RAIRO-Oper. Res. 54 (2020) 287–306
https://doi.org/10.1051/ro/2018102

RAIRO Operations Research
www.rairo-ro.org

INVENTORY AND PRICING DECISIONS FOR IMPERFECT QUALITY ITEMS WITH INSPECTION ERRORS, SALES RETURNS, AND PARTIAL BACKORDERS UNDER INFLATION

Aditi Khanna[1], Aakanksha Kishore[1], Biswajit Sarkar[2,*] and Chandra K. Jaggi[1]

Abstract. In this paper, an optimal replenishment inventory policy for imperfect quality items is presented with a selling price-dependent demand under inflationary conditions using a discounted cash flow (DCF) approach. Due to the presence of defectives in the system, all items go through a 100% inspection process. However, the screening process is also considered to be imperfect and involves errors, namely Type-I and Type-II. In addition, shortages are allowed and are partially backlogged. An optimal solution for the proposed model is derived by maximizing the expected profit function by jointly optimizing three decision variables: selling price, order quantity, and backorder level. To validate the theoretical results, a numerical example along with comprehensive sensitivity analysis is offered. The model has pertinence in industries like textiles, electronics, furniture, footwear, automobiles, and plastics.

Mathematics Subject Classification. 90B05.

Received April 12, 2018. Accepted November 10, 2018.

1. Introduction and literature overview

This section is subdivided into following sections as follows:

1.1. Motivation

In basic inventory analysis, numerous research articles have been based on the assumption that the different costs involved in the inventory system remain constant in the planning horizon. However, this assumption sounds impractical today as many countries experience high annual inflation rates. In consideration of this fact, the present study explores the vital role of DCF approach used for the accuracy of all the costs involved in the inventory system under inflationary conditions. A DCF approach not only permits an accurate recognition of the financial implication of various operating costs in the system, but also provides accuracy for all cash flows in the analysis. However, the financial decisions of the retailer can be inaccurate if the dynamism of demand with respect to pricing is left unaccounted for because it plays a vital role in setting the mark-up price. Generally,demand tends to decrease with an increase in selling price. In addition to the above described

Keywords. Inventory, Screening errors, Time value of money, Partial Backlogging, Selling price-dependent demand.

[1] Department of Operational Research, Faculty of Mathematical Sciences, New Academic Block, University of Delhi, Delhi 110007, India.

[2] Department of Industrial Engineering, Yonsei University, 50 Yonsei-ro, Sinchon-dong, Seodaemun-gu, Seoul, 03722, South Korea.

*Corresponding author: bsbiswajitsarkar@gmail.com

scenario of the classical inventory, many traditional economic order quantity (EOQ) models ignore the possibility of occurrence of imperfect quality items in the received batches. However, due to such realities as improper functioning of machines, design flaws, and imperfect sub parts, this assumption was soon replaced by the rational assumption that defective merchandise occurs in a production lot. This has led to vast improvement in the existing research and modernization in this field. Retailers now prefer to invest time, effort, and money to inspect a whole lot so as to reduce or eliminate defects by a considerable extent. However, it is again not viable to assume the screening process to be perfect due to unavoidable human errors, machine breakdowns, and so on, which leads to the emergence of critical inspection errors, namely Type-I and Type-II. These errors not only lessen the revenue, but also destroy customer trust. Also, to cut down on the inventory holding costs, it is by and large beneficial for the retailer to allow few shortages. However, it is difficult to completely fill the shortages, so partially backordered shortages are a more valid assumption. Consequently, the combination of all of these individually impactful parameters – inflation and time values, price-dependent demand, imperfect quality items, and screening errors – shows interesting results and brings the research closer to reality. This has motivated us to investigate the lesser-researched real-life problems and to develop an economic order quantity model allowing for all of these factors in order to fill the gap between EOQ and realism.

1.2. Literature review

1.2.1. Inflation and time value of money

Inflation and time value of money play crucial roles in the assessment of the cash flows, as they help provide a real estimate of the costs involved. In general, inflation is associated with price increases and decreases in the actual value of money. In the literature on inventory management, the initial studies related to constant inflation rates were those of Buzacott [4], Misra [19], and Misra [20]. Furthermore, Bose [3] investigated the inventory scenario for deteriorating items with two separate inflation rates, namely, the internal inflation rate for the company and the external inflation rate for the general economy. Other interesting research papers in this field are those of Moon and Lee [22] and Jaggi *et al.* [11]. The latter assumed demand to be inflation induced and allowed the model to account for completely backlogged shortages. Soon after, Jaggi and Khanna [12] analyzed the combined effect of permissible delay and credit-linked demand function on the procurement policy of a retailer under inflationary conditions. However, some of the key changes in demand occur due to variations in selling price. Other interesting research papers in this field include Abad and Jaggi [1] and Mondal *et al.* [21]. Sarkar and Moon [30] improved a production model with stochastic demand under inflation and time value of money. Youjun *et al.* [38] constructed an inventory model with advance sales and a price-dependent demand rate. Moreover, Uthayakumar and Palanivel [23] proposed a model to assist the retailer in minimizing the overall inventory costs by considering demand as a function of price and advertisement for non-instantaneously deteriorating items under inflation.

1.2.2. Imperfect quality items

Maintaining high standards of quality is a direct approach to long-term success through customer satisfaction. This need has been analyzed by many researchers in different ways. Porteus [25], Rosenblatt [26], Lee (1987), Schwaller [33], and Zhang and Gerchak [39] initiated vast research in the field of imperfect quality items by creating some significant inventory models. Soon after, Salameh and Jaber [28] extended conventional inventory models by incorporating the concept of imperfect quality items with a random defect percentage and known probability distribution function. Furthermore, Papachristos and Konstantaras [24] investigated the disposal time for imperfect items. In addition, Wee *et al.* [37] advanced the Salameh and Jaber [28] model by allowing for fully backlogged shortages and showed that increasing backordering costs decreased the rate of change in annual profit compared to the Salameh and Jaber [28] model. Eroglu and Ozdemir [7] further extended the research of Salameh and Jaber [28] by evaluating expected profit-per-unit-time using a renewal reward theorem and allowing for fully backlogged shortages. Further extension of the model was achieved by Maddah and Jaber [17], who introduced simpler expressions for the decision variables and also for the profit values. Soon after, Chang and Ho [5] applied a renewal reward theorem to obtain a closed-form solution for optimal lot size, backorder quantity, and net expected profit-per-unit-time by revisiting the work of Wee *et al.* [37]. Roy, Sana, and Chaudhuri [27] constructed an EOQ model that begins with shortages, and their defect percentage of items follows a uniform distribution function. Later, Sana [29] examined the production in an imperfect quality environment for a three-tier supply chain. Tayyab and Sarkar [36] extended this research direction with variable backorder rate under a multi-stage production system.

1.2.3. Inspection plans

Most of the above research articles did not consider any inspection plans for the separation of good- and bad-quality items. One significant work in this direction is that of Duffuaa [6], who proposed an inspection plan for critical components in the system. He also dealt with several misclassification errors during the inspection process. Adding to this, Maddah [18] developed an ordering model under an order overlapping scheme, and Khan *et al.* [14] assumed that the screening process is not free of error. Konstantaras *et al.* [16] considered that the proportion of defectives follows a learning curve that is either S-shaped or takes a power form. Recently, Hsu and Hsu [10] discussed cases when shortages are and are not allowed. For the latter, they used the order overlapping approach as used in Maddah [18] to explore both cases with imperfect quality and imperfect screening scenarios. In this direction, Sarkar and Saren [32] developed an improved inspection policy for the production model.

1.2.4. Inflation, imperfect quality items, inspection plans

Some researchers have contributed by simultaneously analyzing the features of inflation, imperfect quality, and inspection plans. Taheri-Tolgari *et al.* [35] presented an inventory model using a DCF approach for imperfect items and inspection errors under inflationary conditions in a finite time horizon. Furthermore, Jaggi *et al.* [13] investigated the impact of defective items in a lot size model under inflation when both demand and price vary with time. Soon after, Ghoreishi *et al.* [8] dealt with an economic production quantity model under inflationary conditions considering customer returns for non-instantaneous deteriorating items. Later, Ghoreishi *et al.* [9] extended the model by incorporating the dynamism of demand with respect to selling price and inflation. They assumed customer returns to be a function of demand and price. Recently, Sharma [34] conducted a survey on inventory models with inflation and Sarkar [31] introduced a three-stage inspection with some shelf-life products. Kim *et al.* [15] developed a production model under a random controllable lead time.

1.2.5. Novelty in this model

The core problem considered in the current model is that of retail industries facing dynamism in the estimation of all inventory-related costs due to the rise and fall of inflation rates. Additionally, pricing strategies are greatly affected by the fluctuation in demand patterning and the price-sensitive nature of demand. The current study considers price sensitivity in demand along with inflationary conditions. Another challenge faced by retail industries is the presence of defectives in the received lot. To survive in today's cut-throat competitive marker, it becomes profitable for retailers to employ an inspection process to assess the quality of all products before selling them to customers. However, the separation of good- and bad-quality items is also prone to crucial inspection errors, Type-I and Type-II. Another key problem encountered during the process of reducing and eliminating defects is that of shortages, which negatively impact speed to market, customer service, and security. Here, we presume partially backlogged shortages where only some customers are satisfied as per demand, while the rest of the demand is lost. The model is explored to identify the optimal replenishment and backorder quantities in conjunction with the optimal selling price.

2. Model development

To develop this model, some basic assumptions and notation are considered in the following subsection.

2.1. Assumptions

The present model was formulated under the following fundamental assumptions.

(1) Generally, demand is observed to have inverse relationship with mark-up price. So, demand rate $D(p)$ is a function of selling price and of form, $D(p) = bp^{-e}$ where b and e are positive constants.
(2) Screening process is not assumed to be perfect. Thus, it results in Type-I error (salvaging of good products as defectives by mistake) and Type-II error (selling of defectives as good products by mistake). Screening rate is set much higher than the demand rate as aim is to supply only good quality products to end customers.
(3) As a result of Type-II error, some of the defectives get sold to the customers as perfect items by mistake resulting in Sales Returns due to customer dissatisfaction. Defect returns along with accumulated defectives are salvaged at a cheaper price at the end of Inspection process. These defect returns are assumed to occur continuously over complete cycle length beginning and ending at the completion of inspection process.
(4) Discounted Cash Flow (DCF) approach permits an accurate recognition of the financial implication of various operating costs in the system Thus, to get the exact timing of cash flows; this approach is used for better accuracy of results when inflation and time value of money are under consideration as in Jaggi et. al. [11] paper.
(5) Shortages are allowed at the beginning of the inventory cycle and are partially backlogged with backlogging parameter δ. The backordering rate of unsatisfied demand is taken as $\mathrm{e}^{-\delta(t_1-t)}$, an exponential function of shortage building time up to the next replenishment, where t_1 is the waiting time for customers and $t \in [0, t_1]$.
(6) Time period is infinite and lead time is insignificant.

2.2. Notation

The following notations are used for convenience in the paper.

Parameters

$D(p)$	demand rate in units per unit time
b, e	constant demand parameters; $b > 0; e > 0$
λ	screening rate in units per unit time, $\lambda > D(p)$
x	proportion of imperfect items (random variable with known probability density function)
q_1	proportion of Type-I imperfection errors (random variable with known probability density function)
q_2	proportion of Type-II imperfection errors (random variable with known probability density function)
t_1	shortage building time
t_2	shortage fulfillment time
t_3	screening time
T'	inventory down time
T	cycle length
K_0	order cost per cycle at time $t = t_1$
c_0	purchase cost per item ($/item) at time $t = t_1$
s_0	screening cost per item ($/item) at time $t = t_1$
v_0	salvage cost ($<s$) ($/item) at time $t = t_1$
u_0	disposal cost ($/item) at time $t = t_1$
δ	backlog parameter
d	discount rate, representing time value of money
i	inflation rate
R	$d - i$; net discount rate of inflation, a constant
C_{a0}	cost of committing Type-I error ($/item) at time $t = t_1$
C_{r0}	cost of committing Type-II error ($/item) at time $t = t_1$
C_{B0}	shortage cost at time $t = t_1$
C_{L0}	lost sale cost at time $t = 0$

h_0	inventory carrying cost per unit per unit time at time $t = t_1$
h_1	holding cost per defect return per unit time at time $t = t_1$
z_1	maximum on-hand inventory in units when the inspection process ends
z_2	maximum on-hand inventory in units after partial fulfillment of shortages
p.d.f.	probability distribution function

Decision variables

y	order lot size for each cycle (in units)
B	maximum backorder level allowed
p_0	selling price at time $t = t_1$

Functions

$K(t)$	ordering cost per cycle at time t
$C(t)$	purchasing cost per item ($/item) at time t
$S(t)$	screening cost per item ($/item) at time t
$V(t)$	salvage cost ($<s$) ($/item) at time t
$U(t)$	disposal cost ($/ item) at time t
$E_1(t)$	cost of committing Type-I error ($/item) at time t
$E_2(t)$	cost of committing Type-II error ($/item) at time t
$C_B(t)$	shortage cost at time t
$h(t)$	inventory carrying cost per unit per unit time at time t
$P(t)$	selling price at time t
$f(x)$	probability distribution function of defective items
$f(q_1)$	probability distribution function of Type-I error
$f(q_2)$	probability distribution function of Type-II error
$T.C.$	retailer's total cost
$T.R.$	retailer's total revenue
$\pi\,(y, B, p)$	retailer's present total profit, which is a function of three variables: y, B, and p.
$E\,[\pi\,(y, B, p)]$	retailer's expected total profit function

Optimal values

$T*$	optimal cycle length
$B*$	optimal backorder quantity
$y*$	optimal order quantity per cycle
$p*$	optimal selling price
$E\,[\pi\,(y^*, B^*, p^*)]$	retailer's expected optimal totalprofit function

3. Model formulation

In this section, a mathematical model relevant to the above exposed problem, assumptions and description has been formulated. An inventory problem is analyzed in the study in which $x\%$ of defectives exist in every received lot y, where defect proportion x is a random variable with known p.d.f. as $f(x)$. Further, to deliver only good quality products to customers, an inspection process is performed on the ordered lot (y) at a screening rate λ. This results in the bifurcation of the lot (y) into imperfect items (xy) and perfect items $(1 - x)y$. However, screening errors namely Type-I and Type-II errors are generated by the inspection inspector with proportions as $q_1 = P_r$ (*items inspected as defective* | *non- defective item*) and $q_2 = P_r$ (*items not inspected as defective* | *defective item*) $(0 < q_1 < q_2 < 1)$. The p.d.f. of q_1 and q_2 be $f(q_1)$ and $f(q_2)$ respectively. For mathematical simplicity, it is assumed that q_1 and q_2 are independent of defect proportion x. So, all the items involving screening errors are determined inter-dependently in terms of q_1, q_2, and y. Due to Type-I error, some perfect items amounting to total $(1-x)q_1y$ units are sold at a cheaper price and leaving $(1-x)(1-q_1)y$ units to be sold at mark-up price. Whereas, due to Type-II error, some imperfect items are sold as perfect quality viz. (xq_2y)

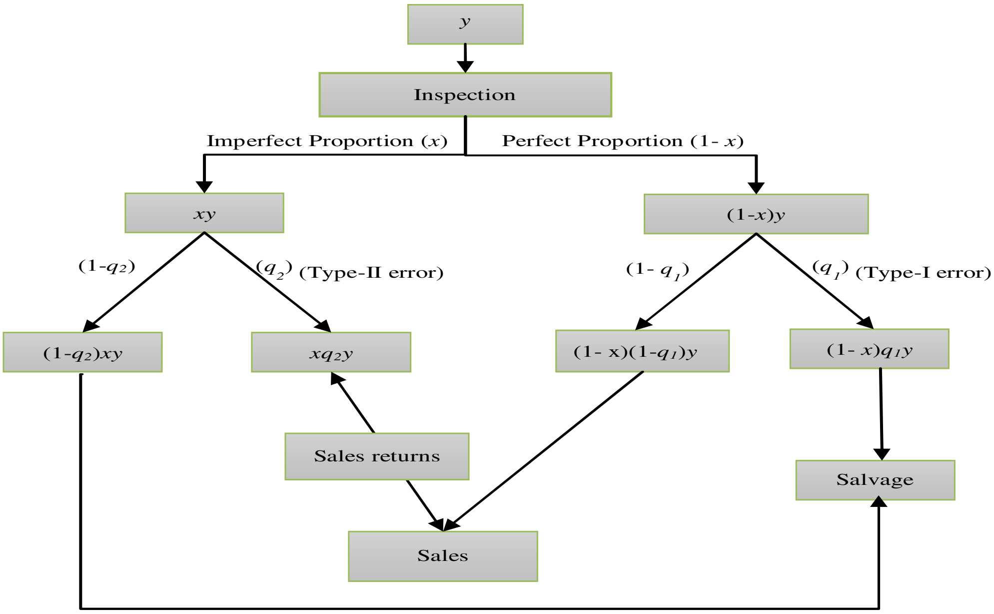

FIGURE 1. Sequence of events in the inventory cycle.

which come back as sales return later while $(1-q_2)xy$ units remain unaffected by the error and get salvaged correctly. Finally, a total of $(1-x)(1-q_1)y+(xq_2y)$ units are sold as perfect items and $(1-x)q_1y+(1-q_2)xy$ units are sold as defective items in the current inventory scenario. A schematic of the above problem description is shown in Figure 1.

The inventory behavior observed from the above mentioned flow of events is exhibited in Figure 2. Here, the inventory system is followed by shortages from the previous cycle. These shortages start building up at the beginning of the inventory cycle i.e. from time 0 to A_1 and these are assumed to be partially backlogged with backlogging parameter δ. We also presume that the backlogging rate of unsatisfied demand is an exponential function of waiting time up to next refill during the stock out period. During the screening process, shortages are met (from A_1 to A_2) parallel to the demand for the perfect items only. So, to elude them during the screening period (from A_2 to A_3), the rate of screening is presumed to be greater than the demand rate: $(\lambda > D(p))$. After the shortages have been eliminated (till time point A_2), screening process continues and ends up in salvaging of the accumulated defectives (say B_1) along with sales returns (say B_2) at time point A_3. All the sales returns are assumed to get accumulated over a complete cycle length T beginning and ending at the completion of inspection process i.e. A_3. After A_3, the inventory depletes to zero as per the demand rate till time point A_4.

From Figure 1, the total outcome of perfect items ready for sale after the integration of inspection errors have been summed up as $(1-x)(1-q_1)y+(xq_2y)$ units which are first used to satisfy the demand and then the remaining perfect items are further used to eliminate shortages. So, from Figure 2, the shortage elimination time can be calculated as:

$$t_2 = \frac{B}{\lambda\left[(1-x)\left(1-q_1\right)+xq_2\right]-D\left(p\right)} = \frac{B}{A\lambda}, \tag{3.1}$$

where

$$A = (1-x)\left(1-q_1\right)+xq_2-\frac{D\left(p\right)}{\lambda}\cdot \tag{3.2}$$

A comprises random variables x, q_1, q_2 and so is also a random variable.

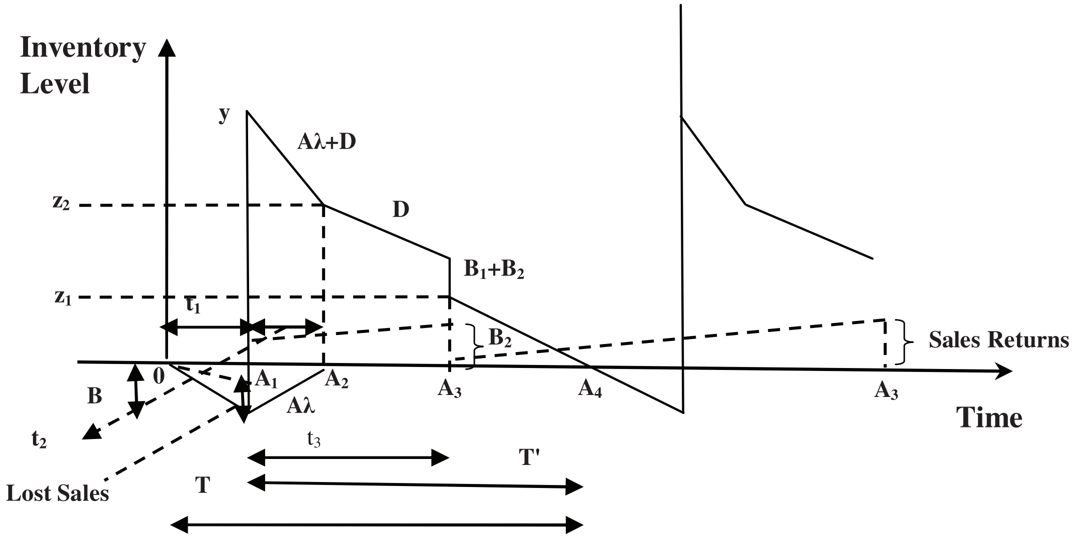

FIGURE 2. Behavior of the inventory system.

Hence,

$$E[A] = (1 - E[x])(1 - [q_1]) + E[x][q_2] - \frac{D(p)}{\lambda} \tag{3.3}$$

t_2 is comprised of random variables x, q_1, q_2 and so is also a random variable. Hence, by using equations (3.1) and (3.3), we get:

$$E[t_2] = \frac{1}{E[A]}\frac{B}{\lambda} = \frac{B}{\lambda\{(1 - E[x])(1 - [q_1]) + E[x][q_2]\} - D(p)}, \tag{3.4}$$

Also, the total screening time can be calculated as

$$t_3 = \frac{y}{\lambda}\cdot \tag{3.5}$$

As the imperfect screening process results in $(1 - q_2)xy$ and $(1 - x)(1 - q_1)xy$ actual defectives and non-defectives, respectively. Thus, the total items sold at the selling price $= [(1 - x)(1 - q_1) + xq_2]\, y$

Therefore, the cycle length is given as

$$T = \frac{[(1 - x)(1 - q_1) + xq_2]\, y}{D(p)} \tag{3.6}$$

T is a combination of random variables, x, q_1, q_2, and so is also a random variable. Therefore, the expected value of cycle length T is given by

$$E[T] = \frac{\{(1 - E[x])(1 - [q_1]) + E[x][q_2]\}\, y}{D(p)}\cdot \tag{3.7}$$

After the end of the imperfect screening process, the total count of defectives includes the sum of actual and falsely classified defectives, calculated as

$$B_1 = [x(1 - q_2) + q_1(1 - x)]\, y. \tag{3.8}$$

Out of the total items sold to customers, wrongly classified perfect items lead to Sales Returns, denoted as

$$B_2 = xq_2y. \tag{3.9}$$

Since, the backlogging rate of unsatisfied demand is an exponential function of backorder building time up to the next refill during the stock out period. The following differential equation shows the change in back-ordering level at any given time:

$$\frac{\mathrm{d}B(t)}{\mathrm{d}t} = -\left[-D(p)\,\mathrm{e}^{-\delta(t_1-t)}\right]; \quad 0 \le t \le t_1. \tag{3.10}$$

Using initial condition $B(0) = 0$, equation (3.10) is solved as

$$B(t) = \frac{D(p)}{\delta}\left[\mathrm{e}^{-\delta(t_1-t)} - \mathrm{e}^{-\delta t_1}\right]. \tag{3.11}$$

Again using boundary condition $B(t_1) = B$, equation (3.11) is solved as

$$t_1 = -\frac{1}{\delta}\ln\left(1 - \frac{B\delta}{D(p)}\right)\cdot \tag{3.12}$$

Due to such reasons as limited shelf space and poor catalog management, customers are not always able to obtain the desired products; this leads to loss of gross profit and sometimes customer loss as well. Here, we presume that all customers are not willing to wait for their orders; thus, the total lost sales are estimated as

$$L(t) = \int_0^{t_1} D(p)\left(1 - \mathrm{e}^{-\delta(t_1-t)}\right)\mathrm{d}t = D(p)\left[t_1 - \frac{1}{\delta}\left(1 - \mathrm{e}^{-\delta t_1}\right)\right]\cdot \tag{3.13}$$

As evident from Figure 2, y units are procured at time A_1, and the whole lot goes through a 100% screening process at the rate of λ units per unit time. From time A_1 to A_2, a portion of good-quality items fulfill the demand and the rest are used to eliminate backorders. The following differential equation shows the change in inventory level at any given time during (A_1, A_2).

$$\frac{\mathrm{d}I_1(t)}{\mathrm{d}t} = -\left[A\lambda + D(p)\right]; \quad t_1 \le t \le t_1 + t_2. \tag{3.14}$$

Using initial condition $I_1(t_1) = y$, equation (3.14) is solved as

$$I_1(t) = y - \left[A\lambda + D(p)\right](t - t_1). \tag{3.15}$$

Again, using boundary condition $I_1(t_1 + t_2) = z_2$, equation (3.15) is solved as

$$z_2 = y - \left[A\lambda + D(p)\right]t_2 = y - \left[A\lambda + D(p)\right]\frac{B}{A\lambda} \tag{3.16}$$

$$z_2 = y - \frac{B\left[xq_2 + (1-x)(1-q_1)\right]}{\left[xq_2 + (1-x)(1-q_1)\right]\lambda - D(p)}\cdot \tag{3.17}$$

From time A_2 to A_3, maximum shortage level allowed has been partially fulfilled while the remaining shortages have been completely lost by A_2. So, all the perfect items coming out of the inspection process are used to satisfy current demand only during this period. Also, all the accumulated defectives (B_1) along with sales returns (B_2) get salvaged at the end of inspection process i.e. at A_3. Sales returns occur as a consequence of customer frustration as they have received faulty products in the name of good quality products by mistake during the inspection process. Here, all the sales returns (B_2) are assumed to get accumulated over a complete cycle length

T beginning and ending at the completion of inspection process i.e. A_3. The following differential equation shows the change in inventory level at any given time in time interval (A_2, A_3).

$$\frac{\mathrm{d}I_2(t)}{\mathrm{d}t} = -D(p); \quad t_1 + t_2 \leq t \leq t_1 + t_3. \tag{3.18}$$

Using initial condition $I_2(t_1 + t_2) = z_2$, equation (3.18) is solved as

$$I_2(t) = z_2 - D(p)(t - t_1 - t_2). \tag{3.19}$$

Again, using boundary condition $I_2(t_1 + t_3) = z_1 + B_1 + B_2$, equation (3.19) is solved as

$$z_1 = z_2 - B_1 - B_2 - [D(p)(t_3 - t_2)] \tag{3.20}$$

$$z_1 = y - \frac{B[xq_2 + (1-x)(1-q_1)]}{[xq_2 + (1-x)(1-q_1)]\lambda - D(p)} - [x(1-q_2) + q_1(1-x)]y - xq_2 y$$
$$- \left[D(p)\left(\frac{y}{\lambda} - \frac{B}{\lambda[(1-x)(1-q_1) + xq_2] - D(p)}\right)\right]. \tag{3.21}$$

After the end of the screening process at A_3, the inventory decreases only as per the demand rate till it finally reaches zero at point A_4. So, in one complete cycle, the demand is satisfied for a total duration of T'. At time point A_3, there is selling off of defectives (B_1) along with sales returns (B_2) at a reduced price in a secondary market. Moreover, the financial impact of sales returns is visible in total cost also (via Type II error cost) as these are entertained with full price refunds in the study. The following differential equation shows the change in inventory level at any given time during period (A_3, A_4).

$$\frac{\mathrm{d}I_3(t)}{\mathrm{d}t} = -D(p); \quad t_1 + t_3 \leq t \leq t_1 + T'. \tag{3.22}$$

Using initial condition $I_3(t_1 + t_3) = z_1 + B_1 + B_2$, equation (3.22) is solved as

$$I_3(t) = z_1 + B_1 + B_2 - D(p)(t - t_1 - t_3). \tag{3.23}$$

Again, using boundary condition $I_3(t_1 + T') = 0$, equation (3.23) is solved as

$$T' = \frac{z_1 + B_1 + B_2}{D(p)} + t_3. \tag{3.24}$$

3.1. Relevant costs

The various cost components integrated in the model are the set-up cost, purchase cost, screening cost, Misclassification error 1 cost, Misclassification error 2 cost, disposal cost, inventory carrying cost, backordering cost, and lost sale cost. Total revenue includes three components: the sale of perfect items, the loss of defect returns, and the sale of scrap items. Because shortages of previous cycles start building up at time $t = 0$, and orders and purchases are made at time $t = t_1$, the effect of inflation can be observed on all of these costs:

$$K(t) = K_0\mathrm{e}^{it};\ C(t) = c_0\mathrm{e}^{it};\ S(t) = s_0\mathrm{e}^{it};\ E_1(t) = c_{a_0}\mathrm{e}^{it};\ E_2(t) = c_{r_0}\mathrm{e}^{it};$$
$$V(t) = v_0\mathrm{e}^{it};\ U(t) = u_0\mathrm{e}^{it};\ C_B(t) = C_{B_0}\mathrm{e}^{it};\ L(t) = C_{L_0}\mathrm{e}^{it};\ h(t) = h_0\mathrm{e}^{it};\ P(t) = p_0\mathrm{e}^{it};$$

Therefore, using the DCF approach, the present worth of various cost components for the first replenishment cycle is estimated as follows:

(a) The present ordering cost is

$$C_a = K_0\mathrm{e}^{-Rt_1}. \tag{3.25}$$

(b) The present purchasing cost is

$$C_P = c_0 y \mathrm{e}^{-Rt_1}. \tag{3.26}$$

(c) So as to deliver only good-quality products to customers, the screening process begins at time $A_1$and continues to time A_3. Thus, the present screening cost is given as

$$C_{\mathrm{scr}} = s_0 y \mathrm{e}^{-Rt_3} \mathrm{e}^{-Rt_1}. \tag{3.27}$$

(d) As a result of Type-I error, a fraction $(1-x)q_1 y$ of the total non-defectives $(1-x)y$ are wrongly classified as defectives, thereby causing monetary loss to the firm by directly losing an opportunity to increase the sales of good-quality items. Thus, the present Type-I error cost is obtained as

$$C_{\mathrm{e}_1} = c_{a_0} (1-x) q_1 y \mathrm{e}^{-Rt_3} \mathrm{e}^{-Rt_1}. \tag{3.28}$$

(e) With the emergence of Type-II error, there are more penalty and goodwill losses to the retailer compared to financial losses because a fraction (xq_2y) of the total defectives (xy) is wrongly classified as non-defectives, resulting in defect returns. The present Type-II error cost is thus determined as

$$C_{\mathrm{e}_2} = c_{r_0} x q_2 y \mathrm{e}^{-RT'} \mathrm{e}^{-Rt_1}. \tag{3.29}$$

(f) The defectives collected after an imperfect inspection process number $((1-q_2)xy + xq_2y + (1-x)q_1y)$, which include the actual defectives, sales returns, and falsely classified defectives, respectively. The present cost incurred by the retailer for disposing of these items is calculated as

$$C_d = u_0 \left[x + q_1 (1-x)\right] y \mathrm{e}^{-R(t_1+t_3)}. \tag{3.30}$$

(g) When the retailer runs out of stock for any particular product in demand, a shortage in the inventory system occurs, and a back-order cost is incurred. So, the present shortage cost is calculated as $C_{\mathrm{sho}} = C_{B_0} \int_0^{t_1+t_2} B \mathrm{e}^{-Rt} \mathrm{d}t$, i.e.

$$C_{\mathrm{sho}} = C_{B_0} \frac{B}{R} \left[1 - \mathrm{e}^{-R(t_1+t_2)}\right]. \tag{3.31}$$

(h) Because all shortages are unable to be filled, there is a penalty cost to the retailer due to the loss of a few sales. The present lost sales cost is estimated as

$$C_{\mathrm{LS}} = C_{L_0} L(t) \mathrm{e}^{-Rt_1} = C_{L_0} D(p) \left[t_1 - \frac{1}{\delta}\left(1 - \mathrm{e}^{-\delta t_1}\right)\right]. \tag{3.32}$$

(i) Inventory holding cost is defined as the cost incurred in carrying all non-defective and defective items plus the sales returns. Due to customer frustration and quality dissatisfaction, wrongly classified defectives sold to customers re-enter the system continuously until the end of screening process. Thus, the present holding cost of all the items held in inventory is given as:

$$
\begin{aligned}
C_h &= h_0 \int_{t_1}^{t_1+T'} I(t)\,\mathrm{e}^{-Rt}\mathrm{d}t + h_1 \int_{t_1+t_3}^{T+t_1+t_3} B_2 \mathrm{e}^{-Rt}\mathrm{d}t \quad \text{i.e.} \\
&= h_0 \left(\int_{t_1}^{t_1+t_2} I_1(t)\mathrm{e}^{-Rt}\mathrm{d}t + \int_{t_1+t_2}^{t_1+t_3} I_2(t)\mathrm{e}^{-Rt}\mathrm{d}t + \int_{t_1+t_3}^{t_1+T'} I_3(t)\mathrm{e}^{-Rt}\mathrm{d}t \right) + h_1 \int_{t_1+t_3}^{T+t_1+t_3} xq_2y\mathrm{e}^{-Rt}\mathrm{d}t \\
&= \frac{h_0}{R} \left\{ y + [A\lambda + D(p)]\, t_1 \right\} \left(\mathrm{e}^{-Rt_1} - \mathrm{e}^{-R(t_1+t_2)} \right) \\
&\quad - \frac{h_0}{R^2} [A\lambda + D(p)] \left\{ \mathrm{e}^{-Rt_1} (1 + Rt_1) - \mathrm{e}^{-R(t_1+t_2)} [1 + R(t_1+t_2)] \right\} \\
&\quad + \frac{h_0}{R} [z_2 + D(p)(t_1+t_2)] \left(\mathrm{e}^{-R(t_1+t_2)} - \mathrm{e}^{-R(t_1+t_3)} \right) \\
&\quad - \frac{h_0}{R^2} D(p) \left\{ \mathrm{e}^{-R(t_1+t_2)} [1 + R(t_1+t_2)] - \mathrm{e}^{-R(t_1+t_3)} [1 + R(t_1+t_3)] \right\} \\
&\quad + \frac{h_0}{R} D(p)(t_1+T') \left(\mathrm{e}^{-R(t_1+t_3)} - \mathrm{e}^{-R\left(t_1+T'\right)} \right) \\
&\quad - \frac{h_0}{R^2} D(p) \left\{ \mathrm{e}^{-R(t_1+t_3)} [1 + R(t_1+t_3)] - \mathrm{e}^{-R\left(t_1+T'\right)} [1 + R(t_1+T')] \right\} \\
&\quad + h_1 \frac{xq_2y}{R} \mathrm{e}^{-R(t_1+t_3)} \left(1 - \mathrm{e}^{-RT} \right)
\end{aligned}
\tag{3.33}
$$

Therefore, by using equations (3.25)–(3.33), the present total cost is calculated as follows: $T.C. = C_a + C_p + C_{\text{scr}} + C_{\text{e}_1} + C_{\text{e}_2} + C_d + C_{\text{sho}} + C_{\text{LS}} + C_{h_0}$ i.e.

$$
\begin{aligned}
&= K_0\mathrm{e}^{-Rt_1} + c_0y\mathrm{e}^{-Rt_1} + s_0y\mathrm{e}^{-Rt_3}\mathrm{e}^{-Rt_1} + c_{r_0}(1-x)\,q_1y\mathrm{e}^{-Rt_3}\mathrm{e}^{-Rt_1} + c_{a_0}xq_2y\mathrm{e}^{-RT'/2}\mathrm{e}^{-Rt_1} \\
&\quad + u_0 [x + (1-x)\,q_1]\, y\mathrm{e}^{-R(t_1+t_3)} \\
&\quad + C_{B_0} \frac{B}{R} \left[1 - \mathrm{e}^{-R(t_1+t_2)} \right] + C_{L_0} D(p) \left[t_1 - \frac{1}{\delta} \left(1 - \mathrm{e}^{-\delta t_1} \right) \right] \\
&\quad + \frac{h_0}{R} \left\{ y + [A\lambda + D(p)]\, t_1 \right\} \left(\mathrm{e}^{-Rt_1} - \mathrm{e}^{-R(t_1+t_2)} \right) \\
&\quad - \frac{h_0}{R^2} [A\lambda + D(p)] \left\{ \mathrm{e}^{-Rt_1} [1 + Rt_1] - \mathrm{e}^{-R(t_1+t_2)} [1 + R(t_1+t_2)] \right\} \\
&\quad + \frac{h_0}{R} [z_2 + D(p)(t_1+t_2)] \left(\mathrm{e}^{-R(t_1+t_2)} - \mathrm{e}^{-R(t_1+t_3)} \right) \\
&\quad - \frac{h_0}{R^2} D(p) \left\{ \mathrm{e}^{-R(t_1+t_2)} [1 + R(t_1+t_2)] - \mathrm{e}^{-R(t_1+t_3)} [1 + R(t_1+t_3)] \right\} \\
&\quad + \frac{h_0}{R} D(p)(t_1+T') \left(\mathrm{e}^{-R(t_1+t_3)} - \mathrm{e}^{-R\left(t_1+T'\right)} \right) \\
&\quad - \frac{h_0}{R^2} D(p) \left\{ \mathrm{e}^{-R(t_1+t_3)} [1 + R(t_1+t_3)] - \mathrm{e}^{-R\left(t_1+T'\right)} [1 + R(t_1+T')] \right\} + h_1 \frac{xq_2y}{R} \mathrm{e}^{-R(t_1+t_3)} \left(1 - \mathrm{e}^{-RT} \right).
\end{aligned}
\tag{3.34}
$$

3.2. Sales revenue

The present total sales revenue consists of three parts. During the cycle, the retailer sells the serviceable/perfect items at a unit price of p_0, generating sales revenue. The present revenue from total demand is given as

$$
R_1 = p_0 \int_{t_1}^{t_1+T'} D(p)\,\mathrm{e}^{-Rt}\mathrm{d}t. \tag{3.35}
$$

(a) Due to consumer returns of falsely inspected defectives, the retailer experiences loss due to full price refunds of the sold defectives. The present revenue from sales returns is calculated as

$$R_2 = -p_0 B_2 \mathrm{e}^{-R(t_1+t_3)}. \tag{3.36}$$

(b) All accumulated defectives are considered as scrap and sold at a cheaper price v_0 than retails price at time point A_3 in a lesser restrictive inventory. Thus, the present revenue from total scrap items is

$$R_3 = v_0 (B_1 + B_2) \mathrm{e}^{-R(t_1+t_3)}. \tag{3.37}$$

Therefore, using equations (3.35)–(3.37),

The present worth of total revenue $(T.R.) = R_1 + R_2 + R_3$

$$T.R. = \frac{p_0}{R} D(p) \left[\left(\mathrm{e}^{-Rt_1} - \mathrm{e}^{-R\left(t_1+T'\right)}\right)\right] - p_0 B_2 \mathrm{e}^{-R(t_1+t_3)} + v_0 (B_1 + B_2) \mathrm{e}^{-R(t_1+t_3)}. \tag{3.38}$$

3.3. Retailer's total profit

The retailer's total profit function $\boldsymbol{\pi}\ (\boldsymbol{y},\ \boldsymbol{B},\ \boldsymbol{p})$ is composed of the following components:

$\boldsymbol{\pi}\ (\boldsymbol{y},\ \boldsymbol{B},\ \boldsymbol{p})$ = Present Total Revenue – Present Ordering Cost – Present Purchase Cost – Present Screening Cost – Present Type-I Error Cost – Present Type-II Error Cost – Present Disposal Cost – Present Shortage Cost – Present Lost Sales Cost – Present Holding Cost

To obtain the present total profit $\boldsymbol{\pi}\ (\boldsymbol{y},\ \boldsymbol{B},\ \boldsymbol{p})$, we subtract equation (3.34) from (3.38), i.e.,

$$\pi(y, B, p) = T.R. - T.C. = T.R. - C_a - C_p - C_{\mathrm{scr}} - C_{\mathrm{e}_1} - C_{\mathrm{e}_2} - C_d - C_{\mathrm{sho}} - C_{\mathrm{LS}} - C_h$$

$$\pi(y,B,p) = \mathrm{e}^{\frac{R}{\delta}\ln\left(1-\frac{B\delta}{D(p)}\right)}$$
$$\left\{\begin{array}{l}
\frac{p_0}{R} D(p)\left(1-\mathrm{e}^{-R\frac{[(1-x)(1-q_1)+xq_2]y-B}{D(p)}}\right) - p_0 x q_2 y^{-R\frac{y}{\lambda}} + (v_0 - u_0)\left[x(1-q_2) + q_1(1-x) + xq_2\right] y\mathrm{e}^{-R\frac{y}{\lambda}} \\
-K_0 - c_0 y - s_0 y\mathrm{e}^{-R\frac{y}{\lambda}} - c_{r_0}(1-x) q_1 y\mathrm{e}^{-R\frac{y}{\lambda}} - c_{a_0} x q_2 y\mathrm{e}^{-R\frac{[(1-x)(1-q_1)+xq_2]y-B}{D(p)}} \\
-C_{B_0}\frac{B}{R}\left[1-\mathrm{e}^{-R\left(\frac{B}{\lambda[xq_2+(1-x)(1-q_1)]-D(p)}\right)}\right] - C_{L_0} D(p)\,\mathrm{e}^{-\frac{R}{\delta}\ln\left(1-\frac{B\delta}{D(p)}\right)} \\
\quad \times\left[-\frac{R}{\delta}\ln\left(1-\frac{B\delta}{D(p)}\right) - \frac{1}{\delta}\left(1-\mathrm{e}^{R\ln\left(1-\frac{B\delta}{D(p)}\right)}\right)\right] \\
-\frac{h_0}{R}\left\{y + \{\lambda[xq_2+(1-x)(1-q_1)]\}\left[-\frac{R}{\delta}\ln\left(1-\frac{B\delta}{D(p)}\right)\right]\right\}\left(1-\mathrm{e}^{-\frac{RB}{\lambda[xq_2+(1-x)(1-q_1)]-D(p)}}\right) \\
+\frac{h_0}{R^2}\{\lambda[xq_2+(1-x)(1-q_1)]\}\left\{\left(1-\frac{R}{\delta}\ln\left(1-\frac{B\delta}{D(p)}\right)\right) - \mathrm{e}^{-\frac{RB}{\lambda[xq_2+(1-x)(1-q_1)]-D(p)}}\right. \\
\quad \left.\times\left[1 + RB\left(-\frac{1}{\delta}\ln\left(1-\frac{B\delta}{D(p)}\right) + \frac{1}{\lambda[xq_2+(1-x)(1-q_1)]-D(p)}\right)\right]\right\} \\
-\frac{h_0}{R}\left[y - \frac{B[xq_2+(1-x)(1-q_1)]}{[xq_2+(1-x)(1-q_1)]\lambda-D(p)} + B\left(1+\frac{D(p)}{\lambda[xq_2+(1-x)(1-q_1)]-D(p)}\right)\right]\left(\mathrm{e}^{-\frac{RB}{\lambda[xq_2+(1-x)(1-q_1)]-D(p)}} - \mathrm{e}^{-R\frac{y}{\lambda}}\right) \\
+\frac{h_0}{R^2} D(p)\left\{\mathrm{e}^{-\frac{RB}{\lambda[xq_2+(1-x)(1-q_1)]-D(p)}}\left[1 + R\left(-\frac{1}{\delta}\ln\left(1-\frac{B\delta}{D(p)}\right) + \frac{B}{\lambda[xq_2+(1-x)(1-q_1)]-D(p)}\right)\right]\right. \\
\quad \left. -\mathrm{e}^{-R\frac{y}{\lambda}}\left[1 + R\left(-\frac{1}{\delta}\ln\left(1-\frac{B\delta}{D(p)}\right) + \frac{y}{\lambda}\right)\right]\right\} \\
-\frac{h_0}{R} D(p)\left(-\frac{1}{\delta}\ln\left(1-\frac{B\delta}{D(p)}\right) + \frac{[(1-x)(1-q_1)+xq_2]y-B}{D(p)}\right)\left(\mathrm{e}^{-R\frac{y}{\lambda}} - \mathrm{e}^{-R\frac{[(1-x)(1-q_1)+xq_2]y-B}{D(p)}}\right) \\
+\frac{h_0}{R^2} D(p)\left\{\mathrm{e}^{-R\frac{y}{\lambda}}\left[1 + R\left(-\frac{1}{\delta}\ln\left(1-\frac{B\delta}{D(p)}\right) + \frac{y}{\lambda}\right)\right] - \mathrm{e}^{-R\frac{[(1-x)(1-q_1)+xq_2]y-B}{D(p)}}\right. \\
\quad \left.\times\left[1 + R\left(-\frac{1}{\delta}\ln\left(1-\frac{B\delta}{D(p)}\right) + \frac{[(1-x)(1-q_1)+xq_2]y-B}{D(p)}\right)\right]\right\} \\
-h_1\frac{xq_2 y}{R}\mathrm{e}^{-R\frac{y}{\lambda}}\left(1-\mathrm{e}^{-R\frac{[(1-x)(1-q_1)+xq_2]y}{D(p)}}\right)
\end{array}\right\}. \tag{3.39}$$

3.4. Retailer's expected total profit

As present total profit is comprised of random variables, x, q_1, q_2, we calculate the expected present total profit function $E\left[\pi\left(y,B,p\right)\right]$ as follows:

$$
\begin{aligned}
&E\left[\pi\left(y,B,p\right)\right] = \mathrm{e}^{\frac{R}{\delta}\ln\left(1-\frac{B\delta}{D(p)}\right)} \\
&\left\{\begin{aligned}
&\tfrac{p_0}{R}D\left(p\right)\left(1-\mathrm{e}^{-R\frac{\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}y-B}{D(p)}}\right)-p_0E\left[x\right]E\left[q_2\right]y^{-R\frac{y}{\lambda}}+\left(v_0-u_0\right)\\
&\quad\times\left\{E\left[x\right]\left(1-E\left[q_2\right]\right)+E\left[q_1\right]\left(1-E\left[x\right]\right)+E\left[x\right]E\left[q_2\right]\right\}y\mathrm{e}^{-R\frac{y}{\lambda}}\\
&-K_0-c_0y-s_0y\mathrm{e}^{-R\frac{y}{\lambda}}-c_{r_0}\left(1-E\left[x\right]\right)E\left[q_1\right]y\mathrm{e}^{-R\frac{y}{\lambda}}-c_{a_0}E\left[x\right]E\left[q_2\right]y\mathrm{e}^{-R\frac{\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}y-B}{D(p)}}\\
&-C_{B_0}D\left(p\right)\tfrac{1}{\delta}\left[\frac{\mathrm{e}^{-\frac{(\delta-R)B}{D(p)}}-1}{(\delta-R)}+\frac{\mathrm{e}^{-\frac{RB}{D(p)}}-1}{R}\right]-C_{L_0}D\left(p\right)\mathrm{e}^{-\frac{R}{\delta}\ln\left(1-\frac{B\delta}{D(p)}\right)}\left[-\tfrac{1}{\delta}\ln\left(1-\tfrac{B\delta}{D(p)}\right)-\tfrac{1}{\delta}\left(1-\mathrm{e}^{-\delta\frac{B}{D(p)}}\right)\right]\\
&-\tfrac{h_0}{R}\left\{y+\left\{\lambda\left\{\left(1-E\left[x\right]\right)\left(1-E\left[q_1\right]\right)+E\left[x\right]E\left[q_2\right]\right\}\right\}\left[-\tfrac{1}{\delta}\ln\left(1-\tfrac{B\delta}{D(p)}\right)\right]\right\}\\
&\quad\times\left(1-\mathrm{e}^{-\frac{RB}{\lambda\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}-D(p)}}\right)\\
&+\tfrac{h_0}{R^2}\left\{\lambda\left\{\left(1-E\left[x\right]\right)\left(1-E\left[q_1\right]\right)+E\left[x\right]E\left[q_2\right]\right\}\right\}\left\{\left(1-\tfrac{R}{\delta}\ln\left(1-\tfrac{B\delta}{D(p)}\right)\right)-\mathrm{e}^{-\frac{RB}{\lambda\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}-D(p)}}\right.\\
&\quad\left.\times\left[1+R\left(-\tfrac{1}{\delta}\ln\left(1-\tfrac{B\delta}{D(p)}\right)+\tfrac{B}{\lambda\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}-D(p)}\right)\right]\right\}\\
&-\tfrac{h_0}{R}\left[y-\tfrac{B\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}}{\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}\lambda-D(p)}+B\left(1+\tfrac{D(p)}{\lambda\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}-D(p)}\right)\right]\\
&\quad\times\left(\mathrm{e}^{-\frac{RB}{\lambda\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}-D(p)}}-\mathrm{e}^{-R\frac{y}{\lambda}}\right)\\
&+\tfrac{h_0}{R^2}D\left(p\right)\left\{\mathrm{e}^{-\frac{RB}{\lambda\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}-D(p)}}\left[1+R\left(-\tfrac{1}{\delta}\ln\left(1-\tfrac{B\delta}{D(p)}\right)+\tfrac{B}{\lambda\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}-D(p)}\right)\right]\right.\\
&\left.-\mathrm{e}^{-R\frac{y}{\lambda}}\left[1+R\left(-\tfrac{1}{\delta}\ln\left(1-\tfrac{B\delta}{D(p)}\right)+\tfrac{y}{\lambda}\right)\right]\right\}\\
&-\tfrac{h_0}{R}D\left(p\right)\left(-\tfrac{1}{\delta}\ln\left(1-\tfrac{B\delta}{D(p)}\right)+\tfrac{\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}y-B}{D(p)}\right)\left(\mathrm{e}^{-R\frac{y}{\lambda}}-\mathrm{e}^{-R\frac{\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}y-B}{D(p)}}\right)\\
&+\tfrac{h_0}{R^2}D\left(p\right)\left\{\mathrm{e}^{-R\frac{y}{\lambda}}\left[1+R\left(-\tfrac{1}{\delta}\ln\left(1-\tfrac{B\delta}{D(p)}\right)+\tfrac{y}{\lambda}\right)\right]-\mathrm{e}^{-R\frac{\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}y-B}{D(p)}}\right.\\
&\quad\left.\times\left[1+R\left(-\tfrac{1}{\delta}\ln\left(1-\tfrac{B\delta}{D(p)}\right)+\tfrac{\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}y-B}{D(p)}\right)\right]\right\}\\
&-h_1\tfrac{E[x]E[q_2]y}{R}\mathrm{e}^{-R\frac{y}{\lambda}}\left(1-\mathrm{e}^{-R\frac{\{(1-E[x])(1-E[q_1])+E[x]E[q_2]\}y}{D(p)}}\right)
\end{aligned}\right\}
\end{aligned}
\tag{3.40}
$$

4. Retailer's optimal policy

The retailer aims to maximize the expected value the total profit function by jointly optimizing the replenishment quantity, the backorder quantity, and the selling price. In this section, concavity of the objective function is proved graphically due to the mathematical complexity of the derivatives.

The result is established in the form of a lemma.

Lemma 4.1. *The function of the retailer's expected total profit is concave.*

Proof. To prove the global optimality of the expected present total profit, comprised of three decision variables, the following sufficient conditions for global optimality must hold:

$$D_1\left(y,B,p\right)<0,\ D_2\left(y,B,p\right)>0,\ \text{and}\ D_3\left(y,B,p\right)<0. \tag{4.1}$$

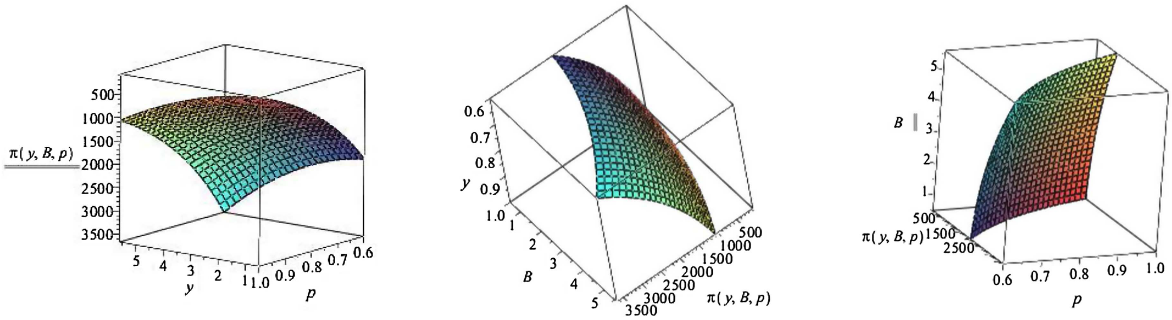

Figure 3. Concavity of the expected value of the present total profit function.

Here, the Hessian matrix H is used in calculating the second-order partial derivatives as follows:

$$H=\begin{bmatrix}\frac{\partial^2 E[\pi(y,B,p)]}{\partial y^2} & \frac{\partial^2 E[\pi(y,B,p)]}{\partial y\partial B} & \frac{\partial^2 E[\pi(y,B,p)]}{\partial y\partial p}\\ \frac{\partial^2 E[\pi(y,B,p)]}{\partial B\partial y} & \frac{\partial^2 E[\pi(y,B,p)]}{\partial B^2} & \frac{\partial^2 E[\pi(y,B,p)]}{\partial B\partial p}\\ \frac{\partial^2 E[\pi(y,B,p)]}{\partial p\partial y} & \frac{\partial^2 E[\pi(y,B,p)]}{\partial p\partial B} & \frac{\partial^2 E[\pi(y,B,p)]}{\partial p^2}\end{bmatrix} \text{ and}$$

$$D_1=\frac{\partial^2 E\left[\pi\left(y,B,p\right)\right]}{\partial y^2}, D_2=\begin{vmatrix}\frac{\partial^2 E[\pi(y,B,p)]}{\partial y^2} & \frac{\partial^2 E[\pi(y,B,p)]}{\partial y\partial B}\\ \frac{\partial^2 E[\pi(y,B,p)]}{\partial B\partial y} & \frac{\partial^2 E[\pi(y,B,p)]}{\partial B^2}\end{vmatrix},$$

$$D_3=\det H=\begin{vmatrix}\frac{\partial^2 E[\pi(y,B,p)]}{\partial y^2} & \frac{\partial^2 E[\pi(y,B,p)]}{\partial y\partial B} & \frac{\partial^2 E[\pi(y,B,p)]}{\partial y\partial p}\\ \frac{\partial^2 E[\pi(y,B,p)]}{\partial B\partial y} & \frac{\partial^2 E[\pi(y,B,p)]}{\partial B^2} & \frac{\partial^2 E[\pi(y,B,p)]}{\partial B\partial p}\\ \frac{\partial^2 E[\pi(y,B,p)]}{\partial p\partial y} & \frac{\partial^2 E[\pi(y,B,p)]}{\partial p\partial B} & \frac{\partial^2 E[\pi(y,B,p)]}{\partial p^2}\end{vmatrix}$$

where D_1, D_2, and D_3 are the minors of the Hessian matrix H. □

Because the derivatives of these minors are very complex, the concavity of the objective function has been proven graphically, as shown in Figure 3.

To determine the optimal values, say y^*, B^*, and p^*, which maximize the function of $E[\pi(y,B,p)]$, the necessary conditions of first-order derivatives must hold:

$$\frac{\partial E\left[\pi\left(y,B,p\right)\right]}{\partial y}=0; \frac{\partial E\left[\pi\left(y,B,p\right)\right]}{\partial B}=0; \frac{\partial E\left[\pi\left(y,B,p\right)\right]}{\partial p}=0 \tag{4.2}$$

which gives the optimal values of y, B and p.

The profit function is highly nonlinear in nature. Thus, the numerical method is used to check the global optimality of the model.

5. Numerical analysis

5.1. Example

This subsection provides validation of the developed model with the help of an example. Here, the optimal replenishment quantity (y^*), optimal backorder quantity (B^*), optimal selling price (p^*), and expected value of total profit $E[\pi^*(y^*,B^*,p^*)]$ are determined using a hypothetical data set.

TABLE 1. Impact of x on the optimal replenishment policy.

$\boldsymbol{x}$	$\boldsymbol{y}^*$	$\boldsymbol{B}^*$	$\boldsymbol{p}^*$(\$)	$\boldsymbol{T}^*$(in days)	$\boldsymbol{D}^*(\boldsymbol{p}^*)$	$\boldsymbol{E}[\boldsymbol{\pi}^*(\boldsymbol{y}^*, \boldsymbol{B}^*, \boldsymbol{p}^*)]$
0.03	1491.42	706.61	61.36	23.06	23 901.23	6 57 909.48
0.05	1393.74	647.47	61.98	21.97	23 422.42	6 57 707.89
0.07	1309.80	596.38	62.62	21.05	22 944.67	6 57 409.59
0.09	1236.93	551.72	63.29	20.29	22 467.93	6 57 006.97
0.11	1172.90	512.28	63.97	19.60	21 991.99	6 56 494.26

TABLE 2. Impact of q_1 on the optimal replenishment policy.

$\boldsymbol{q1}$	$\boldsymbol{y}^*$	$\boldsymbol{B}^*$	$\boldsymbol{p}^*$(\$)	$\boldsymbol{T}^*$(in days)	$\boldsymbol{D}^*(\boldsymbol{p}^*)$	$\boldsymbol{E}[\boldsymbol{\pi}^*(\boldsymbol{y}^*, \boldsymbol{B}^*, \boldsymbol{p}^*)]$
0.00	1343.58	636.97	61.37	20.75	23 895.00	6 93 244.73
0.02	1393.66	647.43	61.98	21.96	23 422.36	6 57 708.01
0.04	1439.18	654.87	62.62	23.15	22 948.49	6 23 477.36
0.06	1478.10	658.52	63.28	24.28	22 474.04	5 90 546.60
0.08	1508.31	657.65	63.96	25.30	21 998.92	5 58 909.88

Defect proportion (x), Type-I error percentage (q_1), and Type-2 error percentage (q_2) are assumed to follow a uniform distribution function with a respective probability distribution function as follows:

$$f(x) = \begin{cases} 50, 0.04 \le x \le 0.06 \\ 0, \quad \text{otherwise} \end{cases} \quad f(q_1) = \begin{cases} 50, 0.01 \le q_1 \le 0.03 \\ 0, \quad \text{otherwise} \end{cases} \quad f(q_2) = \begin{cases} 25, 0.02 \le q_2 \le 0.06 \\ 0, \quad \text{otherwise.} \end{cases}$$

Other parameters assume their values as follows:
$B = 9\,00\,00\,000$, $e = 2$, $\lambda = 50\,000$ units/year, $K_0 = \$100$/order, $c_0 = \$30$/unit, $v_0 = \$20$/unit, $u_0 = \$2$/unit, $s_0 = \$2$/unit, $C_{a0} = \$100$/unit, $C_{r0} = \$500$/unit, $C_{B0} = \$6$/unit, $C_{L0} = \$5$/unit/year, $h_0 = \$5$/unit/year, $x = 0.05$, $q_1 = 0.02$, $q_2 = 0.04$, $R = 0.06$, $\delta = 0.6$.

The optimal production lot size $y^* = 1393$ units/cycle, optimal backorder quantity $B^* = 647$ units/cycle, optimal selling price $p^* = \$61.98$ with corresponding $E\left[\pi^*\left(y^*, B^*, p^*\right)\right] = \$6\,57\,708.01$ units/year, $T^* = 21.96$ days, and $D^*\left(p^*\right) = 23\,422.36$ units/year.

5.2. Sensitivity analysis

A comprehensive sensitivity analysis was performed to analyze the variations in the values of parameters x, q_1, q_2, δ, and R over the decision variables: optimal replenishment quantity y^*, optimal backordering variable B^*, optimal selling price p^*, optimal cycle length T^*, optimal demand $D(p^*)$, and corresponding expected present total profit $E[\pi^*(y^*, B^*, p^*)]$.

Results are summarized in Tables 1–5 using the above values.

From the computational results shown in Tables 1–5, we obtain the following managerial insights.

As shown in Table 1, with a growing proportion of bad-quality items, x, the retailer experiences a decline in optimal profit values ($\boldsymbol{E}[\boldsymbol{\pi}^*(\boldsymbol{y}^*, \boldsymbol{B}^*, \boldsymbol{p}^*)]$) and also a decrement in optimal order quantity ($\boldsymbol{y}^*$), optimal backorder quantity ($\boldsymbol{B}^*$), and optimal demand values ($\boldsymbol{D}^*(\boldsymbol{p}^*)$). However, there is slight increment in the optimal mark-up price ($\boldsymbol{p}^*$). It is explained as with the increase in defectives in the system, the retailer needs to elevate the prices to compensate for the loss of demand and thus the unachieved sales. With lesser demand, the order quantity is bound to decrease as so are the allowable shortages. The decrement in the demand values justifies the purpose of taking price sensitive nature of demand. These trends result in shortening of the cycle length also. In view of this result, it is advisable forthe retailer to analyze the source of imperfections by defect tracking, analyzing

TABLE 3. Impact of q_2 on the optimal replenishment policy.

$q2$	y^*	B^*	p^*($)	T^*(in days)	$D^*(p^*)$	$E[\pi^*(y^*, B^*, p^*)]$
0.00	1285.44	596.07	62.06	20.29	23 364.64	6 83 711.61
0.02	1336.54	620.34	62.02	21.08	23 393.30	6 70 719.98
0.04	1393.66	647.43	61.98	21.96	23 422.36	6 57 708.01
0.06	1458.00	677.93	61.94	22.96	23 451.97	6 44 676.27
0.08	1531.34	712.61	61.90	24.10	23 482.22	6 31 625.89

TABLE 4. Impact of δ on the optimal replenishment policy.

δ	y^*	B^*	p^*($)	T^*(in days)	$D^*(p^*)$	$E[\pi^*(y^*, B^*, p^*)]$
0.40	868.89	404.63	62.06	13.63	23 365.49	6 55 569.93
0.50	1048.36	487.78	62.03	16.48	23 386.08	6 56 521.08
0.60	1393.66	647.43	61.98	21.96	23 422.36	6 57 708.01
0.70	2329.37	1077.81	61.86	36.96	23 514.64	6 59 422.71
0.80	4815.02	2203.20	61.53	77.76	23 771.29	6 62 556.99

complaints, and identifying causes; so as to minimize the proportion of defectives as these efforts can help increase the total profit of the system.

As evident from Table 2, by committing more misclassification errors of Type-I (q_1), an increment in optimal order quantity ($\boldsymbol{y}^*$), optimal backorder quantity ($\boldsymbol{B}^*$), optimal mark-up price ($\boldsymbol{p}^*$) is observed. However, the optimal demand values ($\boldsymbol{D}^*(\boldsymbol{p}^*)$) and optimal profit values ($\boldsymbol{E}[\boldsymbol{\pi}(\boldsymbol{y}^*, \boldsymbol{B}^*, \boldsymbol{p}^*)]$) tend to decrease with this type of error. Due to this error, the retailer is unable to sell all of the perfect items to satisfy the demand and erroneously discards some of them as scrap. This restricts the retailer from achieving maximum possible sales and lowers profit values considerably. Because demand is fulfilled only by perfect items, the retailer ought to order a larger quantity. The loss incurred due to the inspection error is balanced out through a slight increase inthe selling price, which results in a decrease in demand due to its price-sensitive nature. The increase in cycle length is dueto the ordering of a higher order quantity. Thus, it is beneficial for the retailer to reduce the proportion of Type-I error (q_1), as these can be helpful in increasing sales directly.

- Table 3 shows that, as the proportion of Type-II errors (q_2) increases, an increment in optimal order quantity ($\boldsymbol{y}^*$), optimal backorder quantity ($\boldsymbol{B}^*$), optimal demand values ($\boldsymbol{D}^*(\boldsymbol{p}^*)$) is exhibited while the optimal mark-up price ($\boldsymbol{p}^*$) and optimal profit values ($\boldsymbol{E}[\boldsymbol{\pi}^*(\boldsymbol{y}^*, \boldsymbol{B}^*, \boldsymbol{p}^*)]$) show a decreasing trend. With this type of error, there is an increase in defect returns, for which the retailer suffers a penalty cost and goodwill loss. Since some defectives have been wrongly classified as non-defectives by mistake, the increased sales of perfect items increase the order quantity to satisfy the elevated demand. However, due to frustration and quality dissatisfaction, customers might not be willing to purchase more; hence, there is a substantial increase in the value of B^*. Price reduction is justified to enhance sales in such an imperfect environment. Therefore, it is advisable for the retailer to reduce the proportion of Type-II error (q_2), as they have an inverse relation with the total profit value.
- Table 4 reveals that, on increasing the value of backlogging parameter (δ), there is an increment in optimal order quantity ($\boldsymbol{y}^*$), optimal backorder quantity ($\boldsymbol{B}^*$), optimal demand values ($\boldsymbol{D}^*(\boldsymbol{p}^*)$) while the optimal mark-up price ($\boldsymbol{p}^*$) and optimal profit values ($\boldsymbol{E}[\boldsymbol{\pi}^*(\boldsymbol{y}^*, \boldsymbol{B}^*, \boldsymbol{p}^*)]$) tend to decrease. Also, as the value of backlogging parameter (δ) increases, the backlog rate decreases, this means that a lesser amount of shortages will get backlogged. The amount of unsatisfied shortages increases; therefore, to fulfill these, an increase in order quantity is observed along with an eventual increment in cycle length. Order quantity increases,

TABLE 5. Impact of R on the optimal replenishment policy.

$\boldsymbol{R}$	$\boldsymbol{y}^*$	$\boldsymbol{B}^*$	$\boldsymbol{p}^*$(\$)	$\boldsymbol{T}^*$(in days)	$\boldsymbol{D}^*(\boldsymbol{p}^*)$	$\boldsymbol{E}[\boldsymbol{\pi}^*(\boldsymbol{y}^*, \boldsymbol{B}^*, \boldsymbol{p}^*)]$
0.02	6380.16	2920.64	61.53	103.05	23,768.69	6 62 480.56
0.04	2259.97	1047.14	61.90	35.78	23 482.78	6 59 187.44
0.06	1393.66	647.43	61.98	21.96	23 422.36	6 57 708.01
0.08	1077.30	500.94	62.01	16.90	23 400.30	6 56 674.73

bringing higher sales of perfect items, which contributes to higher profit level. The reduction in price helps in increasing the demand rate significantly.

- Further, Table 5 reflects that as the net discount rate (R) increases, the optimal order quantity ($\boldsymbol{y}^*$), optimal backorder quantity ($\boldsymbol{B}^*$), optimal demand values ($\boldsymbol{D}^*(\boldsymbol{p}^*)$), and optimal profit values ($\boldsymbol{E}[\boldsymbol{\pi}^*(\boldsymbol{y}^*, \boldsymbol{B}^*, \boldsymbol{p}^*)]$) tend to decrease while the optimal mark-up price ($\boldsymbol{p}^*$) increases. An increase in net discount rate (R) implies a decrease in the inflation rate. Due to a low inflation rate, the decision maker should order a smaller quantity more frequently as the cycle length decreases, which is helpful in reducing the total cost of inventory. With reduction in order quantity, maximum allowable shortages also exhibit reduction. Moreover, there is a marginal price increase with an eventual decrement in demand so as to balance the diminishing of profit values.

6. Cross interactions of the decision variables

Figure 4 illustrates the relative patterns of different model parameters, x, q_1, q_2, and R on the decision variable y^*. The contribution of R has the greatest impact on the order quantity y^* compared to the other key parameters, x, q_1, and q_2. The rate of inflation decreases with an increase in R, which lowers the prices and hence influences the demand positively, resulting in a higher order quantity. Moreover, the y^* value tends to decrease with rise in the supply of defectives. This is because the retailer will experience reduced demand under imperfect quality environment and hence the will need to order less.

Figure 5 exhibits the changing trends in the values of key parameters $x, q_1, q_{2,}$ and R on the decision variable B^*, which is similar to the effect of these parameters on y^*. The shortage level is observed to increase with rise in the model parameters, $q_{1,}q_2$, and decrease with increase of x, R. Due to a larger number of defects, there is more loss of perfect items as scrap (outcome of Type-I error), and there is an increase in returns/refunds (outcome of Type-II error). As a result, the retailer will have difficulty in satisfying the demand with fewer perfect items, and so won't be able to fulfill the shortages also. Hence, there is no profit in raising the maximum shortage level. Furthermore, the increase in R leads to lower inflation which further decreases the shortage levels.

Figure 6 illustrates the effects of the considered parameters x, q_1, q_2, and R on decision variable $D^*(p)$. It shows that increase in the count of defect-related parameters, x and q_1, the demand shows a declining pattern, which confirms that the retailer cannot compromise on quality issues. Also, with an increasing proportion of sales of falsely classified perfect products, q_2, a slightly upward trend of demand is observed due to the 100% returns/refunds by the retailer, which boosts the retailer's reputation, followed by more customers and increased demand. To balance the losses related to defects, the retailer experiences a small price increase for perfect items in the model so as to maintain profit margins. This also justifies the pupose of using the selling price-dependent demand function because downfall in the demand values is compensated for by relatively higher prices. Furthermore, the decrease in demand is marginal due to an increase in R, which indirectly leads to the lowering of price.

Figure 7 depicts the change in the pattern of model parameters $x, q_1, q_{2,}$ and R on the decision variable $E[\pi^*(y^*, B^*, p^*)]$. Since demand is fulfilled only through perfect items, the total profit values tend to decrease with elevation in the count of defective-related parameters x, q_1, and q_2. This proves that both errors have a

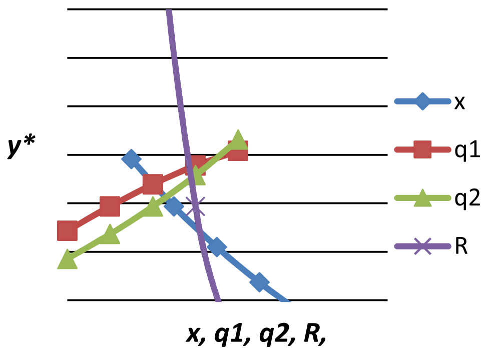

FIGURE 4. Sensitivity of the optimal order quantity y^* w.r.t x, q_1, q_2, R.

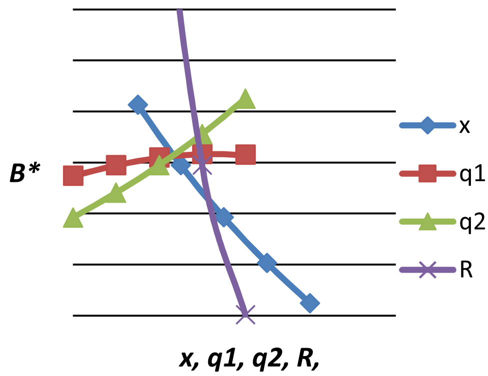

FIGURE 5. Sensitivity of the optimal shortage level B^* w.r.t. x, q_1, q_2, and R.

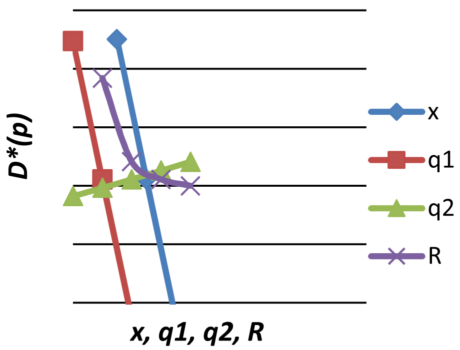

FIGURE 6. Sensitivity of the demand function $D^*(p)$ w.r.t x, q_1, q_2, R.

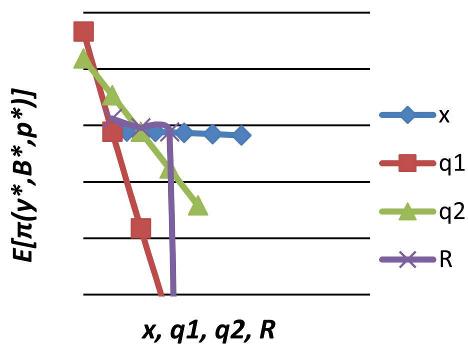

FIGURE 7. Sensitivity of the expected present total profit function $\mathrm{E}[\pi^* \, (y^*, B^*, p^*)]$ w.r.t. x, q_1, q_2, R.

negative effect on the profit values and hence cannot be ignored in the model. It is also seen from the graph that, in the absence of Type-I error (i.e. q_1), more profit is observed compared to that in the absence of Type-II error (i.e., q_2). Thus, it is beneficial for the manager to work on increasing the efficiency of inspection processes so that these errors are less damaging to profits. Moreover, with the increase in the value of R, inflation rate i decreases because $R = d - i$. With the decrease in i, there is a decrease in price and hence profit. Also, with a small change at lower values of R, not much change is observed in profit values; however, when the change in inflation rate becomes large, there is a considerable decline in the profit values.

7. Summary

The application of the model and the outcomes of this research are explained in this section.

7.1. Applicability of this model

The current model is widely applied in many real-world industries like textiles, electronics, furniture, footwear, and plastics, which are at a risk of dynamic pricing due to inflationary conditions. In several developing countries like India, Bangladesh, Bhutan, and Indonesia, where inflation is severe, its effect should not be neglected when determining various inventory policies. Owing to these dynamic economic conditions, there is fluctuation of prices in the market. As a result, the demand for items is also affected by the selling price of products. In addition, the model finds wide applicability in all competitive sectors where the managers are trying to achieve supreme standards of quality and hence cannot afford to overlook quality control issues if they wish to dominate global markets. Though it comes at a relatively higher cost, it pays off tremendously through customer satisfaction, gradually converting into boosted reputation and thus increased revenue.

7.2. Conclusions and future research

The proposed model targets the realistic challenges encountered by the retail industry, one of which is inflation and its effects on inventory policies. To survive in today's competitive market; one cannot ignore the ramifications of inflation as it strongly affects inventory-related costs, pricing strategies, and ultimately customer demand. The results show that consideration of inflation is vital for better accuracy of cash flows and hence should be taken into account. The assumption of static demand also does not seem viable enough to be integrated in the study, and the results indicate that the price-sensitive nature of demand has a considerable impact on performance measures. Retailers also face rigorous competition in terms of quality standards. Because these industries rely on products they do not produce, it is essential for them to integrate a rigorous inspection process to sort out the defectives in the received lot. However, despite modernization in techniques, certain manual errors cause Type-I and Type-II inspection errors. Retailers should identify the causes of errors to eliminate or reduce them or change suppliers. Finally, the model assumes partially backlogged shortages, which have a tendency to reduce the inventory costs. Hence, an economic order model is established to investigate the optimal replenishment quantity, optimal backorder level, and optimal pricing by maximizing the expected value of the present total profit in an imperfect environment. A numerical example along with a comprehensive sensitivity analysis is employed to demonstrate the pragmatism of the model. As per the authors' knowledge, such a real-life set-up has not been presented in the research field of inventory management.

In the future, the present model can be extended under different practical parameters such as deterioration, warehousing, and rework. Another direction might be integrating different forms of trade credit decisions into the present model. The study can also be extended for different demand functions such as time-dependent demand, stock-dependent demand, or both.

Acknowledgements. The authors would like to thank the anonymous reviewers and the editor for their valuable comments and suggestions which helped to improve the present paper.

References

[1] P.L. Abad and C.K. Jaggi, A joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive. *Int. J. Prod. Econ.* **83** (2003) 115–122.

[2] H. Bierman and J. Thomas, Inventory decisions under inflationary conditions. *Decis. Sci.* **8** (1977) 151–155.

[3] S. Bose, A. Goswami and K.S. Chaudhuri, An EOQ model for deteriorating items with linear time-dependent demand rate and shortages under inflation and time discounting. *J. Oper. Res. Soc.* **46** (1995) 771–782.

[4] J.A. Buzacott, Economic order quantities with inflation. *Oper. Res. Q.* **26** (1975) 553–558.

[5] H.C. Chang and C.H. Ho, Exact closed-form solutions for "optimal inventory model for items with imperfect quality and shortage backordering". *Omega* **38** (2010) 233–237.

[6] S.O. Duffuaa and M. Khan, Impact of inspection errors on the performance measures of a general repeat inspection plan. *Int. J. Prod. Res.* **43** (2005) 4945–4967.

[7] A. Eroglu and G. Ozdemir, An economic order quantity model with defective items and shortages. *Int. J. Prod. Econ.* **106** (2007) 544–549.

[8] M. Ghoreishi, A. Mirzazadeh and G.W. Weber, Optimal pricing and ordering policy for non-instantaneous deteriorating items under inflation and customer returns. *Optimization* **63** (2014) 1785–1804.

[9] M. Ghoreishi, A. Mirzazadeh, G.W. Weber and I. Nakhai-Kamalabadi, Joint pricing and replenishment decisions for non-instantaneous deteriorating items with partial backlogging, inflation-and selling price-dependent demand and customer returns. *Management* **11** (2015) 933–949.

[10] J.T. Hsu and L.F. Hsu, An EOQ model with imperfect quality items, inspection errors, shortage backordering, and sales returns. *Int. J. Prod. Econ.* **143** (2013) 162–170.

[11] C.K. Jaggi, K.K. Aggarwal and S.K. Goel, Optimal order policy for deteriorating items with inflation induced demand. *Int. J. Prod. Econ.* **103** (2006) 707–714.

[12] C.K. Jaggi and A. Khanna, The retailer's procurement policy with credit-linked demand under inflationary conditions. *Int. J. Procurement Manage.* **2** (2009) 163–179.

[13] C.K. Jaggi, M. Mittal and A. Khanna, Effects of inspection on retailer's ordering policy for deteriorating items with time-dependent demand under inflationary conditions. *Int. J. Syst. Sci.* **44** (2013) 1774–1782.

[14] M. Khan, M.Y. Jaber and M. Bonney, An economic order quantity (EOQ) for items with imperfect quality and inspection errors. *Int. J. Prod. Econ.* **133** (2011) 113–118.

[15] M.S. Kim, J.S. Kim, B. Sarkar, M. Sarkar and M.W. Iqbal, An improved way to calculate imperfect items during long-run production in an integrated inventory model with backorders. *J. Manuf. Syst.* **47** (2018) 153–167.

[16] I. Konstantaras, K. Skouri and M.Y. Jaber, Inventory models for imperfect quality items with shortages and learning in inspection. *Appl. Math. Model.* **36** (2012) 5334–5343.

[17] B. Maddah and M.Y. Jaber, Economic order quantity for items with imperfect quality: revisited. *Int. J. Prod. Econ.* **112** (2008) 808–815.

[18] B. Maddah, M.K. Salameh and L. Moussawi-Haidar, Order overlapping: a practical approach for preventing shortages during screening. *Comput. Ind. Eng.* **58** (2010) 691–695.

[19] R.B. Misra, A study of inflationary effects on inventory systems. *Logistics Spectr.* **9** (1975) 51–55.

[20] R.B. Misra, A note on optimal inventory management under inflation. *Naval Res. Logistics (NRL)* **26** (1979) 161–165.

[21] B. Mondal, A.K. Bhunia and M. Maiti, An inventory system of ameliorating items for price dependent demand rate. *Comput. Ind. Eng.* **45** (2003) 443–456.

[22] I. Moon and S. Lee, The effects of inflation and time-value of money on an economic order quantity model with a random product life cycle. *Eur. J. Oper. Res.* **125** (2000) 588–601.

[23] M. Palanivel and R. Uthayakumar, Finite horizon EOQ model for non-instantaneous deteriorating items with price and advertisement dependent demand and partial backlogging under inflation. *Int. J. Syst. Sci.* **46** (2015) 1762–1773.

[24] S. Papachristos and I. Konstantaras, Economic ordering quantity models for items with imperfect quality. *Int. J. Prod. Econ.* **100** (2006) 148–154.

[25] E.L. Porteus, Optimal lot sizing, process quality improvement and setup cost reduction. *Oper. Res.* **34** (1986) 137–144.

[26] M.J. Rosenblatt and H.L. Lee, Economic production cycles with imperfect production processes. *IIE Trans.* **18** (1986) 48–55.

[27] M.D. Roy, S.S. Sana and K. Chaudhuri, An economic order quantity model of imperfect quality items with partial backlogging. *Int. J. Syst. Sci.* **42** (2011) 1409–1419.

[28] M.K. Salameh and M.Y. Jaber, Economic production quantity model for items with imperfect quality. *Int. J. Prod. Econ.* **64** (2000) 59–64.

[29] S.S. Sana, A production-inventory model of imperfect quality products in a three-layer supply chain. *Decis. Support Syst.* **50** (2011) 539–547.

[30] B. Sarkar and I. Moon, An EPQ model with inflation in an imperfect production system. *Appl. Math. Comput.* **217** (2011) 6159–6167.

[31] B. Sarkar, Supply chain coordination with variable backorder, inspections, and discounts policy for fixed lifetime products. *Math. Prob. Eng.*, *Article ID. 6318737* (2016) 1–14.

[32] B. Sarkar and S. Saren, Product inspection policy for an imperfect production system with inspection errors and warranty cost. *Eur. J. Oper. Res.* **248** (2016) 263–271.

[33] R.L. Schwaller, EOQ under inspection costs. *Prod. Inventory Manage. J.* **29** (1988) 22–24.

[34] P.K. Sharma, Survey of inventory models with inflation. *PARIPEX-Indian J. Res.* **5** (2016).

[35] J. Taheri-Tolgari, A. Mirzazadeh and F. Jolai, An inventory model for imperfect items under inflationary conditions with considering inspection errors. *Comput. Math. Appl.* **63** (2012) 1007–1019.

[36] M. Tayyab and B. Sarkar, Optimal batch quantity in a cleaner multi-stage lean production system with random defective rate. *J. Cleaner Prod.* **139** (2016) 922–934.

[37] H.M. Wee, J. Yu and M.C. Chen, Optimal inventory model for items with imperfect quality and shortage backordering. *Omega*, (2007) **35** 7–11.

[38] Z. Youjun, Z. Yan and C. Liang, An inventory model with advance sales and demand rate dependent on price. In: Service Systems and Service Management (ICSSSM), 12th International Conference on IEEE (2015) 1–5.

[39] X. Zhang and Y. Gerchak, Joint lot sizing and inspection policy in an EOQ model with random yield. *IIE Trans.* **22** (1990) 41–47.