RAIRO-Oper. Res. 54 (2020) 325–339
https://doi.org/10.1051/ro/2018098

RAIRO Operations Research
www.rairo-ro.org

DEA BASED PRODUCTION PLANNING CONSIDERING TECHNOLOGY HETEROGENEITY WITH UNDESIRABLE OUTPUTS

Changyong Liang[1,*], Binyou Wang[1], Tao Ding[2,*] and Yinchao Ma[1]

Abstract. Many researchers have concentrated on production planning issues by using data envelopment analysis (DEA). However, the assumption made by existing approaches that all decision making units (DMUs) are equipped with the same level of production technology is not realistic. Additionally, with the development in the society, environmental factors have come to play important roles in the production process as well. Thus, undesirable outputs should be considered in production planning problems. Therefore, this paper considers the technology heterogeneity factors and undesirable outputs using the data envelopment analysis-based production planning approach. Two examples containing a numerical example that compare with other method and a real sample that concerns the industrial development of 30 provinces in China are used to validate the applicability of our approach.

Mathematics Subject Classification. 90B30, 90B50.

Received August 4, 2016. Accepted November 10, 2018.

1. Introduction

Data envelopment analysis (DEA), introduced by Charnes *et al.* [1] , is a non-parametric linear programming tool which evaluates the relative efficiency of decision-making units (DMUs) with respect to multiple inputs and outputs. DEA is an efficiency estimation technique ([2],[3]), but it can be used for solving many problems of decision making and management. Recently, many researchers have explored the DEA method and applied it to numerous fields, such as resource allocation and production planning ([4]–[7]).

From the perspective of productivity and efficiency, production planning is an emerging and important issue in DEA research. Recently, several DEA-based studies, such as those of Du *et al.* [8], Hosseinzadeh Lotfi and Moghtaderi [9], Amirteimoori and Kordrostami [10] and Amirteimoori and Kordrostami [11], have investigated the problem of production planning in a centralized situation. Du *et al.* [8] propose two production planning methods in a centralized decision making environment with the aim of forecasting demand in the next production period. Hosseinzadeh Lotfi and Moghtaderi [9] present an output change based on a model proposed earlier by Du *et al.* [8]. Moreover, Amirteimoori and Kordrostami [10] consider the size of operating units to ensure that the production plan matches the production capacity. In another work, Amirteimoori and Kordrostami [11] present

Keywords. Data envelopment analysis, production planning, undesirable outputs, technology heterogeneity.

[1] School of Management, Hefei University of Technology, Hefei, Anhui 230009, PR China.
[2] School of Economics, Hefei University of Technology, Hefei, Anhui 230009, PR China.
[*]Corresponding author: hfutdingtao@126.com

an optimal production planning model, which considers the size of the operation units and the production level for each unit, to identify the corresponding changes of each unit's performance.

However, the above mentioned DEA-based approaches of production planning implicitly assume that all DMUs are equipped with the same level of production technology. Creating a production plan by assuming that all DMUs can achieve a common frontier in practice is both impractical and irrational. Furthermore, failing to consider technology heterogeneity would lead to biased production plans under an inappropriate efficient frontier. Considering technology heterogeneity in their analysis, Battese and Rao [12] propose the meta-frontier production function framework, which utilizes the stochastic frontier approach (SFA), in investigating the environmental efficiency of firms from various groups having different technologies. Furthermore, Battese *et al.* [13] present a modified model containing the deterministic components of the stochastic frontier production functions. In another work, O'Donnell *et al.* [14] construct the meta-frontier based on a DEA method instead of the SFA. Zhang *et al.* [15], taking the influencing factors into consideration, propose two approaches aimed at satisfying the demand changes. They simultaneously sought to forecast the minimum total input consumption to arrange new input–output plans for DMUs when demand changed in the next production season. However, Battese and Rao [12] , Battese *et al.* [13] and O'Donnell *et al.* [14] only focus on the evaluation of DMUs. Meanwhile, the method of Zhang *et al.* [15] assumes all DMUs have done their best in the last production season and efficiency would remain unchanged in new plans is deemed unrealistic. Although the aforementioned studies consider technology heterogeneity, they fail to apply the centralized decision-making environment in the meta-frontier approach which can optimize the average or overall production performance of the entire organization, to the process of production planning.

Since Färe *et al.* [17] introduce the first non-parametric model with undesirable outputs, the DEA literature has featured many other studies on such undesirable outputs. Two main methods of integrating undesirable outputs into the DEA efficiency evaluation have been proposed in the literature. The early approach uses conventional DEA models, but with a particular treatment of undesirable outputs. For example, Dyckhoff and Allen [18] consider the DEA method as a multi-criteria approach that models the undesirable output as the input. Seiford and Zhu [19] propose a linear monotonically decreasing transformation approach, wherein a sufficiently large positive scalar is added to the reciprocal additive transformation of the undesirable, thus making the final values for each DMU positive. Since then, a new method of dealing with undesirable outputs has been extensively explored, that is, establishing DEA models that are particularly tailored to handle undesirable outputs. For example, Färe *et al.* [20] suggest an alternative approach based on the directional distance function, in order to increase desirable outputs and decrease undesirable outputs. Recently, variable methods of handling the undesirable outputs have been applied into practice. For example, Lozano *et al.* [21] propose a directional distance approach to deal with undesirable outputs problems in network structures and apply their model to the problem of modeling and benchmarking airport operations in Spain. Liu *et al.* [22] investigate two-stage DEA models with undesirable input-intermediate-outputs and apply this approach to evaluate the efficiencies of China's listed banks. Zanella *et al.* [23] propose a new composite indicator model based on a directional distance function and study the assessment of Brazilian hydropower plants. Arabi *et al.* [24] incorporate the most endogenous direction among all other possible directions to increase the desirable output and decrease the undesirable outputs simultaneously. Khalili–Damghani and Shahmir [25] propose a customized DEA method for solving the problem in returns to scale in the presence of uncertain data and undesirable outputs; they then apply the approach to a six-year study of 17 combined cycle power plants in Iran. Ignatius *et al.* [3] thoroughly investigate the evaluation problems that ambiguous in the real world and propose a fuzzy number model in DEA.

Thus, taking technology heterogeneity factors and undesirable outputs into consideration, we develop a DEA-based production planning approach to determine new production plans for all the individuals under a central authority. The new centralized model is under the constant returns to scale (CRS) assumptions. Our new approach has two main principles. First, the efficiencies of all DMUs in the next period should be greater than or equal to their relative efficiencies in the current period. Second, the efficiencies of all DMUs in the next period should not exceed their technology level. A distinct feature of the current work is that it focuses not only on the efficiency principle but also on practical feasibility. Through our proposed models, we not only obtain the production planning in the next production season while considering the technology heterogeneity of DMUs, but we also deal with the undesirable outputs in the production plan problems.

The structure of this paper is organized as follows: Section 2 presents the meta-frontier and group frontier concepts, and further constructs meta-efficiency and group efficiency. Section 3 proposes a centralized DEA-based model considering the meta-technology ratio of undesirable outputs to production plans under CRS assumptions. This approach is demonstrated by a numerical example in Section 4. Section 4 also applies this approach to an industrial development plan, which covers 30 provinces in China, and further analyzes the results. Section 5 concludes this paper.

2. Meta-frontier and group frontier analysis

2.1. Meta-frontier efficiency model

In many organizations with a centralized decision-making environment such as factories and supermarket chains with many workshops, production usually involves the participation of more than one individual unit and each unit contributes to part of total production. We assume that there are a set of n DMUs denoted by $\text{DMU}_j (j = 1, \ldots, n)$. Every DMU has m inputs, denoted as $X_j = \{x_{1j}, x_{2j}, \ldots, x_{mj}\}$, s outputs, denoted as $Y_j = \{y_{1j}, y_{2j}, \ldots, y_{sj}\}$. DEA-efficient DMUs represent the most advanced technologies, by which a production frontier can be constructed. Moreover, DMUs grasp the same level of production technology, resulting in a meta-frontier concept ([14]). The meta-frontier describes a production plan that is technologically feasible ([14]) and is defined as follows:

$$T^M = \left\{ (x, y) \middle| x \geq \sum_{j=1}^{n} \lambda x_j, y \leq \sum_{j=1}^{n} \lambda y_j, \lambda \geq 0 \right\}, \tag{2.1}$$

where T^M indicates the meta-frontier, and $\lambda = \{\lambda_1, \lambda_2 \ldots \lambda_n\}$ is the intensity vector.

Similar to many previous studies ([27]–[29]), we also adopt the CRS assumption to construct the production planning model. According to the above definition of meta-frontier, the meta-efficiency of DMU_d (i.e. ω_d^{ME}) is computed by using the following linear programming:

$$\begin{aligned}
&\text{Max } \omega_d^{\text{ME}} = \sum_{r=1}^{s} u_r y_{rd} \\
&s.t. \ \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0, \ j = 1, \ldots, n, \\
&\sum_{i=1}^{m} v_i x_{id} = 1, \\
&u_r, v_i \geq \varepsilon, \ r = 1, 2, \ldots, s; i = 1, 2, \ldots, m.
\end{aligned} \tag{2.2}$$

However, in practice, different DMUs usually have their particular production technologies (i.e. technology heterogeneity), which may be due to various reasons such as geographical location, government policy and socioeconomic conditions. When we make a production plan, it would be inappropriate to mix all DMUs under the same standard. To solve this problem, O'Donnell *et al.* [14] apply the concept of group frontier to the DEA model.

2.2. Group frontier efficiency model

Considering technology heterogeneity among DMUs, we can divide DMUs into $K (K > 1)$ independent groups according to different sources of heterogeneity. DMUs belonging to the same group usually have the same technology level. The number of DMUs in the gth group is N_g, thus $\sum_{g=1}^{K} N_g = n$. The production technology of the gth group frontier is defined as follows:

$$T^g = \left\{ (x, y) \middle| x \geq \sum_{j=1}^{n} \lambda_j x_j, y \leq \sum_{j=1}^{n} \lambda_j y_j, \lambda_j \geq 0, j = 1, 2 \ldots N_g \right\}, \ \forall g = 1, 2 \ldots K. \tag{2.3}$$

For the given DMU_d which belongs to the gth group, its relative group efficiency rating noted $\omega_d^{\mathrm{GE}_g}$ can be computed by the following model in the form of linear programming:

$$\begin{aligned} &\text{Max } \omega_d^{\mathrm{GE}_g} = \sum_{r=1}^{s} u_r y_{rd} \\ &s.t. \ \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0, \ j = 1, 2 \ldots N_g, g = 1, \ldots, K, \\ &\sum_{i=1}^{m} v_i x_{id} = 1, \\ &u_r, v_i \geq \varepsilon, \ r = 1, 2, \ldots, s; i = 1, 2, \ldots, m. \end{aligned} \tag{2.4}$$

Through models (2.2) and (2.4), the meta-efficiency ω_d^{ME} and group efficiency $\omega_d^{\mathrm{GE}_g}$ can be calculated.

Additionally, the meta-technology ratio is denoted by $\alpha_j = \frac{\omega_j^{\mathrm{ME}}}{\omega_j^{\mathrm{GE}}}$, which means that the meta-technology ratio for the gth group can be decomposed, that is, the meta-efficiency is divided by group efficiency. The meta-technology ratio α, also called technology gap ratio, reflects the technology difference between meta-efficiency and group efficiency. Apparently, when α_j is closer to 1, DMU_j is more likely to be meta-efficient in the next period. Conversely, when α_j is closer to 0, DMU_j is less likely to be meta-efficient in the next period.

3. Centralized production planning models with undesirable outputs

3.1. Meta-frontier and Group frontier efficiency models with undesirable outputs

We assume that there exists a set of n DMUs denoted by $\mathrm{DMU}_j (j = 1, \ldots, n)$. Each DMU_j has m inputs, denoted as $X_j = \{x_{1j}, x_{2j}, \ldots, x_{mj}\}$; s desirable outputs, denoted as $Y_j = \{y_{1j}, y_{2j}, \ldots, y_{sj}\}$; and t undesirable outputs, denoted as $Z_j = \{z_{1j}, z_{2j}, \ldots, z_{tj}\}$. The undesirable outputs usually consist of environmental pollution, such as wastewater, waste gas, and solid waste ([30]–[32]). In this case, we need to cut down the undesirable pollution to increase the efficiency of the production. That is, we should treat the undesirable and desirable outputs differently while evaluating the production planning performance. Furthermore, decision-makers want to reduce undesirable outputs. Therefore, the undesirable outputs are treated as inputs in this paper, which is analogous to the previous works ([20], [33]). Based on the CRS assumption, the relevant efficiency rating of a given DMU_d with undesirable outputs can be computed using the CCR model in the following linear programming:

$$\begin{aligned} &\text{Max } \theta_d = \sum_{r=1}^{s} u_r y_{rd} \\ &s.t. \ \sum_{i=1}^{m} v_i x_{ij} + \sum_{k=1}^{t} \eta_k z_{kj} - \sum_{r=1}^{s} u_r y_{rj} \geq 0, \ j = 1, \ldots, n, \\ &\sum_{i=1}^{m} v_i x_{id} + \sum_{k=1}^{t} \eta_k z_{kd} = 1, \\ &u_r, v_i, \eta_k \geq \varepsilon, \ r = 1, 2, \ldots, s; i = 1, 2, \ldots, m; k = 1, 2, \ldots, t. \end{aligned} \tag{3.1}$$

According to models (2.1) and (2.2), for a given DMU_d, its relevant meta-efficiency (θ_d^{ME}) can be calculated by the following CRS model:

$$\begin{aligned}
&\text{Max } \theta_d^{\text{ME}} = \sum_{r=1}^{s} u_r y_{rd} \\
&s.t. \ \sum_{i=1}^{m} v_i x_{ij} + \sum_{k=1}^{t} \eta_k z_{kj} - \sum_{r=1}^{s} u_r y_{rj} \geq 0, \ j = 1, \ldots, n, \\
&\sum_{i=1}^{m} v_i x_{id} + \sum_{k=1}^{t} \eta_k z_{kd} = 1, \\
&u_r, v_i, \eta_k \geq 0, \ r = 1, 2, \ldots, s; i = 1, 2, \ldots, m; k = 1, 2, \ldots, t.
\end{aligned} \tag{3.2}$$

Similar to model (2.4), for the given DMU_d belonging to the gth group, its relative group efficiency rating, denoted as $\theta_d^{\text{GE}_g}$, can be computed using the following model:

$$\begin{aligned}
&\text{Max } \theta_d^{\text{GE}_g} = \sum_{r=1}^{s} u_r y_{rd} \\
&s.t. \ \sum_{i=1}^{m} v_i x_{ij} + \sum_{k=1}^{t} \eta_k z_{kj} - \sum_{r=1}^{s} u_r y_{rj} \geq 0, \ j \in N_g, j = 1, \ldots, n, g = 1, \ldots, K, \\
&\sum_{i=1}^{m} v_i x_{id} + \sum_{k=1}^{t} \eta_k z_{kd} = 1, \\
&u_r, v_i, \eta_k \geq \varepsilon, \ r = 1, 2, \ldots, s; i = 1, 2, \ldots, m; k = 1, 2, \ldots, t.
\end{aligned} \tag{3.3}$$

The meta-technology ratio is also equivalent to the meta-efficiency divided by the group efficiency, that is, $\alpha_j = \frac{\theta_j^{\text{ME}}}{\theta_j^{\text{GE}}}$.

3.2. Centralized DEA models for production planning with undesirable outputs

The production planning problem involves determining the number of products to be produced by each unit in the next season when demand changes can be predicted. Suppose that the demand change for desirable output r ($r = 1, \ldots, s$), emission change for undesirable output k ($k = 1, \ldots, t$), and supply change for input i ($i = 1, \ldots, m$) in the next production season can be forecasted as C_r,D_k, and B_i, respectively, then the corresponding values of B_i, C_r, and D_k are not restricted. That is, the changes may be positive, negative or zero. To meet the supply, demand and emission changes, the centralized authority must determine the most preferred input-output plans for all DMUs.

We use the variables b_{ij} ($i = 1, \ldots, m$), c_{rj} ($r = 1, \ldots, s$), and d_{kj} ($k = 1, \ldots, t$) to represent the supply change of input i, demand change of desirable output r, and emission change of undesirable output k of DMU_j in the next production season. Furthermore, the values of total input ith, desirable output rth, and undesirable output kth in the next production season are denoted by $x'_{ij} = x_{ij} + b_{ij}$, $y'_{rj} = y_{rj} + c_{rj}$, and $z'_{kj} = z_{kj} + d_{kj}$, respectively. Apparently, $\sum_{j=1}^{n} b_{ij} = B_i$, $\sum_{j=1}^{n} c_{rj} = C_r$ and $\sum_{j=1}^{n} d_{kj} = D_k$ are on hold.

In making a production plan, managers usually hope to obtain improved production level in the next production period, that is, the meta-efficiency of DMUs in the next production season should not be lower than that in the current season. Meanwhile, in the next production season, the efficient frontier of the different DMUs may vary due to technology heterogeneity. As mentioned previously, we classify DMUs according to the different sources of heterogeneity. Moreover, DMUs belonging to the same group usually have similar technology levels. Thus, in our study, we assume that no technology heterogeneity exists between DMUs in the same group, suggesting that these DMUs can reach a uniform technology level. However, technology heterogeneity still exists between DMUs in different groups. Thus, the various DMUs in different groups may not reach a uniform technology level. As mentioned above, group efficiency (θ^{GE}) is not less than meta-efficiency (θ^{ME}). When considering undesirable outputs, the group efficiency is also not less than meta-efficiency, that is, $\theta^{\mathrm{GE}} \geq \theta^{\mathrm{ME}}$. Additionally, according to the planning results in Zhang *et al.* [15], all DMUs can achieve group efficiency, that is, group efficiency is equivalent to 1. However, in practice, all the DMUs' efficiency cannot achieve the efficiency frontier when considering technological disparity. The meta-technology ratio α, also called technology gap ratio, can reflect the technology difference between meta-efficiency and group efficiency. Alternatively, we rule that each DMU's efficiency in the next production season should not exceed the meta-technology ratio α and the efficiency of each DMU should not be lower than its meta-efficiency in the current production season. We combine our production planning model with a previous work ([34]) and obtain the following inequalities:

$$
\begin{aligned}
\frac{\sum_{r=1}^{s} u_r(y_{rj}+c_{rj})}{\left[\sum_{i=1}^{m} v_i(x_{ij}+b_{ij})+\sum_{k=1}^{t}\eta_k(z_{kj}+d_{kj})\right]} &\geq \theta_j^{\mathrm{ME}}, j=1,\ldots n,\\
\frac{\sum_{r=1}^{s} u_r(y_{rj}+c_{rj})}{\left[\sum_{i=1}^{m} v_i(x_{ij}+b_{ij})+\sum_{k=1}^{t}\eta_k(z_{ij}+d_{kj})\right]} &\leq \alpha_j, j=1,\ldots,n,\\
\sum_{j=1}^{n} b_{ij} &= B_i, i=1,\ldots,m,\\
\sum_{j=1}^{n} c_{rj} &= C_r, r=1,\ldots,s,\\
\sum_{j=1}^{n} d_{kj} &= D_k, k=1,\ldots,t,\\
u_r, v_i, \eta_k \geq \varepsilon,\ r=1,2,\ldots,s; i&=1,2,\ldots,m; k=1,2,\ldots,t,
\end{aligned} \tag{3.4}
$$

where both α_j and θ_j^{ME} range from 0 to 1. The value of θ_j^{ME} is calculated by using model (3.2). Furthermore, the meta-technology ratio α_j defined in Section 3 is computed through the formula $\frac{\theta_j^{\mathrm{ME}}}{\theta_j^{\mathrm{GE}}}$, where θ_j^{ME} and θ_j^{GE} can be obtained by models (3.2) and (3.3), respectively.

The centralized decision-maker can grasp the resources of all DMUs and improve its overall efficiency, assuming that all DMUs are under the control of the centralized decision-maker. Many past studies have investigated applications to bank branches, schools and hospitals, wherein the organizations manage the resources ([35]–[37]). In such cases, the decision-maker hopes to obtain the maximum overall efficiency rather than the individual efficiency of a certain DMU. Focusing more on the realistic condition and combining the constraints in model (3.4), we establish the following model with maximum overall efficiency as the objective function:

$$
\begin{aligned}
&\text{Max}\ \frac{\sum_{r=1}^{s} u_r \sum_{j=1}^{n}(y_{rj}+c_{rj})}{\sum_{i=1}^{m} v_i \sum_{j=1}^{n}(x_{ij}+b_{ij})+\sum_{k=1}^{t}\eta_k\sum_{j=1}^{n}(z_{kj}+d_{kj})}\\
&s.t.\ \frac{\sum_{r=1}^{s} u_r(y_{rj}+c_{rj})}{\left[\sum_{i=1}^{m} v_i(x_{ij}+b_{ij})+\sum_{k=1}^{t}\eta_k(z_{kj}+d_{kj})\right]} \geq \theta_j^{\mathrm{ME}}, j=1,\ldots,n,
\end{aligned}
$$

$$\frac{\sum_{r=1}^{s} u_r(y_{rj}+c_{rj})}{\left[\sum_{i=1}^{m} v_i(x_{ij}+b_{ij})+\sum_{k=1}^{t}\eta_k(z_{ij}+d_{kj})\right]} \leq \alpha_j, j=1,\ldots,n,$$

$$\phi_i^U x_{ij} \geq b_{ij} \geq -\phi_i^L x_{ij}, i=1,\ldots m, j=1,\ldots,n,$$

$$\delta_r^U y_{rj} \geq c_{rj} \geq -\delta_r^L y_{rj}, r=1,\ldots,s, j=1,\ldots,n,$$

$$\varphi_k^U z_{kj} \geq d_{kj} \geq -\varphi_k^L z_{kj}, k=1,\ldots,t, j=1,\ldots n,$$

$$\sum_{j=1}^{n} b_{ij} = B_i,\ i=1,\ldots,m,$$

$$\sum_{j=1}^{n} c_{rj} = C_r, r=1,\ldots,s,$$

$$\sum_{j=1}^{n} d_{kj} = D_k, k=1,\ldots,t,$$

$$u_r, v_i, \eta_k \geq \varepsilon,\ r=1,2,\ldots,s; i=1,2,\ldots,m; k=1,2,\ldots,t. \tag{3.5}$$

The first constraint of model (3.5) ensures that each DMU preserves its efficiency level and the second formula means that the change in the efficiency of every DMU must be restricted by its own technology level. Thereby, the efficiency of every DMU in the next production season cannot exceed the technology gap ratio. Considering the feasibility in practice, the formulas $\phi_i^U x_{ij} \geq b_{ij} \geq -\phi_i^L x_{ij}$, $\delta_r^U y_{rj} \geq c_{rj} \geq -\delta_r^L y_{rj}$ and $\varphi_k^U z_{kj} \geq d_{kj} \geq -\varphi_k^L z_{kj}$ are added to limit the change proportion of the inputs and desirable and undesirable outputs, where U and$L$ indicate the lower and upper bounds, respectively. Notably, the values of the lower and upper bounds are exogenously given. In addition, ϕ, δ, and φ could be a percentage, such as 20% or 10%. Considering practical feasibility, we add the constraints.

Taking practical situations into consideration, we realize that the lower and upper constraints may not be the same. On top of that, the change of inputs or outputs may have no limitation or just have either an upper or lower restriction. For example, labor, one of the inputs of production plan, can increase or decrease in the next production season with a small and identical upper and lower percentage bound, such as 20% or 10%. In the four sessions of the 12th Chinese People's Political Consultative Conference National Committee on May 7, 2016, Chinese leaders emphasized that the economy should be developed and the national GDP increased without sacrificing the environment. Hence, waste emissions should be decreased in the next production seasons. However, in reality, waste emissions have continued to increase. Therefore, the upper and lower bounds can be 10% and 20%, respectively. If the local or central government develops some areas, then equipment and capital will flow into these areas, thus generating an upper bound. Concerning the actual conditions, regardless of equipment or capital, neither cannot be reduced drastically, and as a result, it just has the lower bound marked not less than 10%.

Note that model (3.5) is a non-linear programming problem because of its fractional form and the non-linear items $v_i b_{ij}$, $u_r c_{rj}$ and $\eta_k d_{kj}$ in the constraints. However, model (3.5) can be converted into a linear programming by using the Charnes–Cooper transformation and $\widehat{b}_{ij} = v_i b_{ij}$, $\widehat{c}_{rj} = u_r c_{rj}$ and $\widehat{d}_{kj} = \eta_k d_{kj}$, respectively substituting the non-linear items.

$$\text{Max } \sum_{r=1}^{s}\sum_{j=1}^{n} u_r y_{rj} + \sum_{r=1}^{s}\sum_{j=1}^{n} \widehat{c}_{rj}$$

$$\sum_{r=1}^{s} u_r y_{rj} + \sum_{r=1}^{s} \widehat{c}_{rj} - \theta_j^{\text{ME}} \left(\sum_{i=1}^{m} v_i x_{ij} + \sum_{i=1}^{m} \widehat{b}_{ij} + \sum_{k=1}^{t} \eta_k z_{kj} + \sum_{k=1}^{t} \widehat{d}_{kj}\right) \geq 0,\ j=1,\ldots,n,$$

$$\sum_{r=1}^{s} u_r y_{rj} + \sum_{r=1}^{s} \widehat{c}_{rj} - \alpha_j \left(\sum_{i=1}^{m} v_i x_{ij} + \sum_{i=1}^{m} \widehat{b}_{ij} + \sum_{k=1}^{t} \eta_k z_{kj} + \sum_{k=1}^{t} \widehat{d}_{kj} \right) \leq 0,\ j = 1, \ldots, n,$$
$$\sum_{i=1}^{m} \sum_{j=1}^{n} v_i x_{ij} + \sum_{i=1}^{m} \sum_{j=1}^{n} \widehat{b}_{ij} + \sum_{k=1}^{t} \sum_{j=1}^{n} \eta_k z_{kj} + \sum_{k=1}^{t} \sum_{j=1}^{n} \widehat{d}_{kj} = 1,$$
$$\widehat{b}_{ij} \geq -v_i \phi_i^L x_{ij}, i = 1, \ldots m, j = 1, \ldots, n,$$
$$\widehat{b}_{ij} \leq v_i \phi_i^U x_{ij}, i = 1, \ldots m, j = 1, \ldots, n,$$
$$\widehat{c}_{rj} \geq -u_r \delta_r^L y_{rj}, r = 1, \ldots, s, j = 1, \ldots, n,$$
$$\widehat{c}_{rj} \leq u_r \delta_r^U y_{rj}, r = 1, \ldots, s, j = 1, \ldots, n,$$
$$\widehat{d}_{kj} \geq -\eta_k \varphi_k^L z_{kj}, k = 1, \ldots, t, j = 1, \ldots n,$$
$$\widehat{d}_{kj} \leq \eta_k \varphi_k^U z_{kj}, k = 1, \ldots, t, j = 1, \ldots n,$$
$$\sum_{j=1}^{n} \widehat{b}_{ij} = v_i B_i,\ i = 1, \ldots, m,$$
$$\sum_{j=1}^{n} \widehat{c}_{rj} = u_r C_r,\ r = 1, \ldots, s\ ,$$
$$\sum_{j=1}^{n} \widehat{d}_{kj} = \eta_k D_k,\ k = 1, \ldots, t.$$
$$u_r, v_i, \eta_k \geq \varepsilon,\ r = 1, 2, \ldots, s; i = 1, 2, \ldots, m; k = 1, 2, \ldots, t. \tag{3.6}$$

After solving model (3.6), the optimal solutions v_i^*, u_r^*, η_k^*, $\widehat{b}_{ij^*}$, $\widehat{c}_{rj}^*$, and $\widehat{d}_{kj}^*$ can be identified, and the production plan solutions b_{ij}^*, c_{rj}^*, and d_{kj}^* can be obtained, respectively. Thus, we can easily design new production planning equations $x'_{ij} = x_{ij} + b_{ij}^*, y'_{rj} = y_{rj} + c_{rj}^*$ and $z'_{kj} = z_{kj} + d_{kj}^*$.

4. Applications

4.1. A numerical example

This subsection illustrates our proposed approach presented in Section 3. We demonstrate our approach by a numerical data set and then compare our results with those generated using an alternative method.

Table 1 presents a numerical example with two inputs and two outputs. We set the first and second outputs to be desirable and undesirable output, respectively. Then, we classify the six DMUs into two groups. Group A contains DMU 1, DMU 2, and DMU 3 and Group B includes the remaining DMUs. Table 2 shows the technology heterogeneity indexes calculated by our approach. The meta-efficiency listed in the seventh column is calculated through model (3.2). The relative group efficiency (θ^{GE}) obtained by model (3.3) is shown in the eighth column, and the meta-technology ratio α_j is calculated by $\alpha_j = \frac{\theta_j^{\mathrm{ME}}}{\theta_j^{\mathrm{GE}}}, j = 1, 2, \ldots, n$, and listed in the last column.

Total demand changes, for both the inputs and outputs are predicated as 1 in the next production season. The amount of adjustment for input 1 and undesirable output barely has a lower bound set as 50%. By contrast, the amount of adjustment for output 1 has an upper bound ($\delta_1^U = 50\%$). Moreover, the amount of adjustment for input 2 is between −50% and 50% . The calculated results are displayed in Table 3. As can be seen, the efficiency score of both DMU2 and DMU4 are increasing. We also use the method proposed by Amirteimoori and Kordrostami [11], as shown in Table 4.

The changing values of every input and output and shown in columns 3–10 in Table 5. In addition, b_1^*, b_2^*, c_1^*, and d_1^* represent the changing values of our model and are listed in columns 3, 5, 7 and 9, respectively. Columns

TABLE 1. Numerical data set.

DMU	Input1	Input2	Output1	Undesirable output as input (Output2)
	x_1	x_2	y_1	z_1
1	4	3	2	1
2	6	2	1	2
3	1	3	1	2
4	2	6	1	1
5	3	1	1	2
6	3	2	2	1

TABLE 2. The results of meta-efficiency, group efficiency and meta-technology ratio.

DMU	Group	Input1	Input2	Output1	Undesirable outputs as inputs	Meta-efficiency	Group efficiency	Meta-technology ratio
		x_1	x_2	y_1	z_1	θ^{ME}	θ^{GE}	α
1	A	4	3	2	1	1	1	1
2	A	6	2	1	2	0.5	0.5	0.6667
3	A	1	3	1	2	1	1	1
4	B	2	6	1	1	0.7143	0.7143	0.9524
5	B	3	1	1	2	1	1	1
6	B	3	2	2	1	1	1	1

TABLE 3. Numerical data results.

DMU	Group	Input1	Input2	Output1	Undesirable output as input	New efficiency
		x_1'	x_2'	y_1'	z_1'	θ^*
1	A	2	4.5	3	4.3792	1
2	A	3	1	0.4511	1	0.6767
3	A	0.5	3.598571.5	1	1	
4	B	1	4.4015	1.5	0.5	1
5	B	1.5	1.5	0.8349	1	1
6	B	12	3	1.714	2.1208	1

2, 4, 6 and 8 are the calculations from the method of Amirteimoori and Kordrostami [11], denoted as $b_1^{**}, b_2^{**}, c_1^{**}$, and d_1^{**}, separately. Both our model and the method of Amirteimoori and Kordrostami [11] generate a production planning in the next period. However, although total changes for each input and output are 1, the changes in every DMU can have negative number in our proposed model. In comparison, the method of Amirteimoori and Kordrostami [11] does not have the power to reach this goal. They rule that it must have a uniform sign with the demand change of all inputs and outputs. However, this assumption may become more unrealistic in reality. For example, if the demand change of labor in the next production planning increases, then most DMUs may also expand the number of scales in labor. However, there may be a small part of DMUs that can reduce their labor. Considering that they might enhance their technology or are limited by the cost, this phenomenon matches our model assumption. We also need to point out our model's innovation point: our model rules the limitation of every changing input and output separately in the next period. Changing values have upper and lower bounds,

Table 4. Comparative experiment.

DMU	Group	Input1	Input2	Output1	Undesirable outputs as inputs	New efficiency
		x_1''	x_2''	y_1''	z_1'	θ^{**}
1	A	4	3.0002	2	1	1
2	A	6	2	1.2223	2	0.6875
2	A	6	2	1.2223	2	0.6875
4	B	2	6	1.5553	1	1
5	B	3	1	1.2224	2	1
6	B	4	2.9998	2	2	1

Table 5. Changing values.

DMU	Group	Input1		Input2		Output1		Undesirable output as input		New efficiency	
		b_1^*	b_1^{**}	b_2^*	b_2^{**}	c_1^*	c_1^{**}	d_1^*	d_1^{**}	θ^*	θ^{**}
1	A	−2	0	1.5	0.0002	1	0	3.3792	0	1	1
2	A	−3	0	−1	0	−0.5489	0.2223	−1	0	0.6767	0.6875
3	A	−0.5	0	0.5985	0	0.5	0	−1	0	1	1
4	B	−1	0	−1.5985	0	0.5	0.5553	−0.5	0	1	1
5	B	−1.5	0	0.5	0	−0.1651	0.2224	−1	0	1	1
6	B	9	1	1	0.9998	−0.286	0	1.1208	1	1	1
Total change		1	1	1	1	1	1	1	1		

or just have either an upper or a lower bound. Furthermore, changing values may not need any restriction. Nonetheless, the model of Amirteimoori and Kordrostami [11] must have an alternative choice, resulting in an unmatched actual situation.

4.2. A real world example

In this application, we choose 30 Chinese provincial administrative regions from the China Statistical Yearbooks Database 2013, China Energy Statistical Yearbook 2013, and Industrial Enterprise Science and Technology Activities Statistical Yearbook 2013. These exclude without Taiwan, Hong Kong, Macao, and Tibet due to the lack of access. Several inputs, including labor x_1 (10 000 persons), capital stock x_2 (billions RMB), coal consumption x_3 (10 000 tons) and water consumption x_4 (billion cubic meters), and GDP y_1 (billions RMB), are modeled as desirable outputs along with three undesirable outputs, namely: solid waste z_1 (10 000 tons), wastewater z_2 (10 000 tons) and SO_2 emissions z_3 (10 000 tons). Data are reported in Table 6.

As mentioned above, the assessed DMUs sometimes have varying production technologies due to differences in geographical location, national policy, and socioeconomic conditions. For example, the industrial technology level in the western provinces of China is far lower than that in the eastern provinces. Hence, classifying DMUs into different groups based on their technology heterogeneity is both necessary and meaningful. According to Miao *et al.* [38], we divide Table 1 containing 30 provincial administrative regions into three groups, namely, Group ER (east regions), Group CR (central regions) and Group WR (west regions). After a careful and accurate analysis of these data, we classify the provinces belonging to the same group. The classification results are presented in the second column.

TABLE 6. (Original data). Chinese thirty provincial administrative regions in 2012.

Province	Group	Inputs				Desirable output	Undesirable outputs as inputs		
		x_1	x_2	x_3	x_4	y_1	z_1	z_2	z_3
Beijing	ER	118.9	291.75	396.73	4.89	1780.1	1104.05	9190	5.93
Tianjin	ER	153.8	459.83	970.08	5.09	1288.52	1820	19117	21.55
Hebei	ER	376.6	1329.48	7422.92	25.23	2657.5	45575.83	122645	123.87
Liaoning	ER	383	1040.16	3889.52	22.96	2480.13	27279.74	87168	97.9
Shanghai	ER	258.1	1102.26	835.84	72.93	2010.13	2198.81	46359	19.34
Jiangsu	ER	1092.6	3474.36	5526.32	193.1	5405.82	10224.44	236094	95.92
Zhejiang	ER	700	2571.43	3050.8	60.74	3460.63	4461.42	175416	61.09
Fujian	ER	396.6	607.95	3276.68	75.74	1970.18	7719.54	106319	35.24
Shandong	ER	909.3	3617.02	11526.63	28.1	5001.32	18342.59	183634	154.38
Guangdong	ER	1370.6	2021.84	5338.21	121.58	5706.79	5965.49	186126	77.15
Hainan	ER	12	98.33	239.23	3.83	285.53	385.72	7465	3.3
Shanxi	CR	214.9	857.38	4374.73	15.5	1211.28	29031.5	48108	119.46
Jilin	CR	151.6	460.49	3748.13	27.09	1193.78	4730.89	44842	35.23
Heilongjiang	CR	138.9	554.49	2414.92	41.7	1369.16	6312.55	58355	39.73
Anhui	CR	281.7	659.36	4603.65	99.25	1721.21	12022.34	67175	46.98
Jiangxi	CR	209	569.51	2515.39	58.72	1294.85	11133.6	67871	55.15
Henan	CR	580.4	1111.97	5925.18	60.51	2981.01	15250.47	137356	112.99
Hubei	CR	294.2	987.50	8542.9	121.64	2225.02	7610.94	91609	54.86
Hunan	CR	295.7	747.30	5342.75	98.12	2215.42	8115.92	97133	59.33
Inner Mongolia	WR	127.1	705.17	2943.17	23.55	1598.83	24225.63	33618	124.15
Guangxi	WR	151.9	375.57	3118.2	51.49	1303.1	7963.96	110671	47.16
Chongqing	WR	151.8	337.82	3234.15	39.44	1145.9	3114.89	30611	50.98
Sichuan	WR	386.3	1083.27	6010.52	54.75	2384.98	13187.3	69984	79.4
Guizhou	WR	84.4	389.18	3538.39	39.67	680.22	7835.25	23399	83.71
Yunnan	WR	95.6	368.46	2852.55	27.81	1030.98	16037.59	42811	62.26
Shaanxi	WR	164	474.50	3362.52	13.35	1445.12	7215.11	38037	74.7
Gansu	WR	60.9	326.06	1448.58	15.7	565.02	6671.17	19188	47.99
Qinghai	WR	18.9	149.64	453.12	2.53	188.45	12301.16	8917	12.91
Ningxia	WR	30.6	149.67	1152.54	4.86	232.66	2960.67	16548	38.44
Xinjiang	WR	62.1	460.03	2606.95	12.38	746.63	7879.72	29738	70.47
Total change		9271.5	27381.78	110661.3	1422.25	57580.27	328678.3	2215504	1911.57

The relative meta-efficiency (θ^{ME}) and group efficiency (θ^{GE}) of each province can be calculated through models (3.3) and (3.4), respectively. Additionally, the meta-technology ratio α_j is computed by $\alpha_j = \frac{\theta_j^{\text{ME}}}{\theta_j^{\text{GE}}}, j = 1, 2, \ldots, n$. The calculation results are shown in Table 7.

The meta-efficiency of every province is shown in the third column in Table 7. We can see that the meta-efficiencies of all provinces, apart from Beijing and Hainan, are not extremely high. The reason is that Beijing is the capital of China, which should be the leader of other provinces in many areas. Thus, the central government pays great attention to it technical support and economic boost through national policies. Although the main development track of Hainan does not focus on its industries, some large companies are still built by the government in order to maintain a stable supply for domestic needs. Moreover, group efficiency can be seen in the fourth column. Both the central and west regions have seven provinces that are able to achieve a full efficiency of 1. In comparison, the remaining provinces are inefficient, with Ningxia having the lowest efficiency score of 0.7213. The efficiency of the east region does not achieve this goal, except for Beijing and Hainan, which are completely efficient. Finally, meta-technology ratio α_j is in the fifth column. Evidently, all provinces of the east region project onto the efficient frontier. By contrast, the central and west regions are inefficient. From Table 7, we can see that the meta-technology ratio of the east region is 1, whereas those of the west and central regions are lower. The east region is more developed than the central and west regions in China, and as such, the provinces in the former have great potential to enhance their efficiency.

To analyze the efficiencies of DMUs that contain undesirable outputs in the next production season, under the situation of centralized production planning, we take the meta-efficiency and meta-technology ratio values that are listed in Table 7 into model (3.6). Here, $B_1 = 0$, and $B_2 = 2700$, $B_3 = -5500$, and $B_4 = 0$ respectively indicate the changes in four inputs, the number of labor, total capital stock, coal consumption, and water consumption. A positive value of B_i represents the increase in demand, whereas a negative value shows a decrease in demand. If B_i is 0, then the total demand in the next production season is the same as that in the current season. Total labor and water consumption will be 0% for inner resource reallocation; total capital stock will increase by 10%; and total coal consumption, which is a non-renewable energy resource, will decrease by 5%. Our aim is to achieve a higher GDP. Although the increase in China's GDP has slowed down, it can still reach over 7% in average. Under such a circumstance, we will assume the rate of GDP increase as 7% for all provinces in this research and we set 7% as the numerical value of GDP ($C_1 = 4000$) in the next production season. Undesirable outputs as inputs of solid waste emissions, wastewater emissions, and SO_2 emission change's values are $D_1 = -16000, D_2 = -110000$, and $D_3 = 0$ in the next production season, respectively. The total solid waste, wastewater, and SO_2 emissions, which are difficult to reduce, decrease by 5%, 5% and 0%, respectively.

TABLE 7. The results of meta-efficiency, group efficiency and meta-technology ratio.

Province	Group	Meta-efficiency	Group efficiency	Meta-technology ratio
Beijing	ER	1.0000	1.0000	1.0000
Tianjin	ER	0.6954	0.6954	1.0000
Hebei	ER	0.4468	0.4468	1.0000
Liaoning	ER	0.4212	0.4212	1.0000
Shanghai	ER	0.5670	0.5670	1.0000
Jiangsu	ER	0.3299	0.3299	1.0000
Zhejiang	ER	0.4811	0.4811	1.0000
Fujian	ER	0.5311	0.5311	1.0000
Shandong	ER	0.4889	0.4889	1.0000
Guangdong	ER	0.5933	0.5933	1.0000
Hainan	ER	1.0000	1.0000	1.0000
Shanxi	CR	0.3533	1.0000	0.3533
Jilin	CR	0.4962	1.0000	0.4962
Heilongjiang	CR	0.5686	1.0000	0.5686
Anhui	CR	0.4278	1.0000	0.4278
Jiangxi	CR	0.4026	0.9158	0.4396
Henan	CR	0.4394	1.0000	0.4394
Hubei	CR	0.4623	1.0000	0.4623
Hunan	CR	0.4967	1.0000	0.4967
Inner Mongolia	WR	0.6987	1.0000	0.6987
Guangxi	WR	0.5719	1.0000	0.5719
Chongqing	WR	0.5559	1.0000	0.5559
Sichuan	WR	0.3981	1.0000	0.3981
Guizhou	WR	0.4426	0.7878	0.5618
Yunnan	WR	0.6298	1.0000	0.6298
Shaanxi	WR	0.5632	1.0000	0.5632
Gansu	WR	0.4929	0.8745	0.5637
Qinghai	WR	0.5566	0.9514	0.5850
Ningxia	WR	0.4065	0.7213	0.5635
Xinjiang	WR	0.6014	1.0000	0.6014

TABLE 8. Production planning results.

Province	Group	Inputs				Desirable output	Undesirable outputs as inputs			New efficiency
		x'_1	x'_2	x'_3	x'_4	y'_1	z'_1	z'_2	z'_3	
Beijing	ER	130.79	1013.39	436.4	5.38	1958.11	1214.45	10109	6.52	1.0000
Tianjin	ER	169.18	617.55	1067.09	5.6	1417.37	1638	21028.7	23.7	1.0000
Hebei	ER	338.94	1196.53	6680.63	22.71	2923.25	41018.45	98116	111.48	0.9158
Liaoning	ER	344.7	936.14	3500.57	20.66	2728.14	24551.77	69734.4	88.11	1.0000
Shanghai	ER	232.29	992.03	752.26	65.64	2211.14	1978.93	37087.2	17.41	1.0000
Jiangsu	ER	983.34	3126.92	4973.69	173.79	5946.4	9202	215112.3	86.33	0.6996
Zhejiang	ER	630	2314.29	2745.72	54.67	3806.69	4015.28	140332.8	54.98	0.7063
Fujian	ER	356.94	547.16	2949.01	68.17	2167.2	6947.59	85055.2	31.72	1.0000
Shandong	ER	818.37	3255.32	10373.97	25.29	5501.45	16508.33	146907.2	138.94	0.7789
Guangdong	ER	1430.53	1819.66	4804.39	109.42	6277.47	5368.94	148900.8	69.43	1.0000
Hainan	ER	13.2	92.71	263.15	4.21	314.08	424.29	8211.5	3.63	1.0000
Shanxi	CR	236.39	1313.9	4812.2	17.05	1211.28	27743.92	52918.8	131.41	0.4030
Jilin	CR	166.76	712.42	3373.32	29.8	1193.78	5203.98	49326.2	38.75	0.5481
Heilongjiang	CR	152.79	545.54	2173.43	45.87	1369.16	5681.3	64190.5	43.7	0.7040
Anhui	CR	309.87	666.41	4143.28	109.18	1721.21	13224.57	73892.5	51.68	0.7164
Jiangxi	CR	229.9	591.18	2505.63	64.59	1294.85	12246.96	74658.1	60.66	0.5959
Henan	CR	638.44	2279.55	6517.7	66.56	2981.01	16775.52	151091.6	124.29	0.4475
Hubei	CR	323.62	888.75	7688.61	132.65	2447.52	6849.85	100769.9	49.37	0.7637
Hunan	CR	325.27	724.91	5877.03	107.93	2215.42	8927.51	106846.3	65.26	0.8107
Inner Mongolia	WR	139.81	862.33	2648.85	25.9	1758.71	21803.07	36979.8	136.57	0.7999
Guangxi	WR	167.09	338.01	2806.38	47.53	1433.41	7167.56	121738.1	45.77	1.0000
Chongqing	WR	166.98	444.86	2910.73	43.38	1145.9	3426.38	33672.1	56.08	0.8006
Sichuan	WR	424.93	2180.78	6611.57	60.22	2384.98	14506.03	76982.4	87.34	0.4521
Guizhou	WR	75.96	350.26	3184.55	35.7	748.24	7051.73	18719.2	75.34	0.7738
Yunnan	WR	105.16	331.61	2567.29	30.59	1072.58	14433.83	47092.1	68.49	0.8871
Shaanxi	WR	180.4	962.71	3698.77	14.68	1445.12	7936.62	41840.7	82.17	0.6304
Gansu	WR	66.99	293.45	1303.72	17.27	621.27	6004.05	21106.8	52.79	0.6818
Qinghai	WR	17.01	134.68	407.81	2.28	207.3	11071.04	7133.6	11.62	0.6800
Ningxia	WR	27.54	134.7	1037.29	4.37	255.93	2664.6	13238.4	34.6	0.6285
Xinjiang	WR	68.31	414.03	2346.26	11.14	821.29	7091.75	32711.8	63.42	0.7182
Final		9271.5	30081.78	105161.3	1422.23	61580.26	312678.3	2105504	1911.56	

For each province of every input, desirable and undesirable outputs are considered as inputs. We will also set up limitations. Labor's (x_1) bound is (ϕ^U_{b1}, ϕ^L_{b1}); coal consumption's (x_3) bound is (ϕ^U_{b3}, ϕ^L_{b3}), and water consumption's (x_4) bound (ϕ^U_{b4} and ϕ^L_{b4}) is 10%. Meanwhile, total capital stock (x_2) only owns lower constraint (ϕ^L_{b2}) given as 10% due to the government intervention in the form of capital investments made by the government. Considering the actual situation, we set each province's GDP between 0% (lower bound δ^L_r) and 10% (upper bound δ^U_r). Based on the central government's policies, waste emissions should be decreased in next production season. But in practice, the waste emissions are likely to add increase. Thus, the upper (ϕ^U_{d1}, ϕ^U_{d3}) and lower bounds (ϕ^L_{d1}, ϕ^L_{d3}) of solid waste emissions and SO_2 emissions can be 10%. The gap of wastewater emissions is so large, approximately 180 000, from 7465 (Hainan) to 186 126 (Guangdong). Thus, we increase 10% of the lower bound (ϕ^L_{d2}) to 20% and maintain the upper bound (ϕ^U_{d2}) as 10%. The centralized production planning model with undesirable outputs (10) is presented here to obtain the changing results b^*_{ij}, c^*_{rj}, and d^*_{kj}. In the end, we could easily record production planning in Table 8 by using formulas $x'_{ij} = x_{ij} + b^*_{ij}$, $y'_{rj} = y_{rj} + c^*_{rj}$ and $z'_{kj} = z_{kj} + d^*_{kj}$.

Each province and group is shown in the columns 1 and 2 in Table 8. Columns 3–10 represent the changes in inputs, desirable output, and undesirable outputs as inputs. The new meta-efficiencies are presented in the last column. The last row presents the total demand of inputs in practice, including labor, total capital stock, coal consumption, water consumption, desirable output covering GDP and undesirable outputs containing solid waste, wastewater, and SO_2 emissions.

The production planning of 30 Chinese provincial administrative regions in the next period is formulated through model (3.6). The new efficiency of each DMU is not reduced, but the otherness of the three groups appears. Group ER has a huge increase in efficiency. Provinces like Tianjin, Liaoning, Shanghai, Fujian, and Guangdong achieve the efficiency frontier, whereas Groups CR and WR have a slight growth except for Guangxi.

We need to point out that Guangxi is the only province that obtains the efficiency frontier from Groups CR and WR. It is evident that technology heterogeneity from different regions exists. Although our approach will not make all DMUs efficient, the efficiency scores are extremely high or equal and approximate to their technology level.

5. Conclusions

In this paper, we proposed a DEA-based approach for future production planning in a centralized decision-making environment considering changes in demand. We had two main contributions in this study. First, we established our model considering the technology heterogeneity of each DMU in the next production season. Second our proposed model could deal with the undesirable outputs in the production planning problems. Furthermore, we paid more attention to actual situations, formulating the limitation of every changing input and output separately in the next period. We proposed the concepts of meta-efficiency and group efficiency in production planning to settle the technology heterogeneity in the east, central and west regions of china. Then, we combined practical situations by assuming that the new efficiency in the next production period should be greater than or equal to its meta-efficiency in the current period, and should not exceed its technology level called meta-technology ratio, instead of the efficiency score one in next production period.

The factors related to government policy, economy, and environment were also proposed in this paper. Waste emissions, including solid waste, wastewater, and SO_2 emissions, were then treated as undesirable outputs in our proposed model. Based on these principles, we suggested considering a centralized production planning model. This centralized system could control the consumption of resources, under the final goal of optimizing the overall production performance instead of individual unit performance. To illustrate our proposed method, we presented two examples, namely, a numerical example compared with other models and a real world example based on real-life data. Finally, we analyzed the result of the numerical and empirical study, both of which demonstrated the feasibility and superiority of our approach compared with the alternative method.

Generally, we proposed our production planning model under the assumption of CRS. Identically, we could also extend this model into the VRS model. Furthermore, our proposed model is static; hence, transforming our model into dynamic production planning can be future research direction.

Acknowledgements. The authors are grateful to Professor Joe Zhu for his suggestions and comments on earlier versions of the paper. This research is financially supported by the National Natural Science Foundation of China (71801068, 71331002, 71828101, 71771075, 71601061, 71771077).

References

[1] A. Charnes, W.W. Cooper and E. Rhodes, Measuring the efficiency of decision making units. *Eur. J. Oper. Res.* **2** (1978) 429–444.

[2] M.R. Ghasemi, J. Ignatius and A. Emrouznejad, A bi-objective weighted model for improving the discrimination power in MCDEA. *Eur. J. Oper. Res.* **233** (2014) 640–650.

[3] J. Ignatius, M.R. Ghasemi, F. Zhang, A. Emrouznejad and A. Hatami-Marbini, Carbon efficiency evaluation: an analytical framework using fuzzy DEA. *Eur. J. Oper. Res.* **253** (2016) 428–440.

[4] P. Zhou, B.W. Ang and K.L. Poh, Measuring environmental performance under different environmental DEA technologies. *Energy Econ.* **30** (2008) 1–14.

[5] J. Du, L. Liang, Y. Chen, W.D. Cook and J. Zhu, A bargaining game model for measuring performance of two-stage network structures. *Eur. J. Oper. Res.* **210** (2011) 390–397.

[6] G. Bi, Y. Luo, J. Ding and L. Liang, Environmental performance analysis of Chinese industry from a slacks-based perspective. *Ann. Oper. Res.* **228** (2012) 65–80.

[7] L. Chen and G. Jia, Environmental efficiency analysis of China's regional industry: a data envelopment analysis (DEA) based approach. *J. Cleaner Prod.* **142** (2017) 846–853.

[8] J. Du, L. Liang, Y. Chen and G.B. Bi, DEA-based production planning. *Omega* **38** (2010) 105–112.

[9] F. Hosseinzadeh Lotfi and S. Moghtaderi, DEA-based production planning changes in general situation. *Appl. Math. Sci.* **71** (2010) 3523–3536.

[10] A. Amirteimoori and S. Kordrostami, Production planning: a DEA-based approach. *Int. J. Adv. Manuf. Technol.* **56** (2011) 369–376.

[11] A. Amirteimoori and S. Kordrostami, Production planning in data envelopment analysis. *Int. J. Prod. Econ.* **140** (2012) 212–218.

[12] G.E. Battese and D.S.P. Rao, Technology gap, efficiency, and a stochastic metafrontier function. *Int. J. Bus. Econ.* **2** (2002) 87–93.

[13] G.E. Battese, D.S.P. Rao and C.J. O'Donnell, A metafrontier production function for estimation of technical efficiencies and technology gaps for firms operating under different technologies. *J. Productivity Anal.* **21** (2004) 91–103.

[14] C.J. O'Donnell, D.S.P. Rao and G.E. Battese, Metafrontier frameworks for the study of firm-level efficiencies and technology ratios. *Empirical Econ.* **34** (2008) 231–255.

[15] Y. Zhang, H. Zhang, R. Zhang, Z. Zeng and Z. Wang, DEA-based production planning considering influencing factors. *J. Oper. Res. Soc.* **66** (2015) 1878–1886.

[16] P. Zhou, B. Ang and K. Poh, A survey of data envelopment analysis in energy and environmental studies. *Eur. J. Oper. Res.* **189** (2008) 1–18.

[17] R. Färe, S. Grosskopf, C.A.K. Lovell and C. Pasurka, Multilateral productivity comparisons when some outputs are undesirable: A nonparametric approach. *Rev. Econ. Stat.* **71** (1989) 90–98.

[18] H. Dyckhoff and K. Allen, Measuring ecological efficiency with data envelopment analysis (DEA). *Eur. J. Oper. Res.* **132** (2001) 312–325.

[19] L.M. Seiford and J. Zhu, Modeling undesirable factors in efficiency evaluation. *Eur. J. Oper. Res.* **42** (2002) 16–20.

[20] R. Färe, S. Grosskopf and C.A. Pasurka Jr., Environmental production functions and environmental directional distance functions. *Energy* **32** (2007) 1055–1066.

[21] S. Lozano, E. Gutiérrez and P. Moreno, Network DEA approach to airports performance assessment considering undesirable outputs. *Appl. Math. Model.* **37** (2013) 1665–1676.

[22] W. Liu, Z. Zhou, C. Ma, D. Liu and W. Shen, Two-stage DEA models with undesirable input-intermediate-outputs. *Omega* **56** (2015) 74–87.

[23] A. Zanella, A.S. Camanho and T.G. Dias, Undesirable outputs and weighting schemes in composite indicators based on data envelopment analysis. *Eur. J. Oper. Res.* **245** (2015) 517–530.

[24] B. Arabi, S. Munisamy and A. Emrouznejad, A new slacks-based measure of Malmquist–Luenberger index in the presence of undesirable outputs. *Omega* **51** (2015) 29–37.

[25] K. Khalili-Damghani and Z. Shahmir, Uncertain network data envelopment analysis with undesirable outputs to evaluate the efficiency of electricity power production and distribution processes. *Comput. Ind. Eng.* **88** (2015) 131–150.

[26] T.C. Koopmans, Efficient allocation of resources. *Econometric Soc.* **19** (1951) 455–465.

[27] T. Sueyoshi and Y. Yuan, Social sustainability measured by intermediate approach for DEA environmental assessment: Chinese regional planning for economic development and pollution prevention. *Energy Econ.* **66** (2017) 154–166.

[28] X. Pan, Q. Liu, and X. Peng, Spatial club convergence of regional energy efficiency in China. *Ecol. Indic.* **51** (2015) 25–30.

[29] M. Song, J. Zhang and S. Wang, Review of the network environmental efficiencies of listed petroleum enterprises in China. *Renew. Sustainable Energy Rev.* **43** (2015) 65–71.

[30] S. You and H. Yan, A new approach in modelling undesirable output in DEA model. *J. Oper. Res. Soc.* **62** (2011) 2146–2156.

[31] H. Yang and M. Pollitt, The necessity of distinguishing weak and strong disposability among undesirable outputs in DEA: environmental performance of Chinese coal-fired power plants. *Energy Policy.* **38** (2010) 4440–4444.

[32] J. Wu, Q. Zhu and J. Chu, Two-stage network structures with undesirable intermediate outputs reused: a DEA based approach. *Comput. Econ.* **46** (2015) 455–477.

[33] A. Hailu and T.S. Veeman, Non-parametric productivity analysis with undesirable outputs: an application to Canadian pulp and paper industry. *Am. J. Agric. Econ.* **83** (2001) 605–616.

[34] T. Ding, Y. Chen, H. Wu and Y. Wei, Centralized fixed cost and resource allocation considering technology heterogeneity: a DEA approach. *Ann. Oper. Res.* **268** (2017) 497–511.

[35] S. Lozano and G. Villa, Centralized resource allocation using data envelopment analysis. *J. Productivity Anal.* **22** (2004) 143–161.

[36] C. Mar-Molinero, D. Prior, M.M. Segovia and F. Portillo, On centralized resource utilization and its reallocation by using DEA. *Ann. Oper. Res.* **221** (2014) 273–283.

[37] L. Fang and H. Li, Centralized resource allocation based on the cost–revenue analysis. *Comput. Ind. Eng.* **85** (2015) 395–401.

[38] Z. Miao, Y. Geng and J. Sheng, Efficient allocation of CO_2 emissions in China: a zero sum gains data envelopment model. *J. Cleaner Prod.* **112** (2016) 4144–4150.