RAIRO-Oper. Res. 53 (2019) 59–79
https://doi.org/10.1051/ro/2018045

RAIRO Operations Research
www.rairo-ro.org

INVENTORY OPTIMIZATION MODEL CONSIDERING CONSUMER SHIFT AND INVENTORY TRANSSHIPMENT IN DUAL-CHANNEL SUPPLY CHAINS*

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Abstract. In this paper we consider a dual-channel supply chain which consists of an online store and multiple independent retail stores. In this system, customer shift induces inventory competition while transshipment brings inventory cooperation, both of which influences inventory optimization and control. Therefore we respectively construct inventory optimization models under the two situations: customer shift and inventory transshipment. Specifically, unilateral customer shift and inventory transshipment are considered, and a one-for-one replenishment strategy is applied. We first solve the equilibrium state probability of on-hand inventory through Markov chain theory, then optimize performance measure (*i.e.*, the total costs) to obtain the optimal basic inventory level. Finally, we analyze the impact of customer shift rate and inventory transshipment rate on the inventory strategies through numerical simulation, and further compare the differences in inventory decisions between the above two situations, which prove that inventory cooperation brought by inventory transshipment is not necessarily better than inventory competition brought by customer shift. In addition, we discuss several insights that are evident from the parametric analysis of the model.

Mathematics Subject Classification. 60J20, 90B05, 90C29.

Received October 15, 2017. Accepted June 19, 2018.

Introduction

With the rapid development of information technology and third-party logistics, e-commerce based on Internet has received increasing attention. More and more enterprises begin to sell goods to consumers through their independent retailers and online store. For such decentralized dual-channel supply chain, since customers who shop through online and traditional retail channels cannot be completely separated, there will inevitably be overlapping of customer groups in the two channels. When one of the two channels is out of stock, some customers will shift to the other channel, leading to inventory competition between the two channels [8]. On the other hand, inventory transshipment cooperation will reduce the stock level of the entire supply chain system, thus these competition and cooperation behaviors between the two channels will certainly affect inventory management. Therefore, for two situations – customer shift and inventory transshipment, how to reduce the

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Keywords. Dual-channel, customer shift, inventory transshipment, inventory optimization

*This work was supported by the National Natural Science Foundation of China (Nos. 71431004, 71572033 and 71172174).

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inventory level of the dual-channel supply chain, to reduce the inventory risk and improve the efficiency of inventory management are the purposes of this paper.

There are several groups of existing literature relevant to our research. The first one deals with inventory management in dual-channel supply chains. Some researches focus on pricing and inventory decisions for dual-channel supply chains. For example, Moon *et al.* [23], Huang *et al.* [14], Hsieh *et al.* [12], Roy *et al.* [26] and He *et al.* [10] studied the pricing and inventory strategies of dual channels under different conditions. There are also some papers only concerning inventory decisions in dual-channel systems. Boyaci [4] constructed a single-period static (newsvendor-type) model to obtain the base-stock levels of the two channels. Yao *et al.* [34] constructed three different inventory strategy models, and found that the inventory strategy of the dual-channel decentralized system kept fewer inventories than other two cases: centralized system and outsourced system. These studies mainly concentrate on single-period order strategy of the dual-channel supply chain. Meanwhile, there are also some researches from a multi-period perspective. Considering a one-for-one replenishment strategy, Teymouri *et al.* [30] simplified a Markov chain model from Chiang and Monahan [6] and proposed a best neighborhood algorithm to find a good solution for inventory. Li *et al.* [19] proposed a multi-period stochastic dynamic programming model to analyze the optimal inventory policy of vendor operating dual channels. Although Teymouri *et al.* [30] and Li *et al.* [19] have studied the multi-period inventory decisions in dual-channel supply chain systems, they do not consider the cases of customer shift and inventory transshipment.

The second stream of related literature consists of papers dealing with customer shift in multiple channels. Firstly, from market perspective, many researches explore the motives and influencing factors of customer migration between multiple channels through empirical analysis. For example, Pookulangara *et al.* [24, 25] examined channel-migration behavior using different theories, *i.e.*, decomposed theory of planned behavior and the theory of reasoned action with crossover effects in different channels. Bell *et al.* [2, 3] discussed the impact of opening an offline showroom channel on the customer migration between Virtual Try-On and Home Try-On. Li *et al.* [17, 20] respectively discussed the impact of cross-channel competition and integration on the customer channel migration. Secondly, there are also some studies that analyze the impact of customer shift on pricing, inventory, and other decisions from the perspective of operations. Kauffman *et al.* [15] proposed two pricing models to examine how consumer channel migration affects pricing strategy of the hybrid firm. Considering a dual-channel supply chain consisting of an online store owned by a manufacturer and a retailer, Chiang and Monahan [6], Takahashi *et al.* [29] and Yang *et al.* [32] discussed the optimal inventory decisions of the two channels under customer shift, and analyzed the impact of customer shift on these decisions. We use the insights from Chiang and Monahan [6] to develop our model. But they only discuss inventory decisions in customer shift situation and do not consider inventory transshipment situation. Moreover, note that they focus on the dual-channel supply chain consisting of an online store and a retail store, and do not extend a dual-channel system consisting of an online store and multiple retail stores.

There is a mature information system literature stream pertaining to inventory transshipment in different supply chain systems. Some researches focus on inventory strategy with transshipment among two or multiple retailers or locations. For example, Huang *et al.* [13] designed a heuristic rule for two competitive retailers to decide whether to transfer their inventory or not, and provided an approximate technique to evaluate the transfer policy. Yousuk and Luong [35] proposed a new method-the expected path approach to minimize the total cost of a two-retailer inventory system with preventive lateral transshipment. Liang *et al.* [21] studied a firm's optimal transshipment problem between two locations taking into consideration the setup costs for transshipment and the demand shape. Arikan and Silbermayr [1] discussed the risk pooling effect of unidirectional transshipments with two independently owned and operated locations. There are also a few papers concerning inventory transshipment problem in dual-channel systems. Seifert *et al.* [28] compared the optimal inventory strategy between an integrated supply chain with inventory transshipment and a dedicated supply chain without transshipment in a dual-channel supply chain consisting of one online store and multiple retail stores; He *et al.* [11] discussed the optimal decisions in a decentralized dual-channel system consisting of an online store and a retail store, and further developed contracts to coordinate the supply chain. All the above literatures focus on single-period decisions and do not study multi-period decisions. Meanwhile, they only analyze inventory decisions under transshipment situation, and do not consider customer shift situation.

Finally, there is very little literature relevant to inventory decisions considering both inventory transshipment and customer shift. Only Çömez *et al.* [7], Zhang *et al.* [36] and Li and Li [18] obtained the optimal order quantity in two competing retailers/distributors system under the above two cases, and analyzed the results under different scenarios. Although these studies explore the single-period inventory decisions under inventory transshipment and customer shift, they do not further study multi-period inventory decisions. Moreover, they focus on the supply chain consisting of two competitive retailers, which is different from our model.

In summary, in this paper we consider a one-for-one replenishment strategy, and construct multi-period inventory optimization models in a dual-channel supply chain consisting of an online store and multiple retail stores under two situations – customer shift and inventory transshipment respectively. The impact of customer shift rate and inventory transshipment rate on the optimal inventory strategies is discussed respectively through numerical analysis. We further compare the optimal inventory decisions between the above two situations and explore the relationship between inventory competition brought by customer shift and inventory cooperation brought by inventory transshipment. In addition, we also analyze the impact of related parameters on the optimal inventory decisions and costs to demonstrate management implications.

1. Model description

This paper focuses on a dual-distribution channel inventory system consisting of an online store and N independent retail stores, where a retailer operates all retail stores and a manufacturer operates the online store and supplies products to the retailer. The manufacturer and the retailer adopt decentralized decision-making with the goal of minimizing their costs. The inventory operation mode of this dual-channel supply chain is that both online store and retail stores hold inventory, and meet demand of all online and offline customers, respectively. When the online store is out of stock, we discuss the following two cases:

1) Customer shift. When the online store is out of stock, some customers from the online store will shift to the retail store.
2) Inventory transshipment. When the online store is out of stock, the retail store's remaining inventory will be transshipped to the online store.

The structures of the above two cases are shown in Figure 1. We do not consider the situation that when a retail store is out of stock, the customers of the retail store will shift to the online store, or the online store's remaining inventory will be transshipped to the retail store. The reasons are as follows:

Arikan and Silbermayr [1] reported that one-way transshipment is a common practice in dual channel supply chains. Moreover, the existing literature explores the one-way transshipment from the retail store to the online store, but the contrary is not allowed [11, 13, 28]. In addition, many enterprises, such as Orvis Company Inc., Systemax Inc.'s CompUSA and Jones Apparel Group, use the inventory in their physical channels to fulfill their online orders when their online warehouses are out of stock, but not *vice versa* [27].

Such phenomena can be reasoned. The application of transshipment is often limited by two factors as follows: firstly, transshipment takes time, and consumers often do not want to wait for transshipment when the retail store is out of stock; secondly, transshipment incurs cost, and it does not occur when the transshipment cost is high. In reality, transshipping from retail stores to the online store is much less affected by these two factors compared to the other way round, due to two reasons. One is that online consumers who buy from the online store have more patience to wait when the online store is stock-out; another is that the retail store can transport its excess goods directly to online consumers when the online store is stock-out, which can greatly save logistics cost [11, 13].

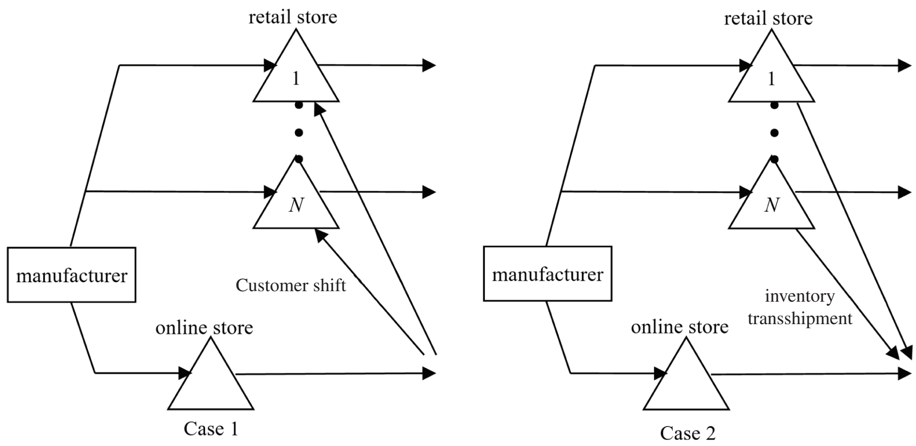

FIGURE 1. Dual-channel supply chain structures considering customer shift and inventory transshipment.

Therefore, this study mainly focuses on inventory transshipment from retail stores to the online store when the online store is out of stock. Meanwhile, in order to compare the optimal inventory strategy between the cases of inventory transshipment and customer shift, we only consider some online customers will shift to the retail stores when the online store is out of stock, but not vice versa. In addition, in this paper we mainly discuss the above two cases between online and retail channels, customer shift and inventory transshipment among retail stores are also not our research content, which already have a mature literature stream (*e.g.*, [7, 9, 18, 35, 36]).

We assume that the retail store is represented by r, the retailer is represented by R, the online store is represented by v, and the manufacturer is represented by M. The market demand is ξ, which obeys Poisson distribution with parameter λ which is customer arrival rate. There are two types of customers on the market: one type of customers prefer traditional retail channel, and the other customers prefer online channel (*e.g.*, [5, 6, 29]). The demands of these two types of customers are independent on each other, and obey Poisson distribution with parameter λ_r and λ_v, respectively. Parameter λ_r is the customer arrival rate of the retail channel, and λ_v is the customer arrival rate of the online channel, $\lambda_r = (1-\alpha)\lambda, \lambda_v = \alpha\lambda$, where $\alpha \in [0,1]$ represents the proportion of customers who prefer the online channel, that is, the preference rate of the online channel. There are many similar examples in reality. For example, some customers do not use the Internet, thus they will shop in the retail stores, while some young people like online shopping [16]. In addition, we assume that the demand of each retail store is independent on each other, and the arrival rate of the ith retail store is λ_{ir} which satisfies $\lambda_r = \sum_{i=1}^{N} \lambda_{ir}$.

We assume that replenishment lead times of the online store and any retail store are independent and follow the exponential distribution, and their mean values are $\frac{1}{\mu_v}$ and $\frac{1}{\mu_r}$, respectively. It means the replenishment rates of the online store and any retail store are μ_v and μ_r respectively. Both the online store and the retail stores use a one-for-one replenishment strategy (*e.g.*, [6, 9, 22, 30]). Under this replenishment inventory strategy, the inventory has remained constant, that is, there exists a basic inventory level. We use y_v to represent the basic inventory level of the online store, and use y_{ir} to represent the basic inventory level of the ith retail store, where $i = 1, 2, \ldots, N$.

Finally, we define some related parameters to construct our model. Parameters c_r and c_v represent unit purchase costs of any retail store and the online store respectively; unit inventory holding costs for the remaining inventory at the end of the period of any retail store and the online store are represented by h_v and h_r, respectively; unit lost sales costs of any retail store and the online store are donated by g_r and g_v, respectively.

TABLE 1. Definition of parameters.

Parameter	Definition	Parameter	Definition
λ	Total customer arrival rate	λ_r	Total customer arrival rate of the retail stores
λ_v	Customer arrival rate of the online store	λ_{ir}	Customer arrival rate of ith retail store
y_v	Basic inventory level of the online store	y_{ir}	Basic inventory level of ith retail store
μ_v	Replenishment rate of the online store	μ_r	Replenishment rate of any retail store
x_v	On-hand inventory of the online store	x_{ir}	On-hand inventory of ith retail store
$\pi_{x_v x_{ir}}$	Equilibrium state probability of on-hand inventory		

2. Inventory optimization model

In this section the inventory optimization model for the online store and the retail stores is established according to Markov chain method and optimization theory. In Section 2.1, we consider the case that online customers shift to the retail store when the online store is out of stock, and calculate the optimal inventory strategy in the decentralized decision; in Section 2.1.1, we calculate the optimal basic inventory levels in another case that inventory transshipment of the retail stores when the online store is out of stock. The parameters used in these models are shown in Table 1.

2.1. The model under customer shift case

We define $\beta_v \in [0,1]$ represents customer shift rate of the online channel, which means the percentage of the online channel's customers who are willing to search and buy products of any retail store when the online store is out of stock. In particular, when $\beta_v = 1$, all customers of the online channel are willing to search and shift to retail stores. First we use Markov chain method to obtain the equilibrium probability of the on-hand inventory state space, and then optimize performance measure (*i.e.*, the costs) to solve the optimal basic inventory levels of the dual-channel supply chain system based on optimization theory.

2.1.1. Equilibrium state probability of on-hand inventory

First, based on the dynamic equilibrium principle of inventory state, we obtain Figure 2 to reflect the inventory state transition, where $(x_v, x_{ir}), i \in N$ represents the state space of on-hand inventory of the online store and the ith retail store.

The state space reflects the dynamic equilibrium state of supply chain inventory at a certain moment. The on-hand inventory state of the dual channels will change, when any of the following four situations occur: 1) a customer arrives at a retail store; 2) an order is met by the online store; 3) a replenishment order arrives at the online store; 4) a replenishment order arrives at a retail store. The equilibrium inventory state illustrates that the demand and supply of the supply chain system are matched. For on-hand inventory, the inventory equilibrium requires that the input and output rates of all states are equal. Figure 2 shows that the input and output items are different in a different on-hand inventory state space, which can be divided into the following four parts:

$$(\mu_v + \mu_r)\pi_{x_v x_{ir}} = \lambda_v \pi_{(x_v+1)x_{ir}} + (\lambda_{ir} + \beta_v \lambda_v)\pi_{x_v(x_{ir}+1)} \tag{2.1}$$

where $x_v = 0, x_{ir} = 0$.

$$\begin{aligned}(\tau_{\mu_v}\mu_v + \mu_r + \lambda_v)\pi_{x_v x_{ir}} = \mu_v \pi_{(x_v-1)x_{ir}} + \psi_{\lambda_v}\lambda_v \pi_{(x_v+1)x_{ir}} \\ + \lambda_{ir}\pi_{x_v(x_{ir}+1)}\end{aligned} \tag{2.2}$$

where $x_v \in \{1, 2, \ldots, y_v - 1, y_v\}, x_{ir} = 0$.

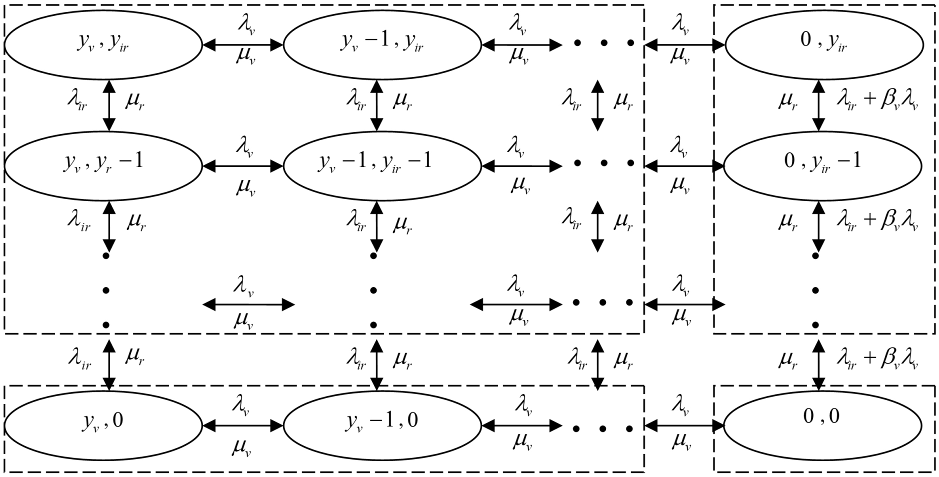

FIGURE 2. Inventory state transition under customer shift case.

$$\tau_{\mu_v} = \begin{cases} 0, & \text{if } x_v = y_v \\ 1, & \text{else} \end{cases}$$

$$\psi_{\lambda_v} = \begin{cases} 0, & \text{if } x_v = y_v \\ 1, & \text{else} \end{cases}$$

$$\begin{aligned}(\mu_v + \tau_{\mu_r}\mu_r + \lambda_{ir} + \beta_v\lambda_v)\pi_{x_v x_{ir}} &= \mu_r \pi_{x_v(x_{ir}-1)} \\ &+ \psi_{\lambda_{ir}}(\lambda_{ir} + \beta_v\lambda_v)\pi_{x_v(x_{ir}+1)} \\ &+ \lambda_v \pi_{(x_v+1)x_{ir}}\end{aligned} \tag{2.3}$$

where $x_v = 0, x_{ir} \in \{1, 2, \ldots, y_{ir}-1, y_{ir}\}$.

$$\tau_{\mu_r} = \begin{cases} 0, & \text{if } x_{ir} = y_{ir} \\ 1, & \text{else} \end{cases}$$

$$\psi_{\lambda_{ir}} = \begin{cases} 0, & \text{if } x_{ir} = y_{ir} \\ 1, & \text{else} \end{cases}$$

$$\begin{aligned}(\tau_{\mu_v}\mu_v + \tau_{\mu_r}\mu_r + \lambda_v + \lambda_{ir})\pi_{x_v x_{ir}} &= \mu_v \pi_{(x_v-1)x_{ir}} + \mu_r \pi_{x_v(x_{ir}-1)} \\ &+ \psi_{\lambda_v}\lambda_v \pi_{(x_v+1)x_{ir}} + \psi_{\lambda_{ir}}\lambda_{ir}\pi_{x_v(x_{ir}+1)}\end{aligned} \tag{2.4}$$

where $x_v \in \{1, 2, \ldots, y_v\}, x_{ir} \in \{1, 2, \ldots, y_{ir}\}$.

$$\tau_{\mu_v} = \begin{cases} 0, & \text{if } x_v = y_v \\ 1, & \text{else} \end{cases}$$

$$\tau_{\mu_r} = \begin{cases} 0, & \text{if } x_{ir} = y_{ir} \\ 1, & \text{else} \end{cases}$$

$$\psi_{\lambda_v} = \begin{cases} 0, & \text{if } x_v = y_v \\ 1, & \text{else} \end{cases}$$

$$\psi_{\lambda_{ir}} = \begin{cases} 0, & \text{if } x_{ir} = y_{ir} \\ 1, & \text{else} \end{cases}$$

Equations (2.1) and (2.3) reflect β_v ratio of online customers will shift to the ith retail store when the online store is out of stock ($x_v = 0$), otherwise no online store's customers will shift to the ith retail store as shown in equations (2.2) and (2.4). The left sides of equations (2.1)–(2.4) reflect the average transition probability outputted from on-hand inventory state (x_v, x_{ir}), *i.e.*, the output item. Equation (2.1) represents the in-process replenishment orders of the online store and the ith retail store at the next moment; the first two items on the left side of equation (2.2) indicate the in-process replenishment orders of the online store and the ith retail store at the next moment, and the last item indicates the demand of the online store at the next moment; the first two items on the left side of equation (2.3) indicate the in-process replenishment orders of the online store and the ith retail store at the next moment, and the last item indicates the demand of the ith retail store at the next moment; the first two items on the left side of equation (2.4) indicate the in-process replenishment orders of the online store and the ith retail store at the next moment, and the latter two items indicate the demands of the online store and the ith retail store at the next moment. Where, τ_{x_v} indicates the online store does not need the in-process replenishment order at the next moment, when it reaches the basic inventory level; $\tau_{x_{ir}}$ indicates the ith retail store does not need the in-process replenishment order at the next moment, when it reaches the basic inventory level; In contrast, the right sides of equations (2.1)–(2.4) reflect the average transition probability input to on-hand inventory state (x_v, x_{ir}), that is, the input item. Equation (2.1) represents the demands of the online store and the ith retail store at the previous moment; the first item on the right side of equation (2.2) indicates the in-process replenishment order of the online store at the previous moment, and the latter two items indicate the demands of the online store and the ith retail store at the previous moment; the first item on the right side of equation (2.3) indicates the in-process replenishment order of the ith retail store at the previous moment, and the latter two items indicate the demands of the online store and the ith retail store at the previous moment; the first two items on the right side of equation (2.4) indicate the in-process replenishment orders of the online store and the ith retail store at the previous moment, and the latter two items indicate the demands of the online store and the ith retail store at the previous moment. Where, ψ_{x_v} indicates no customers of the online store arrive at the previous moment, when the online store reaches the basic inventory level; $\psi_{x_{ir}}$ indicates no customers of the ith retail store arrive at the previous moment, when the ith retail store reaches the basic inventory level.

The equilibrium state probabilities of all on-hand inventories ($\pi_{x_v x_{ir}}$) satisfy the following standardized constraint:

$$\sum_{x_v=0}^{y_v} \sum_{x_{ir}=0}^{y_{ir}} \pi_{x_v x_{ir}} = 1 \tag{2.5}$$

Therefore, by solving equations (2.1)–(2.5), we can obtain the equilibrium state probability of on-hand inventory.

Determining the equilibrium state probability of on-hand inventory can be used as an important criterion for measuring the performance of a supply chain inventory system. It is necessary to further verify the rationality of the above formulas. Next, we take an example to illustrate this rationality. We assume that the basic inventory level of the online store (y_v) is 3, and the basic inventory level of the ith retail store (y_{ir}) is 3. State transition

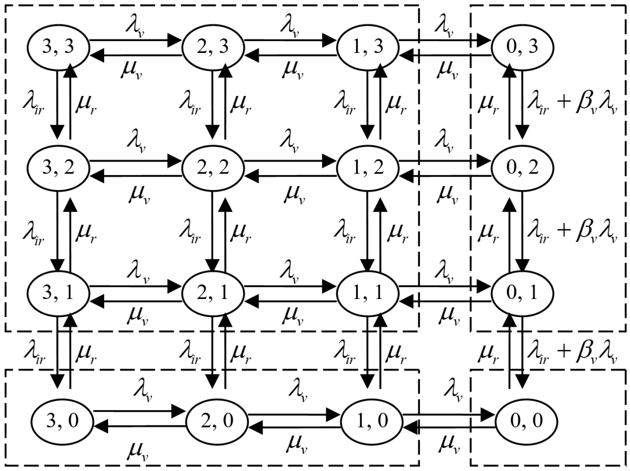

FIGURE 3. State transition of on-hand inventory ($y_v = 3, y_{ir} = 3$).

TABLE 2. Equilibrium state probability of on-hand inventory ($\pi_{x_v x_{ir}}$).

		x_{ir}			
		0	1	2	3
	0	0.0004	0.0027	0.0201	0.1504
	1	0.0005	0.0034	0.0251	0.1880
x_v	2	0.0006	0.0042	0.0313	0.2349
	3	0.0007	0.0052	0.0391	0.2936

Note: $\lambda_v = 2, \lambda_{ir} = 8, \beta_v = 0.5, \mu_v = 15, \mu_r = 10$.

and equilibrium state probability of on-hand inventory in this supply chain system are as shown in Figure 3 and Table 2, respectively. It is not difficult to find that all the results meet the above constraints. Therefore, the formulas for solving the equilibrium state probability we built is reasonable.

2.1.2. Optimal basic inventory level

According to the equilibrium state probability of on-hand inventory discussed above, we use optimization theory to minimize the costs of the manufacturer and the retailer to obtain the optimal basic inventory levels of the dual-channel supply chain in equilibrium state under customer shift case. First, we define inventory holding costs and lost sales costs to obtain the costs of the manufacturer and the retailer as the performance measure:

1) *Average inventory holding costs*
For the given equilibrium state probability, the average inventory of the online store and the ith retail store are given by the following equations:

$$I_{iv} = \sum_{x_v=1}^{y_v} \sum_{x_{ir}=0}^{y_{ir}} x_v \pi_{x_v x_{ir}}, I_v = \sum_{i=1}^{N} I_{iv}$$

$$I_{ir} = \sum_{x_v=0}^{y_v} \sum_{x_{ir}=1}^{y_{ir}} x_{ir} \pi_{x_v x_{ir}}$$

Therefore, the average inventory holding costs of the online store and the ith retail store are as follows:

$$C_{Hv} = h_v I_v, C_{Hir} = h_r I_{ir}$$

where h_v and h_r are unit inventory holding costs of the online store and the retail stores, respectively.

2) *Average lost sales costs*
When any one channel is out of stock, customers in this channel do not want to shift to the other channel, leading to lost sales; when both the retail channel and online channel are out of stock simultaneously, it will also lead to lost sales. The probabilities that the online store and the ith retail store are out of stock are respectively as follows:

$$L_{iv} = \sum_{x_{ir}=1}^{y_{ir}} \pi_{0x_{ir}}, L_v = \sum_{i=1}^{N} L_{iv}$$

$$L_{ir} = \sum_{x_v=1}^{y_v} \pi_{x_v 0}$$

The probability that these two channels are out of stock simultaneously is as follows:

$$L_{ib} = \pi_{00}, L_b = \sum_{i=1}^{N} L_{ib}$$

Therefore, the average lost sales costs of the online store and the ith retail store are as follows:

$$C_{Lv} = g_v(1-\beta_v)L_v\lambda_v + g_v L_b \lambda_v$$
$$C_{Lir} = g_r L_{ir}\lambda_{ir} + g_r L_{ib}\lambda_{ir}$$

where g_v and g_r are unit lost sales costs of the online store and the retail stores, respectively.

3) *Costs of the manufacturer and the retailer*
The costs of the manufacturer and the retailer under the decentralized decision are as follows (recall N retail stores belong to the retailer):

$$TC_M(y_v, y_{ir}) = h_v I_v + g_v(1-\beta_v)L_v\lambda_v + g_v L_b \lambda_v$$
$$TC_R(y_v, y_{ir}) = \sum_{i=1}^{N}(h_r I_{ir} + g_r L_{ir}\lambda_{ir} + g_r L_{ib}\lambda_{ir})$$

Therefore, we can obtain the basic inventory levels of the online store and the retail stores (y_v and $y_{ir}, \forall i \in N$) in equilibrium state under customer shift case through minimizing the above costs, that is, solving the following multi-objective programming problem.

$$\min TC_M(y_v, y_{ir}) = h_v I_v + g_v(1-\beta_v)L_v\lambda_v + g_v L_b \lambda_v$$
$$\min TC_R(y_v, y_{ir}) = \sum_{i=1}^{N}(h_r I_{ir} + g_r L_{ir}\lambda_{ir} + g_r L_{ib}\lambda_{ir}) \tag{2.6}$$

Proposition 2.1. *If the customer arrival rates of N retail stores are the same, then the basic inventory levels of N retail stores are equal, that is, $y_{1r} = y_{2r} = \cdots = y_{Nr} = y_r$, and the corresponding inventory optimization model is as follows:*

$$\min TC_M(y_v, y_r) = h_v I_v + g_v(1-\beta_v)L_v\lambda_v + g_v L_b \lambda_v$$

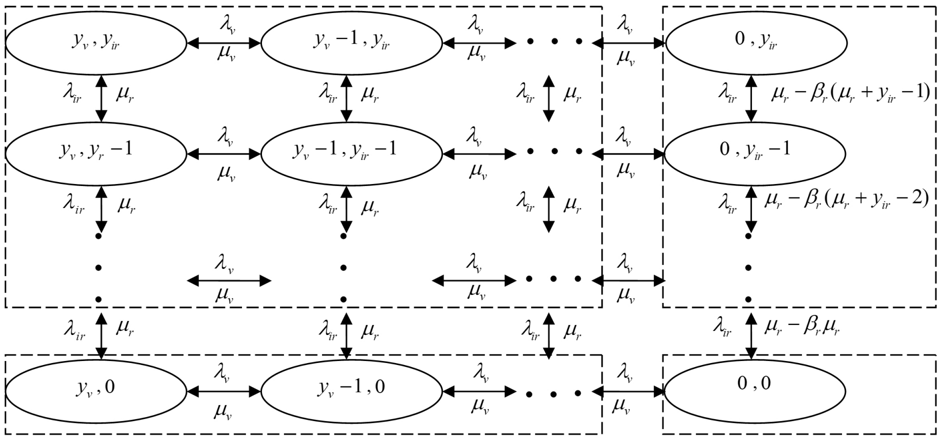

FIGURE 4. Inventory state transition under inventory transshipment case.

$$\min TC_R(y_v, y_r) = Nh_rI_r + g_rL_r\lambda_r + g_rL_b\lambda_r/N$$

where $I_r = I_{ir}, L_r = L_{ir}$.

The proof of Proposition 2.1 is given in Appendix A.

2.2. The model under inventory transshipment case

In the decentralized dual-channel supply chain, the manufacturer and the retailer compete with each other to maximize their own profits. Thus, the transshipment strategy as cooperation between the manufacturer and the retailer must be effectively implemented in accordance with some mechanisms. We assume when the online store is out of stock, any retail store transships the remaining inventory to the online store and the transshipment rate is $\beta_r \in [0,1]$ which satisfies $\mu_r - \beta_r(\mu_r + x_{ir}) \geq 1, \forall x_{ir} \in [0, y_{ir})$ (recall that a one-for-one replenishment strategy is adopted). In order to encourage the retail stores to transfer inventory, the transshipment costs are borne by the manufacturer and unit transshipment cost is c_t ($c_t \leq c_v$). Meanwhile the manufacturer provides transshipment subsidy for the retail stores, and unit transshipment subsidy is t, $0 \leq t \leq c_v - c_t$. The cooperation mechanism between the manufacturer and the retailer based on inventory transshipment has been built. First, based on Markov chain method, inventory state transition figure in inventory transshipment case is obtained and shown in Figure 4.

In order to obtain the equilibrium state probabilities of all on-hand inventories ($\pi_{x_vx_{ir}}$), we need to solve the following formulas:

$$(\mu_v + \mu_r - \beta_r\mu_r)\pi_{x_vx_{ir}} = \lambda_v\pi_{(x_v+1)x_{ir}} + \lambda_{ir}\pi_{x_v(x_{ir}+1)} \tag{2.7}$$

where $x_v = 0, x_{ir} = 0$.

$$\begin{aligned}(\tau_{\mu_v}\mu_v + \mu_r + \lambda_v)\pi_{x_vx_{ir}} &= \mu_v\pi_{(x_v-1)x_{ir}} + \psi_{\lambda_v}\lambda_v\pi_{(x_v+1)x_{ir}} \\ &\quad + \lambda_{ir}\pi_{x_v(x_{ir}+1)}\end{aligned} \tag{2.8}$$

where $x_v \in \{1, 2, \ldots, y_v - 1, y_v\}, x_{ir} = 0$.

$$\tau_{\mu_v} = \begin{cases} 0, & \text{if } x_v = y_v \\ 1, & \text{else} \end{cases}$$

$$\psi_{\lambda_v} = \begin{cases} 0, & \text{if } x_v = y_v \\ 1, & \text{else} \end{cases}$$

$$\begin{aligned} [\tau_{\mu_r}(\mu_r - \beta_r\mu_r - \beta_r x_{ir}) + \mu_v + \lambda_{ir}]\pi_{x_v x_{ir}} = \lambda_v \pi_{(x_v+1)x_{ir}} \\ + \psi_{\lambda_{ir}}\lambda_{ir}\pi_{x_v(x_{ir}+1)} \\ + [\mu_r - \beta_r(\mu_r + x_{ir} - 1)]\pi_{x_v(x_{ir}-1)} \end{aligned} \tag{2.9}$$

where $x_v = 0, x_{ir} \in \{1, 2, \ldots, y_{ir} - 1, y_{ir}\}$.

$$\tau_{\mu_r} = \begin{cases} 0, & \text{if } x_{ir} = y_{ir} \\ 1, & \text{else} \end{cases}$$

$$\psi_{\lambda_{ir}} = \begin{cases} 0, & \text{if } x_{ir} = y_{ir} \\ 1, & \text{else} \end{cases}$$

$$\begin{aligned} (\tau_{\mu_v}\mu_v + \tau_{\mu_r}\mu_r + \lambda_v + \lambda_{ir})\pi_{x_v x_{ir}} = \mu_v \pi_{(x_v-1)x_{ir}} + \mu_r \pi_{x_v(x_{ir}-1)} \\ + \psi_{\lambda_v}\lambda_v \pi_{(x_v+1)x_{ir}} + \psi_{\lambda_{ir}}\lambda_{ir}\pi_{x_v(x_{ir}+1)} \end{aligned} \tag{2.10}$$

where $x_v \in \{1, 2, \ldots, y_v\}, x_{ir} \in \{1, 2, \ldots, y_{ir}\}$.

$$\tau_{\mu_v} = \begin{cases} 0, & \text{if } x_v = y_v \\ 1, & \text{else} \end{cases}$$

$$\tau_{\mu_r} = \begin{cases} 0, & \text{if } x_{ir} = y_{ir} \\ 1, & \text{else} \end{cases}$$

$$\psi_{\lambda_v} = \begin{cases} 0, & \text{if } x_v = y_v \\ 1, & \text{else} \end{cases}$$

$$\psi_{\lambda_{ir}} = \begin{cases} 0, & \text{if } x_{ir} = y_{ir} \\ 1, & \text{else} \end{cases}$$

$$\sum_{x_v=0}^{y_v} \sum_{x_{ir}=0}^{y_{ir}} \pi_{x_v x_{ir}} = 1 \tag{2.11}$$

According to the equilibrium state probability of on-hand inventory discussed above, the inventory optimization model of the dual-channel supply chain in equilibrium state under inventory transshipment case can be established. We define inventory holding costs, lost sales costs, transshipment cost and subsidy to obtain the costs of the manufacturer and the retailer:

1) *Inventory holding costs*
For the given equilibrium state probability, the average inventory of the online store and the ith retail store are given by the following equations:

$$I_{iv} = \sum_{x_v=1}^{y_v} \sum_{x_{ir}=0}^{y_{ir}} x_v \pi_{x_v x_{ir}}, I_v = \sum_{i=1}^{N} I_{iv}$$

$$I_{ir} = \sum_{x_v=0}^{y_v} \sum_{x_{ir}=1}^{y_{ir}} x_{ir} \pi_{x_v x_{ir}}$$

Therefore, the average inventory holding costs of the online store and the ith retail store are as follows:

$$C_{Hv} = h_v I_v, C_{Hir} = h_r I_{ir}$$

where h_v and h_r are unit inventory holding costs of the online store and the retail stores, respectively.

2) *Lost sales costs*
When any one channel is out of stock, customers in this channel do not want to shift to the other channel, leading to lost sales; when both the retail channel and online channel are out of stock simultaneously, it will also lead to lost sales. The probabilities that the online store and the ith retail store are out of stock are respectively as follows:

$$L_{iv} = \sum_{x_{ir}=1}^{y_{ir}} \pi_{0x_{ir}}, L_v = \sum_{i=1}^{N} L_{iv}$$

$$L_{ir} = \sum_{x_v=1}^{y_v} \pi_{x_v 0}$$

The probability that these two channels are out of stock simultaneously is as follows:

$$L_{ib} = \pi_{00}, L_b = \sum_{i=1}^{N} L_{ib}$$

Therefore, the average lost sales costs of the online store and the ith retail store are as follows:

$$C_{Lv} = g_v L_v \lambda_v + g_v L_b \lambda_v$$

$$C_{Lir} = g_r L_{ir} \lambda_{ir} + g_r L_{ib} \lambda_{ir}$$

where g_v and g_r are unit lost sales costs of the online store and the retail stores, respectively.

3) *Transshipment cost and subsidy*
In order to realize transshipment cooperation between the manufacturer and the retailer, the transshipment cost and subsidy provided by the manufacturer are as follows respectively:

$$C_t = c_t \beta_r \mu_r L_v, C_s = t \beta_r \mu_r L_v$$

4) *Costs of the manufacturer and the retailer*
The costs of the manufacturer and the retailer are as follows:

$$TC_M(y_v, y_{ir}) = h_v I_v + g_v L_v \lambda_v + g_v L_b \lambda_v + (c_t + t) \beta_r \mu_r L_v$$

$$TC_R(y_v, y_{ir}) = \sum_{i=1}^{N} (h_r I_{ir} + g_r L_{ir} \lambda_{ir} + g_r L_{ib} \lambda_{ir}) - (c_t + t) \beta_r \mu_r L_v$$

Therefore, we can obtain the optimal basic inventory levels of the two channels in equilibrium state under inventory transshipment case through minimizing the above costs, *i.e.*, solving the following multi-objective programming problem.

TABLE 3. The initial values of parameters.

System parameters	Values	Price parameters	Values
Replenishment rate of online store	$\mu_v = 15$	Unit lost sales cost of online store	$g_v = 80$
Replenishment rate of any retail store	$\mu_r = 10$	Unit lost sales cost of any retail store	$g_r = 100$
Total demand rate	$\lambda = 30$	Unit inventory holding cost of online store	$h_v = 25$
Customer shift rate of online channel	$\beta_v = 0.5$	Unit inventory holding cost of any retail store	$h_r = 40$
Transshipment rate of any retail store	$\beta_r = 0.5$	Unit transshipment cost	$c_t = 15$
Preference rate of online channel	$\alpha = 0.5$	Unit transshipment subsidy	$t = 5$
Number of retail stores	$N = 1$		

$$\min TC_M(y_v, y_{ir}) = h_v I_v + g_v L_v \lambda_v + g_v L_b \lambda_v + (c_t + t)\beta_r \mu_r L_v$$
$$\min TC_R(y_v, y_{ir}) = \sum_{i=1}^{N} (h_r I_{ir} + g_r L_{ir} \lambda_{ir} + g_r L_{ib} \lambda_{ir}) - (c_t + t)\beta_r \mu_r L_v \tag{2.12}$$

Proposition 2.2. *If customer arrival rates of N retail stores are the same, then the basic inventory levels of N retail stores are equal, that is, $y_{1r} = y_{2r} = \cdots = y_{Nr} = y_r$, and the corresponding inventory optimization model is as follows:*

$$\min TC_M(y_v, y_r) = h_v I_v + g_v L_v \lambda_v + g_v L_b \lambda_v + (c_t + t)\beta_r \mu_r L_v$$
$$\min TC_R(y_v, y_r) = N h_r I_r + g_r L_r \lambda_r + g_r L_b \lambda_r / N - (c_t + t)\beta_r \mu_r L_v$$

where $I_r = I_{ir}, L_r = L_{ir}$.

The proof of Proposition 2.2 is similar to that of Proposition 2.1.

3. Numerical simulation

Based on the model discussed above, in this section we respectively analyze the influence of customer shift rate and transshipment rate on the optimal inventory strategies and costs of dual-channel supply chain under customer shift and inventory transshipment cases by numerical simulation. It can further prove the rationality of the established model. In addition, the optimal basic inventory levels and costs in the above two cases are compared, in order to further analyze the interaction between inventory competition and cooperation respectively brought by customer shift and inventory transshipment. The initial values of parameters are shown in Table 3. To simplify the calculation, we assume $\lambda_r = N\lambda_{ir}, \forall i \in N$, then the basic inventory levels of N retail stores are equal according to Propositions 2.1 and 2.2 (*e.g.*, [21, 28]).

3.1. The impact of customer shift rate under customer shift case

The customer shift rate of the online store (β_v) fetches different values from 0 and 1, and its step is 0.1. We consider three different values of the online channel preference rate (α), the corresponding basic inventory levels and costs are shown in Figure 5. Figure 5 shows that when online channel's customer preference rate is low which means that the customer preference rate of retail channel is large, as β_v increases, the basic inventory level of the online store is low and keeps unchanged while that of the retail store is large and keeps unchanged. When the customer preference rate of online channel continues to increase, with the increase of β_v, the basic inventory level of the online store is large but keeps decreasing while that of the retail store is low but has an increasing trend. The corresponding costs also have similar results because they are closely related to inventory levels. The analysis can help companies to establish inventory information systems of this situation. When the inventory of an online store is out of stock, online customers can be informed in time to reduce the costs of customer shift.

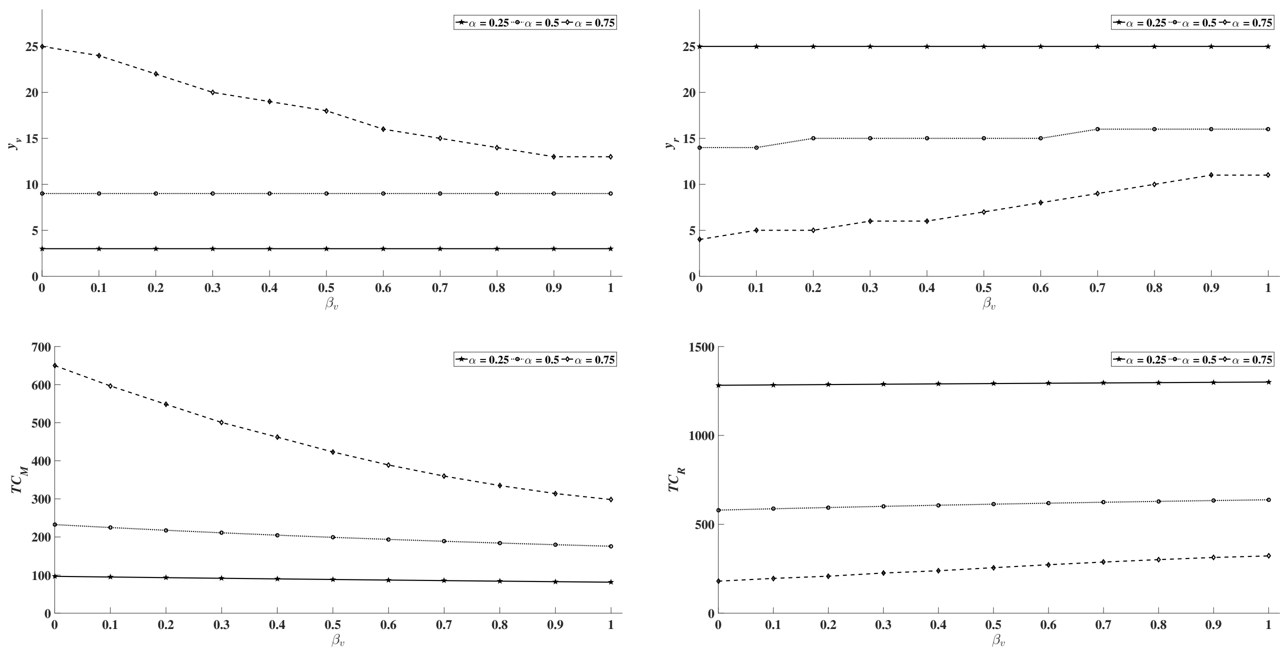

Figure 5. The impact of customer shift rate on the inventory decisions and costs.

3.2. The impact of transshipment rate under inventory transshipment case

The inventory transshipment rate of the retail store (β_r) fetches different values from 0 and 1, and its step is 0.1. We similarly consider three different values of the online channel preference rate (α), the corresponding basic inventory levels and costs are shown in Figure 6. Figure 6 shows that when online channel's customer preference rate is low which means that the customer preference rate of retail channel is large, as β_r increases, the basic inventory level of the online store is low and keeps increasing slowly while that of the retail store is large and keeps unchanged. When the customer preference rate of online channel continues to increase, with the increase of β_r, the basic inventory level of the online store is large and keeps unchanged while that of the retail store is low but has a slow increasing trend. This study can guide companies to optimize inventory scheduling systems to reduce inventory costs.

Next, we mainly discuss the differences of the optimal basic inventory levels and costs in the above two cases by changing the parametric values of the customer preference rate of online channel, retail stores number, demand rate, replenishment rates, holding costs and lost sales costs. Meanwhile, we further analyze the impact of the above related parameters on the optimal inventory decisions and costs to demonstrate sensitivity implications. The outcomes from different scenarios are juxtaposed in Figures 7–12 to illustrate sensitivity implications on the differences between customer shift and inventory transshipment. Note that unless otherwise noted, the same parametric values in Table 3 are used for the study.

3.3. The difference of inventory decisions and costs in the above two cases

1) *The customer preference rate of online channel* (α)

The basic inventory level of the online store in customer shift case is not greater than that in inventory transshipment case while that of the retail store in customer shift case is not less than that in inventory transshipment case, as shown in Figure 7. The corresponding cost of the manufacturer in customer shift case is also not greater than that in inventory transshipment case. However, the cost of the retailer in customer

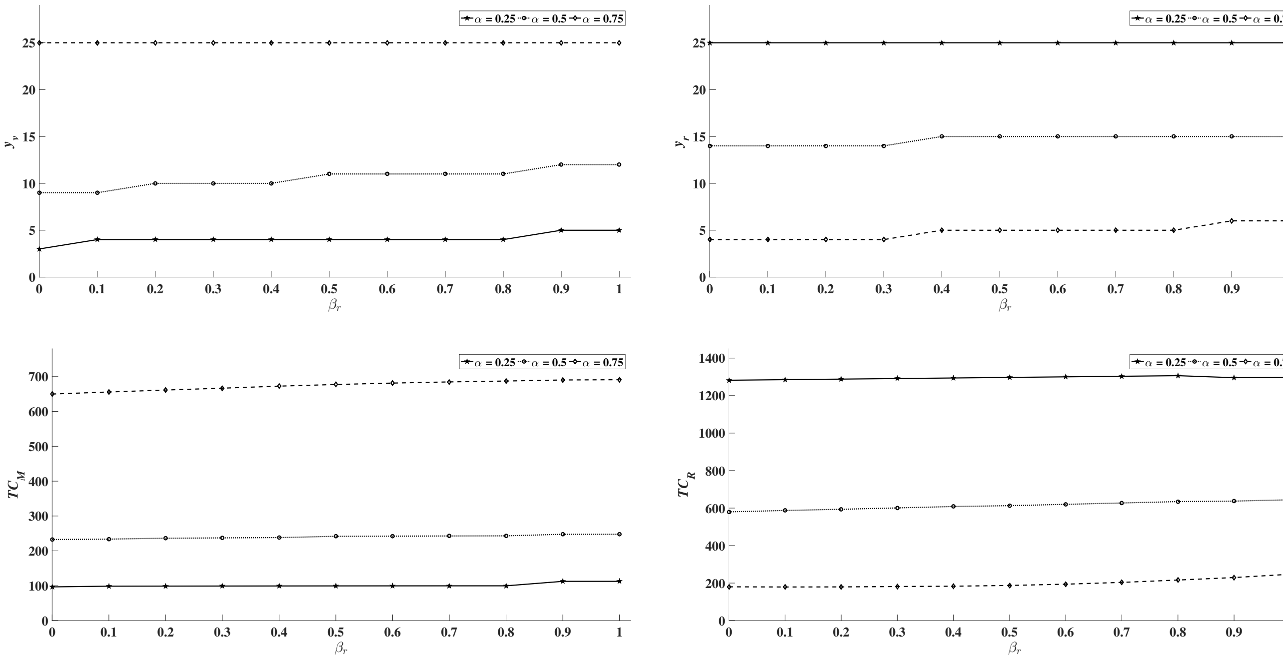

FIGURE 6. The impact of transshipment rate on the inventory decisions and costs.

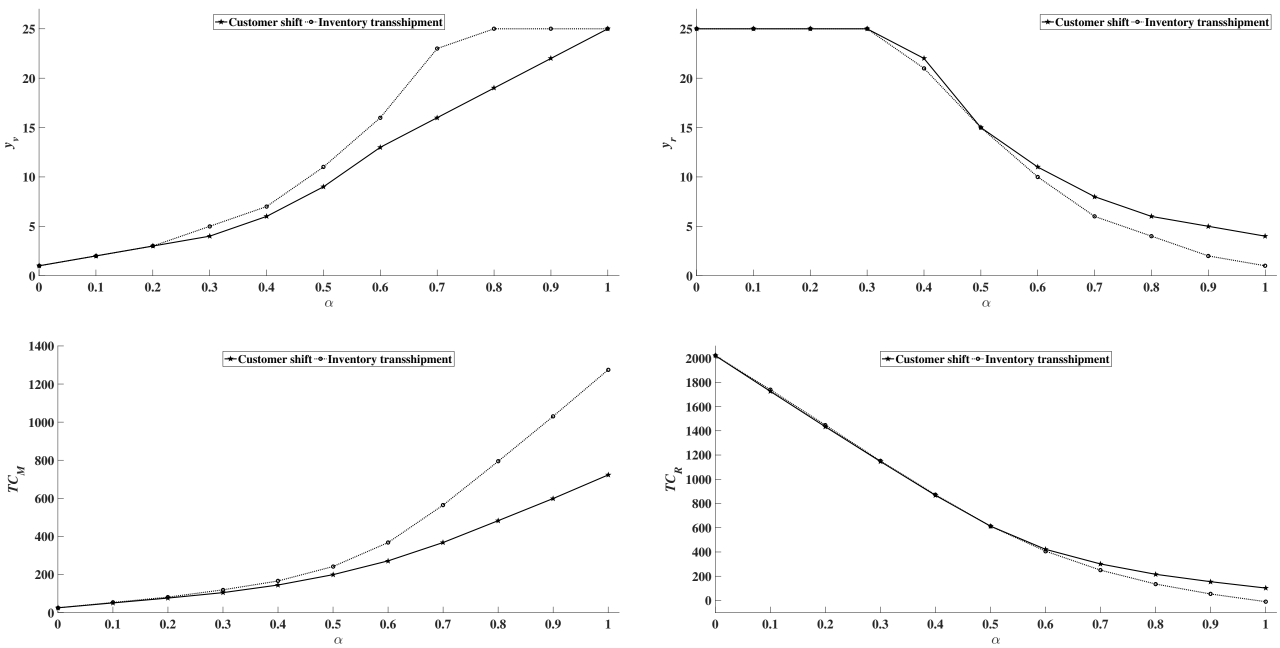

FIGURE 7. Customer preference rate of online channel (α).

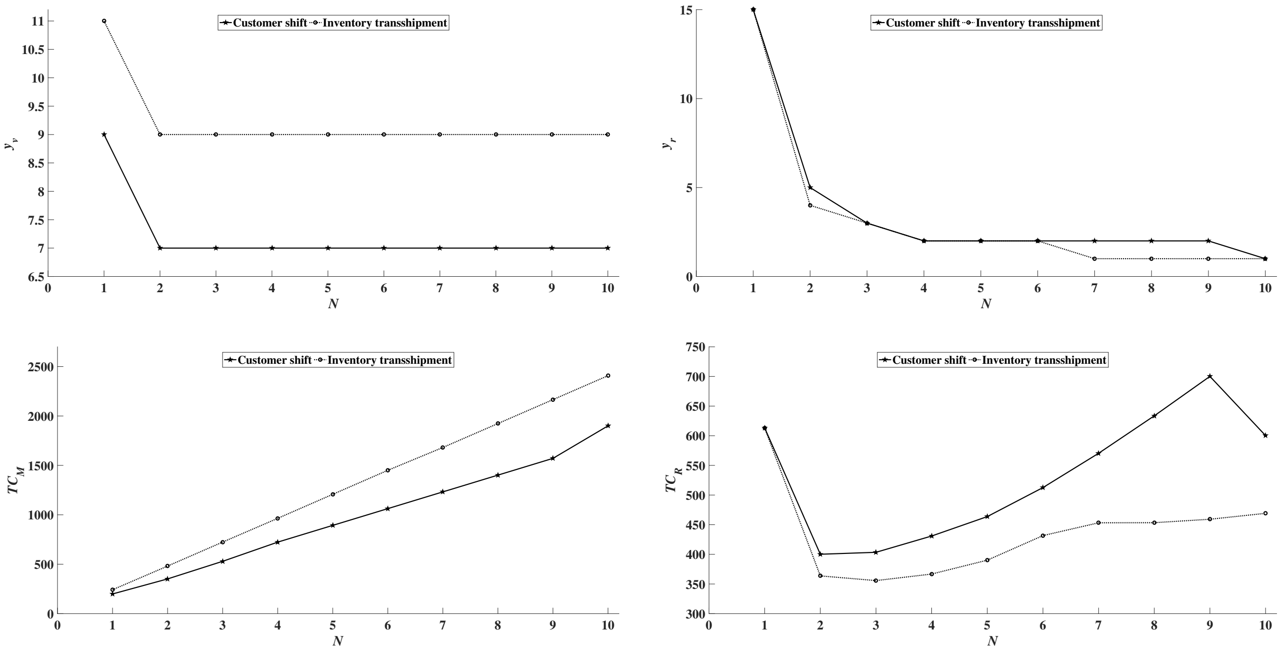

FIGURE 8. Retail stores number (N).

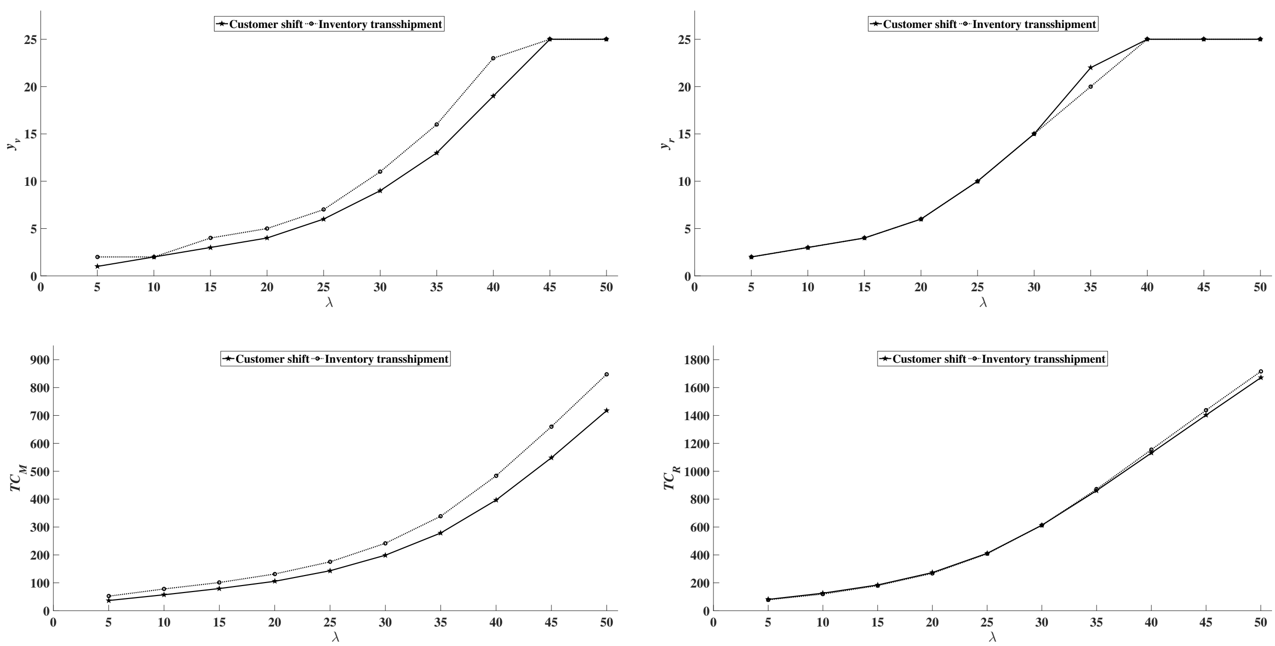

FIGURE 9. Demand rate (λ).

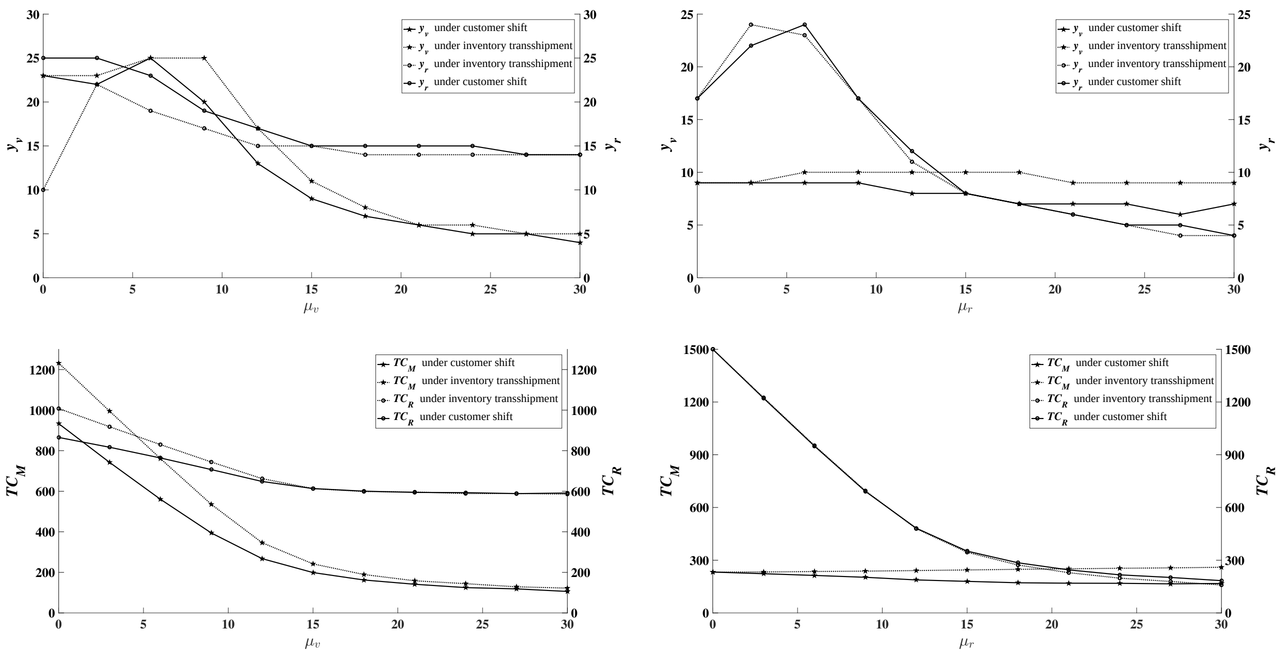

FIGURE 10. Replenishment rates (μ_v, μ_r).

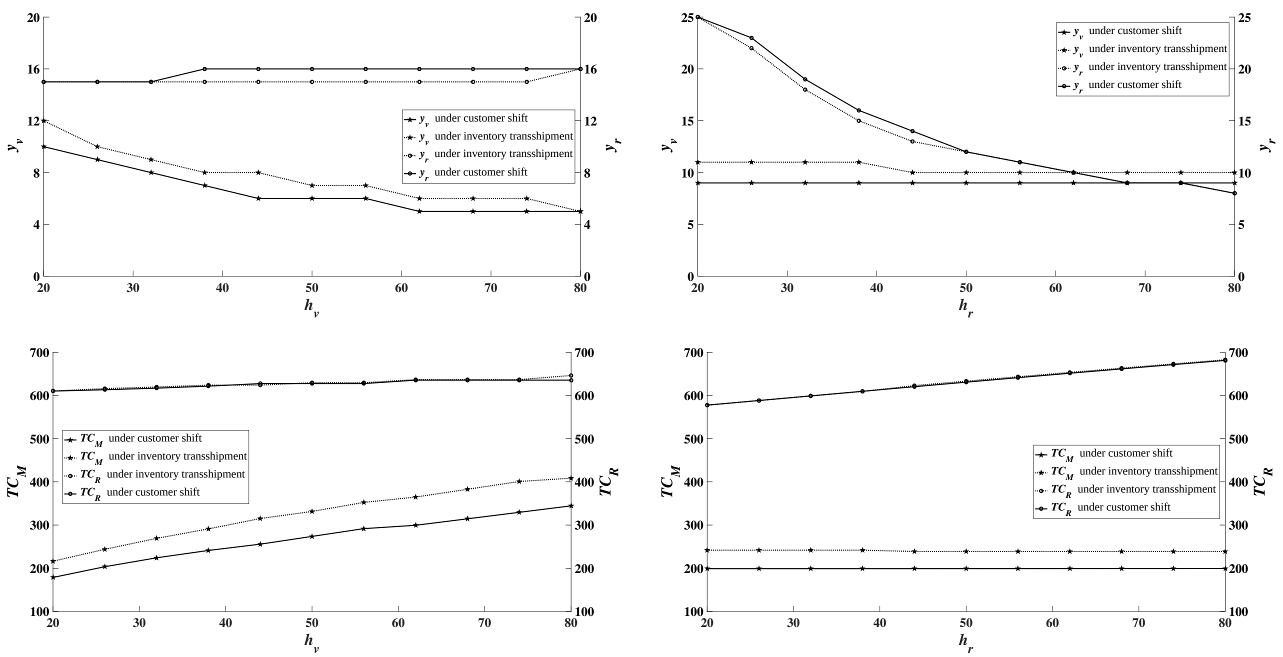

FIGURE 11. Holding costs (h_v, h_r).

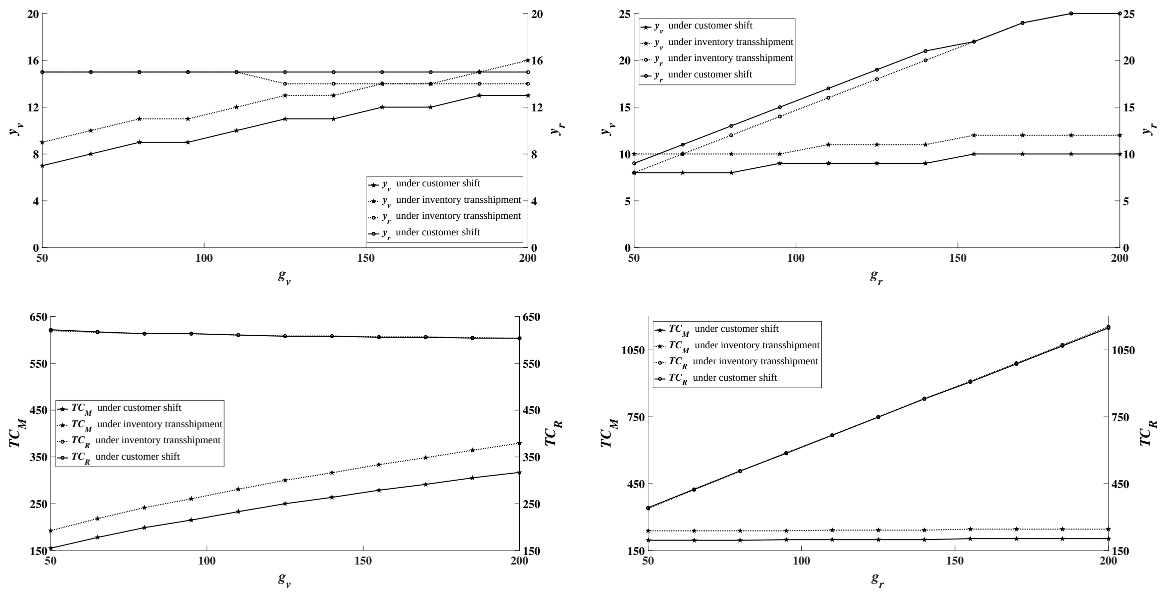

FIGURE 12. Lost sales costs (g_v, g_r).

shift case is less than that in inventory transshipment case when α is small while that of the retailer in customer shift case is greater than that in inventory transshipment case when α is large. In addition, with α increases, the basic inventory level of the online store and the cost of the manufacturer increase while the basic inventory level of the retail store and the cost of the retailer decrease.

2) *The retail stores number* (N)

The basic inventory level of the online store in customer shift case is always less than that in inventory transshipment case while that of the retail store in customer shift case is not less than that in inventory transshipment case, as depicted in Figure 8. The corresponding cost of the manufacturer in customer shift case is also smaller than that in inventory transshipment case while that of the retailer in customer shift case is always greater than that in inventory transshipment case. Moreover, as N increases, although the basic inventory levels of both the online store and the retail store reduce, the cost of the manufacturer increases while that of the retailer first decreases then increases.

3) *The demand rate* (λ)

The basic inventory level of the online store in customer shift case is not greater than that in inventory transshipment case while that of the retail store in customer shift case is almost equal to that in inventory transshipment case, as Figure 9 illustrates. The corresponding cost of the manufacturer in customer shift case is smaller than that in inventory transshipment case. However, the cost of the retailer in customer shift case is greater than that in inventory transshipment case when λ is small while that of the retailer in customer shift case is smaller than that in inventory transshipment case when λ is large. Meanwhile, with λ increasing, the basic inventory levels of the online store and the retail store increase, and the costs of the manufacturer and the retailer also increase.

4) *The replenishment rates* (μ_v, μ_r)

If it is possible to improve the replenishment rate, is it more favorable to have a higher replenishment rate? Figure 10 answers this question. It indicates that the basic inventory level of the online store in customer shift case is not greater than that in inventory transshipment case while that of the retail store in customer shift case is not less than that in inventory transshipment case. The corresponding cost of the manufacturer in customer shift case is smaller than that in inventory transshipment case. However, the difference of the cost of the retailer between customer shift case and inventory transshipment case depends on the replenishment rates. Meanwhile, with the increase of the replenishment rates, both the basic inventory levels and the costs have an approximate downward trend. Therefore, high replenishment rates are desirable if they are available.

5) *The holding costs* (h_v, h_r)
The basic inventory level of the online store in customer shift case is not greater than that in inventory transshipment case while that of the retail store in customer shift case is not less than that in inventory transshipment case, as shown in Figure 11. The corresponding cost of the manufacturer in customer shift case is smaller than that in inventory transshipment case, and that of the retailer in customer shift case is also approximately smaller than that in inventory transshipment case. In addition, with the increase of h_v (resp., h_r), the basic inventory level of the online store (resp., the retail store) reduces while the cost of the manufacturer (resp., the retailer) increases.

6) *The lost sales costs* (g_v, g_r)
The basic inventory level of the online store in customer shift case is always smaller than that in inventory transshipment case while that of the retail store in customer shift case is not less than that in inventory transshipment case illustrated in Figure 12. The corresponding cost of the manufacturer in customer shift case is also smaller than that in inventory transshipment case. However, the cost of the retailer in customer shift case is greater than that in inventory transshipment case when the lost sales costs are small while that of the retailer in customer shift case is smaller than that in inventory transshipment case when the lost sales costs are large. Meanwhile, with the increase of g_v (resp., g_r), the basic inventory level of the online store (resp., the retail store) and the cost of the manufacturer (resp., the retailer) increase.

4. Conclusions

This paper takes unidirectional customer shift and unidirectional inventory transshipment into consideration and establishes inventory optimization models of dual-channel supply chains respectively by Markov theory and optimization theory. By analyzing the impact of customer shift rate of online channel (customers are willingness to switch to the retail channel when a stock-out occurs on online channel) on inventory decisions and costs of dual-channel systems, companies can be guided to optimize inventory information systems, in-time informing online customers about out-of-stock conditions of online channel to reduce customer shift costs. Similarly, by analyzing the effect of inventory transshipment rate of retail channel when online channel is out of stock on inventory decisions and costs, companies can optimize their inventory scheduling systems based on the analysis of this paper to reduce inventory costs.

In this paper, we compare the optimal inventory decisions and costs between customer shift case and inventory transshipment case by changing the values of related parameters. It is not difficult to find that the optimal basic stock level of the online store in customer shift case is not greater than that in inventory transshipment case; the opposite is true for the retail store. The optimal cost of manufacturer in customer shift case is also not greater than that in inventory transshipment, while the cost size of retailer in the both cases depends on the customer preference rate of online channel, the demand rate, the replenishment rates and the lost sales costs. It can be seen that the inventory cooperation brought by inventory transshipment is not necessarily better than the inventory competition brought by customer shift (*e.g.*, [33]).

In addition, we also analyze the impact of related parameters on the optimal inventory decisions and costs to demonstrate sensitivity implications. Obviously, the increase in demand rate will increase the optimal basic inventory levels and costs, while replenishment rates will be the opposite. Therefore, it is important for manufacturer to increase replenishment rate. Retailers should consider replenishment rates as an important factor when choosing a manufacturer. The increase in holding costs leads to a reduction in inventory levels; while the increase in lost sales costs leads to an increase in inventory levels, but the costs both increase. Therefore, companies should try to reduce holding costs and lost sales costs on the basis of strict control of inventory levels. The increase in customer preference rate of online channel will increase online store's inventory levels and costs, while retail stores are decreased. It can be seen that the channel preference rate has a greater impact on inventory decisions. Companies should accurate grasp customer preference in formulating marketing programs so as to avoid inventory shortage or backlog. With the increase in retail stores number, the optimal basic inventory levels of online and retail stores decrease, while the manufacturer's cost increases and the retailer's cost fluctuates. Therefore, retailers need to carefully decide whether to open multiple retail stores.

Because of the complexity of the dynamic inventory problem in this paper, the established models have some potential restrictions. For example, we assume the replenishment inventory strategy is one-for-one, and the demand of customers satisfies a Poisson process. If the replenishment strategy and demand characteristics are different, the results will also be different. In addition, the aim of this paper is to obtain the optimal basic inventory levels under customer shift and inventory transshipment, then the manufacturer can supply to the online store and the retail stores at the beginning period according to this optimal basic inventory levels, which implies there is no restriction on production capacity of the manufacturer. If its production capacity is limited, then the manufacturer will face an inventory allocation problem [31], which is also our future research direction.

Appendix A. Proof of Proposition 2.1

Proof. If customer arrival rates of N retail stores are the same, due to replenishment rate of any retail store and customer shift rate of the online store are the same, then equilibrium state probabilities of N retail stores are the same according to equations (2.1)–(2.5). We already know that unit wholesales price, unit production cost, unit inventory holding cost and unit lost sales cost of different retail stores are the same, respectively, so the basic inventory levels of N retail stores are equal according to equation (2.6), that is, $y_{1r} = y_{2r} = \cdots = y_{Nr} = y_r$.

Correspondingly, the costs of the manufacturer and the retailer are as follows respectively (recall $L_b = \sum_{i=1}^{N} L_{ib}, \lambda_r = \sum_{i=1}^{N} \lambda_{ir}$):

$$TC_M(y_v, y_r) = h_v I_v + g_v(1-\beta_v)L_v\lambda_v + g_v L_b \lambda_v$$

$$\begin{aligned} TC_R(y_v, y_r) &= \sum_{i=1}^{N}(h_r I_r + g_r L_r \lambda_{ir} + g_r L_{ib}\lambda_{ir}) \\ &= N h_r I_r + N g_r L_r \lambda_{ir} + N g_r L_{ib}\lambda_{ir} \\ &= N h_r I_r + g_r L_r \lambda_r + g_r L_b \lambda_{ir} \\ &= N h_r I_r + g_r L_r \lambda_r + g_r L_b \lambda_r / N \end{aligned}$$

where $I_r = I_{ir}, L_r = L_{ir}$.

Therefore, we can obtain the inventory optimization model in this situation.

$$\min TC_M(y_v, y_r) = h_v I_v + g_v(1-\beta_v)L_v\lambda_v + g_v L_b \lambda_v$$

$$\min TC_R(y_v, y_r) = N h_r I_r + g_r L_r \lambda_r + g_r L_b \lambda_r / N$$

□

Acknowledgements. This work was supported by the National Natural Science Foundation of China (Nos. 71431004, 71572033 and 71172174).

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