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A COMMON-WEIGHT DEA MODEL FOR MULTI-CRITERIA ABC INVENTORY CLASSIFICATION WITH QUANTITATIVE AND QUALITATIVE CRITERIA

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Abstract. ABC analysis is a famous technique for inventory classification. However, this technique on the inventory classification only considering one indicator even though other important factors may affect the classification. To address this issue, researchers have proposed multiple criteria inventory classification (MCIC) solutions based on data envelopment analysis (DEA)-like methods. However, previous models almost evaluate items by different weight sets, and the index system only contains quantitative criteria and output indicators. To avoid these shortcomings, we propose an improved common-weight DEA model for MCIC issue. This model simultaneously considers quantitative and qualitative criteria as well as establishes a comprehensive index system that includes inputs and outputs. Apart from its improved discriminating power and lack of subjectivity, this non-parametric and linear programming model provides the performance scores of all items through a single computation. A case study is performed to validate and compare the performance of this new model with that of traditional ABC analysis, DEA–CCR and DEA–CI. The results show that apart from the highly improved discriminating power and significant reduction in computational burden, the proposed model has achieved a more comprehensive ABC inventory classification than the traditional models.

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1. Introduction

ABC analysis is a well-known inventory classification technique based on the Pareto principle that is often used in controlling a large number of inventory items [13, 15, 39]. This technique divides the inventory items or stock-keeping units into three categories, namely, Category A, Category B and Category C. The items in "Category A" create great value for a company and are few in number, those in "Category C" create minimal value to the company and are large in number, and "Category B" is with characteristics that are between Categories A and C.

The traditional ABC analysis technique classifies items into the three categories on the basis of their annual dollar usage (ADU). This technique is easy to understand and use. But it is unreasonable to identify inventory

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items on the basis of a single criterion [15, 16, 27, 39]. Many researchers suggest that in addition to ADU, other important indicators affect the classification, such as critical factor (CF), substitutability, specificity, number of suppliers, consumption amount, inventory cost, out-of-stock loss value, and lead time (LT) [12, 17, 39]. In this sense, inventory classification inherently is a multiple criteria decision making (MCDM) problem, hereafter called multiple criteria inventory classification (MCIC) problem for simplification [33, 39]. Many researchers have proposed solutions in solving MCIC problem that can be classified into three categories, namely, artificial intelligence (AI), MCDM, and mathematical programming (MP). Moreover, some academics employ data envelopment analysis (DEA) into MCIC field. DEA, which was first proposed by Charnes *et al.* [9], is an efficient tool for evaluating the system with multiple inputs and multiple outputs. Since then, DEA has received great attention from researchers [3, 4, 6, 7, 54].

To deal with MCIC problem, this study proposed a common-weight DEA model. The new model distinguishes from previous studies in the evaluation weight set and the evaluation index. Compared with existing approaches, our proposed method possesses several advantages. First, the inventory items are assessed with a common set of weights, which makes the evaluation more fair and provides high discriminating power. Second, the new common-weight DEA model is linear and requires fewer computational efforts, and the assessment result can be obtained just through a single computation. Third, a comprehensive index system is established that simultaneously considers quantitative and qualitative criteria as well as input and output indicators. Fourth, our proposed model can objectively evaluate inventory items. Based on the above advantages, more realistic inventory classification result can be obtained through our new method.

The rest of the paper is organized as follows. Section 2 provides the relevant literatures review. Section 3 introduces the related common-weight MCDM–DEA models introduced in previous studies. Section 4 presents our improved model. Section 5 applies our improved model to a case study of Cui and Lu [12], and results of our model are compared with those of traditional ABC analysis, DEA–CCR and DEA–CI. Section 6 concludes the paper and cites directions for future research.

2. Literature review

In this section, we review the relevant literatures and analyze several deficiencies that need to improve, for example, complex calculation; imperfect index system; subjectivity and not completely fair evaluation. We summarize and compare some existing methods with our proposed model from these aspects at the end of this section.

AI includes many AI-based classification approaches, such as artificial neural networks [36], particle swarm optimization [52], and other techniques cited in [54]. However, these methods have been rarely applied in practice due to their difficulty for inventory managers to understand and use [22, 42].

Multiple attribute decision making (MADM) method is a type of MCDM technique. MADM methods such as the simple additive weighting (SAW) [31], technique for order preference by similarity to ideal solution (TOPSIS) [1, 27], and analysis hierarchy process (AHP) [1, 9, 18, 21, 36] are widely used for MCIC, among which AHP is the most popular method. However, these approaches need more computational effort because they require two steps to classify items into one of the three ABC categories [27]. AHP can consider qualitative criteria, but it has two main deficiencies. The first one is that AHP is a completely subjective approach. It requires a decision maker to make a subjective judgment when comparing pairs of criteria. It is difficult for decision makers to assign accurate values. Accordingly, the classifications may be unrealistic. Second, AHP can only compare a limited number of decision alternatives [20]. When the size of problem (*i.e.*, the number of criteria and inventory items) grows, the computational difficulty will increase.

MP models generate a weight vector or matrix, and its objective function is to maximize or minimize the weighted score of each item [27]. Derived from DEA that is firstly introduced by Charnes *et al.* [9], Ramanathan [39] proposed the R-model, a weighted linear optimization model that can be easily understood and used by inventory managers. However, this model evaluates items with a flexible choice. That is, each item can choose its favorable weights of criteria to maximize its performance score during the evaluation. This unfair feature may lead to inaccurate classification where an item that demonstrates an excellent performance in an irrelevant criterion and does not perform well in other criteria may be classified into Category A. Moreover, the R-model has a poor discriminating power according to Iqbal and Malzahn [25]. To address this shortcoming, Zhou and Fan [56] extended the R-model into the ZF-model that integrates the best and worst performance scores of each item and obtains the final performance score using the control parameter λ, which requires a decision maker to assign the parameter value subjectively. Ng [34] also proposed the NG-model, which converts all criteria measures of an inventory item into a scalar score and flexibly integrates additional information from the decision makers [19]. With a proper transformation, the NG-model can obtain the scores of items without using a linear optimizer. However, in this model, the optimal scores of items are independent of the criteria weights, thereby leading to inappropriate classification. To address this problem, Hadi-Vencheh [19] proposed an improved version of the NG-model that called the H-model. The H-model maintains effects of weights on computing the global score of each item. However, n (we assume that n inventory items must be evaluated) non-linear programs must be calculated and formulated to obtain the performance scores of all items. Decision makers must also rank all criteria for each item before using the NG- and H-models [18,23]. Accordingly, Kaabi and Jabeur [27] proposed a new hybrid weighted optimization model called the ZF–H-model, which integrates the advantages of both the ZF- and H-models. Despite generating reasonable and promising results, the ZF–H-model requires n non-linear programmings, which is difficult to solve. Moreover, the ZF–H-model and the previously mentioned methods only consider quantitative criteria, except Ramanathan [39] took the critical factor (CF) that is a qualitative indicator as: 1 for a very critical item, 0.01 for a non-critical item and 0.5 for a moderately critical item. However, Cui and Lu [12] argued that qualitative indicators, including CF (other literatures also introduced this indicator, *e.g.*, Torabi *et al.* [42] and Hatefi *et al.* [22]), substitutability, and specificity, also influence inventory classification. Besides, these methods assume all criteria are benefit-type criteria that are positively related to the importance level of an item [56] and use the reciprocals to make cost-type indicators as positive indicators. Yet, in reality, the significance order of an item may be negatively affected by several criteria, such as substitutability, specificity, and number of suppliers [12,38].

Several researchers have recently applied DEA methods to ABC inventory classification, such as CCR [12], DEA–CI [19,38], and cross-evaluation [37]. To obtain the performance scores of all decision-making units (DMUs), the objective function in the classical DEA model should be specified to a particular DMU [21,28,29]. Therefore, DEA requires the calculation of n linear programming models, where n denotes the number of DMUs to be evaluated. In this case, DMUs are not evaluated by a common set of weights and can choose their own weights to maximize their performance scores. However, this practice is not suitable for actual management practices because managers usually expect all DMUs to be evaluated by a common set of weights. The traditional DEA model may erroneously determine the effective DMUs because of such flexibility in selecting weights. That is, the well-performing indicators receive extremely high weight values and the poorly performing indicators receive extremely low weight values. These two extremes may ignore the effect of those criteria with small weight values. Furthermore, traditional DEA models also have poor discriminating power because they prevent further discrimination among efficient DMUs with similar performance scores (*i.e.*, 1).

In order to get more fair evaluation to avoid generating unrealistic results and improve the discriminating power of DEA, researchers have developed common-weight MCDM–DEA models where DMUs are evaluated by a common set of weights [2,3,28–30,40]. In previous studies of common-weight DEA method, numerous scholars discussed common-weight DEA method in the presence of imprecise data. For example, Amin and Emrouznejad [5] proposed a new minimax DEA model to deal with the ordinal data and applied it to the advanced manufacturing technology selection problem. Their model used a non-Archimedean epsilon as the lower bound for the inputs and outputs weights, but they did not discussed how to select a suitable epsilon value. Toloo [44] also introduced a mixed integer programming-DEA model to select the most efficient supplier in which case the imprecise data exist. Furthermore, Toloo [47] proposed a new minimax mixed integer linear programming (MILP) model to seek the most efficient DMU with a common set of weights. More recent studies about dealing with the imprecise data in the common-weight DEA field can be referred to Hatami-Marbini and Toloo [24], Toloo and Tavana [49], and so forth. Moreover, to solve the MCIC problem, Hatefi and Torabi [21], Torabi *et al.* [42], Hatefi *et al.* [22], Hatefi and Torabi [23] also applied a common-weight MCDM–DEA model for ABC inventory classification.

Hatefi and Torabi [21] developed a common-weight MCDM–DEA model, that have several advantages compared with Zhou *et al.* [57], for example, high discriminating power and fewer computational efforts. However, they only considered quantitative criteria, while qualitative criteria are very essential for a comprehensive inventory evaluation and classification. Torabi *et al.* [42] proposed a modified model that coped with the quantitative and qualitative criteria derived from the model of Hatefi and Torabi [21] and the imprecise DEA model of Zhu [55]. The modified model demonstrated a higher discriminating power and achieved a more reasonable inventory classification than traditional methods. However, it employed a mixed objective–subjective approach, and is the same with the model developed by Hatefi and Torabi [21], where the value of its control parameter k was obtained via a time-consuming trial-and-error approach. Also, in order to deal with qualitative criteria, this modified model set strongly ordinal relations (SOR) constraints, in which a number of parameters need to be determined exogenously through trial-and-error and sensitivity analysis methods. Hatefi *et al.* [22] also introduced a DEA-like model that considered both qualitative and quantitative indicators. However, the method was not based on a common-weight framework, only used the maximum feasible epsilon parameter to increase its discriminating power, and required the solution of n linear optimization models. Therefore, n different weight sets will be calculated, resulting in unfair results. Hatefi and Torabi [23] proposed a common-weight DEA-like model based on the R-model [39] to eliminate subjectivity and generate a set of common-weights to evaluate all inventory items. This model can facilitate a fair evaluation and provide a more realistic and reasonable ABC inventory classification result than the R-model [39] and H-model [19]. But, this model does not consider qualitative criteria, such as CF, and may obtain more than one efficient item according to Karsak and Ahiska [28–30] and Hatefi and Torabi [21].

In summary, among existing solutions to MCIC, there are several deficiencies need to improve. Firstly, some methods are completely subjective or fixed objective-subjective, such as AHP and ZF-model and so on. Secondly, a few methods take qualitative criteria as well as cost-type criteria into consideration. A complete index system including quantitative and qualitative as well as benefit- and cost-type criteria is essential for comprehensively evaluate and classify inventory items. It is noted that benefit-type indicators means that their value have positive effects on the importance level of an item, so we define that benefit-type indicators are consistent with the outputs in DEA model. Analogously, cost-type indicators are equivalent to inputs. Thirdly, some DEA-like model have poor discriminating power and unfair assessment resulting from no common weights. Finally, some approaches need to compute too many linear or non-linear programming, and the trail-and-error process naturally increase the difficulty of calculation. Table 1 summarizes these shortcomings of different methods and the asterisk indicates the major deficiency.

To alleviate these defects concurrently, we propose an improved DEA model for MCIC problems. This new approach is inspired by three studies, namely, the model of Amin [4] that obtains the most effective DMU, the MCDM-framework of Karsak and Ahiska [29], and the method of converting the non-linear constraints to linear ones in Toloo *et al.* [48]. Similar to Cui and Lu [12] and Hatefi *et al.* [22], we establish a highly comprehensive index system that includes quantitative and qualitative indicators as well as multiple inputs and outputs (*i.e.*, cost-type and benefit-type indicators, respectively). Via the new model, we can estimate the items using a common set of weights to improve the discriminating power and evaluate these items fairly. Furthermore, the proposed model is linear, and can provide efficiencies of all items through a single calculation and requires no additional calculation processes, such as the iterative method and bisection search algorithm used in Hatefi and Torabi [21] and Torabi *et al.* [42]. This model also does not require the subjective judgment and ranking of decision makers. We also list these characteristics of our proposed model in Table 1.

From Table 1, we can see that there are different flaws in different methods, and our approach can alleviate these deficiencies simultaneously. We will validate the performance of our proposed model in Section 4.

TABLE 1. The summary and comparison between some methods.

Methods		Subjectivity	Calculation	Discriminating power	Common weights	
MCDM	AHP	*yes	*Long time			
	R-model	no	*N linear optimization models	*poor	*no	
	ZF-model	*yes	*$2N$ linear optimization models		*no	
MP	NG-model	*yes	Without linear optimizer		*no	
	H-model	*yes	*N non-linear programming		*no	
	ZF-H-model	*yes	*$2N$ non-linear programming		*no	
	Hatefi and Torabi [21]	no	*additional iterative method	high	yes	
	Torabi *et al.* [42]	*yes	*additional iterative method	high	yes	
DEA	Hatefi *et al.* [22]	no	*repeat N times	high	*no	
	Hatefi and Torabi [23]	no	once	high	no	
Our model		**no**	**once**	**high**	**yes**	
Methods		Index system				Other shortcomings
		Qualitative	Benefit-type	Cost-type		
MCDM	AHP	yes	yes	yes	*reciprocals	*Difficulty in calculating large-scale problems
	R-model	yes	critical factor have three categories	yes	*reciprocals	*weights for different items are endogenous and changeable
	ZF-model	yes	no	yes	*reciprocals	
MP	NG-model	yes	*no	yes	*reciprocals	*Performance scores independent on item weights
	H-model	yes	no	yes	*reciprocals	
	ZF-H-model	yes	no	yes	*reciprocals	
	Hatefi and Torabi [21]	yes	*no	yes	*reciprocals	
D	Torabi *et al.* [42]	yes	yes	yes	*reciprocals	*a number of parameters need to exogenously determined
E	Hatefi *et al.* [22]	yes	yes	yes	*reciprocals	
A	Hatefi and Torabi [23]	yes	*no	yes	*reciprocals	*may obtain more than one efficient item
Our model		**yes**	**yes**	**yes**	**initial**	

3. Related common-weight MCDM–DEA models

Our model is developed based on the common-weight MCDM framework of Karsak and Ahiska [29], and the suggestions of the improved integrated model of Amin [4]. Additionally, the non-linear constraints are eliminated by using the method in Toloo *et al.* [48].

The common-weight MCDM framework of Karsak and Ashika [29] focuses on those decision problems with multiple inputs and outputs. As its objective function, this framework minimizes the deviation of efficiency from an ideal efficiency score of 1. This model can improve the discriminating power and obtain the best DMU in consideration. We assume that n DMUs are evaluated under this framework and that each DMU contains s outputs and m inputs. The model is called minimax efficiency model and is presented as follows:

$$\min\ d_{\max} \quad \text{s.t.} \tag{3.1a}$$

$$\begin{cases} d_{\max} - d_j \geq 0,\ \ j = 1, 2 \ldots n, & (3.1b)\\ \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0,\ \ j = 1, 2 \ldots n, & (3.1c)\\ \sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1, & (3.1d)\\ u_r,\ v_i,\ d_j \geq 0,\ \forall r, i, j, & (3.1e) \end{cases}$$

where d_j is the deviation of the efficiency of DMU_j, namely, E_j, from the ideal efficiency score of 1 (*i.e.*, $d_j = 1 - E_j$). Constraint (3.1b) is added to ensure that $d_{\max} = \max\ d_j$, $j = 1, 2 \ldots n$ which means that $d_{\max}$ is the maximum deviation from the ideal efficiency score, u_r and v_i are the weights that are assigned to rth output and ith input, respectively, x_{ij} represents the input $i$used by DMU_j, and y_{rj} denotes the output r produced by DMU_j. The weight restriction constraint (3.1d) is added to limit the sum of the importance weights equals to 1.

Through a single computation of model (3.1), all deviations can be calculated to obtain the efficiencies of all DMUs. This step allows us to assess the relative efficiency of all DMUs based on a common indicator weight set. The minimax efficiency model may sometimes produce more than one efficient DMU, that is, more than one d_j is equivalent to zero. Therefore the model is revised as follows:

$$\min\ d_{\max} - k \sum_{j \in \text{EF}} d_j \quad \text{s.t.} \tag{3.2a}$$

$$\begin{cases} d_{\max} - d_j \geq 0,\ \ j = 1, 2 \ldots n, & (3.2b)\\ d_{\max} - \sum_{j \in \text{EF}} d_j \geq 0,\ \ j = 1, 2 \ldots n, & (3.2c)\\ \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0,\ \ j = 1, 2 \ldots n, & (3.2d)\\ \sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1, & (3.2e)\\ u_r,\ v_i,\ d_j \geq 0,\ \forall r, i, j, & (3.2f) \end{cases}$$

where EF is the set of efficient DMUs determined by the minimax efficiency model. The k is a discriminating parameter, the value of which ranges from 0 to 1. It is assigned by a decision maker with a predetermined step size, from zero until the model produces a single efficient DMU.

The framework of Karsak and Ashika [29] evaluates all DMUs based on a common set of weights and considers both inputs and outputs. However, the initial value of the discriminating parameter k and the iterative step size are determined subjectively, and a time-consuming trial-and-error process is required. Accordingly, Amin [3] proposed an improved MCDM–DEA model that uses a nonlinear constraint to obtain a single efficient DMU ($d_j^* = 0$) by adding binary δ_j and continuous variables β_j. This improved approach of Amin [3], which is derived from the common-weight MCDM–DEA model mixed with the single input and multiple outputs proposed by Karsak and Ashika [29], requires only once computation and is non-iterative. In the similar way, Amin [4] introduced an improved integrated model for the case where multiple inputs and multiple outputs exist. The model in Amin [4] is as follows:

$$\begin{aligned}&\min\ d_{\max}\\&\text{s.t.}\end{aligned} \tag{3.3a}$$

$$\begin{cases}
d_{\max} - d_j \geq 0,\ j = 1, 2 \ldots n & (3.3b)\\
\sum\limits_{r=1}^{s} v_i x_{ij} \leq 1,\ j = 1, 2 \ldots n & (3.3c)\\
\sum\limits_{r=1}^{s} u_r y_{rj} - \sum\limits_{r=1}^{s} v_i x_{ij} + d_j = 0,\ j = 1, 2 \ldots n & (3.3d)\\
\sum\limits_{j=1}^{n} \delta_j = n - 1 & (3.3e)\\
\delta_j - d_j \beta_j = 0,\ j = 1, 2 \ldots n, & (3.3f)\\
\delta_j \in \{0,1\},\ d_j \geq 0,\ \beta_j \geq 1,\ j = 1, 2 \ldots n & (3.3g)\\
u_r \geq \varepsilon,\ r = 1, 2 \ldots s & (3.3h)\\
v_i \geq \varepsilon,\ i = 1, 2 \ldots m & (3.3i)
\end{cases}$$

where constraint (3.3c) is non-geometrically redundant according to the Theorem 2 in Toloo [44]. ε is a very small positive number called the non-Archimedes infinite decimal, which ensures that the weight coefficient of each indicator does not equal to 0 to guarantee the full utilization of all evaluation indicators. In other words, in model (3.3), all weights are strictly positive, which is contrast with models (3.1) and (3.2). This property provides a more fair evaluation and highly improves the discriminating power in DEA approach. Besides, plenty researchers emphasized the crucial role of epsilon value in finding the most efficient DMU. For example, Cook *et al.* [11] pointed that the non-Archimedean value can affect the discriminating power of DEA models and a large value is commonly preferred over a small one. Toloo [43] pointed the most efficient unit may not be found if we ignore the non-Archimedean epsilon. Toloo and Salahi [50] illustrated that Lam [32]'s model may fail to find the most efficient DMU because the value of epsilon is selected unsuitably. Toloo and Salahi [50] introduced a new model to calculate the correct maximum epsilon. Other studies also emphasized the importance of epsilon value, such as Toloo [43], Toloo [46], Salahi and Toloo [41], Toloo and Tavana [49], Toloo and Salahi [50] and Hatami-Marbini and Toloo [24]. In the next section, we will introduce the method of calculating the proper non-Archimedean epsilon value.

Moreover, model (3.3) is non-linear since the multiplier of two variables in formula (3.3f). Recently, Toloo *et al.* [48] proposed a mixed integer linear programming (MILP) that eliminates the non-linear constraint in model (3.3), which significantly reduces the computational burden. Concretely, they replaced the constraints $\delta_j - d_j\beta_j = 0$ and $\beta_j \geq 1$ where $j = 1, 2 \ldots n$ with a linear formula $d_j \leq \delta_j \leq N d_j$ where N indicates an enough large number and $j = 1, 2 \ldots n$. At the same time, other restraints are unchanged. Toloo *et al.* [48] proved that their proposed MILP is equivalent to model (3.3). Accordingly, we will eliminate the non-linear constraints by using the similar method in Toloo *et al.* [48].

4. New common-weight DEA model for MCIC

This section proposes an improved common-weight DEA model for MCIC. Furthermore, we establish a highly comprehensive index system that includes quantitative and qualitative indicators as well as multiple input and output indicators. The new model is described as follows:

$$\min d_{\max} \tag{4.1a}$$

s.t.

$$d_{\max} - d_j \geq 0;\ j = 1, 2, \ldots n \tag{4.1b}$$

$$\sum_{r \in \text{EXO}} u_r y_{rj} + \sum_{r \in \text{ORDO}} w_r \gamma_r(j) - \left[\sum_{i \in \text{EXI}} v_i x_{ij} + \sum_{i \in \text{ORDI}} \mu_i \gamma_i(j)\right] + d_j = 0;\ j = 1, 2, \ldots n \tag{4.1c}$$

$$\sum_{j=1}^{n} \delta_j = n - 1 \tag{4.1d}$$

$$d_j \leq \delta_j \leq N d_j;\ j = 1, 2, \ldots n \tag{4.1e}$$

$$\delta_j \in \{0, 1\}, d_j \geq 0;\ j = 1, 2, \ldots n \tag{4.1f}$$

$$u_r \geq \varepsilon;\ r \in \text{EXO} \tag{4.1g}$$

$$v_i \geq \varepsilon;\ \text{i} \in \text{EXI} \tag{4.1h}$$

$$w_r, \mu_i \in \psi$$

$$\psi = \left\{ \begin{array}{l} w_{rl} - w_{rl+1} \geq \varepsilon, w_{r1} \geq \varepsilon; \mu_{il} - \mu_{il+1} \geq \varepsilon, \mu_{i1} \geq \varepsilon; \\ l = 1, 2, \ldots, L-1;\ \text{i} \in \text{ORDI}; r \in \text{ORDO} \end{array} \right\} \tag{4.1i}$$

where EXO and ORDO are sets of exact and ordinal outputs, respectively, and EXI and ORDI are sets of exact and ordinal inputs, respectively. Formula (4.1b) ensures that $d_{\max}$ denotes the maximum value of the deviations. We assume that n items are under classification. The variable of x_{ij} represents the ith exact input of the jth item, and y_{rj} denotes the rth exact output of the jth item. The ε is the non-Archimedean infinitesimal value that defines the lower bound for the weights of all criteria and the minimum required difference among the successive levels of ordinal criteria. This value also ensures that each importance weight has a certain effect on the assessment. N represents an enough large number. The variable of ψ denotes the set of ordinal relation constraints, which ensures those items rated in the lth place have a higher value than those rated in the l+1st place with respect to ordinal outputs and inputs. The ordinal outputs and inputs are classified based on a common Likert scale (*i.e.*, L). We propose the following theorem for constraints (4.1d)–(4.1f).

Theorem 4.1. *Constraints (4.1d)–(4.1f) ensure that only one d_j is equivalent to 0, that is, a single efficient item.*

Proof. Given constraints $\sum_{j=1}^{n} \delta_j = n - 1$ and $\delta_j \in \{0, 1\}; j = 1, 2, \ldots n$, only one δ is equal to zero. Assuming that $\delta_p = 0$ and $\delta_j = 1$ when $j \neq p,\ j = 1, 2 \ldots n$. With constraints $\beta_j \geq 1$ and $\delta_j - d_j \beta_j = 0$ for all $j = 1, 2 \ldots n$, only one $d_p = 0$ and the other $d_j \leq 1$. □

Theorem 4.1 indicates that our proposed model can get a single efficient item through one computation. In this case, the discriminating power of items can be significantly improved, especially of the efficient ones. Hence, constraints (4.1d)–(4.1f) are critical to get fair classification results.

According to the notation in Cook *et al.* [11], we define L-dimensional unit vectors $\gamma_r(j) = [\gamma_{r1}(j), \gamma_{r2}(j) \ldots \gamma_{rL}(j)]$ and $\gamma_i(j) = [\gamma_{i1}(j), \gamma_{i2}(j) \ldots \gamma_{iL}(j)]$, where

$$\gamma_{rl} = \begin{cases} 1 & \text{if item } j \text{ is rated in } l\text{th place on the } r\text{th ordinal output} \\ 0 & \text{otherwise} \end{cases} \tag{4.2}$$

and

$$\gamma_{il} = \begin{cases} 1 & \text{if item } j \text{ is rated in } l\text{th place on the } i\text{th ordinal input} \\ 0 & \text{otherwise} \end{cases}. \tag{4.3}$$

If $L = 5$ and Item 6 is rated in the second place on the first output criterion, then $\gamma_1(6) = [0, 1, 0, 0, 0]$.

In addition, $u_r, v_i, \forall r \in \text{EXO},\ i \in \text{EXI}$ are the importance weights for the exact outputs and inputs, $w_r, \mu_i, \forall r \in \text{ORDO},\ i \in \text{ORDI}$ are $L-$dimensional worth vectors, such as $w_r = [w_{r1}, w_{r2} \dots w_{rL}]$, and w_{rl} denotes the value of being rated in the lth place with respect to the rth output. Therefore, $w_r\gamma_r(j) = \sum_{l=1}^{L} w_{rl}\gamma_{rl}(j)$ and $\mu_i\gamma_i(j) = \sum_{l=1}^{L} \mu_{il}\gamma_{il}(j)$ denote the worth assigned to the jth item with respect to the rth ordinal output and the ith ordinal input, respectively.

This new model is developed based on these common-weight MCDM–DEA models introduced in Section 3, while is different from previous common-weight DEA models used in the MCIC context. First, the new model is linear and only needs to be computed in one time without additional trail-and-error process. Second, by including multiple inputs and outputs and simultaneously considering exact and ordinal criteria, this new model is a comprehensive model that can effectively solve practical MCIC problems. And it is also flexible. When only quantitative or qualitative indicators are needed, we just need to set $\gamma_i(j) = 0, i \in \text{ORDI}$ and $\gamma_r(j) = 0, r \in \text{ORDO}$ or $x_{ij} = 0, i \in \text{EXI}$ and $y_{rj} = 0, r \in \text{EXO}$. When only input or output indicators are presented, we just need to set $\gamma_r(j) = 0, r \in \text{ORDO}$ and $y_{rj} = 0, r \in \text{EXO}$ or $\gamma_i(j) = 0, i \in \text{ORDI}$ and $x_{ij} = 0, i \in \text{EXI}$. Excepting the differences from existing models, the new model also keeps the original advantages of common-weight model.

The non-Archimedean value can affect the efficiency scores and the discriminating power of DEA models. As mentioned in Section 3, many studies emphasized the important role of epsilon value especially when priority the best efficient DMU. They pointed out that a larger epsilon value is preferred over a smaller one. Simultaneously the epsilon value cannot be too large as it will result in infeasible solutions. Toloo [43] proposed an efficient approach to calculate maximal epsilon value for the model in Amin [4]. Referring the methods in Cook *et al.* [11], Toloo [46] and Toloo *et al.* [48], we establish the following model to calculate the maximum feasible epsilon value, namely, $\varepsilon_{\max}$.

$$\max \varepsilon \tag{4.4a}$$

s.t.

$$\sum_{r\in \text{EXO}} u_r y_{rj} + \sum_{r\in \text{ORDO}} w_r\gamma_r(j) - \left[\sum_{i\in \text{EXI}} v_i x_{ij} + \sum_{i\in \text{ORDI}} \mu_i\gamma_i(j)\right] + d_j = 0;\ j = 1, 2, \dots n \tag{4.4b}$$

$$\sum_{j=1}^{n} \delta_j = n - 1 \tag{4.4c}$$

$$d_j \le \delta_j \le N d_j;\ j = 1, 2, \dots n \tag{4.4d}$$

$$\delta_j \in \{0, 1\}, d_j \ge 0;\ j = 1, 2, \dots n \tag{4.4e}$$

$$\varepsilon - u_r \le 0;\ r \in \text{EXO} \tag{4.4f}$$

$$\varepsilon - v_i \le 0;\ i \in \text{EXI} \tag{4.4g}$$

$$w_r,\ \mu_i \in \psi$$

$$\psi = \left\{ \begin{array}{l} w_{rl} - w_{rl+1} - \varepsilon \ge 0,\ w_{rl} - \varepsilon \ge 0;\ \mu_{il} - \mu_{il+1} - \varepsilon \ge 0,\ \mu_{il} - \varepsilon \ge 0; \\ l = 1, 2, \dots, L-1;\ i \in \text{ORDI};\ r \in \text{ORDO} \end{array} \right\}. \tag{4.4h}$$

Different with model (4.1), in model (4.4), the non-Archimedes infinite decimal ($i.e.$, ε) is a decision variable. Furthermore, the objective function of model (4.4) is the maximum value of the epsilon rather than the minimum D as presented in model (4.1). Therefore, the constraint (4.1b) is removed in model (4.4). To obtain a single efficient DMU, constraints (4.4b)–(4.4h) are applied similarly as in model (4.1). In model (4.4), the optimal objective function value $\varepsilon_{\max}$ belongs to $(0, \infty)$. If we want to limit epsilon within a small region and obtain a smaller feasible epsilon than $\varepsilon_{\max}$, we may add a weight normalization constraint into the model. After the maximum feasible epsilon value is calculated, taking $\varepsilon_{\max}$ into model (4.1) can obtain reasonable performance scores without subjectivity. Besides, the discriminating power can be highly improved.

5. Illustrative example

As mentioned above, a few articles take qualitative criteria and cost-type indicators into account. To validate the performance of our model, we utilize the modified data derived from initial data from Cui and Lu [12], which includes quantitative and qualitative as well as benefit- and cost-type indicators. We compare its inventory classification results with those of traditional ABC analysis, DEA–CCR and DEA–CI.

5.1. Inventory classification evaluation criteria and initial data

As mentioned above, ABC inventory classification is essentially a MCIC problem, and the classification is affected by quantitative and qualitative criteria, such as CF and LT. Moreover, not all indicators positively affect the importance level of inventory items because several criteria are negatively related to their performance scores, including substitutability, specificity, and number of suppliers. The new modified common-weight model for the MCIC problem includes quantitative and qualitative criteria as well as inputs and outputs in the DEA model. The index system introduced by Cui and Lu [12] is discussed as follows.

CF is a categorical and discontinuous criterion. Therefore, many researchers exclude CF when calculating performance scores of inventory items. However, the ordinal values of this qualitative criterion reflect the degree of importance of inventory items and it is defined as input in Cui and Lu [12]. In other words, an item with a smaller ordinal value warrants more attention.

As another qualitative indicator, substitutability (SUB) refers to the degree to which an inventory item can be replaced by other similar items. A SUB value of 1 indicates that the item cannot be replaced, 3 indicates that the item can be replaced but influences the effect, and 5 indicates that the item can be completely replaced. Therefore, an item with a lower SUB has a higher importance.

Specificity (SPE) evaluates the degree of specialization of the items. An item with a smaller SPE has a higher importance. An SPE of 1 indicates highest specificity, 3 indicates moderate specificity, and 5 indicates weak specificity.

Number of suppliers (NOS) is a quantitative index. An item with the fewest suppliers has the highest importance.

These four criteria negatively affect the importance level of the items. That is, an item with a smaller value of any of these criteria warrants more attention. Conversely, four other indicators positively influence the significance ranking of items.

First, the value of consumption amount (CA) is equal to unit price multiplied by annual consumption quantity. Second, inventory cost (IC) represents the inventory maintenance cost of the item for one year. Third, out-of-stock loss value (SL) refers to the loss resulting from the shortage of inventory items. Fourth, lead time (LT) refers to the required time from order to storage, with a longer LT indicating a higher chance for stock depletion or a higher tendency for losses to occur.

Cui and Lu [12] and Ping and Du [38] identified several criteria which smaller values increase the importance of an item as inputs (*i.e.*, CF, SUB, SPE and NOS), on the contrary, others as outputs (*i.e.*, CA, IC, SL and LT). To test our model, we modify the raw data as follows. First, we take the opposite ranking values of CF in Cui and Lu [12], and then treat the result as an output that can make both the inputs and outputs contain qualitative and quantitative criteria simultaneously. In other words, we take 1 when the ordinal value of CF in Cui and Lu [12] is 9, 2 when the initial CF is 8, and all the way up to 9. Second, we use the same Likert scale (*i.e.*, $L = 9$) to sort the qualitative inputs and outputs. Table 2 presents the modified data.

Table 2. Sample of spare parts of a beer enterprise.

Item	Inputs			Outputs				
	SUB	SPE	NOS	CF	CA	IC	SL	LT
1	1	3	4	3	406 272	13 995.45	870	7
2	3	5	13	7	199 017.3	37 985.12	2 500	5
3	5	3	6	4	170 765	3 300	1 200	10
4	3	3	28	7	101 444.4	28 654	5 600	15
5	5	5	18	3	97 692	11 523.69	820	3
6	3	5	50	6	71 400	21 420	1 000	2
7	3	1	2	9	67 035	18 523	25 300	30
8	3	5	8	8	60 546	18 163.8	3 600	10
9	5	5	22	4	56 760	10 590.33	180	4
10	3	1	1	6	40 000	11 456.98	550	30
11	3	3	5	7	35 100	7 856	3200	5
12	5	3	4	4	18 400	5 520	560	8
13	5	5	28	2	15 750	4 725	130	1
14	3	3	8	4	12 993	3 126.48	300	3
15	3	5	12	5	11 897.52	3 569.256	420	7
16	3	5	18	8	11 120	3 336	4800	2
17	5	5	12	3	10 943.46	4 423.56	1300	4
18	3	3	8	4	9 752.7	3 123	300	7
19	5	5	43	1	8 047.7	2 414.31	50	1
20	3	3	16	5	4 012.1	1 098.63	260	2
21	5	5	50	3	3 631.2	799.25	66	2
22	5	5	55	3	2 573.2	689.13	40	2
23	3	5	30	3	1 661.52	486.37	80	2
24	1	3	4	2	1 169.28	324	100	5
25	5	5	60	1	1 140.98	342.25	12	2
26	3	3	22	7	676.5	181.2	120	2
27	5	3	120	1	400	114.3	2.5	1
28	5	5	100	1	316.2	105.36	1.2	1
29	5	5	50	1	262.5	78.75	30	1
30	1	1	3	5	231	189.24	280	7

5.2. Results and analysis

We experiment with the data in Table 2 by running Lingo 11 on a laptop computer with Intel(R) Core(TM) i5-3210M CPU @2.50 GHz and 4.00 GB RAM. The runtime of model (4.1) is 1.0 s, and the total solver iterations are 191.

Set $N = 10\,000$, we calculated the appropriate maximum value of the epsilon using model (4.4). Afterward, we use $\varepsilon_{\max}$ as the non-Archimedean value in model (4.1) and obtain all DMUs' deviations by calculating model (4.1). Then, efficiencies of all DMUs can be obtained according to the formula $d_j = 1 - E_j$. For ease of comparison with other methods, we use the same classification distribution as those employed in previous studies, that is, 6 for class A, 9 for class B, and 15 for class C. All results are presented in Table 3. The second column shows the classification of these items from our proposed method. The third, fourth, and fifth columns present the classification results of traditional ABC analysis, DEA–CCR and DEA–CI, respectively.

Table 3 shows that 11 of 30 items classifications are exactly same as generated by the three previous methods, and only 4 items (items 8, 4, 6, 17) out of other 19 items are reclassified by our model because of comprehensive evaluation. That is, the only 4 items' classification are completely different from all previous methods. For example, the CA of Item 8 is lower than those of Item 5, while other outputs (*i.e.*, IC, SL and LT) of Item 8 are

Table 3. Inventory classification results of different methods.

Item	Our proposed method	Traditional ABC	DEA–CCR	DEA–CI
3	A	A	B	B
1	A	A	A	A
2	A	A	A	A
7	A	B	A	A
10	A	B	A	A
8	A	B	B	B
12	B	B	C	C
11	B	B	B	A
5	B	A	B	C
14	B	B	C	B
18	B	C	B	B
17	B	C	C	C
4	B	A	A	A
15	B	B	B	C
9	B	B	B	C
20	C	C	C	B
16	C	C	C	C
26	C	C	C	B
13	C	B	C	C
23	C	C	C	C
6	C	A	B	B
30	C	C	A	B
24	C	C	B	B
19	C	C	C	C
21	C	C	C	C
29	C	C	C	C
22	C	C	C	C
25	C	C	C	C
28	C	C	C	C
27	C	C	C	C

higher than Item 5 and all inputs of Item 8 are smaller than Item 5. In other words, only 1 out of the 8 criteria of Item 8 are dominated by Item 5. Therefore, Item 8 is classified to Category A, while Item 5 is classified to Category B. However, this classification result is different from that of the other three methods. Moreover, Item 30 has small inputs, and both DEA–CCR and DEA–CI classify this item to Categories A and B. If we consider the inputs and outputs simultaneously, we find that CA, IC, LV and LT (particularly CA and IC) of Item 30 only have small and negligible values in all assessment items. Furthermore, the CO, IC, SL and LT of Item 17 are larger than those of Item 20. Therefore, we must place Items 17 and 20 in Categories B and C, respectively, to achieve a comprehensive consideration of all indicators. Moreover, compared with the previous approaches, a few items in our proposed model are ranked in the back, such as Items 4 and 6, but these classifications can be explained reasonably. For example, Item 4 has a high value for certain output indicators yet has 28 suppliers, which is considered too large in all evaluated items. Meanwhile, Items 8, 11, 12 and 5 only have 8, 5, 4 and 18 suppliers, respectively. Therefore, it is reasonable that Item 4 is less important than Items 8, 11, 12 and 5. Similarly, given its large number of suppliers and small LT, we place Item 6 in Category C.

Generally, our model generates different classifications for 14 of the 30 items compared with DEA–CI, and reclassifies only 9 of the 30 items compared with the traditional ABC classification and DEA–CCR, thereby fully validating the effectiveness of our proposed method. These comparisons and analyses also show that our proposed model can evaluate items by considering all the eight criteria simultaneously. Given that the criteria contain quantitative and positive indicators as well as qualitative and negative indicators, the proposed model achieves a comprehensive and reasonable ABC inventory classification. Moreover, given that the new model uses a common set of weights to assess the 30 inventory items, all these items are not allowed to select favorable weight sets. This feature allows us to compute fair performance scores and achieve a high discriminating power. Furthermore, the performance scores of all items can be computed through a single linear programming after obtaining the maximum epsilon, thereby significantly saving the computational efforts. Therefore, the proposed model can obtain the most efficient item and improve the discriminating power of all items through a single computation. The calculation does not involve any subjectivity, which indicates that the results are highly objective and reasonable. In sum, compared with the traditional ABC analysis, DEA–CCR and DEA–CI, the new model has many advantages.

6. Conclusions

To address the MCIC problem that is a MCDM issue, many researchers have proposed DEA-like models. However, previous methods almost evaluated items with different weight sets, and just considered quantitative and output indicator, which lead to the incomplete and unfair evaluation of items. This study proposed an improved common-weight DEA model that evaluates inventory items using a common set of weights and combines quantitative and qualitative criteria as well as inputs and outputs simultaneously. Compared with previous research, our proposed model has following advantages:

(i) Complete index system. Our model considers multiple inputs and outputs and contains quantitative and qualitative indicators. These factors are necessary for a comprehensive and realistic inventory classification.
(ii) Lower computational burden. Our proposed model is linear, it requires only a single calculation, and does not require the trial and error or bisection approaches.
(iii) No subjectivity. The proposed model is a nonparametric programming and can avoid subjective selection of epsilon value, which involves no subjectivity and ensures a fair evaluation.
(iv) Fairer assessment. We evaluate inventory items using a common set of weights. In this way, we prevent any item from being evaluated by its own favorable weights. Thus, the proposed approach can generate fair, essential results that coincide with actual management practices.
(v) Higher discriminating power. Our model is derived from the common-weight DEA approaches and uses the maximum feasible epsilon value. In this sense, the discriminating power of the proposed model is increased.

Future studies may pursue other interesting research directions. First, the evaluation indicators can be further improved to make the evaluation and classification results closer to reality. Second, we can also incorporate the preferences of inventory managers into a new model. When assessing and classifying inventory items, considering subjective opinions and intuitive senses of decision makers is an interesting further research direction [51].

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