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A GENERALIZED DEA MODEL FOR INPUTS (OUTPUTS) ESTIMATION UNDER INTER-TEMPORAL DEPENDENCE

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Abstract. This paper extended the inverse Data Envelopment Analysis (DEA) to the framework of dynamic DEA. The following question is studied under inter-temporal dependence assumption: among a set of decision making units (DMUs), to what extent should the input (output) levels of the DMU change if the efficiency index of a DMU remains unchanged, yet the output (input) levels change? This question is answered using (periodic weak) Pareto solutions of multiple-objective linear programming (MOLP) problems in the framework of dynamic DEA. In this study, unlike other proposed methods, the simultaneous increase and decrease of the various input (output) levels are considered under inter-temporal dependence. In addition, a numerical example with real data is provided to illustrate the objective of this research.

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1. Introduction

Dynamic Data Envelopment Analysis (DEA) is a nonparametric technique based on mathematical programming for evaluating the performance of Decision Making Units (DMUs) in presence of time factor and inter-temporal dependence of input-output levels. This technique is an interesting research issue in DEA field because of its importance in evaluating the performance of a company or organization in the assessment window. Fallah-Fini *et al.* [10] reported that the situations at which inter-temporal dependence of input-output levels may occur can be divided into five cases as follows: (i) production delays; (ii) inventories; (iii) capital or quasi-fixed factors; (iv) adjustment costs; and (v) incremental improvement and learning models. This paper deals with the case of dynamic DEA where the inter-temporal dependence takes place by changing the capital stock among various production periods. In this case, Emrouznejad [6] and Emrouznejad and Thanassoulis [7] proposed a linear programming (LP) model for evaluating the performance of DMUs. This model was revised by Jahanshahloo *et al.* [25]. Emrouznejad and Thanassoulis [8] provided a dynamic DEA model for estimating the dynamic Malmquist index. Another case of the inter-temporal dependence refers to the production processes in which some of the output levels produced in a time period are used as inputs in the next period. This kind of the inter-temporal dependence has been studied in many theoretical and applied publications, including Färe

Keywords. Data Envelopment Analysis (DEA), inverse DEA, multiple-objective linear programming (MOLP), inter-temporal dependence, (Periodic weak) Pareto solution.

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and Grosskopf [11], Sengupta [29], Nemoto and Goto [28], Sueyoshi and Sekitani [31], Tone and Tsutsui [32,33], Soleimani-Damaneh [30], and Ghobadi *et al.* [16]. Note that, the topic discussed in this paper under this kind of the inter-temporal dependence can be worth studying as well, though we do not pursue them in the present study.

A class of DEA models is called inverse DEA. The aim of a conventional DEA model is to estimate the efficiency score of a specific DMU with certain inputs and outputs, while the basic concept in an inverse DEA is to estimate the input and output levels for a special DMU to achieve a given efficiency target. An important general question in the field of inverse DEA is posed by Hadi-Vencheh and Foroughi [17] as follows: among a set of DMUs, to what extent should the input (output) levels of the DMU change if the efficiency index of a DMU remains unchanged, yet the output (input) levels change? Its question in a special case is studied by Wei *et al.* [34]. In Wei *et al.* [34] the input (output) increases of a specific DMU are estimated for its given output (input) increases under preserving the efficiency score. The question introduced by Wei *et al.* [34] in the traditional DEA is extended to the dynamic DEA of framework by Jahanshahloo *et al.* [24]. In the mentioned work, preserving the performance index under only increase some or all of the inputs (outputs) have been investigated.

In this paper, the above general question introduced by Hadi-Vencheh and Foroughi [17] in the field of inverse DEA is extended to dynamic DEA of framework. Note that, inputs (outputs) change means that some of the inputs (outputs) would be increased and some others decreased or remain the same. Therefore, answering this question in the framework of dynamic DEA is more general and includes Jahanshahloo *et al.* [24] as a special case. The problem of change in inputs (outputs) with maintaining the performance index, under inter-temporal dependent data, has not been studied, yet. This paper considers the arbitrary change in input (output) levels, while the other offered method [24] fail to simultaneously consider the arbitrary change of the various input (outputs) levels.

The result of this study can help policy makers to take better decisions for any change in the resources/products of a particular unit preserving the efficiency criterion.

The rest of the paper is organized as follows: Section 2 reviews literature on inverse DEA. Section 3 gives some preliminaries from inter-temporal dependence between input and output levels. Section 4 is devoted to the main results of the paper. A general model to estimate input (output) levels in the inverse DEA is extended in the framework of dynamic DEA. A numerical example with real data is presented to confirm the credibility (substantiate the accuracy) and applicability of our method in Section 5. Section 6 presents the conclusions of the paper.

2. Literature review on inverse DEA

The idea of inverse DEA for the first time was appeared in Zhang and Cui [36] to estimate inputs of a DMU under increasing outputs and preserving the efficiency score. Since then, some of the researches in the field of DEA from both theoretical and practical viewpoints have focused on inverse DEA. The general question introduced by Hadi-Vencheh and Foroughi [17] in the traditional DEA is answered based on Pareto solutions of MOLP problems by themselves, though its question in a special case is answered by Wei *et al.* [34]. In fact, they considered this question: among a set of decision making units (DMUs), if the efficiency index of a DMU remains unchanged, yet the output (input) levels increase, to what extent should the output (output) levels of the DMU increase? To answer this question, Wei *et al.* [34] proposed a MOLP and LP model where the DMU is inefficient and weakly efficient, respectively. The question introduced by Wei *et al.* [34] in the traditional DEA is extended to the dynamic DEA of framework by Jahanshahloo *et al.* [24]. In the mentioned work, preserving the performance index under only increase some or all of the inputs (outputs) have been investigated. In addition, this question is extended to the fuzzy data under inter-temporal dependence assumption by Ghobadi *et al.* [15]. The inverse DEA models can be used for sensitivity analysis [23], firms restructuring [3] setting revenue target [4,27], preserve (improve) efficiency values [21,22,26,34,35], resource allocation [18], and merging the banks [1,2,12]. Other studies on inverse DEA include: Jahanshahloo *et al.* [20], Ghobadi and Jahangiri [14], Hadi-vencheh *et al.* [19], Ghobadi [13], and Emrouznejad *et al.* [9].

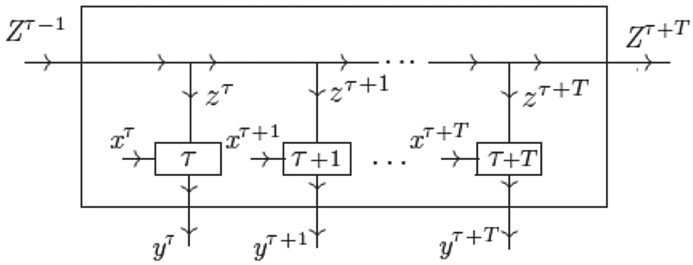

FIGURE 1. Production flow.

3. DEA DYNAMIC

In this section, a mathematical programming model is reviewed to estimate the efficiency score of DMU under inter-temporal dependent data.

Suppose that there exist a set of n observations of DMUs, whose performance is assessed in a time horizon, say $t = 1, 2, \ldots$. Furthermore, a window of periods $w = \{t \mid t = \tau, \tau+1, \ldots, \tau+T\}$ is considered as the assessment window. Assume that each DMU uses two kinds of inputs in each time period: period-specific inputs (denoted by x) and capital inputs (denoted by z) to produce a kind of output in each time period (denoted by y). The initial stock inputs supply capital inputs in any time period of assessment window. Since τ is considered as the initial time period in assessing window, so the initial stock input is represented by $Z^{\tau-1}$. Because the DMU survives after the terminal time period in the assessing window, so having more terminal capital stock is desirable. The terminal capital stock is denoted by $Z^{\tau+T}$, because $\tau+T$ is considered as the terminal time period in assessing window.

According to the above discussion, the set of inputs of DMU_j $(j = 1, 2, \ldots, n)$ is as follows:

period-specific input paths: $x_j^w = (x_{ij}^{\tau}, x_{ij}^{\tau+1}, \ldots, x_{ij}^{\tau+T} : \ \forall i \in I_1)$,
change in stock paths: $z_j^w = (z_{ij}^{\tau}, z_{ij}^{\tau+1}, \ldots, z_{ij}^{\tau+T} : \ \forall i \in I_2)$,
initial-stock inputs: $Z_j^{\tau-1} = (Z_{ij}^{\tau-1} : \ \forall i \in I_2)$,

where the set of inputs, $I = \{1, 2, \ldots, m\}$ is divided into two subsets $I_1, I_2 \subset I$, such that $I_1 \cup I_2 = I$ and $I_1 \cap I_2 = \phi$.

The set of outputs of DMU_j is as follows:

output paths: $y_j^w = (y_{rj}^{\tau}, y_{rj}^{\tau+1}, \ldots, y_{rj}^{\tau+T} : \ \forall r \in O = \{1, 2, \ldots, s\})$,
terminal-stock inputs as outputs: $Z_j^{\tau+T} = (Z_{ij}^{\tau+T} : \ \forall i \in I_2)$.

It is clear that

$$Z_{ij}^{\tau+T} = Z_{ij}^{\tau-1} - \textstyle\sum_{t\in w} z_{ij}^t \qquad \forall i \in I_2. \tag{3.1}$$

Figure 1 indicates a production flow in the assessment window:

To clarify the above discussion, it can be assumed that the evaluation of the performance of a set of research organizations (as DMUs) is under study. The main products of a research organization is invention, discovery, publications, etc. To produce these products, each research organization uses various resources including the resources received from the government, facilities (personnel, the research spaces, etc.) and equipment. In addition to usual inputs (resources), the research organizations have some capital grants which can be used to unpredictable expenses when the managers are not able to continue by the usual inputs. These inputs are called capital inputs and denoted by z^t.

The input matrixes $X^t_{|I_1|\times n}$ and $Z^t_{|I_2|\times n}$ and output matrix $Y^t_{s\times n}$, for each $t \in w$, can be represented as

$$X^t = [x_1^t, x_2^t, \ldots, x_n^t], \quad Z^t = [z_1^t, z_2^t, \ldots, z_n^t], \quad Y^t = [y_1^t, y_2^t, \ldots, y_n^t].$$

Also, the initial-stock input matrix $Z^{\tau-1}_{|I_2|\times n}$ and terminal-stock input matrix $Z^{\tau+T}_{|I_2|\times n}$ can be represented as

$$Z^{\tau-1} = [Z_1^{\tau-1}, Z_2^{\tau-1}, \ldots, Z_n^{\tau-1}], \quad Z^{\tau+T} = [Z_1^{\tau+T}, Z_2^{\tau+T}, \ldots, Z_n^{\tau+T}].$$

Suppose that $(x_j^w, z_j^w, y_j^w, Z_j^{\tau-1}, Z_j^{\tau+T})$ denote the assessment path of $\text{DMU}_j;\ j = 1, 2, \ldots, n$, in the assessment window w. Emrouznejad and Thanassoulis [7] introduced an LP model to estimate the efficiency score of the assessment path corresponding to a DMU. This model has been improved by Jahanshahloo *et al.* [25]. They proposed the following LP to evaluate the performance of the assessment path of $(x_o^w, z_o^w, y_o^w, Z_o^{\tau-1}, Z_o^{\tau+T})$, $o \in \{1, 2, \ldots, n\}$:

$$\begin{aligned}
\rho_o = \min\ & \frac{\sum_{t=\tau}^{T+\tau} \theta^t}{T+1} \\
\text{s.t. } & X^t\lambda \le \theta^t x_o^t, && \forall t \in w, \\
& Z^t\lambda \le \theta^t z_o^t, && \forall t \in w, \\
& Y^t\lambda \ge y_o^t, && \forall t \in w, \\
& Z^{\tau+T}\lambda \ge Z_o^{\tau+T}, \\
& Z^{\tau-1}\lambda \le Z_o^{\tau-1}, \\
& \theta^t \le 1, && \forall t \in w, \\
& \lambda \in \Omega,
\end{aligned} \tag{3.2}$$

where

$$\Omega = \left\{\lambda | \lambda \in R^n_{\ge 0},\ \delta_1(e\lambda + \delta_2(-1)^{\delta_3}\nu) = \delta_1,\ \nu \ge 0,\ e = (1, 1, \ldots, 1) \in R^n\right\}.$$

Here, δ_1, δ_2, and δ_3 are parameters with $0{-}1$ values. It is obvious that:

(i) If $\delta_1 = 0$, then model (3.2) is under a constant returns to scale (CRS) assumption of the production technology.
(ii) If $\delta_1 = 1$ and $\delta_2 = 0$, then model (3.2) is under a variable returns to scale (VRS) assumption of the production technology.
(iii) If $\delta_1 = \delta_2 = 1$ and $\delta_3 = 0$, then model (3.2) is under a non-increasing returns to scale (NIRS) assumption of the production technology.
(iv) If $\delta_1 = \delta_2 = \delta_3 = 1$, then model (3.2) is under a non-decreasing returns to scale (NDRS) assumption of the production technology.

In model (3.2), (λ, θ^w) is variables vector. ρ_o in model (3.2) is called the input-oriented efficiency score of the assessment path corresponding to DMU_o. It is not difficult to see that $\rho_o \le 1$.

The output-oriented version of model (3.2) is as follows:

$$\begin{aligned}
\Phi_o = \max\ & \frac{\sum_{t=\tau}^{T+\tau} \varphi^t}{T+1} \\
\text{s.t. } & X^t\lambda \le x_o^t, && \forall t \in w, \\
& Z^t\lambda \le z_o^t, && \forall t \in w, \\
& Y^t\lambda \ge \varphi^t y_o^t, && \forall t \in w, \\
& Z^{\tau+T}\lambda \ge Z_o^{\tau+T}, \\
& Z^{\tau-1}\lambda \le Z_o^{\tau-1}, \\
& \varphi^t \ge 1, && \forall t \in w, \\
& \lambda \in \Omega.
\end{aligned} \tag{3.3}$$

In the above model $(\lambda, \varphi^w = \varphi^{\tau,\tau+1,\dots,\tau+T})$ is variables vector. Φ_o in model (3.3) is called the output-oriented efficiency score of the assessment path corresponding to DMU_o. It is obvious that $\Phi_o \geq 1$.

The assessment path corresponding to DMU_o is called input-(res. output-) oriented weakly efficient if $\rho_o = 1$ (res. $\Phi_o = 1$).

4. Inverse DEA under inter-temporal dependence

In this section, the questions introduced by Hadi-Vencheh and Foroughi [17] in the traditional DEA are extended to dynamic DEA of framework. The basic concept in an inverse DEA is to estimate the input and output levels for a special DMU to achieve a given efficiency target, while the aim of a traditional DEA model is to estimate the efficiency score of a DMU with certain inputs and outputs.

In the beginning, the example mentioned in Section 2 is considered to illustrate the motivation of our work. Suppose that some research organizations define targets for some criterion, which are wished to get in future. For example, it is possible that the general policy in a research organization is decreasing the resources received from the government up to 10% (reducing government dependency) and increasing the research possibility up to 5% in the next 5 years preserving the efficiency criterion. The question is that to what extent the products should be changed to achieve this aim. The inverse DEA specifies to what extent the products should change. This section deals with this questions.

At first, the following question studied under inter-temporal dependence.

Question 1. If the efficiency index ρ_o remains unchanged, but the outputs change, to what extent should the inputs of DMU_o change?

To attain this goal, suppose the outputs of DMU_o are changed from y_o^w to $\beta_o^w = y_o^w + \Delta y_o^w$ in which $\Delta y_o^w \in R^s$. We find $(\alpha_o^{w*}, \eta_o^{w*}, \Gamma_o^{\tau-1*})$ provided that the efficiency score of DMU_o is still ρ_o. In fact,

$$(\alpha_o^{w*}, \eta_o^{w*}) = (x_o^w, z_o^w) + (\Delta x_o^w, \Delta z_o^w), \quad (\Delta x_o^w, \Delta z_o^w) \in R^{|I_1|} \times R^{|I_2|}, \tag{4.1}$$

$$\Gamma_o^{\tau-1*} = Z_o^{\tau-1} + \Delta Z_o^{\tau-1}, \qquad \Delta Z_o^{\tau-1} \in R^{|I_2|}. \tag{4.2}$$

Note that, since z_o^w value may change, according to (3.1), the initial and/or terminal capital stocks may also change. Since $Z_o^{\tau+T}$ is considered as output here, we supply the changes of z_o^w value by changing the initial capital stock, $Z_o^{\tau-1}$.

Suppose DMU_{n+1} represents the unit generated after changing the input and output vectors. The following model is proposed to measure the efficiency score of DMU_{n+1}:

$$\begin{aligned} \rho_{n+1} = \min\ & \frac{\sum_{t=\tau}^{T+\tau} \theta^t}{T+1} \\ \text{s.t. } & X^t\lambda + \lambda_{n+1}\alpha_o^{t*} \leq \theta^t \alpha_o^{t*}, && \forall t \in w, \\ & Z^t\lambda + \lambda_{n+1}\eta_o^{t*} \leq \theta^t \eta_o^{t*}, && \forall t \in w, \\ & Y^t\lambda + \lambda_{n+1}\beta_o^{t} \geq \beta_o^{t}, && \forall t \in w, \\ & Z^{\tau+T}\lambda + \lambda_{n+1}Z_o^{\tau+T} \geq Z_o^{\tau+T}, \\ & Z^{\tau-1}\lambda + \lambda_{n+1}\Gamma_o^{\tau-1*} \leq \Gamma_o^{\tau-1*}, \\ & \theta^t \leq 1, && \forall t \in w, \\ & (\lambda, \lambda_{n+1}) \in \Omega^+, \end{aligned} \tag{4.3}$$

where

$$\Omega^+ = \left\{(\lambda, \lambda_{n+1}) | \lambda \in R^n_{\geq 0},\ \delta_1(e\lambda + \lambda_{n+1} + \delta_2(-1)^{\delta_3}\nu) = \delta_1,\ \nu \geq 0,\ \lambda_{n+1} \geq 0\right\}.$$

The variable vector of the above model is $(\lambda, \lambda_{n+1}, \theta^\tau, \theta^{\tau+1}, \dots, \theta^{\tau+T})$.

If the optimal values of models (3.2) and (4.3) are equal, then the efficiency score of DMU_o remains unchanged, *i.e.*,

$$\text{eff}(\alpha_o^{w*}, \eta_o^{w*}, \beta_o^w, \Gamma_o^{\tau-1*}, Z_o^{\tau+T}) = \text{eff}(x_o^w, z_o^w, y_o^w, Z_o^{\tau-1}, Z_o^{\tau+T})^2.$$

To solve the question, the following MOLP model is proposed;

$$\begin{array}{lll}
\min & (\alpha_o^t \in R^{|I_1|} \ \& \ \eta_o^t \in R^{|I_2|}) & \forall t \in w \\
\text{s.t.} & X^t\lambda \leq \theta^{t*}\alpha_o^t, & \forall t \in w \\
& Z^t\lambda \leq \theta^{t*}\eta_o^t, & \forall t \in w \\
& Y^t\lambda \geq \beta_o^t, & \forall t \in w \\
& Z^{\tau+T}\lambda \geq Z_o^{\tau+T}, & \\
& Z^{\tau-1}\lambda \leq \Gamma_o^{\tau-1}, & \\
& \eta_o^t + Z_o^{\tau+1} = \Gamma_o^{\tau-1}, & \\
& \lambda \in \Omega. &
\end{array} \tag{4.4}$$

$(\lambda,\ \alpha_o^w,\ \eta_o^w, \Gamma_o^{\tau-1})$ is the variables vector in MOLP (4.4). $(\theta^{t*} :\ \forall t \in w)$ is the optimal solution of LP (3.2).

Definition 4.1. Let $\Lambda = (\lambda, \alpha_o^w, \eta_o^w, \Gamma_o^{\tau-1})$ be a feasible solution to MOLP (4.4). Then

(a) Λ is called a weak pareto (efficient) solution to MOLP (4.4) if there does not exist another feasible solution $\overline{\Lambda} = (\overline{\lambda},\ \overline{\alpha}_o^w,\ \overline{\eta}_o^w, \overline{\Gamma}_o^{\tau-1})$ such that $(\overline{\alpha}_o^w,\ \overline{\eta}_o^w) < (\alpha_o^w,\ \eta_o^w)$.

(b) Λ is called a periodic weakly pareto (efficient) solution to MOLP (4.4) if there does not exist another feasible solution $\overline{\Lambda} = (\overline{\lambda},\ \overline{\alpha}_o^w,\ \overline{\eta}_o^w, \overline{\Gamma}_o^{\tau-1})$ and some $p \in w$ such that $(\overline{\alpha}_o^p,\ \overline{\eta}_o^p) < (\alpha_o^p,\ \eta_o^p)$ and $(\overline{\alpha}_o^t,\ \overline{\eta}_o^t) \leq (\alpha_o^t,\ \eta_o^t)$ for each $t \in w - \{p\}$.

(c) Λ is called a pareto (efficient) solution to MOLP (4.4) if there does not exist another feasible solution $\overline{\Lambda} = (\overline{\lambda},\ \overline{\alpha}_o^w,\ \overline{\eta}_o^w, \overline{\Gamma}_o^{\tau-1})$ such that
  (i) $\alpha_{io}^t \leq \overline{\alpha}_{io}^t$ $(\forall i \in I_1)$ and $\eta_{io}^t \leq \overline{\eta}_{io}^t$ $(\forall i \in I_2)$ for each $t \in w$;
  (ii) for some $t \in w$, $\alpha_{io}^t < \overline{\alpha}_{io}^t$ for some $i \in I_1$ or $\eta_{io}^t < \overline{\eta}_{io}^t$ for some $i \in I_2$.

Suppose that the sets X_E, X_W, and X_{PWP} are denoted pareto, weak pareto, periodic weakly pareto solutions to MOLP (4.4), respectively. It is not difficult to see that $X_E \subseteq X_{\text{PWP}} \subseteq X_W$. See Jahanshahloo *et al.* [24]. The following theorem characterizes the periodic weak Pareto solutions of MOLP (4.4) using the same procedure of Theorem 4.5 in [24].

Theorem 4.2. *Let* $\Lambda = (\lambda, \alpha_o^w, \eta_o^w, \Gamma_o^{\tau-1})$ *be a feasible solution to MOLP (4.4).* Λ *is a periodic weakly Pareto solution of MOLP (4.4) if and only if there exist nonzero nonnegative weight row-vectors* $(u^\tau, v^\tau), (u^{\tau+1}, v^{\tau+1}), (u^{\tau+T}, v^{\tau+T}) \in R^{|I_1|} \times R^{|I_2|}$ *such that* Λ *is an optimal solution to the following LP:*

$$\begin{array}{ll}
\min & \sum_{t \in w} u^t\alpha_o^t + \sum_{t \in w} v^t\eta_o^t \\
\text{s.t.} & \text{The constraints of MOLP (4.4).}
\end{array} \tag{4.5}$$

The following Theorem shows how the above MOLP can be used for input estimation.

Theorem 4.3. *Suppose that the efficiency score of* DMU_o *is* $\rho_o = 1$. *If* $\Lambda = (\lambda^*, \alpha_o^{w*}, \eta_o^{w*}, \Gamma_o^{\tau-1*})$ *is an pareto solution to (4.4), in which* $(\alpha_o^{w*}, \eta_o^{w*}) = (x_o^w, z_o^w)$ *or* $x_o^t \neq \alpha_o^{t*}$ *for* $t \in w$, *then*

$$\text{eff}(\alpha_o^{w*}, \eta_o^{w*}, \beta_o^w, \Gamma_o^{\tau-1*}, Z_o^{\tau+T}) = \text{eff}(x_o^w, z_o^w, y_o^w, Z_o^{\tau-1}, Z_o^{\tau+T}).$$

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[2]Hereafter, we use the notation "eff" instead of "efficiency" for simplicity.

Proof. To prove the theorem, $\rho_{n+1} = \rho_o = 1$ should be shown. By contradiction assume that $\rho_{n+1} < \rho_o = 1$. On the other hand, $\theta^{t+} \leq 1$ for each $t \in w$. Therefore, $\theta^{t+} < 1$ for some $t \in w$.
Suppose $((\overline{\lambda} = (\lambda_1^+, \ldots, \lambda_n^+), \lambda_{n+1}^+), \theta^{\tau+}, \ldots, \theta^{\tau+T+})$ is an optimal solution of problem (4.3). Therefore,

$$X^t\overline{\lambda} + \lambda_{n+1}^+ \alpha_o^{t*} \leq \theta^{t+}\alpha_o^{t*}, \qquad \forall t \in w, \tag{4.6}$$

$$Z^t\overline{\lambda} + \lambda_{n+1}^+ \eta_o^{t*} \leq \theta^{t+}\eta_o^{t*}, \qquad \forall t \in w, \tag{4.7}$$

$$Y^t\overline{\lambda} + \lambda_{n+1}^+ \beta_o^{t} \geq \beta_o^{t}, \qquad \forall t \in w, \tag{4.8}$$

$$Z^{\tau+T}\overline{\lambda} + \lambda_{n+1}^+ Z_o^{\tau+T} \geq Z_o^{\tau+T}, \tag{4.9}$$

$$Z^{\tau-1}\overline{\lambda} + \lambda_{n+1}^+ \Gamma_o^{\tau-1*} \leq \Gamma_o^{\tau-1*}, \tag{4.10}$$

$$\lambda^+ = (\overline{\lambda}, \lambda_{n+1}^+) \in \Omega^+. \tag{4.11}$$

On the other hand, Λ is a feasible solution of MOLP (4.4) and $\theta^{t*} = 1$ for each $t \in w$ because $\rho_o = 1$. Therefore, we have

$$X^t\lambda^* \leq \theta^{t*}\alpha_o^{t*} = \alpha_o^{t*}, \qquad \forall t \in w, \tag{4.12}$$

$$Z^t\lambda^* \leq \theta^{t*}\eta_o^{t*} = \eta_o^{t*}, \qquad \forall t \in w, \tag{4.13}$$

$$Y^t\lambda^* \geq \beta_o^{t}, \qquad \forall t \in w, \tag{4.14}$$

$$Z^{\tau+T}\lambda^* \geq Z_o^{\tau+T}, \tag{4.15}$$

$$Z^{\tau-1}\lambda^* \leq \Gamma_o^{\tau-1*}, \tag{4.16}$$

$$\lambda^* \in \Omega. \tag{4.17}$$

□

According to (4.12)–(4.17), it is obvious that $((\lambda^*, 0), \theta^t = 1; \forall t \in w)$ is a feasible solution of LP (4.3) and hence $\rho_{n+1} \leq 1$. Now according to (4.6) and (4.12),

$$\theta^{t+}\alpha_o^{t*} \geq X^t\overline{\lambda} + \lambda_{n+1}^+\alpha_o^{t*} \geq X^t\overline{\lambda} + \lambda_{n+1}^+ X^t\lambda^* = (\overline{\lambda} + \lambda_{n+1}^+\lambda^*)X^t, \ \forall t \in w. \tag{4.18}$$

Let $\tilde{\lambda} = \overline{\lambda} + \lambda_{n+1}^+\lambda^*$, and write (4.18) as

$$\theta^{t+}\alpha_o^{t*} \geq X^t\tilde{\lambda}, \ \forall t \in w. \tag{4.19}$$

Using a similar method we have

$$\theta^{t+}\eta_o^{t*} \geq Z^t\tilde{\lambda}, \quad \forall t \in w, \tag{4.20}$$

$$\beta_o^t \leq Y^t\tilde{\lambda}, \quad \forall t \in w, \tag{4.21}$$

$$Z_o^{\tau+T} \leq Z^{\tau+T}\tilde{\lambda}, \tag{4.22}$$

$$\Gamma_o^{\tau-1*} \geq Z^{\tau-1}\tilde{\lambda}, \tag{4.23}$$

$$\tilde{\lambda} \in \Omega. \tag{4.24}$$

If $(\alpha_o^{w*}, \eta_o^{w*}) = (x_o^w, z_o^w)$, then by (4.19) and (4.20) we have

$$X^t\tilde{\lambda} \leq \theta^{t+}\alpha_o^{t*} = \theta^{t+}x_o^t, \;\; \forall t \in w, \tag{4.25}$$

$$Z^t\tilde{\lambda} \leq \theta^{t+}\eta_o^{t*} = \theta^{t+}z_o^t, \quad \forall t \in w. \tag{4.26}$$

In addition,

$$\Gamma_o^{\tau-1*} = Z_o^{\tau+T} - \sum_{t\in w}\eta_o^{t*} = Z_o^{\tau+T} - \sum_{t\in w} z_o^t = Z_o^{\tau-1}. \tag{4.27}$$

Hence, equations (4.21)–(4.27) imply that $(\tilde{\lambda}, \theta^{t+};\; \forall t \in w)$ is a feasible solution to LP (3.2). Therefore, $\rho_o \leq \frac{\sum_{t=\tau}^{\tau+T}\theta^{t+}}{T+1} < 1$. But this is against the assumption that $\rho_o = 1$ is the optimal value of LP (3.2).

Now, suppose $x_o^t \neq \alpha_o^{t*}$ for $t \in w$. According to contradiction assume and without loss of generality, we assume that $\theta^{p+} < 1$. By equations (4.19) and (4.20), we have

$$X^t\tilde{\lambda} \leq \theta^{t+}\alpha_o^{t*} \leq \alpha_o^{t*} = \theta^{t*}\alpha_o^{t*}, \;\; \forall t \in w - \{p\}, \tag{4.28}$$

$$X^p\tilde{\lambda} \leq \theta^{p+}\alpha_o^{p*} < \alpha_o^{t*} = \theta^{p*}\alpha_o^{p*}. \tag{4.29}$$

There exists a $0 < \mu < 1$ such that

$$X^p\tilde{\lambda} \leq \theta^{p*}(\mu\alpha_o^{p*}). \tag{4.30}$$

Now, define $\bar{\Gamma}_o^{\tau-1} = \Gamma_o^{\tau-1*}$, $\bar{\eta}_o^w = \eta_o^{w*}$, and

$$\bar{\alpha}_o^t = \begin{cases} \alpha_o^{t*} & \text{if} \quad t \in w - \{p\}, \\ \mu\alpha_o^{p*} & \text{if} \quad t = p. \end{cases}$$

Feasibility of Λ to MOLP (4.4) and definition of $\bar{\Gamma}_o^{\tau-1}$ and $\bar{\eta}_o^t$, implies

$$\bar{\Gamma}_o^{\tau-1} - \sum_{t\in w}\bar{\eta}_o^t = \Gamma_o^{\tau-1*} - \sum_{t\in w}\eta_o^{t*} = Z_o^{\tau+T}. \tag{4.31}$$

By (4.20) and (4.23), we have

$$Z^t\tilde{\lambda} \leq \theta^{t+}\eta_o^{t*} \leq \eta_o^{t*} = \theta^{t*}\eta_o^{t*}, \quad \forall t \in w, \tag{4.32}$$

$$Z^{\tau-1}\tilde{\lambda} \leq \Gamma_o^{\tau-1*} = \bar{\Gamma}_o^{\tau-1}. \tag{4.33}$$

$\bar{\Lambda} = (\tilde{\lambda}, \bar{\alpha}_o^w, \bar{\eta}_o^w, \bar{\Gamma}_o^{\tau-1})$ is a feasible solution to MOLP (4.4) by equations (4.21), (4.22), (4.24), (4.28), and (4.30)–(4.33). It is obvious $(\bar{\alpha}_o^w, \bar{\eta}_o^w) \leq (\alpha_o^{w*}, \eta_o^{w*})$ and $(\bar{\alpha}_o^w, \bar{\eta}_o^w) \neq (\alpha_o^{w*}, \eta_o^{w*})$. But this is impossible because Λ is a pareto solution of MOLP (4.4).

Remark 4.4. If min $(\alpha_o^t \in R^{|I_1|} : \forall t \in w)$ replaces the objective function of MOLP (4.4), then Theorem 4.3 will remain valid.

The proof of the following theorem is similar to the proof given by Theorem 4.2 in [24]. This theorem is a converse version of Theorem 4.3.

Theorem 4.5. *Suppose that* $\Lambda = (\lambda, \alpha_o^w, \eta_o^w, \Gamma_o^{\tau-1})$ *is a feasible solution to (4.4) in which*

$$\text{eff}(\alpha_o^w, \eta_o^w, \beta_o^w, \Gamma_o^{\tau-1}, Z_o^{\tau+T}) = \text{eff}(x_o^w, z_o^w, y_o^w, Z_o^{\tau-1}, Z_o^{\tau+T}).$$

Then Λ *is a (periodic) weak Pareto solution to (4.4).*

Now consider this question:

Question 2. If the efficiency index φ_o remains unchanged, but the inputs change, to what extent should the outputs of DMU_o change?

In fact, if the efficiency index φ_o remains unchanged, but the input levels of DMU_o are changed from (x_o^w, z_o^w) to $(\alpha_o^w, \eta_o^w) = (x_o^w, z_o^w) + (\Delta x_o^w, \Delta z_o^w)$ in which $(\Delta x_o^w, \Delta z_o^w) \in R^{|I_1|} \times R^{|I_2|}$, how much should the output levels of DMUo change? Note that, according to (3.1), the initial stock input has to change to $\Gamma_o^{\tau-1} = Z_o^{\tau+T} + \sum_{t\in w} \eta_o^t$.

The following MOLP model is proposed to estimate the output vector $\beta_o^w = y_o^w + \Delta y_o^w$ in which $\Delta y_o^w \in R^s$, provided that the efficiency score of DMUo is unchanged.

$$
\begin{aligned}
\max \quad & \beta_o^w = (\beta_{ro}^t;\ \forall t \in w,\ \forall r \in O) \\
\text{s.t.} \quad & X^t\lambda \le \alpha_o^t, && \forall t \in w \\
& Z^t\lambda \le \eta_o^t, && \forall t \in w \\
& Y^t\lambda \ge \varphi^{t*}\beta_o^t, && \forall t \in w \\
& Z^{\tau+T}\lambda \ge Z_o^{\tau+T}, \\
& Z^{\tau-1}\lambda \le \Gamma_o^{\tau-1}, \\
& \lambda \in \Omega.
\end{aligned} \tag{4.34}
$$

$(\lambda,\ \beta_o^w)$ is the variables vector in MOLP (4.34). $(\varphi^{t*} :\ \forall t \in w)$ is the optimal solution of LP (3.2).

Suppose DMU_{n+1} represents DMU_o after changing the input and output levels. The following model is proposed to measure the efficiency score of DMU_{n+1}:

$$
\begin{aligned}
\varphi_{n+1} = \max \ & \frac{\sum_{t=\tau}^{T+\tau} \varphi^t}{T+1} \\
\text{s.t.}\ & X^t\lambda + \lambda_{n+1}\alpha_o^t \le \alpha_o^t, && \forall t \in w, \\
& Z^t\lambda + \lambda_{n+1}\eta_o^t \le \eta_o^t, && \forall t \in w, \\
& Y^t\lambda + \lambda_{n+1}\beta_o^{t*} \ge \varphi^t\beta_o^{t*}, && \forall t \in w, \\
& Z^{\tau+T}\lambda + \lambda_{n+1}Z_o^{\tau+T} \ge Z_o^{\tau+T}, \\
& Z^{\tau-1}\lambda + \lambda_{n+1}\Gamma_o^{\tau-1*} \le \Gamma_o^{\tau-1*}, \\
& \varphi^t \ge 1, && \forall t \in w, \\
& (\lambda, \lambda_{n+1}) \in \Omega^+.
\end{aligned} \tag{4.35}
$$

The variables vector of the above model is $(\lambda, \lambda_{n+1}, \varphi^\tau, \varphi^{\tau+1}, \ldots, \varphi^{\tau+T})$.

If the optimal values of Model (3.3) and Model (4.35) are equal, then the efficiency score of DMUo remains unchanged, *i.e.*,

$$\mathrm{eff}(\alpha_o^w, \eta_o^w, \beta_o^{w*}, \Gamma_o^{\tau-1}, Z_o^{\tau+T}) = \mathrm{eff}(x_o^w, z_o^w, y_o^w, Z_o^{\tau-1}, Z_o^{\tau+T}).$$

The following theorem solves the above question. The proof of this theorem is similar to the proof given by Theorem 4.3 with some minor modifications.

Theorem 4.6. *Suppose that the efficiency score of* DMU_o *is* $\Phi_o = 1$. *If* $\Lambda = (\lambda^*, \beta_o^{w*})$ *is a (periodic weak) Pareto solution to (4.34), in which* $y_o^t \ne \beta_o^{t*}$ *for* $t \in w$, *then*

$$\mathrm{eff}(\alpha_o^w, \eta_o^w, \beta_o^{w*}, \Gamma_o^{\tau-1}, Z_o^{\tau+T}) = \mathrm{eff}(x_o^w, z_o^w, y_o^w, Z_o^{\tau-1}, Z_o^{\tau+T}).$$

The following theorem is a converse version of Theorem 4.6. The proof of this theorem is similar to the proof given by Theorem 4.5.

Theorem 4.7. *Suppose $\Lambda = (\lambda, \beta_o^w)$ is a feasible solution to (4.34) in which*

$$\mathrm{eff}(\alpha_o^w, \eta_o^w, \beta_o^w, \Gamma_o^{\tau-1}, Z_o^{\tau+T}) = \mathrm{eff}(x_o^w, z_o^w, y_o^w, Z_o^{\tau-1}, Z_o^{\tau+T}).$$

Then Λ is a (periodic) weak Pareto solution to (4.34).

5. A numerical illustration

In this section, we consider the dataset of 20 branches of an Iranian commercial bank in Isfahan province in a three-month period. The dataset is picked up from the Central Branch Data Center and given in Table A.1 in the appendix. Generally, there are two approaches to determine the inputs and outputs of a bank: the production approach and the intermediation approach. In the production approach, the bank is considered as a manufacturer which is exactly the same manufacturer in the product market. Therefore, inputs and outputs are considered physical entities (labour and capital) and all deposits, respectively. However, the selection is based on the assets and liabilities of the bank in the intermediation approach, so purchased funds, borrowed funds (time deposits and other borrowed funds), core deposits, labour, and capital are considered usually as input and total loans, securities and other earning assets are considered usually as outputs. In this study, input and output items are defined according to the intermediation approach.

Here, the inter-temporal dependence occurred in 3-month period ($w = \{1, 2, 3\}$). For each time, two period-specific inputs, a capital input, and three outputs are defined. Period-specific inputs consist of employees score (x_1) and deferred claims (x_2). In addition to the usual resources, each branch of the bank receives financial assistance from the central branch when the managers are not able to continue by the usual resources. Such grants are considered as capital stock. According to the level and rank of the branch, the maximum amount of donation is fixed in a given time period (assessment window). Therefore, we consider it as an initial capital or overall fund ($Z^{\tau-1}$). During this period, initial capital is divided between branches for different periods of time, the remaining amount is represented by $Z^{T+\tau}$ as terminal-stock. Outputs consist of the loans (y_1), deposit (y_2) and profit (y_3). The related data is listed in Table A.1 in the appendix.

Using Model (3.2), under constant returns to scale (CRS) assumption, the efficiency score for each of the branches of the bank is obtained and reported in Table 1.

From Table 1, for example, it can be seen that B04 is efficient ($\rho_{04}^* = 1$). An inverse DEA case is illustrated in the following: Consider B04 defines targets for some criteria, which are wished to be achieved in the future. In other words, suppose that B04 aims to investigate this issue: among banks under study, to what extent should the input levels (usual and capital) change if its efficiency index remains unchanged, yet the output levels (loans, deposit, and profit) change from y^w to β^w as in Table 2. More precisely, the percentage of expected changes in the deposit (y_1), loans (y_2), and income commission (y_3) are given in Table 2 and Figure 2.

As it can be seen from Table 2, outputs y_1^1, y_1^2, y_2^2, y_1^3, and y_2^3 have increased up to $5\%, 10\%, 5\%, 10\%$, and 5%, respectively, while outputs y_2^1, y_3^1, and y_3^2 have decreased up to $10\%, 5\%$, and 10%, respectively. In addition, y_3^3 and terminal stock have not changed.

Considering MOLP (4.4) corresponding to B04 and using the weight-sum method [5], two Pareto solutions are generated to estimate the input vector (usual, capital, and initial stock) as reported in Table 3. More precisely, the percentage of necessary changes of the input levels (usual and stock inputs) should be similar to Table 3.

Existing inputs and proposed inputs for B04 are shown in Figure 3.

Therefore, due to Theorem 4.3, when the output levels of B04 change from y^w to β^w, the input levels (usual and input stock) should change to (α^w, η^w) if we would like to preserve the efficiency score of this bank. In other words, in order to preserve the efficiency index, B04 can choose one of the following two approaches:

(i) The period-specific input paths x_1^1, x_1^2, and x_1^3 should increase up to $1.58\%, 1.72\%$, and 1.75%, respectively, while all the period-specific input paths of x_2^1, x_2^2, and x_2^3 should decrease up to 31.99%. In addition, the stock input z^1 should increase up to 19.75% and the stock inputs z^2 and z^3 should decrease up to 11.48% and 0.47%, respectively. Consequently, the initial capital stock should increase up to 16.42%.

TABLE 1. The efficiency score of 20 bank branches in the three-month period.

DMUs	B01	B02	B03	B04	B05	B06	B07	B08	B09	B10
Efficiency in 1th period (θ^{1*})	0.4129	1.0000	1.0000	1.0000	1.0000	0.9392	1.0000	1.0000	1.0000	0.9313
Efficiency in 2th period (θ^{2*})	0.5997	1.0000	0.6173	1.0000	1.0000	0.7621	1.0000	1.0000	1.0000	0.7348
Efficiency in 3th period (θ^{3*})	0.6005	1.0000	0.6445	1.0000	1.0000	0.7628	1.0000	1.0000	1.0000	0.7357
Efficiency Score (ρ^*)	0.5377	1.0000	0.7539	1.0000	1.0000	0.8214	1.0000	1.0000	1.0000	0.8006
DMUs	B11	B12	B13	B14	B15	B16	B17	B18	B19	B20
Efficiency in 1th period (θ^{1*})	1.0000	1.0000	0.9631	1.0000	1.0000	1.0000	1.0000	1.0000	0.9853	1.0000
Efficiency in 2th period (θ^{2*})	1.0000	1.0000	0.9797	1.0000	1.0000	1.0000	1.0000	1.0000	0.8403	1.0000
Efficiency in 3th period (θ^{3*})	1.0000	1.0000	0.8698	1.0000	1.0000	1.0000	1.0000	1.0000	0.8392	1.0000
Efficiency Score (ρ^*)	1.0000	1.0000	0.9375	1.0000	1.0000	1.0000	1.0000	1.0000	0.8883	1.0000

TABLE 2. The percentage of expected changes in the outputs B04.

Period	$t=1$			$t=2$			$t=3$			$t=34$
Outputs	Loans (y_1)	Deposit (y_2)	Profit (y_3)	Loans (y_1)	Deposit (y_2)	Profit (y_3)	Loans (y_1)	Deposit (y_2)	Profit (y_3)	Terminal stock (Z)
Old output (y^w)	65 413 851	101 707 340	3 898 218	69 347 301	100 617 551	3 917 817	73 820 219	100 647 643	3 924 818	10 348 292
New output (β^w)	68 684 544	91 536 606	3 703 307	76 282 031	105 648 429	3 526 035	81 202 241	105 680 025	3 924 818	10 348 292
Percentage changes	5%	–10%	–5%	10%	5%	–10%	10%	5%	0%	0%

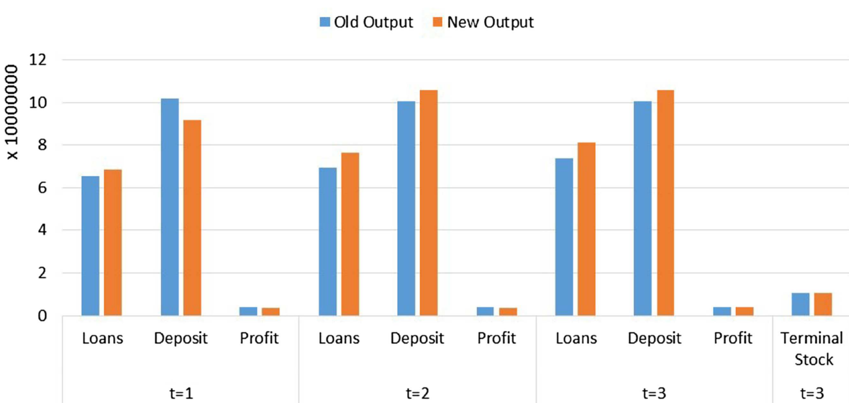

FIGURE 2. The old and new outputs B04 in the three-month period.

TABLE 3. The percentage of necessary changes in the inputs B04.

Period	$t=1$			$t=2$			$t=3$			Initial stock
Inputs	Employees Score (x_1)	Deferred Claims (x_2)	Change Stock (z)	Employees Score (x_1)	Deferred Claims (x_2)	Change Stock (z)	Employees Score (x_1)	Deferred Claims (x_2)	Change Stock (z)	Overall Fund (Z)
New Inputs-1 (α^w, η^w)	15.146	704 050	8 398 000	7.822	704 050	5 454 400	7.8344	702 780	9 804 700	34 005 392
Percentage Changes	1.58%	–31.99%	19.75%	1.72%	–31.99%	–11.84%	1.75%	–31.99%	–0.47%	16.42%
New Inputs-2 (α^w, η^w)	16.401	1 138 700	4 140 200	8.459	1 138 700	6 805 600	8.47	1 136 500	10 837 000	32 131 092
Percentage Changes	10%	10%	46.65%	8.14%	10%	10%	8.11%	10%	10%	10%

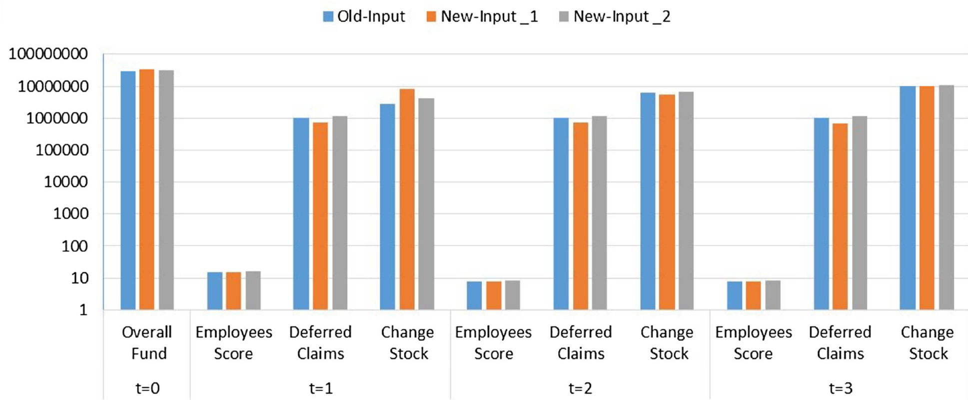

FIGURE 3. The old and proposed inputs B04 in the three-month period.

(ii) The period-specific input paths x_1^1, x_1^2, x_1^3, x_2^1, x_2^2, and x_2^3 should increase up to 10%, 10%, 8.14%, 10%, 8.11%, and 10%, respectively. The stock inputs z^1, z^2, and z^3 should increase up to 46.65%, 10%, and 10%, respectively. In this case, the initial capital stock should increase up to 10%.

6. CONCLUSION

This paper extended the following question in the field of inverse DEA to the dynamic DEA: how should decision maker control the changes in inputs (outputs) of a given DMU in which the efficiency score of the DMU is preserved? This question is answered using periodic weak Pareto solutions to MOLP problems under inter-temporal dependence.

In this study, the simultaneous increase and the decrease of the various inputs (outputs) are considered while other methods are studied estimate outputs (inputs) for a given DMU when some or all inputs (or outputs) are increased. In other words, the given results in this paper are more general and includes Jahanshahloo *et al.* [24] as a special case. To illustrate the provided inverse DEA method, an application in banking sector is discussed to achieve a given efficiency target.

The given results are important practically, because these can be used for firms restructuring, merging the banks, sensitivity analysis, resource allocation, and setting revenue target. These can help policy makers to take better decisions for any change in the resources/products of a particular DMU preserving the efficiency criterion.

Here, following research topics are suggested:

– Similar models can be investigated for the other case of inter-temporal dependence (when some of the output levels produced in a time period is used as inputs in the next period).
– Similar models can be investigated for dynamically inefficient DMUs.
– Similar models can be developed in presence of stochastic or negative data.

Appendix A.

Table A.1. The data of 20 bank branches in the three-month period.

Period	Bank	B01	B02	B03	B04	B05	B06	B07	B08	B09	B10
	Employees Score (x_1)	19.83	7.08	4.01	14.91	5.33	12.84	15.72	10.94	13.08	15.97
	Deferred Claims (x_2)	4 603 910	9547	136 115	1 035 215	1 030 194	2 664 633	1 086 083	225 665	7 480 348	3 486 536
$t=1$	Change Stock (z)	25 415 944	631 223	39 350 793	2 823 106	3 536 597	56 976 524	260 385 226	22 036 925	310 092	163 475 721
	Loans (y_1)	75 097 467	25 258 238	60 530 507	65 413 851	45 667 593	157 015 854	462 186 659	105 618 280	54 863 663	274 163 028
	Deposit (y_2)	80 023 776	40 413 775	48 420 589	101 707 340	67 411 796	163 070 472	397 280 289	130 611 124	90 586 108	229 181 190
	Profit (y_3)	2 211 465	767 801	1 770 276	3 898 218	1 291 753	4 334 957	16 849 485	4 072 227	2 335 849	5 869 352
	Employees Score (x_1)	9.38	5.66	4.76	7.69	3.67	11	11.04	7.7	10.44	12.24
	Deferred Claims (x_2)	4 258 676	9547	136 115	1 035 215	837 568	1 217 053	406 113	22 915	7 479 093	3 674 116
$t=2$	Change Stock (z)	33 423 425	2 635 237	42 532 282	6 186 940	7 640 404	73 818 634	237 525 938	21 522 051	3 651 731	193 337 240
	Loans (y_1)	77 540 480	28 099 162	63 386 263	69 347 301	51 678 971	165 665 373	457 271 922	111 799 242	63 460 385	300 159 210
	Deposit (y_2)	75 205 729	40 501 342	49 871 113	100 617 551	69 103 984	158 442 184	408 482 255	141 070 488	96 511 315	241 263 111
	Profit (y_3)	2 372 091	767 801	1 779 213	3 917 817	1 387 823	4 786 804	17 214 726	4 089 487	2 467 889	6 667 268
	Employees Score (x_1)	9.39	5.67	4.76	7.7	3.68	11.01	11.06	7.71	10.46	12.25
	Deferred Claims (x_2)	4 164 067	9 547	136 115	1 033 215	786 287	1 267 053	395 522	225 915	7 477 837	3 657 016
$t=3$	Change Stock (z)	34 302 366	6 014 493	39 988 455	9 851 458	13 105 122	70 323 601	229 207 401	17 014 036	10 980 081	213 492 432
	Loans (y_1)	79 100 108	31 026 240	64 986 239	73 820 219	58 194 638	174 946 921	465 979 022	116 557 642	79 367 959	308 891 608
	Deposit (y_2)	76 524 475	39 083 163	53 471 773	100 647 643	70 522 315	174 602 767	426 766 238	156 249 562	107 494 554	239 236 387
	Profit (y_3)	2 608 693	767 801	1 936 626	3 924 818	1 400 253	4 898 347	17 976 626	4 304 301	2 469 807	7 726 739
$t=0$	Overall Fund (Z^0)	117 244 609	26 566 357	158 665 116	29 209 796	39 252 881	267 062 245	1 037 209 839	87 622 951	30 043 956	729 364 697
$t=3$	Terminal Stock (Z^3)	24 102 874	17 285 404	36 793 586	10 348 292	14 970 758	65 943 486	310 091 274	27 049 939	15 102 052	159 059 304

Table A.1. Continued.

Period	Bank	B11	B12	B13	B14	B15	B16	B17	B18	B19	B20
	Employees Score (x_1)	4.53	3.88	14.6	12.19	9.93	3.2	19.25	6.18	6	7.86
	Deferred Claims (x_2)	1 531 195	106 162	430 201	288 733	79 572	23 274	655 170	70 840	5 771 009	604 842
$t=1$	Change Stock (z)	1 978 878	2 484 718	55 740 403	45 231 323	3 673 025	10 685 654	45 300 766	63 930 945	25 973 395	5 532 201
	Loans (y_1)	27 052 607	32 767 317	164 983 122	142 754 970	48 940 586	30 547 469	178 762 561	88 958 994	49 562 263	40 493 880
	Deposit (y_2)	40 188 125	48 484 615	175 915 348	155 702 255	72 499 127	32 161 835	209 467 807	67 461 848	44 092 348	54 972 684
	(y_3)	557 744	1 044 161	6 675 662	4 650 278	2 145 795	923 161	9 683 107	4 552 123	695 511	1 539 726
	Employees Score (x_1)	3.22	4.07	11.19	9.09	5.14	3.37	12.02	6.33	5.64	4.1
	Deferred Claims (x_2)	1 512 675	106 162	430 201	288 733	79 572	23 274	654 770	68 785	152 093	604 842
$t=2$	Change Stock (z)	4 830 066	3 853 539	50 458 213	63 179 340	2 663 053	16 251 577	36 316 407	61 785 788	29 507 223	2 316 070
	Loans (y_1)	28 785 809	37 246 308	174 059 672	147 933 524	56 425 243	32 096 496	186 258 838	92 482 607	51 036 143	44 447 422
	Deposit (y_2)	37 491 612	52 947 228	195 596 771	144 033 211	87 184 148	29 011 554	234 294 279	72 931 204	43 318 395	68 159 592
	Profit (y_3)	559 513	1 059 440	6 683 103	5 174 718	2 518 669	925 370	10 570 787	4 761 568	695 511	1 563 397
	Employees Score (x_1)	3.22	4.07	11.2	9.1	5.15	3.37	12.04	6.33	5.65	4.1
	Deferred Claims (x_2)	1 512 675	106 162	430 201	277 182	79 572	23 274	654 770	68 211	148 329	604 842
$t=3$	Change Stock (z)	8 979 634	5 806 457	60 083 361	91 013 826	6 205 102	14 326 858	39 962 610	61 511 021	31 628 113	6 803 359
	Loans (y_1)	31 425 356	40 643 483	175 858 151	153 347 359	60 613 709	33 842 858	215 702 229	93 642 456	54 119 292	47 543 624
	Deposit (y_2)	35 479 015	54 711 936	186 753 718	128 483 906	86 298 673	33 071 151	274 686 310	74 504 425	45 692 855	63 980 358
	Profit (y_3)	579 594	1 069 796	6 685 865	5 174 718	2 764 479	966 104	10 576 311	5 066 401	698 963	1 571 893
$t=0$	Overall Fund (Z^0)	34 213 206	34 626 288	218 101 708	282 355 290	27 056 255	51 708 898	234 220 827	254 370 473	134 860 848	25 822 016
$t=3$	Terminal Stock (Z^3)	18 424 628	22 481 574	51 819 731	82 930 801	14 515 075	10 444 809	112 641 044	67 142 719	47 752 117	11 170 386

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References

[1] G.R. Amin and S. Al-Muharrami, A new inverse data envelopment analysis model for mergers with negative data. *IMA J. Manage. Math.* **29** (2016) 137–149.
[2] G.R. Amin, A. Emrouznejad and S. Gattoufi, Minor and major consolidations in inverse DEA: definition and determination. *Comput. Ind. Eng.* **103** (2017) 193–200.
[3] G.R. Amin, A. Emrouznejad and S. Gattoufi, Modelling generalized firms' restructuring using inverse DEA. *J. Prod. Anal.* **48** (2017) 51–61.
[4] L. Dong Joon, Inverse DEA with frontier changes for new target setting. *Eur. J. Oper. Res.* **254** (2016) 510–516.
[5] M. Ehrgott, Multicriteria Optimization. Springer Berlin, Berlin (2005).
[6] A. Emrouznejad, An alternative DEA measure: a case of oecd countries. *Appl. Econ. Lett.* **10** (2003) 779–782.
[7] A. Emrouznejad and E. Thanassoulis, A mathematical model for dynamic efficiency using data envelopment analysis. *Appl. Math. Comput.* **160** (2005) 363–378.
[8] A. Emrouznejad and E. Thanassoulis, Measurement of productivity index with dynamic DEA. *Int. J. Oper. Res.* **8** (2010) 247–260.
[9] A. Emrouznejad, G.-L. Yang and G.R. Amin, A novel inverse DEA model with application to allocate the CO_2 emissions quota to different regions in chinese manufacturing industries. *J. Oper. Res. Soc.* **70** (2019) 1079–1090.
[10] S. Fallah-Fini, K. Triantis and A.L. Johnson, Reviewing the literature on non-parametric dynamic efficiency measurement: state-of-the-art. *J. Prod. Anal.* **41** (2014) 51–67.
[11] R. Färe and S. Grosskopf, Intertemporal production Frontiers: With Dynamic DEA. Kluwer Academic Publishers, Dordrecht (1996).
[12] S. Gattoufi, G.R. Amin and A. Emrouznejad, A new inverse DEA method for merging banks. *IMA J. Manage. Math.* **25** (2014) 73–87.
[13] S. Ghobadi, Inputs and outputs estimation in inverse DEA. *Iran. J. Optim.* **9** (2017) 119–129.
[14] S. Ghobadi and S. Jahangiri, Inverse DEA: review, extension and application. *Int. J. Technol. Decis. Making* **14** (2015) 805–824.
[15] S. Ghobadi, G.R. Jahanshahloo, F. Hoseinzadeh Lotfi and M. Rostami-malkhalifeh, Dynamic inverse DEA in the presence of fuzzy data. *Adv. Environ. Biol.* **8** (2014) 139–151.
[16] S. Ghobadi, G.R. Jahanshahloo, F. Hoseinzadeh Lotfi and M. Rostami-malkhalifeh, Efficiency measure under inter-temporal dependence. *Int. J. Technol. Decis. Making* **17** (2018) 657–675.
[17] A. Hadi-Vencheh and A.A. Foroughi, A generalized DEA model for inputs/outputs estimation. *Math. Comput. Model.* **43** (2006) 447–457.
[18] A. Hadi-Vencheh, A.A. Foroughi and M. Soleimani-Damaneh, A DEA model for resource allocation. *Econ. Model.* **25** (2008) 983–993.
[19] A. Hadi-vencheh, A. Hatami-marbini, Z. Ghelej Beigi and K. Gholami, An inverse optimization model for imprecise data envelopment analysis. *Optimization* **64** (2015) 2441–2452.
[20] G.R. Jahanshahloo, F. Hoseinzadeh Lotfi, M. Rostami-malkhalifeh and S. Ghobadi, Using enhanced russell model to solve inverse data envelopment analysis problems. *Sci. World J.* **2014** (2014) 1–10.
[21] G.R. Jahanshahloo, F.H. Lotfi, N. Shoja, G. Tohidi and S. Razavyan, Input estimation and identification of extra inputs in inverse DEA models. *Appl. Math. Comput.* **156** (2004) 427–437.
[22] G.R. Jahanshahloo, F.H. Lotfi, N. Shoja, G. Tohidi and S. Razavyan, The outputs estimation of a dmu according to improvement of its efficiency. *Appl. Math. Comput.* **147** (2004) 409–413.
[23] G.R. Jahanshahloo, F.H. Lotfi, N. Shoja, G. Tohidi and S. Razavyan, Sensitivity of efficiency classifications in the inverse DEA models. *Appl. Math. Comput.* **169** (2005) 905–916.
[24] G.R. Jahanshahloo, M. Soleimani-damaneh and S. Ghobadi, Inverse DEA under inter-temporal dependence using multiple-objective programming. *Eur. J. Oper. Res.* **240** (2015) 447–456.
[25] G.R. Jahanshahloo, M. Soleimani-damaneh, and M. Reshadi, On the pareto (dynamically) efficient paths. *Int. J. Comput. Math.* **63** (2006) 629–633.
[26] S. Lertworasirikul, P. Charnsethikul and S.C. Fang, Inverse data envelopment analysis model to preserve relative efficiency values: the case of variable returns to scale. *Comput. Ind. Eng.* **61** (2011) 1017–1023.
[27] H.T. Lin, An efficiency-driven approach for setting revenue target. *Decis. Support Syst.* **49** (2010) 311–317.
[28] J. Nemoto and M. Goto, Dynamic data envelopment analysis: modeling intertemporal behavior of a firm in the presence of productive inefficiencies. *Econ. Lett.* **64** (1999) 51–56.
[29] J.K. Sengupta, Dynamics of data envelopment analysis: theory of systems efficiency. *J. Oper. Res. Soc.* **47** (1996) 14–21.
[30] M. Soleimani-Damaneh, An enumerative algorithm for solving nonconvex dynamic DEA models. *Optim. Lett.* **7** (2013) 101–115.
[31] T. Sueyoshi and K. Sekitani, Returns to scale in dynamic DEA. *Eur. J. Oper. Res.* **16** (2005) 536–544.
[32] K. Tone and M. Tsutsui, Ddynamic DEA: a slacks-based measure approach. *Omega* **38** (2010) 145–156.
[33] K. Tone and M. Tsutsui, Dynamic DEA with network structure: a slacksbased measure approach. *Omega* **42** (2014) 124–131.

[34] Q.L. Wei, J.Z. Zhang and X.S. Zhang, An inverse DEA model for inputs/outputs estimate. *Eur. J. Oper. Res.* **121** (2000) 151–163.

[35] H. Yan, Q.L. Wei and G. Hao, DEA models for resource reallocation and production input/output estimation. *Eur. J. Oper. Res.* **136** (2002) 19–31.

[36] X. Zhang and J. Cui, A project evaluation system in the state economic information system of china: an operation research practice in public sectore. *Int. Trans. Oper.* **6** (1999) 441–452.