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RANDOMIZED CONTROL OF ARRIVALS IN A FINITE-BUFFER GI/M/1 SYSTEM WITH STARTING FAILURES

Dong-Yuh Yang[1,*], Jau-Chuan Ke[2] and Chia-Huang Wu[3]

Abstract. This paper proposes a randomized policy for the control of arrivals in a finite-buffer $GI/M/1$ system with a single server. When the capacity of system is full, no new arrivals are permitted to enter the system. When the number of customers in the system decreases to threshold F, a new arriving customer is allowed to join the system with probability p. The system requires an exponential startup time before allowing customers to enter the system. The startup process may not be successful and is then restarted once again. Using the supplementary variable technique in a recursive process, we obtain the stationary distribution of the system size. Various performance measures of the system are developed. We also create a cost model based on the system performance measures and cost elements. The optimal threshold, optimal capacity and optimal startup rate of the system are determined to minimize the expected cost per unit time. Finally, we provide numerical examples to conduct a sensitivity analysis.

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1. Introduction

The queueing model under F-policy has been studied by numerous researchers due to its wide applicability in production systems, computer systems, and transportation systems. The F-policy was introduced by [7] to facilitate the control of arrivals at a queueing system. Moreover, [7] also obtained the interrelationship between the F-policy and N-policy $M/M/1/K$ queues with exponential startup times. Wang and Yang [18] extended the results in [7] to the F-policy $M/M/1/K$ queue with server breakdowns. Wang *et al.* [16] and [17] applied the method of supplementary variable to study the F-policy $M/G/1/K$ and F-policy $G/M/1/K$ queues with exponential startup times, respectively. Yang *et al.* [27] discussed the optimal management problem for the F-policy $M/M/1/K$ queue with single working vacation, wherein the server takes a vacation as soon as the system becomes empty. However, rather than completely stopping the service during the vacation period, the server continuous working, albeit at a lower service rate. The interrelationship between the N-policy $M/G/1/K$ queue with exponential startup times and the F-policy $G/M/1/K$ queue with exponential startup times was

Keywords. Cost optimization, (p, F)-policy, GI/M/1 system, supplementary variable technique.

[1] Institute of Information and Decision Sciences, National Taipei University of Business, Taipei, Taiwan.
[2] Department of Applied Statistics, National Taichung University of Science and Technology, Taichung, Taiwan.
[3] Department of Industrial Engineering and Management, National Chiao Tung University, Hsinchu, Taiwan.
*Corresponding author: yangdy@ntub.edu.tw

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Table 1. Literature on queueing systems with randomized control policy.

Randomized control policy	Study
Randomized control of N-policy	[5, 6, 14, 19, 20, 23–25]
Randomized control of T-policy	[12, 19, 21, 26]
Randomized control of F-policy	[3, 4, 6, 24, 29]

examined by [13]. For the F-policy $M/H_2/1/K$ queue with server breakdowns and exponential startup times, [1] formulated an optimization problem to determine the optimal capacity, the optimal threshold, the optimal startup rate, and the optimal repair rate at the minimum expected cost per unit time. In addition, [10] considered the $M/M/2/K$ queue with server breakdowns, multi-optional phase repair, and start-up under an (N, F)-policy in which the first server is turned on when N or more customers appear in the system, and the second server is turned on when the system becomes full. For the $M/M/2/K$ queue under the triadic (F, Q, M)-policy, [8] derived explicit expressions of stationary probability distributions of the number of customers in the system. They established a cost model and employed a genetic algorithm to solve the problem of cost minimization. Chang *et al.* [2] investigated the machine repair problem with warm standbys under F-policy, wherein no additional failed machines are permitted to enter the maintenance facility when the capacity of the maintenance facility is reached. Until the capacity of the maintenance facility drops to the threshold value F, failed machines are permitted to enter the facility to be repaired. Jain *et al.* [11] used the successive over-relaxation method to obtain the steady-state probability distribution of the failed number of machines in the system, to deal with the machine repair problem with server breakdowns under the F-policy. Yang and Chang [22] analyzed the F-policy queueing system by using fuzzy set theory. They formulated a class of nonlinear functions to find the α-cuts and constructed the membership function of the system size from various values of α. Jain and Sanga [9] applied the supplementary variable technique to analyze the multi-component machining system, in which an arriving failed unit finding the repairman busy joins the retrial orbit. The failed unit in orbit makes an attempt for repair after a random time that follows a general distribution. The recent work of Yang and Yang [28] focused on steady-state analysis of the F-policy $M/M/2/K$ queue with heterogeneous servers.

Queueing systems with randomized control policy have been studied by numerous researchers. We summarize the literature on queueing systems with randomized control policy in Table 1. It is notable that there are few reports to study queueing systems with randomized control of F-policy. This motivates us to investigate the queueing system with the randomized control of F-policy and starting failures. In such model, a random time is required for setup before customers are allowed to enter the system. Upon a starting failure, the system is restarted to allow customers to enter the system, which occurs frequently in daily life. The remainder of the paper is organized as follows. In Section 2, we describe the proposed queueing model and outline the assumptions. In Section 3, we formulate the mathematical model of the system using the supplementary variable technique. The recursive method is used to derive the stationary distribution of the number of customers in the system. In Section 4, we develop various system performance measures and deal with the problem of cost optimization. In Section 5, sensitivity analysis is performed to examine the effects of system parameters on the expected number of customers in the system, optimal threshold, optimal capacity of the system, optimal startup rate, and minimum expected cost. Finally, conclusions are drawn in Section 6.

2. Description of the model

In this study, we consider the finite-buffer $GI/M/1$ system with starting failures under (p, F)-policy.

2.1. Assumptions

The system is implemented under the following assumptions:

(i) The inter-arrival times are independent and identically distributed (i.i.d.) random variables with distribution function $A(u)$, probability density function $a(u)$, Laplace–Stieltjes transform (LST) $a^*(s) = \int_0^\infty e^{-su}\mathrm{d}A(u) = \int_0^\infty e^{-su}a(u)\mathrm{d}u$, and mean inter-arrival time $a_1 = -a^{*(1)}(0)$, where $a^{*(j)}(\psi)$ denotes the jth ($j \geq 1$) order derivative of $a^*(s)$ at $s = \psi$.

(ii) The server can only serve one customer at a time, and service time is exponentially distributed with mean rate μ.

(iii) Customers arriving at the system form a single waiting line and are served on a first-come first-served (FCFS) basis.

(iv) When the number of customers in the system reaches the maximum capacity of the system K, newly arriving customers are not permitted entry into the system with probability p (or probability $1 - p$) until the number of customers in the system decreases to the threshold value F (or threshold value $F - 1$), where $0 < F \leq K - 1$.

(v) The system is blocked for new arrivals if a customer arrives at a full system or is not allowed to enter the system. Then, the new arriving customer does not receive service and is lost.

(vi) A startup time is required before allowing customers to enter the system, which follows an exponential distribution with parameter γ. As long as the system is started successfully (with probability θ), the system behaves normally. Otherwise, if the system is started unsuccessfully (with probability $1 - \theta$), the customer is still not permitted to enter the system. The system is then restarted once again, the process of which is assumed to be successful.

(vii) The inter-arrival times, service times, and startup times are mutually independent of each other.

2.2. Notations

The following notations are used throughout this paper:

K: system capacity

F: threshold

p: probability that newly arriving customers are permitted to join the queue

$A(u)$: distribution function of inter-arrival time

$a(u)$: probability density function of inter-arrival time

$a^*(s)$: LST of inter-arrival time

$a^{*(l)}(s)$: the lth order derivative of $a^*(s)$ with respect to s

a_1: mean inter-arrival time

μ: mean service rate

γ: mean startup rate

θ: probability that the system is started successfully

$P_{0,n}(t)$: probability that there are n customers in the system at time t when the arrivals are not permitted to enter the system and the system is started up, where $n = 0, 1, \ldots, F$

$P_{1,n}(t)$: probability that there are n customers in the system at time t when the arrivals are permitted to enter the system and the system is in operation, where $n = 0, 1, \ldots, K - 1$

$P_{2,n}(t)$: probability that there are n customers in the system at time t when the arrivals are not permitted to enter the system and the system is in operation, where $n = F, F + 1, \ldots, K$

$P_{3,n}(t)$: probability that there are n customers in the system at time t when the startup is not successful at once and then the system is started again, where $n = 0, 1, \ldots, F$

$P_{0,n}$: steady-state probability that there are n customers in the system when the arrivals are not permitted to enter the system and the system is started up, where $n = 0, 1, \ldots, F$

$P_{1,n}$: steady-state probability that there are n customers in the system when the arrivals are permitted to enter the system and the system is in operation, where $n = 0, 1, \ldots, K - 1$

$P_{2,n}$: steady-state probability that there are n customers in the system at time t when the arrivals are not permitted to enter the system and the system is in operation, where $n = F, F + 1, \ldots, K$

$P_{3,n}$: steady-state probability that there are n customers in the system when the startup is not successful at once and then the system is started again, where $n = 0, 1, \dots, F$.

2.3. Practical application of the model

Consider a manufacturing system with one production machine in charge of assembling semi-finished products. In order to prevent system overload, a control policy is applied. When the available production space is exhausted, system closes off the entrance gate to avoid system overload. Newly arriving semi-finished products are not permitted to enter the system until the number of semi-finished products drops to threshold F with probability p or to threshold $F-1$ with probability $1-p$. Then the system requires startup time to open the entrance gate to permit semi-finished products entering the system. The startup process is not always perfect, and a restart is implemented. Thus, the queueing model studied in the paper can be applied to this manufacturing system.

3. Analysis of the model

We first set up the differential equations in steady state by treating remaining inter-arrival time as the supplementary variable. Let $N(t)$ be the number of customers in the system and $U(t)$ be remaining inter-arrival time for the next arrival at time t. The system state at time t is denoted by

$$Y(t) = \begin{cases} 0, & \text{if the arrivals are not permitted to enter the system and the system is started up,} \\ 1, & \text{if the arrivals are permitted to enter the system and the system is in operation,} \\ 2, & \text{if the arrivals are not permitted to enter the system and the system is in operation,} \\ 3, & \text{if the startup is not successful at once and then the system is started again.} \end{cases}$$

We define

$$\begin{aligned}
P_{0,n}(u,t)\mathrm{d}u &= \Pr\{N(t)=n,\ Y(t)=0,\ u < U(t) \le u+\mathrm{d}u\}, \quad u \ge 0, \quad n = 0,1,\dots,F, \\
P_{1,n}(u,t)\mathrm{d}u &= \Pr\{N(t)=n,\ Y(t)=1,\ u < U(t) \le u+\mathrm{d}u\}, \quad u \ge 0, \quad n = 0,1,\dots,K-1, \\
P_{2,n}(t) &= \Pr\{N(t)=n,\ Y(t)=2\}, \quad n = F, F+1, \dots, K, \\
P_{3,n}(u,t)\mathrm{d}u &= \Pr\{N(t)=n,\ Y(t)=3,\ u < U(t) \le u+\mathrm{d}u\}, \quad u \ge 0, \quad n = 0,1,\dots,F, \\
P_{0,n}(t) &= \int_0^\infty P_{0,n}(u,t)\mathrm{d}u, \quad n = 0,1,\dots,F, \\
P_{1,n}(t) &= \int_0^\infty P_{1,n}(u,t)\mathrm{d}u, \quad n = 0,1,\dots,K-1, \\
P_{3,n}(t) &= \int_0^\infty P_{3,n}(u,t)\mathrm{d}u, \quad n = 0,1,\dots,F.
\end{aligned}$$

In steady-state, we define

$$\begin{aligned}
P_{0,n} &= \lim_{t\to\infty} P_{0,n}(t), \quad n = 0,1,\dots,F, \\
P_{1,n} &= \lim_{t\to\infty} P_{1,n}(t), \quad n = 0,1,\dots,K-1, \\
P_{2,n} &= \lim_{t\to\infty} P_{2,n}(t), \quad n = F, F+1,\dots,K, \\
P_{3,n} &= \lim_{t\to\infty} P_{3,n}(t), \quad n = 0,1,\dots,F, \\
P_{0,n}(u) &= \lim_{t\to\infty} P_{0,n}(u,t), \quad n = 0,1,\dots,F, \\
P_{1,n}(u) &= \lim_{t\to\infty} P_{1,n}(u,t), \quad n = 0,1,\dots,K-1, \\
P_{3,n}(u) &= \lim_{t\to\infty} P_{3,n}(u,t), \quad n = 0,1,\dots,F,
\end{aligned}$$

and further define $P_{i,n}(u) = P_{i,n}a(u)$ for $i = 0, 3$ and $n = 0, 1, \ldots, F$.

Relating the states of the system at two consecutive time epochs, t and $t + \mathrm{d}t$, and taking $\mathrm{d}t \to 0$, we get the following steady-state differential equations:

$$0 = -\gamma P_{0,0} + \mu P_{0,1} \tag{3.1}$$
$$0 = -(\gamma+\mu)P_{0,n} + \mu P_{0,n+1}, \quad n = 1, 2, \ldots, F-2, \tag{3.2}$$
$$0 = -(\gamma+\mu)P_{0,F-1} + \mu P_{0,F} + \mu P_{2,F}, \tag{3.3}$$
$$0 = -(\gamma+\mu)P_{0,F} + \mu p P_{2,F+1}, \tag{3.4}$$
$$-\frac{d}{\mathrm{d}u}P_{1,0}(u) = \theta\gamma P_{0,0}a(u) + \gamma P_{3,0}a(u) + \mu P_{1,1}(u), \tag{3.5}$$
$$-\frac{d}{\mathrm{d}u}P_{1,n}(u) = -\mu P_{1,n}(u) + \theta\gamma P_{0,n}a(u) + \gamma P_{3,n}a(u) + P_{1,n-1}(0)a(u) + \mu P_{1,n+1}(u), \quad n = 1, 2, \ldots, F, \tag{3.6}$$
$$-\frac{d}{\mathrm{d}u}P_{1,n}(u) = -\mu P_{1,n}(u) + P_{1,n-1}(0)a(u) + \mu P_{1,n+1}(u), \quad n = F+1, F+2, \ldots, K-2, \tag{3.7}$$
$$-\frac{d}{\mathrm{d}u}P_{1,K-1}(u) = -\mu P_{1,K-1}(u) + P_{1,K-2}(0)a(u), \tag{3.8}$$
$$0 = -\mu P_{2,F} + \mu(1-p)P_{2,F+1}, \tag{3.9}$$
$$0 = -\mu P_{2,n} + \mu P_{2,n+1}, \quad n = F+1, F+2, \ldots, K-1, \tag{3.10}$$
$$0 = -\mu P_{2,K} + P_{1,K-1}(0), \tag{3.11}$$
$$0 = -\gamma P_{3,0} + (1-\theta)\gamma P_{0,0} + \mu P_{3,1}, \tag{3.12}$$
$$0 = -(\gamma+\mu)P_{3,n} + (1-\theta)\gamma P_{0,n} + \mu P_{3,n+1}, \quad n = 1, 2, \ldots, F-1, \tag{3.13}$$
$$0 = -(\gamma+\mu)P_{3,F} + (1-\theta)\gamma P_{0,F}. \tag{3.14}$$

Let us define the LST of $P_{1,n}(u)$ as follows:

$$P^*_{1,n}(s) = \int_0^\infty e^{-su} P_{1,n}(u)\mathrm{d}u, \quad s \ge 0,$$

so that

$$P_{1,n} = P^*_{1,n}(0) = \int_0^\infty P_{1,n}(u)\mathrm{d}u.$$

Multiplying equations (3.5)–(3.8) by e^{-su} on both sides and integrating over u, it yields

$$-sP^*_{1,0}(s) = \theta\gamma P_{0,0}a^*(s) + \gamma P_{3,0}a^*(s) + \mu P^*_{1,1}(s) - P_{1,0}(0), \tag{3.15}$$
$$(\mu - s)P^*_{1,n}(s) = \theta\gamma P_{0,n}a^*(s) + \gamma P_{3,n}a^*(s) + P_{1,n-1}(0)a^*(s) + \mu P^*_{1,n+1}(s) - P_{1,n}(0), n = 1, 2, \ldots, F, \tag{3.16}$$
$$(\mu - s)P^*_{1,n}(s) = P_{1,n-1}(0)a^*(s) + \mu P^*_{1,n+1}(s) - P_{1,n}(0), n = F+1, F+2, \ldots, K-2, \tag{3.17}$$
$$(\mu - s)P^*_{1,K-1}(s) = P_{1,K-2}(0)a^*(s) - P_{1,K-1}(0). \tag{3.18}$$

Now using equations (3.1) and (3.2), we have

$$P_{0,n} = \frac{1-\delta}{\delta^n}P_{0,0}, \quad n = 1, 2, 3, \ldots, F-1, \tag{3.19}$$

where $\delta = \mu/(\mu+\gamma)$.

From equations (3.3)–(3.4) and (3.9), we obtain

$$P_{0,F} = \frac{p(1-\delta)}{\delta^{F-1}(1-p+p\delta)} P_{0,0}, \tag{3.20}$$

$$P_{2,F} = \frac{(\gamma+\mu)(1-p)}{\mu p} P_{0,F}, \tag{3.21}$$

$$P_{2,F+1} = \frac{(1-\delta)}{\delta^{F}(1-p+p\delta)} P_{0,0}. \tag{3.22}$$

From equation (3.10), it yields

$$P_{2,n} = \frac{(1-\delta)}{\delta^{F}(1-p+p\delta)} P_{0,0}, \quad n = F+2, F+3, \ldots, K. \tag{3.23}$$

From equation (3.11), $P_{1,K-1}(0)$ can be written as

$$P_{1,K-1}(0) = \phi P_{0,0}, \tag{3.24}$$

where

$$\phi = \frac{\gamma}{\delta^{F-1}(1-p+p\delta)}.$$

It follows from equations (3.14) and (3.20) that

$$P_{3,F} = \frac{p(1-\delta)^2(1-\theta)}{\delta^{F-1}(1-p+p\delta)} P_{0,0}. \tag{3.25}$$

Combing equation (3.13) with equations (3.19) and (3.25) results in the following:

$$P_{3,n} = \frac{(1-\theta)(1-\delta)^2[p\delta + (1-p+p\delta)(F-n)]}{(1-p+p\delta)\delta^{n}} P_{0,0}, \quad n = 1, 2, \ldots, F-1. \tag{3.26}$$

From equation (3.12), we have

$$P_{3,0} = (1-\theta)\left[\frac{p\delta(1-\delta) + (1-p+p\delta)(F(1-\delta)+\delta)}{(1-p+p\delta)}\right] P_{0,0}. \tag{3.27}$$

Substituting $s = \mu$ into equation (3.18) and in conjunction with equation (3.24), we get

$$P_{1,K-2}(0) = \frac{\phi}{a^*(\mu)} P_{0,0}. \tag{3.28}$$

By letting $s = \mu$ in equations (3.16) and (3.17), we obtain

$$P_{1,n}(0) = \begin{cases} \frac{P_{1,n+1}(0) - \theta\gamma P_{0,n+1} a^*(\mu) - \gamma P_{3,n+1} a^*(\mu) - \mu P^*_{1,n+2}(\mu)}{a^*(\mu)}, & n = 0, 1, \ldots, F-1, \\ \frac{P_{1,n+1}(0) - \mu P^*_{1,n+2}(\mu)}{a^*(\mu)}, & n = F, F+1, \ldots, K-4, K-3. \end{cases} \tag{3.29}$$

To determine $P^*_{1,n}(\mu)$ $(2 \le n \le K-1)$, we first differentiate equation (3.18) j times with respect to s and set $s = \mu$ to obtain

$$P^{*(j-1)}_{1,K-1}(\mu) = -\frac{1}{j} \cdot \frac{a^{*(j)}(\mu)}{a^*(\mu)} \phi P_{0,0}, \tag{3.30}$$

where $P^{*(0)}_{1,K-1}(\mu) = P^*_{1,K-1}(\mu)$.

Differentiating equations (3.16)–(3.17) j times with respect to s and then substituting $s=\mu$, we obtain

$$P_{1,n}^{*(j-1)}(\mu)=\begin{cases}-\frac{1}{j}\left[\left(\theta\gamma P_{0,n}+\gamma P_{3,n}+P_{1,n-1}(0)\right)a^{*(j)}(\mu)+\mu P_{1,n+1}^{*(j)}(\mu)\right], & n=1,2,\ldots,F,\\ -\frac{1}{j}\left(P_{1,n-1}(0)a^{*(j)}(\mu)+\mu P_{1,n+1}^{*(j)}(\mu)\right), & n=F+1,F+2,\ldots,K-2,\end{cases}\tag{3.31}$$

where $j\geq 1$ and $P_{1,n}^{*(0)}(\mu)=P_{1,n}^{*}(\mu)$.

From equations (3.30)–(3.31), it yields

$$P_{1,n}^{*}(\mu)=\begin{cases}\frac{1}{\mu}\sum\limits_{i=1}^{F-n+1}\frac{(-\mu)^i}{i!}a^{*(i)}(\mu)\left(\theta\gamma P_{0,n+i-1}+\gamma P_{3,n+i-1}\right)+\frac{1}{\mu}\sum\limits_{i=1}^{K-n-1}\frac{(-\mu)^i}{i!}a^{*(i)}(\mu)P_{1,n-2+i}(0)\\ \quad-\frac{(-\mu)^{K-n-1}}{(K-n)!}\cdot\frac{a^{*(K-n)}(\mu)}{a^{*}(\mu)}\phi\cdot P_{0,0},\ n=2,3,\ldots,F,\\ \frac{1}{\mu}\sum\limits_{i=1}^{K-n-1}\frac{(-\mu)^i}{i!}a^{*(i)}(\mu)P_{1,n-2+i}(0)-\frac{(-\mu)^{K-n-1}}{(K-n)!}\cdot\frac{a^{*(K-n)}(\mu)}{a^{*}(\mu)}\phi\cdot P_{0,0},\ n=F+1,F+2,\ldots,K-1.\end{cases}\tag{3.32}$$

Inserting equation (3.32) into equation (3.29) gives us the following:

$$P_{1,n}(0)=\begin{cases}\frac{P_{1,n+1}(0)}{a^{*}(\mu)}-\gamma(\theta P_{0,n+1}+P_{3,n+1})-\frac{\mu}{a^{*}(\mu)}\left[\frac{1}{\mu}\sum\limits_{i=1}^{F-n-1}\frac{(-\mu)^i}{i!}a^{*(i)}(\mu)\left(\theta\gamma P_{0,n+i+1}+\gamma P_{3,n+i+1}\right)\right.\\ \left.\quad+\frac{1}{\mu}\sum\limits_{i=1}^{K-n-3}\frac{(-\mu)^i}{i!}a^{*(i)}(\mu)P_{1,n+i}(0)-\frac{(-\mu)^{K-n-3}}{(K-n-2)!}\cdot\frac{a^{*(K-n-2)}(\mu)}{a^{*}(\mu)}\phi\cdot P_{0,0}\right],\ n=0,1,2,\ldots,F-1,\\ \frac{P_{1,n+1}(0)}{a^{*}(\mu)}-\frac{\mu}{a^{*}(\mu)}\left[\frac{1}{\mu}\sum\limits_{i=1}^{K-n-3}\frac{(-\mu)^i}{i!}a^{*(i)}(\mu)P_{1,n+i}(0)-\frac{(-\mu)^{K-n-3}}{(K-n-2)!}\cdot\frac{a^{*(K-n-2)}(\mu)}{a^{*}(\mu)}\phi\cdot P_{0,0}\right],\\ n=F,F+1,\ldots,K-3.\end{cases}\tag{3.33}$$

Substituting $s=0$ into equations (3.15)–(3.18), it follows

$$P_{1,n}^{*}(0)=\begin{cases}\frac{1}{\mu}[P_{1,n-1}(0)-\gamma\sum\limits_{i=0}^{n-1}(\theta P_{0,i}+P_{3,i})],\ n=1,2,\ldots,F+1,\\ \frac{1}{\mu}[P_{1,n-1}(0)-\gamma\sum\limits_{i=0}^{F}(\theta P_{0,i}+P_{3,i})],\ n=F+2,F+3,\ldots,K-1.\end{cases}\tag{3.34}$$

Thus, using equations (3.19)–(3.20), (3.25)–(3.26) and (3.33), $P_{1,n}^{*}(0)$ can be expressed in terms of $P_{0,0}$.

Finally, we differentiate equations (3.15)–(3.18) with respect to s and then substitute $s=0$ to obtain

$$P_{1,0}^{*}(0)=-\gamma a^{*(1)}(0)(\theta P_{0,0}+P_{3,0})-\mu P_{1,1}^{*(1)}(0),\tag{3.35}$$

$$P_{1,n}^{*(1)}(0)=\frac{1}{\mu}\left(P_{1,n}^{*}(0)+\theta\gamma P_{0,n}a^{*(1)}(0)+\gamma P_{3,n}a^{*(1)}(0)+P_{1,n-1}(0)a^{*(1)}(0)+\mu P_{1,n+1}^{*(1)}(0)\right),\quad n=1,2,\ldots,F,\tag{3.36}$$

$$P_{1,n}^{*(1)}(0)=\frac{1}{\mu}\left(\sum_{i=n}^{K-1}P_{1,i}^{*}(0)+a^{*(1)}(0)\sum_{i=n-1}^{K-2}P_{1,i}(0)\right),\quad n=F+1,F+2,\ldots,K-2,\tag{3.37}$$

$$P_{1,K-1}^{*(1)}(0)=\frac{1}{\mu}\left(P_{1,K-1}^{*}(0)+P_{1,K-2}(0)a^{*(1)}(0)\right).\tag{3.38}$$

Using equations (3.36)–(3.38), $P_{1,1}^{*(1)}(0)$ can be evaluated using the recursive technique. Thus, equation (3.35) becomes

$$P_{1,0}^{*}(0)=-\left(a^{*(1)}(0)\gamma\sum_{i=0}^{F}(\theta P_{0,i}+P_{3,i})+\sum_{i=1}^{K-1}P_{1,i}^{*}(0)+a^{*(1)}(0)\sum_{i=0}^{K-2}P_{1,i}(0)\right).\tag{3.39}$$

Applying the normalization condition

$$\sum_{n=0}^{F} P_{0,n} + \sum_{n=0}^{K-1} P_{1,n}^{*}(0) + \sum_{n=F}^{K} P_{2,n} + \sum_{n=0}^{F} P_{3,n} = 1, \tag{3.40}$$

we can determine $P_{0,0}$. Then, $P_{0,n} \quad (1 \le n \le F)$can be obtained using equations (3.19)–(3.20), $P_{1,n}^{*}(0)$ $(0 \le n \le K-1)$ using equations (3.34) and (39), $P_{2,n}$ $(F \le n \le K)$ using equations (3.21)–(3.23), and $P_{3,n}$ $(0 \le n \le F)$ using equations (3.25)–(3.27).

3.1. Special cases

We outline two queueing models available in literature as special cases of our model by selecting the specific values of system parameters.

Case 1. When $p = 1$ and $\theta = 1$, the proposed model coincides with the model presented by [17].
Case 2. By setting $\theta = 1$ and considering the exponential inter-arrival time distribution, the proposed model can be reduced to the $(p,\, F)$-policy $M/M/1/K$ queue with exponential startup times studied by [24].

3.2. Computational algorithm

Based on the above analysis, an algorithm is formulated to compute the stationary probability distribution of the number of customers in the finite-buffer $GI/M/1$ system with starting failures under $(p,\, F)$-policy. Given the values of μ, γ, θ, p, F, K, and the LST of inter-arrival time distribution $a^{*}(s)$, we present a step-by-step procedure for the computational algorithm as follows:

Step 1: Set $P_{0,0} = 1$.
Step 2: For $n = 1, 2, \ldots, F$, compute $P_{0,n}$ using equations (3.19)–(3.20).
Step 3: For $n = F, F+1, \ldots, K$, compute $P_{2,n}$ using equations (3.21)–(3.23).
Step 4: For $n = 0, 1, \ldots, F$, compute $P_{3,n}$ using equations (3.25)–(3.27).
Step 5: For $n = 0, 1, \ldots, K-1$, compute $P_{1,n}(0)$ using equations (3.24), (3.28), (3.33).
Step 6: For $n = 1, 2, \ldots, K-1$, compute $P_{1,n}^{*}(0)$ using equation (3.34).
Step 7: Compute $P_{1,0}^{*}(0)$ using equation (3.39).
Step 8: Compute $\Omega = P_{0,0} + \sum_{n=1}^{F} P_{0,n} + \sum_{n=0}^{K-1} P_{1,n}^{*}(0) + \sum_{n=F}^{K} P_{2,n} + \sum_{n=0}^{F} P_{3,n}$.
Step 9: For $n = 0, 1, \ldots, F$, compute $P_{0,n} = P_{0,n}/\Omega$.
Step 10: For $n = 0, 1, \ldots, K-1$, compute $P_{1,n} = P_{1,n}^{*}(0)/\Omega$.
Step 11: For $n = F, F+1, \ldots, K$, compute $P_{2,n} = P_{2,n}/\Omega$.
Step 12: For $n = 0, 1, \ldots, F$, compute $P_{3,n} = P_{3,n}/\Omega$.

4. Optimal management policy

We develop a number of important system performance measures and establish a cost model. We also introduce a cost optimization problem with the aim of determining the optimal threshold, optimal capacity of the system, and optimal startup rate capable of minimizing the expected cost function per unit time.

4.1. System performance measures

The measurement of system performance is important in the construction of a cost model. Once the stationary distribution of the system size has been obtained, we develop various performance measures of the system as follows:

- Expected number of customers in the system:

$$L_S = \sum_{n=1}^{F} nP_{0,n} + \sum_{n=1}^{K-1} nP_{1,n} + \sum_{n=F}^{K} nP_{2,n} + \sum_{n=1}^{F} nP_{3,n}. \tag{4.1}$$

- Probability that the server is busy:

$$P_B = \sum_{n=1}^{F} P_{0,n} + \sum_{n=1}^{K-1} P_{1,n} + \sum_{n=F}^{K} P_{2,n} + \sum_{n=1}^{F} P_{3,n}. \tag{4.2}$$

- Probability that the system is started up:

$$P_S = \sum_{n=0}^{F} P_{0,n} + \sum_{n=0}^{F} P_{3,n}. \tag{4.3}$$

- Probability that the system is blocked:

$$P_L = \sum_{n=0}^{F} P_{0,n} + \sum_{n=F}^{K} P_{2,n} + \sum_{n=0}^{F} P_{3,n}. \tag{4.4}$$

4.2. Cost model and optimization

In the following, we define cost elements:
$C_h \equiv$ holding cost per unit time for each customer present in the system;
$C_b \equiv$ cost per unit time when the server is busy;
$C_s \equiv$ cost per unit time as the system is started up;
$C_l \equiv$ cost per lost customer;
$C_k \equiv$ cost per unit time for the capacity of the system;
$C_r \equiv$ cost per customer when the system is started by the mean startup rate γ.

Based on the system performance measures and cost elements, the expected cost function per unit time is given as follows:

$$TC(F, K, \gamma) = C_h L_S + C_b P_B + C_s P_S + C_l \lambda P_L + C_k K + C_r \gamma, \tag{4.5}$$

where F, K and γ are decision variables. We aim at determining the optimal threshold (F^*), optimal capacity of the system (K^*), and optimal startup rate (γ^*) to minimize the expected cost function in equation (4.5). It should be noted that in practical applications, the system capacity K is finite and bounded, i.e., $2 \le K^L \le K \le K^U$. Thus, the cost minimization problem can be expressed as

$$TC(F^*, K^*, \gamma^*) = \min_{\substack{0 < F \le K-1 \\ 2 \le K^L \le K \le K^U}} \left\{ C_h L_S + C_b P_B + C_s P_S + C_l \lambda P_L + C_k K + C_r \gamma \right\}. \tag{4.6}$$

Due to the highly non-linear and complex nature of the cost function in equation (4.5), it is difficult to obtain the analytical solutions of F^*, K^*, and γ^*. Thus, we solve the cost optimization problem in equation (4.6) numerically. One may notice that the decision variables F and K are discrete variables, and γ is a continuous variable. Suppose that the cost function TC is a convex function of the variable γ for each combination of F and K. We first focus on the optimization of γ for the given values of (F, K). To this end, we employ three heuristic algorithms to search for γ^* based on the golden-section search, bisection search, and gradient descent methods. Each method is described below:

Golden-section search (GS) method

The golden-section search (see [15]) is a technique for finding the extreme of a strictly unimodal function. The GS method only uses the values of the function and does not rely on the derivatives of the function. It maintains the values of the function for triples of points whose distances form a golden ratio, which is an efficient way to reduce progressively the interval locating the minimum.

Bisection search (BS) method

The BS method is a root-finding method, which is simple and robust. It bisects an interval into two subintervals, and then selects a subinterval in which a root exists. The bisection process is repeated until a required error tolerance is archived. Then, the midpoint serves as the approximate root of the function.

Gradient descent (GD) method

The GD method is by far the most popular optimization technique. It is an optimization algorithm based on the first-order derivative. The method starts with an initial solution and moves in the steepest descent direction until the (approximated) gradient is sufficiently small.

After γ^* has been found for each combination of F and K, we obtain (F^*,K^*) at the minimum expected cost per unit time. The procedure for searching the approximate optimum solution consists of the following steps:
Step 1: Initialize $K = K^L$, $F = 1$ and $(F^*, K^*, \gamma^*, TC^*) = (1, K^L, \infty, \infty)$.
Step 2: Implement *GS*, *BS* and *GD* methods to obtain the optimal value $\gamma^*(F, K)$ and its corresponding minimum cost $TC(F, K, \gamma^*(F, K))$.
Step 3: If $TC(F, K, \gamma^*(F, K)) < TC^*$, set $F^* = F$, $K^* = K$, $\gamma^* = \gamma^*(F, K)$, and $TC^* = TC(F, K, \gamma^*(F, K))$.
Step 4: Set $F = F + 1$ and repeat Steps 2–3 until $F = K - 1$.
Step 5: Set $K = K + 1$ and repeat Steps 2–4 until $K = K^U$.
Step 6: Output $(F^*, K^*, \gamma^*, TC(F^*, K^*, \gamma^*))$.

5. Sensitivity analysis

In this section, we provide some numerical examples to study the sensitivity of the system parameters on the expected number of customers in the system, the optimal values of (F, K), and the corresponding minimum expected cost. We consider three inter-arrival time distributions: exponential (M) distribution with $a^*(s) = \lambda/(\lambda + s)$, three-stage Erlang ($E_3$) distribution with $a^*(s) = (3\lambda/(3\lambda + s))^3$, and deterministic (D) distribution with $a^*(s) = e^{-s/\lambda}$, where λ is the mean arrival rate. For computational purposes, the values of system parameters are chosen to be $\lambda = 5.0$, $\mu = 4.0$, $\gamma = 3.0$, $\theta = 0.6$, and $p = 0.5$. Each system parameter is varied at a time while the others held constant.

5.1. Effects of the system parameters on L_S

We first present numerical results to explore the influences of various system parameters on the expected number of customers in the system (L_S). The results are exhibited in Figures 1–5, and we fix $F = 8$ and $K = 15$ in all the graphs. In Figure 1, we plot the impact of the mean arrival rate λ on L_S for different inter-arrival time distributions. It can be observed that L_S increases as λ increases for all inter-arrival time distributions. When λ ranges from 1.0 to 4.0, L_S in the case of the exponential distribution is higher than in the three-stage Erlang and deterministic distributions. Moreover, when λ is larger than 4.0, L_S in the case of the deterministic distribution is higher as compared to that in the exponential and three-stage Erlang distributions.

Figure 2 illustrates the effect of the mean service rate μ on L_S for different inter-arrival time distributions. As we can see in this figure, L_S shows a decreasing trend when μ increases. We also observe that L_S in the case of the deterministic distribution is higher than in the three-stage Erlang and exponential distributions when μ is between 1.0 and 5.0. Further, if μ is larger than 5.0, L_S in the case of the exponential distribution is higher as compared to that in the three-stage Erlang and deterministic distributions.

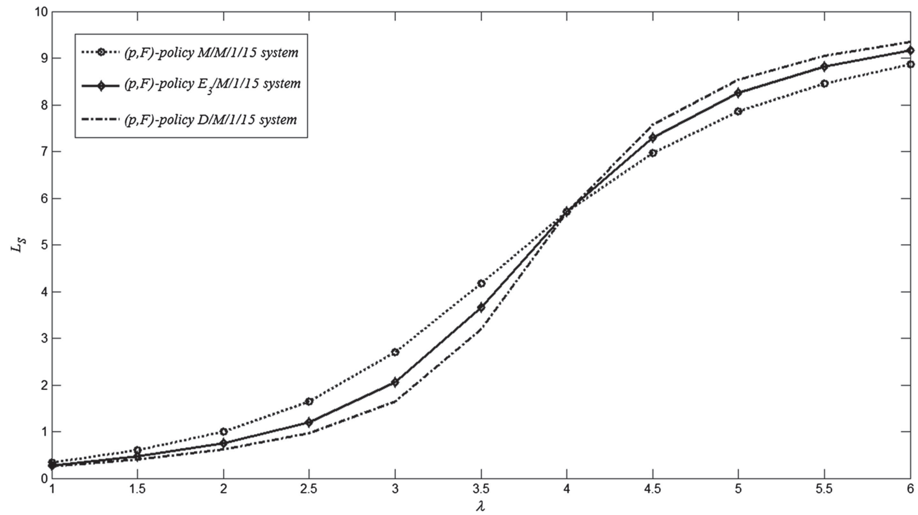

FIGURE 1. Effect of λ on L_S for different inter-arrival time distributions.

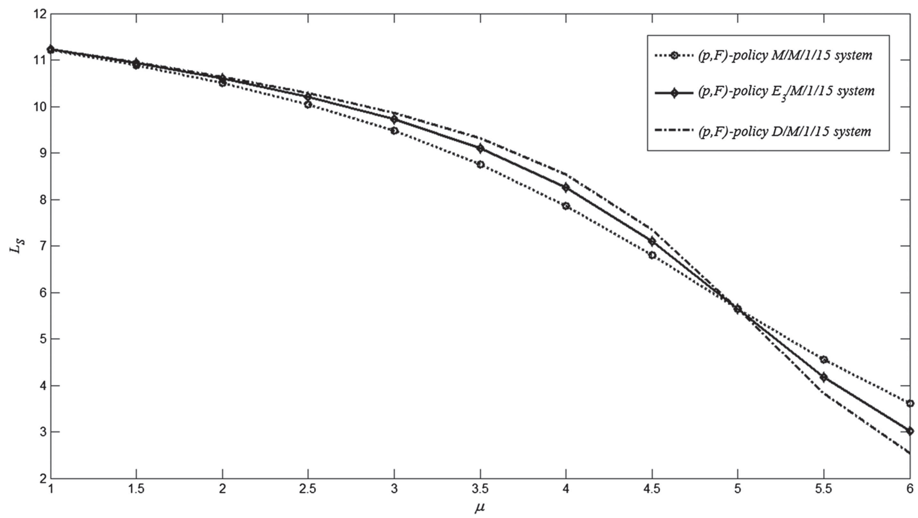

FIGURE 2. Effect of μ on L_S for different inter-arrival time distributions.

In Figures 3 and 4, we show the behavior of L_S for different inter-arrival time distributions when the mean startup rate γ and the probability θ vary. One can easily see that L_S increases with increasing the values of γ and θ. Moreover, the curve corresponding to the deterministic distribution is higher than the other two curves.

Finally, Figure 5 plots L_S versus the probability p for different inter-arrival time distributions. In Figure 5, it is found that L_S increases with increasing values of p. In addition, L_S in the case of the deterministic distribution is higher as compared to that in the exponential and three-stage Erlang distributions.

5.2. Effects of the system parameters on (F^*, K^*, γ^*) and $TC(F^*, K^*, \gamma^*)$

This subsection is to study the effects of various parameters on the optimal threshold (F^*), optimal capacity of the system (K^*), optimal mean startup rate (γ^*), and corresponding minimum expected cost $(TC(F^*, K^*, \gamma^*))$.

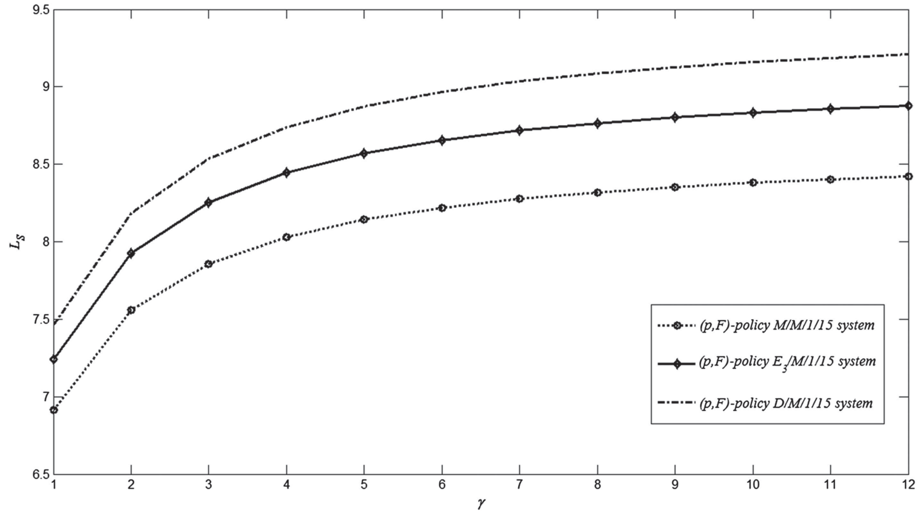

FIGURE 3. Effect of γ on L_S for different inter-arrival time distributions.

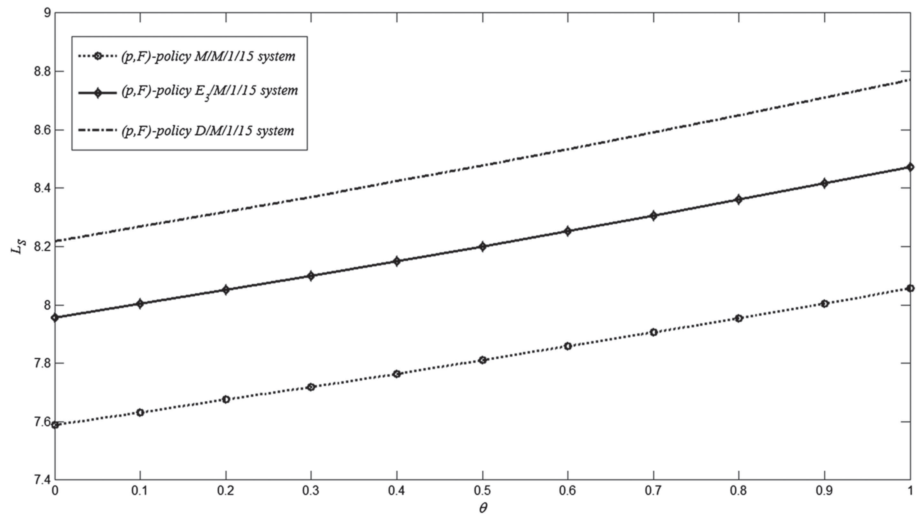

FIGURE 4. Effect of θ on L_S for different inter-arrival time distributions.

We consider the cost elements: $C_h = 8$, $C_b = 150$, $C_s = 250$, $C_l = 300$, $C_k = 3$, and $C_r = 100$. The lower and upper bounds of the system capacity are set as $K^L = 3$ and $K^U = 20$, respectively. In Tables 2–5, the numerical results of F^*, K^*, and γ^* are obtained using the solution procedure in Section 4.2. It is noted that "CPU Time" denotes the summation of execution times of the three algorithms.

In Table 2, we illustrate the effect of the mean arrival rate λ on (F^*, K^*, γ^*) and $TC(F^*, K^*, \gamma^*)$ for different inter-arrival time distributions. For the exponential inter-arrival time distribution, (F^*, K^*) increases when λ increases from 2.0 to 3.0 and decreases when λ increases from 3.0 to 6.0. For the three-stage Erlang and deterministic inter-arrival time distributions, we observe that (F^*, K^*) increases when λ increases from 2.0 to 4.0 and decreases when λ increases from 4.0 to 6.0. Moreover, γ^* increases as λ increases, and $TC(F^*, K^*, \gamma^*)$ shows an increasing trend with an increase in λ.

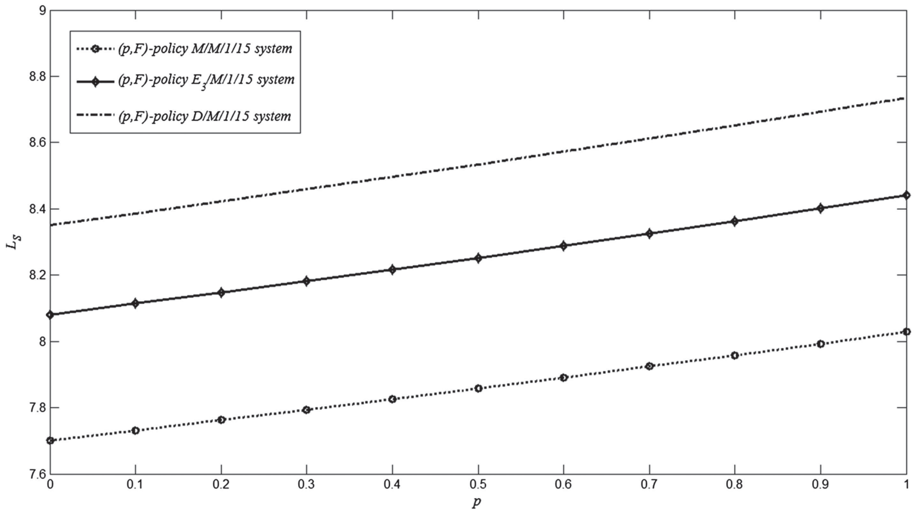

FIGURE 5. Effect of p on L_S for different inter-arrival time distributions.

TABLE 2. Effect of λ on the optimal values of (F, K, γ) and corresponding minimum expected cost for the (p, F)-policy $GI/M/1/K$ system with starting failures.

Inter-arrival time distribution	λ	2.0	3.0	4.0	5.0	6.0
Exponential distribution	(F^*, K^*)	(12, 13)	(19, 20)	(17, 20)	(12, 18)	(10, 17)
	γ^*	0.0360	0.1056	0.4874	0.8119	0.9948
	$TC(F^*, K^*, \gamma^*)$	129.243	217.500	424.337	741.429	1062.38
	CPU Time	32.662	21.882	20.947	20.757	20.773
Three-stage Erlang distribution	(F^*, K^*)	(8, 9)	(15, 16)	(17, 20)	(11, 17)	(10, 17)
	γ^*	0.0333	0.0721	0.4179	0.8219	0.9758
	$TC(F^*, K^*, \gamma^*)$	114.658	191.714	392.597	730.023	1057.36
	CPU Time	30.122	25.397	19.598	19.619	19.663
Deterministic distribution	(F^*, K^*)	(6, 7)	(13, 14)	(16, 19)	(11, 16)	(9, 16)
	γ^*	0.0267	0.0448	0.3929	0.8330	1.0169
	$TC(F^*, K^*, \gamma^*)$	106.381	176.680	373.431	724.761	1055.33
	CPU Time	37.789	28.101	19.012	18.775	19.021

Table 3 illustrates the effect induced by changes in the mean service rate μ on (F^*, K^*, γ^*) and $TC(F^*, K^*, \gamma^*)$ for different inter-arrival time distributions. As shown in Table 3, (F^*, K^*) increases under the exponential inter-arrival time distribution if μ ranges from 3.0 to 7.0. For the three-stage Erlang time distribution, (F^*, K^*) increases when μ ranges from 3.0 to 6.0. For the deterministic inter-arrival time distribution, (F^*, K^*) increases when μ ranges from 3.0 to 5.0. Under three inter-arrival time distributions, it is found that γ^* decreases as μ increases, and $TC(F^*, K^*, \gamma^*)$ tends to decrease with an increase in μ.

With respect to the changes in the probabilities θ and p, numerical results of (F^*, K^*, γ^*) and $TC(F^*, K^*, \gamma^*)$ are shown in Tables 4 and 5, respectively. From Table 4, it is clear that the larger θ is, the smaller F^*, K^* and γ^* are for different inter-arrival time distributions. In addition, $TC(F^*, K^*, \gamma^*)$ decreases as θ increases. In Table 5, under the exponential and three-stage Erlang inter-arrival time distributions, it shows that (i) F^*

TABLE 3. Effect of μ on the optimal values of (F, K, γ) and corresponding minimum expected cost for the (p, F)-policy GI/M/1/K system with starting failures.

Inter-arrival time distribution	μ	3.0	4.0	5.0	6.0	7.0
Exponential distribution	(F^*, K^*)	(8, 15)	(12, 18)	(18, 20)	(19, 20)	(19, 20)
	γ^*	0.8789	0.8119	0.6082	0.3191	0.1240
	$TC(F^*, K^*, \gamma^*)$	1030.66	741.429	462.161	289.571	212.442
	CPU Time	5.863	6.035	5.868	5.975	6.120
Three-stage Erlang distribution	(F^*, K^*)	(8, 15)	(11, 17)	(18, 20)	(19, 20)	(16, 17)
	γ^*	0.8670	0.8219	0.5221	0.1540	0.0581
	$TC(F^*, K^*, \gamma^*)$	1027.42	730.023	422.753	243.153	183.896
	CPU Time	5.989	5.979	6.147	6.302	8.205
Deterministic distribution	(F^*, K^*)	(7, 14)	(11, 16)	(18, 20)	(18, 19)	(13, 14)
	γ^*	0.9108	0.8330	0.4647	0.0840	0.0409
	$TC(F^*, K^*, \gamma^*)$	1026.07	724.761	399.489	219.980	168.533
	CPU Time	5.750	5.749	5.697	6.064	8.621

TABLE 4. Effect of θ on the optimal values of (F, K, γ) and corresponding minimum expected cost for the (p, F)-policy $GI/M/1/K$ system with starting failures.

Inter-arrival time distribution	θ	0.1	0.3	0.5	0.7	0.9
Exponential distribution	(F^*, K^*)	(14, 19)	(14, 19)	(13, 18)	(12, 18)	(11, 17)
	γ^*	0.8626	0.8281	0.8256	0.7837	0.7552
	$TC(F^*, K^*, \gamma^*)$	757.440	752.220	745.533	736.854	726.023
	CPU Time	6.667	5.902	5.836	5.818	6.007
Three-stage Erlang distribution	(F^*, K^*)	(13, 17)	(12, 17)	(12, 17)	(11, 16)	(10, 16)
	γ^*	0.8996	0.8802	0.8351	0.8227	0.7653
	$TC(F^*, K^*, \gamma^*)$	746.527	741.017	734.182	725.336	713.840
	CPU Time	6.035	5.998	6.002	6.036	5.991
Deterministic distribution	(F^*, K^*)	(12, 16)	(11, 16)	(11, 16)	(10, 15)	(9, 15)
	γ^*	0.9273	0.9070	0.8598	0.8479	0.7895
	$TC(F^*, K^*, \gamma^*)$	741.521	736.007	728.872	719.915	708.235
	CPU Time	5.731	5.719	5.721	5.743	5.732

decreases when p increases from 0.1 to 0.3; (ii) F^* is unchanged when p increases from 0.3 to 0.9; and (iii) K^* remains unchanged with increasing value of p. For the deterministic distribution, one can see that (i) (F^*, K^*) is not affected as p locates between 0.1 and 0.5; (ii) F^* decreases and K^* remains unchanged as p increases from 0.5 to 0.7; and (iii) (F^*, K^*) remains unchanged as p increases from 0.7 to 0.9. Moreover, we also observe that γ^* and $TC(F^*, K^*, \gamma^*)$ are insensitive to p under three inter-arrival time distributions.

Finally, in order to compare the effectiveness of the three algorithms, we plot the curve of the average CPU time among three inter-arrival time distributions for each algorithm versus different system parameters, as shown in Figure 6. It can be seen that the BS method is better than the GS method, but in some situations is worse than the GD method. However, the running time of the GD method is unstable. This is because that a poor initial solution leads to a poor efficiency. In general, a good initial solution is difficult to be determined. Therefore, the BS method is suggested to find γ^*.

TABLE 5. Effect of p on the optimal values of (F, K, γ) and corresponding minimum expected cost for the (p, F)-policy $GI/M/1/K$ system with starting failures.

Inter-arrival time distribution	p	0.1	0.3	0.5	0.7	0.9
Exponential distribution	(F^*, K^*)	(13, 18)	(12, 18)	(12, 18)	(12, 18)	(12, 18)
	γ^*	0.8051	0.8142	0.8120	0.8197	0.8074
	$TC(F^*, K^*, \gamma^*)$	741.371	741.470	741.429	741.384	741.337
	CPU Time	6.494	5.859	5.862	5.869	5.909
Three-stage Erlang distribution	(F^*, K^*)	(12, 17)	(11, 17)	(11, 17)	(11, 17)	(11, 17)
	γ^*	0.8142	0.8244	0.8219	0.8194	0.8168
	$TC(F^*, K^*, \gamma^*)$	729.978	730.061	730.023	729.982	729.938
	CPU Time	6.191	6.012	6.135	6.251	5.986
Deterministic distribution	(F^*, K^*)	(11, 16)	(11, 16)	(11, 16)	(10, 16)	(10, 16)
	γ^*	0.8382	0.8356	0.8330	0.8436	0.8409
	$TC(F^*, K^*, \gamma^*)$	724.574	724.667	724.761	724.675	724.578
	CPU Time	5.749	5.756	5.713	5.744	5.753

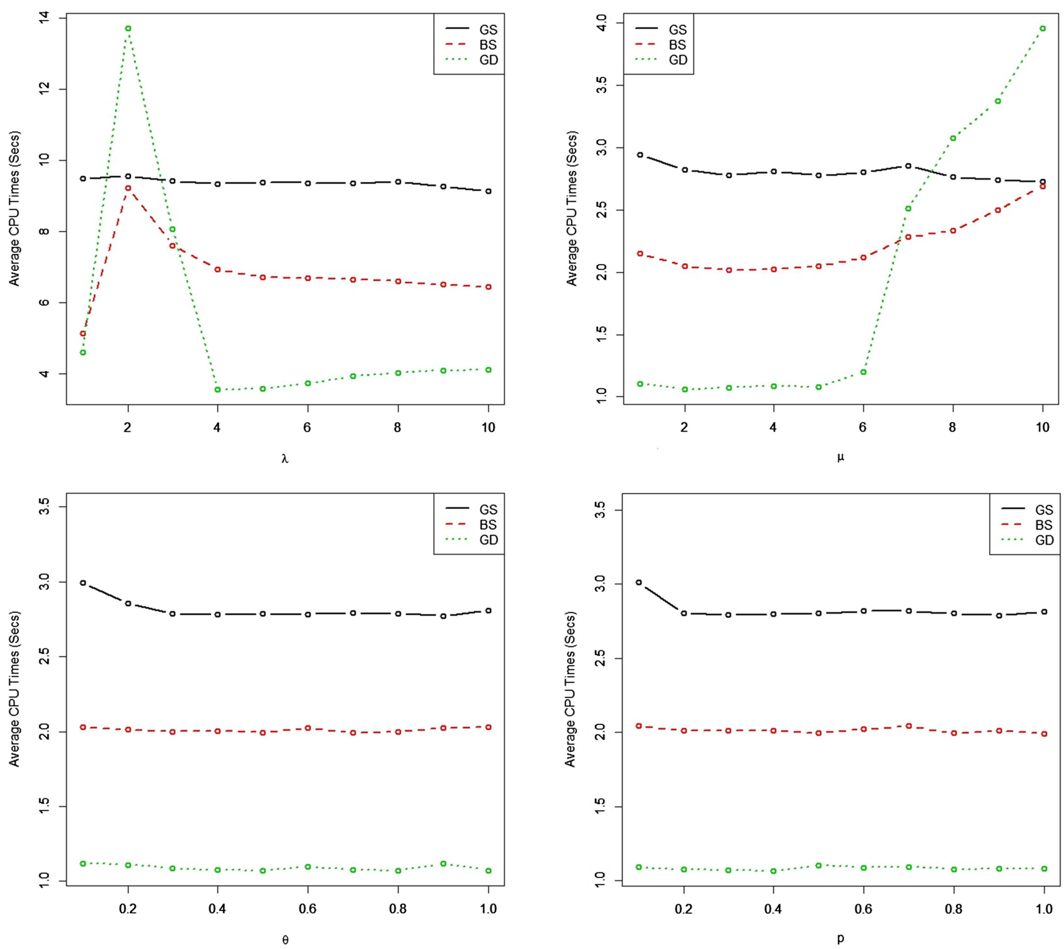

FIGURE 6. Average CPU times for the GS, BS, and GD methods.

6. Conclusions

This paper studied the steady-state analysis of the finite-buffer $GI/M/1$ system with starting failures under (p, F)-policy using the supplementary variable technique. A computational algorithm was proposed to facilitate the computation of the stationary probability distribution of the system size. We then developed a variety of measures by which to evaluate system performances. A cost optimization problem was formulated to minimize the expected cost per unit time. Sensitivity analysis was performed to evaluate the effects of various system parameters on L_S, (F^*, K^*, γ^*) and $TC(F^*, K^*, \gamma^*)$ with the help of numerical examples. In future, we can further extend the service process in our queueing model to the Markovian service process. Since this paper focused on the reliable server, another future work is to include an unreliable server.

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