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THE EFFECT OF FULL OR PARTIAL PRICING INTEGRATION ON SUPPLY CHAIN MANAGEMENT

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Abstract. A previous paper proposed a supply chain model, comprised of a retailer and manufacturer, in which the manufacturer uses product pricing to maximize the profit of the entire supply chain. The increased profits gained from integration are then shared among all the supply chain members. The optimal pricing strategy was shown to be "products on consignment" for sale. The present study extends this simple two-layer supply chain model to a more complicated three-layer model, in which the supply chain comprises not only the retailer and manufacturer, but also an intermediate distributor. In contrast to the previous model, the present model not only considers the role of the distributor, but also the effects of product nonconformance at each facility in the supply chain. The profit function of each facility in the supply chain is established, including the sales revenue, procurement cost, and quality control cost. The investment cost at the retailer to improve the service level is also considered. It is shown that the total profit of the supply chain is maximized when the retailer's optimal service level is adopted, where this service level is adjusted in accordance with the distributor's unit sale price. Furthermore, after price integration, the overall profit of the supply chain is found to equal the retailer's profit. In other words, the total profit of the manufacturer and distributor is equal to zero. Numerical examples are given to illustrate the proposed pricing integration model under different quality environments. The results are contrasted with those obtained using a traditional pricing model, namely the "make up on cost" model. Overall, the present results show that the manufacturer is always the winner under partial price integration (*i.e.*, only the retailer and distributor join the integration). Furthermore, partial integration is far less profitable for the retailer and distributor than full integration.

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1. Introduction

Increasing market demand has always been an important issue for retailers. Kurata and Namb [12] studied the impact of after-sales service levels on product sales, and found that upgrading the after-sales service can increase sales. Choi [7] (or see [6]) investigated the decision of retailers to invest in "service level" (hereafter

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referred to as SL) on the impact of sales. Notably, the term SL was taken to refer to factors such as the layout of the retail store, the packaging and display of the goods, the attitude of the service staff, and the professional knowledge and responses of the sales staff to customer questions rather than to the probability of the retailer satisfying customer orders (see *e.g.*, [20]). That is, if better conditions of sale are provided for the same goods at the same sales price, increased sales will result. Based on the hypothesis proposed by Choi [7], the investment function for achieving a SL of $s \geq 0$ is given by $K(s) = \frac{1}{2}ks^2$, where $k > 0$. The resulting average sales of the product are then given as $D(s) = a + bs$, where $a > 0$ represents the base market demand and $b > 0$ indicates the increase in sales resulting from an increase in the SL (see [22]). In other words, when the retail price is fixed, the market demand depends on the retailer's service level. It is noted that this research scenario is different from that of Chen *et al.* [2], who considered market demand to be driven by retail pricing, or that of Chen *et al.* [3], who contended that the market demand in different periods depends mainly on current pricing and previous pricing. It differs also from the premise of Cheung *et al.* [5] that the relationship between pricing and market demand distribution is unclear.

According to Choi [7], the optimal value of the SL depends on the product purchase price of the retailer. To some extent this accounts for the fact that the facilities immediately upstream of the relater often use pricing to increase the quantity of retailer orders (see [14]). Choi [7] showed that the optimal SL can be achieved by adopting a strategy of selling products on consignment, which reduces the risk of retailers in holding the goods (also see, *e.g.*, [15–17]); particularly when the retailer faces restrictions on funds for purchasing (see, *e.g.*, [21]).

Choi [7] considered a two-layer supply chain consisting of a retailer and a manufacturer. The present study extends this model to include an intermediate distributor, and uses the resulting three-layer model to investigate the impact of pricing for the distributor and manufacturer on the whole profit of the supply chain, given that each member of the supply chain possesses a certain nonconforming rate of the product. Notably, the issue of product quality was ignored by Choi [7]. However, in the model proposed in this study, it is assumed that product inspections are performed to ensure that the quality level meets the downstream requirement. Furthermore, inspection errors are assumed to exist (see also [25]). After pricing integration, the change (increase) of the profit for the entire supply chain is investigated and the increased profit is allocated among all the supply chain facilities (see, *e.g.*, [11, 26]).

In the following section, the mathematical model for the studied problem is established. In Section 3, the integrated pricing and its benefits are investigated. Concluding remarks are offered in Section 4.

2. Problem description and mathematical model

This paper considers fashion apparel that has a short sales period. The clothing industry is labor-intensive; as such, undesirable nonconforming problems often caused by the human factor (refer to [1]) are inevitable. These include, for example, the use of the wrong color threads on a garment, dropped stitches, etc. (*e.g.*, see [23]). A number of strategies can be employed by the manufacturer for the disposal of these nonconforming apparel items. There are reworks or discounted sales (*e.g.*, see [18]) or they can be recycled by the manufacturer through reverse logistics (see, *e.g.*, [10, 24]). However, the integration of production and distribution is an issue when nonconforming products do exist (*e.g.*, refer to [13]). In the following discussion, we first investigate the effect of the nonconforming rate on members of the supply chain on their profits, as well as the whole profit on the supply chain.

Consider a supply chain comprised of the facilities of manufacturer, distributor and retailer, where the distributor plays the role of packaging products, inspecting broken needles, storing products for the retailer, and so on. Assume that the nonconforming rates of the products possessed by the retailer, distributor and manufacturer are x_0, y_0 and z_0, respectively, where $0 \leq x_0 \leq y_0 \leq z_0 \leq 1$. Let $\frac{1}{1-x_0} = 1 + x$, $\frac{1}{1-y_0} = 1 + y$ and $\frac{1}{1-z_0} = 1 + z$, which represent the required number of products when the retailer, distributor or manufacturer provide a conforming product for their downstream customer. Note that $x = \frac{x_0}{1-x_0}$, $y = \frac{y_0}{1-y_0}$ and $z = \frac{z_0}{1-z_0}$. Let $f(m) = \frac{m}{1-m}$. Since the first derivative of $f(m)$ is given by $\frac{1}{(1-m)^2} > 0$, $f(m)$ increases with m, and hence $0 \leq x \leq y \leq z$.

Normally, apparel products only have Type II inspection errors, that is, the inspection error caused when inspectors ignore defects in the product, such as broken needles, loose yarns and other defects. For any given batch, the relationship between the required number of inspections and a desired output nonconforming rate is proposed as follows.

Proposition 2.1. *Consider a batch with a size of N, which has a nonconforming rate of $a\%$. To reduce the batch of its nonconforming rate from $a\%$ to $b\%$ by inspections, with only a Type II inspection error (denoted by β), those inspected nonconforming products will be discarded. Then, the required number that should be inspected in the batch is given by $\frac{N(a\%-b\%)}{a\%(1-b\%)(1-\beta)}$, where $\frac{N(a\%-b\%)}{1-b\%}$ nonconforming products are inspected and disposed of.*

The proof of Proposition 2.1 is given in the Appendix. Proposition 2.1 also reveals that the proportion of inspection for any batch is $\frac{a\%-b\%}{a\%(1-b\%)(1-\beta)}$.

The profit function for the members of the supply chain is analyzed as follows (also refer to Fig. 1). Suppose that the retailer considers having a service level, $s \geq 0$. Then, it is necessary for the retailer to invest $K(s) = 0.5ks^2$, where $k > 0$, and the market demand becomes $D(s) = a + bs$, where $a, b > 0$. In this case, the retailer purchases $\frac{D(s)}{1-x_0} = (1+x)D(s)$ from the distributor with a unit purchase cost $w_0(x_0)$. Note that $w_0(x_0)$ (or denoted by w_0 for simplification) is regarded as the unit price with nonconforming rate x_0, which differs from providing different product warranty periods for different prices (see, *e.g.* [4]). To achieve a zero nonconforming rate, the required number of inspections can be obtained by applying Proposition 2.1 as $\frac{(1+x)D(s)(x_0-0)}{x_0(1-0)(1-\beta_1)}$, where β_1 represents the probability of Type II inspection errors for the retailer. Here, it is assumed that $\beta_1 = 0$ to ensure that each product conforms before sale, which means that the retailer has to carefully inspect each product. After inspections are performed, there are $\frac{(1+x)D(s)(x_0-0)}{1-0} = xD(s)$ nonconforming products which will be inspected and disposed of, where $1 + x = \frac{1}{1-x_0}$ and $\frac{x_0}{1-x_0} = x$ are used. The unit disposal cost is denoted by c'_R. If the nonconforming goods still have residual value, they can be returned to the manufacturer for remanufacturing (see [19]), and are hence salvaged. In this case, one has $c'_R < 0$; otherwise, $c'_R \geq 0$. For the other $D(s)$ conforming products, they will be sold to the market at unit price r. Accordingly, the profit function of the retailer can be written by

$$\pi_R(s) = (r - w_0)D(s) - (c'_R + w_0)xD(s) - K(s) - c_1\frac{(1+x)D(s)}{1-\beta_1}, \tag{2.1}$$

where c_1 represents the unit inspection cost for the retailer.

To meet the retailer's demand, $(1+x)D(s)$, distributor purchases $\frac{D(s)}{1-y_0} = (1+y)D(s)$ from the manufacturer, with unit purchase price $m_0(y_0)$ (or denoted by m_0 for simplification). To reduce the nonconforming rate from y_0 to x_0, the required number of inspections is given by (see Prop. 2.1) $\frac{(1+y)D(s)(y_0-x_0)}{y_0(1-x_0)(1-\beta_2)}$, where β_2 represents the probability of Type II inspection errors by the distributor. Proposition 2.1 also says that there are $\frac{(1+y)D(s)(y_0-x_0)}{1-x_0} = \frac{D(s)(y_0-x_0)}{(1-x_0)(1-y_0)} = (y-x)D(s)$ nonconforming products to be inspected and disposed of, where $x = \frac{x_0}{1-x_0}$ and $y = \frac{y_0}{1-y_0}$ are used. These nonconforming products will be disposed of with unit cost c'_{DC}. Note that if the nonconforming goods still have residual value, then $c'_{\mathrm{DC}} < 0$; otherwise, $c'_{\mathrm{DC}} \geq 0$. The other $(1+x)D(s)$ products, which have a nonconforming rate x_0, will satisfy the retailer's demand with unit price $w_0(x_0)$. The resulting expected total profit of the distributor is given by

$$\pi_{\mathrm{DC}}(s) = (w_0 - m_0)(1+x)D(s) - (c'_{\mathrm{DC}} + m_0)(y-x)D(s) - c_2\frac{(1+y)D(s)(y_0-x_0)}{y_0(1-x_0)(1-\beta_2)}, \tag{2.2}$$

where c_2 represents the unit inspection cost for the distributor.

To meet the distributor's demand, $(1+y)D(s)$, the manufacturer needs to produce $\frac{D(s)}{1-z_0} = D(s)(1+z)$ with unit manufacturing cost $\underline{m}$. To reduce the nonconforming rate from z_0 to y_0, Proposition 2.1 indicates that the required number of inspections is $\frac{(1+z)D(s)(z_0-y_0)}{z_0(1-y_0)(1-\beta_3)}$, where β_3 represents the probability of Type II inspection errors for the manufacturer. In addition, there are $\frac{(1+z)D(s)(z_0-y_0)}{1-y_0} = \frac{D(s)(z_0-y_0)}{(1-z_0)(1-y_0)} = (z-y)D(s)$ nonconforming

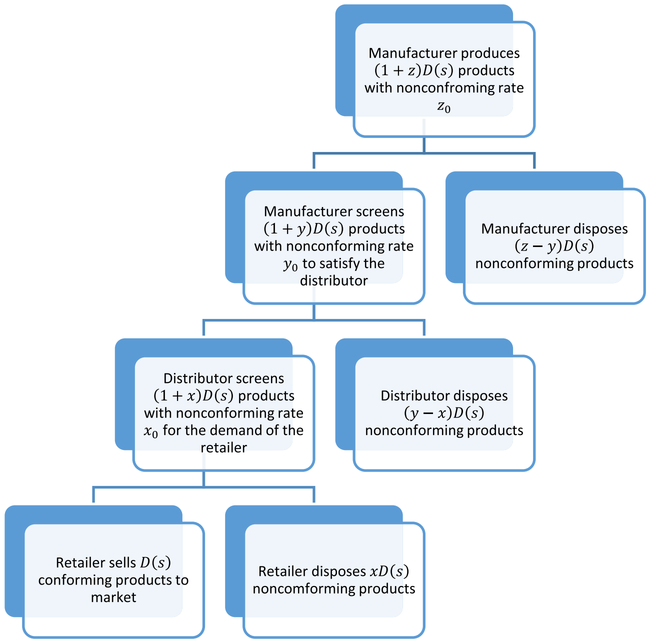

FIGURE 1. Product quantity possessed by each facility in the supply chain.

products that will be inspected and disposed of, where $y = \frac{y_0}{1-y_0}$ and $z = \frac{z_0}{1-z_0}$ are used. The nonconforming goods will be disposed of with unit cost c'_M. If the nonconforming goods still have residual value, then $c'_M < 0$; otherwise, $c'_M \geq 0$. The other $(1+y)\,D\,(s)$ products, which have nonconforming rate y_0, will be used to satisfy the demand of the distributor. Denoting the unit cost of manufacturing a product as $\underline{m}\,(z_0)$ (or denoted by $\underline{m}$ for simplification), then, the expected total profit of manufacturer is given by

$$\pi_M\,(s) = (m_0 - \underline{m})\,(1+y)\,D\,(s) - (c'_M + \underline{m})\,(z-y)\,D\,(s) \qquad (2.3)$$
$$- c_3 \frac{(1+z)\,D\,(s)\,(z_0 - y_0)}{z_0\,(1-y_0)\,(1-\beta_3)},$$

where c_3 represents the unit inspection cost for the manufacturer.

Let $\rho_R = \frac{x_0 - 0}{1-x_0}$, $\rho_{\rm DC} = \frac{y_0 - x_0}{1-y_0}$ and $\rho_M = \frac{z_0 - y_0}{1-z_0}$ denote the percentage of the increased conforming rate to achieve the promise for their downstream customer, for the retailer, distributor and manufacturer, respectively. These can be regarded as the quality responsibilities for these supply chain facilities. The relationship between the quality responsibility and the number required for obtaining a conforming product is expressed by the following proposition.

Proposition 2.2. *When the retailer, distributor and manufacturer provide a conforming product to their downstream customer, it is necessary for them to hold the expected number of products,* $1+\rho_R$, $(1+\rho_{\rm DC})\,(1+\rho_R)$ *and* $(1+\rho_M)\,(1+\rho_{\rm DC})\,(1+\rho_R)$, *respectively, or equivalently, to hold the additional number of products,* $x = \rho_R$, $y = \rho_{\rm DC}\,(1+\rho_R) + \rho_R$ *and* $z = \rho_M\,(1+\rho_{\rm DC})\,(1+\rho_R) + \rho_{\rm DC}\,(1+\rho_R) + \rho_R$, *respectively.*

Proposition 2.2 indicates that the facility located more upstream of the supply chain needs more products to meet the demand of their downstream customer. See the Appendix for proof of Proposition 2.2. On the other hand, if ρ_R, $\rho_{\rm DC}$ and ρ_M are given, the corresponding nonconforming rate for the retailer, distributor and manufacturer can be obtained by $x_0 = \frac{\rho_R}{1+\rho_R}$, $y_0 = \frac{\rho_{\rm DC} + \frac{\rho_R}{1+\rho_R}}{\rho_{\rm DC}+1}$, $z_0 = \frac{\rho_M + \frac{\rho_{\rm DC} + \frac{\rho_R}{1+\rho_R}}{\rho_{\rm DC}+1}}{\rho_M + 1}$, respectively.

The profit function given in equations (2.1)–(2.3) can also be expressed by a function of the quality responsibilities, as shown below:

Substituting $x = \rho_R$ and $x_0 = \frac{\rho_R}{1+\rho_R}$ into equation (2.1) yields

$$\pi_R(s) = (r - w_0) D(s) - (c'_R + w_0) \rho_R D(s) - K(s) - c_1 \frac{(1+\rho_R) D(s)}{1-\beta_1}. \tag{2.4}$$

Using $x = \rho_R$, $1 + y = \frac{1}{1-y_0}$, $\rho_{\text{DC}} = \frac{y_0 - x_0}{1-y_0}$, $x_0 = \frac{\rho_R}{1+\rho_R}$ and $y_0 = \frac{\rho_{\text{DC}} + \frac{\rho_R}{1+\rho_R}}{\rho_{\text{DC}}+1} = \frac{\rho_{\text{DC}}(1+\rho_R)+\rho_R}{(\rho_{\text{DC}}+1)(1+\rho_R)}$ into equation (2.2) yields

$$\begin{aligned}\pi_{\text{DC}}(s) = {} & (w_0 - m_0)(1+\rho_R) D(s) - (c'_{\text{DC}} + m_0) \rho_{\text{DC}} (1+\rho_R) D(s) \\ & - c_2 \frac{(\rho_{\text{DC}}+1)(1+\rho_R)^2 \rho_{\text{DC}}}{[\rho_{\text{DC}}(1+\rho_R)+\rho_R](1-\beta_2)} D(s).\end{aligned} \tag{2.5}$$

Integrating $1 + y = (1+\rho_{\text{DC}})(1+\rho_R)$, $1 + z = \frac{1}{1-z_0}$, $\rho_M = \frac{z_0 - y_0}{1-z_0}$, $y_0 = \frac{\rho_{\text{DC}} + \frac{\rho_R}{1+\rho_R}}{\rho_{\text{DC}}+1}$ and $z_0 = \frac{\rho_M + \frac{\rho_{\text{DC}} + \frac{\rho_R}{1+\rho_R}}{\rho_{\text{DC}}+1}}{\rho_M + 1}$ into equation (2.3) yields

$$\begin{aligned}\pi_M(s) = {} & (m_0 - \underline{m})(1+\rho_{\text{DC}})(1+\rho_R) D(s) - (c'_M + \underline{m}) \rho_M (1+\rho_{\text{DC}})(1+\rho_R) D(s) \\ & - c_3 \frac{(1+\rho_R)^2 (1+\rho_{\text{DC}})^2 \rho_M}{[\rho_M(1+\rho_{\text{DC}})(1+\rho_R) + \rho_{\text{DC}}(1+\rho_R) + \rho_R](1-\beta_3)} D(s).\end{aligned} \tag{2.6}$$

Adding the expressions of equations (2.4)–(2.6) obtains the expected total profit function of the whole supply chain, as follows:

From equation (2.4), it is easy to see that:

$$\begin{aligned}\pi_{\text{SC}}(s) = {} & \{r - c'_R \rho_R - [c'_{\text{DC}} \rho_{\text{DC}} + ((c'_M + \underline{m}) \rho_M + \underline{m})(1+\rho_{\text{DC}})](1+\rho_R)\} D(s) \\ & - K(s) \\ & - \left\{ c_1 \frac{1+\rho_R}{1-\beta_1} + c_2 \frac{(\rho_{\text{DC}}+1)(1+\rho_R)^2 \rho_{\text{DC}}}{[\rho_{\text{DC}}(1+\rho_R)+\rho_R](1-\beta_2)} \right. \\ & \left. + c_3 \frac{(1+\rho_R)^2 (1+\rho_{\text{DC}})^2 \rho_M}{[\rho_M(1+\rho_{\text{DC}})(1+\rho_R) + \rho_{\text{DC}}(1+\rho_R) + \rho_R](1-\beta_3)} \right\} D(s).\end{aligned} \tag{2.7}$$

Solving $\mathrm{d}\pi_R(s)/\mathrm{d}s = 0$ yields

$$s_R^*(w_0) = \frac{b}{k} \left[r - w_0(1+\rho_R) - c'_R \rho_R - c_1 \frac{1+\rho_R}{1-\beta_1} \right]. \tag{2.8}$$

Since $\mathrm{d}^2\pi_{\text{SC}}(s)/\mathrm{d}s^2 = -k < 0$, the retailer has the maximal profit if SL is set to be the right-hand-side of equation (2.8). From equation (2.8), one observes that the smaller the value of ρ_R, the larger the value of $s_R^*(w_0)$. Note that the overall profit of the supply chain given in equation (2.7) is also dependent on $s_R^*(w_0)$.

3. Benefits from price integration

The following section analyzes how to achieve the maximal profitability of the supply chain through the integration of distributor and manufacturer in their product prices. Note that product price r may cause the changes of the market demand (see [27]); however, the scope of application of this study does not include the decision of r (*i.e.* r is treated as a constant, which is different from the treatment of Choi *et al.* [9] who studied how to make decisions on e-commerce retail prices, or that of Zhang *et al.* [28]), who considered how to exploit dynamic pricing strategies to gain more profit for the supply chain). From equation (2.4), after the price integration of the supply chain, the expected profit for the retailer is written by

$$\pi_R(s) = \left\{ r - w_1(1+\rho_R) - c'_R\rho_R - c_1\frac{1+\rho_R}{1-\beta_1} \right\} D(s) - K(s), \tag{3.1}$$

where w_1, a decision variable, represents the unit purchase price from the distributor after performing price integration for the supply chain.

From equation (3.1), one has $\mathrm{d}\pi_R(s)/\mathrm{d}s = (r - w_1(1+\rho_R) - c'_R\rho_R)\,b - ks$ and $\mathrm{d}^2\pi_R(s)/\mathrm{d}s^2 = -k < 0$. Solving $\mathrm{d}\pi_R(s)/\mathrm{d}s = 0$ yields:

$$s^*_R(w_1) = \frac{b}{k}\left[r - w_1(1+\rho_R) - c'_R\rho_R - c_1\frac{1+\rho_R}{1-\beta_1} \right] \tag{3.2}$$

that maximizes the retailer's profit given in equation (3.1). Denote m_1 as the unit purchase price from the manufacturer after the price integration for the supply chain is proposed, which is also a decision variable. From equations (2.5) and (2.6), the expected profit of the distributor and the manufacturer can be obtained by

$$\begin{aligned}\pi_{\mathrm{DC}}(s) = {} & (w_1 - m_1)(1+\rho_R)D(s) - (c'_{\mathrm{DC}} + m_1)\rho_{\mathrm{DC}}(1+\rho_R)D(s) \\ & - c_2\frac{(\rho_{\mathrm{DC}}+1)(1+\rho_R)^2\rho_{\mathrm{DC}}}{[\rho_{\mathrm{DC}}(1+\rho_R)+\rho_R](1-\beta_2)}D(s)\end{aligned} \tag{3.3}$$

and

$$\begin{aligned}\pi_M(s) = {} & (m_1 - \underline{m})(1+\rho_{\mathrm{DC}})(1+\rho_R)D(s) - (c'_M + \underline{m})\rho_M(1+\rho_{\mathrm{DC}})(1+\rho_R)D(s) \\ & - c_3\frac{(1+\rho_R)^2(1+\rho_{\mathrm{DC}})^2\rho_M}{[\rho_M(1+\rho_{\mathrm{DC}})(1+\rho_R)+\rho_{\mathrm{DC}}(1+\rho_R)+\rho_R](1-\beta_3)}D(s),\end{aligned} \tag{3.4}$$

respectively, adding equations (3.1), (3.3) and (3.4) obtains the profit of the whole supply chain, as follows:

$$\begin{aligned}\pi_{\mathrm{SC}}(s) = {} & \{r - c'_R\rho_R - [c'_{\mathrm{DC}}\rho_{\mathrm{DC}} + ((c'_M + \underline{m})\rho_M + \underline{m})(1+\rho_{\mathrm{DC}})](1+\rho_R)\}D(s) \\ & - K(s) \\ & - \left\{ c_1\frac{1+\rho_R}{1-\beta_1} + c_2\frac{(\rho_{\mathrm{DC}}+1)(1+\rho_R)^2\rho_{\mathrm{DC}}}{[\rho_{\mathrm{DC}}(1+\rho_R)+\rho_R](1-\beta_2)} \right. \\ & \left. + c_3\frac{(1+\rho_R)^2(1+\rho_{\mathrm{DC}})^2\rho_M}{[\rho_M(1+\rho_{\mathrm{DC}})(1+\rho_R)+\rho_{\mathrm{DC}}(1+\rho_R)+\rho_R](1-\beta_3)} \right\} D(s).\end{aligned} \tag{3.5}$$

From equation (3.5), one has

$$\begin{aligned}\mathrm{d}\pi_{SC}(s)/\mathrm{d}s = {} & \Bigg\{ r - c'_R\rho_R - [c'_{\mathrm{DC}}\rho_{\mathrm{DC}} + ((c'_M + \underline{m})\rho_M + \underline{m})(1+\rho_{\mathrm{DC}})](1+\rho_R) \\ & - \left[c_1\frac{1+\rho_R}{1-\beta_1} + c_2\frac{(\rho_{\mathrm{DC}}+1)(1+\rho_R)^2\rho_{\mathrm{DC}}}{[\rho_{\mathrm{DC}}(1+\rho_R)+\rho_R](1-\beta_2)} \right. \\ & \left. + c_3\frac{(1+\rho_R)^2(1+\rho_{\mathrm{DC}})^2\rho_M}{[\rho_M(1+\rho_{\mathrm{DC}})(1+\rho_R)+\rho_{\mathrm{DC}}(1+\rho_R)+\rho_R](1-\beta_3)} \right] \Bigg\} b - ks.\end{aligned} \tag{3.6}$$

TABLE 1. Degree of integration for the members in the supply chain.

Degree of integration	Retailer	Distributor	Manufacturer	Prices set
Full integration	◯	◯	◯	w_1 and m_1 are obtained by equation (3.8) and equation (3.9), respectively
Partial integration	◯	◯	X	w_1 is obtained by equation (3.8) and $m_1 = m_0$.
Integration failure	◯	X	◯	$w_1 = w_0$ and $m_1 = m_0$
	◯	X	X	
	X	◯	◯	
	X	◯	X	
	X	X	◯	
	X	X	X	

Moreover, $\mathrm{d}^2\pi_{\mathrm{SC}}(s)/\mathrm{d}s^2 = -k < 0$. Therefore, solving $\mathrm{d}\pi_{\mathrm{SC}}(s)/\mathrm{d}s = 0$ obtains

$$s^*_{\mathrm{SC}} = \frac{b}{k}\Bigg\{r - c'_R\rho_R - [c'_{\mathrm{DC}}\rho_{\mathrm{DC}} + ((c'_M + \underline{m})\rho_M + \underline{m})(1+\rho_{\mathrm{DC}})](1+\rho_R) - \Bigg[c_1\frac{1+\rho_R}{1-\beta_1} + c_2\frac{(\rho_{\mathrm{DC}}+1)(1+\rho_R)^2\rho_{\mathrm{DC}}}{[\rho_{\mathrm{DC}}(1+\rho_R)+\rho_R](1-\beta_2)} + c_3\frac{(1+\rho_R)^2(1+\rho_{\mathrm{DC}})^2\rho_M}{[\rho_M(1+\rho_{\mathrm{DC}})(1+\rho_R)+\rho_{\mathrm{DC}}(1+\rho_R)+\rho_R](1-\beta_3)}\Bigg]\Bigg\}, \tag{3.7}$$

maximizing $\pi_{\mathrm{SC}}(s)$. It is easy to see that s^*_{SC} is a constant but depends on the quality responsibilities, ρ_R, ρ_{DC} and ρ_M.

If $s^*_R(w_1) = s^*_{\mathrm{SC}}$ (pls. refer to Eqs. (3.2) and (3.7)); one then achieves the maximum profit for the entire supply chain:

$$w_1 = c'_{\mathrm{DC}}\rho_{\mathrm{DC}} + ((c'_M + \underline{m})\rho_M + \underline{m})(1+\rho_{\mathrm{DC}}) + c_2\frac{(\rho_{\mathrm{DC}}+1)(1+\rho_R)\rho_{\mathrm{DC}}}{[\rho_{\mathrm{DC}}(1+\rho_R)+\rho_R](1-\beta_2)} + c_3\frac{(1+\rho_R)(1+\rho_{\mathrm{DC}})^2\rho_M}{[\rho_M(1+\rho_{\mathrm{DC}})(1+\rho_R)+\rho_{\mathrm{DC}}(1+\rho_R)+\rho_R](1-\beta_3)}. \tag{3.8}$$

(Please also refer to Tab. 1, where "◯" and "X" indicate that the member agrees while does not agree to participate in price integration.) If the retailer agrees with price integration, it means that: (i) the retailer agrees to use all of the saved procurement costs to improve the service levels in order to increase sales volumes of the market, which increases the order quantity from the distributor. (ii) The retailer agrees to share the increased profit from pricing integration with other members of the supply chain. On the other hand, if the distributor agrees with price integration, it means that the distributor is willing to sell the product to the retailer at a price of w_1 given in equation (3.8). For the manufacturer, if the manufacturer agrees to share the increased profits, which result from integrated pricing, to other members of the supply chain; it then means that the manufacturer is willing to participate in this price consolidation, yet is reluctant. It is worth noting that when the retailer and distributor cooperate in price integration, but the manufacturer is unwilling to participate, the manufacturer can still increase profits because the integration of the retailer and distributor increases product sales. Therefore, in terms of volume, this also increases the amount ordered from the manufacturer by the distributor; thus, the manufacturer's profit increases.

The following proposition shows the change of the profit for each facility in the supply chain after partial price integration; that is, only the distributor and the retailer proceed with pricing integration.

Proposition 3.1 (Partial integration). *When w_1 adopts equation (3.8) and its resulting $s_R^*(w_1)$ given in equation (3.2) is non-negative, the profit of the whole supply chain $\pi_{\mathrm{SC}}(s)$ given in equation (3.5) achieves maximum at $s_{\mathrm{SC}}^* = s_R^*(w_1)$. Furthermore,*

$$\begin{aligned}\pi_{\mathrm{SC}}\left(s_{\mathrm{SC}}^*\right) = \pi_R\left(s_R^*\left(w_1\right)\right) \textit{ and} \\ -\pi_{\mathrm{DC}}\left(s_R^*\left(w_1\right)\right) = \pi_M\left(s_R^*\left(w_1\right)\right) \\ = \Big\{\left(m_1 - \underline{m} - \left(c_M' + \underline{m}\right)\rho_M\right)\left(1+\rho_{\mathrm{DC}}\right)\left(1+\rho_R\right) \\ -c_3 \frac{\left(1+\rho_R\right)^2\left(1+\rho_{\mathrm{DC}}\right)^2\rho_M}{\left[\rho_M\left(1+\rho_{\mathrm{DC}}\right)\left(1+\rho_R\right)+\rho_{\mathrm{DC}}\left(1+\rho_R\right)+\rho_R\right]\left(1-\beta_3\right)}\Big\} D\left(s_R^*\left(w_1\right)\right).\end{aligned}$$

See the Appendix for proof of Proposition 3.1. Proposition 3.1 indicates that the higher the unit price the manufacturer offers to the distributor, the greater the loss for the distributor (which is exactly equal to the revenue of the manufacturer). The loss for the distributor is zero if one sets:

$$m_1 = c_M' + \underline{m}\left(1+\rho_M\right) + c_3 \frac{\left(1+\rho_R\right)^2\left(1+\rho_{\mathrm{DC}}\right)^2\rho_M}{\left[\rho_M\left(1+\rho_{\mathrm{DC}}\right)\left(1+\rho_R\right)+\rho_{\mathrm{DC}}\left(1+\rho_R\right)+\rho_R\right]\left(1-\beta_3\right)}. \tag{3.9}$$

Therefore, when both equations (3.8) and (3.9) are adopted, it can be regarded as a case of full integration. When full integration in pricing is successful, as shown by Choi [8], reassigning the increased profit from the price integration of the supply chain, can be described as follows. It is denoted that the proportions of the assignments for the whole supply chain's profit for the manufacturer, distributor and retailer are λ_M, λ_{DC} and $1-\lambda_M-\lambda_{\mathrm{DC}}$, respectively. Sharing the profit among the manufacturer, distributor and retailer should satisfy $\lambda_M \pi_R\left(s_R^*\left(w_1\right)\right) \geq \pi_M\left(s_R^*\left(w_0\right)\right)$, $\lambda_{\mathrm{DC}}\pi_R\left(s_R^*\left(w_1\right)\right) \geq \pi_{\mathrm{DC}}\left(s_R^*\left(w_0\right)\right)$ and $\left(1-\lambda_M-\lambda_{\mathrm{DC}}\right)\pi_R\left(s_R^*\left(w_1\right)\right) \geq \pi_R\left(s_R^*\left(w_0\right)\right) \geq 0$. Note that for the case of partial integration, one sets $\lambda_M = 0$.

4. Numerical examples

In the following illustrative numerical example, the pricing model before price integration for the members of the supply chain is based on a modified "make up on cost model":

$$\begin{aligned}\text{Unit price (or unit manufacturing cost)} = \text{Unit purchase cost (or 1)} \times [1 + \text{Additional percentage} \\ \times (1 - \text{nonconforming rate})^2\right].\end{aligned}$$

The following pricing formulae are used: $\underline{m} = 1 \times \left[1 + 10\% \times \left(1-z_0\right)^2\right]$, $m_0 = \underline{m} \times \left[1 + 25\% \times \left(1-y_0\right)^2\right]$, $w_0 = m_0 \times \left[1 + 50\% \times \left(1-x_0\right)^2\right]$ and $r = w_0 \times (1 + 100\%)$. In addition, setting $c_1 = 0.05$, $c_2 = 0.03$, $c_1 = 0.01$, $\beta_1 = 0$, $\beta_2 = 0.01$, $\beta_3 = 0.02$, $c_M' = -0.15$, $c_{\mathrm{DC}}' = -0.15$, $c_R' = -0.15$, $k = 100$, $a = 70$ and $b = 100$. After the partial integration of pricing, one computes the value of w_1 by equation (3.8) and sets $m_1 = m_0$. The impact of ρ_R, ρ_{DC} and ρ_M (or equivalently, x_0, y_0 and z_0, respectively) on the profit of the facilities of the supply chain are summarized in Table 2, where for each case (Cases A–H), their related results before and after the price integration of the supply chain are recorded in its first and second rows, respectively. We denote $\Delta\mathrm{SC} = \pi_{\mathrm{SC}}\left(s_R^*\left(w_1\right)\right) - \pi_{\mathrm{SC}}\left(s_R^*\left(w_0\right)\right)$ and $\Delta M = \pi_M\left(s_R^*\left(w_1\right)\right) - \pi_M\left(s_R^*\left(w_0\right)\right)$ represent the increased profit for the whole supply chain and the manufacture, respectively, after the partial integration of pricing.

From Table 2, before performing the integration of pricing for the supply chain, the effect of quality responsibilities/nonconforming rates on the profits are observed as follows:

- The three cases, A, B and F, indicate that the quality responsibility of manufacturer, distributor and the retailer in the supply chain are equal. However, Case A has the most profit for the whole supply chain since it has the lowest quality responsibility among them.

TABLE 2. The effect of partial prices integration on the supply chain management.

	ρ_R	ρ_{DC}	ρ_M	w_0	$s_R^*(w_0)$	Before/after the prices integration					
	x_0	y_0	z_0	w_1	$s_R^*(w_1)$	π_R	π_{DC}	π_M	π_{SC}	$\frac{\Delta SC}{\pi_{SC}(s_R^*(w_0))}$	$\frac{\Delta M}{\Delta SC}$
A	0.0001	0.0001	0.0001	2.062	2.012	343.22	182.26	100.72	626.19		
	0.0001	0.0002	0.0003	1.019	3.055	680.63	−139.47	139.47	680.63	8.69%	71.19%
B	0.0050	0.0050	0.0050	2.042	1.982	335.25	176.53	96.92	608.70		
	0.0050	0.0099	0.0149	1.028	3.002	660.66	−133.76	133.76	660.66	8.54%	70.89%
C	0.0050	0.0100	0.0200	2.031	1.971	332.33	171.83	90.53	594.69		
	0.0050	0.0148	0.0341	1.054	2.954	642.91	−123.80	123.80	642.91	8.11%	69.01%
D	0.0050	0.0100	0.0300	2.028	1.968	331.44	171.33	87.00	589.77		
	0.0050	0.0148	0.0435	1.064	2.936	636.64	−118.58	118.58	636.64	7.95%	67.36%
E	0.0050	0.0150	0.0300	2.022	1.963	329.97	168.17	86.43	584.57		
	0.0050	0.0197	0.0482	1.071	2.919	630.29	−117.47	117.47	630.29	7.82%	67.90%
F	0.0100	0.0100	0.0100	2.022	1.953	327.43	170.88	93.15	591.46		
	0.0099	0.0197	0.0294	1.037	2.948	640.99	−128.10	128.10	640.99	8.37%	70.56%
G	0.0100	0.0100	0.0300	2.016	1.946	325.67	169.88	85.90	581.45		
	0.0099	0.0197	0.0483	1.059	2.913	628.16	−117.28	117.28	628.16	8.03%	67.17%
H	0.0100	0.0150	0.0300	2.010	1.941	324.24	166.62	85.31	576.17		
	0.0099	0.0245	0.0529	1.066	2.895	621.67	−116.13	116.13	621.67	7.90%	67.73%

– For Cases C and D (or Cases F and G), their values of ρ_R and ρ_{DC} are equal. When ρ_M is smaller, the manufacturer has a larger profit, as is the overall profit of the supply chain.
– For Cases D and E (or Cases G and H), where ρ_R and ρ_M are equal. It is easy to see that when ρ_{DC} is smaller, the resulting profit for the distributor is larger and the overall profit of the supply chain is also larger.
– With the same value of ρ_{DC} and ρ_M, when ρ_R is smaller, the profits of both the retailer and the supply chain are larger (see Cases D and G (or Cases E and H)).

After performing the integration of pricing for the supply chain, some observations are made as follows:

– After the pricing integration, the retailer has a larger service level than expected, that is, $s_R^*(w_1) > s_R^*(w_0)$.
– After the integration of pricing, the overall profit of the supply chain improves. That is, $\pi_{SC}(s_R^*(w_1)) > \pi_{SC}(s_R^*(w_0))$. In addition, one has $\pi_M(s_R^*(w_1)) = -\pi_{DC}(s_R^*(w_1))$, as shown in Proposition 3.1.

5. Conclusions

To meet a certain market demand, the existence of nonconforming products results in the need for either larger procurement or expanded production at the facilities in the supply chain, thereby decreasing profits. This paper has thus investigated the effect of price integration on the improvement of profit for the entire supply chain given the existence of nonconforming products.

The benefits of price integration can briefly be stated as follows: (i) The retailer can save procurement costs to invest instead in the service level; thereby increasing sales and revenue for the whole supply chain. (ii) Increased sales enable the supply chain to gain greater profits than without price integration, and give the opportunity to use a profit-sharing mechanism to increase the profit of each member of the supply chain. These benefits are exemplified by the following observations. Referring to Case A in Table 2, if full integration is achieved, the retailer, distributor and manufacturer all agree to redistribute their profits. Assume that the retailer receives a 3% increase in profit. The retailer's profit thus increases from 343.22 to 353.52. Furthermore, assume that the manufacturer is given a 10% increase in profit, which is equal to 110.79. Under these conditions, the profit of the distributor increases from 182.26 to 216.32, *i.e.*, an increase of 18.69%. On the other hand, if partial integration is achieved, only the retailer and distributor agree to join the integration and redistribute their profits. Suppose that the retailer again experiences an increase of 3% in profit after partial integration, (*i.e.*, the retailer has a profit of 353.52 after integration). The profit earned by the manufacturer is 139.47, *i.e.*, the profit increase is around 38.47%. Thus, the profit of the distributor increases from 182.26 to 187.6434; an increase of just 2.95%. If the manufacturer's increased profit (supposed to be 60.43) exceeds the overall increase in profit (*i.e.*, 54.44) after partial integration, then such a partial integration fails. In such a case, the members of the supply chain should consider conducting full integration.

It is important to identify the applicable timing for partial or full integration, or integration failure, as explored below. Let ΔM^F denote the change in profit for the manufacturer after full integration. Then,

$\Delta \mathrm{SC} > \Delta M$ (Partial integration): In this case, if $\Delta M > \Delta M^F$, then the manufacturer will be reluctant to join the integration in order to prevent a reduction in his profits. Furthermore, if $\Delta M \leq \Delta M^F$, then the retailer and distributor will refuse to allow the manufacturer to participate in the integration since partial integration affords them a greater aggregated profit, *i.e.*, $\Delta \mathrm{SC} - \Delta M \geq \Delta \mathrm{SC} - \Delta M^F$.

$\Delta \mathrm{SC} \leq \Delta M$ (Integration failure or full integration): In this case, the retailer and distributor are less willing to integrate prices (*i.e.*, integration fails). Obviously, if the retailer and distributor fail to integrate, the manufacturer cannot achieve any increase in profit. Therefore, it can be inferred that an astute manufacturer should pursue price integration as a clear profit maximization strategy, and will presumably find favor from the retailer and distributor in doing so.

The present results have several important managerial implications. Firstly, in the face of partial integration, the manufacturer is always a winner since even if he does not participate in the integration, his profit increases. Secondly, partial integration may not always be beneficial to the retailer and distributor. For example, profits may be owned only by the manufacturer, and the aggregated profit of the retailer and the distributor may therefore sometimes decrease rather than increase. Thirdly, full integration provides an increased profit for every facility in the supply chain.

Appendix

Proof of Proposition 2.1

Consider a batch that has N products with the nonconforming rate of $a\%$. Thus, there are $N(1-a\%)$ conforming products, and one wants to reduce the nonconforming rate to $b\%$ (*i.e.* $b\% < a\%$) for the batch. The required number to be inspected is assumed to be n. After inspecting n products from N products, there is an average of $na\%(1-\beta)$ nonconforming products that will be inspected and discarded, where β represents the type II inspection error. In this case, $N - na\%(1-\beta)$ will remain, where there is $N(1-a\%)$ conforming products. Consequently, setting $\frac{N(1-a\%)}{N-na\%(1-\beta)} = 1-b\%$ yields $n = \frac{N(a\%-b\%)}{a\%(1-b\%)(1-\beta)}$. □

Proof of Proposition 2.2

Since $\frac{x_0}{1-x_0} = x$, $\frac{y_0}{1-y_0} = y$ and $\frac{z_0}{1-z_0} = z$, one has $x_0 = \frac{x}{1+x}$, $y_0 = \frac{y}{1+y}$ and $z_0 = \frac{z}{1+z}$. Therefore, $\rho_R = \frac{x_0}{1-x_0} = \frac{x-0}{1+0}$, $\rho_{\mathrm{DC}} = \frac{y_0-x_0}{1-y_0} = \frac{y-x}{1+x}$ and $\rho_M = \frac{z_0-y_0}{1-z_0} = \frac{z-y}{1+y}$. These imply that $\rho_R = x$, $y = x + (1+x)\rho_{\mathrm{DC}}$ and $z = y + (1+y)\rho_M$. In addition, to provide a conforming product for their downstream customers, the retailer, distributor and manufacturer need to hold the number of products at $1+x$, $1+y$ and $1+z$, respectively. It is easy to see that $1+x = 1+\rho_R$. Furthermore, $1+y = 1+x+(1+x)\rho_{\mathrm{DC}} = (1+x)(1+\rho_{\mathrm{DC}}) = (1+\rho_{\mathrm{DC}})(1+\rho_R)$ and $1+z = 1+(1+y)\rho_M = (1+y)(1+\rho_M) = (1+\rho_M)(1+\rho_{\mathrm{DC}})(1+\rho_R)$. □

Proof of Proposition 3.1

Integrating equation (3.8) in equation (3.2) yields equation (3.7), that is, $s_R^*(w_1) = s_{\mathrm{SC}}^*$. Furthermore, placing equation (3.8) in equation (3.1) yields equation (3.5). This shows that $\pi_R(s_R^*(w_1)) = \pi_{\mathrm{SC}}(s_{\mathrm{SC}}^*)$.

When equation (3.8) is used in equation (3.3), one has $\pi_{\mathrm{DC}}(s_R^*(w_1)) = -\pi_M(s_R^*(w_1))$ (see Eq. (3.4)). □

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