RAIRO-Oper. Res. 53 (2019) 1709–1720
https://doi.org/10.1051/ro/2018096

RAIRO Operations Research
www.rairo-ro.org

OPTIMAL MULTI-PRODUCT SUPPLIER SELECTION UNDER STOCHASTIC DEMAND WITH SERVICE LEVEL AND BUDGET CONSTRAINTS USING LEARNING VECTOR QUANTIZATION NEURAL NETWORK

Hajar HormozzadehGhalati[1], Alireza Abbasi[2]
and Abolghasem Sadeghi-Niaraki[3,*]

Abstract. In today's competitive marketplace demand, evaluation and selection of suppliers are pivotal for firms, and therefore decision makers need to select suppliers and the optimal order quantities when outsourcing. However, there is uncertainty and risk due to lack of precise data for supplier selection. Uncertainty can impose shortage or overstocks, because of stochastic demand, to firms; in this case, considering inventory control is essential. In this research, an appropriate spatial model is developed for a multi-product supplier selection model with service level and budget constraints. Learning Vector Quantization Neural Network is used to find the optimal number of decision variables with the goal of maximizing the expected profit of supply chains. By analyzing a practical example and conducting sensitivity analysis, we find that corporate profit will be maximized if the optimal integration of suppliers and the optimal order quantities from each supplier is determined. In addition, budget and service level should be considered in the process of finding the best result.

Mathematics Subject Classification. 90B30, 90B15, 90C30, 92B20.

Received March 15, 2018. Accepted October 20, 2018.

1. Introduction

Competition in global marketplace demand has obliged many enterprises to pay more attention to supply chain management [5]. Supply chain management (SCM) covers all parts of an organization or firm, which attempts to utilize its own materials, information and property to satisfy consumer needs with the goal of accessing competitive supply chains [35]. SCM, as a component of each firm, tries to find the best integration of suppliers for supplying customer's demand [8] and also reducing the purchasing cost [31]. This can be formulated as a supplier selection problem which can be classified into two types [14]: (1) single sourcing: selecting a single supplier to provide all required resources for the buyer; and (2) multiple sourcing: selecting several suppliers, which all together, provide the resources that the buyer is requesting [38]. A tendency towards single sourcing

Keywords. Supply chain management, multi-supplier selection, stochastic demand, Learning Vector Quantization (LVQ) neural network, nonlinear programming optimization model.

[1] Department of Economics and Management, College of Management, Shiraz Branch, Islamic Azad University, Shiraz, Iran.

[2] School of Engineering and Information Technology, The University of New South Wales (UNSW), Canberra, Australia.

[3] Department of Computer Science and Engineering, Sejong University, Seoul, Republic of Korea, Geoinformation Technology Center of Excellence, Faculty of Geodesy and Geomatics Engineering, K.N. Toosi University of Technology, Tehran, Iran.

[*] Corresponding author: a.sadeghi.ni@gmail.com

Article published by EDP Sciences

has been reported [31] and [34] to improve communication and cooperation among buyers and sellers. The significant point is that the single sourcing can shorten product lifecycle and change the rate of technology [26].

In supplier selection, decision makers are required to find an appropriate approach to answer the following two questions: (1) Which suppliers should be selected? and (2) What is the optimal order quantity allocated to each of the selected suppliers? [8] and [38]. In order to answer these questions, this paper regards three parameters; stochastic demand, service level, and available budget. All these three factors lead us to consider inventory control.

In the real scenario to find a better solution for supplier selection problem, stochastic demand plays an important role. Uncertainty in demands of consumers put SCM in a complex circumstance. In this case, two possible conditions can be occurred due to uncertain demand; shortage and overstocks [24]. Therefore, managers have to take into consideration inventory control in order to meet the market demand.

Inventory control considers commodity flow from upstream to downstream of supply chain and conducting service level (*i.e.* product accessibility for consumers). The service level has a direct relationship with inventory level [7]. For example, increasing in the extent of stock in order to cover the stochastic demand leads to increased optimal service level [24]. Inventory control also has an effect on optimal order quantity, price, and gives higher total expected profit of the supply chain management [25].

The foremost point is that a proper section of SCM, in which this section interacts with consumers' need directly, should be responsible for controlling inventory. Supporting this notion, a manufacturer as a main part of the supply chain, who knows an amount of the demand lower and upper bound for product j, has more information to handle the inventory control rather than suppliers. Since, if the role of controlling inventory shifts from manufacturer to suppliers, the company is charged with the costs of product's shortage and overstock [44]. Therefore, manufacturer can figure out the right amount of budget to reach the optimal service level and providing goods for consumers.

Supplier selection problem has been formulated as a structured optimization problem, and many methods are proposed to solve it over the last two decades. For instance, Zhang and Zhang [45] used a "branch and bound" algorithm for supplier selection and order allocation under stochastic demand. Kumar De and Sana [10] proposed centralized and decentralized supply chain model under stochastic demand and used Fuzzy Hausdroff metric in the components of fuzzy set to solve their model. Zouggari and Benyoucef [46] solved group multi-criteria supplier selection problem by presenting a new decision making approach. At first, they used Fuzzy-AHP for supplier selection. After that, a Fuzzy TOPSIS is used to find out the weights for assigning order between selected suppliers. Modak and Kelle [25] examined two-echelon dual-channel supply chain considering the price and delivery-time sensitive, as well as stochastic demand and used a distribution-free approach for analyzing both centralized and decentralized systems with random variables. In terms of comparing centralized and decentralized systems in modifying the behaviour of system Stackelberg and Nash game was used; for coordinated decision-making, a hybrid all-unit quantity discount with a franchise fee contract was employed. They applied the generalized Nash bargaining solution technique to divide surplus profit between the manufacturer and the retailer. Poon *et al.* [28] proposed a RFID-GA-based warehouse resource allocation system for a stochastic production material demand problem. Türk *et al.* [39] proposed an integrated approach to the supplier selection and inventory planning and solved this problem *via* two steps. Firstly, the suppliers were ranked *via* Interval Type-2 Fuzzy Sets. Secondly, Multi-objective Evolutionary was applied to select the supplier and solve the inventory planning problem. Büyüközkan and Çifçi [4] proposed a novel approach for supplier selection with imperfect information and used Fuzzy ANP (analytical network process) in a multi-person decision-making structure. The result shows that their work was novel for selecting sustainable supplier. Taleizadeh *et al.* [37] employed a harmony search algorithm for multi-buyer multi-vendor supply chains problem under stochastic demand. Liao and Rittscher [21] studied a multi-objective supplier selection under stochastic demand using a genetic algorithm. Aggarwal *et al.* [1] presented a multi-objective vendor selection and order allocation problem considering time dependant and stochastic demand, which is solved using Preemptive Goal programming and weighted sum Aggregate Objective Function. Chai *et al.* [6] presented a systematic and comprehensive literature review of several decision-making techniques used for supplier selection; they selected 123 journal articles and found 26 techniques, which were used in these articles in order to solve supplier selection.

Gencer and Gürpinar [13] proposed a model using Analytic Network Process (ANP) to examine the influence of selection criteria on supplier selection and to rank the importance of the criteria. They claimed that firms could use their modified ANP model in order to have strong relationships with suppliers. Ramanathan [30] applied total cost of ownership (TCO) and analytical hierarchy process (AHP) to consider quantitative (*i.e.*, cost) and qualitative factors (*e.g.*, quality, technology, service) respectively, and then used three types of data envelopment analysis (DEA) models (*i.e.*, tradition DEA model; super efficiency model; and assurance region model) to mix the result of TCO and AHP for selecting the suppliers. Duan and Ventura [11] proposed a new dynamic supplier selection and inventory management model for a serial supply chain system. They used MATLAB's mixed-integer linear programming (MILP) solver to gain minimal purchasing cost. Ting and Cho [38] used AHP for identifying a set of nominee suppliers and then formulated multi-objective linear programming (MOLP) with multiple objectives and system constraints for allocating the optimal order quantities to the nominee suppliers. Azadeh and Alem [2] and Ho *et al.* [15] reviewed the literature on the multi-criteria decision-making (MCDM) approaches for selecting and evaluating the performance of suppliers and showed approaches using many or few users and the evaluating criteria with most tendencies. Dursun and Karsak [12] used quality function deployment (QFD)-based fuzzy MCDM approach in a situation that a group of experts faced multi-criteria with vagueness and imprecision for supplier selection.

Mendoza and Ventura [23] proposed mixed-integrated nonlinear programming model in order to select suppliers for providing order quantity. They used purchasing, inventory and transportation costs in their model, which resulted in selecting of actual suppliers and order quantity prepared by them. Ljubojević *et al.* [22] presented a hybrid approach in order to select the transport service provider. They did their research in two steps: (1) based on selected criteria and sub-criteria, ideal weights has been shown; and (2) the real weights were determined and compared with the result of step 1. Based on this comparison, providers were evaluated and selected.

From the preceding review, different supplier selection problems had different conditions, requirements and constraints which are different from the one selected for this article. Although, different papers considered these problems as a selection problem and various techniques have been used to select the suppliers, we used a different perspective, *i.e.* classification way, to solve the problem. To best of our knowledge of authors, this is the first time this approach is used: selecting suppliers *via* classification method. The problem under examination in this research considers multi-suppliers, a single buyer with multi-products and stochastic demand in a single period with service levels and budget constraints, only one supplier (as an output, target class) should be dedicated to each product (as an input, sub-class) in our model. This condition is shown as a binary variable in the mathematical model. After a deep survey, an artificial neural network came to be known as the perfect technique to solve this problem.

Neural networking is a method in artificial intelligence in which computer models attempt to imitate the capabilities of human brains in prediction and classification problems [19]. This method is an effective and powerful tool, which can be used in different fields, and in particular, in supplier selection due to its characteristics like nonlinearity, self-organization and fault-tolerance [42].

Learning Vector Quantization Neural Network (LVQNN) has great advantages over other supplier selection approaches to select the best supplier for preparing order quantity of product. The first and the main advantage of LVQNN is the lowest classification errors rate in complicated situations. The second benefit is its local learning ability. This method does not need any previous data and is learnt just by any given data automatically [20]. Third positive point of this method is easy implementation for multi-class classification problems. The forth one is the ability to adjust the complexity of problem during training [41]. The fifth advantage is an intuitive interpreting and the robust behavior of LVQNN as the prototypes represents their classes [40]. Through the help of LVQNN, with its properties such as self-learning, and being competitive (winner-takes-all that gives a single output as a winner, with score one, to each input vector), a solution for our supplier selection problem will be found [3], which cannot be supported by classical neural network. Therefore, we use LVQNN, for the first time, to maximize firm's profit.

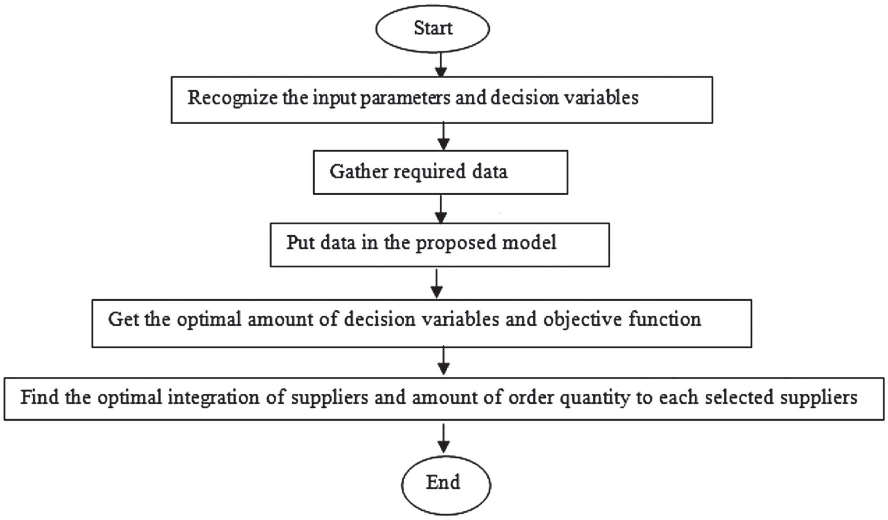

Figure 1. The processes for implementing our proposed model.

The nonlinear programming optimization model under uncertainty of demand, in which there are supplier, buyer, and consumers exist, is applicable in different domains such as agriculture, water supply planning and resource management, production planning, location, inventory management, transportation problem, and resource allocation in financial systems [9, 18, 32, 33, 43].

The objective of this paper is to propose a spatial decision-making approach to answer the main aforementioned questions; selecting an optimal suppliers and allocating order quantity of product j. In order to solve these questions a nonlinear programming optimization model is presented and LVQNN is applied to find the optimal solution for the proposed model. In supplier selection process, the company's aim is to select the best supplier to provide each product. The remainder of the paper is as follows: Section 2 explains the research process for supplier selection, which follows by providing a numerical example and conducting a sensitivity analysis in Sections 3 and 4. Conclusion and further research are discussed in Section 5.

2. The Proposed Model for multi-product supplier selection

When reviewing important issues in the current marketplace and the challenges for firms, we identified the need for further examination of "supplier selection" as the main challenge. Then, literatures on "supplier selection" problem was investigated by reviewing the existing methods, the models addressing this issue, and the effective variables in the models. In order to design and propose an appropriate model for this problem, we found LVQNN, as an "artificial neural network" model, and "nonlinear programming" model could help us in finding the optimal integration of suppliers with the right order quantity for each supplier. We also proposed a schematic model. Figure 1 demonstrates the order of processes required for verifying the applicability and effectiveness of our proposed model.

2.1. Mathematical modelling

In the first step, we develop a mathematical model, define a set of indices, parameters and decision variables and use the same assumptions used in reference [43].

Indices:

i: set of suppliers $i = 1, 2, 3, \ldots, m$;
j: set of products $j = 1, 2, 3, \ldots, n$;

Parameters:

x_j: stochastic demand of product j;
$f_j(x_j)$: demand probability density function for product j;
a_j: demand lower bound for product j;
b_j: demand upper bound for product j;
P_{ij}: selling price per unit of product j supplied by supplier i;
C_{ij}: purchase cost per unit of product j supplied by supplier i;
L_{ij}: salvage value of product j supplied by supplier i at the end of the selling season;
S_{ij}: shortage cost per unit of product j supplied by supplier i;
SL: service level;
AB: available budget;
PC (purchase cost) $= \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}Q_jY_{ij}$;
SC (shortage cost) $= \sum_{i=1}^{m}\sum_{j=1}^{n} Y_{ij}S_{ij}\int_{Q_j}^{b_j} (x_j - Q_j)\, f_j(x_j)\mathrm{d}x_j$;
SV (salvage value) $= \sum_{i=1}^{m}\sum_{j=1}^{n} Y_{ij}L_{ij}\int_{a_j}^{Q_j} (Q_j - x_j)\, f_j(x_j)\,\mathrm{d}x_j$;
SR (the expected sale revenue) $= \sum_{i=1}^{m}\sum_{j=1}^{n} Y_{ij}P_{ij}\left[\int_{Q_j}^{b_j} Q_jf_j(x_j)\,\mathrm{d}x_j + \int_{a_j}^{Q_j} x_jf_j(x_j)\,\mathrm{d}x_j\right]$;
EP: expected profit;
ROI (Return on investment) = EP / Budget;
U (uniform distribution) $= \frac{1}{bj-aj}$ for $(aj \leq xj \leq bj)$.

Decision variables:

Y_{ij}: {1, if product j is supplied by supplier i; 0, else};
Q_j : order quantity of product j.

Objective function:

The objective function of problem is to maximize the expected profit.
Expected profit (EP) is achieved from bellowing function.
EP = SR − PC − SC + SV.
Therefore, a nonlinear programming model for selecting appropriate suppliers can be defined as follows:

$$\text{Max EP} = \text{EP}(Q_j, Y_{ij})$$

Constraints:

1. $\int_{a_j}^{Q_j} f_j(x_j)\,\mathrm{d}x_j \geq \text{minimum SL required}, j = 1,\ldots,n$
2. $PC \leq AB$
3. $\sum_{i=1}^{m} Y_{ij} = 1$
4. $Y_{ij} = \begin{cases} 1, \text{ if product } j \text{ is supplied by supplier } i\,; \\ 0, \text{ otherwise.}\end{cases}$
5. $Q_j \geq 0 j = 1,\ldots,n.$

Constraint 1 reflects that stochastic demand for product j must meet the minimum service level; therefore, when Q_j is ordered, the firm can insure to have actual service level for product j. The second constraint shows that the firm should consider its available budget when purchasing the products. Thus, the total purchasing payment cannot become more than the available budget. Constraint 3 demonstrates that each product should

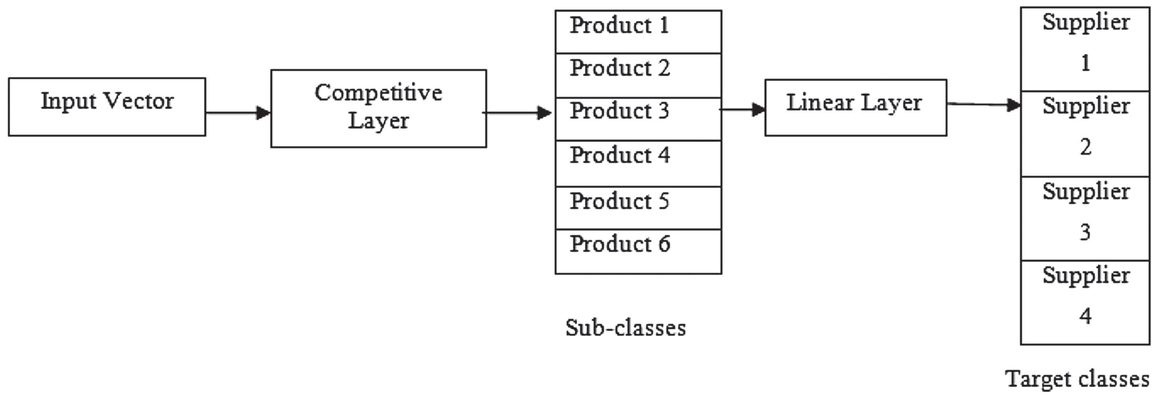

Figure 2. Schematic model: LVQNN structure.

be only supplied by a single supplier. Constraint 4 represents a binary variable, which is determined by our Learning Vector Quantization Neural Network. Constraint 5 determines how much product the selected supplier should provide.

2.2. Schematic model

In this section, the main goal is to give the whole perspective about an adequate design based on LVQ for our case study data set.

For designing our neural network model, the following three phases need to be conducted.

Phase 1: Design the component

First, we should define the architecture of the network (inputs, outputs, hidden layer and nodes). Inputs are parameters required for decision-making. In this study, the integration of suppliers and the number of allocation order are outputs of the model. The product information (*e.g.*, C_{ij}, P_{ij}, L_{ij}, S_{ij}, a_j, b_j), obtained from the suppliers, are the network's input.

Phase 2: Network's training

In order to train the neural network, the type should be selected. We use a LVQNN with two layers (*i.e.*, competitive and linear layers), proposed by Kohonen [17], which combines competitive learning with supervision. The competitive layer implements classification *via* learning: each neuron with the learning attribute can be used to recognize prototype vectors and classifies the area of the input space. In order to achieve the classification, we should calculate the distances between the input vectors and the prototype vectors. After determining the classification of input vectors, the linear layer converts the information achieved from competitive layer into the target classes, which is defined by the user and shows the outputs. The neuron with the value one (1) shows that weight vector of its neuron is closest to the input vectors and in this case, the other neurons get value zero (0).

As shown in Figure 2, the output of the competitive layer is called sub-classes, which in this study represent the firm's products. Another class in LVQ's architecture is called target classes as an output of linear layer that in this study represents our suppliers using approach in reference [29]. In other words, LVQNN allocates sub-classes (products) to target classes (suppliers) in order to maximize the firm's profit.

After identifying the components of LVQNN, our network needed to be prepared for training with regard to the target value.

Phase 3: Implementation

Network implementation is done by simulation to show how the LVQNN model structure works properly. In order to lead correct decision, an accurate LVQNN topology should be designed. According to Figure 2, products

TABLE 1. Designed LVQ.

Parameters	Details
Type of neural network	Learning vector quantization
Learning function	Learning quantization 2
Training epochs	100
Input nodes	6
Hidden nodes	13
output nodes	4

TABLE 2. The best value for each supplier.

	Supplier 1	Supplier 2	Supplier 3	Supplier 4
Product 1	$y_{11} = 1$	$y_{21} = 0$	$y_{31} = 0$	$y_{41} = 0$
Product 2	$y_{12} = 0$	$y_{22} = 1$	$y_{32} = 0$	$y_{42} = 0$
Product 3	$y_{13} = 0$	$y_{23} = 0$	$y_{33} = 0$	$y_{43} = 1$
Product 4	$y_{14} = 0$	$y_{24} = 1$	$y_{34} = 0$	$y_{44} = 0$
Product 5	$y_{15} = 0$	$y_{25} = 0$	$y_{35} = 1$	$y_{45} = 0$
Product 6	$y_{16} = 1$	$y_{26} = 0$	$y_{36} = 0$	$y_{46} = 0$

are our inputs and suppliers are our outputs. Based on our firm's activity on outsourcing six products considering four suppliers for supplying its material, our LVQNN has six input nodes (representing the outsourced products). The information about these six products is provided in Appendix A. LVQNN has a good performance with one hidden layer [16, 27]. Using the equation $(2n + 1)$ for finding the hidden nodes, where n is the number of input nodes, with $n = 6$, the hidden nodes are 13 [36]. We applied learning vector quantization 2 as the learning function. Table 1 demonstrates the designed LVQ training parameters and their values.

3. Implementing the Model: Numerical example

For evaluating the performance of our proposed model, it is surveyed with MATLAB (2013) for the set of data provided by an industrial manufacturing plant, shown in Appendix A. The result has been achieved after training the LVQNN for 100 epochs considering a stochastic demand with uniform distribution.

3.1. Getting the optimal values of decision variables

After implementing the LVQ NN, the optimal values of decision variables are obtained, as follows (Tab. 2):

As mentioned, products of the case study are known as sub-classes of the LVQNN. Suppliers are known as target classes, based on 4 suppliers (presented as four columns in LVQ network). The LVQ network allocates 6 subclasses (products) through 4 neurons in the linear layer to 4 target classes (suppliers). Considering the values of the binary variable y_{ij}= 1, the output of LVQ network for products 1–6 should be supplied by supplier 1, 2, 4, 2, 3, 1, respectively.

3.2. The optimal number of products

The LVQNN, formulating a nonlinear programming model with the objective of maximizing the EP, calculates the optimal order quantity for each product (j), as follows:

$$Q_1 = 232 \quad , Q_2 = 286, \quad Q_3 = 428, \quad Q_4 = 279, \quad Q_5 = 182, \quad Q_6 = 193.$$

Maximum EP based on the optimal values of Q_j and y_{ij}, which have been shown above, is: $34,377.96.

3.3. Comparative Analysis

As discussed in the introduction section, different methods such as QFD Fuzzy MCDM approach, Mixed-integer linear programming, AHP, and ANP are proposed to solve supplier selection problems. Based on reviewing the existing literature about supplier selection problem, it seems that there is not any survey, which considers the supplier selection problem as a classification case. Considering supplier selection as classification problem leads us to use the unique method such as LVQNN for the first time.

The role of inventory control in achieving expected profit was not mentioned in the existing literature. The findings can contribute to the existing literature on the significance impact of shortage and overstocks on rise or fall the expected profit.

4. Sensitivity Analysis

After designing the model and getting the selection of suppliers, sensitivity of the model to changes in constraints are analyzed. It is important for managers to know how the variation in budget and service level can change the EP and ROI.

4.1. Expected Profit (EP) based on different budget constraints

Without considering and changing the amount of service level constraint, we change the amount of budget. The relationship between budget and expected profit is shown in Table 3.

Based on Table 3, the firm in our case study will have the highest profit (*i.e.*, \$34,377.96) under a budget of \$96,312. The optimum profit calculation is based on optimum variables in the previous parts. The important point is that the optimal SL corresponding to the optimal expected profit is 0.6752. As shown in the next section the average of all SL for all products is equal to the optimal SL, therefore, SL for each product in the optimal result is close to the optimal SL, which means the lowest dispersion and the best statistical condition.

4.2. The optimal order quantity given the optimal budget

Table 4 shows the optimal integration of suppliers with the optimal value of order quantity for each one considering the optimal budget and different SL.

4.3. Return on Investment (ROI) based on the change of Service Level (SL)

For sensitivity analysis on service level, this variable as an input is varied between 0 (not given service) and 1 (completely given service) while other inputs are fixed. Expected profit is computed based on this variation needed for calculating ROI. Return on investment (ROI) is calculated by dividing expected profit by budget.

Table 5 shows the relationship between ROI (which is calculated as EP/budget) and SL. As shown in Table 3, the maximum EP (∼\$34K) occurred in SL = 0.68, while the maximum ROI is occurred when the SL is 0.5. If

Table 3. Expected profit and budget change.

Budget(\$)	Expected profit (EP) (\$)	Service level (SL)
65,562	15,654.68	0
79,642	26,874.43	0.2512
92,842	3,764.05	0.6015
96,312	**34,377.96**	0.6752
95,952	32,761.89	0.7021
105,823	30,011.41	0.8151
121,210	28,001.83	1

Table 4. the optimal order quantity of products

Product j	Supplier i	Q_j	SL	Mean	
1	1	232	0.6633	$\overline{SL}_{=}$	0.6752
2	2	286	0.6735		
3	4	428	0.6990		
4	2	279	0.6503		
5	3	182	0.6777		
6	1	193	0.9638		

Table 5. EP and ROI *versus* SL.

SL	EP ($)	Budget needed ($)	ROI = EP/budget (%)
0	18,879.32	66,831	28.2493
0.1	20,019.98	70,287	28.4832
0.2	22,117.37	74,561	29.6634
0.3	27,879.38	80,013	34.8436
0.4	30,116.93	84,650	35.5782
0.5	31,697.23	87,127	**36.3805**[1]
0.6	32,995.86	91,110	36.2154
3.68	**34325.71**[2]	96719	35.4901
0.7	32,522.88	94,986	34.2397
0.8	31,872.31	100,852	31.6031
0.9	30,610.76	108,012	28.3401
1	28,996.82	115,982	25.0011

[1] Optimal value for ROI.
[2] Optimal value for EP.

managers and/or decision makers in firms are trying to return the initial investment in the earliest time, they should pay attention to ROI, which is 36.38% in this case.

5. Conclusion and further research

In this paper, we study multi-suppliers' selection problem, considering a single buyer requiring multi-products with stochastic demand, to find the optimal integration of suppliers. Stochastic demand can bring shortage and overstocks to firms. Therefore, inventory control was considered in this article. Both optimal EP and return on investment (ROI) can help managers to control their inventory properly. This has been formulated as a nonlinear programming optimization model, with the objective of maximizing the expected profit subject to budget and service level constraints. LVQ neural network model as a self-learning method, which combines competitive learning with supervision, is used for the first time to solve this problem.

The findings show that the service level for the optimal return on investment (ROI) is different from the service level for the optimal expected profit (EP). Based on the results, the following suggestions are recommended for managers and decision makers: (1) to get higher profit, managers should find and control appropriate service level and budget in order to maximize the profit; this situation occurs when decision maker know how to control the firm's inventory. Based on facing to stochastic demand, they have to determine optimal budget for preparing enough goods in order to respond to the market demand and (2) when maximizing return on investment, the optimal ROI, the service level and budget might be less than the optimal service level and budget (in the case used in this study, the optimal ROI leads to a non-optimal profit).

For further research, the supplier selection spatial model presented in this article can be applied with other decision-making methods such as Tabu search, simulated annealing, and ant colony. In this case, the performance of different methodologies can be compared in order to gain a better solution. Also, instead of selecting single supplier for providing each product, scholars can solve this problem for multiple suppliers. In addition, to minimize the uncertainty related to stochastic demand, fuzzy theory can be applied in future works to model the supplier selection problem. Besides, single period can be substituted with multiple periods; in this case, the lead-time incorporates to our model.

Appendix A. Case study data (parameters)

Product		Supplier1	Supplier 2	Supplier 3	Supplier 4
Product 1	c_{ij}	109	100	103	106
	P_{ij}	149	126	132	139
	L_{ij}	89	81	78	83
	S_{ij}	69	62	64	66
	a_j	150	150	150	150
	b_j	280	280	280	280
Product 2	c_{ij}	95	99	88	91
	P_{ij}	114	127	101	114
	L_{ij}	74	85	67	74
	S_{ij}	55	60	53	55
	a_j	210	210	210	210
	b_j	330	330	330	330
Product 3	c_{ij}	52	57	60	59
	P_{ij}	71	78	83	78
	L_{ij}	40	48	50	48
	S_{ij}	29	36	39	36
	a_j	320	320	320	320
	b_j	480	480	480	480
Product 4	c_{ij}	70	62	63	60
	P_{ij}	92	87	87	80
	L_{ij}	59	55	55	50
	S_{ij}	44	40	40	38
	a_j	217	217	217	217
	b_j	317	317	317	317
Product 5	c_{ij}	125	137	115	121
	P_{ij}	171	180	147	162
	L_{ij}	113	118	97	104
	S_{ij}	84	86	76	79
	a_j	130	130	130	130
	b_j	210	210	210	210
Product 6	c_{ij}	110	103	105	100
	P_{ij}	161	156	147	140
	L_{ij}	80	75	73	70
	S_{ij}	51	48	46	42
	a_j	120	120	120	120
	b_j	230	230	230	230

Acknowledgement. This research was supported by Korean MSIT (Ministry of Science and ICT) under the ITRC support program (IITP-2017-2016-0-00312) supervised by the IITP.

References

[1] R. Aggarwal, S.P. Singh and P.K. Kapur, Integrated dynamic vendor selection and order allocation problem for the time dependent and stochastic data. *Benchmarking An Int. J.* **25** (2018) 777–796.

[2] A. Azadeh and S.M. Alem, A flexible deterministic, stochastic and fuzzy Data Envelopment Analysis approach for supply chain risk and vendor selection problem: Simulation analysis. *Expert Syst. Appl.* **37** (2010) 7438–7448.

[3] M. Biehl, A. Ghosh and B. Hammer, Dynamics and generalization ability of LVQ algorithms. *J. Mach. Learn. Res.* **8** (2007) 323–360.

[4] G. Büyüközkan and G. Çifçi, A novel fuzzy multi-criteria decision framework for sustainable supplier selection with incomplete information. *Comput. Ind.* **62** (2011) 164–174.

[5] F. Çebi and D. Bayraktar, An integrated approach for supplier selection. *Logist. Inf. Manag.* **16** (2003) 395–400.

[6] J. Chai, J.N.K. Liu and E.W.T. Ngai, Application of decision-making techniques in supplier selection: A systematic review of literature. *Expert Syst. Appl.* **40** (2013) 3872–3885.

[7] V. Chaudhary, R. Kulshrestha and S. Routroy, State-of-the-art literature review on inventory models for perishable products. *J. Adv. Manag. Res.* **15** (2018) 306–346.

[8] Y.J. Chen, Structured methodology for supplier selection and evaluation in a supply chain. *Inf. Sci. (NY)* **181** (2011) 1651–1670.

[9] L. Cheng, E. Subrahmanian and A.W. Westerberg, Design and planning under uncertainty: issues on problem formulation and solution. *Comput. Chem. Eng.* **27** (2003) 781–801.

[10] S. Kumar De and S.S. Sana, Two-layer supply chain model for Cauchy type Stochastic demand under fuzzy environment. *Int. J. Intell. Comput. Cybern.* **11** (2018) 285–308.

[11] L. Duan and J.A. Ventura, A dynamic supplier selection and inventory management model in a serial supply chain with a novel supplier price break scheme and flexible time periods. *Eur. J. Oper. Res.* **272** (2019) 979–998.

[12] M. Dursun and E.E. Karsak, A QFD-based fuzzy MCDM approach for supplier selection. *Appl. Math. Model.* **37** (2013) 5864–5875.

[13] C. Gencer and D. Gürpinar, Analytic network process in supplier selection: A case study in an electronic firm. *Appl. Math. Model.* **31** (2007) 2475–2486.

[14] S.H. Ghodsypour and C. O'brien, The total cost of logistics in supplier selection, under conditions of multiple sourcing, multiple criteria and capacity constraint. *Int. J. Prod. Econ.* **73** (2001) 15–27.

[15] W. Ho, X. Xu and P.K. Dey, Multi-criteria decision making approaches for supplier evaluation and selection: A literature review. *Eur. J. Oper. Res.* **202** (2010) 16–24.

[16] S. Karsoliya, Approximating number of hidden layer neurons in multiple hidden layer BPNN architecture. *Int. J. Eng. Trends Technol.* **3** (2012) 714–717.

[17] T. Kohonen, Self-Organizing Maps. Springer, Berlin (1995).

[18] R. Kouwenberg, Scenario generation and stochastic programming models for asset liability management. *Eur. J. Oper. Res.* **134** (2001) 279–292.

[19] R.J. Kuo, Y.C. Wang and F.C. Tien, Integration of artificial neural network and MADA methods for green supplier selection. *J. Clean. Prod.* **18** (2010) 1161–1170.

[20] M. Kurimo, Using self-organizing maps and learning vector quantization for mixture density hidden Markov models (1997) CiteSeer.

[21] Z. Liao and J. Rittscher. A multi-objective supplier selection model under stochastic demand conditions. *Int. J. Prod. Econ.* **105** (2007) 150–159.

[22] S. Ljubojević, D. Pamučar, D. Jovanović and V. Vešović, Outsourcing transport service: a fuzzy multi-criteria methodology for provider selection based on comparison of the real and ideal parameters of providers. *Oper. Res. Int. J.* **19** (2019) 399–433.

[23] A. Mendoza and J.A. Ventura, Modeling actual transportation costs in supplier selection and order quantity allocation decisions. *Oper. Res.* **13** (2013) 5–25.

[24] N.M. Modak, Exploring Omni-channel supply chain under price and delivery time sensitive stochastic demand. *Sup. Chain. For. Int. J.* **18** (2017) 218–230.

[25] N.M. Modak and P. Kelle, Managing a dual-channel supply chain under price and delivery-time dependent stochastic demand. *Eur. J. Oper. Res.* **272** (2019) 147–161.

[26] R.L Nydick and R.P. Hill, Using the analytic hierarchy process to structure the supplier selection procedure. *J. Supply Chain Manag.* **28** (1992) 31.

[27] G. Panchal, A. Ganatra, Y.P. Kosta and D. Panchal, Behaviour analysis of multilayer perceptronswith multiple hidden neurons and hidden layers. *Int. J. Comput. Theory Eng.* **3** (2011) 332–337.

[28] T.C. Poon, K.L. Choy, C.K. Cheng, S.I. Lao and H.Y. Lam, Effective selection and allocation of material handling equipment for stochastic production material demand problems using genetic algorithm. *Expert Syst. Appl.* **38** (2011) 12497–12505.

[29] H. Pujara and K. Prasad, Image segmentation using learning vector quantization of artificial neural network. *Int. J. Adv. Res. Artif. Intell.* **2** (2013) 51–55.

[30] R. Ramanathan, Supplier selection problem: integrating DEA with the approaches of total cost of ownership and AHP. *Supply Chain Manag. Int. J.* **12** (2007) 258–261.

[31] R. Farzipoor Saen, Suppliers selection in the presence of both cardinal and ordinal data. *Eur. J. Oper. Res.* **183** (2007) 741–747.

[32] N.V. Sahinidis, Optimization under uncertainty: state-of-the-art and opportunities. *Comput. Chem. Eng.* **28** (2004) 971–983.

[33] M. Sakawa, I. Nishizaki and Y. Uemura, A decentralized two-level transportation problem in a housing material manufacturer: Interactive fuzzy programming approach. *Eur. J. Oper. Res.* **141** (2002) 167–185.

[34] H. Shin, D.A. Collier and D. Wilson, Supply management orientation and supplier/buyer performance. *J. Oper. Manag.* **18** (2000) 317–333.

[35] H. Stadtler, Supply chain management and advanced planning – basics, overview and challenges. *Eur. J. Oper. Res.* **163** (2005) 575–588.

[36] D. Stathakis, How many hidden layers and nodes? *Int. J. Remote Sens.* **30** (2009) 2133–2147.

[37] A.A. Taleizadeh, S.T. Akhavan Niaki and F. Barzinpour, Multiple-buyer multiple-vendor multi-product multi-constraint supply chain problem with stochastic demand and variable lead-time: a harmony search algorithm. *Appl. Math. Comput.* **217** (2011) 9234–9253.

[38] S.-C. Ting and D.I. Cho, An integrated approach for supplier selection and purchasing decisions. *Supply Chain Manag. Int. J.* **13** (2008) 116–127.

[39] S. Türk, E. Özcan and R. John, Multi-objective optimisation in inventory planning with supplier selection. *Expert Syst. Appl.* **78** (2017) 51–63.

[40] T. Villmann, A. Bohnsack and M. Kaden, Can learning vector quantization be an alternative to SVM and deep learning?-recent trends and advanced variants of learning vector quantization for classification learning. *J. Artif. Intell. Soft Comput. Res.* **7** (2017) 65–81.

[41] A. Witoelar, M. Biehl and B. Hammer, Learning vector Quantization: generalization ability and dynamics of competing prototypes. In *Dagstuhl Seminar Proceedings*, *Schloss Dagstuhl-Leibniz-Zentrum für Informatik* (2007).

[42] J. Wu, LVQ neural network based classification decision approach to mechanism type in conceptual design. *Artif. Intell. Comput. Intell.* **7004** (2011) 378–384.

[43] P.C. Yang, H.M. Wee, S. Pai and Y.F. Tseng, Solving a stochastic demand multi-product supplier selection model with service level and budget constraints using Genetic Algorithm. *Expert Syst. Appl.* **38** (2011) 14773–14777.

[44] Z. Qu, H. Raff and N. Schmitt, Incentives through Inventory Control in Supply Chains. *Int. J. Ind. Organ.* **59** (2018) 486–513.

[45] J.L. Zhang and M.Y. Zhang, Supplier selection and purchase problem with fixed cost and constrained order quantities under stochastic demand. *Int. J. Prod. Econ.* **129** (2011) 1–7.

[46] A. Zouggari and L. Benyoucef, Simulation based fuzzy TOPSIS approach for group multi-criteria supplier selection problem. *Eng. Appl. Artif. Intell.* **25** (2012) 507–519.