nu_d=1.63>1$, and $\mu_e+\eta_e+\nu_e=1.02>1$ can be obtained, which can not conform to the definition of $0\leq\mu_A\left(x\right)+\eta_A\left(x\right)+\nu_A\left(x\right)\leq 1$ for PFNs in Definition 2.1. Therefore, Example 2.3 demonstrates that the existing operations of PFNs are unreasonable.

2.3. Generalized picture distance measure

Distance measure is an important tool to reflect the difference between two different objects and it has been widely introduced in various fuzzy environment [7,30]. Under picture fuzzy environment, Cuong and Kreinovich [6] first proposed a basic picture distance measure. Subsequently, Le [17] proposed a generalized picture distance as follows.

Definition 2.4 ([17])**.** Let A and B be two PFSs in X, then the generalized picture distance measure can be defined as follows:

$$d_G(A,B) = \frac{\left(\frac{1}{n}\sum_{i=1}^{n}(\Delta)\right)^{1/p}}{\left(\frac{1}{n}\sum_{i=1}^{n}(\Delta)\right)^{1/p} + \left(\max_i\left\{\Phi_i^A,\Phi_i^B\right\} + \frac{1}{n}\left|\Phi_i^A - \Phi_i^B\right|\right)^{1/p} + 1}, \tag{2.1}$$

where $\Delta = \frac{\Delta\mu_i^p+\Delta\eta_i^p+\Delta\nu_i^p}{3} + \max\{\Delta\mu_i^p,\Delta\eta_i^p,\Delta\nu_i^p\}$, $\Delta\mu_i = |\mu_A(x_i) - \mu_B(x_i)|$, $\Delta\eta_i = |\eta_A(x_i) - \eta_B(x_i)|$, $\Delta\nu_i = |\nu_A(x_i) - \nu_B(x_i)|$, $\Phi_i^A = |\mu_A(x_i) + \eta_A(x_i) + \nu_A(x_i)|$ and $\Phi_i^B = |\mu_B(x_i) + \eta_B(x_i) + \nu_B(x_i)|$.

2.4. Convex combination of two PFNs

The convex combination operation is first used in two linguistic terms by Delgado *et al.* [8]. Then, Wei *et al.* [41] proposed the convex combination operation for hesitant fuzzy linguistic term sets. Afterwards, Peng *et al.* [27] applied the convex combination operation to probability multi-valued neutrosophic environments. In addition, Cuong and Kreinovich [6] firstly defined convex combination of PFS with some simple propositions in the following.

Definition 2.5. Let $A = \{\alpha_1,\alpha_2,\ldots,\alpha_n\}$ be a PFS, α_i and α_j are two PFNs in A. Then the picture fuzzy convex combination operation of α_i and $\alpha_j$is defined as

$$\mathrm{PFC}_2(w_1,\alpha_i,w_2,\alpha_j) = w_1\otimes\alpha_i \oplus w_2\otimes\alpha_j = \langle w_1\mu_i + w_2\mu_j, w_1\eta_i + w_2\eta_j, w_1\nu_i + w_2\nu_j\rangle \tag{2.2}$$

where $w_1 + w_2 = 1$ and $0 \le w_1, w_2 \le 1$. It is obvious that the result obtained by equation (2.2) is also a PFN.

Example 2.6. Let $\alpha_1 = \langle 0.2, 0.3, 0.4\rangle$, and $\alpha_2 = \langle 0.3, 0.4, 0.2\rangle$ be two PFNs, $w_1 = 0.4$, and $w_2 = 0.6$. Then, there is

$$\begin{aligned}\mathrm{PFC}_2(w_1,\alpha_i,w_2,\alpha_j) &= w_1\otimes\alpha_1 \oplus w_2\otimes\alpha_2\\ &= \langle 0.4\times0.2 + 0.6\times0.3, 0.4\times0.3 + 0.6\times0.4, 0.4\times0.4 + 0.6\times0.2\rangle\\ &= \langle 0.26, 0.36, 0.28\rangle\end{aligned}$$

3. Some picture fuzzy aggregation operators based on convex combination

In this section, some aggregation operators will be proposed to aggregate individual evaluations into overall evaluations based on the convex combination of PFS.

3.1. Picture fuzzy weighted average operator

First, PFWA operator is defined based on the convex combination of two PFNs as follows.

Definition 3.1. Let $\alpha_i = \langle\mu_i,\eta_i,\nu_i\rangle\,(i = 1,2,\ldots,n)$ be a collection of PFNs, then the PFWA operator is defined as

$$\begin{aligned}\mathrm{PFWA}(\alpha_1,\alpha_2,\ldots,\alpha_n) &= \mathrm{PFC}_n(w_i,\alpha_i, i = 1,2,\ldots,n)\\ &= w_1\otimes\alpha_1 \oplus (1-w_1)\otimes \mathrm{PFC}_{n-1}\left(\frac{w_g}{\sum_{i=2}^{n} w_i},\alpha_g, g = 2,\ldots,n\right),\end{aligned} \tag{3.1}$$

where $w = (w_1,w_2,\ldots,w_n)$ is the weight vector of α_i, with $0 \le w_i \le 1$ and $\sum_{i=1}^{n} w_i = 1$.

Theorem 3.2. Let $\alpha_i = \langle\mu_i,\eta_i,\nu_i\rangle\,(i = 1,2,\ldots,n)$ be a collection of PFNs, then the value aggregated by equation (3.1) is also a PFN, and

$$\begin{aligned}\mathrm{PFWA}\left(\alpha_1,\alpha_2,\ldots,\alpha_n\right) &= \mathrm{PFC}_n\left(w_i,\alpha_i,i=1,2,\ldots,n\right)\\ &= w_1\otimes\alpha_1\oplus\left(1-w_1\right)\mathrm{PFC}_{n-1}\left(\frac{w_g}{\sum_{i=2}^n w_i},\alpha_g,g=2,\ldots,n\right)\\ &= \left\langle\sum_{i=1}^n w_i\mu_i,\sum_{i=1}^n w_i\eta_i,\sum_{i=1}^n w_i\nu_i\right\rangle. \end{aligned}\tag{3.2}$$

Proof. (1) When $n=2$, the following equation can be obtained based on Definition 2.5,

$$\begin{aligned}\mathrm{PFWA}\left(\alpha_1,\alpha_2\right) &= \mathrm{PFC}_2\left(w_1,\alpha_1,w_2,\alpha_2\right)=w_1\otimes\alpha_1\oplus w_2\otimes\alpha_2\\ &= \left\langle w_1\mu_1+w_2\mu_2,w_1\eta_1+w_2\eta_2,w_1\nu_1+w_2\nu_2\right\rangle\\ &= \left\langle\sum_{i=1}^2 w_i\mu_i,\sum_{i=1}^2 w_i\eta_i,\sum_{i=1}^2 w_i\nu_i\right\rangle.\end{aligned}$$

That is, when $n=2$, equation (3.2) is true.

(2) Assuming equation (3.2) holds for $n=l$, then

$$\begin{aligned}\mathrm{PFWA}\left(\alpha_1,\alpha_2,\ldots,\alpha_l\right) &= \mathrm{PFC}_l\left(w_i,\alpha_i,i=1,2,\ldots,l\right)\\ &= w_1\otimes\alpha_1\oplus\left(1-w_1\right)\otimes\mathrm{PFC}_{l-1}\left(\frac{w_g}{\sum_{i=2}^n w_i},\alpha_g,g=2,\ldots,l\right)\\ &= \left\langle\sum_{i=1}^l w_i\mu_i,\sum_{i=1}^l w_i\eta_i,\sum_{i=1}^l w_i\nu_i\right\rangle.\end{aligned}$$

(3) When $n=l+1$, we can acquire the following result

$$\begin{aligned}\mathrm{PFWA}\left(\alpha_1,\alpha_2,\ldots,\alpha_l,\alpha_{l+1}\right) &= \mathrm{PFC}_{l+1}\left(w_i,\alpha_i,i=1,2,\ldots,l,l+1\right)\\ &= w_1\otimes\alpha_1\oplus\left(1-w_1\right)\otimes\mathrm{PFC}_l\left(\frac{w_g}{\sum_{i=2}^n w_i},\alpha_g,g=2,\ldots,l,l+1\right)\\ &= \left\langle\sum_{i=1}^{l+1} w_i\mu_i,\sum_{i=1}^{l+1} w_i\eta_i,\sum_{i=1}^{l+1} w_i\nu_i\right\rangle.\end{aligned}$$

That is, equation (3.2) also holds for $n=l+1$. Therefore, equation (3.2) is true for all n.

□

Theorem 3.3 (Monotonicity). *Let* $\alpha_i=\langle\mu_{\alpha_i},\eta_{\alpha_i},\nu_{\alpha_i}\rangle\,(i=1,2,\ldots,n)$ *and* $\beta_i=\langle\mu_{\beta_i},\eta_{\beta_i},\nu_{\beta_i}\rangle\,(i=1,2,\ldots,n)$ *be two collections of PFSs. If* $\alpha_i\le\beta_i$ *for all* i, *that* $\mu_{\alpha_i}\le\mu_{\beta_i}$, $\eta_{\alpha_i}\le\eta_{\beta_i}$ *and* $\nu_{\alpha_i}\ge\nu_{\beta_i}$ *for all* i, *then*

$$\mathrm{PFWA}\left(\alpha_1,\alpha_2,\ldots,\alpha_n\right)\le\mathrm{PFWA}\left(\beta_1,\beta_2,\ldots,\beta_n\right).\tag{3.3}$$

Theorem 3.4 (Boundedness). *Let* $\alpha_i=\langle\mu_{\alpha_i},\eta_{\alpha_i},\nu_{\alpha_i}\rangle\,(i=1,2,\ldots,n)$ *be a collection of PFSs. Thereafter, we have*

$$\mathrm{PFWA}\left(\alpha^-,\alpha^-,\ldots,\alpha^-\right)\le\mathrm{PFWA}\left(\alpha_1,\alpha_2,\ldots,\alpha_n\right)\le\mathrm{PFWA}\left(\alpha^+,\alpha^+,\ldots,\alpha^+\right)$$

where

$$\begin{aligned}\alpha^+ &= \langle\mu_{\alpha^+},\eta_{\alpha^+},\nu_{\alpha^+}\rangle\\ &= \langle\max\left(\mu_{\alpha_1},\mu_{\alpha_2},\ldots,\mu_{\alpha_n}\right),\min\left(\eta_{\alpha_1},\eta_{\alpha_2},\ldots,\eta_{\alpha_n}\right),\min\left(\nu_{\alpha_1},\nu_{\alpha_2},\ldots,\nu_{\alpha_n}\right)\rangle;\\ \alpha^- &= \langle\mu_{\alpha^-},\eta_{\alpha^-},\nu_{\alpha^-}\rangle\\ &= \langle\min\left(\mu_{\alpha_1},\mu_{\alpha_2},\ldots,\mu_{\alpha_n}\right),\min\left(\eta_{\alpha_1},\eta_{\alpha_2},\ldots,\eta_{\alpha_n}\right),\max\left(\nu_{\alpha_1},\nu_{\alpha_2},\ldots,\nu_{\alpha_n}\right)\rangle.\end{aligned}$$

Theorem 3.5 (Idempotency). *Let* $\{\alpha_1,\alpha_2,\ldots,\alpha_n\}$ *be a collection of PFSs. If* $\alpha_i=\alpha\;(i=1,2,\ldots,n)$, *then* $\mathrm{PFWA}\left(\alpha_1,\alpha_2,\ldots,\alpha_n\right)=\alpha$.

3.2. Laplace distribution picture fuzzy order weighted average operator

The Laplace distribution assigns the highest weights to the middle scores and the smallest weights to the highest and lowest values, so it can handle outliers stably and robustly [21]. Recently, Mohammed *et al.* [22] proposed a new method for obtaining the weight vector of the OWA operator based on the Laplace distribution and utilized the proposed operator to aggregate the uncertain information about the breast tumors. Meanwhile, the usefulness of the proposed operator was illustrated by comparing the proposed operator with other operators. Then, the weight vector of OWA based on the Laplace distribution can be calculated based on the equations (3.4) and (3.5) as follows.

$$W_i = \left(\frac{1}{2\lambda_k} e^{-\frac{\left|i-\frac{1+k}{2}\right|}{\lambda_k}}\right) \Big/ \left(\sum_{i=1}^{i=k} \frac{1}{2\lambda_k} e^{-\frac{\left|i-\frac{1+k}{2}\right|}{\lambda_k}}\right), \quad i = 1, 2, \ldots, k. \tag{3.4}$$

$$\sqrt{2}\lambda_k = \sigma = \sqrt{\frac{1}{k} \sum_{i=1}^{i=k} \left(i - \frac{1+k}{2}\right)^2} \tag{3.5}$$

where k is the number of DMs, σ is the standard deviation, and λ_k is the scale of the Laplace distribution.

Based on the characteristics of Laplace distribution and the idea of Mohammed *et al.* [22], we will propose LD-PFOWA operator to reduce motivational bias and inconsistency described in Section 1.

Definition 3.6. A LD-PFOWA operator of dimension n is a mapping $F : R^n \to R$, then

$$\begin{aligned} \text{LD-PFOWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) &= \text{PFC}_n\left(W_i, \alpha_{(i)}, i = 1, 2, \ldots, n\right) \\ &= W_1 \otimes \alpha_{(1)} \oplus (1 - W_1) \otimes \text{PFC}_{n-1}\left(\frac{W_g}{\sum_{i=2}^{n} W_i}, \alpha_{(g)}, g = 2, \ldots, n\right) \end{aligned} \tag{3.6}$$

where $\alpha_{(i)}$ is the ith largest PFN in $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$, and $W = (W_1, W_2, \ldots, W_n)$ calculated by equations (3.4) and (3.5) is the weight vector of $\left\{\alpha_{(1)}, \alpha_{(2)}, \ldots, \alpha_{(n)}\right\}$.

Theorem 3.7. *Let $\alpha_i = \langle \mu_i, \eta_i, \nu_i \rangle$ $(i = 1, 2, \ldots, n)$ be a collection of PFNs, then the value aggregated by equation (3.6) is also a PFN, and*

$$\begin{aligned} \text{LD-PFOWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) &= \text{PFC}_n\left(W_i, \alpha_{(i)}, i = 1, 2, \ldots, n\right) \\ &= W_1 \otimes \alpha_{(1)} \oplus (1 - W_1) \otimes \text{PFC}_{n-1}\left(\frac{W_g}{\sum_{i=2}^{n} W_i}, \alpha_{(g)}, g = 2, \ldots, n\right) \\ &= \left\langle \sum_{i=1}^{n} W_i \mu_{(i)}, \sum_{i=1}^{n} W_i \eta_{(i)}, \sum_{i=1}^{n} W_i \nu_{(i)} \right\rangle. \end{aligned} \tag{3.7}$$

The proof of Theorem 3.7 is similar to Theorem 3.2, thus, the proof is omitted here.

Theorem 3.8 (Monotonicity)**.** *Let $\alpha_i = \langle \mu_{\alpha_i}, \eta_{\alpha_i}, \nu_{\alpha_i} \rangle$ $(i = 1, 2, \ldots, n)$ and $\beta_i = \langle \mu_{\beta_i}, \eta_{\beta_i}, \nu_{\beta_i} \rangle$ $(i = 1, 2, \ldots, n)$ be two collections of PFSs. If $\alpha_{(i)} \leq \beta_{(i)}$ for all i, that $\mu_{\alpha_{(i)}} \leq \mu_{\beta_{(i)}}$, $\eta_{\alpha_{(i)}} \leq \eta_{\beta_{(i)}}$ and $\nu_{\alpha_{(i)}} \geq \nu_{\beta_{(i)}}$ for all i, then*

$$\text{LP-PFOWA}\left(\alpha_{(1)}, \alpha_{(2)}, \ldots, \alpha_{(n)}\right) \leq \text{LP-PFOWA}\left(\beta_{(1)}, \beta_{(2)}, \ldots, \beta_{(n)}\right). \tag{3.8}$$

Theorem 3.9 (Boundedness)**.** *Let*

$$\alpha_i = \langle \mu_{\alpha_i}, \eta_{\alpha_i}, \nu_{\alpha_i} \rangle \, (i = 1, 2, \ldots, n)$$

be a collection of PFSs. Thereafter, we have

$$\begin{aligned} \text{LP-PFOWA}\left(\alpha^-, \alpha^-, \ldots, \alpha^-\right) &\leq \text{LP-PFOWA}\left(\alpha_{(1)}, \alpha_{(2)}, \ldots, \alpha_{(n)}\right) \\ &\leq \text{LP-PFOWA}\left(\alpha^+, \alpha^+, \ldots, \alpha^+\right) \end{aligned}$$

where

$$\alpha^+ = \langle \mu_{\alpha^+}, \eta_{\alpha^+}, \nu_{\alpha^+} \rangle = \left\langle \begin{array}{l} \max\left(\mu_{\alpha_{(1)}}, \mu_{\alpha_{(2)}}, \ldots, \mu_{\alpha_{(n)}}\right), \\ \min\left(\eta_{\alpha_{(1)}}, \eta_{\alpha_{(2)}}, \ldots, \eta_{\alpha_{(n)}}\right), \min\left(\nu_{\alpha_{(1)}}, \nu_{\alpha_{(2)}}, \ldots, \nu_{\alpha_{(n)}}\right) \end{array} \right\rangle;$$

...

$$\alpha^- = \langle \mu_{\alpha^-}, \eta_{\alpha^-}, \nu_{\alpha^-} \rangle = \left\langle \begin{array}{l} \min\left(\mu_{\alpha_{(1)}}, \mu_{\alpha_{(2)}}, \ldots, \mu_{\alpha_{(n)}}\right), \\ \min\left(\eta_{\alpha_{(1)}}, \eta_{\alpha_{(2)}}, \ldots, \eta_{\alpha_{(n)}}\right), \max\left(\nu_{\alpha_{(1)}}, \nu_{\alpha_{(2)}}, \ldots, \nu_{\alpha_{(n)}}\right) \end{array} \right\rangle.$$

Theorem 3.10 (Idempotency)**.** *Let* $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ *be a collection of PFSs. If* $\alpha_i = \alpha$ $(i = 1, 2, \ldots, n)$*, then* LP-PFOWA $(\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha$.

4. TOPSIS-based QUALIFLEX method with PFNs

In this section, a novel TOPSIS-based QUALIFLEX method is develop with the generalized picture distance measure. In the following, we first propose the picture fuzzy TOPSIS method. Further, TOPSIS-based QUALIFLEX method with PFNs will be developed.

4.1. Picture fuzzy TOPSIS method

TOPSIS method is a useful method to rank the feasible alternatives, whose basic principle is that the optimum alternative should have the shortest distance from the PIS and the farthest distance from NIS [2]. Then, the steps of picture fuzzy TOPSIS method is described in the following.

(1) A decision-making problem refers to m alternatives under n criteria. The evaluation value $\sigma_{ij} = \langle \mu_{ij}, \eta_{ij}, \nu_{ij} \rangle$ of alternative X_i under criterion C_j is in the form of PFN. In addition, the PIS and NIS can be defined as PIS $= f^+ = \langle 1, 0, 0 \rangle$ and NIS $= f^- = \langle 0, 0, 1 \rangle$, respectively.

(2) The closeness coefficient of TOPSIS based on the generalized picture distance measure in Definition 4.1 can be calculated as

$$CC\left(X_i\right) = \frac{d_G\left(\sigma_{ij}, \sigma^-\right)}{d_G\left(\sigma_{ij}, \sigma^+\right) + d_G\left(\sigma_{ij}, \sigma^-\right)}, i = 1, 2, \ldots, m. \tag{4.1}$$

(3) The higher value of $CC\left(X_i\right)$ indicates that the alternative X_i is closer to PIS and farther from NIS simultaneously, and then, the alternative X_i is better.

4.2. TOPSIS-based QUALIFLEX method with PFNs

In this subsection, the TOPSIS-based QUALIFLEX method under picture fuzzy environment is proposed, which uses closeness coefficient to identify the corresponding concordance/discordance index. And the proposed method can obtain the best permutation according to the pair-wise comparisons of all alternatives. Then, the steps of the TOPSIS-based QUALIFLEX method is described in the following.

Step 1. For a set X with m alternatives, then $m!$ permutations of the ranking for all alternatives will be exist. Assuming that P_l denotes the lth permutation, then $P_l = \left(\ldots, X_i, \ldots, X_k, \ldots\right), \ l = 1, 2, \ldots, m!$, where $X_i, X_k \in X$ and the alternative X_i is ranked higher than or equal to X_k.

Step 2. The concordance/discordance index $\xi_l^j(X_i, X_k)$ based on the closeness coefficient for each pair of alternatives (X_i, X_k) $(X_i, X_k \in X)$ with respect to C_j is defined as follows.

$$\begin{aligned}\xi_l^j(X_i, X_k) &= CC(X_i) - CC(X_k)\\ &= \frac{d_G(\sigma_{ij}, \sigma^-)}{d_G(\sigma_{ij}, \sigma^+) + d_G(\sigma_{ij}, \sigma^-)} - \frac{d_G(\sigma_{kj}, \sigma^-)}{d_G(\sigma_{kj}, \sigma^+) + d_G(\sigma_{kj}, \sigma^-)},\end{aligned} \tag{4.2}$$

where $\xi_l^j(X_i, X_k) \in [-1, 1]$. Based on the closeness coefficient of PFNs, it is easily obtained from equation (4.2) that

(1) If $\xi_l^j(X_i, X_k) > 0$, that is $CC(X_i) > CC(X_k)$, then X_i is ranked over X_k under C_j. Thus, there exists concordance between the extended closeness coefficient-based ranking orders and the preorder of $X_i$and X_j under P_l.
(2) If $\xi_l^j(X_i, X_k) = 0$, that is $CC(X_i) = CC(X_k)$, then X_i and X_k have the same ranking under C_j. Thus, there exists ex aequo between the extended closeness coefficient-based ranking orders and preorder of $X_i$and X_j under P_l.
(3) If $\xi_l^j(X_i, X_k) < 0$, that is $CC(X_i) < CC(X_k)$, then X_k is ranked over X_i under C_j. Thus, there exists discordance between the extended closeness coefficient-based ranking orders and preorder of $X_i$and X_j under P_l.

For convenience, the concordance/discordance index $\xi_l^j(X_i, X_k)$ can be rewritten as

$$\xi_l^j(X_i, X_k) = \begin{cases} CC(X_i) - CC(X_k) > 0 \Leftrightarrow \text{there is concordance}\\ CC(X_i) - CC(X_k) = 0 \Leftrightarrow \text{there is ex aequo}\\ CC(X_i) - CC(X_k) < 0 \Leftrightarrow \text{there is discordance}\end{cases}. \tag{4.3}$$

Step 3. The weighted concordance/discordance index $\xi_l(X_i, X_k)$ based on the closeness coefficient for each pair of alternatives (X_i, X_k) $(X_i, X_k \in X)$ with respect to all criteria and P_l can be defined as

$$\xi_l(X_i, X_k) = \sum_{j=1}^{n} w_j \xi_l^j(X_i, X_k) = \sum_{j=1}^{n} w_j (CC(X_i) - CC(X_k)) \tag{4.4}$$

where w_j is the weight of criterion C_j.

Step 4. The comprehensive concordance/discordance index ξ_l for P_l can be obtained as

$$\xi_l = \sum_{X_i, X_k \in X} \sum_{j=1}^{n} w_j \xi_l^j(X_i, X_k) = \sum_{X_i, X_k \in X} \sum_{j=1}^{n} w_j (CC(X_i) - CC(X_k)). \tag{4.5}$$

Step 5. The final ranking order of all alternatives can be obtained by the comprehensive concordance/discordance index of each permutation. The bigger ξ_l is, the more reliable P_l is. Therefore, the optimal ranking of the alternatives P^* can be determined by the following equation.

$$\xi^* = \max_{l=1}^{n!} \{\xi_l\}. \tag{4.6}$$

5. Procedure of the picture fuzzy MCGDM method

In this section, a picture fuzzy MCGDM method is developed through combining the LD-PFOWA operator and TOPSIS-based QUALIFLEX method to effectively operate PFNs.

For a MCGDM problem with picture fuzzy information that consists of m alternatives under n criteria by DM d_k. And the evaluation value of alternative X_i $(i = 1, 2, \ldots, m)$ associated with criterion C_j $(j = 1, 2, \ldots, n)$ provided by DM d_k is in the form of PFN denoting as $\sigma_{ij}^k = \langle \mu_{ij}^k, \eta_{ij}^k, \nu_{ij}^k \rangle$. Then, the picture fuzzy decision information can be constructed as $D^k = [\sigma_{ij}^k]_{m\times n}$. Assuming that the weight vector of criteria is $w = (w_1, w_2, \ldots, w_m)$, where $w_j \in [0, 1]$ and $\sum_{j=1}^{m} w_j = 1$.

Based on the above analysis, the procedure of the proposed method for solving the picture fuzzy MCGDM problem can be summarized as follows.

Step 1. Normalize the evaluation information.
Two types of criteria exist in the decision information: maximizing criteria and minimizing criteria. To reduce the influence of different dimensions, we first transform the minimizing function into maximizing function using equation (5.1), and obtain the normalized evaluation information.

$$x_{ij} = \begin{cases} r_{ij} = \langle \mu_{ij}, \eta_{ij}, \nu_{ij} \rangle, C_j \text{is a maximizing criterion} \\ r_{ij}^c = \langle \nu_{ij}, \eta_{ij}, \mu_{ij} \rangle, C_j \text{is a minimizing criterion} \end{cases}. \tag{5.1}$$

Step 2. Obtain the weight vector of DMs based on Laplace distribution.
(1) Calculate the scale λ_k of Laplace distribution based on equation (3.5).
(2) Calculate the weight vector of DMs using equation (3.4).

Step 3. Reorder the evaluation values.
Determine the PIS f_j^+ and NIS f_j^-, and calculate the closeness coefficient $CC\left(\sigma_{ij}^k\right)$ of TOPSIS method by equation (4.1). The value of $CC\left(\sigma_{ij}^k\right)$ is greater, σ_{ij}^k is larger. Then, the evaluation values can be reordered based on closeness coefficient $CC\left(\sigma_{ij}^k\right)$.

Step 4. Obtain the overall information.
Utilize LD-PFOWA operator to synthesize individual evaluation information into the overall information based on equation (5.1).

Step 5. List all possible permutations for all alternatives.
All possible $m!$ permutations of the ranking for m alternatives are listed and P_l indicates the lth permutation.

Step 6. Calculate concordance/discordance index.
The concordance/discordance index $\xi_l^j(X_i, X_k)$ based on the closeness coefficient for each pair of alternatives (X_i, X_k) $(X_i, X_k \in X)$ with respect to C_j can be calculated by equation (4.2).

Step 7. Compute the weighted concordance/discordance index.
The weighted concordance/discordance index $\xi_l(X_i, X_k)$ based on the closeness coefficient for each pair of alternatives (X_i, X_k) $(X_i, X_k \in X)$ with respect to all criteria and P_l can be computed by equation (4.4).

Step 8. Obtain the comprehensive concordance/discordance index.
The comprehensive concordance/discordance index ξ_l for the permutation P_l can be obtained by equation (4.5).

Step 9. Identify the optimal ranking of alternatives.
The optimal ranking of alternatives P^* can be identified by equation (4.6). Then, we can obtain the final ranking of all alternatives.

6. Case study

The aim of this section is to illustrate the application and effectiveness of the proposed MCGDM method with picture fuzzy information. A case study about hotel BEER project selection problem for ESCO will be presented. Moreover, the method's efficiency is proved through comparative analysis with other methods. Figure 1 shows the scheme for this case study.

6.1. Background

With the arrival of the era of mass tourism, hotel industry as a key part of the tourism gained rapid development. However, the characters of hotel industry are high investment, high energy consumption and serious pollution, which are not only bad for environment, but also increase the cost of hotel owner. Thus, hotel energy

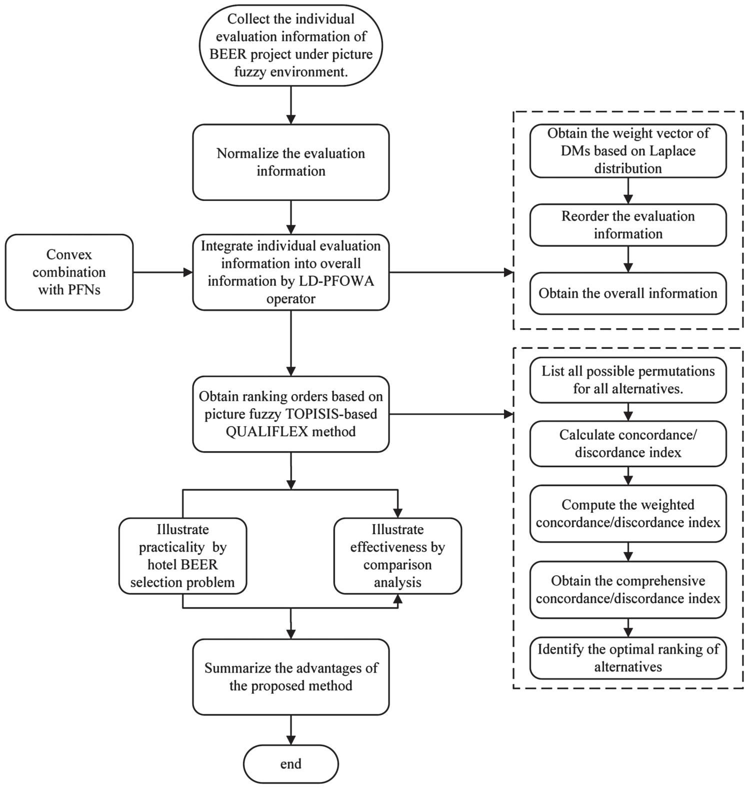

Figure 1. Scheme for this case study.

efficiency retrofit is necessary. In recent years, with the development of EPC, more and more hotels select to conduct energy efficiency retrofit by ESCO.

Four hotels all announcement of tender for conducting BEER. They are X_1 in Beijing, China, X_2 in Shandong, China, X_3 in Shenzhen, China, and X_4 in Jiangsu, China. An ESCO in Shandong, China wants to select one of the hotels for bidding. Therefore, five professional teams denoted by $\{d_1, d_2, d_3, d_4, d_5\}$ are invited to help ESCO select an optimal BEER project. Each professional team is made up of ten people, such as, building energy conservation engineers, risk analysts, finance experts, environmental experts, and hotel managers. Moreover, six main factors and their weight (shown in Tab. 1) will be considered through literature review [53]. Considering the fuzziness and uncertainty in the selection process, PFNs are employed to describe the evaluation information, and Tables 2–6 list the decision-making information of d_k. The meaning of PFNs will be explained as follows. For example, the value of X_1 under criterion C_6 provided by the first team d_1 is denoted as $\sigma_{16}^1 = \langle \mu_{16}^1, \eta_{16}^1, \nu_{16}^1 \rangle = \langle 0.4, 0.3, 0.1 \rangle$, where $\mu_{16}^1 = 0.4$ means that four people in the first team consider that the hotel X_1 has a good external economics; $\eta_{16}^1 = 0.2$ means that two people in the first team consider that the hotel X_1 has a medium external economics; $\nu_{16}^1 = 0.1$ means that one person in the first team considers hotel X_1 has a bad external economic; and $\pi_{16}^1 = 0.3$ means that three people in the first team refuse to provide an evaluation for X_1 under the criterion C_6.

6.2. Illustration of the proposed method

Step 1. Normalize the evaluation information.

Table 1. Criteria and their weights for evaluating.

Criteria	Weight
Project management	0.27
Project financing	0.2
Knowledge and innovation of EPC, sustainable development (SD) and measurement and verification	0.15
Formulation of sustainable development strategy	0.15
Contracting	0.13
External economics	0.1

Table 2. The decision-making information of d_1.

	C_1	C_2	C_3	C_4	C_5	C_6
X_1	$\langle 0.7, 0.2, 0.1\rangle$	$\langle 0.3, 0.2, 0.5\rangle$	$\langle 0.4, 0.2, 0.1\rangle$	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.5, 0.2, 0.3\rangle$	$\langle 0.4, 0.3, 0.1\rangle$
X_2	$\langle 0.4, 0.1, 0.5\rangle$	$\langle 0.4, 0.2, 0.2\rangle$	$\langle 0.5, 0.3, 0.2\rangle$	$\langle 0.4, 0.2, 0.2\rangle$	$\langle 0.2, 0.1, 0.6\rangle$	$\langle 0.5, 0.1, 0.1\rangle$
X_3	$\langle 0.3, 0.3, 0.1\rangle$	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.7, 0.1, 0.2\rangle$	$\langle 0.3, 0.3, 0.2\rangle$	$\langle 0.4, 0.3, 0.1\rangle$	$\langle 0.3, 0.5, 0.1\rangle$
X_4	$\langle 0.5, 0.2, 0.3\rangle$	$\langle 0.5, 0.3, 0.2\rangle$	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.4, 0.2, 0.1\rangle$	$\langle 0.6, 0.2, 0.1\rangle$	$\langle 0.5, 0.2, 0.1\rangle$

Table 3. The decision-making information of d_2.

	C_1	C_2	C_3	C_4	C_5	C_6
X_1	$\langle 0.4, 0.2, 0.3\rangle$	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.4, 0.2, 0.1\rangle$	$\langle 0.5, 0.1, 0.2\rangle$
X_2	$\langle 0.3, 0.2, 0.3\rangle$	$\langle 0.6, 0.1, 0\rangle$	$\langle 0.6, 0.2, 0.1\rangle$	$\langle 0.5, 0.1, 0.1\rangle$	$\langle 0.3, 0.2, 0.3\rangle$	$\langle 0.6, 0.2, 0.1\rangle$
X_3	$\langle 0.5, 0.1, 0.1\rangle$	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.5, 0.1, 0.2\rangle$	$\langle 0.1, 0.4, 0.5\rangle$	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.2, 0.2, 0.3\rangle$
X_4	$\langle 0.3, 0.1, 0.3\rangle$	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.5, 0.1, 0.2\rangle$	$\langle 0.3, 0.5, 0.1\rangle$	$\langle 0.5, 0.3, 0.1\rangle$	$\langle 0.6, 0.2, 0.1\rangle$

Table 4. The decision-making information of d_3.

	C_1	C_2	C_3	C_4	C_5	C_6
X_1	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.1, 0.2, 0.4\rangle$	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.3, 0.3, 0.1\rangle$	$\langle 0.5, 0.1, 0.1\rangle$	$\langle 0.3, 0.3, 0.2\rangle$
X_2	$\langle 0.3, 0.2, 0.3\rangle$	$\langle 0.3, 0.2, 0.2\rangle$	$\langle 0.5, 0.1, 0.3\rangle$	$\langle 0.3, 0.5, 0.1\rangle$	$\langle 0.4, 0.2, 0.2\rangle$	$\langle 0.5, 0.2, 0.3\rangle$
X_3	$\langle 0.5, 0.2, 0.3\rangle$	$\langle 0.4, 0.3, 0.2\rangle$	$\langle 0.6, 0.1, 0.1\rangle$	$\langle 0.2, 0.3, 0.2\rangle$	$\langle 0.6, 0.2, 0.2\rangle$	$\langle 0.4, 0.1, 0.2\rangle$
X_4	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.4, 0.2, 0.1\rangle$	$\langle 0.5, 0.2, 0.3\rangle$	$\langle 0.4, 0.2, 0.1\rangle$	$\langle 0.7, 0.2, 0.1\rangle$	$\langle 0.5, 0.2, 0.1\rangle$

Table 5. The decision-making information of d_4.

	C_1	C_2	C_3	C_4	C_5	C_6
X_1	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.2, 0.7, 0.1\rangle$	$\langle 0.4, 0.3, 0.1\rangle$	$\langle 0.4, 0.3, 0.2\rangle$	$\langle 0.6, 0.2, 0.1\rangle$	$\langle 0.5, 0.2, 0.1\rangle$
X_2	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.5, 0.1, 0.2\rangle$	$\langle 0.5, 0.2, 0.3\rangle$	$\langle 0.4, 0.2, 0.1\rangle$
X_3	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.4, 0.1, 0.4\rangle$	$\langle 0.5, 0.1, 0.2\rangle$	$\langle 0.2, 0.2, 0.3\rangle$	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.5, 0.2, 0.1\rangle$
X_4	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.3, 0.3, 0.1\rangle$	$\langle 0.3, 0.2, 0.2\rangle$	$\langle 0.2, 0.2.0.4\rangle$	$\langle 0.6, 0.2, 0.2\rangle$	$\langle 0.6, 0.2, 0.1\rangle$

TABLE 6. The decision-making information of d_5.

	C_1	C_2	C_3	C_4	C_5	C_6
X_1	$\langle 0.2, 0.3, 0.5\rangle$	$\langle 0.3, 0.2, 0.2\rangle$	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.5, 0.2, 0.2\rangle$	$\langle 0.4, 0.2, 0.1\rangle$
X_2	$\langle 0.4, 0.2, 0.1\rangle$	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.5, 0.2, 0.3\rangle$	$\langle 0.3, 0.2, 0.2\rangle$	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.5, 0.3, 0.1\rangle$
X_3	$\langle 0.3, 0.1, 0.4\rangle$	$\langle 0.5, 0.2, 0.3\rangle$	$\langle 0.7, 0.1, 0\rangle$	$\langle 0.4, 0.2, 0.2\rangle$	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.4, 0.1, 0.2\rangle$
X_4	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.4, 0.2, 0.3\rangle$	$\langle 0.5, 0.1, 0.3\rangle$	$\langle 0.5, 0.1, 0.1\rangle$	$\langle 0.4, 0.3, 0.1\rangle$	$\langle 0.4, 0.2, 0.1\rangle$

TABLE 7. The overall information.

	C_1	C_2	C_3	C_4	C_5	C_6
X_1	$\langle 0.443, 0.139, 0.282\rangle$	$\langle 0.25, 0.449, 0.212\rangle$	$\langle 0.393, 0.15, 0.189\rangle$	$\langle 0.389, 0.243, 0.182\rangle$	$\langle 0.489, 0.2, 0.163\rangle$	$\langle 0.418, 0.207, 0.125\rangle$
X_2	$\langle 0.314, 0.193, 0.318\rangle$	$\langle 0.407, 0.157, 0.205\rangle$	$\langle 0.5, 0.162, 0.2686\rangle$	$\langle 0.4, 0.23, 0.175\rangle$	$\langle 0.375, 0.175, 0.252\rangle$	$\langle 0.489, 0.232, 0.114\rangle$
X_3	$\langle 0.4, 0.155, 0.207\rangle$	$\langle 0.432, 0.132, 0.255\rangle$	$\langle 0.6, 0.1, 0.137\rangle$	$\langle 0.225, 0.282, 0.175\rangle$	$\langle 0.525, 0.212, 0.206\rangle$	$\langle 0.375, 0.187, 0.182\rangle$
X_4	$\langle 0.468, 0.175, 0.232\rangle$	$\langle 0.407, 0.237, 0.132\rangle$	$\langle 0.493, 0.125, 0.268\rangle$	$\langle 0.375, 0.248, 0.12\rangle$	$\langle 0.575, 0.225, 0.15\rangle$	$\langle 0.518, 0.2, 0.1\rangle$

Taking into account the six criteria belong to the maximizing type, the normalized evaluation information is retained in Tables 2–6.

Step 2. Obtain the weight vector of DMs based on Laplace distribution.

(1) The scale λ_k of Laplace distribution can be calculated based on equation (3.5). Then, $\sqrt{2}\lambda_5 = \sigma = \sqrt{\frac{1}{5}\sum_{i=1}^{i=5}\left(i - \frac{1+5}{2}\right)^2} \to \lambda_5 = 1$.

(2) The weight vector $W = (0.068, 0.183, 0.498, 0.183, 0.068)$ is obtained using equation (3.4).

Step 3. Reorder the evaluation values.

Determined the PIS $f_j^+ = \langle 1, 0, 0\rangle$ and NIS $f_j^- = \langle 0, 0, 1\rangle$ and calculate closeness coefficient $CC\left(\sigma_{ij}^k\right)$ by equation (4.1). For example, $CC\left(\sigma_{11}^1\right) = 0.68$, $CC\left(\sigma_{11}^2\right) = 0.527$, $CC\left(\sigma_{11}^3\right) = 0.622$, $CC\left(\sigma_{11}^4\right) = 0.528$, $CC\left(\sigma_{11}^5\right) = 0.42$ are obtained utilizing equation (4.1). Then the evaluation values are reordered in the following: $CC\left(\sigma_{11}^{(1)}\right) = CC\left(\sigma_{11}^1\right) = 0.68$, $CC\left(\sigma_{11}^{(2)}\right) = CC\left(\sigma_{11}^3\right) = 0.622$, $CC\left(\sigma_{11}^{(3)}\right) = CC\left(\sigma_{11}^4\right) = 0.528$, $CC\left(\sigma_{11}^{(4)}\right) = CC\left(\sigma_{11}^2\right) = 0.527$, $CC\left(\sigma_{11}^{(5)}\right) = CC\left(\sigma_{11}^5\right) = 0.42$.

Step 4. Obtain the overall information.

The overall information is synthesized utilizing LD-PFOWA operator based on equation (5.1). For example,

$$\begin{aligned}\sigma_{11} &= 0.068 \times \sigma_{11}^{(1)} + 0.183 \times \sigma_{11}^{(2)} + 0.498 \times \sigma_{11}^{(3)} + 0.183 \times \sigma_{11}^{(4)} + 0.068 \times \sigma_{11}^{(5)} \\ &= \langle 0.443, 0.139, 0.282\rangle .\end{aligned}$$

Then, the overall information is shown in Table 7.

Step 5. List all possible permutations for all alternatives. All possible $m!$ permutations of the ranking for m alternatives are listed as follows.

$$\begin{aligned}&P_1 = (X_1, X_2, X_3, X_4), P_2 = (X_1, X_2, X_4, X_3), P_3 = (X_1, X_3, X_2, X_4), \\ &P_4 = (X_1, X_3, X_4, X_2), P_5 = (X_1, X_4, X_2, X_3), P_6 = (X_1, X_4, X_3, X_2), \\ &P_7 = (X_2, X_1, X_3, X_4), P_8 = (X_2, X_1, X_4, X_3), P_9 = (X_2, X_3, X_1, X_4), \\ &P_{10} = (X_2, X_3, X_4, X_1), P_{11} = (X_2, X_4, X_1, X_3), P_{12} = (X_2, X_4, X_3, X_1), \\ &P_{13} = (X_3, X_1, X_2, X_4), P_{14} = (X_3, X_1, X_4, X_2), P_{15} = (X_3, X_2, X_1, X_4),\end{aligned}$$

TABLE 8. The values of concordance/discordance indices for each pair of alternatives.

	C_1	C_2	C_3	C_4	C_5	C_6
$\xi_l^j(X_1,X_2)$	0.1084	−0.0626	−0.0825	−0.0095	−0.0095	0.0993
$\xi_l^j(X_1,X_3)$	0.0310	−0.0799	−0.1774	0.1194	0.1194	−0.0251
$\xi_l^j(X_1,X_4)$	−0.0237	−0.0688	−0.0773	0.0076	0.0076	−0.0688
$\xi_l^j(X_2,X_1)$	−0.1084	0.0626	0.0825	0.0095	0.0095	−0.0993
$\xi_l^j(X_2,X_3)$	−0.0773	−0.0173	−0.0949	0.1289	0.1289	−0.1244
$\xi_l^j(X_2,X_4)$	−0.1321	−0.0062	0.0051	0.0171	0.0171	−0.1681
$\xi_l^j(X_3,X_1)$	−0.0310	0.0799	0.1774	−0.1194	−0.1194	0.0251
$\xi_l^j(X_3,X_2)$	0.0773	0.0173	0.0949	−0.1289	−0.1289	0.1244
$\xi_l^j(X_3,X_4)$	−0.0548	0.0111	0.1000	−0.1118	−0.1118	−0.0438
$\xi_l^j(X_4,X_1)$	0.0237	0.0688	0.0773	−0.0076	−0.0076	0.0688
$\xi_l^j(X_4,X_2)$	0.1321	0.0062	−0.0051	−0.0171	−0.0171	0.1681
$\xi_l^j(X_4,X_3)$	0.0548	−0.0111	−0.1000	0.1118	0.1118	0.0438

$$P_{16}=(X_3,X_2,X_4,X_1),P_{17}=(X_3,X_4,X_1,X_2),P_{18}=(X_3,X_4,X_2,X_1),$$
$$P_{19}=(X_4,X_1,X_2,X_3),P_{20}=(X_4,X_1,X_3,X_2),P_{21}=(X_4,X_2,X_1,X_3),$$
$$P_{22}=(X_4,X_2,X_3,X_1),P_{23}=(X_4,X_3,X_1,X_2),P_{24}=(X_4,X_3,X_2,X_1).$$

Step 6. Calculate concordance/discordance index. The concordance/discordance index $\xi_l^j(X_i,X_k)$ based on the closeness coefficient for each pair of alternatives (X_i,X_k) $(X_i,X_k\in X)$ with respect to C_j and P_l can be calculated by equation (4.2). Then, the values of concordance/discordance indices for each pair of alternatives are shown in Table 8.

Step 7. Compute the weighted concordance/discordance index.
The weighted concordance/discordance index $\xi_l(X_i,X_k)$ based on the closeness coefficient for each pair of alternatives (X_i,X_k) $(X_i,X_k\in X)$ with respect to all criteria and P_l can be computed by equation (4.4).
For example,

$$\begin{aligned}\xi_l(X_1,X_2)&=0.27\times 0.1084+0.2\times(-0.0626)+0.15\times(-0.0825)+0.15\times(-0.0095)\\&\quad+0.13\times(-0.0095)+0.1\times 0.0993\\&=0.011633.\end{aligned}$$

Then, the weighted values of concordance/discordance indices for each pair of alternatives are shown in Table 9.

Step 8. Obtain the comprehensive concordance/discordance index. The comprehensive concordance/discordance index ξ^l for P_l can be obtained by equation (4.5).
For example,

$$\begin{aligned}\xi_1&=\xi_l(X_1,X_2)+\xi_l(X_1,X_3)+\xi_l(X_1,X_4)+\xi_l(X_2,X_1)+\xi_l(X_2,X_3)+\xi_l(X_2,X_4)\\&\quad+\xi_l(X_3,X_1)+\xi_l(X_3,X_2)+\xi_l(X_3,X_4)+\xi_l(X_4,X_1)+\xi_l(X_4,X_2)+\xi_l(X_4,X_3)\\&=-0.12451.\end{aligned}$$

Then, the values of comprehensive concordance/discordance indices are shown in Table 10.

Step 9. Identify the optimal ranking of alternatives.
Based on equation (4.6), we can obtain $\xi^*=\max_{l=1}^{n!}\{\xi_l\}=\xi_{23}$ and $P^*=P_{23}=X_4,X_3,X_1,X_2$. Thus, the final ranking is $X_4\succ X_3\succ X_1\succ X_2$.

TABLE 9. The weighted values of concordance/discordance indices for each pair of alternatives.

$\xi_l(X_i, X_k)$	Value	$\xi_l(X_i, X_k)$	Value
$\xi_l(X_1, X_2)$	0.011633	$\xi_l(X_3, X_1)$	0.003281
$\xi_l(X_1, X_3)$	−0.00328	$\xi_l(X_3, X_2)$	0.014913
$\xi_l(X_1, X_4)$	−0.03653	$\xi_l(X_3, X_4)$	−0.03325
$\xi_l(X_2, X_1)$	−0.01163	$\xi_l(X_4, X_1)$	0.036533
$\xi_l(X_2, X_3)$	−0.01491	$\xi_l(X_4, X_2)$	0.048166
$\xi_l(X_2, X_4)$	−0.04817	$\xi_l(X_4, X_3)$	0.033253

TABLE 10. The values of comprehensive concordance/discordance indices.

P_l	ξ_l	Value	P_l	ξ_l	Value
$P_1 = X_1, X_2, X_3, X_4$	ξ_1	−0.12451	$P_{13} = X_3, X_1, X_2, X_4$	ξ_{13}	−0.08813
$P_2 = X_1, X_2, X_4, X_3$	ξ_2	−0.05801	$P_{14} = X_3, X_1, X_4, X_2$	ξ_{14}	0.008207
$P_3 = X_1, X_3, X_2, X_4$	ξ_3	−0.09469	$P_{15} = X_3, X_2, X_1, X_4$	ξ_{15}	−0.11139
$P_4 = X_1, X_3, X_4, X_2$	ξ_4	0.001645	$P_{16} = X_3, X_2, X_4, X_1$	ξ_{16}	−0.03832
$P_5 = X_1, X_4, X_2, X_3$	ξ_5	0.038324	$P_{17} = X_3, X_4, X_1, X_2$	ξ_{17}	0.081274
$P_6 = X_1, X_4, X_3, X_2$	ξ_6	0.068151	$P_{18} = X_3, X_4, X_2, X_1$	ξ_{18}	0.058009
$P_7 = X_2, X_1, X_3, X_4$	ξ_7	−0.14778	$P_{19} = X_4, X_1, X_2, X_3$	ξ_{19}	0.111391
$P_8 = X_2, X_1, X_4, X_3$	ξ_8	−0.08127	$P_{20} = X_4, X_1, X_3, X_2$	ξ_{20}	0.141217
$P_9 = X_2, X_3, X_1, X_4$	ξ_9	−0.14122	$P_{21} = X_4, X_2, X_1, X_3$	ξ_{21}	0.088125
$P_{10} = X_2, X_3, X_4, X_1$	ξ_{10}	−0.06815	$P_{22} = X_4, X_2, X_3, X_1$	ξ_{22}	0.094687
$P_{11} = X_2, X_4, X_1, X_3$	ξ_{11}	−0.00821	$P_{23} = X_4, X_3, X_1, X_2$	ξ_{23}	0.147779
$P_{12} = X_2, X_4, X_3, X_1$	ξ_{12}	−0.00165	$P_{24} = X_4, X_3, X_2, X_1$	ξ_{24}	0.124514

6.3. Comparative analyses and discussion

This subsection conducs two comparative analyses with other relevant methods to validate the effectiveness and advantages of the proposed method. To illustrate the effectiveness of the proposed method, we first prove the effectiveness of the proposed LD-PFOWA operator. Then, we conduct the validation of the proposed method by comparing with other representative MCDM methods. Meanwhile, the comparison analyses are based on the same case study described previously.

Case 1. Comparative analysis of the LD-PFOWA operator.

To verify the effectiveness of the proposed LD-PFOWA operator, we compare it with other picture fuzzy operators under the same case study. The first one is the PFOWA operator proposed by Wei [44]. The second is the PFOWG operator proposed by Wang [34]. The final method is utilized LD-PFOWA operator proposed in our paper. Each kind of aggregation operator will aggregate evaluation information separately twice. Firstly, aggregation operators are used to synthesize individual evaluation information into the overall information. And secondly, they are used to aggregate criteria evaluation information into a comprehensive value of each alternative. Finally, the ranking order of alternatives are obtained through comparing the comprehensives values based on a comparison method proposed by Wang [34]. Table 11 exhibits the rankings of the above four methods.

From Table 11, it is obvious that the ranking orders of the three aggregation operators are equal. That can illustrate the effectiveness of the proposed LD-PFOWA operator. Then, the comparisons will be demonstrated in the following. PFOWA operator is proposed based on operations of PFNs proposed by Wei [44], but this operations have some drawbacks. PFOWG operators is proposed based on the operation of PFNs proposed by Wang [34]. However, this operation is not as simple to calculate and easy to understand as the convex combination of PFNs. LD-PFOWA operator, proposed based on convex combination of PFNs, can reduce the

Table 11. Comparisons of the different operators with the case study.

Method	Ranking orders
PFOWA [44]	$X_4 \succ X_3 \succ X_1 \succ X_2$
PFOWG [34]	$X_4 \succ X_3 \succ X_1 \succ X_2$
LD-PFOWA	$X_4 \succ X_3 \succ X_1 \succ X_2$

Table 12. Comparisons of the different methods with the same numerical examples.

Method	Ranking orders
Picture fuzzy cross-entropy method [43]	$A_4 \succ A_3 \succ A_2 \succ A_1$
TOPSIS	$A_4 \succ A_3 \succ A_1 \succ A_2$
TOPSIS-based QUALIFLEX method	$A_4 \succ A_3 \succ A_1 \succ A_2$

motivational bias and minimize the inconsistency in alternative assessment in MDGDM. Therefore, the proposed LD-PFOWA operator is valid and more useful in the applications than other operators.

Case 2. Comparative analysis of the proposed method.

To further illustrate the validity and feasibility of the proposed TOPSIS-based QUALIFLEX method, we compared it with other methods and the discussion is based on the same case study described above with picture fuzzy information. In Case 1, we have illustrated the effectiveness of the proposed LD-PFOWA operator. Thus, we use the overall information obtained by the above case study to analyze with some other comparable MCDM methods, which include the picture fuzzy cross-entropy method [43] and TOPSIS method.

The picture fuzzy cross-entropy method [43] was proposed by Wei *et al.* and it is the first MCDM method under picture fuzzy environment. The values of picture fuzzy weighted cross entropy between PFNs and PIS are utilized to rank the alternatives and select the best one. TOPISIS based on picture fuzzy distance method has been described in Section 3.1. Then, Table 12 shows the comparisons of this two methods and the proposed method.

From Table 12, the best project is A_4 for the three methods. The worst project is A_1 for the picture fuzzy cross-entropy method, while A_2 is the worst one for the rest two methods. The main reason that brought the differences mainly lies in the picture fuzzy cross-entropy just considers PIS, while other two methods not only consider the PIS, but also consider the NIS. Therefore, it is reasonable that the ranking order of the picture fuzzy cross-entropy may not be the same as the proposed method. Although the TOPSIS method and the proposed method have the same ranking orders, the proposed method is more comprehensive. Because, the proposed TOPSIS-based QUALIFLEX method considers the outranking of the pair-wise comparisons of all alternatives. From the analyses above, we can conclude that the obtained results of the proposed method is valid and superior to those listed methods.

In summary, the proposed MCGDM method based on LD-PFOWA operator and TOPSIS-based QUALIFLEX has the following advantages. Frist, the proposed method overcomes the drawbacks of the research [47]. Second, the proposed method uses LD-PFOWA operator to deal with the motivational bias produced by DMs' subjective preferences. Third, cognitive bias derived from the different knowledge backgrounds of DMs are overcome by MCGDM method. Moreover, the proposed method consider not only the influence of PIS, but also the influence of NIS. Finally, the proposed method is based on an outranking method, which considers the pair-wise comparisons of all alternatives.

7. Conclusions

More and more BEER projects have been implemented by ESCO to improve energy efficiency and protect environment. Selecting an optimum BEER project can be regarded as a MCGDM problem under picture fuzzy environment. In this research, PFWA operator and LD-PFOWA operator were proposed based on the convex combination of two PFNs to aggregate the DMs' evaluation information. Meanwhile, a TOPSIS-based QUALIFLEX method with picture fuzzy information was developed to rank BEER projects. Furthermore, the procedure of the proposed MCGDM method was presented. Finally, a case study of hotel BEER project selection was conducted to illustrate the application and effectiveness of the proposed method.

Then, the primary contributions of this research can be summarized as follows:

(1) Hotel BEER project selection was described as an MCGDM problem with PFNs in order to handle the complexity, uncertainty and fuzziness of the selection process.
(2) LD-PFOWA operator was proposed based on the convex combination of PFNs to aggregate PFNs by overcoming motivational bias.
(3) A TOPSIS-based QUALIFLEX method under picture fuzzy environment was developed in order to rank the hotel BEER projects.

Finally, we assure several directions for future studies as follows.

(1) Utilizing picture fuzzy information to describe the evaluation information is more understandable, appropriate and sufficient. It also can be applied to supplier selection, economic risk evaluation, medical diagnosis. Thus, it is significant for researchers to conduct more studies on PFSs.
(2) In this study, we ignore the interrelationships among the criteria. Researchers can devote themselves to improve this method in future work to overcome this defect.
(3) The proposed method does not take into account the influence of DMs' psychological behavioral characteristics. In the future, scholars can introduce the psychological behavior of DMs into the proposed method.

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