

EQUILIBRIUM AND INCENTIVES FOR SUPERVISOR–POSTGRADUATE COLLABORATIONS: A GAME-THEORETIC APPROACH

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Abstract. In most universities, supervisors collaborate with their postgraduate students in writing papers. As a consequence, the relationship between supervisors and postgraduates in the collaborative work becomes the most important one among various relationships between them. In this paper, using a game model, we show that in the current educational system of China, there is a dilemma between supervisors and their postgraduates for their collaborative work – in most cases, either the supervisor or the students will not spend any effort in their joint work. After that, we also investigate whether the two common incentive strategies, *i.e.*, (i) incentives to students, and (ii) incentives to faculties, can solve this dilemma. Our results show that a university can solve the problem by either (i) just using strong incentives to postgraduate students, or (ii) by using a combination of a normal incentive to students and a strong incentive to faculties. Also, we find that when the incentives to the students and to the faculties are below a certain level, all incentives will be just in vain – neither can they improve the serious relationship between supervisors and their postgraduates, nor can they improve the paper quality.

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1. INTRODUCTION

Postgraduate cultivation is vital for constructing a world-class university, which is highly correlated with the university’s mission of academic progress, talent training and serving the society. For academic postgraduates (especially the doctoral candidates), publishing a certain amount of papers with a certain level of quality is stipulated as a necessary condition for graduation in most universities. The graduate education among many countries such as China adopts the supervisor-responsible system in scientific research.

Ideally, the supervisor-responsible system should benefit both the supervisors and the postgraduates. Specifically, when supervisors and their postgraduates cooperate on the research and write papers based on their research, the supervisors should assist the postgraduates to become independent researchers by improving their creative thinking, self-directed continuous learning, and problem-solving skills, and the students should also

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work hard, trying their best to produce high-quality research. Supervisors and their postgraduates should also have a strong synergistic effect with which both of them can be improved from their collaboration.

While the ideal indeed describes an ideal state, we can always observe that there are many disharmonies between supervisors and postgraduates in practice. On the side of supervisors, some complained that their postgraduates were too lazy, some others felt painful because their excellent students eventually entered the industries after they trained their research skills for several years. Similarly, on the other side of postgraduate students, some grumbled that their supervisor had not provided them any instruction, some swallowed their anger to their tough supervisors.

In this paper, we proposed a game model to explain these disharmonies. In particular, we consider a system where a supervisor and one of his/her student collaborate in writing a paper. While the quality of the paper depends on both the supervisor and the student's effort, their utilities of the paper differ and are related to the quality. Within a Stackelberg game framework where the supervisor is assumed as the leader and the student as the follower, we consider that both of them are rational and are to maximize their net utilities. The equilibrium results demonstrate that in most cases there is a dilemma between supervisors and their postgraduates for their collaborative work – either the supervisor or the students will not spend any effort in their joint work. This dilemma explains most disharmonies between supervisors and their postgraduates.

Facing the above dilemma, we investigate whether it can be solved by two common incentive strategies, *i.e.*, (i) incentives to students, and (ii) incentives to faculties. Our results show that universities or schools can solve it by either (i) just using strong incentives to postgraduate students, or (ii) by using a combination of a normal incentive to students and a strong incentive to faculties. But when the incentives to the students and the faculties below a certain level, all incentives will just in vain – neither can they improve the serious relationship between supervisors and their postgraduates, nor can they improve the paper quality.

Contributions of this paper include the following. First, as far as we know, it is the first paper that investigates the relationship between supervisors and their students within a game model framework. This provides a new approach to study the educational system. Second, our results could be a guide for postgraduate students when they are choosing or selecting their supervisors, as well as a guide for supervisors when selecting their students. Third, our results also provide some general suggestions to universities or schools when they are planning their various incentive programs.

The rest of this paper is organized as follows. Section 2 reviews the literature related to our paper. The basic model is developed in Section 3. Section 4 derives and explains the equilibrium solutions. The dilemma between supervisors and their postgraduate students is also found in this section. The two common incentive strategies are investigated in Section 5. Some numerical examples are presented in Section 6. Conclusions and future research are in Section 7.

2. LITERATURE REVIEW

Literature related to this paper is mainly on the two streams, *i.e.*, the relationship between the supervisors and the students and game theory.

2.1. The supervisor–student relationship

The relationship between supervisors and postgraduate students has attracted much attention from the field of educational research. We make a brief review of them on the nature of the supervisor–student relationship and the conflict between supervisors and postgraduates.

It is a consensus that the supervisor–postgraduate relationship is not only the relationship between teaching and learning but also actually a kind of cooperative one [16,16,20,27,31,38,47,56]. Levinson [33] characterizes the supervisor–student relationship as long-term, complex, and multifaceted. For postgraduates, the relationships with supervisors are much closer than those for undergraduates [30,45]. Hardy [23] comments that “Mentoring is different from advising because it involves a personal relationship. Mentors are close trusted colleagues and guides”. Busch Judith and Candace Garrett [6] argue that the ideal relationship between supervisors and

postgraduate include mutuality and comprehensiveness. Similarly, in the view of [28], the mentoring is usually characterized by a more emotional flavor: this relationship “may be complex and deeply connected friendships that are inherently mutual”. Johnson and Huwe [27] say that supervisors commonly enact any or all of the following roles: academic advisor, career counselor, course instructor, coach, and friend. They further to point out that the strongest mentoring relationship is marked by the mentor’s execution of multiple roles. Lechuga [32] also makes similar arguments and further elaborates on these roles.

Previous research indicates that conflicts occur in a significant number of supervisors and postgraduates [1, 3, 9, 17, 21, 27, 39]. Such conflicts may hurt their relationships, with negative consequences for both of them [21, 42], especially for the postgraduate student. Chiste [9] identifies sources of the conflicts, including lack of information, lack of feedback or useful feedback, lack of time, excessive control on the part of a supervisor, discrimination or unfair treatment based on cultural background, lack of openness or honesty, and gender-based discrimination. Also, some researchers discuss how to conduct effective management and/or prevent conflicts [3, 29]. Facing these disharmonies, some literature addressed on how to alleviate them and improve the cooperation between the supervisor and the postgraduate [2, 8, 18, 26, 40, 44, 50]. More recently, [36] study the perceptions of ethical issues in supervision among doctoral students and supervisors. Howells *et al.* [24] explore the role of gratitude in enhancing the relationship between students and their supervisors.

Because the supervisor-responsible system of China has many problems in practice, the issue of the relationship between supervisors and postgraduates also arouses consideration among Chinese researchers. For instance, [19, 51–53, 55] describe various conflicts between supervisors and postgraduates and provide various suggestions for the education reform in China. Liu [35] indicates that it is important to coordinate the relationship between supervisors and postgraduates in the current supervisor-responsible system of China. Liu [34] discusses the supervisor–postgraduate relationship from the aspect of “academic guidance” and suggests that the supervisors have responsibilities to instruct the students in doing research. Hu [25] investigates this problem from the aspect of “research collaboration” and points out that the supervisor–postgraduate relationship should be a kind of cooperative one in research.

We should note that all the previous papers mentioned above are based on either speculative approach or empirical approach. The game theory model proposed in this paper actually provides a new perspective in analyzing the relationship between supervisors and postgraduates.

2.2. Game theory

Game theory was developed in the 1950s as a means of using mathematical models to analyze conflicting economic phenomena and to find possible cooperation modes between two or more actors [43, 49]. Game theory has been widely applied in many fields, including economics [7, 41, 49], business [54], engineering [46] and sociology [4, 5, 10, 13, 37].

However, in the education literature, very few studies have employed game theory to explore interactions of the supervisors (or teachers) and the students. Hamburger [22] is the first to apply game theory to it. It examines the interaction between one teacher and multiple students, where each actor is assumed rational in making her/his decision with a reasonable expectation of the actions of other actors. A similar problem is also investigated by Correa and Gruver [15], where the assumption in [22] that both the students and the teacher have only finite number strategies have been changed into that the two members have an infinite and continuous number of strategies. Later, Correa [11, 12, 14] also incorporate various factors such as the class size, teacher’s capabilities and work ethics into this research, to investigate their influences on the interactions between teachers and students.

While previous papers focus on a general form of interaction between teachers and students, our work concentrates just on a more specific one, *i.e.*, their relationship in collaborating to write a paper. Our objective is to examine the performance of a university’s possible incentive strategies.

3. MODEL DEVELOPMENT

Consider the scenario that a supervisor collaborates with one of his/her postgraduate students on a research paper. We suppose that the quality of the collaborative paper is determined by their efforts. In specific, let u_T denote the supervisor's effort and u_S denote the student's effort, the quality of the paper is assumed to be

$$Q(u_T, u_S) = \alpha u_T + \beta u_S + \gamma u_T u_S, \quad (3.1)$$

where α , β and γ are all positive parameters. While equation (3.1) indicates that the quality of the paper is linear to the supervisor's and the student's effort, the term $\gamma u_T u_S$ in it also implies that a kind of synergistic effect will occur in their collaboration. Also, a large α (or β) implies that the supervisor (or the student) has a high efficiency in doing the research.

When facing a paper with a quality of Q , the postgraduate student and the supervisor have different utility. For most postgraduate students in China, they are required to publish at least one paper in some academic journals in a certain list before they can apply for their master's thesis defense. While the journal list varies across schools and universities, once there is a journal list, it actually forms the minimum requirement for the paper's quality. Let Q_S denote this minimum requirement. If the paper's quality is lower than Q_S , the paper cannot be accepted by any journal in the list, implying the student cannot apply his/her thesis defense and cannot obtain his/her master degree. On the contrary, once the quality is higher than Q_S so that the paper can be accepted by one journal in the list, the student can obtain his/her master degree in time, and the master degree is really favorable in finding a good job. With the above facts, for most postgraduate students, their utility function related to the paper is

$$R_S = \begin{cases} 0 & \text{if } Q < Q_S, \\ a & \text{if } Q \geq Q_S. \end{cases} \quad (3.2)$$

Similarly, while the supervisor's utility increases in the quality of the paper in general, the supervisor also has a minimum quality requirement Q_T for his/her research papers. In fact, for most supervisors, publishing papers in low quality academic journals implies a loss of their reputations. Thus, the supervisor will not join the work if the collaborative paper's quality is lower than his/her requirement Q_T . With the above consideration, we suppose that the supervisor's utility function related to the paper is

$$R_T = \begin{cases} 0 & \text{if } Q < Q_T, \\ b(Q - Q_T) & \text{if } Q \geq Q_T. \end{cases} \quad (3.3)$$

In equation (3.3), the parameter b measures the supervisor's marginal utility from the improved quality of the paper. Usually, a larger b will occur when the supervisor is more enthusiastic about research or his/her university has stronger incentives for his/her research.

When the supervisor and the student input their efforts in the research, they also suffer a kind of cost, which could be either physical or mental cost. Without loss of generality, we suppose that the supervisor and the student occur a quadratic cost of their own efforts, *i.e.*,

$$C(u_i) = \frac{1}{2} u_i^2, i = \{T, S\}, \quad (3.4)$$

to indicate the fact that their efforts have an increasing and diminishing effect on improving the paper's quality. Such an assumption is common in literature, readers may refer [48, 54]. Then, the net utility of the supervisor is

$$\pi_T = R_T(Q) - C(u_T), \quad (3.5)$$

and that of the student is

$$\pi_S = R_S(Q) - C(u_S), \quad (3.6)$$

where $R_S(Q)$ and $R_T(Q)$ are given by equations (3.2) and (3.3) respectively, and $C(u_T)$ and $C(u_S)$ are given by equation (3.4).

We suppose the supervisor and the postgraduate student are rational, *i.e.*, they optimally decide their efforts to maximize their own net utility. In addition, we suppose the sequence of events as follows. The supervisor acts as the Stackelberg leader, determines his/her effort u_T firstly. Then, the student, as the Stackelberg follower, makes his/her effort u_S based on the supervisor's effort u_T .

4. THE STACKELBERG EQUILIBRIUM

To obtain the Stackelberg Equilibrium we use a backward induction approach. Specifically, we first derive the student's optimal u_S for any given u_T to maximize his/her net utility given by equation (3.5), then we substitute the student's best response into the supervisor's net utility function and then derive his/her optimal u_T .

For any given the supervisor's effort u_T , the postgraduate student's best response is given by Proposition 4.1.

Proposition 4.1. *For any given the supervisor's effort u_T , the postgraduate student's optimal effort is*

$$u_S = \begin{cases} \frac{Q_S - \alpha u_T}{\beta + \gamma u_T} & \text{if } 0 \leq u_T < \frac{Q_S}{\alpha} \\ 0 & \text{if } u_T \geq \frac{Q_S}{\alpha}, \end{cases} \quad (4.1)$$

under which the corresponding quality of the paper is

$$Q(Q_T) = \begin{cases} Q_S & \text{if } 0 \leq u_T < \frac{Q_S}{\alpha}, \\ \alpha u_T & \text{if } u_T \geq \frac{Q_S}{\alpha}. \end{cases} \quad (4.2)$$

The proofs of all propositions are provided in the appendix. Proposition 4.1 demonstrates that the student's effort decreases in the supervisor's effort. The more effort that the supervisor inputs in their collaborative paper, the less effort the student will input. Specifically, when the supervisor's effort is higher enough, *i.e.*, $u_T \geq Q_S/\alpha$, the student will not make any effort.

Substituting the student's best response function into the supervisor's objective function given by equation (3.5), we maximize the new objective function and then obtain the supervisor's optimal decision as well as the equilibrium results, which is given by Proposition 4.2.

Proposition 4.2. *When the supervisor collaborates with the student in writing a paper, if the supervisor is the Stackelberg leader and the student is the follower, the supervisor's equilibrium effort is*

$$u_T^* = \begin{cases} b\alpha & \text{if } \max(Q_S, Q_T) < \frac{b\alpha^2}{2} \\ 0 & \text{if } \max(Q_S, Q_T) \geq \frac{b\alpha^2}{2}, \end{cases} \quad (4.3)$$

and the postgraduate student's equilibrium effort is

$$u_S^* = \begin{cases} 0 & \text{if } \max(Q_S, Q_T) < \frac{b\alpha^2}{2} \\ \frac{Q_S}{\beta} & \text{if } \max(Q_S, Q_T) \geq \frac{b\alpha^2}{2}, \end{cases} \quad (4.4)$$

under which the equilibrium quality of the research paper is

$$Q^* = \begin{cases} b\alpha^2 & \text{if } \max(Q_S, Q_T) < \frac{b\alpha^2}{2} \\ Q_S & \text{if } \max(Q_S, Q_T) \geq \frac{b\alpha^2}{2}. \end{cases} \quad (4.5)$$

TABLE 1. Equilibrium solutions for the game between the supervisor and the postgraduate.

	Conditions	The supervisor's equilibrium effort and his net utility	The student's equilibrium effort and his net utility	Paper quality Q
Case A	$Q_S < Q_T$ and $Q_T < b\alpha^2/2$	$u_T^* = b\alpha$ and $\pi_T^* = b^2\alpha^2/2 - bQ_T$	$u_S^* = 0$ and $\pi_S^* = a$	$b\alpha^2$
Case B	$Q_S < Q_T$ and $Q_T \geq b\alpha^2/2$	$u_T^* = 0$ and $\pi_T^* = 0$	$u_S^* = Q_S/\beta$ and $\pi_S^* = a - Q_S^2/2\beta^2$	Q_S
Case C	$Q_S \geq Q_T$ and $Q_S < b\alpha^2/2$	$u_T^* = b\alpha$ and $\pi_T^* = b^2\alpha^2/2 - bQ_S$	$u_S^* = 0$ and $\pi_S^* = a$	$b\alpha^2$
Case D	$Q_S \geq Q_T$ and $Q_S \geq b\alpha^2/2$	$u_T^* = 0$ and $\pi_T^* = b(Q_S - Q_T)$	$u_S^* = Q_S/\beta$ and $\pi_S^* = a - Q_S^2/2\beta^2$	Q_S

To make it easy for our explanation, we rewrite the equilibrium results of Proposition 4.2 into four different cases shown in Table 1.

From Table 1 we found that the equilibrium solutions are mainly dependent on two dimensions, *i.e.*, (i) whether the supervisor's personal requirement on the paper's quality is higher than that of the student, and (ii) whether the supervisor is efficient enough in writing papers. The answers of those two dimensions eventually turn into four different cases.

Case A. $Q_S < Q_T$ and $Q_T < b\alpha^2/2$. The condition $Q_S < Q_T$ means that the supervisor's basic requirement on the paper's quality is higher than the university's minimum requirement on the student's paper, and the condition $Q_T < b\alpha^2/2$ means that either the supervisor is efficient in writing papers (*i.e.*, a large α) and/or the supervisor has a high motivation in writing high-quality papers (*i.e.*, a large b). Under such a condition, the supervisor makes much effort in the research paper but the student's effort falls into zero. Since in most universities of China, the supervisors are asked to pay the postgraduate research fee, for supervisors with such characteristics, the postgraduate seems to be just a burden. This result explains why most young scholars returned from famous overseas universities, especially in research areas such as management science or marketing science, would not likely to recruit postgraduate student but to do research all themselves.

Case B. $Q_S < Q_T$ and $Q_T \geq b\alpha^2/2$. In this case, while the supervisor's requirement on the paper's quality is higher than the student's requirement (this is just the same as Case A), compared with his/her high-quality standard, the supervisor has either a low efficiency in research (a small α) or low motivation in doing research (a small b). Under such a condition, the supervisor has no interest to collaborate with the student, thus the student has to do the research all himself/herself, which eventually leads the paper's quality just equaling to the university minimum standard.

This situation usually happens in senior faculties. For most overseas scholars, because of the incentive from the coming tenured position (a large b), they perform efficiently enough in publishing papers before they obtain their tenured position. But once the tenured position is achieved, the value of b decreases suddenly but their intrinsic academic standard Q_T keeps as a constant. These two facts eventually make the second inequality, *i.e.*, $Q_T \geq b\alpha^2/2$ holds. As a consequence, they are no longer interested in publishing papers. Despite that they have postgraduate students, in most cases, the students have to survive themselves.

Case C. $Q_S \geq Q_T$ and $Q_T < b\alpha^2/2$. Despite the equilibrium results of this case is the same as those in Case A, *i.e.*, the supervisor makes enough effort in the collaborative work but the student hitchhikes, the two conditions differ. In particular, although the supervisor's requirement on the paper's quality is lower than that of the students, once the supervisor is efficient enough in writing papers and/or has enough motivation in doing research, his effort in the joint work will be high enough so that the students will spend no effort. Combining the results of Case A, we see that no matter whether the supervisor's requirement on the paper's quality is higher or lower than that of the student's, once the supervisor is efficient and has a high motivation in writing

papers, the student can always lie down on the job. This is why smart students would like a young professor to be their supervisor – the young scholars usually have a high motivation for doing research.

Case D. $Q_S \geq Q_T$ and $Q_S \geq b\alpha^2/2$. The two conditions here imply that: (i) the university's paper quality standard for the students is higher than that for the faculties (*i.e.*, $Q_S \geq Q_T$), and (ii) the supervisor is either inefficient or has a low motivation in doing research (*i.e.*, $Q_S \geq b\alpha^2/2$). While the equilibrium results of the case are just the same as those in Case B, *i.e.*, the supervisor will invest no effort in the collaborative work and the students have to survive themselves, the difference in the supervisor's net utility will make the two cases as far apart as heaven and earth. In detail, without inputting any effort, the supervisor's net utility in Case B is zero, implying that the supervisor has no self-incentive to instruct postgraduate. However, for the current case, because the minimum requirement of the student is higher than the supervisor's, the supervisor always has a positive net utility without any effort. Thus, once such a condition occurs, the supervisor will recruit postgraduate students as many as possible but does not provide any instruction to them. This will hurt the supervisor-student relationship deeply, but unfortunately, this situation does popularly exist in the current supervisor-responsible system of China [19, 55].

Summarizing the above 4 cases, we find there is a dilemma in the current supervisor responsible system of China. Because the supervisors and the postgraduate students have different valuations on the paper's quality, the supervisors and their students cannot form a real collaboration in doing research – either the supervisor or the student will make no effort in the collaborative work. The harmonious relationship between supervisors and students seems to be a luxury here.

5. THE ROLE OF TWO COMMON INCENTIVES

In Section 4 we find that there is a dilemma between the supervisors and their postgraduate students. When facing such a dilemma, an interesting question is whether and in which condition the common incentive strategies in practice can solve this problem. Usually, universities and their schools have two different incentive strategies, one is to the student orientation incentives and the other is faculty orientation incentives. The former is to encourage the postgraduate student doing better research with high-award scholarships. For instance, in University of Science and Technology of China, postgraduate students who have completed high-quality papers, can not only obtain a high award scholarship and a certain research fund, but also have a chance to enter their doctoral study without an entrance examination. The latter is to encourage their faculties with a certain award. Most universities in China provides a high award to their faculties when they published papers in high-quality journals such as Nature, Science, PRL and so on.

Despite the difference between the two strategies, both of them are to build up a certain relationship between the award and the quality of their papers. Owing to such a relationship, the postgraduate's utility function turns into the following, *i.e.*,

$$R_S = \begin{cases} 0 & \text{if } Q < Q_S, \\ a + b(Q - Q_S) & \text{if } Q \geq Q_S. \end{cases} \quad (5.1)$$

In equation (5.1), while the parameter Q_S and a are the same as those in equation (3.2), the parameter $b_1 > 0$ measures the degree of the incentive intensity to students. A higher b_1 means that the university offers more incentives to the students for their research work.

As for the supervisors, with research incentives from their university or school, their utility function usually keeps the same form as that in equation (3.3), but the value of the parameter b increases.

Similar to Section 4, we suppose that the supervisor is the Stackelberg leader and the student is the follower, as well as that they try to maximize their own net utilities given by equations (3.5) and (3.6). The only difference is that the student's utility function changes from equation (3.2) to (5.1).

Under this new setting, the student's best response to a given supervisor's effort u_T is given by Proposition 5.1.

Proposition 5.1. *Giving the supervisor's effort u_T , the student's optimal effort is*

$$u_S(u_T) = \begin{cases} b_1(\beta + \gamma u_T) & \text{if } F(u_T) \geq Q_S \\ \frac{Q_S - \alpha u_T}{\beta + \gamma u_T} & \text{if } F(u_T) < Q_S, \end{cases} \quad (5.2)$$

under which the paper's quality is

$$Q(u_T, u_S) = \begin{cases} F(u_T) & \text{if } F(u_T) \geq Q_S \\ Q_S & \text{if } F(u_T) < Q_S, \end{cases} \quad (5.3)$$

where $F(u_T) = \alpha u_T + b_1(\beta + \gamma u_T)^2$.

Proposition 5.2 illustrates that incentives to the students can mitigate the serious relationship between supervisors and their postgraduate students as shown in Section 4. Still, the student's effort decreases in the supervisor's effort u_T when $\alpha u_T + b_1(\beta + \gamma u_T)^2 < Q_S$ holds. But when $\alpha u_T + b_1(\beta + \gamma u_T)^2 \geq Q_S$ satisfies, the student would like to make more efforts if the supervisor pays more effort. We should note that the condition $\alpha u_T + b_1(\beta + \gamma u_T)^2 \geq Q_S$ can be satisfied more easily when a larger b_1 (or β) occurs. This means that incentives to students will work better when the incentives degree is higher (or when the student is more efficient in doing research). Similarly, if there is a higher synergistic effect (*i.e.*, a larger γ) between the supervisor and the student, the incentives to the student can also do a better job.

Substitute the results of Proposition 5.1 into the supervisor's net utility function, *i.e.*, equation (3.3), then by maximizing the new function, we obtain the supervisor's equilibrium effort, as given by Proposition 5.2.

Proposition 5.2. *Let*

$$I(Q_i) = \sqrt{\alpha^2 + 4\alpha b_1 \beta \gamma + 4b_1 \gamma^2 Q_i}, \quad i = T, S, \quad (5.4)$$

(i) *When $b_1 \geq \max(Q_T, Q_S)/\beta^2$, the supervisor's equilibrium effort on the collaborative paper is*

$$u_T^* = \frac{b\alpha + 2bb_1\gamma\beta}{1 - 2bb_1\gamma^2}, \quad (5.5)$$

the student's equilibrium effort is

$$u_S^* = \frac{b_1(\beta + \gamma b b_1 \alpha)}{1 - 2bb_1\gamma^2}, \quad (5.6)$$

under which the paper's quality is $Q^ = \alpha u_T^* + b_1(\beta + \gamma u_T^*)^2 > \max(Q_T, Q_S)$;*

(ii) *When $0 < b_1 < \max(Q_T, Q_S)/\beta^2$, and*

$$b \geq \max\left(\frac{2(Q_T - b_1\beta^2)}{I^2(Q_T)}, \frac{2(Q_S - b_1\beta^2)}{I^2(Q_S)}\right), \quad (5.7)$$

the supervisor's equilibrium effort on the collaborative paper is

$$u_T^* = \frac{b\alpha + 2bb_1\gamma\beta}{1 - 2bb_1\gamma^2}, \quad (5.8)$$

the student's equilibrium effort is

$$u_S^* = \frac{b_1(\beta + \gamma b b_1 \alpha)}{1 - 2bb_1\gamma^2}, \quad (5.9)$$

under which the paper's quality is $Q^ = \alpha u_T^* + b_1(\beta + \gamma u_T^*)^2 > \max(Q_T, Q_S)$;*

(iii) When $0 < b_1 < \max(Q_T, Q_S)/\beta^2$, and

$$0 < b < \max\left(\frac{2(Q_T - b_1\beta^2)}{I^2(Q_T)}, \frac{2(Q_S - b_1\beta^2)}{I^2(Q_S)}\right), \quad (5.10)$$

the supervisor's equilibrium effort on the collaborative paper is $u_T^* = 0$, and the student's equilibrium effort is $u_S^* = Q_S/\beta$, under which the paper quality is $Q^* = Q_S$.

Proposition 5.2 demonstrates that three different equilibrium could exist under various conditions. Specifically, we see that the relationship between the supervisor and the student is in a harmonious state in the former two scenarios – both the supervisor and the student would like to make their own contribution on the collaborative paper, and the paper's quality will be significantly improved. But for the third scenario, all the incentives are in vain – still, the supervisor makes no effort and the student has to survive himself/herself with a minimum quality paper.

We should explain the managerial difference between the first two scenarios here. For the first scenario, the condition $b_1 \geq \max(Q_T, Q_S)/\beta^2$ means that the university or/and the school's incentives to postgraduates are strong enough. If we measure a university or a school's research level by the quality of paper, this equilibrium demonstrates that a university or school can improve its research level by simply offering sufficient incentives to its postgraduate or doctoral students. Note that the threshold $\max(Q_T, Q_S)/\beta^2$ decreases in β and the paper's equilibrium quality increases in β , we know that this strategy can be easily applied and will perform well if the students are more efficient in doing research. Once we have a belief that excellent universities such as Tsinghua University and Peking University can enroll excellent graduate students with high efficiency in doing research, we should also believe that they can easily develop a brand new research area or enhance a weak discipline even without excellent faculties. However, for most universities which cannot attract excellent students, such a strategy will be costly and inefficient.

As for the second scenario, while the condition $0 < b_1 < \max(Q_T, Q_S)/\beta^2$ means that there is just a moderate or a low incentive to postgraduate, the other condition which is given by equation (5.7) implies that incentives to faculties are strong enough. By using a combination of a moderate incentive to its students with a high incentive to its faculties, a university can also improve its research level and improve the relationship between supervisors and postgraduates. Noting that the threshold in equation (5.7) decreases in b_1 , we see that there is a substitutive effect between incentives to faculties and those to students. If the incentives to students are high enough, those to faculties can be reduced to a certain degree. In addition, the equilibrium quality of the paper, *i.e.*, $Q^* = \alpha u_T^* + b_1(\beta + \gamma u_T^*)^2$, is highly related to the incentives to students. This means that it is still necessary to offer the students with strong incentives to improve the paper quality. For most universities without excellent students, this combined incentive strategy should be a good alternative. The equilibrium results of the third scenario demonstrate that there is a threshold for its incentives to faculties. When the incentives to faculties below this threshold level, all the spending related to incentives are just in vain. Neither does it have effect in improving the supervisor-student relationship, nor is it helpful in improving the paper quality. A university should try to avoid such a dilemma.

6. NUMERICAL EXAMPLES

In this section, we briefly report the results of our numerical experimentation. We did this experiment for two reasons: one is to verify our analytical findings, and the other is to obtain more insights into the incentive strategies that improve the cooperation relationship between the supervisor and the postgraduate.

To have a better understanding of Proposition 5.2, we draw the areas of the three scenarios of Proposition 5.2 in Figure 1. In the three sub-figures of Figure 1, the x -axis is b_1 (the degree of the incentive intensity to students) and the y -axis is b (*i.e.*, the degree of the incentive intensity to supervisors). While parameters are fixed as $\alpha = 2$, $\gamma = 0.1$, $Q_T = 4$ and $Q_S = 3$, the parameter β that reflects the student's efficiency in doing research is different in the three sub-figures, taking values in 1, 1.5 and 2 correspondingly.

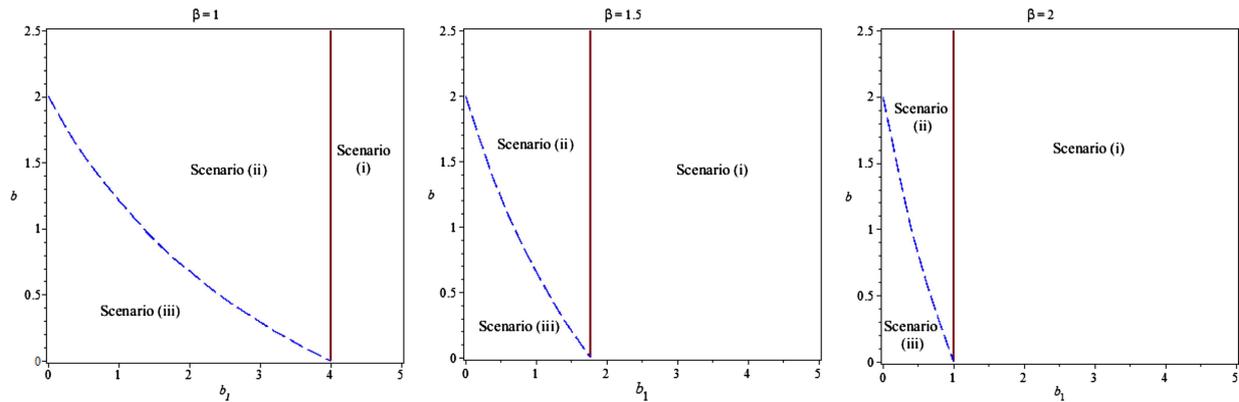


FIGURE 1. The regions for the three scenarios of Proposition 5.2.

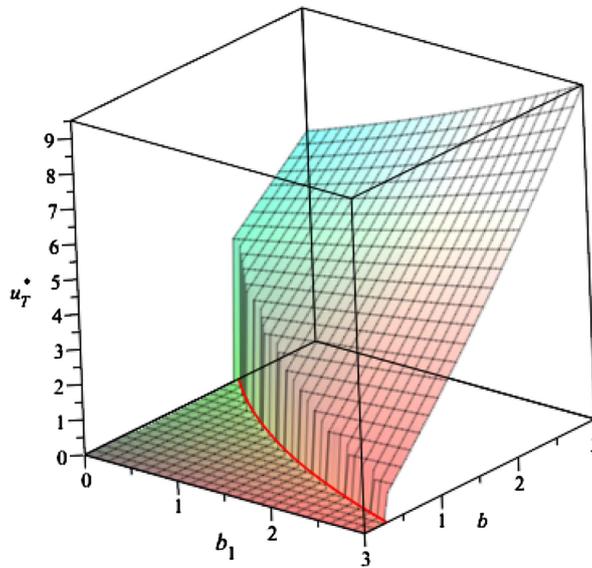


FIGURE 2. Supervisor's effort u_T^* vs. various degrees of incentive intensity to the supervisor b and degrees of incentive intensity to the student b_1 .

Comparison of these three sub-figures demonstrates the following results. First, as the student's efficiency β increases, the area of Scenario (iii) shrinks. This implies that it will be easier for famous universities with excellent graduate students to avoid the incentive "black hole", but for most universities, the situation will be worse. Second, in all the three sub-figures, the dilemma situation can be entirely avoided if and only if b is large enough. Thus, to avoid the dilemma mentioned above, universities should offer stronger incentives to students and supervisors when the students are less efficient.

To explore the impacts of the two common incentives on the equilibrium results, we change the values of b (*i.e.*, the degree of the incentive intensity to supervisors) and b_1 (the degree of the incentive intensity to students), and plot the supervisor's equilibrium effort, the student's equilibrium effort and the equilibrium quality of the collaborated paper in Figures 2-4, respectively. Just similar to Figure 1, other parameters are fixed as $\alpha = 2$, $\gamma = 0.1$, $Q_S = 4$, $Q_T = 3$ and $\beta = 1$.

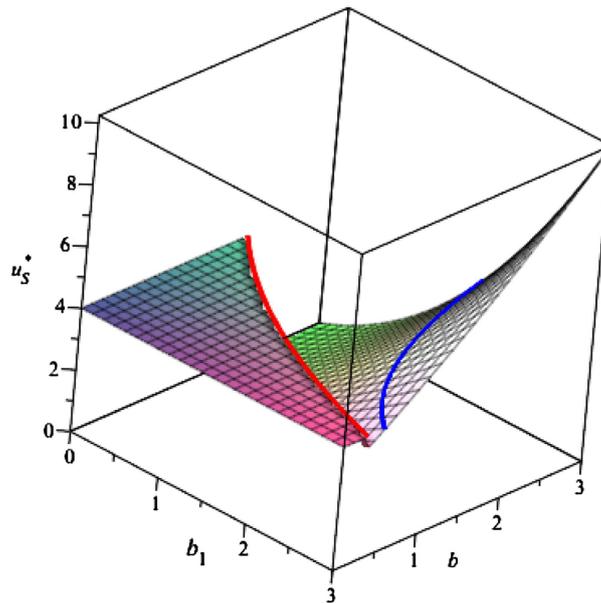


FIGURE 3. Student's effort u_s^* vs. various degrees of incentive intensity to the supervisor b and degrees of incentive intensity to the student b_1 .

These three figures verify the existence of the threshold for the degrees of the incentive intensity to supervisors and students. When b_1 and b are beneath the threshold level (the red lines in the three figures), incentives to the supervisor or/and to the student are just in vain: the supervisor's effort is zero, the student's effort is fixed at Q_S/β and the paper's quality is fixed at its lowest level Q_S . But once b_1 and b exceed the threshold, from Figures 2 and 4 we can see that the supervisor's effort will be suddenly increased and the paper's quality will be improved significantly.

Besides the above managerial implication, Figure 3 illustrates a new dilemma: the student's effort may be suddenly decreased when b_1 and b exceed the red line. In practice, such a dilemma may be presented as the following phenomenon, for some postgraduate students with high quality publications, the quality of their thesis is unexpectedly low. This phenomenon occurs because the publication is a collaborative work of the supervisor and the student, but the thesis is usually mainly the student's work. Because the student spends little effort in the cooperative work, the student's academic ability cannot be trained efficiently, and this eventually results in a low quality thesis. Such a phenomenon will be more serious in some famous universities where the joint publication is in English but the students' thesis is in Chinese. As shown in Figure 3, to avoid the new dilemma, b_1 and b should exceed the blue line, it is necessary for a university to offer stronger incentives to both students and supervisors.

7. CONCLUSION

The relationship between supervisors and postgraduate students is a most important one for most research orientation universities. In this paper, we focus on such relationship when a supervisor collaborates with his student in writing an academic paper under the current supervisor-responsible system.

Utilizing the Stackelberg game method, we derive the equilibrium efforts of the two members in writing a paper. Results show that there is a dilemma in the current supervisor responsible system, *i.e.*, either the supervisor or the student will invest no effort in their collaborating work but just to hitchhike, in spite that they can achieve a higher quality paper if both work harder.

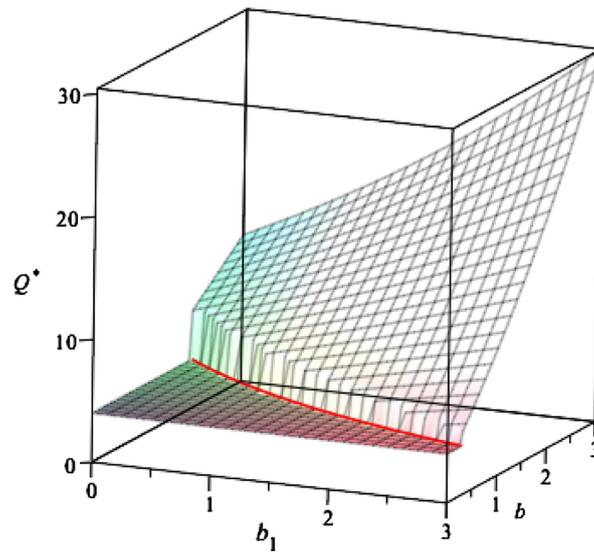


FIGURE 4. Paper quality Q^* vs. various degrees of incentive intensity to the supervisor b and degrees of incentive intensity to the student b_1 .

Also, we examine the performance of two different incentive strategies, *i.e.*, incentives to students and incentives to faculties. We discover that while universities can improve the supervisor–student relationship and increase the paper quality by offering either strong student orientation incentives, or a combination of strong faculty-orientation incentives and weak student-orientation incentives, high incentives to students will significantly improve the paper quality. Additionally, we find that there is a threshold effect in the incentives of a university. All the incentives could be just in vain when incentives are below a certain level. These findings could help universities to make a proper plan in encouraging their faculties and students to do research.

Limitations and future research related to this topic include the following. First, for brevity, we consider that the interaction between the supervisor and the student follows a Stackelberg game. In practice, the interactive process between them is dynamic and is more complex, and thus dynamic models could be investigated later. Second, in this paper, we just consider the relationship between the supervisor and the student when they collaboratively write a paper, more relationships between them could be considered later. Third, despite that our model incorporates the relationship between supervisors and their postgraduates, the model can also be used or modified to analyze how should a scholar seek a proper co-author, or how should a firm choose a partner for a joint R&D research program.

APPENDIX A. ALL PROOFS

A.1. Proof of Proposition 4.1

Given the supervisor's effort u_T , when u_T is sufficiently large such that the quality of the paper is always not less than the minimum graduation requirements Q_S , the net utility of the postgraduate is $a - u_S^2/2$. When the supervisor and the postgraduate make a joint effort and the quality of the paper achieves Q_S , the net utility of the postgraduate is also $a - u_S^2/2$. When the efforts of supervisor and the postgraduate are not large enough such that the quality of the paper is lower than Q_S , the net utility of the postgraduate is then $-u_S^2/2$.

Therefore, the net utility of the postgraduate can be written as

$$\pi_S(u_S|u_T) = \begin{cases} a - \frac{u_S^2}{2} & \text{if } u_T \geq \frac{Q_S}{\alpha} \\ -\frac{u_S^2}{2} & \text{if } u_S < \frac{Q_S - \alpha u_T}{\beta + \gamma u_T}, u_T < \frac{Q_S}{\alpha} \\ a - \frac{u_S^2}{2} & \text{if } u_S \geq \frac{Q_S - \alpha u_T}{\beta + \gamma u_T}, u_T < \frac{Q_S}{\alpha}. \end{cases} \quad (\text{A.1})$$

The optimal decisions of the postgraduate can be obtained easily from equation (A.1), *i.e.*,

$$u_S^*(u_T) = \begin{cases} 0 & \text{if } u_T \geq \frac{Q_S}{\alpha} \\ u_S < \frac{Q_S - \alpha u_T}{\beta + \gamma u_T} & \text{if } u_T < \frac{Q_S}{\alpha}. \end{cases} \quad (\text{A.2})$$

In reality, even though the supervisor makes no effort, the postgraduate can still make the paper quality meet graduation requirements by his own efforts. The positive utility comes from successful graduation should not be less than his cost, so we can assume $(a - Q_S^2/2\beta^2) \geq 0$. Substituting the optimal decisions of the postgraduate into the paper quality function shown in equation (3.1), we can have

$$Q(u_T, u_S) = \begin{cases} \alpha u_T & \text{if } u_T \geq \frac{Q_S}{\alpha} \\ Q_S & \text{if } u_T < \frac{Q_S}{\alpha}. \end{cases} \quad (\text{A.3})$$

A.2. Proof of Proposition 4.2

Substituting equation (A.3) into equation (3.5) yields following two cases:

(i) When $Q_S < Q_T$, the net utility function of the supervisor is

$$\pi_T = \begin{cases} -\frac{u_T^2}{2} & \text{if } u_T < \frac{Q_S}{\alpha} \\ -\frac{u_T^2}{2} & \text{if } \frac{u_S}{\alpha} \leq u_T < \frac{Q_T}{\alpha} \\ b\alpha u_T - bQ_T - \frac{u_T^2}{2} & \text{if } u_T \geq \frac{Q_T}{\alpha}. \end{cases} \quad (\text{A.4})$$

Simplifying (A.4), we can have

$$\pi_T = \begin{cases} b\alpha u_T - bQ_T - \frac{u_T^2}{2} & \text{if } u_T \geq \frac{Q_T}{\alpha} \\ -\frac{u_T^2}{2} & \text{if } u_T < \frac{Q_T}{\alpha}. \end{cases} \quad (\text{A.5})$$

According to equation (A.5), we can get that, when $u_T < Q_T/\alpha$, the optimal decision of the supervisor is $u_T^* = 0$, and the corresponding utility is 0. When $u_T \geq Q_T/\alpha$, then if $b\alpha \geq Q_T/\alpha$, the optimal decision of the supervisor is $u_T^* = b\alpha$, and the corresponding net utility of the supervisor is $b^2\alpha^2/2 - bQ_T$; if $b\alpha < Q_T/\alpha$, the optimal decision of the supervisor is $u_T^* = Q_T/\alpha$, and the corresponding net utility of the supervisor is $-Q_T^2/2\alpha^2$. Comparing the net utility of the supervisor under the condition $u_T < Q_T/\alpha$, and choosing the larger one as the optimal decision of the supervisor, we obtain the following results:

- (1) If $b\alpha \geq Q_T/\alpha$ and $b^2\alpha^2/2 - bQ_T > 0$, *i.e.*, $Q_T < b\alpha^2/2$, the optimal decision of supervisor is $u_T^* = Q_T/\alpha$;
- (2) If $b\alpha \geq Q_T/\alpha$ and $b^2\alpha^2/2 - bQ_T \leq 0$, the optimal decision of the supervisor is $u_T^* = 0$;
- (3) If $b\alpha < Q_T/\alpha$, the optimal decision of the supervisor is $u_T^* = 0$.

Combining results (2) and (3), we can find that, when $Q_T \geq b\alpha^2/2$, the optimal decision of the supervisor is $u_T^* = 0$.

(ii) When $Q_S \geq Q_T$, the net utility function of the supervisor is

$$\pi_T = \begin{cases} b\alpha u_T - bQ_T - \frac{u_T^2}{2} & \text{if } u_T \geq \frac{Q_S}{\alpha} \\ b(Q_S - Q_T) - \frac{u_T^2}{2} & \text{if } u_T < \frac{Q_S}{\alpha}. \end{cases} \quad (\text{A.6})$$

According to equation (A.6), we can get that, when $u_T < \frac{Q_S}{\alpha}$, the optimal decision of the supervisor is $u_T^* = 0$, and the corresponding net utility of the supervisor is $b(Q_S - Q_T)$. When $u_T \geq \frac{Q_S}{\alpha}$, then if $b\alpha \geq Q_S/\alpha$, the optimal decision of the supervisor is $u_T^* = b\alpha$, and the corresponding net utility of the supervisor is $b^2\alpha^2/2 - bQ_T$; if $b\alpha < Q_S/\alpha$, the optimal decision of the supervisor is $u_T^* = Q_S/\alpha$, and the corresponding net utility of the supervisor is $b(Q_S - Q_T) - Q_T^2/2\alpha^2$. Comparing the net utility of the supervisor under the condition $u_T \geq Q_S/\alpha$ with that under the condition $u_T < Q_S/\alpha$ and choosing the larger one as the optimal decision of the supervisor, we obtain the following results:

- (1) If $Q_S < b\alpha^2/2$, the optimal decision of the supervisor is $u_T^* = b\alpha$;
- (2) If $b\alpha^2/2 \leq Q_S \leq b\alpha^2$, the optimal decision of the supervisor is $u_T^* = 0$;
- (3) If $Q_S > b\alpha^2$, the optimal decision of the supervisor is $u_T^* = 0$.

Combining the results (2) and (3), we can find that if $Q_S \geq b\alpha^2/2$, the optimal decision of the supervisor is $u_T^* = 0$.

Combining the results of above two situations that $Q_S < Q_T$ and $Q_S \geq Q_T$, the equilibrium decision of the supervisor is obtained. Substituting u_T^* into equations (4.1) and (4.2), we get the postgraduate’s effort and the corresponding paper quality. The details are given by Proposition 4.2.

A.3. Proof of Proposition 5.1

Given u_T , let $G(u_T) = \alpha u_T + \beta u_S + \gamma u_T u_S - Q_S$, the net utility function of the postgraduate can be written as

$$\pi_S(u_S|u_T) = \begin{cases} a + b_1 G(u_T) - \frac{u_S^2}{2} & \text{if } u_T \geq \frac{Q_S}{\alpha} \\ -\frac{u_S^2}{2} & \text{if } u_S < \frac{Q_S - \alpha u_T}{\beta + \gamma u_T}, u_T < \frac{Q_S}{\alpha} \\ a + b_1 G(u_T) - \frac{u_S^2}{2} & \text{if } u_S \geq \frac{Q_S - \alpha u_T}{\beta + \gamma u_T}, u_T < \frac{Q_S}{\alpha}. \end{cases} \tag{A.7}$$

From equation (A.7), we obtain the optimal decision of the postgraduate and the corresponding paper quality Q . Let $\Delta = a + b_1\alpha u_T + b_1^2(\beta + \gamma u_T)^2/2 - b_1 Q_S$, the results are shown as follows:

- (1) When $u_T \geq Q_S/\alpha$, because $\Delta - a \geq 0$ always holds, we get the optimal effort of the postgraduate from the first order condition of $\pi_S(u_S|u_T)$, i.e., $u_S^* = b_1(\beta + \gamma u_T)$. The postgraduate student’s net utility is Δ , and the corresponding paper quality is $Q^* = \alpha u_T + b_1(\beta + \gamma u_T)^2$.
- (2) When $u_S < (Q_S - \alpha u_T)/(\beta + \gamma u_T)$ and $u_T < Q_S/\alpha$, $u_S^* = 0$, the postgraduate student net utility is 0.
- (3) When $u_S \geq (Q_S - \alpha u_T)/(\beta + \gamma u_T)$ and $u_T < Q_S/\alpha$ hold simultaneously, the first order condition yields $u_S = b_1(\beta + \gamma u_T)$. By considering the boundary condition, we can obtain that: (i) if $b_1(\beta + \gamma u_T) \geq (Q_S - \alpha u_T)/(\beta + \gamma u_T)$ satisfies, the optimal effort of the postgraduate student is $u_S^* = b_1(\beta + \gamma u_T)$, under which her/his net utility is $\Delta > 0$ and the corresponding paper quality is $Q^* = \alpha u_T + b_1(\beta + \gamma u_T)^2$; and (ii) if $b_1(\beta + \gamma u_T) < (Q_S - \alpha u_T)/(\beta + \gamma u_T)$ holds, the optimal effort of the postgraduate student is $u_S^* < (Q_S - \alpha u_T)/(\beta + \gamma u_T)$, under which her/his net utility of the postgraduate is $a - (Q_S - \alpha u_T)^2/2(\beta + \gamma u_T)^2 \geq a - (Q_S^2/2\beta^2) \geq 0$ and the corresponding paper quality is $Q^* = Q_S$. This case dominates the second one where the net utility of the postgraduate student is 0.

From above (1)–(3), we derive the optimal decision of the postgraduate

$$u_S^*(u_T) = \begin{cases} b_1(\beta + \gamma u_T) & \text{if } u_T \geq \frac{Q_S}{\alpha} \\ \frac{Q_S - \alpha u_T}{\beta + \gamma u_T} & \text{if } F(u_T) < Q_S, u_T < \frac{Q_S}{\alpha} \\ b_1(\beta + \gamma u_T) & \text{if } F(u_T) \geq Q_S, u_T < \frac{Q_S}{\alpha}. \end{cases} \tag{A.8}$$

Substituting the optimal decisions of the postgraduate into paper quality function, we can get the corresponding paper quality

$$Q^*(u_T) = \begin{cases} F(u_T) & \text{if } u_T \geq \frac{Q_S}{\alpha} \\ Q_S & \text{if } F(u_T) < Q_S, u_T < \frac{Q_S}{\alpha} \\ F(u_T) & \text{if } F(u_T) \geq Q_S, u_T < \frac{Q_S}{\alpha}. \end{cases} \tag{A.9}$$

Simplifying equations (A.8) and (A.9), the optimal decisions of the postgraduate and the corresponding paper quality can be obtained as shown in Proposition 5.1.

A.4. Proof of Proposition 5.2

Substituting equation (5.1) into equation (3.5) yields following two cases:

(i) When $Q_S < Q_T$, the net utility function of the supervisor is

$$\pi_T(u_T) = \begin{cases} H(u_T) & \text{if } u_T \geq \frac{Q_S}{\alpha}, F(u_T) \geq Q_T \\ -\frac{u_T^2}{2} & \text{if } u_T \geq \frac{Q_S}{\alpha}, F(u_T) < Q_T \\ -\frac{u_T^2}{2} & \text{if } F(u_T) < Q_S, u_T < \frac{Q_S}{\alpha} \\ H(u_T) & \text{if } F(u_T) \geq Q_S, u_T < \frac{Q_S}{\alpha}, F(u_T) \geq Q_T \\ -\frac{u_T^2}{2} & \text{if } F(u_T) \geq Q_S, u_T < \frac{Q_S}{\alpha}, F(u_T) < Q_T, \end{cases} \quad (\text{A.10})$$

where $H(u_T) = b\alpha u_T + bb_1(\beta + \gamma u_T)^2 - bQ_T - \frac{u_T^2}{2}$.

Simplifying equation (A.10), we can have

$$\pi_T(u_T) = \begin{cases} H(u_T) & \text{if } F(u_T) \geq Q_T \\ -\frac{u_T^2}{2} & \text{if } F(u_T) < Q_T. \end{cases} \quad (\text{A.11})$$

According to equation (A.11), we can get the optimal decisions of supervisor:

(1) When $b_1 \geq Q_T/\beta^2$, the condition $\alpha u_T + b_1(\beta + \gamma u_T)^2 \geq Q_T \geq Q_S$ holds, the supervisor's net utility is $b\alpha u_T + bb_1(\beta + \gamma u_T)^2 - bQ_T - \frac{u_T^2}{2}$. The first order derivative of the supervisor's net utility function yields $u_T^* = (b\alpha + 2bb_1\gamma\beta)/(1 - 2bb_1\gamma^2)$. Substituting it into equations (5.1) and (5.3), we get $u_S^* = [b_1(\beta + bb_1\gamma\alpha)]/(1 - 2bb_1\gamma^2)$, and $Q_T^* = \alpha u_T^* + b_1(\beta + \gamma u_T^*)^2 \geq b\alpha^2 > Q_T > Q_S$.

(2) When $b_1 < Q_T/\beta^2$, the net utility of the supervisor can be written as

$$\pi_T(u_T) = \begin{cases} H(u_T) & \text{if } u_T \geq \frac{-\alpha - 2b_1\beta\gamma + I(Q_T)}{2b_1\gamma^2} > 0 \\ -\frac{u_T^2}{2} & \text{if } 0 \leq u_T < \frac{-\alpha - 2b_1\beta\gamma + I(Q_T)}{2b_1\gamma^2}, \end{cases} \quad (\text{A.12})$$

where $I(Q_T) = \sqrt{\alpha^2 + 4\alpha b_1\beta\gamma + 4b_1\gamma^2 Q_T}$.

The optimal decisions of the supervisor and the postgraduate and the corresponding paper quality can be obtained from equation (A.12):

(2.1) If

$$\frac{b\alpha + 2bb_1\gamma\beta}{1 - 2bb_1\gamma^2} \geq \frac{-\alpha - 2b_1\beta\gamma + I(Q_T)}{2b_1\gamma^2} \quad (\text{A.13})$$

and

$$\pi_T \left(\frac{b\alpha + 2bb_1\gamma\beta}{1 - 2bb_1\gamma^2} \right) > 0, \quad (\text{A.14})$$

in which equations (A.13) and (A.14) can be simplified to

$$b \geq \frac{2(Q_T - b_1\beta^2)}{I^2(Q_T)}, \quad (\text{A.15})$$

the equilibrium decision of the supervisor is $u_T^* = (b\alpha + 2bb_1\gamma\beta)/(1 - 2bb_1\gamma^2)$, the equilibrium decision of the postgraduate is $u_S^* = [b_1(\beta + bb_1\gamma\alpha)]/(1 - 2bb_1\gamma^2)$, and the quality of the paper satisfies $Q^* \geq b\alpha^2 \geq Q_T > Q_S$.

(2.2) If

$$\frac{b\alpha + 2bb_1\gamma\beta}{1 - 2bb_1\gamma^2} \geq \frac{-\alpha - 2b_1\beta\gamma + I(Q_T)}{2b_1\gamma^2} \quad (\text{A.16})$$

and

$$b < \frac{2(Q_S - b_1\beta^2)}{I^2(Q_T)} \left(\text{i.e., } \pi_T \left(\frac{b\alpha + 2bb_1\gamma\beta}{1 - 2bb_1\gamma^2} \right) > 0 \right), \quad (\text{A.17})$$

in which, equations (A.16) and (A.17) are equivalent to

$$0 \leq \frac{1}{2} - \frac{\alpha - 2\beta\gamma b_1}{2I(Q_T)} \leq b < \frac{2(Q_T - b_1\beta^2)}{I^2(Q_T)}, \quad (\text{A.18})$$

the optimal decisions of the supervisor and the postgraduate are $u_T^* = 0$, $u_S^* = Q_S/\beta$, and the paper quality is equal to $Q^* = Q_S$.

(2.3) If

$$0 < b < \frac{1}{2} - \frac{\alpha - 2\beta\gamma b_1}{2I(Q_T)}, \quad (\text{A.19})$$

the optimal decisions of the supervisor and the postgraduate are $u_T^* = 0$, $u_S^* = Q_S/\beta$, and the paper quality is equal to $Q^* = Q_S$.

(ii) When $Q_S \geq Q_T$, the utility function of the supervisor is

$$\pi_T(u_T) = \begin{cases} H(u_T) & \text{if } F(u_T) \geq Q_S \\ b(Q_S - Q_T) - \frac{u_T^2}{2} & \text{if } F(u_T) < Q_S. \end{cases} \quad (\text{A.20})$$

According to equation (A.20), the optimal decisions of the supervisor can be obtained:

(1) When $b_1 \geq Q_S/\beta^2 \geq Q_T/\beta^2$, the condition $Q = \alpha u_T + b_1(\beta + \gamma u_T)^2 \geq Q_S \geq Q_T$ holds. The supervisor's net utility is $\pi_T(u_T) = b\alpha u_T + bb_1(\beta + \gamma u_T)^2 - bQ_T - u_T^2/2$. The first order derivative of the supervisor's net utility function yields $u_T^* = (b\alpha + 2bb_1\gamma\beta)/(1 - 2bb_1\gamma^2) > b\alpha$. Substituting it into equations (5.1) and (5.3), we can get the optimal effort of the postgraduate $u_S^* = [b_1(\beta + b\gamma\alpha)]/(1 - 2bb_1\gamma^2)$, and the paper quality satisfies $Q^* = \alpha u_T^* + b_1(\beta + \gamma u_T^*)^2 \geq b\alpha^2 > Q_S > Q_T$.

(2) When $b_1 < Q_S/\beta^2$, the net utility function of the supervisor is

$$\pi_T(u_T) = \begin{cases} H(u_T) & \text{if } u_T \geq \frac{-\alpha - 2b_1\beta\gamma + I(Q_T)}{2b_1\gamma^2} \\ b(Q_S - Q_T) - \frac{u_T^2}{2} & \text{if } 0 \leq u_T < \frac{-\alpha - 2b_1\beta\gamma + I(Q_T)}{2b_1\gamma^2}. \end{cases} \quad (\text{A.21})$$

The equilibrium decisions of the supervisor and the postgraduate and the corresponding paper quality can be obtained from equation (A.21):

(2.1) If

$$\frac{b\alpha + 2bb_1\gamma\beta}{1 - 2bb_1\gamma^2} \geq \frac{-\alpha - 2b_1\beta\gamma + I(Q_S)}{2b_1\gamma^2} \quad (\text{A.22})$$

and

$$\pi_T \left(\frac{b\alpha + 2bb_1\gamma\beta}{1 - 2bb_1\gamma^2} \right) > b(Q_S - Q_T), \quad (\text{A.23})$$

in which equations (A.22) and (A.23) can be simplified to

$$b \geq \frac{2(Q_S - b_1\beta^2)}{I^2(Q_S)}, \quad (\text{A.24})$$

where $I(Q_S) = \sqrt{\alpha^2 + 4\alpha b_1\beta\gamma + 4b_1\gamma^2 Q_S}$, the equilibrium decision of the supervisor is $u_T^* = (b\alpha + 2bb_1\gamma\beta)/(1 - 2bb_1\gamma^2) > b\alpha$, the equilibrium decision of the postgraduate is $u_S^* = [b_1(\beta + bb_1\gamma\alpha)]/(1 - 2bb_1\gamma^2) \geq Q_S/\beta$, and the quality of the paper satisfies $Q^* \geq b\alpha^2 \geq Q_S$.

(2.2) If

$$\frac{b\alpha + 2bb_1\gamma\beta}{1 - 2bb_1\gamma^2} \geq \frac{-\alpha - 2b_1\beta\gamma + I(Q_S)}{2b_1\gamma^2} \quad (\text{A.25})$$

and

$$\pi_T \left(\frac{b\alpha + 2bb_1\gamma\beta}{1 - 2bb_1\gamma^2} \right) \leq b(Q_S - Q_T), \quad (\text{A.26})$$

in which equations (A.25) and (A.26) can be simplified to

$$0 \leq \frac{1}{2} - \frac{\alpha - 2\beta\gamma b_1}{2I(Q_S)} \leq b < \frac{2(Q_S - b_1\beta^2)}{I^2(Q_S)}, \quad (\text{A.27})$$

the equilibrium decision of the supervisor is $u_T^* = 0$, the corresponding net utility of the supervisor is $b(Q_S - Q_T)$, $u_S^* = Q_S/\beta$, and the paper quality is equal to $Q^* = Q_S$.

(2.3) If

$$0 < b < \frac{1}{2} - \frac{\alpha - 2\beta\gamma b_1}{2I(Q_S)}, \quad (\text{A.28})$$

the equilibrium decision of the supervisor is $u_T^* = 0$. Substituting it into equations (5.2) and (5.3), we can find that the optimal effort of the postgraduate is $u_S^* = Q_S/\beta$, and the paper quality is equal to $Q^* = Q_S$.

To sum up, when $b_1 \geq \max(Q_T, Q_S)/\beta^2$, the equilibrium efforts of the supervisor and the postgraduate are $u_T^* = (b\alpha + 2bb_1\gamma\beta)/(1 - 2bb_1\gamma^2)$ and $u_S^* = [b_1(\beta + bb_1\gamma\alpha)]/(1 - 2bb_1\gamma^2)$, and the quality of the paper is $Q^* = \alpha u_T^* + b_1(\beta + \gamma u_T^*)^2 \geq b\alpha^2 > \max(Q_T, Q_S)$.

When $b_1 < \max(Q_T, Q_S)/\beta^2$,

(i) If

$$b \geq \max \left(\frac{2(Q_S - b_1\beta^2)}{I^2(Q_S)}, \frac{2(Q_T - b_1\beta^2)}{I^2(Q_T)} \right),$$

the equilibrium efforts of the supervisor and the postgraduate are $u_T^* = (b\alpha + 2bb_1\gamma\beta)/(1 - 2bb_1\gamma^2)$ and $u_S^* = [b_1(\beta + bb_1\gamma\alpha)]/(1 - 2bb_1\gamma^2)$, respectively, under which the quality of the paper is $Q^* = \alpha u_T^* + b_1(\beta + \gamma u_T^*)^2 \geq b\alpha^2 > Q_S$.

(ii) If

$$0 < b < \max \left(\frac{2(Q_S - b_1\beta^2)}{I^2(Q_S)}, \frac{2(Q_T - b_1\beta^2)}{I^2(Q_T)} \right)$$

holds, the equilibrium efforts of the supervisor and the postgraduate are $u_T^* = 0$ and $u_S^* = Q_S/\beta$, and the paper quality is $Q^* = Q_S$. Then, the Proposition 5.2 is proved.

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