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CROSS-BRAND AND CROSS-CHANNEL ADVERTISING STRATEGIES IN A DUAL-CHANNEL SUPPLY CHAIN

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Abstract. The economic value of private brands and online marketing channel have been widely recognized in literature and practical life. Besides, studies show that advertising, as one of the major factors, can affect consumer attitudes and has significant effects on demand and profit. On the existing basis, this paper analyzes the advertising strategies under competition between a national brand manufacturer and a retailer with private brand, where the national brand can be sold both through a direct channel and a store channel but the retailer brand can be sold only through a store channel. We study the best advertising investment strategies and the balanced profits of the national brand manufacturer and retailer in the disintegrated system and the integrated system. Specially, in the disintegrated system, we discuss the best decision-making issues for national brand manufacturer and retailer in two special cases which there is only have brand competition or channel spillover effects. We discuss the impacts of the spillover effect and brand competition on the chain members and advertising strategies of different channels. In addition, we design a unilateral advertising subsidy contract to coordinate the supply chain. The results in this paper offer structural and quantitative insights into the interplay between the manufacturer and retailer in the dual channel supply chain and can be used as a reference for choosing the optimal advertising strategy.

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1. Introduction

The development of more and more private label products is becoming a tool for retailers to attract more customers and create differentiated competitive advantages. Own brand refers to the product that is developed and produced directly by the physical retailer and the factory, and is branded by the retail company and sold in the store. For example, in 1993, Wal-Mart's own brand "Great Value" was born in the United States, and currently has nearly 2000 products sold in China.

Retailers' own brand has entered a stage of rapid development and occupies more and more market shares. Meanwhile, the retailers' brands are threatening the national brands manufacturers. According to data released by Wal-Mart, in the first quarter of 2018, omni-channel sales of Great Value products increased by nearly 40%

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year-on-year. On February 19, 2017, Gaoxin Retail released its 2016 financial report, which showed that the sales volume of RT-Mart's own brand actual increased by 88%, and the sales of luggage and travel goods brand Airport and sports brand Cup's doubled compared with last year. Many of the existing theoretical research focus on the model of price competition between brands after introduced retailer brands [10, 21, 33].

But, a lot of experience suggests that the marketing strategies also played a key role in the brand competition. Advertising is one of the most commonly used marketing tools. Manufacturers and retailers can use advertising to improve their brand awareness and loyalty. Advertising expenditure represents an important part of firms' marketing budgets. In the first half of 2016, China's four largest dairy companies spent 7.742 billion RMB yuan on advertising, which was 1.65 times the total net profit, with an average daily expenditure of 42 430 000 yuan. (The new blue network report, 2016). IEG predicted that global sponsorship AD spending in 2017 will grow from $60.1 billion in 2016 to $62.8 billion, up 4.5% from a year earlier, 2017 American sponsored advertising spending is expected to increase 4.1%, to $23.2 billion. (IEG report, 2017). However, most researches are focused on product pricing. Few scholars have studied the advertising investment strategies of retailers' own brands (SB) and national brand manufacturers (NB). This is still worth studying.

At the same time, as the popularization of internet, more and more manufacturers explore the application of e-channel as a direct channel to sell their products. Many big manufacturers, such as Haier and Huawei, have not only built their direct stores on TMALL, JD.COM, and other platforms, but also kept their own online stores. Haier Group has established an online enterprise to sell appliances online. Network marketing has brought huge benefits to enterprises, and it has also accelerated the development speed of enterprises. 2017 Tmall double eleven "100 million yuan club" list shows that Haier ranked third. Compared with the physical channels, the direct online channel can provide wider coverage with lower operating costs and easy access to vast consumers [5, 20, 36].

However, most of the existing research on direct sales online of brand manufacturers focuses on pricing and channel competition. Few articles have studied the advertising investment strategies of retailers and national brand manufacturers while considering retailers' own brands and national brand manufacturers' online direct sales channels. This paper will focus on this issue, and consider the advertising investment strategies of retailers and national brand manufacturers to enrich and supplement the existing literature while considering brand competition and channel competition.

This paper differs from previous studies in two aspects. Firstly, previous works have explored the research areas of brand advertising, local advertising and dual-channel supply chain management in isolation. This work attempts to integrate these three streams of research into one framework. Secondly, we consider the brand competition and channel competition simultaneously using the demand model. Finally, we analyze the impact of various factors on the advertising expenditures and profits of the manufacturer and the retailer.

Specifically, this article builds a supply chain model consisting of a retailer and a national brand manufacturer. The retailer produces and sells its' own brand products through physical stores, and the national brand manufacturer sells its' products through online direct sales channel and the retailer channel. National brand manufacturer launches national advertising for its' products, and retailer place local advertising on the products of its' own brand and national brand. Considering brand competition and channel competition, we will study the best advertising investment strategies and the balanced profits of the national brand manufacturer and retailer in the disintegrated system and the integrated system. Specially, in the disintegrated system, we will discuss the best decision-making issues for national brand manufacturer and retailer in two special cases which there is only have brand competition or channel spillover effects. We will discuss the impacts of the spillover effect and brand competition on the chain members and advertising strategies of different channels, so that enrich and supplement existing literature. This paper will also design a unilateral advertising subsidy contract to coordinate the supply chain. The conclusions of this paper will provide recommendations for national brand manufacturers and retailers in the competition and advertising investment strategies in the dual-channel supply chain, which can be used as a reference for selecting the best advertising investment strategy.

The remainder of the study is organized as follows. Section 2 is devoted to a literature review. In Section 3, we introduce the supply structure, notations and key assumptions. General models are proposed in Section 4. Follows, we coordinate the supply chain in Section 5 and summary conclusions in Section 6.

2. Literature review

This paper is related to three streams of researches in the literature: the first one examines the perspective of dual-channel supply chain, the second one studies retailer private brands, and the last one studies advertising strategies.

For the first stream, the introduction of online direct marketing as a new distribution channel has an inevitable conflict with the traditional offline distribution channel. The conflict is a headache for firms committed to the development of e-commerce enterprises. Most studies pay attention on the pricing competition between the members, but not consider retailer private brand and the effect of advertising strategies at the same time [3, 7, 11, 16–18, 23, 25, 28, 31, 36–38, 40, 42, 45]. For instance, Huang *et al.* [17] introduce demand disturbance management and studies the joint decision-making problem in the production and pricing of dual channel supply chain. Ryan *et al.* [36] obtain optimal price of the dual channel supply chain and compare the optimal price of the integrated supply chain under the double channel. Li *et al.* [23] and Liu *et al.* [28] study the pricing strategies and the influence of risk aversion in a dual-channel supply chain from the point of risk-adverse retailer and asymmetric information respectively. Zhao *et al.* [45] investigate the pricing problem of complementary products in a dual-channel supply chain with two manufacturers and one retailer. Modak [31] develops a two-level Omni-channel supply chain under price and delivery time sensitive additive stochastic demand. The decision variables are price, stocking and delivery lead time. Saha *et al.* [38] explore characteristics of three different channel structures of three-echelon supply chain, and analyze the optimal pricing policies of all the proposed models.

The rapid development of retailer brand has aroused the concern of the academia. By launching its' own brand product, retailers have changed into rivals from the product demanders for the manufacturers. This will produce competition issues between retailer brand and the similar manufacturer brand. The second stream of literature primarily examines retailer private brands from the points of consumer perceptions, pricing competition and so on, but few studies have considered the competition between the retailer's own brand and national brand from the perspective of advertising and channel competition [1,4,6,8–10,21,24,29,33,35,39]. For instance, Chen *et al.* [6] examine the differential impact of various upstream supply arrangements for store brand products or the strategic motives for store brand supply. Seenivasan *et al.* [39] investigate the relationship between store brand loyalty and store loyalty. Chung and Lee [9] construct a game theoretic model composed of one or two national brand manufacturers and a retailer, and analyze the retailer's store brand quality decision in vertically differentiated product categories. Choi [8] builds a game-theoretic model of price competition between a national brand manufacturer and a retailer that also sells its private label and examines a national brand's strategy of building brand premium in the context of channel coordination. Chung and Lee [10] conduct an empirical investigation of the impact of store brand introductions on the price leadership relations in a distribution channel between a retailer and national brand manufacturers. Li *et al.* [24] build a supply chain with a store brand and a national brand, and examine the retailer's decision on returns policies for the two brands and the effects of returns policies on the competition between the two brands.

Besides, a large number of empirical studies show that the marketing also played a key role in the brand competition. Advertising strategy is one of the most commonly used methods. Manufacturers and retailers can use advertising to further improve consumer's brand awareness and loyalty in order to gain more profits [2,12–15,19,22,26,27,30,32,34,41,43,44]. For instance, Xie and Wei [41] address channel coordination by seeking optimal cooperative advertising strategies and equilibrium pricing in a two-member distribution channel. Liu *et al.* [27] build a supply chain model contains two competing manufacturer–retailer, and examine the efficacy of cost sharing. Lu *et al.* [30] establish an optimization model to maximize the firm's total profit by making a joint pricing and advertising strategy. Lin *et al.* [26] build an analytical model to study generic and brand advertising competition under an inter-industry competitive framework. Zhang *et al.* [44] consider a Stackelberg game between one manufacturer and one online retailer, and examine the effects of dynamic pricing and advertising effort on supply chain performance. Nguyen *et al.* [32] test the assertion that co-branded advertising should ideally benefit both brands.

In summary, most of the existing research on dual-channel supply chain focuses on pricing competition among supply chain members, and rarely considers the impact of retailers' own brand and advertising investment strategies. As retailers become more and more concerned about the differentiated competitive advantages brought by their own brand, retailers' own brand have an increasingly greater impact on the cooperation and competition between manufacturer and retailer. So, it is necessary to consider the impact of retailers' own brand in the dual-channel supply chain research. In addition, the current research on retailers' own brand mainly focuses on consumer cognition and pricing competition. Few studies have introduced online direct sales channel to study the competition between retailers' own brand and national brand from the perspective of channel competition and advertising investment. In order to be more realistic and to supplement these previous papers, we consider the competition between retailers' own brand and national brand and the competition between online direct sales channel and retailers' channel in a dual-channel supply chain model consisting of a retailer and a national brand manufacturer, and study the advertising investment strategies and profitability of retailer and national brand manufacturer in different situations.

The main contributions of this paper to the extant literature are threefold: Firstly, we attempt to integrate the three streams of, as yet, rather disjointed research work: brand advertising, local advertising and dual-channel supply chain management. In the previous studies, these three factors are scant investigated simultaneously. The second achievement is that this study focus on the brand competition and channel competition simultaneously using the demand model. Thirdly, we analyze the impact of brand competition and channel competition on the advertising expenditures and profits of the manufacturer and the retailer. The conclusions of this paper will provide recommendations for large-scale brand manufacturers and retailers in the competition and advertising investment strategies in the dual-channel supply chain, which can be used as a reference for selecting the best advertising investment strategy.

3. Model overview

Consider a two-stage supply chain which consists of a national brand manufacturer and a retailer with its own brand. The national brand manufacturer can sell its product through two channels – the direct channel (*i.e.*, selling by himself) and the indirect channel (*i.e.*, selling through the retailer). The retailer also harbors and sells its own brand. For simplicity, we call the national brand manufacturer he and the retailer she in this paper. Figure 1 below depicts our supply chain framework clearly. This framework confirms many practical selling structures. For example, Haier sells its electrical appliances both through his own on-line store and the 3PL retailer such as Suning. Meanwhile, Suning may sell electrical appliances of its own brand.

In this framework, the national brand manufacturer decides his advertisement investments on the direct channel (*i.e.*, A), the retailer decides her advertisement investment for her own brand (*i.e.*, g) and for the national brand (*i.e.*, a).

We consider both the retailer's own brand and the direct channel in this paper, so we need a description of brand and channel competition.

The national brand manufacturer's national advertising has a positive impact on the demand of the national brand both in the direct channel and the indirect channel, but has a negative impact on the demand of the retailer's own brand. The local advertising for the national brand in the store also has a positive impact on the demand of the national brand both in the direct channel and the indirect channel, but has a negative impact on the demand of the retailer's own brand. The local advertising for the retailer's brand has a negative impact on the demand of the national brand both in the direct channel and the indirect channel, but has a positive impact on the demand of the retailer's own brand.

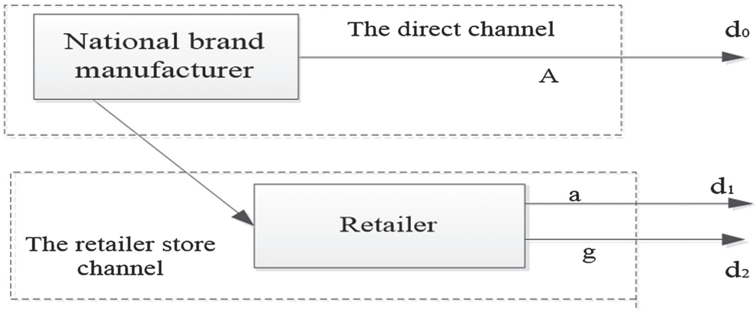

FIGURE 1. Dual-channel supply chain.

TABLE 1. List of notations.

Symbol	Description
π_{NB}	Profit of the national brand manufacturer (NB)
π_{SB}	Profit of the retailer
π_I	Total profit for the integrated system
a	Advertising effort of the retailer for NB in retailer store
A	Advertising effort of NB for itself in direct channel
g	Advertising effort of retailer for itself in store channel
d_i	Demand of brand i
a_i	Potential market size of brand i
m_i	Marginal revenue of brand i
b_i	Self-advertising sensitivity of brand i
β_0	Cross-advertising sensitivity between NB at direct channel and indirect channel
β_1	Cross-advertising sensitivity between NB at direct channel and the retailer brand
β_2	Cross-advertising sensitivity between NB at store channel and the retailer brand
Subscripts	$i = 0$ for an NB at direct channel $i = 1$ for an NB at store channel $i = 2$ for retailer at store channel

In this paper, we assume that parameter β_0 captures the promoting effect degree between direct channel and the indirect channel of the advertising for national brand. Parameters β_1 and β_2 capture consumers' brand choice behavior. The more store brand products which are similar to the national brand products, β_1 and β_2 have greater values. Besides, parameters $b_i,\ i = 0, 1, 2$ represent the self-advertising sensitivity. We assume that $b_i > \beta_i,\ i = 0, 1, 2$ can be interpreted as the influence of the self-advertising is larger than that of other advertising. All parameters are summarized in Table 1.

The following notations in Table 1 are used for the development of the mathematical models.

4. Model development

We will study the best advertising investment strategies and the balanced profits of the national brand manufacturer and retailer in the disintegrated system and the integrated system. Specially, in the disintegrated system, we will discuss the best decision-making issues for national brand manufacturer and retailer in two special cases which there is only have brand competition or channel spillover effects.

In this article, the superscripts "C" indicate the case that there is only channel spillover effect, the superscripts "B" indicate the case that there is only brand competition.

4.1. The disintegrated system

In this subsection, we consider the case when there exists both the brand competition between the two brands and the spillover effect across the two channels. For example, the electric appliance brand CHANGHONG not only sells products through the retail channel of Suning Tesco, but also establishes its own online direct sales channel to sell products, and its products are sold online and offline, and the same models are available online and offline. At the same time, Suning's own brand Whirlpool also produces and sells electrical appliances such as refrigerators, air conditioners and washing machines. Therefore, there is a competitive relationship between CHANGHONG brand and Suning's own brand Whirlpool. The advertisement of brand manufacturer CHANGHONG in the direct sales channel will also affect Suning's sales of CHANGHONG brand products and Whirlpool brand products. Similarly, Suning's advertisements for CHANGHONG brand and Whirlpool brand in retail channels will also affect the sales of CHANGHONG brand products on direct sales channels.

Specifically, the national advertising campaign can not only increase the customers' demand for the national brand at the direct channel but also the indirect channel (*i.e.*, the retailer store). Similarly, the retailer's advertising campaign for the national brand can increase the customers' demand both at the retailer store and the direct channel. Here we adopt a same spillover effect ($\beta_0 \neq 0$) of the national advertising on the retailer store and the retailer's advertising on the direct channel. Besides, we assume the negative impact between the sale of the national brand on the direct channel and the sale of the retailer's own brand is β_1 ($\beta_1 \neq 0$). The negative impact between the sale of the national brand on the retailer's channel and the sale of the retailer's own brand is β_2($\beta_2 \neq 0$).

The national brand manufacturer devotes an advertising effort of A on the direct channel. The retailer devotes an advertising effort of a for the national brand and g for his own brand.

We assume a Nash game: both the national brand manufacturer and the retailer simultaneously decide their optimal advertising effort so as to maximize profit. Note that, this decision process reflects an actual situation in which a retailer is a giant that can produce its own private brand and has a strong market position. For example, large retailers such as Suning and Wal-Mart have advantages in scale, brand and management. In the strong and weak relationship with large-scale brand manufacturers (such as Haier, GREE, etc.), they are basically evenly matched and have strong bargaining power. Similar assumption is used in Kurata *et al.* [21].

The demand function for the national brand at the direct channel is

$$d_0 = a_0 + b_0 A + \beta_0 a - \beta_1 g. \tag{4.1}$$

The demand function for the national brand at the retailer store is

$$d_1 = a_1 + b_1 a + \beta_0 A - \beta_2 g. \tag{4.2}$$

The demand function for the retailer's own brand at the retailer store is

$$d_2 = a_2 + b_2 g - \beta_1 A - \beta_2 a. \tag{4.3}$$

The profit functions under NB *vs.* SB competition are formulated as follows:
The profit function for the national brand manufacturer is

$$\pi_{\mathrm{NB}} = m_0 d_0 + m_3 d_1 - \frac{1}{2}\eta_0 A^2. \tag{4.4}$$

The profit function for the retailer is

$$\pi_{\mathrm{SB}} = m_1 d_1 + m_2 d_2 - \frac{1}{2}\eta_1 a^2 - \frac{1}{2}\eta_2 g^2. \tag{4.5}$$

The objective function for the national brand manufacturer is

$$\underset{A}{\text{Max}}\, \pi_{\text{NB}}|a, g = m_0 d_0 + m_3 d_1 - \frac{1}{2}\eta_0 A^2. \tag{4.6}$$

The objective function for the retailer is

$$\underset{a,g}{\text{Max}}\, \pi_{\text{SB}}|A = m_1 d_1 + m_2 d_2 - \frac{1}{2}\eta_1 a^2 - \frac{1}{2}\eta_2 g^2. \tag{4.7}$$

Property 4.1 shows the concavity of π_{NB} (π_{SB}) with respect to A (a and g) which guarantees the existence of the unique maximum point of the profit function.

Property 4.1. (1) The profit function for national brand manufacturer, π_{NB}, is concave in A.
(2) The profit function for retailer,π_{SB}, is joint concave in a and g.

Solving the optimization models (4.6) and (4.7), we can get the equilibrium advertising investment: A^* for the direct channel, a^*, g^* for the retailer store channel.

Proposition 4.2. *The optimal advertising spending, A^*, a^* and g^* are determined as:*

$$\begin{bmatrix} A^* \\ a^* \\ g^* \end{bmatrix} = X^{-1}Y, \text{ where } X \equiv \begin{bmatrix} \eta_0 & 0 & 0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & \eta_2 \end{bmatrix}, Y \equiv \begin{bmatrix} b_0 m_0 + \beta_0 m_3 \\ b_1 m_1 - \beta_2 m_2 \\ b_2 m_2 - \beta_2 m_1 \end{bmatrix}.$$

Hence, we can derive that $A^* = \frac{b_0 m_0 + \beta_0 m_3}{\eta_0}, a^* = \frac{b_1 m_1 - \beta_2 m_2}{\eta_1}, g^* = \frac{b_2 m_2 - \beta_2 m_1}{\eta_2}$.

The profits of national brand manufacturer and retailer are:

$$\begin{aligned} \pi^*_{\text{NB}} &= m_0 a_0 + m_3 a_1 + \frac{(b_0 m_0 + \beta_0 m_3)^2}{2\eta_0} + \frac{(b_1 m_1 - \beta_2 m_2)(m_0 \beta_0 + m_3 b_1)}{\eta_1} \\ &\quad - \frac{(b_2 m_2 - \beta_2 m_1)(m_0 \beta_1 + m_3 \beta_2)}{\eta_2} \\ \pi^*_{\text{SB}} &= m_1 a_1 + m_2 a_2 + \frac{(b_0 m_0 + \beta_0 m_3)(m_1 \beta_0 - m_2 \beta_1)}{\eta_0} + \frac{(b_1 m_1 - \beta_2 m_2)^2}{2\eta_1} \\ &\quad + \frac{(b_2 m_2 - \beta_2 m_1)^2}{2\eta_2}. \end{aligned}$$

4.1.1. Sensitivity analysis

In the real marketing environment, many factors can affect firms' advertising spending, such as the consumers' brand perception and the stores' sales promotion activities. In this chapter, we discuss the sensitivity analysis of the equilibrium profit with respect to six parameters: $\beta_0, \beta_1, \beta_2, b_0, b_1, b_2$. Proposition 4.3 summarizes the sensitivities with respect to the equilibrium advertising spending. Propositions 4.4 and 4.5 show the sensitivities of the optimal profit for each firm.

Proposition 4.3. *The sensitivity of the optimal advertising spending with respect to the parameters are obtained as:*

$$\begin{aligned} &\frac{\partial A^*}{\partial b_0} \geq 0, \frac{\partial a^*}{\partial b_0} = 0, \frac{\partial g^*}{\partial b_0} = 0; \frac{\partial A^*}{\partial b_1} = 0, \frac{\partial a^*}{\partial b_1} \geq 0, \frac{\partial g^*}{\partial b_1} = 0; \\ &\frac{\partial A^*}{\partial b_2} = 0, \frac{\partial a^*}{\partial b_2} = 0, \frac{\partial g^*}{\partial b_2} \geq 0; \frac{\partial A^*}{\partial \beta_0} \geq 0, \frac{\partial a^*}{\partial \beta_0} = 0, \frac{\partial g^*}{\partial \beta_0} = 0; \\ &\frac{\partial A^*}{\partial \beta_1} = 0, \frac{\partial a^*}{\partial \beta_1} = 0, \frac{\partial g^*}{\partial \beta_1} = 0; \frac{\partial A^*}{\partial \beta_2} = 0, \frac{\partial a^*}{\partial \beta_2} \leq 0, \frac{\partial g^*}{\partial \beta_2} \leq 0. \end{aligned}$$

Proposition 4.4. *The sensitivity to the profit function π_{NB} of national brand manufacturer:*

$$\frac{\partial \pi^*_{\mathrm{NB}}}{\partial b_0} \geq 0; \frac{\partial \pi^*_{\mathrm{NB}}}{\partial b_1} \geq 0; \frac{\partial \pi^*_{\mathrm{NB}}}{\partial b_2} \leq 0; \frac{\partial \pi^*_{\mathrm{NB}}}{\partial \beta_0} \geq 0; \frac{\partial \pi^*_{\mathrm{NB}}}{\partial \beta_1} \leq 0; \frac{\partial \pi^*_{\mathrm{NB}}}{\partial \beta_2} \leq 0.$$

From Propostion 4.4, we know that when there is simultaneously exist brand competition and channel spillover effects, the balanced profits of national brand manufacturer π^*_{NB} is both increasing with b_0, b_1 and β_0. This is because, the more b_0 the more efficient the national brand manufacturers' advertising in the direct sales channel, the more significant the promotion of the sales of national brands in the direct sales channel; The bigger b_1 the more efficient the retailer's advertising for national brand products, the more it can promote the sales of national brand products in the retailer channel; The greater the channel spillover effect (β_0), the greater the promotion of the advertising investment of national brand in the direct sales channel (retailer channel) to the sales of national brand products in the retail channel (direct sales channel).

Correspondingly, since the national brand products sold in the direct sales channel competes with the retailer's own brand products (the competition coefficient is β_1), the national brand products sold in the retailer channel also competes with the retailer's own brand products (the competition coefficient is β_2), the greater the competition, the more unfavorable the sale of products. Therefore π^*_{NB} is a decreasing function for both β_1 and β_2. The greater the b_2, the higher the advertising efficiency of the retailer's own brand, thus promoting the sales of its own branded products, and enhance the crowding effect on the national brand products of the direct selling channel and the retailer channel. Therefore, π^*_{NB} is a decreasing function of b_2.

Proposition 4.5. *The sensitivity to the profit function π_{SB} of the retailer with respect to b_0 is:*

$$\frac{\partial \pi^*_{\mathrm{SB}}}{\partial b_0} = (m_1\beta_0 - m_2\beta_1)\frac{\partial A^*}{\partial b_0},$$

(1) if $(m_1\beta_0 - m_2\beta_1) \geq 0$, then $\frac{\partial \pi^*_{\mathrm{SB}}}{\partial b_0} \geq 0$
(2) if $(m_1\beta_0 - m_2\beta_1) \leq 0$, then

$$\frac{\partial \pi^*_{\mathrm{SB}}}{\partial b_0} \leq 0; \frac{\partial \pi^*_{\mathrm{SB}}}{\partial b_1} \geq 0, \frac{\partial \pi^*_{\mathrm{SB}}}{\partial b_2} \geq 0, \frac{\partial \pi^*_{\mathrm{SB}}}{\partial \beta_0} \geq 0, \frac{\partial \pi^*_{\mathrm{SB}}}{\partial \beta_1} \leq 0 \text{ and } \frac{\partial \pi^*_{\mathrm{SB}}}{\partial \beta_2} \leq 0.$$

From Proposition 4.5, the profit of the retailer π^*_{SB} can be either increase or decrease with the b_0. When the spillover effect across the two channels dominates the brand competition (*i.e.*, $m_1\beta_0 - m_2\beta_1 \geq 0$), the advertising effort of the national brand on the direct channel can bring benefit for the retailer. The reverse is true when the brand competition dominates the spillover effect across channels ($m_1\beta_0 - m_2\beta_1 \leq 0$).

Moreover, the larger b_1, the more the retailer's advertisements for national brand products can promote its sales in the retailer channel; the larger b_2, the more the retailer's own brand advertising can promote the sales of its own brand products; The larger the channel spillover effect β_0, the advertising investment of national brand in direct sales channel (retailer channel) can more promote the sales of national brand products in retail channel (direct sales channel). Therefore, π^*_{SB} is an increasing function with respect to b_1, b_2, β_0. The greater the competition between national brand products and retailers' own brand products, the more unfavorable with the product sales, so π^*_{SB} is a decreasing function for both β_1 and β_2.

Very interestingly, we know from Proposition 4.3 that the advertising investment of national brand manufacturer and retailer does not change with β_1: $\frac{\partial A^*}{\partial \beta_1} = 0$, $\frac{\partial a^*}{\partial \beta_1} = 0$, $\frac{\partial g^*}{\partial \beta_1} = 0$; Advertising investment by national brand manufacturer does not change with changes in β_2: $\frac{\partial A^*}{\partial \beta_2} = 0$, retailer's advertising investment in private label and national brand products decreases with the increase of β_2: $\frac{\partial a^*}{\partial \beta_2} \leq 0$, $\frac{\partial g^*}{\partial \beta_2} \leq 0$. And, by Propositions 4.4 and 4.5 we find that $\frac{\partial \pi^*_{\mathrm{SB}}}{\partial \beta_1} \leq 0$, $\frac{\partial \pi^*_{\mathrm{SB}}}{\partial \beta_1} \leq 0$. That is, the stronger the competitive relationship between the state-owned brand products sold and the retailer's own brand products in the direct sales channel, the lower the profit of the two companies; and $\frac{\partial \pi^*_{\mathrm{NB}}}{\partial \beta_2} \leq 0$, $\frac{\partial \pi^*_{\mathrm{SB}}}{\partial \beta_2} \leq 0$, that is, the stronger the competitive relationship between the state-owned brand products sold by retailers and the retailer's own brand products, the lower the profits of the two companies. Therefore, in summary, national brand manufacturer will not change his advertising strategies in direct sales channel because of the competitive between brands. For retailer, she will not change the advertising strategy because of the competitive relationship between national brand products sold on direct sales channel and retailer's own brand products. However, when the national brand products sold in the retailer's store compete with her own brand products, she will also reduce the advertising investment of national brand and private brand in retail channel to reduce the intensity of competition.

In addition, Proposition 4.3 shows that when there is both brand competition and product competition between national brand manufacturer and retailer (*i.e.*, $\beta_i \neq 0, i = 0, 1, 2$), the best advertising investment of national brand manufacturer is increasing with the channel spillover effect: $\frac{\partial A^*}{\partial \beta_0} \geq 0$, while retailers' advertising investment on national brand products does not change with channel spillover effects: $\frac{\partial a^*}{\partial \beta_0} = 0, \frac{\partial g^*}{\partial \beta_0} = 0$. And, by Propositions 4.4 and 4.5, we know that $\frac{\partial \pi_{\mathrm{NB}}^*}{\partial \beta_0} \geq 0$ and $\frac{\partial \pi_{\mathrm{SB}}^*}{\partial \beta_0} \geq 0$, that is, the profits of national brand manufacturer and retailer all increase with the increase of spillover effects. Therefore, we know that the advertising investment of national brand in the direct sales channel has positive externalities to the retailer's economic activities, that is, there is a phenomenon that retailers "free riders".

4.1.2. Special Scenario 1: Advertising campaign with spillover effect across channels

In this special scenario, we focus on the case when the advertising campaign for the same product has spillover effect across different channels ($\beta_0 \neq 0$). We consider there is no brand competition between the national brand and the retailer's own brand ($\beta_1 = \beta_2 = 0$). Thus, the advertising campaign for the national brand (the retailer's own brand) has no impact on the sale of the retailer's own brand (the national brand). This can happen when the product differentiation between the two products is huge. For example, Wal-Mart is a very large chain retailer with a wide range of products, including food, toys, new clothing, cosmetics, household appliances, daily necessities, meat and fruit, and more. Consumers can buy a refrigerator produced by Haier, a large home appliance brand, in a large supermarket in Wal-Mart. They can also purchase the refrigerator they want directly at Haier's online store. Then, Haier's advertising on its own products and the promotion of Haier refrigerators by Wal-Mart's large supermarkets will promote consumer buying behavior. At the same time, Wal-Mart also pays great attention to the development of its own brands, such as the daily necessities brand Great Value. Since the products of Great Value and Haier are very different, there is no competition between the brands. Therefore, Wal-Mart's advertisement for Great Value products will not affect the sales of Haier brand products.

When only brand spillover effect across channels exists, the preferable marketing strategies for the retailer and national brand manufacturer are obtained as follows:

Proposition 4.6. *Considering the channel spillover effect, π_{NB}^C is concave in A^C, π_{SB}^C is joint concave in a^c and g^c. The best advertising investment and profit of national brand manufacturer and retailer are:*

$$A^{C^*} = \frac{m_0 b_0 + m_3 \beta_0}{\eta_0}, a^{C^*} = \frac{m_1 b_1}{\eta_1}, g^{C^*} = \frac{m_2 b_2}{\eta_2}$$
$$\pi_{\mathrm{NB}}^{C^*} = m_0 a_0 + m_3 a_1 + \frac{m_1 b_1}{\eta_1}(m_0 \beta_0 + m_3 b_1) + \frac{(m_3 \beta_0 + m_0 b_0)^2}{2\eta_0}$$
$$\pi_{\mathrm{SB}}^{C^*} = m_1 a_1 + m_2 a_2 + m_1 \beta_0 \frac{m_0 b_0 + m_3 \beta_0}{\eta_0} + \frac{m_1^2 b_1^2}{2\eta_1} + \frac{m_2^2 b_2^2}{2\eta_2}.$$

Proposition 4.7. *(Sensitivity of the optimal profit when there is only channel competition between national brand and the retailer's own brand) For the national brand manufacturer:*

$$\frac{\partial \pi_{\mathrm{NB}}^{C^*}}{\partial b_0} \geq 0, \frac{\partial \pi_{\mathrm{NB}}^{C^*}}{\partial b_1} \geq 0, \frac{\partial \pi_{\mathrm{NB}}^{C^*}}{\partial b_2} = 0, \frac{\partial \pi_{\mathrm{NB}}^{C^*}}{\partial \beta_0} \geq 0.$$

For the retailer:

$$\frac{\partial \pi_{\text{SB}}^{C^*}}{\partial b_0} \geq 0, \frac{\partial \pi_{\text{SB}}^{C^*}}{\partial b_1} \geq 0, \frac{\partial \pi_{\text{SB}}^{C^*}}{\partial b_2} \geq 0, \frac{\partial \pi_{\text{SB}}^{C^*}}{\partial \beta_0} \geq 0.$$

The sensitivity of the balanced advertising investment about the parameters are:

$$\frac{\partial A^{C^*}}{\partial b_0} = \frac{m_0}{\eta_0} > 0, \frac{\partial a^{C^*}}{\partial b_0} = 0, \frac{\partial g^{C^*}}{\partial b_0} = 0; \frac{\partial A^{C^*}}{\partial b_1} = 0, \frac{\partial a^{C^*}}{\partial b_1} = \frac{m_1}{\eta_1} > 0, \frac{\partial g^{C^*}}{\partial b_1} = 0;$$

$$\frac{\partial A^{C^*}}{\partial b_2} = 0, \frac{\partial a^{C^*}}{\partial b_2} = 0, \frac{\partial g^{C^*}}{\partial b_2} = \frac{m_2}{\eta_2} > 0; \frac{\partial A^{C^*}}{\partial \beta_0} = \frac{m_3}{\eta_0} > 0, \frac{\partial a^{C^*}}{\partial \beta_0} = 0, \frac{\partial g^{C^*}}{\partial \beta_0} = 0.$$

From Proposition 4.7, $\pi_{\text{NB}}^{C^*}$ and $\pi_{\text{SB}}^{C^*}$ are increasing with β_0. This represents that the spillover effect of advertising across channels can always benefit the national brand and the store brand. This is because when there is a big difference between the national brand products and the retailer's own brand products, there is no competition between the two brands. Moreover, advertising information obtained from direct sales channel and retail channel will all encourage consumers to purchase products of nation brand. Then, national brand manufacturer can sell more products through direct channel, and retailer can also sell more products in retail channel. Therefore, in order to get more profit,the national brand manufacturer and the retailer can deliver their advertising campaign both on the direct channel and the retail channel to make the most use of the spillover effect.

Interestingly, it is known by Proposition 4.7, $\frac{\partial A^{C^*}}{\beta_0} > 0$, $\frac{\partial a^{C^*}}{\partial \beta_0} = 0$, $\frac{\partial g^{C^*}}{\partial \beta_0} = 0$. Therefore, the greater the channel spillover effect, the more ads the national brand manufacturer should place in the direct channel, and the retailer do not need to increase the effort. But the retailer's profit and the manufacturer's profit are both increasing functions with regard to channel spillover effects. So, the manufacturer's advertising investment in the direct channel benefits the retailer, and have a positive externality for the retailer's economic activities, called the retailer "free rides". Therefore, when national brand goods do not compete with retailer's own brand products, retailer should support national brand manufacturer to sell goods from direct channel and place more advertisements in direct channel.

Different with Proposition 4.4, Proposition 4.7 reveals that $\pi_{\text{SB}}^{C^*}$ is an increasing function of b_0. That's because a bigger b_0 can directly leads a larger advertising investment of the national brand on the direct channel (*i.e.*, A). With the increasing of A, the demand for the national brand on the retailer channel is increased, while the demand for the retailer's own brand doesn't change because there is no brand competition between the two brands. Therefore, a bigger b_0 can indirectly bring benefits to the retailer.

Moreover, from Proposition 4.7, we know the profit of the national brand is not related to b_2 when there is only spillover effect, but Proposition 4.4 tells that the profit of the national brand decreases with b_2 when there is both spillover effect and brand competition. This is because when the retailer's own brand and the national brand products are very different, there is no competition between the brands, and the advertisement of the retailer's own brand products will not affect the sales of the national brand products and will not affect the profit of national brand manufacturer.

4.1.3. Special Scenario 2: Advertising campaign with brand competition

In this special scenario, we focus on the case when there is only competition between the national brand and the retailer's own brand ($\beta_0 = 0, \beta_1 \neq 0, \beta_2 \neq 0$). In this situation, the advertising campaign for the national brand at the direct channel and the retailers' channel both have negative impacts on the sales of the retailer's own brand. Correspondingly, the advertising campaign for the retailer's brand has a negative impact on the sales of the national brand at both of the channels. This situation can find a lot of examples in the actual market. In order to avoid competition between products in the same brand, many national brand manufacturers deliberately distinguish between online and offline products, they take different ways to sell outdated products online. At the same time, they design and produce products specially designed for network sale, so as to avoid competing against each other. For example, when Combo planned to implement the online strategy, it developed a set of products for the online, which was the later Prosignia series of computers, which were designed for commercial computers for small and medium-sized enterprises. Li Ning's offline flagship store specializes in selling regular-priced new products. Some stores sell inventory goods, while the official e-commerce platform mainly sells limited edition products, including star signatures and other items that can be collected. The models of electric hoods sold online and offline such as FOTILE and SACON are different, to avoid online and offline competition for the same brand products. But, at the same time, large retailers such as Suning will also have their own kitchen brand to produce range hoods. For example, Suning's own brand, Whirlpool, its products include washing machines, microwave ovens, refrigerators, air conditioners, cooker range hoods, dishwashers and home kitchen garbage disposal machines. Therefore, there is a competitive relationship between the online and offline sales of the range hood produced by FOTILE and the hood produced by Whirlpool. The advertising of the FOTILE brand on the online model range hood will not affect the sales of the offline models, but will have a negative impact on the sales of the range hoods produced by Whirlpool. The advertisement of the Whirlpool range hood will have a negative impact on the FOTILE online and offline range hoods.

Proposition 4.8. *In the case of considering brand competition, π^B_{NB} is concave in A^B, π^B_{SB} is joint concave in a^B and g^B. The best advertising investment strategy and the profit of national brand manufacturer and retailer are:*

$$A^{B^*}=\frac{m_0b_0}{\eta_0}, a^{B^*}=\frac{m_1b_1-m_2\beta_2}{\eta_1}, g^{B^*}=\frac{m_2b_2-m_1\beta_2}{\eta_2}.$$

$$\pi^{B^*}_{\mathrm{NB}}=m_0a_0+m_3a_1+\frac{m_0m_0b_0b_0}{2\eta_0}+m_3b_1\frac{m_1b_1-m_2\beta_2}{\eta_1}-\frac{m_2b_2-m_1\beta_2}{\eta_2}(m_0\beta_1+m_3\beta_2),$$

$$\pi^{B^*}_{\mathrm{SB}}=m_1a_1+m_2a_2-m_2\beta_1\frac{m_0b_0}{\eta_0}+\frac{(m_1b_1-m_2\beta_2)^2}{2\eta_1}+\frac{(m_2b_2-m_1\beta_2)^2}{2\eta_2}.$$

Corollary 4.9. *Compare these three situations: only considering channel spillover effect, only considering brand competition, and considering both the channel spillover effect and brand competition, we can get:*

$$A^*=A^{C^*}>A^{B^*}, a^*=a^{B^*}<a^{C^*}, g^*=g^{B^*}<g^{C^*}$$
$$\pi^{B^*}_{\mathrm{NB}}<\pi^*_{\mathrm{NB}}<\pi^{C^*}_{\mathrm{NB}}, \pi^{B^*}_{\mathrm{SB}}<\pi^*_{\mathrm{SB}}<\pi^{C^*}_{\mathrm{SB}}.$$

From Corollary 4.9 we know that competition between national brand and retailer-owned brand has a negative impact on both the sales of the national brand manufacturer and retailer. In the case that only have channel spillover effects, national brand manufacturer and retailer can get more profit, that is $\pi^{B^*}_{\mathrm{NB}}<\pi^*_{\mathrm{NB}}<\pi^{C^*}_{\mathrm{NB}}, \pi^{B^*}_{\mathrm{SB}}<\pi^*_{\mathrm{SB}}<\pi^{C^*}_{\mathrm{SB}}$; Moreover, at this time, national brand manufacturer and retailer will be more active in advertising, namely: $A^*=A^{C^*}>A^{B^*}, a^*=a^{B^*}<a^{C^*}, g^*=g^{B^*}<g^{C^*}$. Therefore, in the actual market, national brand manufacturer and retailer should negotiate to avoid or reduce competition between brands. For example, a retailer can produce and sell a product that is different from a national brand, or a national brand manufacturer chooses a retailer that does not have its own brand or produces a different product to sell its product offline.

In addition, the results of the study also show that channel spillovers can make national brand manufacturer and retailer more profitable, so unlike traditional knowledge (some retailers do not allow manufacturers to sell the same products in the direct sales channel and the store channel) national brand manufacturer selling the same products and advertising in the direct sales channel and retailer channel will make national brand manufacturer and retailer more profitable. Moreover, from the above research, when there is a channel spillover effect, the advertising behavior of the national brand manufacturer in the direct selling channel has a positive externality to the retailer's economic activities, that is, retailer "free rides". Therefore, in the actual market, retailers should not prevent national brand manufacturers from selling the same products sold offline in the direct sales channel.

Proposition 4.10. *Sensitivity of the optimal profit when there is only brand competition. For the national brand manufacturer,*

$$\frac{\partial\pi^{B^*}_{\mathrm{NB}}}{\partial b_0}\geq 0, \frac{\partial\pi^{B^*}_{\mathrm{NB}}}{\partial b_1}\geq 0, \frac{\partial\pi^{B^*}_{\mathrm{NB}}}{\partial b_2}\leq 0, \frac{\partial\pi^{B^*}_{\mathrm{NB}}}{\partial \beta_1}\leq 0, \frac{\partial\pi^{B^*}_{\mathrm{NB}}}{\partial \beta_2}\leq 0.$$

For the retailer,

$$\frac{\partial \pi_{\text{SB}}^{B^*}}{\partial b_0} \leq 0, \frac{\partial \pi_{\text{SB}}^{B^*}}{\partial b_1} \geq 0, \frac{\partial \pi_{\text{SB}}^{B^*}}{\partial b_2} \geq 0, \frac{\partial \pi_{\text{SB}}^{B^*}}{\partial \beta_1} \leq 0, \frac{\partial \pi_{\text{SB}}^{B^*}}{\partial \beta_2} \leq 0.$$

The monotony of the balanced advertising investment of the retailer and the national brand manufacturer are:

$$\begin{aligned}
&\frac{\partial A^{B*}}{\partial b_0} = \frac{m_0}{\eta_0} > 0, \frac{\partial a^{B*}}{\partial b_0} = 0, \frac{\partial g^{B*}}{\partial b_0} = 0; \frac{\partial A^{B*}}{\partial b_1} = 0, \frac{\partial a^{B*}}{\partial b_1} = \frac{m_1}{\eta_1} > 0, \frac{\partial g^{B*}}{\partial b_1} = 0;\\
&\frac{\partial A^{B*}}{\partial b_2} = 0, \frac{\partial a^{B*}}{\partial b_2} = 0, \frac{\partial g^{B*}}{\partial b_2} = \frac{m_2}{\eta_2} > 0; \frac{\partial A^{B*}}{\partial \beta_1} = 0, \frac{\partial a^{B*}}{\partial \beta_1} = 0, \frac{\partial g^{B*}}{\partial \beta_1} = 0;\\
&\frac{\partial A^{B*}}{\partial \beta_2} = 0, \frac{\partial a^{B*}}{\partial \beta_2} = \frac{-m_2}{\eta_1} < 0, \frac{\partial g^{B*}}{\partial \beta_2} = \frac{-m_1}{\eta_2} < 0.
\end{aligned}$$

Different from Proposition 4.7 where the profit of the national brand $\frac{\partial \pi_{\text{NB}}^{C*}}{\partial b_2} = 0$, Proposition 4.10 reveals that π_{NB}^{B*} also decreases with the marginal value of advertising effort for the retailer's brand b_2. This is leaded by the competition between the two brands.

Moreover, in Proposition 4.5, we know that, if $m_1\beta_0 - m_2\beta_1 \geq 0$, then $\frac{\partial \pi_{\text{SB}}^{*}}{\partial b_0} \geq 0$, and if $m_1\beta_0 - m_2\beta_1 \leq 0$, then $\frac{\partial \pi_{\text{SB}}^{*}}{\partial b_0} \leq 0$. However, in Proposition 4.10, the retailer's profit is decreasing with b_0, $\frac{\partial \pi_{\text{SB}}^{*}}{\partial b_0} \leq 0$. That's because the advertising for the national brand on the direct channel has no spillover effect on the retailers' profit, but harms the retailer for the brand competition.

4.1.4. Verifying the model with close-to-really data and discussions

Here we graphically explain the change of the equilibrium by presenting several numerical examples to facilitate our analytical discussion. We use comparative statics in our analysis: that is, equilibrium changes due to a parameter value change, so that our focus is not to decide the exact parameter estimates but to understand how equilibrium behaves when exposed to environmental change. We examine the effect of change of b_i and β_i for $i = 0, 1, 2$.

Our parameter setting is: $m_0 = 5.35, m_1 = 3.68, m_2 = 3.16, m_3 = 3, a_0 = 20, a_1 = 20, a_2 = 20, \eta_0 = 0.2, \eta_1 = 0.2, \eta_2 = 0.2, b_0 = 2.0, b_1 = 3.0, b_2 = 3.5, \beta_0 = 0.4, \beta_1 = 0.2, \beta_2 = 1.2$.

In particular, we change each parameter by either +10% or −10%. In our numerical setting, the optimal profits are determined as $A^* = 59.50, a^* = 36.24, g^* = 33.22$.

Figure 2 shows the sensitivities of the national brand manufacturer's profit. In contrast, Figure 3 shows how parameter change influences the profit of the retailer.

From Propositions 4.4 and 4.5 and the numerical example, we know: π_{NB}^* is an increasing function about b_0, b_1 and β_0, and it's a reduction function about b_2, β_1 and β_2. This shows that the advertising spending for national brand is favorable for the national brand manufacturer both in the direct channel and store channel. But the advertising investment for retailer brand will have a bad effect on the profits of the national brand manufacturer. Therefore, for the national brand manufacturer, the development of its own brand awareness and loyalty is very important. They needn't enhance consumer's channel loyalty, but still need to sale goods in the retailer store.

Also, in this study, we change each parameter by either +10% or −10%, to study the question how the equilibrium changes due to a parameter value change. From Figure 2, we can also find that, the growth speed of π_{NB}^* about b_1 is faster than that of b_0, and both are faster than about β_0. This shows that the channel spillover effect on the profit of national brand manufacturer is not significant compared to direct investment in national brand. Moreover, under the same conditions, for national brand manufacturer, the impact of the advertising investment of national brand in the retail channel is more significant than that of national brand in the direct sales channel. Meanwhile, from Figure 2, we find that the decrease speed of π_{NB}^* about β_1 is smaller than about b_2 and β_2. That is to say, for national brand manufacturer, the negative impact of the competition between the two brands in the retail channel is stronger than the impact of the competition between the national brand of

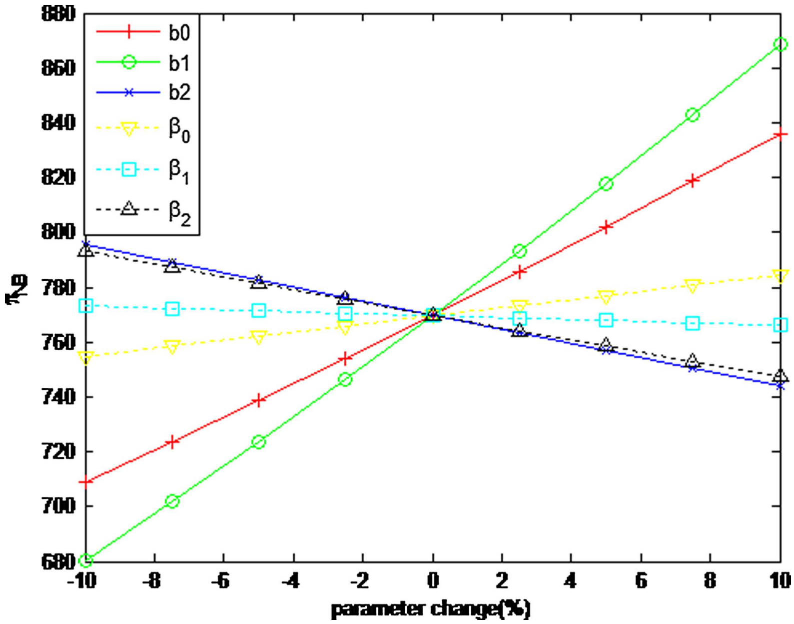

FIGURE 2. Sensitivities of national brand manufacturer's profit.

the direct selling channel and the retailers' own brand. Therefore, national brand manufacturer should pay more attention to the sales of products in the retail channel, and actively seek cooperation with retailers to obtain more profits. At the same time, manufacturers can also actively develop offline physical stores, directly contact the consumer market, and more rationally advertise. For example, manufacturers of large electrical appliances such as Haier, Midea, and GREE have established many physical stores online. Haier has 143 330 sales outlets around the world, with users in more than 100 countries and regions around the world (Euromonitor report, 2016). In China, Haier has established nearly 40 000 specialty stores, with a coverage rate of 98.7.

For retailer, from Figure 3, π^*_{SB} is an increasing function about b_0, b_1, b_2 and β_0, and is a decreasing function about β_1 and β_2. Combined with Proposition 4.5, this shows that the channel spillover effect dominates the competition between the national brand and the retailer's own brand (*i.e.*, $m_1\beta_0 - m_2\beta_1 \geq 0$), the state-owned brand ads in the direct channel have advantageous effect on retailer. And, the advertising for its own brand and state-owned brand in retail channel are favorable to the retailer. So, for retailer, it is important to develop state-owned brand and its' own brand's loyalty. Besides, they should enhance customers' channel loyalty.

In addition, from Figure 3, we can also find, the growth speed of π^*_{SB} about b_0 is lower than about β_0, b_2 and b_1. This shows that even in the case of channel spillover effect dominating brand competition, for retailers, the role of advertising investment on national brands in direct sales channels is very limited. The role of retailer's advertising investment of national brand and private brand in the retailer channel is very significant. Meanwhile, from Figure 3, the decrease speed of π^*_{SB} about β_2 is lager that about β_1. This shows that the competition between the two brands in the retail channel has a greater negative impact on retailers. In summary, retailers should coordinate the competitive relationship between the national brands on the retail channel and retailer's own brands, and reasonably place advertisements to obtain more profits. Retailers can develop products that differ greatly with national brand products, or develop products complement with national brand products, to reduce the intensity of competition between brands and promote the sale of goods. For example, Wal-Mart's own branded products are rich in variety and specifications. Through different designs, private brands and mainstream manufacturers' brands complement each other, avoiding direct competition between the two parties, and effectively maintaining good relations with manufacturers.

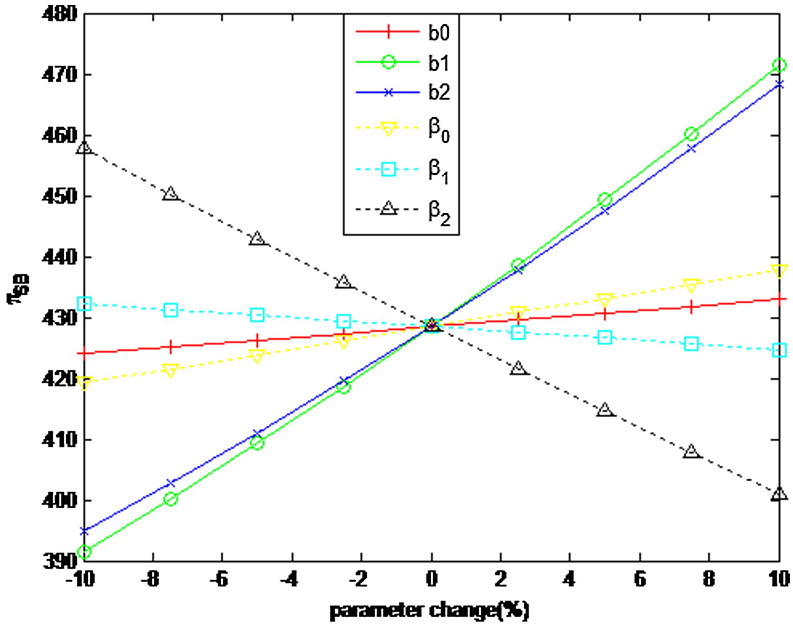

FIGURE 3. Sensitivities of retailer's profit.

4.2. The integrated system

In an integrated supply chain system, all the supply chain members are unified and one decision maker optimizes the performance of the total system. This is equivalent to a large brand manufacturer in the market environment who has both online and offline franchise stores, or when national brand manufacturer and retailer fully cooperate. Due to the diversification of consumer demand, it is impossible to have only one sales model to meet all needs, and online channels and offline channels need to form a common interest. Consumers can look at the goods offline, order online, pay for the goods offline, and enjoy the corresponding services, which can make consumers more assured and more convenient. For example, Nordstrom, an American clothing company founded in 1901, sells its products through specialty stores and Internet. In 2008, Nordstrom effectively integrated the inventory system and physical store inventory system of the e-commerce platform. On the basis of the company's online and offline channels with a unified inventory system, the company began to launch the "online shopping store pick-up" service, forming an online and offline convergence advantage.

In our modeling framework, the optimal advertising spending for the integrated system can be obtained by maximizing the profit of the system:

$$\begin{aligned}\pi_I = {} & m_0(a_0 + b_0A + \beta_0 a - \beta_1 g) + m_1(a_1 + b_1 a + \beta_0 A - \beta_2 g) \\ & + m_2(a_2 + b_2 g - \beta_1 A - \beta_2 a) + m_3(a_1 + b_1 a + \beta_0 A - \beta_2 g) \\ & - \frac{1}{2}\eta_0 A^2 - \frac{1}{2}\eta_1 a^2 - \frac{1}{2}\eta_2 g^2. \end{aligned} \tag{4.8}$$

Property 4.11 proves the existence of a unique optimal solution for the profit function, and Proposition 4.12 shows the equilibrium advertising spending for the integrated system.

Property 4.11. The profit function for the integrated system is joint concave with respect to A, a and g.

Proposition 4.12. *For the centralized system, optimal advertising inputs are obtained as:*

$$A_I^* = \frac{m_0 b_0 + m_1\beta_0 - m_2\beta_1 + m_3\beta_0}{\eta_0},$$

$$a_I^* = \frac{m_0\beta_0 + m_1 b_1 - m_2\beta_2 + m_3 b_1}{\eta_1}$$
$$g_I^* = \frac{m_2 b_2 - m_0\beta_1 - m_1\beta_2 - m_3\beta_2}{\eta_2}.$$

The total profit is:

$$\pi_I^* = m_0 a_0 + m_1 a_1 + m_2 a_2 + m_3 a_1 + \frac{(m_0 b_0 + m_1\beta_0 - m_2\beta_1 + m_3\beta_0)^2}{2\eta_0} + \frac{(m_0\beta_0 + m_1 b_1 - m_2\beta_2 + m_3 b_1)^2}{2\eta_1} + \frac{(m_2 b_2 - m_0\beta_1 - m_1\beta_2 - m_3\beta_2)^2}{2\eta_2}.$$

On the other hand, the total profit for the disintegrated system can be defined as:

$$\pi_D^* = \pi_{\text{NB}}^* + \pi_{\text{SB}}^*.$$

Proposition 4.13 summarizes the relationship between the two systems.

Proposition 4.13. (1) *The profit of integrated system is larger than that of disintegrated system.* $\pi_I^* > \pi_D^*$

(2) *The relationship between the national brand manufacturer's input in the integrated system and the disintegrated system is:*

$$A_I^* - A^* = \frac{m_0 b_0 + m_1\beta_0 - m_2\beta_1 + m_3\beta_0}{\eta_0} - \frac{b_0 m_0 + \beta_0 m_3}{\eta_0} = \frac{m_1\beta_0 - m_2\beta_1}{\eta_0}.$$

if $m_1\beta_0 - m_2\beta_1 \geq 0$, *then* $A_I^* \geq A^*$; *otherwise,* $A_I^* < A^*$.

(3) *In the integrated system, the retailer's advertising spending for national brand in retailer store channel is larger than that in the disintegrated system.* $a_I^* > a^*$.

(4) *In the integrated system, the retailer's advertising spending for its own brand in retailer store channel is smaller than that in the disintegrated system.* $g_I^* < g^*$.

Compared with the disintegrated case, the integrated system can be seen as an integrated company making decisions as long as the total profit can be maximized. According to Proposition 4.5, we can obtain the relationship between A_I^* and A^*. Furthermore, as the state-owned brand's investment in the direct sales channel and retailer channel will cause overflow, increasing the investment of the state-owned brand merchandise advertising in the retail channel so that the effect of overflow increases and the profits of state-owned brand increase, thus we get the conclusion $a_I^* > a^*$ under the centralized case. Besides, if reduce the investment of the retailer's brand, competition between state-owned brand and retailer brand will ease up and effect of overflow between the two channels enhances so that the gross profits increase, thus $g_I^* < g^*$. Furthermore, we know from Proposition 4.12 that the total profit of integrated system is larger than that of disintegrated system. Then, the two firms can sign some contracts for more profits. We will try to coordinate the supply chain in Section 5.

5. Supply chain coordination

Note that the profit for the integrated system is always higher than that for the decentralized system. We wonder whether there exists a supply chain contract to coordinate the system to improve the performance of each member.

The unilateral advertising investment subsidy can be a potential contract to improve the perform of each member in the decentralized system. Suppose the national brand manufacturer shares a part of t $(0 < t < 1)$ for the advertising investment in the retailer channel and the retailer shares the left part $1 - t$. The profit function of the national brand manufacturer is:

$$\pi_{\text{NB}}^u = m_0 d_0 + m_3 d_1 - \frac{1}{2}\eta_0 A^2 - \frac{1}{2}\eta_1 t a^2. \tag{5.1}$$

The profit function of the retailer is

$$\pi_{\text{SB}}^{u} = m_1 d_1 + m_2 d_2 - \frac{1}{2}\eta_1(1-t)a^2 - \frac{1}{2}\eta_2 g^2. \tag{5.2}$$

Proposition 5.1 shows the result for our model.

Proposition 5.1. *The unilateral advertising spending subsidy contract can improve the perform of the national brand manufacturer and retailer, we have*

$$t = \frac{-2\beta_0 b_1 m_0 m_1 + 2\beta_0\beta_2 m_0 m_2 - 2b_1^2 m_1 m_3 + 2b_1\beta_2 m_2 m_3 - (b_1 m_1 - \beta_2 m_2)^2}{-2\beta_0 b_1 m_0 m_1 + 2\beta_0\beta_2 m_0 m_2 - 2b_1^2 m_1 m_3 + 2b_1\beta_2 m_2 m_3 + (b_1 m_1 - \beta_2 m_2)^2} > 0.$$

We reveal that the unilateral advertising spending subsidy coordination can coordinate the system. It's a Pareto improvement compared the case without this contract. That's because with the advertising sharing mechanism, the retailer has more incentive to improve a, which approaches the optimal advertising effort when they are in an integrated system.

6. Conclusion

Advertising decisions are very important for upstream and downstream entities in supply chain. In this paper, we investigate the optimal decisions for brand advertising and local advertising in a dual-channel supply chain using the Nash game. We develop a game theoretic model that integrates brand advertising, local advertising and dual-channel supply chain management into the same framework, and explore the equilibrium advertising strategies that maximize the profits of the manufacturer and the retailer. Then, we design a unilateral advertising subsidy contract to coordinate the supply chain. Our results can provide a decision making aid for the upstream and downstream entities in supply chain when considering brand competition and channel competition.

The main novels of this paper are twofold. (i) Different from most previous studies, we incorporate brand advertising, local advertising and dual-channel supply chain management into a same model framework to investigate the impact of various factors on the advertising expenditure and profits of the manufacturer and retailer. (ii) We focus on the brand competition and channel competition simultaneously using the demand model. In the previous studies, these two factors are scant investigated simultaneously.

Through model calculations and theoretical analysis, we have some very interesting conclusions.

Firstly, in the disintegrated system, when channel spillover effects and brand competition exist simultaneously, we find through calculation and model analysis: (1) National brand manufacturer will not change their advertising strategies in direct sales channel because of the competition between brands. (2) For retailers, they will not change the advertising strategy because of the competitive relationship between national brand products sold by direct sales channel and their own brand products; However, when the national brand products sold by retailer compete with retailer's own branded products, the retailer will also reduce the advertising investment of national brand and private brand in retail channel to reduce the intensity of competition. (3) Propositions 4.3–4.5, we can see that the profits of national brand manufacturer and retailer increase with the increase of channel spillover effects. National brand manufacturer will increase his advertising investment in direct sales channel as channel spillovers increase, but retailers' advertising investment strategies will not change with channel spillover effects. It can be seen that the advertising investment of national brand manufacturer in the direct sales channel has positive externalities to the retailer's economic activities, that is, retailers "free riders".

Secondly, in the disintegrated system, we analyzed two special cases and found that: (1) when there is only channel spillover effect, the spillover effect of advertising across channels can always benefit the national brand and the store brand. We found that it was the channel spillover effect that produced the phenomenon of retailers "free riders". So in order to get more profit,the national brand manufacturer and the retailer can deliver their advertising campaign both on the direct channel and the retail channel to make the most use of the spillover effect. In the actual market, retailers should not blindly prevent national brand manufacturers from selling the same products in the direct sales channel. (2) Comparing the advertising investment strategy and the equilibrium profit under the three situations (both channel overflow effect and brand competition, only channel spillover effect, and only brand competition), we found that the competition between national brand and retailer's own brand has a negative impact on national brand manufacturer and retailer. And, with only channel spillovers, national brand manufacturer and retailer can get more profit (that is; $\pi_{\mathrm{NB}}^{B*} < \pi_{\mathrm{NB}}^{*} < \pi_{\mathrm{NB}}^{C*}$, $\pi_{\mathrm{SB}}^{B*} < \pi_{\mathrm{SB}}^{*} < \pi_{\mathrm{SB}}^{C*}$). At this time, national brand manufacturer and retailer will be more active in advertising (that is: $A^* = A^{C*} > A^{B*}; a^* = a^{B*} < a^{C*}; g^* = g^{B*} < g^{C*}$). Therefore, in the actual market, national brand manufacturer and retailer should negotiate to avoid or reduce competition between brands.

This work, however, has some limitations which can be addressed by considering future research directions. First, we assume that both the manufacturer and retailer share the same information. This assumption may not cope with realistic situations. Future research can investigate the issues with private information. Second, we only explore advertising competition of the dual-channel supply chain with channel competition. Future research can add price or service into the model as decision variables. Thirdly, to simplify the analysis, we employ a linear advertising response function. It would be interesting to investigate nonlinear advertising response function in channel analysis.

Appendix A.

Proof of Property 4.1

Substituting d_0, d_1, d_2 into π_{NB}, we get: $\pi_{\mathrm{NB}} = m_0(a_0 + b_0A + \beta_0 a - \beta_1 g) + m_3(a_1 + b_1 a + \beta_0 A - \beta_2 g) - \frac{1}{2}\eta_0 A^2$.

The first derivative of π_{NB} is: $\frac{\partial \pi_{\mathrm{NB}}}{\partial A} = b_0 m_0 + \beta_0 m_3 - \eta_0 A$.

Thus, the second derivative of π_{NB} is: $\frac{\partial^2 \pi_{\mathrm{NB}}}{\partial A^2} = -\eta_0 < 0$. Hence, π_{NB} is concave in A.

Substituting d_0, d_1, d_2 into π_{SB}, we get: $\pi_{\mathrm{SB}} = m_1(a_1 + b_1 a + \beta_0 A - \beta_2 g) + m_2(a_2 + b_2 g - \beta_1 A - \beta_2 a) - \frac{1}{2}\eta_1 a^2 - \frac{1}{2}\eta_2 g^2$.

The first derivative of π_{SB} is $\frac{\partial \pi_{\mathrm{SB}}}{\partial a} = b_1 m_1 - \beta_2 m_2 - \eta_1 a$, $\frac{\partial \pi_{\mathrm{SB}}}{\partial g} = b_2 m_2 - \beta_2 m_1 - \eta_2 g$. Thus, the second derivative of π_{SB} is $\frac{\partial^2 \pi_{\mathrm{SB}}}{\partial a^2} = -\eta_1 < 0$, $\frac{\partial^2 \pi_{\mathrm{SB}}}{\partial g^2} = -\eta_2 < 0$, $\frac{\partial^2 \pi_{\mathrm{SB}}}{\partial a \partial g} = \frac{\partial^2 \pi_{\mathrm{SB}}}{\partial g \partial a} = 0$.

Let H be a Hessian of π_{SB}: $H = \begin{bmatrix} -\eta_1 & 0 \\ 0 & -\eta_2 \end{bmatrix}$. Since $\det(H) = \eta_1\eta_2 > 0$. Hence, π_{SB} is jointly concave in a and g.

Proof of Proposition 4.2

$$\frac{\partial \pi_{\mathrm{NB}}}{\partial A} = b_0 m_0 + \beta_0 m_3 - \eta_0 A \tag{A.1}$$

$$\frac{\partial \pi_{\mathrm{SB}}}{\partial a} = b_1 m_1 - \beta_2 m_2 - \eta_1 a \tag{A.2}$$

$$\frac{\partial \pi_{\mathrm{SB}}}{\partial g} = b_2 m_2 - \beta_2 m_1 - \eta_2 g. \tag{A.3}$$

We can rewrite (A.1)–(A.3) as a following matrix form:

$$\begin{bmatrix} \eta_0 & 0 & 0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & \eta_2 \end{bmatrix} \begin{bmatrix} A \\ a \\ g \end{bmatrix} = \begin{bmatrix} b_0 m_0 + \beta_0 m_3 \\ b_1 m_1 - \beta_2 m_2 \\ b_2 m_2 - \beta_2 m_1 \end{bmatrix}.$$

Set $X \equiv \begin{bmatrix} \eta_0 & 0 & 0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & \eta_2 \end{bmatrix}, Y \equiv \begin{bmatrix} b_0 m_0 + \beta_0 m_3 \\ b_1 m_1 - \beta_2 m_2 \\ b_2 m_2 - \beta_2 m_1 \end{bmatrix}, AD \equiv \begin{bmatrix} A \\ a \\ g \end{bmatrix}$.

Obviously, matrix X is nonsingular and positive definite. As a result, the optimal advertising expenditures can be expressed as:

$$\begin{bmatrix} A \\ a \\ g \end{bmatrix} = X^{-1}Y$$

which is: $A^* = \frac{b_0m_0+\beta_0m_3}{\eta_0}, a^* = \frac{b_1m_1-\beta_2m_2}{\eta_1}, g^* = \frac{b_2m_2-\beta_2m_1}{\eta_2}$. Substituting A^*, a^*, g^* into π_{NB} and π_{SB}, we can get:

$$\pi^*_{\text{NB}} = m_0a_0 + m_3a_1 + \frac{(b_0m_0+\beta_0m_3)^2}{2\eta_0} + \frac{(b_1m_1-\beta_2m_2)(m_0\beta_0+m_3b_1)}{\eta_1} - \frac{(b_2m_2-\beta_2m_1)(m_0\beta_1+m_3\beta_2)}{\eta_2}$$

$$\pi^*_{\text{SB}} = m_1a_1 + m_2a_2 + \frac{(b_0m_0+\beta_0m_3)(m_1\beta_0-m_2\beta_1)}{\eta_0} + \frac{(b_1m_1-\beta_2m_2)^2}{2\eta_1} + \frac{(b_2m_2-\beta_2m_1)^2}{2\eta_2}.$$

Proof of Proposition 4.3

Substituting A^*, a^*, g^* into $\frac{\partial\pi_{\text{NB}}}{\partial A} = b_0m_0+\beta_0m_3-\eta_0A$, $\frac{\partial\pi_{\text{SB}}}{\partial a} = b_1m_1-\beta_2m_2-\eta_1a$, $\frac{\partial\pi_{\text{SB}}}{\partial g} = b_2m_2-\beta_2m_1-\eta_2g$. From the first-order conditions, we have

$$\eta_0A^* = b_0m_0+\beta_0m_3 \tag{A.4}$$
$$\eta_1a^* = b_1m_1-\beta_2m_2 \tag{A.5}$$
$$\eta_2g^* = b_2m_2-\beta_2m_1. \tag{A.6}$$

Taking the derivative of (A.4)–(A.6) with respect to b_0, we have

$$\eta_0\frac{\partial A^*}{\partial b_0} = m_0 \tag{A.7}$$
$$\eta_1\frac{\partial a^*}{\partial b_0} = 0 \tag{A.8}$$
$$\eta_2\frac{\partial g^*}{\partial b_0} = 0. \tag{A.9}$$

We can rewrite (A.7)–(A.9) as a following matrix form:

$$X\begin{bmatrix} \frac{\partial A^*}{\partial b_0} \\ \frac{\partial a^*}{\partial b_0} \\ \frac{\partial g^*}{\partial b_0} \end{bmatrix} = \begin{bmatrix} m_0 \\ 0 \\ 0 \end{bmatrix}.$$

Applying Cramer's rule, $\frac{\partial A^*}{\partial b_0}$ can be expressed as:

$$\frac{\partial A^*}{\partial b_0} = \frac{1}{\det(X)}\det\begin{bmatrix} m_0 & 0 & 0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & \eta_2 \end{bmatrix} = \frac{m_0}{\eta_0} \geq 0.$$

Similarly, we can get:

$$\frac{\partial a^*}{\partial b_0} = \frac{1}{\det(X)}\det\begin{bmatrix} \eta_0 & m_0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \eta_2 \end{bmatrix} = 0$$

$$\frac{\partial g^*}{\partial b_0} = \frac{1}{\det(X)} \det \begin{bmatrix} \eta_0 & 0 & m_0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0.$$

So we can obtain $\frac{\partial A^*}{\partial b_0} \geq 0$, $\frac{\partial a^*}{\partial b_0} = 0$, $\frac{\partial g^*}{\partial b_0} = 0$.

By the same way, we can get the sensitivity of the optimal advertising spending with respect to the parameters: $b_1, b_2, \beta_0, \beta_1, \beta_2$ that Proposition 4.3 showed.

Proof of Propositions 4.4 and 4.5

We apply an envelop theorem: the derivative of the optimal value with respect to the parameter is equivalent to the partial derivative of the objective function when the derivative is evaluated by the optimal solution. For example, $\frac{\partial \pi^*_{\text{NB}}}{\partial b_0} = \frac{\partial \pi_{\text{NB}}}{\partial b_0}|_{A^*,a^*,g^*}$, π_{NB} is a function of A. Thus, the derivative of π_{NB} with respect to b_0 is $\frac{\partial \pi_{\text{NB}}}{\partial b_0}|_{A^*,a^*,g^*} = m_0 A$, $\frac{\partial \pi^*_{\text{NB}}}{\partial b_0} \geq 0$.

Applying the same logic to the other parameters and functions, we can prove of Propositions 4.4 and 4.5.

Proof of Proposition 4.6

The first-order partial derivative of π^C_{NB}:

$$\frac{\partial \pi^C_{\text{NB}}}{\partial A^C} = m_0 b_0 + m_3 \beta_0 - \eta_0 A^C. \tag{A.10}$$

The second-order partial derivative is: $\frac{\partial^2 \pi^C_{\text{NB}}}{\partial A^{C2}} = -\eta_0 < 0$. Therefore, π^C_{NB} is concave in A^C.

The first-order partial derivatives of π^C_{SB} about a^C and g^C:

$$\frac{\partial \pi^C_{\text{SB}}}{\partial a^C} = m_1 b_1 - \eta_1 a^C \tag{A.11}$$

$$\frac{\partial \pi^C_{\text{SB}}}{\partial g^C} = m_2 b_2 - \eta_2 g^C. \tag{A.12}$$

The second-order partial derivative and the second-order mixed partial derivative:

$$\frac{\partial^2 \pi^C_{\text{SB}}}{\partial a^{C2}} = -\eta_1, \frac{\partial^2 \pi^C_{\text{SB}}}{\partial a^C \partial g^C} = 0, \frac{\partial^2 \pi^C_{\text{SB}}}{\partial g^{C2}} = -\eta_2, \frac{\partial^2 \pi^C_{\text{SB}}}{\partial g^C \partial a^C} = 0.$$

The Heather matrix: $H = \begin{bmatrix} -\eta_1 & 0 \\ 0 & -\eta_2 \end{bmatrix}$, the first order master subtype is: $|H_1| = -\eta_1 < 0$, the second order master subtype is: $|H_2| = \eta_1 \eta_2 > 0$. Therefore, π^C_{SB} is joint concave in a^C and g^C.

Let the formulas (A.10)–(A.12) be zero, and the parallel solution can be obtained:

$$A^{C*} = \frac{m_0 b_0 + m_3 \beta_0}{\eta_0}, a^{C*} = \frac{m_1 b_1}{\eta_1}, g^{C*} = \frac{m_2 b_2}{\eta_2}.$$

So the profits of national brand manufacturer and retailer are:

$$\pi^{C*}_{\text{NB}} = m_0 a_0 + m_3 a_1 + \frac{m_1 b_1}{\eta_1}(m_0 \beta_0 + m_3 b_1) + \frac{(m_3 \beta_0 + m_0 b_0)^2}{2\eta_0}$$

$$\pi^{C*}_{\text{SB}} = m_1 a_1 + m_2 a_2 + m_1 \beta_0 \frac{m_0 b_0 + m_3 \beta_0}{\eta_0} + \frac{m_1^2 b_1^2}{2\eta_1} + \frac{m_2^2 b_2^2}{2\eta_2}.$$

Proof of Proposition 4.7

Using the same logic as Propositions 4.4 and 4.5, we determine the derivative of the maximum profit with respect to the parameters as follows:

$$\frac{\partial \pi_{\mathrm{NB}}^{C*}}{\partial b_1} = \frac{\partial \pi_{\mathrm{NB}}^{C}}{\partial b_1}|_{A^{C*},a^{C*},g^{C*}} = m_3 a^{C*} + m_3 b_1 \frac{\partial a^{C*}}{\partial b_1} + m_0 \beta_0 \frac{\partial a^{C*}}{\partial b_1}.$$

If $\beta_1 = \beta_2 = 0$, then

$$\begin{bmatrix} \eta_0 & 0 & 0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & \eta_2 \end{bmatrix} \begin{bmatrix} A^C \\ a^C \\ g^C \end{bmatrix} = \begin{bmatrix} b_0 m_0 + \beta_0 m_3 \\ b_1 m_1 \\ b_2 m_2 \end{bmatrix}$$
$$\eta_0 A^{C*} = b_0 m_0 + \beta_0 m_3$$
$$\eta_1 a^{C*} = b_1 m_1$$
$$\eta_2 g^{C*} = b_2 m_2$$
$$\frac{\partial A^{C*}}{\partial b_1} = 0, \frac{\partial a^{C*}}{\partial b_1} = \frac{m_1}{\eta_1} \geq 0, \frac{\partial g^{C*}}{\partial b_1} = 0.$$

Thus, we finally conclude: $\frac{\partial \pi_{\mathrm{NB}}^{C*}}{\partial b_1} = m_3 a^{C*} + m_3 b_1 \frac{\partial a^{C*}}{\partial b_1} + m_0 \beta_0 \frac{\partial a^{C*}}{\partial b_1} \geq 0$. Using the same logic to the remaining parameters, we can prove Proposition 4.7.

Proof of Proposition 4.8

The first-order partial derivative of π_{NB}^{B}:

$$\frac{\partial \pi_{\mathrm{NB}}^{B}}{\partial A^B} = m_0 b_0 - \eta_0 A^B. \tag{A.13}$$

The second-order partial derivative is: $\frac{\partial^2 \pi_{\mathrm{NB}}^{B}}{\partial A^{B2}} = -\eta_0 < 0$.

Hence, π_{NB}^{B} is concave in A^B.

The first-order partial derivatives of π_{SB}^{B} About a^B and g^B:

$$\frac{\partial \pi_{\mathrm{SB}}^{B}}{\partial a^B} = m_1 b_1 - m_2 \beta_2 - \eta_1 a^B \tag{A.14}$$
$$\frac{\partial \pi_{\mathrm{SB}}^{B}}{\partial g^B} = m_2 b_2 - m_1 \beta_2 - \eta_2 g^B. \tag{A.15}$$

The second-order partial derivative and the second-order mixed partial derivative:

$$\frac{\partial^2 \pi_{\mathrm{SB}}^{B}}{\partial a^{B2}} = -\eta_1, \frac{\partial^2 \pi_{\mathrm{SB}}^{B}}{\partial a^B \partial g^B} = 0, \frac{\partial^2 \pi_{\mathrm{SB}}^{B}}{\partial g^{B2}} = -\eta_2, \frac{\partial^2 \pi_{\mathrm{SB}}^{B}}{\partial g^B \partial a^B} = 0.$$

The Heather matrix is: $H = \begin{bmatrix} -\eta_1 & 0 \\ 0 & -\eta_2 \end{bmatrix}$, the first order master subtype is: $|H_1| = -\eta_1 < 0$, the second order master subtype is: $|H_2| = \eta_1 \eta_2 > 0$. Therefore, π_{SB}^{B} is joint concave in a^B and g^B.

Let the formulas (A.13)–(A.15) be zero, and the parallel solution can be obtained:

$$A^{B*} = \frac{m_0 b_0}{\eta_0}, a^{B*} = \frac{m_1 b_1 - m_2 \beta_2}{\eta_1}, g^{B*} = \frac{m_2 b_2 - m_1 \beta_2}{\eta_2}.$$

Hence, the profits of national brand manufacturer and retailer are:

$$\pi_{\rm NB}^{B*} = m_0 a_0 + m_3 a_1 + \frac{m_0 m_0 b_0 b_0}{2\eta_0} + m_3 b_1 \frac{m_1 b_1 - m_2\beta_2}{\eta_1} - \frac{m_2 b_2 - m_1\beta_2}{\eta_2}(m_0\beta_1 + m_3\beta_2)$$

$$\pi_{\rm SB}^{B*} = m_1 a_1 + m_2 a_2 - m_2\beta_1 \frac{m_0 b_0}{\eta_0} + \frac{(m_1 b_1 - m_2\beta_2)^2}{2\eta_1} + \frac{(m_2 b_2 - m_1\beta_2)^2}{2\eta_2}.$$

Proof of Corollary 4.9

$$A^* - A^{C*} = \frac{b_0 m_0 + \beta_0 m_3}{\eta_0} - \frac{m_0 b_0 + m_3\beta_0}{\eta_0} = 0, A^* - A^{B*} = \frac{\beta_0 m_3}{\eta_0} > 0.$$

So we have $A^* = A^{C*} > A^{B*}$.

$$a^* - a^{C*} = \frac{b_1 m_1 - \beta_2 m_2}{\eta_1} - \frac{m_1 b_1}{\eta_1} = \frac{-\beta_2 m_2}{\eta_1} < 0$$

$$a^* - a^{B*} = \frac{b_1 m_1 - \beta_2 m_2}{\eta_1} - \frac{m_1 b_1 - m_2\beta_2}{\eta_1} = 0.$$

So we have $a^* = a^{B*} < a^{C*}$.

$$g^* - g^{C*} = \frac{b_2 m_2 - \beta_2 m_1}{\eta_2} - \frac{m_2 b_2}{\eta_2} = \frac{-\beta_2 m_1}{\eta_2} < 0$$

$$g^* - g^{B*} = \frac{b_2 m_2 - \beta_2 m_1}{\eta_2} - \frac{m_2 b_2 - m_1\beta_2}{\eta_2} = 0.$$

So we have $g^* = g^{B*} < g^{C*}$.

$$\pi_{\rm NB}^* - \pi_{\rm NB}^{C*} = \frac{-\beta_2 m_2(m_0\beta_0 + m_3 b_1)}{\eta_1} - \frac{(b_2 m_2 - \beta_2 m_1)(m_0\beta_1 + m_3\beta_2)}{\eta_2} < 0$$

$$\pi_{\rm NB}^* - \pi_{\rm NB}^{B*} = \frac{\beta_0 m_3(2b_0 m_0 + \beta_0 m_3)}{2\eta_0} + \frac{m_0\beta_0(b_1 m_1 - \beta_2 m_2)}{\eta_1} > 0.$$

So we have $\pi_{\rm NB}^{B*} < \pi_{\rm NB}^* < \pi_{\rm NB}^{C*}$.

$$\pi_{\rm SB}^* - \pi_{\rm SB}^{C*} = \frac{-m_2\beta_1(b_0 m_0 + \beta_0 m_3)}{\eta_0} + \beta_2 m_2 \frac{-2b_1 m_1 + \beta_2 m_2}{2\eta_1} + \beta_2 m_1 \frac{(-2b_2 m_2 + \beta_2 m_1)}{2\eta_2} < 0$$

$$\pi_{\rm SB}^* - \pi_{\rm SB}^{B*} = \frac{\beta_0(b_0 m_0 m_1 + \beta_0 m_1 m_3 - \beta_1 m_2 m_3)}{\eta_0} > 0.$$

So we have $\pi_{\rm SB}^{B*} < \pi_{\rm SB}^* < \pi_{\rm SB}^{C*}$.

Proof of Proposition 4.10

Using the same logic as Propositions 4.4 and 4.5, we determine the derivative of the optimal profit with respect to b_1 as:

$$\frac{\partial \pi_{\rm NB}^{B*}}{\partial b_1} = \frac{\partial \pi_{\rm NB}^{B}}{\partial b_1}|_{A^{B*}, a^{B*}, g^{B*}} = -m_0\beta_1 \frac{\partial g^{B*}}{\partial b_1} + m_3 a^{B*} + m_3 b_1 \frac{\partial a^{B*}}{\partial b_1} - m_3\beta_2 \frac{\partial g^{B*}}{\partial b_1}.$$

If $\beta_0 = 0$, we have:

$$\begin{bmatrix} \eta_0 & 0 & 0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & \eta_2 \end{bmatrix} \begin{bmatrix} A^{B*} \\ a^{B*} \\ g^{B*} \end{bmatrix} = \begin{bmatrix} b_0 m_0 \\ b_1 m_1 - \beta_2 m_2 \\ b_2 m_2 - \beta_2 m_1 \end{bmatrix}$$

$$\eta_0 A^{B*} = b_0 m_0$$

$$\eta_1 a^{B*} = b_1 m_1 - \beta_2 m_2$$

$$\eta_2 g^{B*} = b_2 m_2 - \beta_2 m_1$$

$$\frac{\partial A^{B*}}{\partial b_1} = 0, \frac{\partial a^{B*}}{\partial b_1} = \frac{m_1}{\eta_1} \geq 0, \frac{\partial g^{B*}}{\partial b_1} = 0.$$

Thus, we conclude $\frac{\partial \pi_{\mathrm{NB}}^{B*}}{\partial b_1} \geq 0$. Using the same logic to the remaining parameters, we can prove Proposition 4.10.

Proof of Property 4.11

Taking the derivatives of π_I with respect to A, a and g, we obtain:

$$\frac{\partial \pi_I}{\partial A} = m_0 b_0 + m_1 \beta_0 - m_2 \beta_1 + m_3 \beta_0 - \eta_0 A$$

$$\frac{\partial \pi_I}{\partial a} = m_0 \beta_0 + m_1 b_1 - m_2 \beta_2 + m_3 b_1 - \eta_1 a$$

$$\frac{\partial \pi_I}{\partial g} = -m_0 \beta_1 - m_1 \beta_2 + m_2 b_2 - m_3 \beta_2 - \eta_2 g.$$

From the second-order conditions, we obtain:

$$\frac{\partial^2 \pi_I}{\partial A^2} = -\eta_0 < 0, \frac{\partial^2 \pi_I}{\partial A \partial a} = 0, \frac{\partial^2 \pi_I}{\partial A \partial g} = 0$$

$$\frac{\partial^2 \pi_I}{\partial \mathrm{a}^2} = -\eta_1 < 0, \frac{\partial^2 \pi_I}{\partial a \partial A} = 0, \frac{\partial^2 \pi_I}{\partial a \partial g} = 0$$

$$\frac{\partial^2 \pi_I}{\partial g^2} = -\eta_2 < 0, \frac{\partial^2 \pi_I}{\partial g \partial A} = 0, \frac{\partial^2 \pi_I}{\partial g \partial a} = 0.$$

Hence, π_I is jointly concave in A, a and g.

Proof of Proposition 4.12

Taking the derivatives of π_I with respect to A, a and g, we obtain:

$$\frac{\partial \pi_I}{\partial A} = m_0 b_0 + m_1 \beta_0 - m_2 \beta_1 + m_3 \beta_0 - \eta_0 A$$

$$\frac{\partial \pi_I}{\partial a} = m_0 \beta_0 + m_1 b_1 - m_2 \beta_2 + m_3 b_1 - \eta_1 a$$

$$\frac{\partial \pi_I}{\partial g} = -m_0 \beta_1 - m_1 \beta_2 + m_2 b_2 - m_3 \beta_2 - \eta_2 g.$$

From the first-order conditions, we have:

$$A_I^* = \frac{m_0 b_0 + m_1 \beta_0 - m_2 \beta_1 + m_3 \beta_0}{\eta_0}$$

$$a_I^* = \frac{m_0 \beta_0 + m_1 b_1 - m_2 \beta_2 + m_3 b_1}{\eta_1}$$

$$g_I^* = \frac{m_2 b_2 - m_0 \beta_1 - m_1 \beta_2 - m_3 \beta_2}{\eta_2}.$$

So, the total profit is:

$$\pi_I^* = m_0a_0 + m_1a_1 + m_2a_2 + m_3a_1 + \frac{(m_0b_0 + m_1\beta_0 - m_2\beta_1 + m_3\beta_0)^2}{2\eta_0} + \frac{(m_0\beta_0 + m_1b_1 - m_2\beta_2 + m_3b_1)^2}{2\eta_1} + \frac{(m_2b_2 - m_0\beta_1 - m_1\beta_2 - m_3\beta_2)^2}{2\eta_2}.$$

Proof of Proposition 4.13

The profit function of the integrated system is:

$$\begin{aligned}\pi_I^* = {}& m_0\left(a_0 + b_0\frac{m_0b_0 + m_1\beta_0 - m_2\beta_1 + m_3\beta_0}{\eta_0} + \beta_0\frac{m_0\beta_0 + m_1b_1 - m_2\beta_2 + m_3b_1}{\eta_1}\right.\\ &\left. - \beta_1\frac{m_2b_2 - m_0\beta_1 - m_1\beta_2 - m_3\beta_2}{\eta_2}\right) + m_1\left(a_1 + b_1\frac{m_0\beta_0 + m_1b_1 - m_2\beta_2 + m_3b_1}{\eta_1}\right.\\ &\left. + \beta_0\frac{m_0b_0 + m_1\beta_0 - m_2\beta_1 + m_3\beta_0}{\eta_0} - \beta_2\frac{m_2b_2 - m_0\beta_1 - m_1\beta_2 - m_3\beta_2}{\eta_2}\right)\\ & + m_2\left(a_2 + b_2\frac{m_2b_2 - m_0\beta_1 - m_1\beta_2 - m_3\beta_2}{\eta_2} - \beta_1\frac{m_0b_0 + m_1\beta_0 - m_2\beta_1 + m_3\beta_0}{\eta_0}\right.\\ &\left. - \beta_2\frac{m_0\beta_0 + m_1b_1 - m_2\beta_2 + m_3b_1}{\eta_1}\right) + m_3\left(a_1 + b_1\frac{m_0\beta_0 + m_1b_1 - m_2\beta_2 + m_3b_1}{\eta_1}\right.\\ &\left. + \beta_0\frac{m_0b_0 + m_1\beta_0 - m_2\beta_1 + m_3\beta_0}{\eta_0} - \beta_2\frac{m_2b_2 - m_0\beta_1 - m_1\beta_2 - m_3\beta_2}{\eta_2}\right)\\ & - \frac{1}{2}\eta_0\left[\frac{m_0b_0 + m_1\beta_0 - m_2\beta_1 + m_3\beta_0}{\eta_0}\right]^2 - \frac{1}{2}\eta_1\left[\frac{m_0\beta_0 + m_1b_1 - m_2\beta_2 + m_3b_1}{\eta_1}\right]^2\\ & - \frac{1}{2}\eta_2\left[\frac{m_2b_2 - m_0\beta_1 - m_1\beta_2 - m_3\beta_2}{\eta_2}\right]^2.\end{aligned}$$

The profit function of the disintegrated system is:

$$\begin{aligned}\pi_D^* &= \pi_{\mathrm{NB}}^* + \pi_{\mathrm{SB}}^*\\ &= m_0\left(a_0 + b_0\frac{b_0m_0 + \beta_0m_3}{\eta_0} + \beta_0\frac{b_1m_1 - \beta_2m_2}{\eta_1} - \beta_1\frac{b_2m_2 - \beta_2m_1}{\eta_2}\right)\\ &\quad + m_3\left(a_1 + b_1\frac{b_1m_1 - \beta_2m_2}{\eta_1} + \beta_0\frac{b_0m_0 + \beta_0m_3}{\eta_0} - \beta_2\frac{b_2m_2 - \beta_2m_1}{\eta_2}\right)\\ &\quad + m_1\left(a_1 + b_1\frac{b_1m_1 - \beta_2m_2}{\eta_1} + \beta_0\frac{b_0m_0 + \beta_0m_3}{\eta_0} - \beta_2\frac{b_2m_2 - \beta_2m_1}{\eta_2}\right)\\ &\quad + m_2\left(a_2 + b_2\frac{b_2m_2 - \beta_2m_1}{\eta_2} - \beta_1\frac{b_0m_0 + \beta_0m_3}{\eta_0} - \beta_2\frac{b_1m_1 - \beta_2m_2}{\eta_1}\right)\\ &\quad - \frac{1}{2}\eta_0\left[\frac{b_0m_0 + \beta_0m_3}{\eta_0}\right]^2 - \frac{1}{2}\eta_1\left[\frac{b_1m_1 - \beta_2m_2}{\eta_1}\right]^2 - \frac{1}{2}\eta_2\left[\frac{b_2m_2 - \beta_2m_1}{\eta_2}\right]^2.\end{aligned}$$

Then, we can get:

$$\pi_I^* - \pi_D^* = \frac{(m_1\beta_0 - m_2\beta_1)^2}{2\eta_0} + \frac{(m_0\beta_0 + m_3b_1)^2}{2\eta_1} + \frac{(m_0\beta_1 + m_3\beta_2)^2}{2\eta_2} > 0.$$

Moreover, the optimal advertising investments in the integrated system are:

$$A^* = \frac{b_0m_0 + \beta_0m_3}{\eta_0}$$
$$a^* = \frac{b_1m_1 - \beta_2m_2}{\eta_1}$$
$$g^* = \frac{b_2m_2 - \beta_2m_1}{\eta_2}.$$

Hence,

$$A_I^* - A^* = \frac{m_0b_0 + m_1\beta_0 - m_2\beta_1 + m_3\beta_0}{\eta_0} - \frac{b_0m_0 + \beta_0m_3}{\eta_0} = \frac{m_1\beta_0 - m_2\beta_1}{\eta_0}$$
$$a_I^* - a^* = \frac{m_0\beta_0 + m_1b_1 - m_2\beta_2 + m_3b_1}{\eta_1} - \frac{b_1m_1 - \beta_2m_2}{\eta_1} = \frac{m_0\beta_0 + m_3b_1}{\eta_1} > 0$$
$$g_I^* - g^* = \frac{m_2b_2 - m_0\beta_1 - m_1\beta_2 - m_3\beta_2}{\eta_2} - \frac{b_2m_2 - \beta_2m_1}{\eta_2} = \frac{-m_0\beta_1 - m_3\beta_2}{\eta_2} < 0.$$

So, we can prove Proposition 4.13.

Proof of Proposition 5.1

The first-order condition: $\frac{\partial \pi_{\mathrm{NB}}^u}{\partial A} = b_0m_0 + \beta_0m_3 - \eta_0A$. Thus, the second-order condition: $\frac{\partial^2 \pi_{\mathrm{NB}}^u}{\partial A^2} = -\eta_0 < 0$. Hence, π_{NB}^u is concave in A.

The first-order condition:

$$\frac{\partial \pi_{\mathrm{SB}}^u}{\partial a} = b_1m_1 - \beta_2m_2 - \eta_1(1-t)a, \frac{\partial \pi_{\mathrm{SB}}^u}{\partial g} = b_2m_2 - \beta_2m_1 - \eta_2g.$$

Thus, the second-order condition: $\frac{\partial^2 \pi_{\mathrm{SB}}^u}{\partial a^2} = -\eta_1(1-t) < 0$, $\frac{\partial^2 \pi_{\mathrm{SB}}^u}{\partial g^2} = -\eta_2 < 0$, $\frac{\partial^2 \pi_{\mathrm{SB}}^u}{\partial a \partial g} = \frac{\partial^2 \pi_{\mathrm{SB}}^u}{\partial g \partial a} = 0$ So, the Hessian of π_{SB}^u is:

$$H = \begin{bmatrix} -\eta_1(1-t) & 0 \\ 0 & -\eta_2 \end{bmatrix}.$$

Since, $\det(H) = (1-t)\eta_1\eta_2 > 0$. Hence, π_{SB}^u is joint concave in a and g.

Then, we can obtain the optimal advertising investments:

$$A^{**} = \frac{b_0m_0 + \beta_0m_3}{\eta_0}$$
$$a^{**} = \frac{b_1m_1 - \beta_2m_2}{\eta_1(1-t)}$$
$$g^{**} = \frac{b_2m_2 - \beta_2m_1}{\eta_2}.$$

So, the profits of the two firms are:

$$\pi_{\mathrm{NB}}^u = m_0\left(a_0 + b_0A + \beta_0\frac{b_1m_1 - \beta_2m_2}{\eta_1(1-t)} - \beta_1g\right) + m_3\left(a_1 + b_1\frac{b_1m_1 - \beta_2m_2}{\eta_1(1-t)} + \beta_0A - \beta_2g\right)$$
$$- \frac{1}{2}\eta_0A^2 - \frac{1}{2}\eta_1t\left[\frac{b_1m_1 - \beta_2m_2}{\eta_1(1-t)}\right]^2$$
$$\pi_{\mathrm{SB}}^u = m_1\left(a_1 + b_1\frac{b_1m_1 - \beta_2m_2}{\eta_1(1-t)} + \beta_0A - \beta_2g\right) + m_2\left(a_2 + b_2g - \beta_1A - \beta_2\frac{b_1m_1 - \beta_2m_2}{\eta_1(1-t)}\right)$$
$$- \frac{1}{2}\eta_1(1-t)\left[\frac{b_1m_1 - \beta_2m_2}{\eta_1(1-t)}\right]^2 - \frac{1}{2}\eta_2g^2.$$

Taking the derivative of $\pi_{\rm NB}^u$ and $\pi_{\rm SB}^u$ with respect to t, we obtain:

$$\frac{\partial \pi_{\rm NB}^u}{\partial t} = \frac{(-2\beta_0 b_1 m_0 m_1 + 2\beta_0\beta_2 m_0 m_2 - 2b_1^2 m_1 m_3 + 2b_1\beta_2 m_2 m_3)(1-t) - (1+t)(b_1 m_1 - \beta_2 m_2)^2}{2\eta_1(1-t)^3}$$

$$\pi_{\rm SB}^u = \frac{(b_1 m_1 - \beta_2 m_2)^2}{2\eta_1(1-t)} > 0.$$

So, we get

$$t = \frac{-2\beta_0 b_1 m_0 m_1 + 2\beta_0\beta_2 m_0 m_2 - 2b_1^2 m_1 m_3 + 2b_1\beta_2 m_2 m_3 - (b_1 m_1 - \beta_2 m_2)^2}{-2\beta_0 b_1 m_0 m_1 + 2\beta_0\beta_2 m_0 m_2 - 2b_1^2 m_1 m_3 + 2b_1\beta_2 m_2 m_3 + (b_1 m_1 - \beta_2 m_2)^2}.$$

Take the value of t into $\pi_{\rm NB}^u$ and $\pi_{\rm SB}^u$. We can get $\pi_{\rm NB}^u > \pi_{\rm NB}$ and $\pi_{\rm SB}^u > \pi_{\rm SB}$.

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