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A THREE-ECHELON SUPPLY CHAIN MODEL WITH PRICE AND TWO-LEVEL QUALITY DEPENDENT DEMAND

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Abstract. This paper considers a three-echelon supply chain model with one supplier, one manufacturer and one retailer for trading a single product. We assume that the market demand at the retailer's end is stochastic, but dependent on price and quality of the product. The final product's quality depends on the manufacturing process and the raw material's quality. We first develop models for centralized and decentralized scenarios. Then we try to coordinate the decentralized system with some contract mechanism. We show that revenue sharing contract is not able to coordinate the system, but a composite contract comprised of sales rebate and penalty (SRP) with return is able to coordinate the system. Finally, we illustrate the developed model with a numerical example and show the efficiency of SRP with return policy. We graphically show the effects of various model-parameters on the optimal decisions. Most of the existing literature's focus on the quality of the finished product, but in this model we incorporate the quality of the raw material as a decision variable along with the finished product quality. We also able to coordinate the three echelon model with a composite contract which is seldom addressed in the existing literatures.

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1. Introduction

It is not possible for every manufacturer to produce raw material as well beside producing finished good and sell the finished good to end customers. So, there exists the need of a raw material supplier or a finished goods seller. Need of several entities to smoothly run a business set up gives birth to supply chain. Supply chain is a system involving several entities like raw material supplier, manufacturer, distributor, retailer, end customer, etc. When we buy some product from the market, it is the result of effort put in by several supply chain members. This process start with raw material supplier supplying raw material to the manufacturer and end with retailer selling the finished product to the end customer. Supply chains can be categorized in two kinds – centralized and decentralized. In the centralized supply chain, there exists a single decision maker who takes all required decisions on behalf of the supply chain members. In reality, it is quite hypothetical to consider a single decision maker for the entire supply chain. In the decentralized supply chain, every member tries to optimize his/her own profit without considering the entire supply chain's profit. As a result, the profit of the

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supply chain reduces. To improve supply chain's efficiency and profit, members are aligned with some contracts such as, quantity discount, buy-back, revenue sharing etc. These contracts can coordinate the supply chain and present win-win situation for every participating member.

Proper demand analysis is another important aspect of improving supply chain profit. There are many factors such as price of the commodity, after sales service, advertisement, quality of the product, etc which can influence market demand. For instance, in buying a mobile phone or a laptop, price as well as durability play an important role in potent customers mind. In today's market, Apple is well known for its product quality but high product price. On the other hand, Samsung and Lenovo are renowned for their relatively low price and feature reach products. Customers who prefer quality over price opt for Apple's products. Customers who prefer features and pricing compromise with the product quality and buy Samsung or Lenovo products. Product quality not only depends on manufacturer's production process but also raw material quality of the supplier. As a result, if the manufacturer wants to produce high quality product, he needs both improved manufacturing process and good quality raw material. As for example, we can say Lenovo uses low cost components while assembling laptops to cut down overall pricing while Apple uses very high quality components in Macbook and as a result the overall price increases. It is, therefore, necessary to struck a balance between price and quality to optimize profit. We want to incorporate price and quality dependency of demand into our supply chain model where the finished product quality is dependent on both raw material quality of the supplier and the manufacturer's production process.

In the literature, a large number of papers on price-dependent demand is available but literatures on quality aspect of the product combined with pricing is rather limited. Here we mention a few papers which consider quality aspect of the product in supply chain. Reyniers and Tapiero [38] modeled the effects of price rebate and after-sales warranty on the choice of a supplier from the quality perspective. Forker [13] linked quality management with process optimization to address both effectiveness and efficiency concerns. Banker *et al.* [2] considered quality competition in Nash environment. Baiman *et al.* [1] developed a model considering contracting and quality cost. Lin *et al.* [26] formulated a structural equation model of supply chain quality management and organizational performance. Robinson and Malhotra [39] defined the concept of supply chain quality management and its relevance to academic and industrial practice. Bernstein and Federgruen [3] considered coordination mechanisms for supply chains under price and service competition. Foster [14] developed a supply chain model focusing on quality management. In many industries, competition is shifting from price to quality in specific segments of the market (Gans [16]; Ren and Zhou [37]). Franca *et al.* [15] tried to evaluate tradeoff between profit and quality. Karipidis [25] studied market evolutions of dimensions of design quality. Rong *et al.* [40] followed an optimization approach for managing fresh food quality throughout the supply chain. Xie *et al.* [49] discussed about quality investment and pricing decision in a risk-averse supply chain. Tse and Tan [47] proposed a supply chain product quality risk management framework, integrating both the incremental calculus and marginal analysis. Wang and Li [48] discussed the impact of the accuracy of quality or shelf-life indicator and timing and frequency of discount in a selling period on retailing performance. Yu and Ma [50] considered the impact of decision sequence of pricing and quality investment in decentralized assembly system. Bhunia and Shaikh developed two inventory models for deteriorating items one considering selling price dependent demand [4] and other two-warehouse [6]. Maiti and Giri [30] studied a closed loop supply chain considering retail price and quality dependent demand. Darwish *et al.* [11] developed a vendor managed inventory model for single-vendor multi-retailer with quality consideration. Liu *et al.* [29] found out service capability procurement decision in logistics service supply chain under demand updating and quality guarantee. Bhunia *et al.* [5, 7–9] studied several single and two-warehouse inventory models for perishable goods. Taleizadeh *et al.* [43] optimizes price and warranty tenure in a competitive duopoly supply chain model. Cao *et al.* [10] analysed inventory control and pricing decisions in the regret-averse newsvendor setup. Tiwari *et al.* [44, 45] and Mishra *et al.* [32] proposed optimal pricing decisions for perishable product inventory models. Tiwari *et al.* [46] developed a sustainable inventory model with deteriorating and imperfect quality items considering carbon emission. Panda *et al.* [34] discussed a two-warehouse inventory model with deteriorating items where the demand is price and stock dependent.

TABLE 1. Recent literatures related to current study.

Refernces	Echelon	Demand	Coordination mechanism
Xie *et al.* [49]	Two	Price, Quality, Uncertain	–
He and Zhao [20]	Three	Uncertain	–
Seifert *et al.* [42]	Three	Uncertain	Sub-Supply Chain
Wang and Li [48]	Two	Price, Quality, Deterministic	Price Markdown
Yu and Ma [50]	Two	Price, Quality, Uncertain	–
Saha [41]	Three	Price, Deterministic	Combine Downward Direct Discount
Maiti and Giri [30]	Two	Price, Quality, Deterministic	–
Cao *et al.* [10]	One	Price, Uncertain	–
Current	Three	Price, Two level Quality, Uncertain	SRP with Return

In the existing literature, there are rich number of works about two-echelon supply chain model, but very few models involve three or more echelons. Munson and Rosenblatt [33] tried to coordinate a three-echelon supply chain model with quantity discounts, assuming deterministic demand. Giannoccaro and Pontrandolfo [18] constructed a three-echelon supply chain model with revenue-sharing contract which is used by both up-stream and down-stream supply chain members. Jaber *et al.* [22] studied a three-echelon supply chain model with price discounts, price dependent demand, and profit-sharing. Ding and Chen [12] proposed flexible buyback contracts which can fully coordinate a three-echelon supply chain. Jaber and Goyal [21] investigated the coordination of order quantities among the players in a three-echelon supply chain with a centralized decision process, allowing more than one player at each echelon. Jaber *et al.* [23] considered a three-echelon supply chain model and coordinated it with learning based continuous improvement and constant demand. Khouja [24], and Lee and Moon [27] assumed deterministic demand and employed EOQ model. Huang and Huang [19] developed a multi-echelon model with price competition and deterministic demand. Georgiadis *et al.* [17] considered a four-echelon supply chain network under uncertainties in time and demand. Mirzapour *et al.* [31] considered a three-echelon multi-product multi-period SC problem that would face with uncertainties in cost parameters and demands. He and Zhao [20] considered a three-echelon supply chain with both demand and supply uncertainties. Seifert *et al.* [42] developed a three-echelon supply chain model with price-only contracts and sub-supply chain coordination. Liu *et al.* [28] analyzed the impact of partial information sharing in a three-echelon supply chain. Saha [41] studied channel characteristics and coordination in three-echelon dual-channel supply chain. Pasandideh *et al.* [35] optimized a bi-objective multi-product multi-period three-echelon supply chain network with warehouse reliability.

All the above mentioned literatures have some limitations in one way or other. Most literatures considering quality of the finished product neglected the importance of raw material quality. In this paper, we consider a three-echelon supply chain model with price and quality dependent demand, where the manufacturer's product quality depends on both supplier's raw material quality and manufacturer's production process. We have considered supplier's raw material quality and manufacturer's quality decision parameter as decision variables along with the retail price of the product. Finally, we try to coordinate the system with a composite contract. We have provided a comparison between our work and existing literatures in Table 1. The paper is organized as follows: In Section 2, notations and modeling assumptions are provided. Section 3 deals with model development and procedure for finding the optimal decisions. Section 4 is devoted to numerical analysis. Managerial insights and conclusions are presented in Section 5.

2. Notations and assumptions

We use the following notations to develop the proposed model:

i	:	supply chain entity, $i = s$ (supplier), m (manufacturer), r (retailer).
p	:	unit selling price of the retailer (decision variable).
x_s	:	raw material quality of the supplier (decision variable).
α	:	quality decision parameter of the manufacturer (decision variable).
c_s	:	unit production cost of the supplier.
c_m	:	unit production cost of the manufacturer.
w_i	:	unit wholesale price of the entity i, where $i = s, m$.
$D(> 0)$	:	total market demand.
a	:	deterministic part of market demand, $a > 0$.
b	:	sensitivity of market demand with price, $b > 0$.
β	:	sensitivity of market demand with quality of the product, $\beta > 0$.
Q	:	order quantity of the retailer.
ε	:	random variable representing the stochastic part of the demand with positive support, density function $f(.)$, cumulative distribution function $F(.)$ and mean μ.
z	:	stocking factor when demand is stochastic.
v	:	salvage value per unit for leftover inventory.
s	:	lost sale cost per unit at the retailer's end due to stock out.
η_i	:	per unit cost for quality improvement, where $i = s, m$.
g_i	:	goodwill lost cost due to quality reason, where $i = s, m$.

The following assumptions are made for developing the proposed model:

(1) We assume that the supply chain is comprised of three entities – a raw material supplier, a manufacturer and a retailer for trading a single product.

(2) The product quality is dependent on the raw material quality and the manufacturing process of the manufacturer. The supplier decides it's raw material quality x_s. As raw material is not 100% pure in quality, $0 < x_s < 1$. The supplier can improve raw material quality subject to additional cost. The manufacturer's product quality depends on the raw material quality, We assume, $x_m = \alpha x_s$, where $\alpha(>0)$ = quality decision parameter of the manufacturer. Since, the manufacturer's product quality is also not 100% pure, so $0 < x_m < 1$. Due to quality dissatisfaction, the supplier and the manufacturer may lose their goodwill, which incurs a cost (goodwill loss cost) at a rate g_i, where $i = s, m$.

(3) Set up cost of the supplier and the manufacturer are c_s and c_m, respectively. We assume, $c = c_s + c_m$.

(4) There is a linear price dependency among the supplier, manufacturer and retailer. The wholesale price of the supplier is $w_s > c_s$, the wholesale price of the manufacturer is $w_m > w_s + c_m$ and retail price $p > w_m$.

(5) We assume, that the market demand of the product is $D = D(p, x_m) + \varepsilon$, where ε is the stochastic part of the demand and $D(p, x_m)$ is the deterministic part of the demand. $D(p, x_m)$ is dependent on retail price and quality of the product. We further assume, $D(p, x_m) = a - bp + \beta x_m$, where b = price sensitivity parameter and β = quality sensitivity parameter.

(6) We consider shortage and overage in our model. The unsold quantity is sold by the retailer at salvage value v $(v < p)$ per unit.

(7) There is no capacity constraint or lead time for the manufacturer or the supplier.

3. Model formulation and analysis

It is very well known fact that the centralized supply chain model is the most efficient one. However, in most of the real life situations, it is not possible to have centralized set up in which only one decision maker takes decisions on behalf of all the entities. So, supply chain models need to be developed and studied considering decentralized scenario. In this section, we first develop centralized model and then decentralized model. Further, a contract will be implemented to coordinate the decentralized supply chain and achieve win-win outcome.

3.1. Centralized supply chain

In this case, the sole decision maker decides the optimal raw material quality (x_s), quality decision factor (α), retail price (p) of the product and order quantity (Q). The expected profit for the centralized system is given by

$$\begin{aligned}\Pi_c(\alpha, x_s, p, Q) = & E\{p(Q \wedge D) - s(D-Q)^+ + v(Q-D)^+ - cQ\} - \eta_s x_s^2 - \eta_m x_m^2 - g_s(1-x_s) \\ & -g_m(1-x_m),\end{aligned} \tag{3.1}$$

where $X \wedge Y = \min\{X, Y\}$ and $(X)^+ = \max\{x, 0\}$. We define the stocking factor z as $z = Q - D(p, x_m)$ to cover the randomness of demand. This stocking factor act as order-up-to level in newsvendor model, to separate the random component of the demand from deterministic trend. Then the expected profit becomes

$$\begin{aligned}\Pi_c(\alpha, x_s, p, z) = & E\{p[D(p, x_m) + (\varepsilon \wedge z)] - s(\varepsilon - z)^+ + v(z-\varepsilon)^+\} - c[z + D(p, x_m)] - \eta_s x_s^2 \\ & -\eta_m x_m^2 - g_s(1-x_s) - g_m(1-x_m) \\ = & (p-c)[D(p, x_m) + \mu] - (c-v)\Lambda(z) - (p+s-c)\Theta(z) - \eta_s x_s^2 - \eta_m x_m^2 \\ & -g_s(1-x_s) - g_m(1-x_m),\end{aligned} \tag{3.2}$$

where $\Lambda(z) = \int_0^z (z-\xi)dF(\xi)$ and $\Theta(z) = \int_z^\infty (\xi - z)dF(\xi)$ are expected overage or stock out quantity.

The first order partial derivative of $\Pi_c(\alpha, x_s, p, z)$ with respect to α, x_s, p and z are given by

$$\frac{\partial \Pi_c(\alpha, x_s, p, z)}{\partial z} = -(c-v) + (p+s-c)\big(1 - F(z)\big) \tag{3.3}$$

$$\frac{\partial \Pi_c(\alpha, x_s, p, z)}{\partial p} = -b(p-c) + D(p, x_m) + \mu - \Theta(z) \tag{3.4}$$

$$\frac{\partial \Pi_c(\alpha, x_s, p, z)}{\partial \alpha} = \beta x_s(p-c) - 2\eta_m {x_s}^2 \alpha + g_m x_s \tag{3.5}$$

$$\frac{\partial \Pi_c(\alpha, x_s, p, z)}{\partial x_s} = \beta\alpha(p-c) - 2\eta_m x_s \alpha^2 - 2\eta_s x_s + g_m \alpha + g_s. \tag{3.6}$$

If the quality of the product is fixed we see that,

$$\frac{\partial^2 \Pi_c(\alpha, x_s, p, z)}{\partial z^2} = -(p+s-c)f(z) < 0$$

i.e., system profit $\Pi_c(\alpha, x_s, p, z)$ is concave with respect to stocking factor (z) as $p > c$. Now, from the first order optimality condition, we get the optimal value of stocking factor (z) for given p as

$$z(p) = F^{-1}\left(1 - \frac{c-v}{p+s-c}\right) \tag{3.7}$$

and for given stocking factor (z) and product quality (x_m), we have

$$\frac{\partial^2 \Pi_c(\alpha, x_s, p, z)}{\partial p^2} = -2b < 0$$

i.e., system profit $\Pi_c(\alpha, x_s, p, z)$ is concave with respect to retail price (p). Now, from the first order optimality condition, we get the optimal retail price as

$$p(z, x_s, \alpha) = \frac{a + bc + \mu + \beta x_s \alpha - \Theta(z)}{2b}. \tag{3.8}$$

Proposition 3.1. *Retail price in stochastic scenario is always greater than that in deterministic scenario.*

Proof. We define p^0 is the retail price when the demand is certain. Now, we can write

$$p = p^0 - \frac{\Theta(z)}{2b},\ where\ p^0 = \frac{a + bc + \mu + \beta x_s \alpha}{2b}\cdot$$

It is clear from the above that $p < p^0$. □

Proposition 3.2. *The optimal stocking factor increases with retail price.*

Proof. Taking first order partial derivative of stocking factor (z) with respect to retail price (p), we have

$$\frac{\mathrm{d}z}{\mathrm{d}p} = \frac{(c-v)}{f(z)(p+s-v)^2} > 0.$$

So, we can say that when price increases, the optimal stocking factor increases to cover the uncertainty of demand. □

Proposition 3.3. *For known retail price given in equation (3.8), the profit of the system is concave with respect to raw material quality x_s and manufacturer's quality decision parameter α.*

Proof. Taking second order partial derivative of $\Pi_c(\alpha, x_s, p, z)$ with respect to x_s and α respectively, we get

$$\frac{\partial^2 \Pi_c(\alpha, x_s, p, z)}{\partial \alpha^2} = -2\eta_m {x_s}^2 < 0,\ \frac{\partial^2 \Pi_c(\alpha, x_s, p, z)}{\partial^2 x_s} = -2\eta_m \alpha^2 - 2\eta_s < 0,$$

since $\alpha > 0,\ x_s > 0,\ \eta_m > 0$ and $\eta_s > 0$.

When the value of stocking factor (z) is given, we have from equations (3.5), (3.6) and (3.8)

$$\alpha^c = \frac{2\eta s\big(b(2g_m - c\beta) + \beta(a - Oz + \mu)\big)}{g_s(4b\eta m - \beta^2)} \tag{3.9}$$

$$x_s^c = \frac{g_s}{2\eta_s}\cdot \tag{3.10}$$

We can see from equation (3.10) that raw material quality of the supplier is dependent only on his costs. Now, using the optimal values of x_s^c and α^c in equation (3.8) we get,

$$p^c(z) = \frac{c(\beta^2 - 2b\eta_m) - g_m\beta - 2\eta_m(a - \Theta z + \mu)}{\beta^2 - 4b\eta_m}\cdot \tag{3.11}$$

□

Theorem 3.4. *Under linear additive demand function, the single-period stochastic centralized supply chain's solution is to set quality to x_s^c, quality decision parameter to α^c, price to p^c, and order $a - bp^c + \beta x_s^c + z^c$, subject to:*

(i) *if $F(.)$ is arbitrary distribution, the entire support must be searched to find z^c;*
(ii) *if $F(.)$ satisfies $2r(z)^2 + (dr(z)dz) > 0$ where $r(z) = f(z)/(1 - F(z))$ is the hazard rate, then z^c is the largest z satisfying the first-order condition.*

Proof. The theorem can be proved in manner similar to Petruzzi and Dada [36]. □

3.2. Decentralized supply chain

In the decentralized scenario, every supply chain entity tries to optimize his/her profit. Here, we consider the Stackelberg game where the supplier is the Stackelberg leader. At first the supplier decides his raw material quality (x_s) and wholesale price (w_s), depending on it the manufacturer decides his final product quality (α) and wholesale price (w_m) and finally the retailer decides his order quantity (Q) and retail price (p). The profit function of the supplier, manufacturer and the retailer are given by

$$\Pi_s(x_s) = (w_s - c_s)q - \eta_s x_s^2 - g_s(1 - x_s) \tag{3.12}$$
$$\Pi_m(\alpha) = (w_m - w_s - c_m)q - \eta_m x_m^2 - g_m(1 - x_m) \tag{3.13}$$
$$\Pi_r(p, Q) = E\{p(Q \wedge D) - s(D - Q)^+ + v(Q - D)^+ - w_m q\}.$$

Now, substituting $z = Q - D(p, x_m)$, like the previous section, in the retailer's profit function and simplifying, we get

$$\Pi_r(p, z) = (p - w_m)[D(p, x_m) + \mu] - (w_m - v)\Lambda(z) - (p + s - w_m)\Theta(z). \tag{3.14}$$

Now, differentiating partially we have

$$\frac{\partial \Pi_r(p, z)}{\partial z} = -(w_m - v) + (p + s - w_m)\big(1 - F(z)\big) \tag{3.15}$$
$$\frac{\partial \Pi_r(p, z)}{\partial p} = -b(p - w_m) + D(p, x_m) + \mu - \Theta(z) \tag{3.16}$$
$$\frac{\partial \Pi_m(\alpha)}{\partial \alpha} = (w_m - w_s - c_m)\beta x_s - 2\eta_m {x_s}^2 \alpha + g_m x_s \tag{3.17}$$
$$\frac{\partial \Pi_s(x_s)}{\partial x_s} = (w_s - c_s)\beta\alpha - 2\eta_m x_s \alpha^2 - 2\eta_s x_s + g_m \alpha + g_s. \tag{3.18}$$

As shown in the previous section, we can easily see that Π_r is concave w.r.t. p and z and for fixed order quantity of the retailer, profit functions of the manufacturer and the supplier are also concave with respect to quality decision parameter (α) and raw material quality (x_s), respectively. Now, the optimal stocking factor of the retailer for given raw material quality and retail price is given by

$$z^d(p) = F^{-1}\left(1 - \frac{w_m + h}{p + s + h}\right) \tag{3.19}$$

and for given z, α and x_s, we have

$$p^d(z, \alpha, x_s) = \frac{a + bw_m + \mu + \beta\alpha x_s - \Theta(z)}{2b}. \tag{3.20}$$

Proposition 3.5. *Retail price in stochastic scenario is always greater than that in deterministic scenario. Also,* $p^d > p^c$.

Proof. We define p^{d0} is the retail price when the demand is certain. Now, we can write

$$p^d = p^{d0} - \frac{\Theta(z)}{2b}, \ \textit{where } p^{d0} = \frac{a + bw_m + \mu + \beta x_s \alpha}{2b}.$$

It is clear from the above that $p^d < p^{d0}$. We can also see that as $p^d > p^c$, $w_m > c$. □

Proposition 3.6. *The optimal stocking factor increases with the retail price.*

Proof is similar to that of Proposition 3.2.

Proposition 3.7. *For known retail price given in equation (3.20), the profit of the system is concave with respect to raw material quality x_s and manufacturer's quality parameter α.*

The proof is omitted as it is similar to that given in the previous section.

Now, the manufacturers product quality decision parameter after finding out the retailer's optimal response is

$$\alpha^d = \frac{2g_m - (c_m - w_m + w_s)\beta}{4x_s\eta_m}\cdot \tag{3.21}$$

The supplier's raw material quality after knowing the retailer's and the manufacturer's responses is

$$x_s^d = \frac{g_s}{2\eta_s}\cdot \tag{3.22}$$

From above, we see that the supplier's raw material quality is the same as that of the decentralized scenario. The difference lies in the manufacturer's quality decision parameter.

3.3. Spanning revenue sharing contract

With spanning revenue sharing contract, the retailer shares a portion of his sales revenue with the manufacturer and the supplier. In exchange, the supplier and the manufacturer charges lower wholesale price for their products. Let ϕ_s and ϕ_m are the portions of sales revenue shared by the retailer with the supplier and the manufacturer, respectively. The retailer keeps the remaining portion (say, $\phi = 1 - \phi_s - \phi_m$) of the revenue, where $0 < \phi < 1$. In this scenario, the supplier's, manufacturer's and the retailer's profit functions are

$$\Pi_s^{rs} = (\phi_s p + w_s - c_s)q - \eta_s x_s^2 - g_s(1 - x_s) \tag{3.23}$$
$$\Pi_m^{rs} = (\phi_m p + w_m - w_s - c_m)q - \eta_m x_m^2 - g_m(1 - x_m) \tag{3.24}$$
$$\Pi_r^{rs} = (\phi p - w_m)[D(p, x_m) + \mu] - (w_m - v)\Lambda(z) - (\phi p + s - w_m)\Theta(z). \tag{3.25}$$

Differentiating partially, we get

$$\frac{\partial \Pi_r^{rs}}{\partial z} = -(w_m - v) + (\phi p + s - w_m)\big(1 - F(z)\big) \tag{3.26}$$
$$\frac{\partial \Pi_r^{rs}}{\partial p} = -(\phi p - w_m)b + \phi D(p, x_m) + \phi\mu - \Theta(z) \tag{3.27}$$
$$\frac{\partial \Pi_m^{rs}}{\partial \alpha} = (\phi_m p + w_m - w_s - c_m)\beta x_s - 2\eta_m {x_s}^2\alpha + g_m x_s \tag{3.28}$$
$$\frac{\partial \Pi_s^{rs}}{\partial x_s} = (\phi_s p + w_s - c_s)\beta\alpha - 2\eta_m x_s\alpha^2 - 2\eta_s x_s + g_m\alpha + g_s. \tag{3.29}$$

Comparing equations (3.5) and (3.6) with (3.28) and (3.29) we get $w_m = \phi c$ and $w_s = c_s - \phi_s c$. The values of w_m and w_s satisfy equation (3.27) but not able to satisfy equation (3.26). So, spanning revenue sharing contract can not be applied in this case.

3.4. Sales rebate and penalty (SRP) with return policy

Under SRP with return contract, the manufacturer sets up a sales target T for the retailer. If the retailer exceeds the sales target T then the manufacturer will give him a rebate: a reward of τ for each unit of product sold above T. Otherwise, the retailer will need to pay the manufacturer a penalty of τ per unit of unsold product below T. In return, the manufacturer and the supplier produce high quality product and raw material which are similar to those of the centralized system. The manufacturer also orders T units of raw material from the supplier. Now, the profit function of the retailer is given by

$$\Pi_r^{\text{srp}} = (p+\tau-w_m)(D(p,x_m)+\mu) - (w_m-r)\Lambda(z) - (p+\tau+s-w_m)\Theta(z) - \tau T \tag{3.30}$$

The manufacturer and the supplier produce their products matching with the qualities of those products in the centralized system.

Proposition 3.8. *In order to achieve similar profit as the centralized system, the manufacturer and the supplier have to set quality decision parameter to α^c and raw material quality to x_s^c respectively and contract parameters must be*

$$\begin{cases} t = w_m - c_m - c_s \\ r = w_m - c_m - c_s + v \end{cases} \tag{3.31}$$

Proof. Taking first order partial derivative of Π_r^{srp} w.r.t. p and z we get

$$\frac{\partial \Pi_r^{\text{srp}}}{\partial z} = -(w_m - r) + (p + t_m + s - w_m)\big(1 - F(z)\big) \tag{3.32}$$

$$\frac{\partial \Pi_r^{\text{srp}}}{\partial p} = -b(p + t_m - w_m) + D(p, x_m) + \mu - \Theta(z). \tag{3.33}$$

Now comparing equations (3.3) and (3.4) with (3.32) and (3.33) we see that t and r satisfies the condition

$$\begin{cases} t = w_m - c_m - c_s \\ r = w_m - c_m - c_s + v. \end{cases}$$

Now putting the values of equation (3.31) into (3.30) we get

$$\begin{aligned} \Pi_r^{\text{srp}}(\alpha^c, x_s^c, r, t) = \Pi_c(\alpha^c, x_s^c, p^c, Q^c) - (w - c_s - c_m)T - \eta_s x_s^{c2} - \eta_m {x_m^c}^2 - g_s(1 - x_s^c) \\ -g_m(1 - x_m^c). \end{aligned} \tag{3.34}$$

In this scenario, the supplier and the manufacturers profit becomes

$$\begin{aligned} \Pi_s(x_s^c) &= (w_s - c_s)T - \eta_s x_s^{c2} - g_s(1 - x_s^c) \\ &= \Pi_c(\alpha^c, x_s^c, p^c, Q^c) - \Pi_r^{\text{srp}}(\alpha^c, x_s^c, r, t) - \Pi_m(\alpha^c) \end{aligned} \tag{3.35}$$

$$\begin{aligned} \Pi_m(\alpha^c) &= (w_m - w_s - c_m)T - \eta_m {x_m^c}^2 - g_m(1 - x_m^c) \\ &= \Pi_c(\alpha^c, x_s^c, p^c, Q^c) - \Pi_r^{\text{srp}}(\alpha^c, x_s^c, r, t) - \Pi_s(x_s^c). \end{aligned} \tag{3.36}$$

Thus we can say, profit can be allocated among supply chain members by varying T. □

4. Numerical analysis

In this section, we illustrate our model by considering the following parameter-values: $a = 500$, $b = 5$, $c_s = 35\$$, $w_s = 50\$$, $c_m = 15\$$, $w_m = 75\$$, $\beta = 1$, $g_s = 7\$$, $\eta_s = 5\$$, $\eta_m = 25\$$, $g_m = 15\$$, $v = 8\$$, $s = 1\$$

TABLE 2. Optimal results under different scenarios.

Optimal decisions	Centralized (c)	Decentralized (d)	SRP contract (srp)
x_s	0.7	0.7	0.7
α	1.37	0.57	1.37
x_m	0.95	0.4	0.95
p	82.83	93.32	82.83
Q	181.62	98.72	181.66
Π_s	–	1476.3	1795.45
Π_m	–	974.23	1176.47
Π_r	–	1285.87	1741.75
Π	4713.67	3736.4	4713.67

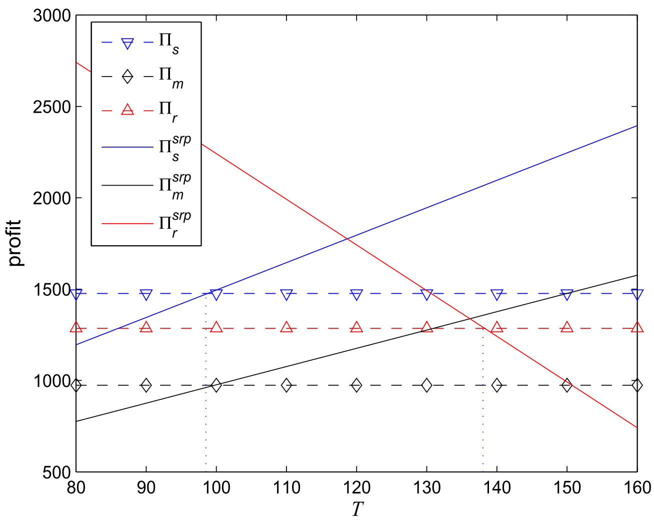
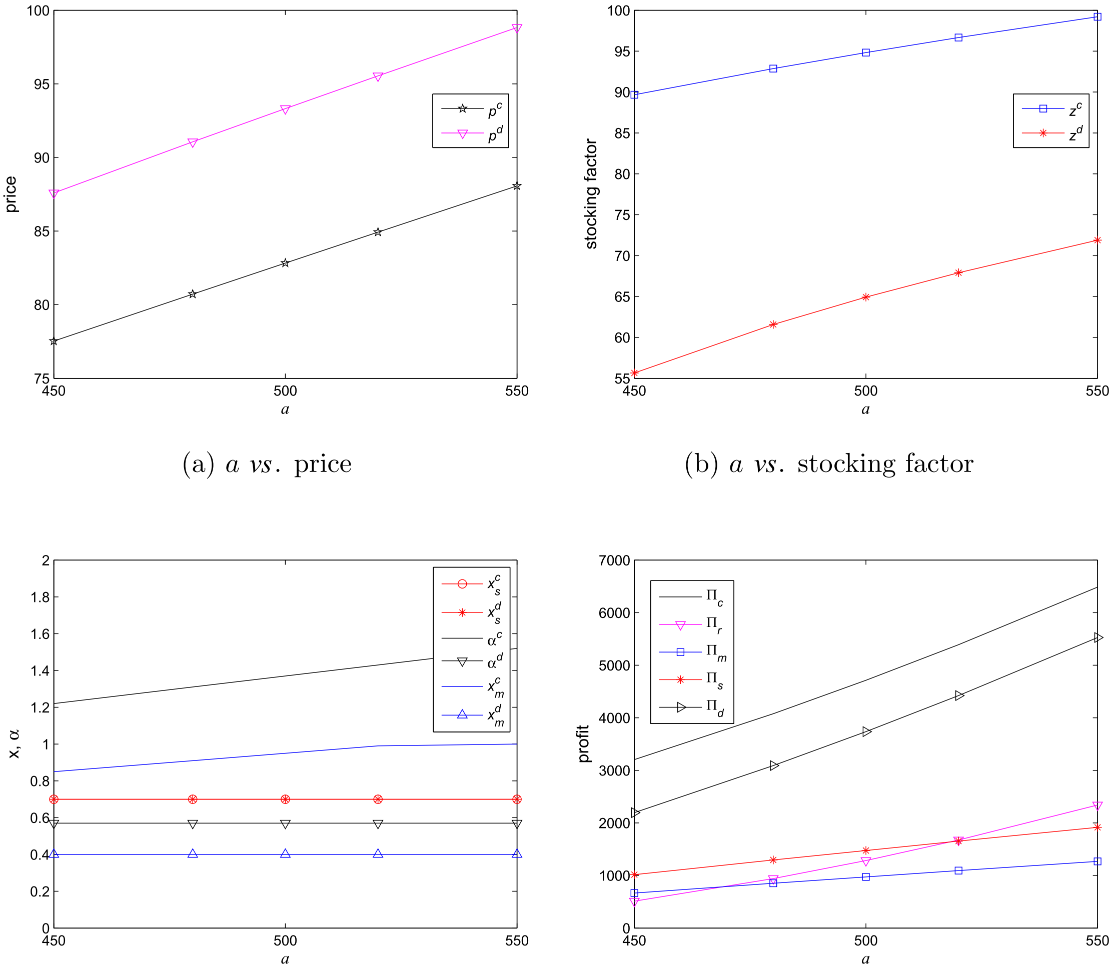


FIGURE 1. Win-win situation for the retailer, manufacturer and the supplier.

and $T = 120$. We also assume that the randomness of the demand follows normal distribution *i.e.*, $f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, x > 0$ with mean $\mu = 100$ and variance $\sigma = 50$. We summarize the results obtained in Table 2.

From Table 2 we see that SRP contract with return policy can achieve the performance of the centralized system which is the benchmark scenario. In decentralized scenario, the total profit of the system is much less than that of the centralized scenario. It is due to the fact that in decentralized scenario, product quality is much lesser and retail price is higher. We also observe that if SRP with return contract is implemented, individual profits of the supplier, manufacturer and the retailer increase and make it a win-win situation for each of them.

From Figure 1 we observe that if we vary the contract parameter T then the total profit of the system varies among the supplier, manufacturer and the retailer accordingly. We also see that there exists a threshold value and limiting value of T. If $T < 98.5$ then the supplier and the manufacturer may not be encouraged to implement the SRP with return contract. In this case, their profits will be lower than those in decentralized scenario. On the other hand, if $T > 138$ then the retailer's profit will be lesser than that in case of decentralized scenario. In this case, the retailer may not be interested in the contract. When $98.5 < T < 138$, everybody will benefit from this contract. Now we analyze the affect of changing various parameter-values on the optimal results.



(a) a *vs*. price

(b) a *vs*. stocking factor

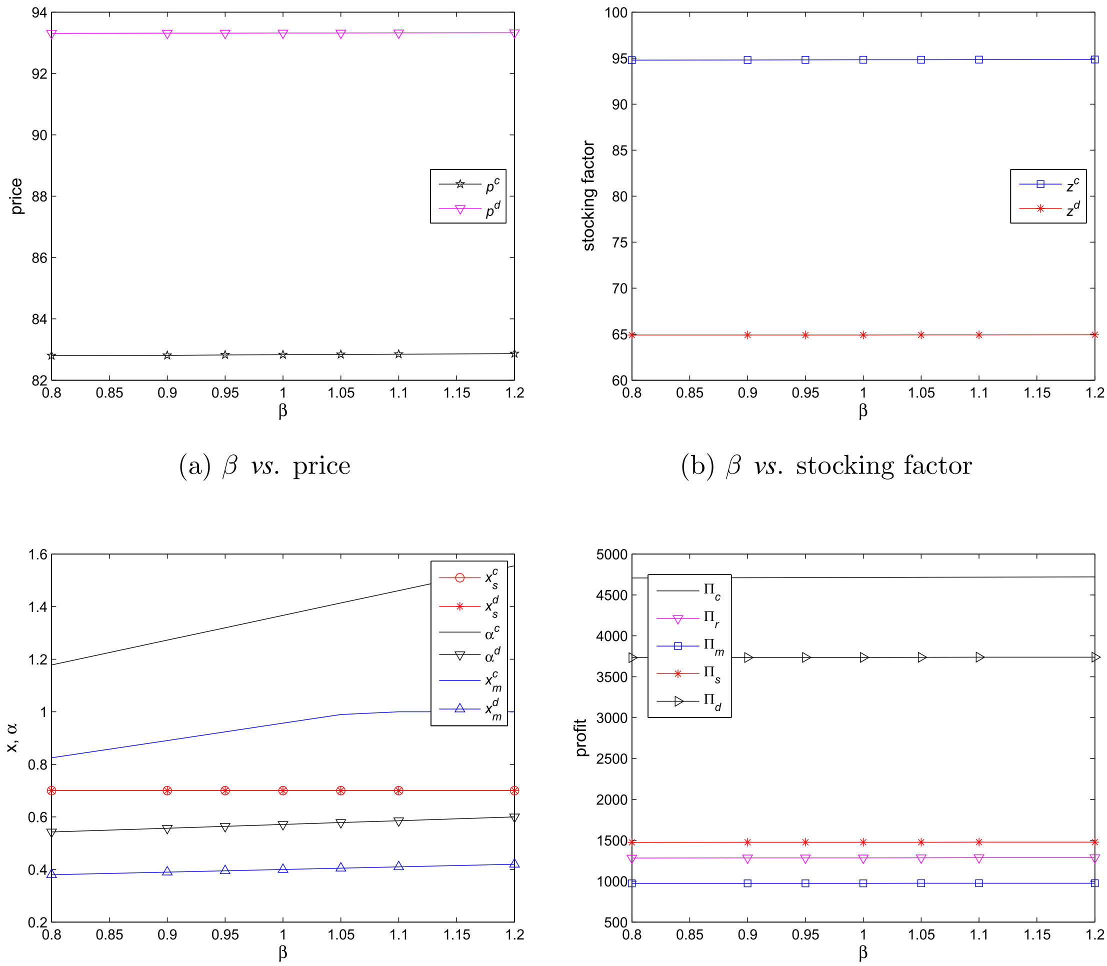
(c) a *vs*. quality decisions

(d) a *vs*. profit

FIGURE 2. Sensitivity of fixed part of demand on the optimal decisions.

Figures 2–4 depict the dependency of retail price, raw material quality, manufacturer's quality decision parameter with the change in deterministic part of market demand (a), retail price sensitivity parameter (b) and quality sensitivity parameter (β), respectively. Figures 5 and 6 show the dependency of the supplier's raw material quality and the manufacturer's quality decisions over the loss of goodwill and product improvement cost.

(i) With the increase in fixed part of the market demand (a), retail price, stocking factor and profit of the system increase (see Fig. 2(a)–(c)). When the market is large, the retailer has to stock more; he/she may be encouraged to increase the retail price. As demand is higher in this case, negative effect of higher pricing is nullified by the higher market demand and supply chain members achieve higher overall profit. Raw material quality remains unchanged in both scenarios and finished product quality increases as quality decision parameter increases with the increasing market in the centralized scenario (Fig. 2(d)). The finished product quality can be improved upto a certain level. After reaching that point, it is meaningless for the manufacturer to increase his/her quality decision parameter further. In decentralized scenario, as the market



(a) β *vs.* price

(b) β *vs.* stocking factor

(c) β *vs.* quality decisions

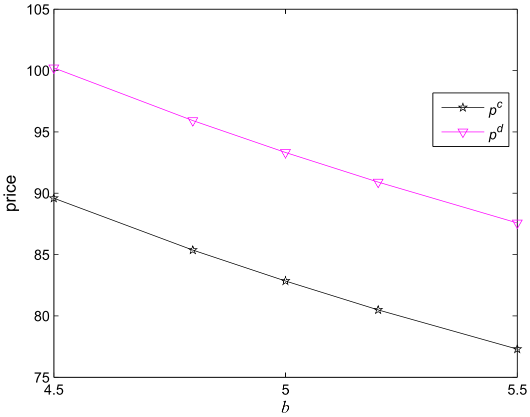
(d) β *vs.* profit

FIGURE 3. Sensitivity of the parameter β on the optimal results.

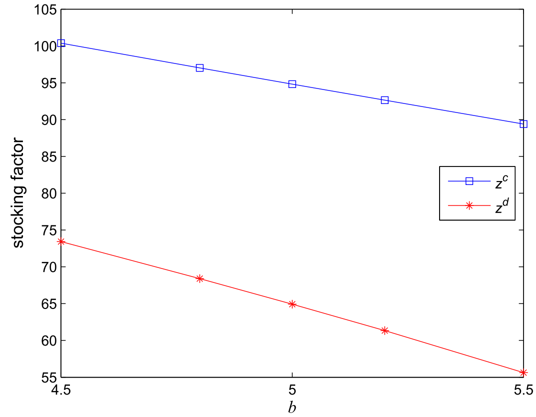
size has no direct effect on the manufacturer or the supplier, raw material quality of the supplier and quality decision parameter of the manufacturer remain fixed.

(ii) With the increase in quality sensitivity parameter β, retail price and stocking factor of the retailer remain almost same (see Fig. 3(a) and (b)) and raw material quality of the supplier remains unchanged but quality decision parameter of the manufacturer increases steadily (see Fig. 3(c)). To cope with the higher quality dependency of demand, the manufacturer has to improve product quality. We also see that in decentralized scenario, as the manufacturer is less affected by the changing nature of market demand, this increase in quality decision parameter is lesser. Here also, there exists a limiting product quality. When the quality reaches that limit, the manufacturer do not invest to improve product quality. In the decentralized scenario, this limiting value is much higher. Figure 3(d) shows slight increase in profit of the system in each scenario. As the effect of quality is less on profit, change in profit is also lesser.

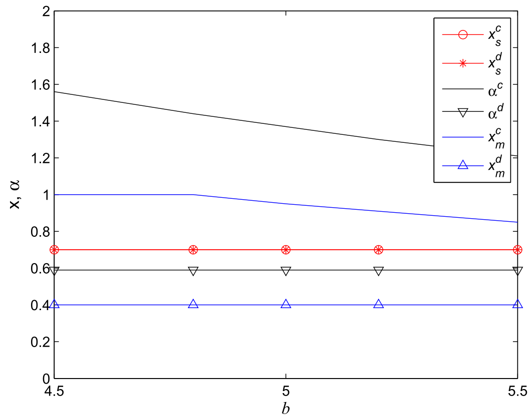
(iii) Increase of price dependency parameter b results in reduced retail price, stocking factor and profit for each scenario (see Fig. 4(a)–(d)). As the demand decreases with the increasing price dependency, the



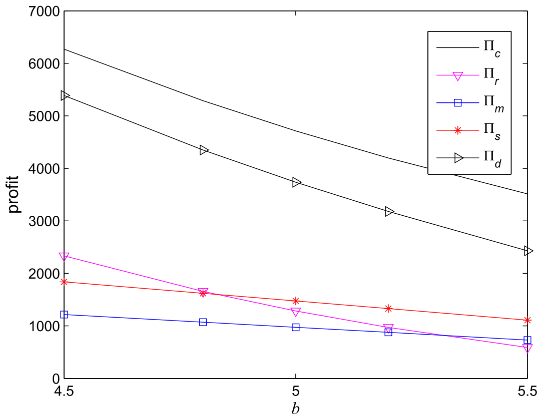
(a) b *vs.* price



(b) b *vs.* stocking factor



(c) b *vs.* quality decisions



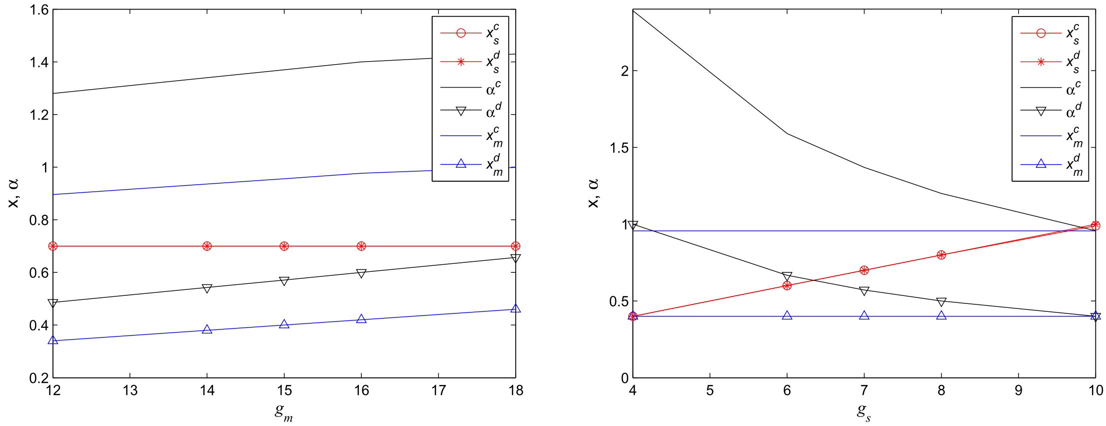
(d) b *vs.* profit

FIGURE 4. Sensitivity of the parameter b on the optimal results.

retailers decisions and the profit of each member of supply chain decrease. Here also raw material quality remains unchanged but the quality decision parameter decreases. As a result, product quality decreases. (see Fig. 4(c)).

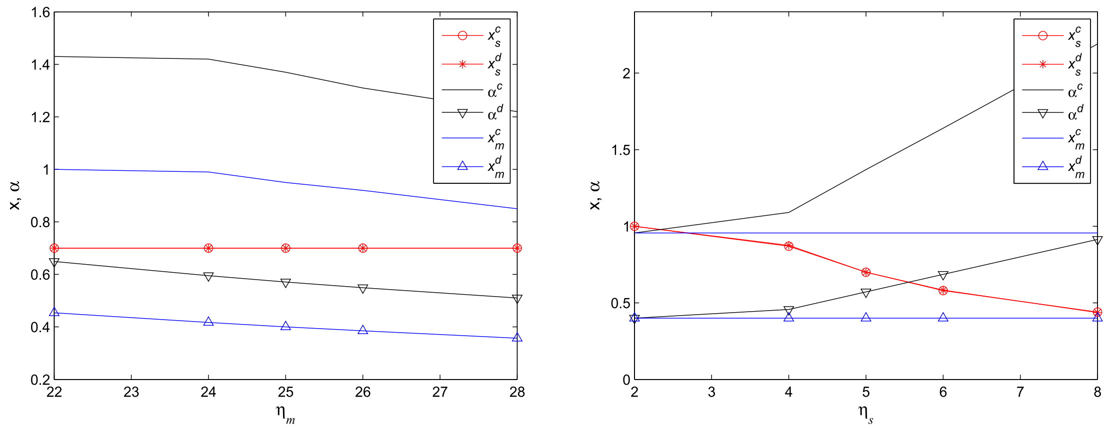
(iv) Figure 5(a) and (b) describe that the supplier is immune to the manufacturer's goodwill loss cost. The manufacture has to improve the end product on its own to keep reputation in the market. On the other hand, higher goodwill loss cost encourages the supplier to produce high quality raw materials. In such situations, the manufacturer has to put less effort to maintain same quality products.

(v) Figure 6(a) and (b) show that, if the manufacturer's quality improvement cost increases, he/she becomes reluctant to improve product quality further. As a result, overall product quality goes down. Again, in case of the supplier higher cost means lower raw material quality, the manufacturer has to improve more to retain certain product quality.



(a) g_m *vs.* quality decisions (b) g_s *vs.* quality decisions

FIGURE 5. Sensitivity of goodwill loss cost on quality aspect.



(a) η_m *vs.* quality decisions (b) η_s *vs.* quality decisions

FIGURE 6. Sensitivity of quality improvement cost on quality aspect.

5. MANAGERIAL IMPLICATION AND CONCLUSION

The managerial aspects of the proposed supply chain model are undeniable. In our model, the supplier and the manufacturer have to maintain balance between cost and quality. The retailer has to predict market demand and stock accordingly. He/She has to set their pricing right for maximizing his/her profit. In our model, we try to depict a picture how the supplier and the manufacturer can optimize their product quality and analyze various aspects which may affect their product quality. Our observations show that the upstream members of the supply chain face less heat from the changing nature of the market than their downstream counterparts. The supplier can produce raw materials, balancing its reputation and cost. The manufacturer has to monitor the supplier and the retailers' decisions and play accordingly. The supplier and the manufacturer are less susceptible to market vulnerabilities due to their upstream positions in the supply chain. In this paper, we try to analyze, what is the best pricing and stocking amount of the retailer. Moreover, we try to implement a contract to coordinate the supply chain. A modified sales rebate and return contract makes the situation more profitable for each party. Also, it can match the centralized system in profits.

There are very few works present in the existing literatures which addresses the issue of raw material quality as well as the finished product quality in a supply chain model. So, there are plenty of rooms for future work in this direction. In this model, we have discussed linear demand scenario which can be extended to a non-linear demand scenario. We only consider a raw material supplier, a manufacturer and a retailer for trading a single product. It can be further generalized by incorporating multiple suppliers, manufacturers or retailers instead of one. Multiple products also can be a viable option for future studies. One can also generalize this model by taking multi-period scenario instead of single-period. Disruption or yield uncertainty issues are not addressed in this model which can be a topic of future interest.

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