

A ROBUST POSSIBILISTIC PROGRAMMING MODEL FOR WATER ALLOCATION PROBLEM

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Abstract. Over the past few years, water allocation problem has increasingly spotlighted by governments, researchers and practitioners. As water plays an important role in people's life and business environment, the problem of water allocation should be considered carefully to properly satisfy demand of water consumers. In the real world applications, problems like water allocation are uncertain owing to long-term planning horizon of such problems. Therefore, employing efficient methods for tackling uncertainty of parameters should be regarded by field researchers. In this regard, this paper proposes a bi-objective mathematical programming model for water distribution network design. The extended model maximizes total profit of water distribution as well as maximizing priority of water transferring among water customer zones. Then, to cope effectively with uncertainty of parameters, a novel robust possibilistic programming method is applied. Then, fuzzy and robust fuzzy programming models are compared against each other and output results confirm superiority and effective performance of the robust fuzzy model in the water allocation problem. Also, output results of the extended model show its accurate performance that results in applicability of the model as a strong planning tool in real world cases.

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1. INTRODUCTION

Water resources allocation is a complex problem that has only become more important in the recent years due to population growth and climate changes. Water allocation could be defined as process of delivering different types of waters (*i.e.*, drinking, non-drinking) from different water resources to different demand zones (*i.e.*, agricultural, rural, urban, industrial) [10, 45]. During the recent decades, field experts have warned impeccable war of water owing to water resources scarcity. The noted matter is heightened by political changes, resources mismanagement and climatic anomalies [28]. These factors can trigger massive upheavals as well as demographic and developmental transformations, all of which, in turn, contribute to significant socio-economic differentiations. Also, the constantly increasing demand for water in terms of both sufficient quantity and satisfying quality has forced planners to contemplate comprehensive, complex and ambitious plans for water resources management systems [13, 24]. In this regard, proper allocation of water resources to different consumers at different

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regions of the countries is an important issue that can prevent communities from increasing population intensity in big cities and also helps to sustainable development of countries.

Most of the water companies are dependent on budget assigned by governments because governments subsidize water companies to serve consumers on-time and with the highest quality of delivered water. Accordingly, water companies should set their long-term plans based on allocated budget that could be regarded as a big limitation on the water resources allocation plans. In this regard, most of the researchers have strived to minimize total costs of water allocation problem in the presented models (see [15, 18, 29]). However, optimization of amount of allocated budget to each zone is an important subject that is disregarded by field researchers. Other important issue is that cost minimization is just related to economic aspects of water allocation problem. Although, there are other performance measures in the water allocation problem that can improve quality of provided services for consumers. Water leakage minimization, minimization of greenhouse gas emissions and energy usage are some of objective functions employed in design of municipal water supply systems beside cost minimization (see [40, 41]). Notably, different regions of the countries need different types of waters. For example, industrial zones need more non-drinking water in comparison to municipal demand zones [5]. Therefore, priority of distributing different types of waters in divergent regions of the countries is different. Accordingly, designing a water resource allocation model that enables company decision makers to allocate water resources to demand zones based on their priority could be regarded as an interesting research line. It could result in applicability of output results in real world water allocation cases and prevents from immigration of people to big cities.

The notable matter in water allocation problem is that managers of water companies should plan water allocation for long-term horizons. However, climate changes and changing water consumption trend result in uncertainty of planning problem as well as unreliability of output decisions [2, 12, 29]. It should be noted that uncertainty is an undeniable part of water allocation problem owing to wasteful consumption of water resources and their inaccurate allocation to consumer zones. Disregarding uncertainties in the design of water distribution networks could result in customer dissatisfaction, unsatisfied demand and waste of water resources [26, 28]. Therefore, effective modelling of uncertainties in the water allocation problem could stop great losses of companies and preserves water resources for future generations. Also, appropriate usage and priority-based allocation of water resources could improve social and economic aspects of life in different municipal regions and stops environmental damages caused by inaccurate water allocation plans.

Many researchers have strived to model uncertainty of parameters in the water resource allocation scope. One of the most used methods for controlling uncertainty of parameters is stochastic programming. This method employs scenario-based programming method to model uncertain parameters. Each scenario is defined based on available historical data and their corresponding probability distribution [1, 4, 11]. As an example, Watkins *et al.* [47] developed a stochastic programming model to manage water supplies from highland lakes to lower Colorado River. The formulated model maximizes expected profit of selling water to customer zones as well as maximizing recreational benefit of selling water. They employed benders decomposition approach to solve the proposed two-stage stochastic programming model. Li *et al.* [22] proposed a hybrid interval-parameter multi-stage stochastic programming model for water resources management. They analyzed different uncertainty scenarios to achieve the best water distribution policy. Housh *et al.* [15] proposed a novel limited stochastic programming method for water distribution network planning. They applied a new scenario reduction approach to decrease number of uncertainty scenarios. Li and Guo [21] rendered a scenario-based stochastic programming model for water resources allocation in China. They hybridized probabilistic chance-constrained programming, semi-infinite programming, integer programming and interval linear programming methods to effectively cope with imprecision of parameters. Total profit of selling water resources and total environmental and social effects of water distribution networks on societies are maximized in the presented model. Xu *et al.* [48] presented a multi-period reverse supply chain network based on case study of collecting solid wastes in Dalian. They minimized costs of network design to collect solid wastes in a cost efficient manner. They proposed landfill, incineration and composting as strategic choices to minimize adverse environmental impacts of solid wastes. To cope with uncertainty of parameters and control risk-aversion of output decisions, they have suggested a robust possibilistic programming approach. The rendered model minimizes the gap between the best and the worst

values of objective function to control optimality robustness of the output results. Notably, in such network design problems, deviations of uncertain parameters could result in higher values of objective function and the biggest problem is that objective function is minimized based on expected value of uncertain parameters. Therefore, to control optimality robustness of results, the gap between the worst value of the objective function and its expected value should be minimized that is the weakness of the proposed model. Naderi and Pishvaei [28] proposed a two stage stochastic programming model to manage water and wastewater supply systems. They applied benders decomposition approach to solve the rendered complex optimization model. Notably, using stochastic programming approach has some deficiencies. Most of the time, enough historical data is not available in industrial cases. Therefore, defining different scenarios is not possible owing to insufficiency of historical data. Also, even if historical data is available, defining lots of uncertainty scenarios is a time-consuming and difficult task. Additionally, enhancement of number of uncertain parameters heightens the noted obstacles [14, 36].

Possibilistic programming (PP) method is another applicable approach for controlling ambiguity of parameters (see [9, 33, 39, 49]). Fuzzy programming applies possibility distribution of uncertain parameters and their related fuzzy membership function to model imprecise parameters of the model. Prominent points of membership function could be determined based on opinion of managers and company decision makers in the fuzzy programming approaches [35, 46]. In this regard, the need to historical data would be solved via using PP approaches. Many researchers have strived to model uncertainty of parameters in water allocation problem via applying the PP method. As an example, Jairaj and Vedula [16] presented a multi-reservoir water allocation model. They used fuzzy set theory to control uncertainty of parameters. Notably, total deviations of water irrigation withdrawals are minimized as objective function of the suggested model. Li *et al.* [23] proposed a multi-stage fuzzy stochastic programming model to cope with imprecision of parameters in the water allocation problem. The formulated model enables decision makers to analyze various uncertainty scenarios and find the best water distribution policy. Lu *et al.* [25] suggested an interval-based fuzzy programming model for managing water distribution systems. Effectiveness of the presented model is analyzed based on case study of managing agricultural water distribution system. Shi *et al.* [41] proposed a two-phase programming method to manage water resources in a rural regions of China. A fuzzy inexact interval programming method is employed to effectively tackle uncertainty of parameters. Shibu and Reddy [42] proposed a bi-objective programming model for water allocation problem. They applied theory of fuzzy random variables to cope with uncertainty of parameters. Total costs of water distribution network are minimized besides maximizing reliability of network. Noteworthy, fuzzy programming methods have some deficiencies that can adversely affect their accurate performance. Firstly, fuzzy programming methods need time consuming simulations to determine minimum satisfaction level of uncertain parameters. In this regard, decision makers should change satisfaction levels up to finding satisfactory outputs based on their opinion. However, output results of the PP method are not guaranteed to be optimal. Reason of the noted matter is assignment of value of satisfaction levels based on opinion of decision makers that could be full of human errors. Notably, increasing the number of uncertain parameters could heighten toughness of assigning minimum satisfaction levels [34, 37]. Using robust programming could help to control risk-aversion level of output results (see [6, 29]). Robust programming method tends to optimize risk-aversion of output decisions based on preferences of company managers [27]. Therefore, hybridizing possibilistic and robust programming methods that results in extension of a robust possibilistic programming model is an important issue that enables decision makers to control output results and makes them applicable in most of the industrial cases.

With regard to enumerated matters, this paper aims to extend a robust fuzzy programming model to maximize total profit of water transferring and priority of water transferring to demand zones. Notably, the extended robust fuzzy programming model is capable of controlling risk-aversion of output decisions of the model based on opinion of field experts and company decision makers. The model will demonstrate how robust credibility-based fuzzy chance constrained programming model can be used to obtain a risk-averse optimal solutions in the water allocation problem regarding different robustness measures called optimality robustness and feasibility robustness. The other important aim of the extended model is designing a water allocation model that is able

to optimizing assigned budget to each region to establish water distribution infrastructures and improve fair distribution of water resources between different municipal consumer regions.

This paper is organized as follows. Section 2 presents problem definition and formulation of the proposed model. To cope with uncertainty of parameters, the robust possibilistic water allocation model is extended in Section 3. Section 4 corresponds to implementation and evaluation of the proposed robust fuzzy programming model. Some managerial implications are rendered in this section. Finally, conclusions and some future research guidelines are presented in Section 5.

2. PROBLEM DEFINITION AND MODEL FORMULATION

In this section, a comprehensive problem definition is presented to formulate the proposed water allocation network design model. Also, the real world oriented assumptions that are applied to formulate objective functions and constraints of the model are rendered. Finally, based on the presented nomenclatures (*i.e.*, indices, parameters and decision variables), model formulation of the water allocation problem is presented.

2.1. Problem definition

In the presented model, different types of water resources (*i.e.*, potable, non-potable, sweet, salty and wastewater) should be transferred to different demand zones (*i.e.*, municipal, industrial and agricultural). Each demand zone has a predefined priority for different types of waters. Therefore, amount of transferred water from different reservoirs to different regions should be determined based on the water transferring costs and priority of consumers. Aim of the extended network is maximizing total profit of water transferring as well as satisfying demand of consumers regarding their priority for water usage. Notably, water demand can be conditioned based on different uses and preference of consumers. Modern presented approaches for water allocation are mostly founded on complex rules to deal with available variability in such distribution networks and balancing the environmental, social, political and economic implications of different water allocation scenarios. Rather than a simple set of fixed rules, modern water allocation plans include different scenarios that project how water usage may affect climate change, shifting economies, water pricing incentives and options to share the benefits of water usage. Therefore, caring about priority of water transferring to user zones is an important issue that is modeled as a new objective function in this paper. Figure 1 shows various stages in water allocation process. Notable issue that affect power of water companies for water transferring and satisfying priority of consumers is budget assignment to different consumer zones. In this regard, the extended model determines amount of assigned budget to each demand region and also prohibits violation of total assigned budget from total available budget.

Based on the enumerate matters, the extended bi-objective model optimizes total amount of water transferred to different demand zones. Also, assigned budget to different regions for water transferring is optimized in the extended model. Total profit of water transferring to demand zones should be maximized as well as maximizing priority of water transferring to demand regions.

2.2. Model formulation

Based on the presented problem definition, Following nomenclatures are defined to formulate the water allocation model. Imprecise parameters are presented with a tilde on.

2.2.1. Indices

- i Set of available resources for water harvesting ($i = 1, \dots, I$)
- j Set of applicant cities for water ($j = 1, \dots, J$)
- k Set of variety of consumed water (potable, non-potable, sweet, salty, wastewater, etc.) ($k = 1, \dots, K$)
- f Set of manufactured products ($f = 1, \dots, F$).

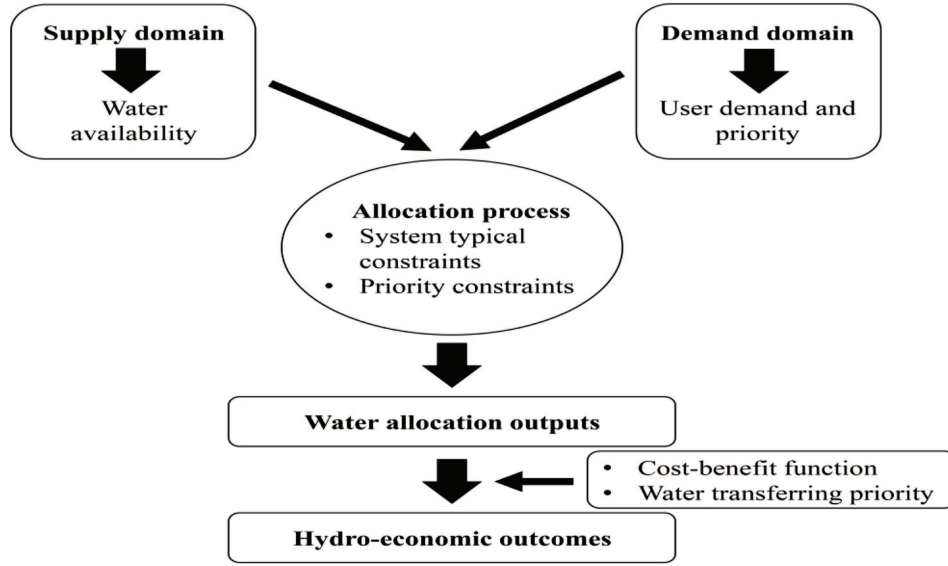


FIGURE 1. Graphical representation of water allocation process.

2.2.2. Parameters

$\tilde{D}_{j k f}$	Demand of city j from water type k for product f
$A_{j k f}$	Priority of city j from water type k for product f
$\widetilde{\text{Cap}}_{i k}$	Capacity of reservoir i for water type k
$\tilde{C}_{i j}$	Cost of water transferring from reservoir i to city j
$\text{Max} B$	Maximum available budget
S_f	Amount of product f that is harvested per liter of consumed water
\tilde{L}_f	Price of product f
\aleph	Percentage of capacity of each reservoir that is transferable to customer zones.

2.2.3. Decision variables

$X_{i j k f}$	Amount of water type k for produce product f that is transferred from reservoir i to city j
B_j	Amount of budget assigned to the city j .

2.2.4. Objective function and constraints

Regarding the presented sets, parameters and decision variables, the water allocation model could be formulated as follows.

$$\text{Max } Z_1 = \sum_i^I \sum_j^J \sum_k^K \sum_f^F S_f \cdot \tilde{L}_f \cdot X_{i j k f} \quad (2.1)$$

$$\text{Max } Z_2 = \sum_i^I \sum_j^J \sum_k^K \sum_f^F ((J.K.F) + 1 - A_{j k f}) \cdot X_{i j k f} \quad (2.2)$$

$$\text{s.t. } \sum_i^I X_{i j k f} \geq \tilde{D}_{j k f} \quad \forall j, k, f \quad (2.3)$$

$$\sum_j^J \sum_f^F X_{i j k f} \leq \aleph \left(\widetilde{\text{Cap}}_{i k} \right) \quad \forall i, k \quad (2.4)$$

$$\sum_i^I \sum_k^K \sum_f^F \tilde{C}_{ij} \cdot X_{ijkf} \leq B_j \quad \forall j \quad (2.5)$$

$$\sum_j^J B_j \leq \text{Max}B \quad (2.6)$$

$$X_{ijkf} \geq 0 \quad \forall i, j, k, f \quad (2.7)$$

$$B_j \geq 0 \quad \forall j \quad (2.8)$$

Objective function (2.1) maximizes total profit of water transferring to demand zones. Objective function (2.2) maximizes priority of water transferring with regard to priority of each demand region. It is noteworthy that regions with higher priority have lower priority number in the objective function (2.2). Constraint (2.3) assures that the amount of transferred water to each demand region should be greater than or equal to the demand of that region. Constraint (2.4) guarantees that maximum amount of transferable water to each demand region, should be less than or equal to maximum storage capacity level of water resources. Notably, at each water resource region, owing to environmental regulations, a predefined percent of capacity of water resources could be transferred to demand zones. Constraint (2.5) determines amount of assigned budget to each demand zone with regard to volume of the needed water at each demand region. As objective function (2.1) maximizes total profit of water distribution network and it depends on amount of transferred water to consumer zones, and also with regard the limited available budget, constraint (2.5) determines the necessary budget for transferring water to each demand region. Constraint (2.6) assures that total budget assigned to different cities for water transferring should be less than or equal to maximum available budget. Constraints (2.7) and (2.8) represent non-negative decision variables of the model.

3. PROPOSED ROBUST FUZZY MATHEMATICAL PROGRAMMING MODEL

Uncertainty is an inseparable part of supply chains owing to variable condition of routine life. In this regard, there are two types of uncertainties that can affect accuracy of output decisions of extended models in the supply chain network design scope. The first type of uncertainty is related to ambiguity of parameters of models. PP models could be applied to cope with this kind of uncertainty (see [7, 17, 38, 43]). The second kind of uncertainty is related to flexibility of constraints and target value of objective functions. Flexible programming models could be employed to tackle flexibility of soft constraints (see [32, 34, 44]). Uncertain parameters of water allocation planning problem belong to the first type of uncertainty and accordingly they could be modeled via opinion of decision makers and insufficient available historical data. However, as mentioned by Pishvaei *et al.* [37] and Pishvaei and Fazli Khalaf [34], PP methods have some deficiencies and are not enough intelligent to automatically adjust risk-aversion degree of output decisions. In other words, they are not capable of controlling robustness of output decisions. Extending robust possibilistic programming models could help to optimize risk-aversion level of output decisions and also solve different weaknesses of possibilistic programming methods.

Robust programming methods are capable of finding risk-averse results based on opinion of field experts. Robustness concept could be defined in two separate parts of mathematical programming models. In other words, “feasibility robustness” and “optimality robustness” are two important key factors that adjust risk-aversion level of outputs decisions in mathematical programming models that include uncertain parameters. Feasibility robustness means that optimal decision variables are most of the time near to optimal condition regarding different uncertainty scenarios. It minimizes total infeasibility and violation of constraints comprising uncertain parameters. In other words, value of uncertain parameters could be changed and accordingly there is possibility of constraint violations. In this regard, feasibility robustness helps to risk-averse adjustment of output decisions in network design problems. Optimality robustness means that value of objective function is almost kept near its optimal value regarding different uncertainty scenarios [34, 37]. In other words, it controls deviations of objective function. Based on the enumerated matters, robust programming methods could be classified into

three types (*i.e.*, hard worst case, soft worst case and realistic robust programming). Hard worst case robust programming methods seek to create maximum risk-aversion in output decisions. In other words, worst value of parameters would be regarded in objective function and constraints to find optimum value of decision variables of the model. Hard worst case robust programming approach is applicable in conditions that any change in decisions makes great losses for companies [3, 8]. Soft worst case robust programming method adds capability of controlling constraints violations to mathematical programming models. However, objective function should be optimized based worst value of available imprecise parameters. Hard worst case robust programming approach is a limited version of soft worst case robust programming method (see [19, 30]). Realistic robust programming methods have capability of constraints and objective function violations. In other words, level of risk-aversion of output decisions could be optimized based on risk-aversion level of company decision makers. Noteworthy, soft and hard worst case robust programming methods are special cases of robust realistic programming method (see [20, 31]). In this regard, inspired by robust possibilistic programming models presented by Pishvae and Fazli Khalaf [34], a new robust credibility-base fuzzy chance constrained programming model is proposed in this paper via hybridizing robust realistic programming and PP methods. The extended model is capable of optimizing risk-aversion level of output decisions based on opinion of field experts.

3.1. Possibilistic programming model

In this section, credibility-base fuzzy chance-constrained programming model extended by Pishvae *et al.* [37] is applied to cope with uncertainty of parameters in objective function and constraints of the model. It is assumed that parameters tainted with uncertainty have trapezoidal possibility distribution and value of their four prominent points could be determined by opinion of field experts and via using insufficient existing data. In this method, expected value operator and chance constrained modelling method are employed to model uncertain parameters of objective function and constraints, respectively. With regard to the noted matters, equivalent crisp model could be rendered as follows:

$$\text{Max } Z_1 = \sum_i^I \sum_j^J \sum_k^K \sum_f^F S_f \cdot \left(\frac{L_f^{(1)} + L_f^{(2)} + L_f^{(3)} + L_f^{(4)}}{4} \right) \cdot X_{ijkf} \quad (3.1)$$

$$\text{Max } Z_2 = \sum_i^I \sum_j^J \sum_k^K \sum_f^F ((J.K.F) + 1 - A_{jkf}) \cdot X_{ijkf} \quad (3.2)$$

$$\text{s.t. } \sum_i^I X_{ijkf} \geq \left[(2 - 2\alpha_1) D_{jkf}^{(3)} + (2\alpha_1 - 1) D_{jkf}^{(4)} \right] \quad \forall j, k, f \quad (3.3)$$

$$\sum_j^J \sum_f^F X_{ijkf} \leq \left[(2\alpha_2 - 1) \text{Cap}_{ik}^{(1)} + (2 - 2\alpha_2) \text{Cap}_{ik}^{(2)} \right] \quad \forall i, k \quad (3.4)$$

$$\sum_i^I \sum_k^K \sum_f^F \left[(2 - 2\alpha_3) C_{ij}^{(3)} + (2\alpha_3 - 1) C_{ij}^{(4)} \right] \cdot X_{ijkf} \leq B_j \quad \forall j \quad (3.5)$$

$$\sum_j^J B_j \leq \text{Max}B \quad (3.6)$$

$$X_{ijkf} \geq 0 \quad \forall i, j, k, f \quad (3.7)$$

$$B_j \geq 0 \quad \forall j. \quad (3.8)$$

Where parameters $\alpha_1, \alpha_2, \alpha_3$ are minimum satisfaction levels of chance constraints comprising uncertain parameters that their value should be subjectively determined based on opinion of decision makers and their

risk-averseness level (*i.e.*, $0.5 < \alpha_1, \alpha_2, \alpha_3 \leq 1$). Interactive adjustment of satisfaction levels is a time-consuming procedure that relies on experience and dexterity of decision makers. In other words, value of satisfaction levels would be adjusted with regard to output objective function value. Even, final chosen value of satisfaction levels could not be regarded as the best satisfaction levels owing to complexity of adjusting the value of satisfaction levels in interactive manner. Also, objective function (3.2) is modeled based on expected value of uncertain parameters. It is not capable of controlling deviations over and under planned expected objective value that makes output results unreliable. Noted matters are deficiencies of credibility-base fuzzy chance-constrained programming model which could lead to poor performance of the extended PP method. Applying robust programming concepts could help to solve aforementioned gaps and extend an intelligent model that is capable of concurrent controlling of feasibility and optimality robustness of the extended model. In this regard, a robust credibility-base fuzzy chance-constrained programming model is proposed in following section.

3.2. Proposed robust credibility-base fuzzy chance constrained programming model

Robust programming methods seek to achieve to two important goals: feasibility and optimality robustness. Aim of feasibility robustness is finding outcome feasible solutions for almost all possible values of parameters tainted with uncertainty. Optimality robustness concept seek to to keep objective functions' value near to optimal for almost all possible values of ambiguous parameters. Application of aforementioned concepts in extended PP model could eliminate its imperfections. Accordingly, with regard to presented concepts and inspired by robust fuzzy programming models presented by Pishavee and Fazli Khalaf [34], robust credibility-base fuzzy chance-constrained programming model could be proposed as follows:

$$\begin{aligned} \text{Max } Z = & \sum_i^I \sum_j^J \sum_k^K \sum_f^F S_f \cdot \left(\frac{L_f^{(1)} + L_f^{(2)} + L_f^{(3)} + L_f^{(4)}}{4} \right) \cdot X_{ijkf} \\ & - \pi \left[\sum_i^I \sum_j^J \sum_k^K \sum_f^F S_f \cdot \left(\frac{L_f^{(1)} + L_f^{(2)} + L_f^{(3)} + L_f^{(4)}}{4} \right) \cdot X_{ijkf} - \sum_i^I \sum_j^J \sum_k^K \sum_f^F S_f \cdot L_f^{(1)} \cdot X_{ijkf} \right] \\ & - \gamma \left[D_{jkf}^{(4)} - \left\{ (2 - 2\alpha_1) D_{jkf}^{(3)} + (2\alpha_1 - 1) D_{jkf}^{(4)} \right\} \right] \\ & - \mu \left[\left\{ (2\alpha_2 - 1) \text{Cap}_{jk}^{(1)} + (2 - 2\alpha_2) \text{Cap}_{jk}^{(2)} \right\} - \text{Cap}_{jk}^{(1)} \right] \\ & - \theta \left[C_{ij}^{(4)} - \left\{ (2 - 2\alpha_3) C_{ij}^{(3)} + (2\alpha_3 - 1) C_{ij}^{(4)} \right\} \right] \end{aligned} \quad (3.9)$$

$$\text{s.t.} \quad \sum_i^I X_{ijkf} \geq \left[(2 - 2\alpha_1) D_{jkf}^{(3)} + (2\alpha_1 - 1) D_{jkf}^{(4)} \right] \quad \forall j, k, f \quad (3.10)$$

$$\sum_j^J \sum_f^F X_{ijkf} \leq \aleph \left[(2\alpha_2 - 1) \text{Cap}_{ik}^{(1)} + (2 - 2\alpha_2) \text{Cap}_{ik}^{(2)} \right] \quad \forall i, k \quad (3.11)$$

$$\sum_i^I \sum_k^K \sum_f^F \left[(2 - 2\alpha_3) C_{ij}^{(3)} + (2\alpha_3 - 1) C_{ij}^{(4)} \right] \cdot X_{ijkf} \leq B_j \quad \forall j \quad (3.12)$$

$$\sum_j^J B_j \leq \text{Max} B \quad (3.13)$$

$$0.5 < \alpha_1, \alpha_2, \alpha_3 \leq 1 \quad (3.14)$$

$$X_{ijkf} \geq 0 \quad \forall i, j, k, f \quad (3.15)$$

$$B_j \geq 0 \quad \forall j. \quad (3.16)$$

TABLE 1. Data generation style.

Parameter	Data generation style	Unit
$D_{j,k,f}^{(1)}$	$U [20, 50]$	thousand m ³
$D_{j,k,f}^{(2)}, D_{j,k,f}^{(3)}, D_{j,k,f}^{(4)}$	$D_{j,k,f}^{(1)} + U [10, 25] \forall \mathfrak{I} = 2, 3, 4$	thousand m ³
$\text{Cap}_{i,k}^{(1)}$	$U [1000, 3500]$	thousand m ³
$\text{Cap}_{i,k}^{(2)}, \text{Cap}_{i,k}^{(3)}, \text{Cap}_{i,k}^{(4)}$	$\text{Cap}_{i,k}^{(1)} + U [150, 500] \forall \mathfrak{I} = 2, 3, 4$	thousand m ³
$C_{i,j}^{(1)}$	$U [5, 20]$	Thousand
$C_{i,j}^{(2)}, C_{i,j}^{(3)}, C_{i,j}^{(4)}$	$C_{i,j}^{(1)} + U [5, 15] \forall \mathfrak{I} = 2, 3, 4$	Thousand
$L_f^{(1)}$	$U [10, 30]$	Monetary unit
$L_f^{(2)}, L_f^{(3)}, L_f^{(4)}$	$L_f^{(1)} + U [5, 20] \forall \mathfrak{I} = 2, 3, 4$	Monetary unit

Where the first term of objective function (3.9) maximizes expected value of objective function. Deviations under expected value of objective function is minimized via the second term of objective function. It helps to control optimality robustness via minimizing deviations of objective function and keeping it near to its expected value regarding most of the uncertainty scenarios. Notably, violations of objective function is penalized via parameter π (*i.e.*, optimality robustness control parameter). Remaining terms of the objective function (3.9) control feasibility robustness of the model. They calculate possible violations of constraints based on determined satisfaction levels. In other words, they determine interval between worst value of uncertain parameters and their regarded value in constraints. Accordingly, they calculate total penalty costs of possible violations of uncertain parameters in the model constraints. Higher penalty costs (*i.e.*, γ, μ, θ) could result in lower violations of constraints and also risk-averse adjustment of satisfaction levels. It is worthy to mention that presented terms optimize satisfaction levels based on penalty costs of violations of uncertain parameters which eliminated the need to time-consuming simulations to find appropriate value of satisfaction levels.

4. IMPLEMENTATION AND EVALUATION

In this section, the extended model is solved and its outputs are analyzed. In this regard, to assess performance and usefulness of the extended possibilistic and robust possibilistic programming models, data is generated randomly based on defined uniform distributions. Style of data generation for different parameters of the model is presented in Table 1. Notably, the extended models include lots of parameters and presenting all parameters in the manuscript is not possible. Therefore, value of some parameters is rendered in the paper and value of other parameters could be provided via request of interested readers. Table 2 presents demand of customer zones and priority of satisfying the needed amount of water by different consumer regions.

In Table 1, data is generated via using uniform distributions. In other words, to form trapezoidal possibility distribution of the uncertain parameters, first of all, a random number is generated via applying uniform distribution to find the lowest prominent point of trapezoidal membership function (*i.e.*, $D_{j,k,f}^{(1)} = U [20, 50]$). Then, a different uniform distribution is defined to find the second prominent point of trapezoidal membership function. In other words, the generated random number would be added to the previous prominent point of trapezoidal possibility distribution (*i.e.*, $D_{j,k,f}^{(2)} = D_{j,k,f}^{(1)} + U [10, 25]$). This process would be continued up to finding four prominent points of trapezoidal membership function. Notably, all presented parameters in the Table 1 are generated based on the defined parameter generation scheme. Aim of such parameter generation method is finding divergent parameters to assess performance of the extended model under random realizations and show its efficient performance regarding different uncertainty scenarios.

Maximum available budget (*i.e.*, $\text{Max}B$) is considered 200 000 monetary units.

TABLE 2. The demand and priority of each city.

City (j)	Type of water (k)	Product (f)	Demand ($D_{j,k,f}^{(1)}, D_{j,k,f}^{(2)}, D_{j,k,f}^{(3)}, D_{j,k,f}^{(4)}$)	Priority ($P_{j,k,f}$)
(1)	(1)	(1)	(44,67,82,97)	6
		(2)	(34,57,81,98)	3
		(3)	(25,43,65,77)	16
	(2)	(1)	(33,45,55,72)	11
		(2)	(26,43,59,76)	7
		(3)	(28,51,67,87)	17
(2)	(1)	(1)	(41,56,81,106)	14
		(2)	(45,56,79,94)	8
		(3)	(20,44,64,78)	5
	(2)	(1)	(30,44,65,75)	15
		(2)	(28,44,63,74)	1
		(3)	(27,51,76,92)	2
(3)	(1)	(1)	(37,53,64,75)	4
		(2)	(23,39,49,71)	18
		(3)	(44,60,83,97)	13
	(2)	(1)	(21,34,59,83)	9
		(2)	(50,60,78,102)	10
		(3)	(31,45,70,84)	12

To find efficient solutions of the extended multi-objective programming model, ε -constraint method is employed. Utilizing this method can help to find Pareto-frontier solutions based on preference of company executives. In other words, it helps to prove conflict of objective functions. It means that maximizing priority of demand satisfaction results in lowering profit of water distribution network. In the primary phase of this method, extreme points of objective functions should be found (*i.e.*, the best and worst values of each objective function). Therefore, each objective function should be regarded in the model as main objective and the model should be solved as a single-objective model under nominal data. Then, optimal value of decision variables of each objective function should be put in other objective to find the worst value of other conflicting objective function. Finally, to assess the effect of enhancing the value of the second objective function on value of the first objective function, the second objective function would be added to constraints. By changing value of the second objective function between its worst and best values, value of the first objective would be optimized based on defined epsilon values. Changing the value of epsilon parameter results in changing the value of objective function. It shows the effect of changing the value of one objective function on the other one. Extended model based on the rendered multi-objective programming method could be formulated as follows:

$$\text{Max} \sum_i^I \sum_j^J \sum_k^K \sum_f^F S_f \cdot \tilde{L}_f \cdot X_{ijkf} \quad (4.1)$$

$$\text{s.t.} \quad \sum_i^I \sum_j^J \sum_k^K \sum_f^F ((J.K.F) + 1 - A_{jkf}) \cdot X_{ijkf} \geq \varepsilon \quad (4.2)$$

$$x \in F(x). \quad (4.3)$$

TABLE 3. The performance of the fuzzy model under nominal data.

PP							
$\alpha_{1,2,3} = 0.6$		$\alpha_{1,2,3} = 0.7$		$\alpha_{1,2,3} = 0.8$		$\alpha_{1,2,3} = 0.9$	
Z_1	Z_2	Z_1	Z_2	Z_1	Z_2	Z_1	Z_2
716271.79	76730.41	687766.36	73386.76	660950.14	69967.85	635748.80	66754.65
715374.35	80335.42	682265.93	76449.52	657091.03	72693.59	634056.25	69318.57
710543.88	83940.44	676236.34	79512.29	650896.82	75419.33	628375.91	71882.5
701454.54	87545.45	669258.03	82575.05	643366.55	78145.07	621086.33	74446.42
656991.42	91150.47	623319.05	85637.82	618067.06	80870.82	594365.23	77010.35

TABLE 4. The performance of the robust model under nominal data.

Robust	
Z_1	Z_2
762115.71	78083.29
759302.33	82909.66
753180.8	87736.03
743528.06	92562.4
702359.14	97388.77

Where $F(x)$ corresponds to feasible region of optimization model that includes the constraints of the original model (*i.e.*, (2.3)–(2.8)). There is a specific set goal to produce different Pareto-optimal solutions where the estimation of epsilon is shifted systemically in the changing range of the second objective function value. Notably, this range is portioned into equivalent intervals that determines different points used for generating pareto-optimal solutions. As decision makers are mostly concerned about the robustness of the profit maximization objective function, without loss of simplification, we just utilize the robust possibilistic programming definitions for the first objective function and robustness concept is disregarded in the second objective function. Outputs of implementing multi-objective programming method and achieved pareto solutions of the possibilistic water allocation and the robust possibilistic water allocation models are presented in Tables 3 and 4, respectively.

As it obvious, increasing total priority of satisfying demand of customer zones in the PP model has led to decrease of profit maximization objective function. In other words, output results of applied multi-objective programming method approve conflicting performance of objective functions regarding different minimum satisfaction levels. Also, increasing minimum satisfaction levels has led to increase of total profit objective function and total value of priority maximization objective function. Reason of the noted matter is that enhancement of satisfaction levels results in risk-averse performance of the model. Accordingly, amount of demand of customer zones would be increased and total needed water resources would be increased in the water distribution network. Output results of the robust possibilistic programming model confirm accurate performance of the extended model. It shows conflicting performance of the objective functions regarding different epsilon values. It should be noted that in the robust possibilistic programming model, increasing the value of penalties of feasibility robustness terms in the objective function results in increase of the value of minimum satisfaction levels. In other words, increasing penalties causes minimum violation of constraints. Therefore, the value of satisfaction levels would be set one and total violation of constraints would be zero. Accordingly, total penalty costs of constraints violations would be zero. It should be noted that increasing penalty costs results in maximizing flow of water in the network. Therefore, total profit would be maximized based on the required water resources.

To evaluate output solutions achieved by solving the proposed possibilistic and robust possibilistic programming models under nominal data, 10 arbitrary realizations are randomly generated. For example, if

$\tilde{\xi} = (\xi_1, \xi_2, \xi_3, \xi_4)$ is an uncertain parameter with trapezoidal possibility distribution, the realization is performed via producing an arbitrary uniform number between upper and lower limits of distribution membership function (*i.e.*, $\xi_{\text{real}} \sim [\xi_1, \xi_4]$). Uniform data generation should be performed for all uncertain parameters of the extended model. Aim of the noted matter is simulating different scenarios to assess performance of the robust possibilistic water distribution network design model against the possibilistic programming model. Parameter generation should be repeated 10 times to find divergent uncertainty scenarios. Then, the solutions achieved by solving models under nominal data (X^*, B^*) will be put in a linear programming model as parameters. Therefore, decision variables of the model are only related to violation of constraints that include uncertain parameters. Notably, designing different uncertainty scenarios could help to find total violation of constraints and their related penalty costs in different probable conditions of the water distribution network. The elucidated linear programming model could be formulated as follows:

$$\text{Max} \sum_i^I \sum_j^J \sum_k^K \sum_f^F S_f \cdot L_f^{\text{real}} \cdot X_{ijkf}^* - \pi R_1 - \mu R_2 - \delta R_3 \quad (4.4)$$

$$\text{s.t.} \quad \sum_i^I \sum_j^J \sum_k^K \sum_f^F ((J.K.F) + 1 - A_{jkf}) \cdot X_{ijkf}^* \geq \varepsilon \quad (4.5)$$

$$\sum_i^I X_{ijkf}^* + R_1 \geq D_{jkf}^{\text{real}} \quad \forall j, k, f \quad (4.6)$$

$$\sum_j^J \sum_f^F X_{ijkf}^* - R_2 \leq \aleph \left(\text{Cap}_{ik}^{\text{real}} \right) \quad \forall i, k. \quad (4.7)$$

$$\sum_i^I \sum_k^K \sum_f^F C_{ij}^{\text{real}} \cdot X_{ijkf}^* - R_3 \leq B_j^* \quad \forall j \quad (4.8)$$

$$\sum_j^J B_j^* \leq \text{Max} B \quad (4.9)$$

$$R_1, R_2, R_3 \geq \quad (4.10)$$

In the rendered linear programming model (4.4)–(4.10), decision variables R_1 , R_2 and R_3 determine amount of violation of chance constraints under random realization. Therefore, violations of constraints would be penalized in the objective function (4.4) as decision variables. Finally, average and standard deviation of the objective function values under random realizations are utilized as performance measures to assess the proposed robust possibilistic programming model and compare it against the PP model. In other words, value of total violation of constraints and total costs of water distribution network design under each random realization should be calculated. Then, after finding the optimal value of the objective functions of ten realizations, average and standard deviation of ten objective functions should be calculated. It helps to find the expected value of objective function and its possible deviations under random realizations. Finally, average and standard deviation of the models should be compared to find the best model with the highest average profit and the lowest standard deviation. The results of model examinations are presented in Table 5 and Figure 2.

As it is clearly understood from Table 5 and Figure 2, the average of total profit maximization objective function under random realizations in the robust model is better than the fuzzy programming model regarding different confidence levels. Also, standard deviation of the robust possibilistic programming model is lower than the PP model regarding different minimum satisfaction levels. Therefore, the robust possibilistic programming model outperforms the PP model regarding both performance measures (*i.e.*, average and standard deviation). Reason of the noted matters is that the robust possibilistic programming model works in risk-averse manner and

TABLE 5. The performance of the models under realizations.

No. of realization	PP	PP	PP	PP	Robust
	$\alpha_{1,2,3} = 0.6$	$\alpha_{1,2,3} = 0.7$	$\alpha_{1,2,3} = 0.8$	$\alpha_{1,2,3} = 0.9$	
1	570016.82	558006.47	546912.69	532775.67	584541.78
2	713949.97	687711.93	662287.94	638270.18	742529.72
3	906912.21	868727.05	832736.79	798844.84	909744.82
4	564473.41	543586.01	523989.72	505627.74	588094.6
5	499034.12	484822.10	470965.03	457450.78	582378.38
6	515796.49	496404.05	477250.07	459351.11	536551.68
7	769742.76	741057.38	714136.52	688902.55	772164.32
8	502072.84	481427.66	461912.82	443549.20	594530.32
9	927113.24	892471.35	859891.20	859891.20	936510.58
10	482327.32	467270.90	452269.83	438111.62	548518.38
Average	645143.92	622148.5	600235.3	582277.5	679556.5
Standard deviation	171754.897	163659.3	156283.9	155464.3	150274.3

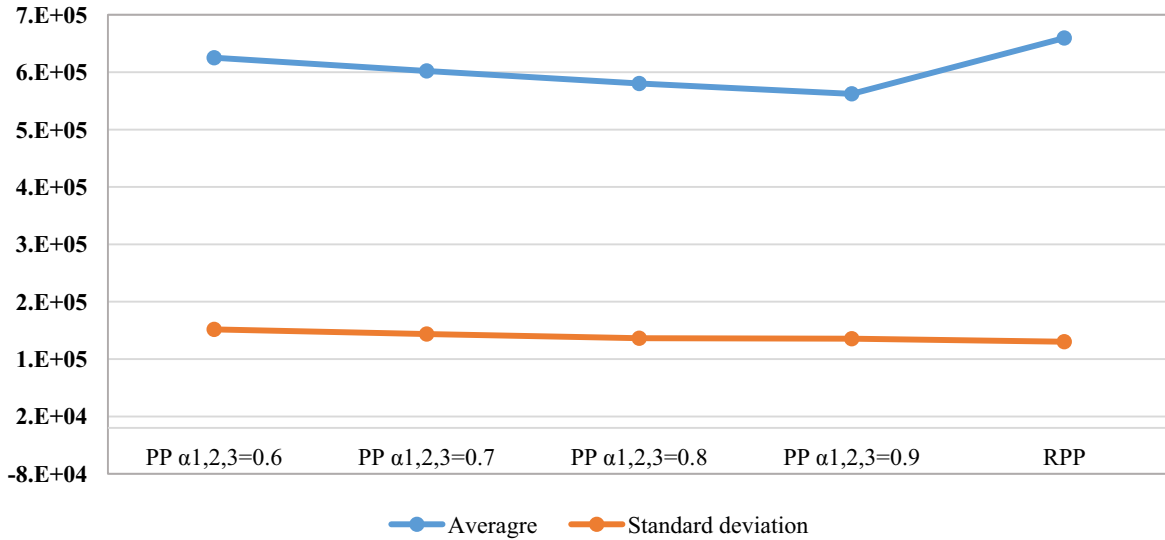


FIGURE 2. Comparison of the average and standard deviation of the models under realizations.

accordingly total violations of constraints in the robust possibilistic programming model is lower than the PP model. Therefore, total penalty of constraints violations in the robust possibilistic programming model is lower in comparison to the PP model. It should be noted that risk-averse performance of the model results in the lowest violation of the constraints. Therefore, the robust possibilistic programming model would be penalized less than the PP model regarding different uncertainty generated scenarios. Notably, as the extended robust possibilistic programming model is an extension of the possibilistic chance-constrained programming model, output results of the proposed model is compared to the traditional PP method. In other words, the suggested robust possibilistic programming model solves weaknesses of the PP model. Therefore, in the implementation and evaluation section, the aforementioned methods and their performance are compared to each other to show better performance of the robust fuzzy programming model. Output results confirm that the robust possibilistic programming model outperforms the possibilistic programming model that could be regarded as the biggest advantage of the

extended model. It should be noted that the extended robust possibilistic programming model includes some more decision variables and terms in constraints and objective function. The noted issue increases complexity of the model. However, after solving the model, output results confirm that the extended model is not complex and could be solved in a reasonable time by CPLEX solver of GAMS optimization software. In other words, the PP model is solved in 0.78 s and the robust possibilistic programming model is solved in 0.81 s. Presented results approve that the extended model is not a complex mathematical programming model and could be solved by optimization software. Accordingly, it could be concluded that the robust possibilistic programming model is a better decision making tool because it helps decision makers to make risk-averse decisions. Also, it enables decision makers to adjust risk-aversion level of output decisions based on their preference. It should be noted that the extended robust possibilistic programming model could be applicable in the real world water allocation problem because achieved outputs confirm its accurate and reliable performance against the traditional fuzzy programming models.

5. CONCLUSION

The water allocation problem is an important problem because water resources are scarce and presented models enable company managers to appropriately distribute different water resources between various demand regions. Accordingly, in this paper, a bi-objective water allocation model is proposed that the objectives maximize total profit of transferring water to demand regions and priority of the cities for transferring water. As an obvious matter, the model has some imprecise parameters so that robust credibility-based fuzzy chance constrained programming method is extended to tackle uncertainty of parameters. Notably, the extended model is capable of optimizing assigned budget to each region for water transferring based on their priority. The fuzzy and robust fuzzy models are compared against each other and performed comparison indicates that the robust model outperforms the PP model regarding average and standard deviation performance measures. An important issue is that output results of the proposed model are risk-averse and decision makers are capable of adjusting risk-aversion of results. Noted matter could be regarded as an advantage of the extended model. Also, reliable outputs of the model confirm that the model could be employed as an applicable decision making tool in real world problems.

As future research guideline it should be noted that other robust programming approaches could be used in water allocation scope to cope with uncertainty of parameters. Also, an efficient meta-heuristic algorithm can be proposed to solve the extended models with large size problem instances. Notably, some other factors such as social responsibility and greenness of the network could be used to improve profitability and social image of the designed water distribution network.

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