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INFLUENCE OF CONTROLLABLE LEAD TIME, PREMIUM PRICE, AND UNEQUAL SHIPMENTS UNDER ENVIRONMENTAL EFFECTS IN A SUPPLY CHAIN MANAGEMENT

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Abstract. Recently, carbon emission becomes a major issue during transportation of products from one player to another player. Due to the increasing number of single-setup-multi-delivery (SSMD) policies by several industries, fixed and variable transportation cost and carbon emission cost are considered. The aim of the model is to reduce the total cost of supply chain for controlling the lead time and to diminish setup cost by a discrete investment. A premium cost is introduced and Stackelberg game policy is employed to obtain the analytical solution. Some numerical examples are given to validate the model. Sensitivity analysis and managerial insights are given to show the applicability of the model. Finally, the outcomes show that the model minimizes the optimum cost at the optimal values of the decision variables. It is found that the total cost is minimized when the multi-buyer is leader and vendor is follower.

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1. Introduction

Recently, the main issue of each supply chain is to control environmental problems. Several researchers like Sarkar *et al.* [23] and Sarkar and Saren [31] developed two different models to solve these issues. But both the models are deterministic type with negligible lead time. But due to uncontrolled lead time, those models cannot give proper results. Thus, an improved model is needed to solve the environmental impact with controllable lead time in supply chain management. Therefore, attainability is adopted in the study of supply chains to make an always profitable supply chain. Sarkar [22] assumed some basic assumptions *i.e.*, that a supply chain consists of a single-vendor and single-buyer. In present marketing situation, this is unrealistic. Thus, this model utilizes single-vendor with a multi-buyer by using just-in-time strategy. Dolgui *et al.* [5] introduced a basic inventory model with least setup cost along transfer line design. The authors designed the sequentially operating process. Recently, Sarkar *et al.* [23] incorporated the concept of unequal lot size in the SSMD policy within the supply

Keywords. Supply chain management, unequal shipment size, discrete setup cost reduction, transportation cost, carbon emission cost, Stackelberg game.

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chain environment as SSMUD (single-setup-multi-unequal-delivery). This unequal lot size idea is utilized in proposed model. It is also assumed that the vendor may send finished products sequentially to all buyers in unequal shipment sizes. There are several integrated inventory or supply chain models in which some continuous investments were employed to reduce the setup cost of the model (for instance, Sarkar and Moon [24], Sarkar and Majumder [27]), which is sometime unrealistic in reality. Thus, this model considers a discrete investment to control the setup cost of the supply chain model.

Generally, the lead time was assumed as negligible or constant in all basic inventory models. There are various existing models (Ouyang *et al.* [13], Sarkar and Mahapatra [25]), that considered a stochastic lead time, as in reality it is common that lead time is not constant. Hence, this proposed model also considers a stochastic lead time with the reorder point as a function of the lead time as well as the safety factor. A crashing cost is used to reduce this random lead time, (see Ref. Shin *et al.* [33]).

Due to multiple shipments, there is a possibility of an increasing carbon emission cost. Thus, to reduce this cost, both fixed and variable transportation costs as well as fixed and variable carbon emission costs are utilized within the supply chain model (refer to Sarkar *et al.* [26]). This model develops a two-echelon single-vendor multi-buyer supply chain management by considering SSMUD policy, a controllable lead time, a crashing cost to reduce the lead time, and a discrete investment to reduce setup cost. This model is determined analytically by assuming equality and inequality of power within the supply chain model using game theory. This paper is designed as follows: a literature review is given in Section 2. Section 3 explains the problem definition, notation, and assumptions. Section 4 describes the mathematical model, and the solution methodology is given in Section 5. Numerical experiments and a sensitivity analysis are given in Section 6. Finally, concluding remarks are given in Section 7.

2. Literature review

The setup cost always plays an important role in any production system, but many supply chain models consider the setup cost as fixed or constant. Porteus [14] first considered that a setup cost can be easily reduced by some continuous investment function. This idea is widely used among in the inventory models, but Ouyang *et al.* [13] was the first researcher to introduce this concept within an inventory model. Annadurai and Uthaykumar [1] developed the (Q, R, L) model, where the setup cost reduction was described by a continuous investment with considering defective items. Sarkar and Majumder [27] considered a setup cost reduction by a continuous investment, considering two models with either a normal distribution or an unknown distribution with known mean and standard deviation for the lead time demand in a single-vendor single-buyer model. Sarkar and Moon [24] extended the model of Ouyang *et al.* [13] using the same setup cost reduction technique, but with a variable lead time dependent backorder rate. Shin *et al.* [33] expanded the same setup cost reduction by transportation discounts and service level constrains.

Cárdenas-Barrón [3] wrote a note on supply chain model with transportation cost. Shahrestani *et al.* [30] formulated a heuristic method to solve bi-objective shop scheduling problem. Two-stage hybrid system is constructed in this model but without environmental effect. Taleizadeh *et al.* [35] explained about three-echelon supply chain model with lead time aggregation. Yang and Wee [45] considered an integrated inventory model in this direction. Wahab *et al.* [44] developed a two-level supply chain model with coordination between players. The model stands for perfect and imperfect production under environmental effecting. Jaber *et al.* [8] discussed carbon dioxide emission reduction as a major contribution within the supply chain model. Zanoni *et al.* [49] discussed price-dependent demand which is effected by atmosphere in their model. Koupaei *et al.* [9] explained the multi-objective evolutionary system by an efficient algorithm for flexible manufacturing system. Kim and Son [10] developed a modeling based on agent policy and traffic simulation, where controllable lead time or environmental effect assumption is relayed. Jauhari *et al.* [7] extended the supply chain model by adding unequal shipments policy with cooperative policy and presence of defective items. Bazan *et al.* [2] explained carbon emission and energy effects on manufacturer-retailer integrated inventory model. Recently, Wangsa [41] developed an incentive policy and greenhouse gas penalty system in a supply chain to ensure the environmental effect with reduced total cost. In this direction, Sarkar *et al.* [20, 28, 29] developed several models on setup cost reduction and quality improvement of products.

Inspection is an important process for maintaining the brand image of the manufacturing industry as after inspection, by which it can only be confirmed that the product is perfect or imperfect. If the product is imperfect it can be reworked, and if it is perfect, it can be transported to the market for sale. Teng *et al.* [37] developed a three-stage inspection process to separate scrap products for constructing an integrated vendor-buyer model for the economic lot size by using an algebraic approach, which is extended from the model of Wee and Chung [42]. Taleizadeh *et al.* [36] developed a deterministic multi-production single machine economic production quantity model with single-stage production to ensure the quality of products. The same three-stage inspection strategy is used in Sarkar *et al.* [19], where he used a fixed lifetime product in both centralized and decentralized supply chain models. Based on this situation, to verify the quality of products, this proposed model considers a single-stage inspection process to reduce the existing three-stage inspection cost. Zhou *et al.* [46] established the model based on maintenance planning and energy consumption control. The energy consumption facility is affected by the operation condition, which is closely connected with the associated maintenance policy. Two types of maintenance activities are implemented for the server, *i.e.*, the planned maintenance and the reactive maintenance.

Petajisto [15] introduced an index premium costs for index funds. In real life conditions, there is competition between every business in obtaining more profit than the others. Thus, a premium cost is included in this proposed model. Li *et al.* [12] considered an uncertain premium with respect to the distortion function. Premium cost is included in the model via cell technology. The newly formulated model is for single items with the assumption of premium prices. Shao *et al.* [32] provided a multi-factor model with variable time, where in the computational gas market, risk premiums are included to increase cost value. Chien and Naknoi [4] added a risk premium and global imbalances to reduce the entire system cost. Quaia *et al.* [17] described the economic two-echelon technique known as premium power. Li [11] considered a stock market in the business system as well as a maturity premium to earn more profit at the optimum level. Wang and Huo [43] introduced a premium pricing policy in the fruits-market to reduce the total system cost. Park [16] proposed a premium price construction from Korea's energy efficiency grade label (KEEGL). The Korean government began energy certification of televisions by setting to analyze a possible price-effect of the new label.

In the business market, order quantities are not always equal. They order either unequal quantities or same, number of shifting process equal for all customers. Siajadi *et al.* [34] used only a multiple shipment policy for the supply chain in the distribution system but with equal shipments. Zhou and Wang [47] discussed a single-vendor single-buyer integrated model under an equal shipment policy. The proposed model is developed for a single-vendor multi-buyer system with unequal lot sizes and shipment policy. Roy *et al.* [18] considered an integrated model for imperfect items under the assumption of shortage. Hariga *et al.* [6] developed a vendor's inventory system by assuming unequal shipments as well as a space constraint.

Every manufacturing industry is formulated on the basis of three main purposes: collection of raw materials, manufacturing of finished products, and distribution of products to customers. Therefore, transporting ways play a vital role for all production systems. Vroblefski *et al.* [40] discussed several transportation policies for consequently delivering to the warehouse with equal shipments. Zhao *et al.* [48] considered an improved algorithm by mentioning fixed transportation cost. It was found that previous research considered either fixed or variable transportation costs. Variable transportation may arise when the transportation system fails during service time due to uncertain disruptions. The proposed model highlights both fixed and variable transportation costs. Based on this concept, Sarkar [21] discussed a two-layer supply chain model with fixed and variable transportation costs. In his model, he did not consider carbon emissions due to transportation during multi-delivery. Sarkar *et al.* [23] continued the same idea of variable and fixed transportation with unequal lot sizes within a deterministic environment. Tiwari *et al.* [38] investigated the application in supply chain management of the six years details. They provided managerial beneficiates to industries from their research. The authors survived new strategies to investigate how business managers can produce and organize data. A integrated inventory model was discussed recently in this paper Tiwari *et al.* [39]. The paper considered imperfect items with deterioration cost. Further,

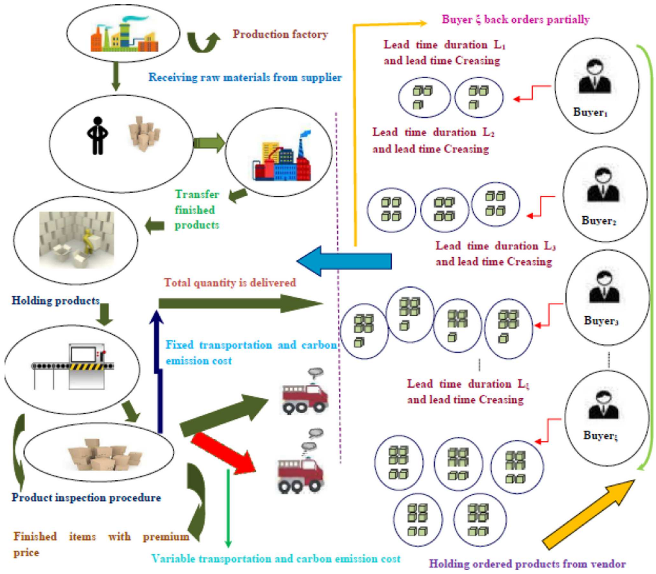

FIGURE 1. Diagram of single-vendor-multi-buyer two-echelon supply chain model.

TABLE 1. Contribution of different authors.

Author(s)	Discrete setup cost	Inspection cost	Premimum cost	Unequal delivery lot size	Transportation cost	Carbon emission cost
Chien and Naknoi *et al.* [4]			√			
Li [11]			√			
Li *et al.* [12]			√			
Ouyang *et al.* [13]	√					
Petajisto [15]			√			
Roy *et al.* [18]				√		
Sarkar *et al.* [29]	√	√		√		√
Quaia *et al.* [36]			√			
Sarkar *et al.* [42]					√	√
Wang and Huo [43]			√			
This paper	√	√	√	√	√	√

it was introduced some environmental costs to reduce impacts of the industry and profits of two parties were calculated with considering variables. See Table 1 for different author's contributions.

3. Problem definition, notation, and assumptions

The following section consists of problem definition, notation, and assumptions.

3.1. Problem definition

The aim of this model is to reduce the total system cost of a two-echelon supply chain with a single-vendor multi-buyer under the effects of environmental issues. The vendor produces products and transports the ordered products to the multi-buyer according to the SSMUD policy after inspection of all products. Those products, which are considered defective after inspection, are discarded from the system. Due to multi-delivery, there is an increasing cost for transportation and carbon emission, thus this model considers both fixed and variable costs for these to show the effect of reality. The lead time is considered as random variable, which follows a normal distribution and in the second model, it is considered with unknown distribution with known mean and variance. To reduce the setup cost, discrete investment is used and the optimum investment is calculated. The players in the supply chain are always unequal in power, thus Stackelberg game policy is utilized to solve this model. A major contribution of this model is that it considers the premium cost, which is the best matching strategy for attaching customers with their products.

3.2. Notation

3.2.1. Index

i component for lead time with minimum duration ($i = 1, 2, \ldots, n$)
j component for lead time with normal duration ($j = 1, 2, \ldots, n$)
ξ number of buyers ($\xi = 1, 2, \ldots, m$)
ζ number of investment ($\zeta = 1, 2, \ldots, y$)

3.2.2. Decision variables

K_ζ investment of the vendor per setup ($/setup)
n_ξ unequal number lots for multi-buyer in one cycle (positive integer)
q_ξ quantity ordered per delivery (units)
α increasing rate of shipment lot size (positive integer)
L_ξ length of the lead time for a buyer-ξ (weeks)
k_ξ safety factor

3.2.3. Parameters

Q_ξ total quantity ordered by multi-buyer (units)
S_0 initial setup cost of the vendor per setup ($/setup)
T cycle length (year)
C_v unit production cost paid by the vendor ($/unit)
r_v holding cost of the vendor per unit per unit time ($/unit/unit time)
P replenishment rate per unit time (units/year)
P_r premium cost of vendor (units/year)
S_b fixed carbon emission cost ($/shipment)
$V_{b\xi}$ variable carbon emission cost ($/unit)
I_s inspection cost of the vendor ($/units)
F fixed transportation cost ($/shipment)
V_ξ variable transportation cost ($/unit)
R_ξ reorder point of the buyer ξ (units)
D_ξ average demand per unit time of the buyer ξ (units/year)
A_ξ ordering cost of the buyer ξ per order ($/order)
$C_{b\xi}$ unit purchasing cost paid by ξ-buyers ($/unit)
$r_{b\xi}$ holding cost of the buyer ξ per unit per unit time ($/unit/unit time)
Π_{jai} unit backorder cost for the buyer ξ ($/unit shortage)
σ_ξ standard deviation of the lead time demand per time
X lead time demand
$E(.)$ mathematical expectation
$x^{(+)}$ maximum value of x and 0

3.3. Assumptions

The following assumptions are considered for this model.

1. A two-echelon single-vendor multi-buyer supply chain model is considered for single-type of item.
2. The vendor sequentially sends to the ordered products to buyer $\xi; (\xi = 1, 2, \ldots, m)$.
3. During production, the vendor starts with an initial setup cost of S_0. This cost can be reduced by using a capital investment. It is natural that the investment needed to reduce the setup cost is discrete in nature. The discrete setup cost with the investment function is expressed as $S = S_0 e^{-rK_\zeta}$, where K_ζ $(\zeta = 1, 2, \ldots, y)$ is a strictly decreasing function with $K_0 = 0$ and r is a known parameter.
4. The vendor inspects the total manufacturing lot before delivering items in a single cycle. At the end of the inspection process, the vendor delivers only perfect products to m-buyers, and defective products are discarded from the system with some negligible cost.
5. There is a tendency for buyers to expect item's quality as with to the brand image of the vendor. To maintain the quality of each product, a premium cost is utilized by the vendor for each product.
6. For carrying finished goods, some transportation costs are necessary. Therefore, fixed transportation cost is assumed due to fixed transportation. If the amount of transportation is increased due to sudden disruption during loading-unloading time or transporting time, a variable transportation cost is included. In a similar way, a fixed carbon emission cost is considered to calculate the carbon emission with respect to a fixed transportation time. During a variable transportation time, the variable carbon emission is considered. Therefore, both fixed and variable transportation and carbon emission costs are included.
7. The vendor produces a total of Q_ξ items with respect to the number of buyers ξ at a finite production rate P $(P > D_\xi)$ at a single setup. Then, these quantities Q_ξ are delivered with unequal shipments to the ξ buyers according to the (SSMUD) policy.
8. A transportation process of lot size Q_ξ is considered at n_ξ distinct times with unequal lots within each shipment q_ξ to the ξ buyers. An increasing delivery rate in the SSMUD policy is considered, denoted by α.
9. The reorder point for the ξth buyer is given by $R_\xi = D_\xi L_\xi + k_\xi \sigma_\xi \sqrt{L_\xi}$. Here, $D_\xi L_\xi$ is the expected demand during the lead time, k_ξ is a safety factor, and $k_\xi \sigma \sqrt{L_\xi}$ is a safety stock.
10. Shortages due to partial backordering are considered.
11. The lead times L_m are mutually independent components from each other and c_m is the crashing cost corresponding to the ξth buyer. For m components, a_m = minimum duration, b_m = normal duration, and c_m = crashing cost per unit time with the condition $c_1 \leq c_2 \leq \ldots \leq c_m$ are considered.
12. $L_0 \equiv \sum_{j=1}^{m} b_j$. If L_i is the length of the lead time with components lowered to their minimum duration, then L_i can be expressed as $L_i = L_0 - \sum_{j=1}^{i}(b_j - a_j)$. The lead time crashing cost per cycle $C_\xi(L_\xi)$ is expressed as $C_\xi(L_\xi) = c_i(L_{i-1} - L_\xi) + \sum_{j=1}^{i-1} c_j(b_j - a_j)$.

4. Mathematical model

The mathematical model is formulated for two players (vendor and multi-buyer) which is given bellow.

4.1. Mathematical model of vendor

The production house of vendor has received raw materials from suppliers and produced finish products and the vendor sends those finished products for the multi-buyer. Therefore, the vendor produces Q_ξ lots corresponding to the multi-buyer in one cycle. A single-setup-multi-unequal-delivery (SSMUD) policy is used to transfer those products. A product's order-delivery is considered with n_ξ unequal shipments and an increasing rate α. Further, quantities are also in different shipment lots q_ξ.

For the buyer ξ, the n_ξth delivery is $(n_\xi - 1)\alpha q_\xi$, $n_\xi > 1$, *i.e.*, the second shipment lot size is αq_ξ. After that, the delivery lot sizes are $2\alpha q_\xi$, $3\alpha q_\xi$, and so on.

Therefore, per production cycle, the total quantities are transported from the vendor to the buyer ξ, summing the shipment lots as follows:

$$q_\xi + \alpha q_\xi + 2\alpha q_\xi + \ldots + (n_\xi - 1)\alpha q_\xi = q_\xi + \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2}. \tag{4.1}$$

Now, the cycle length of the vendor is obtained as

$$\sum_{\xi=1}^{m} \frac{q_\xi}{P} + \sum_{\xi=1}^{m} \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2(P - D_\xi)} = \sum_{\xi=1}^{m} \left[\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2(P - D_\xi)} \right]. \tag{4.2}$$

The following costs are calculated to obtain the total cost for the vendor.

4.1.1. Setup cost

The vendor considers an initial setup cost K_ζ during production. Therefore, the total setup cost of the vendor is

$$\sum_{\zeta=1}^{y} \sum_{\xi=1}^{m} \frac{K_\zeta}{\left[\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2(P - D_\xi)} \right]}. \tag{4.3}$$

Again, it can be calculated after the effect of capital investment on the initial setup cost. Therefore, to reduce the setup cost per setup, the discrete investment is $S_0 e^{-rK_\zeta}$, where K_ζ is a strictly decreasing function, $K_0 = 0$, and r is a known parameter.

Therefore, the discrete setup cost is $\sum_{\zeta=1}^{y} \sum_{\xi=1}^{m} \frac{S_0 e^{-rK_\zeta}}{\left[\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2(P - D_\xi)} \right]}$.

Thus, the total initial setup cost of the vendor along with investment is

$$\sum_{\zeta=1}^{y} \sum_{\xi=1}^{m} \left(\frac{K_\zeta}{\left[\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2(P - D_\xi)} \right]} + \frac{S_0 e^{-rK_\zeta}}{\left[\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2(P - D_\xi)} \right]} \right). \tag{4.4}$$

4.1.2. Holding cost

The vendor generally makes Q_ξ items corresponding to ξ buyers and these products are kept in stock. The products are not divided equally for distribution to the m buyers. Both the number of shipments and the lot size are distinct, where each buyer receives a quantity $Q_\xi = \frac{2q_\xi + q_\xi \alpha n_\xi (n_\xi - 1)}{2}$ from the vendor.

Therefore, the inventory level average of the vender is obtained by subtract the ξ-buyers' total inventory from the vendor's total inventory.

From the Figure 2, the dotted area is calculated by dividing it into three parts. The vendor has $\sum_{\xi=1}^{m} Q_\xi$ finished products, during a unit of time $\frac{Q_\xi}{P}$ and the remaining $(n_\xi - 1)$ number of cycles.

The above part in Figure 2 shows a rectangular area and therefore, the area is

$$\left[\sum_{\xi=1}^{m} Q_\xi \right] \left[\sum_{\xi=1}^{m} \frac{q_\xi}{P} + \sum_{\xi=1}^{m} \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2D_\xi} \right]. \tag{4.5}$$

In following triangular section is calculated as

$$\begin{aligned} &= \frac{1}{2} \left[\sum_{\xi=1}^{m} Q_\xi \right] \left[\sum_{\xi=1}^{m} \frac{Q_\xi}{P} \right] \\ &= \frac{1}{2P} \left(\sum_{\xi=1}^{m} Q_\xi \right)^2. \end{aligned} \tag{4.6}$$

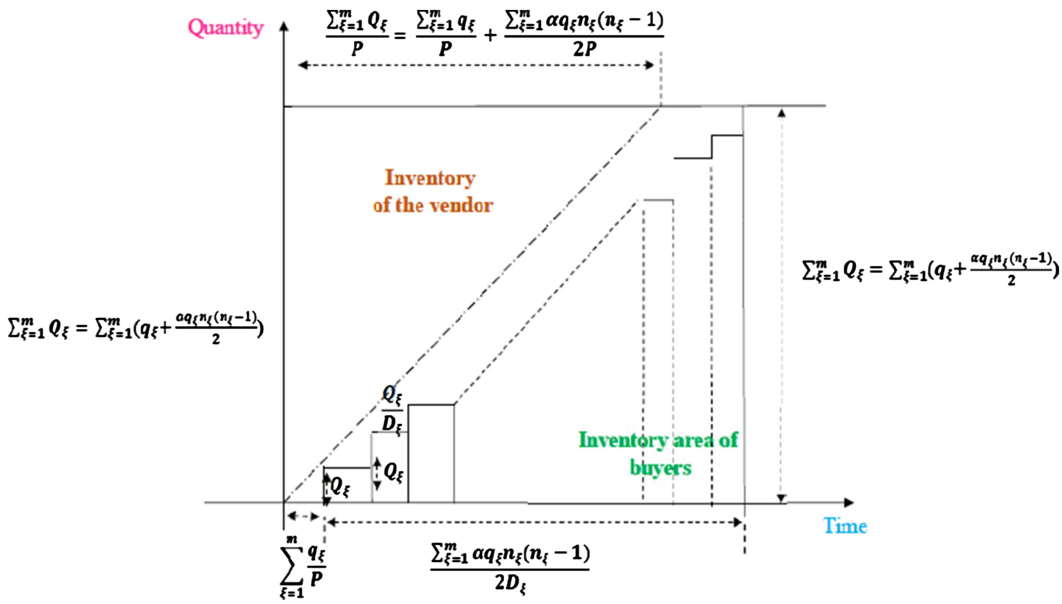

FIGURE 2. Supply chain system of SSMUD policy.

Accumulated inventory by shaded area is

$$= \frac{1}{2D_\xi}\left(\sum_{\xi=1}^{m} q_\xi\right)^2 + \frac{\alpha^2}{2D_\xi}\left(\sum_{\xi=1}^{m} q_\xi\right)^2 + \frac{(2\alpha)^2}{2D_\xi}\left(\sum_{\xi=1}^{m} q_\xi\right)^2 \ldots \frac{((n_\xi-1)\alpha)^2}{2D_\xi}\left(\sum_{\xi=1}^{m} q_\xi\right)^2$$

$$= \frac{1}{2D_\xi}\left(\sum_{\xi=1}^{m} q_\xi\right)^2 + \frac{\alpha^2}{2D_\xi}\left[1+2^2+3^2+\ldots(n_\xi-1)^2\right]$$

$$= \frac{1}{2D_\xi}\left(\sum_{\xi=1}^{m} q_\xi\right)^2 + \frac{\alpha^2}{12D_\xi}\left(\sum_{\xi=1}^{m} q_\xi\right)^2 n_\xi(n_\xi-1)(2n_\xi-1)$$

$$= \frac{1}{2D_\xi}\left(\sum_{\xi=1}^{m} q_\xi\right)^2\left[1+\frac{\alpha^2 n_\xi(n_\xi-1)(2n_\xi-1)}{6}\right]. \tag{4.7}$$

The vendor's total holding cost per unit time is obtained

$$= \frac{r_v C_v}{\left(\sum_{\xi=1}^{m}\frac{q_\xi}{P}+\sum_{\xi=1}^{m}\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\left(\left[\sum_{\xi=1}^{m} Q_\xi\right]\left[\sum_{\xi=1}^{m}\frac{q_\xi}{P}+\sum_{\xi=1}^{m}\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2D_\xi}\right] - \frac{1}{2P}\left(\sum_{\xi=1}^{m} Q_\xi\right)^2 - \frac{1}{2D_\xi}\left(\sum_{\xi=1}^{m} q_\xi\right)^2\left[1+\frac{\alpha^2 n_\xi(n_\xi-1)(2n_\xi-1)}{6}\right]\right). \tag{4.8}$$

4.1.3. Inspection cost

After the production of items is finished, the vendor investigates every finished product before distributing them to m buyers. This is because, there may be a chance of not fulfilling buyers' orders without inspection. We consider an inspection cost I_s and the total inspection cost is obtained by multiplying the quantity by I_s and the inspection cost per unit cycle is

$$\sum_{\xi=1}^{m}\left(I_s\frac{Q_\xi}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\right). \tag{4.9}$$

4.1.4. Premium cost

Finally, preparing products after inspection, the vendor introduces a minimum cost to highlight these products to m buyers as well as to obtain more profit. It is a business technique to understand a buyers' mind. The vendor marks a premium cost P_r and the total premium cost is simplified by multiplying the quantity by P_r. Therefore, the premium price per unit cycle is

$$\sum_{\xi=1}^{m}\left(P_r\frac{Q_\xi}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\right). \tag{4.10}$$

4.1.5. Transportation cost

To transport the ordered products, fixed transportation costs $n_\xi F$ within n_ξ shipments during a fixed transporting time are considered. Then, the fixed transportation cost per unit cycle is

$$\sum_{\xi=1}^{m}\left(\frac{n_\xi F}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\right).$$

When there is any obstruction during transportation or delivery, there is a variable transportation cost V_ξ. Thus, the cost for variable transportation is obtained per cycle as $\sum_{\xi=1}^{m} V_\xi Q_\xi$.

Now, both fixed and variable transportation costs are given as

$$\sum_{\xi=1}^{m}\left(\frac{n_\xi F}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}+\frac{V_\xi Q_\xi}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\right). \tag{4.11}$$

4.1.6. Carbon emission cost

In a fixed transportation time, for n_ξ shipments, a fixed carbon emission cost $n_\xi S_b$ is allowed for the entire production cycle and the corresponding variable carbon emission cost is $V_{b\xi}$. The variable carbon emission cost per shipment is $\sum_{\xi=1}^{m} V_{b\xi} Q_\xi$.

Then, the fixed and variable carbon emission costs are expressed as

$$\sum_{\xi=1}^{m}\left(\frac{n_\xi S_b}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}+\frac{V_{b\xi} Q_\xi}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\right). \tag{4.12}$$

Therefore, the vendor's total cost function $\mathrm{TVC}(n_\xi, q_\xi, \alpha, K_\zeta)$ is given by

$$\begin{aligned}\mathrm{TVC}(n_\xi, q_\xi, \alpha, K_\zeta) = \sum_{\zeta=1}^{y}\sum_{\xi=1}^{m}\Bigg[\Bigg(&\frac{K_\zeta}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}+\frac{S_0 e^{-rK_\zeta}}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\Bigg)\\
&+\frac{r_v C_v}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\Bigg(\left(\sum_{\xi=1}^{m}Q_\xi\right)\Big(\sum_{\xi=1}^{m}\frac{q_\xi}{P}\\
&+\sum_{\xi=1}^{m}\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2D_\xi}\Big)-\frac{1}{2P}\left(\sum_{\xi=1}^{m}Q_\xi\right)^2\\
&-\frac{1}{2D_\xi}\Big(\sum_{\xi=1}^{m}q_\xi\Big)^2\Big[1+\frac{\alpha^2 n_\xi(n_\xi-1)(2n_\xi-1)}{6}\Big]\Bigg)\\
&+I_s\frac{Q_\xi}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}+P_r\frac{Q_\xi}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\\
&+\left(\frac{n_\xi F}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}+\frac{V_\xi Q_\xi}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\right)\\
&+\left(\frac{n_\xi S_b}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}+\frac{V_{b\xi} Q_\xi}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\right)\Bigg].\end{aligned} \tag{4.13}$$

4.2. Multi-buyers' model

There are m buyers whose order sizes are different. The vendor sends products with unequal shipments. This is due to the controllable lead time, which follows a normal distribution. Shortages exist and shipments are partially backordered when a lead time crashing cost is used to reduce. During the final products are delivered to m buyers, vendor sends q_1 shift in first lot. Then, the second shipment lot size is αq_2 at the increasing rate of α. Therefore, number of shipment lotsizes are $2\alpha q_3$, $3\alpha q_4$, and so on. Thus, n_ξth delivery is $(n_\xi - 1)\alpha q_\xi$ $(n_\xi > 1)$. Summing the shipment lots as follows:

$$q_1 + \alpha q_2 + 2\alpha q_3 + \ldots + (n_\xi - 1)\alpha q_\xi = q_\xi + \frac{\alpha q_\xi n_\xi(n_\xi - 1)}{2}. \tag{4.14}$$

Now, the production cycle length of the multi-buyer is obtained as

$$\frac{q_\xi + \alpha\frac{n_\xi(n_\xi-1)}{2}q_\xi}{D_\xi} = \frac{(2q_\xi + q_\xi\alpha n_\xi(n_\xi - 1))}{2D_\xi}. \tag{4.15}$$

4.2.1. Ordering cost

The m buyers' production cycle time length is expressed as $\frac{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))}{2D_\xi}$. The m buyers consider their ordering costs given by A_ξ. Therefore, the ordering cost per unit time is

$$\sum_{\xi=1}^{m}\frac{2A_\xi D_\xi}{(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))}. \tag{4.16}$$

4.2.2. Holding cost

When the inventory level reaches at reorder points R_ξ, the order quantities of the buyer ξ $\frac{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))}{2}$ are shifted. Before receiving an order, the expected inventory level is $R_\xi - D_\xi L_\xi$ and the expected inventory level after a delivery of $\frac{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))}{2}$ is $\frac{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))}{2} + (R_\xi - D_\xi L_\xi)$. Then, the average inventory in one cycle is $\frac{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))}{4} + (R_\xi - D_\xi L_\xi)$.

Also, the ξ buyers' holding cost per unit time is

$$\sum_{\xi=1}^{m} r_{b\xi} C_{b\xi}\left[\frac{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))}{4} + (R_\xi - D_\xi L_\xi)\right]. \tag{4.17}$$

4.2.3. Backordering cost

The lead time demand X follows a normal distribution with mean $D_\xi L_\xi$ corresponding to the buyer ξ and a standard deviation of $\sigma_\xi\sqrt{L_\xi}$. X has a cumulative distribution function F and the reorder point $R_\xi = D_\xi L_\xi + k_\xi \sigma_\xi \sqrt{L_\xi}$. If $X > R_\xi$, partially backorder occur. Therefore, the shortage at the end of the cycle is $E(X - R_\xi)^+ = \int_{R_\xi}^{\infty}(x - R_\xi)\mathrm{d}F(x)$.

The expected shortage costs per unit time is $\frac{2\Pi_{\text{jai}} D_\xi E(X-R_\xi)^+}{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))}$.

The expression $E(X - R_\xi)^+$ is calculated as follows:

$$\begin{aligned} E(X-R_\xi)^+ &= \int_{R_\xi}^{\infty}(x-R_\xi)\mathrm{d}F(x) \\ &= \sigma_\xi\sqrt{L_\xi}\psi_\xi(k_{\text{jai}}). \end{aligned} \tag{4.18}$$

where $\psi_\xi(k_{\text{jai}}) = \phi_\xi(k_{\text{jai}}) - k_\xi[1 - \Phi_\xi(k_{\text{jai}})]$, ϕ_ξ = the standard normal probability density functions, and Φ_ξ = the cumulative distribution functions of the normal distribution of buyer ξ. The safety factor to be a decision variable with respect to R_ξ.

4.2.4. Lead time crashing cost

The lead time crashing cost per unit cycle is

$$\frac{2D_\xi C_\xi(L_{\text{jai}})}{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))}. \tag{4.19}$$

This cost is used to reduce the lead time.

Therefore, the m buyers' total cost function is

$$\begin{aligned} TB_mC(n_\xi, q_\xi, \alpha, k_\xi, L_\xi) = \sum_{\xi=1}^{m}\Bigg(&\frac{2A_\xi D_\xi}{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))} \\ &+ r_{b\xi}C_{b\xi}\left(\frac{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))}{4} + k_\xi\sigma_\xi\sqrt{L_\xi}\right) \\ &+ \frac{2\Pi_{\text{jai}} D_\xi \sigma_\xi\sqrt{L_\xi}\psi_\xi(k_{\text{jai}})}{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))} \\ &+ \frac{2D_\xi C_\xi(L_{\text{jai}})}{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))}\Bigg). \end{aligned} \tag{4.20}$$

To solve the model, there are two possible cases the players of the supply chain have equal power or have unequal power. The model is solved using the Stackelberg game policy within unequal power, and there are two cases: the first case is that the vendor is the leader and the multi-buyer is the follower, and the second case is the reverse.

5. Solution methodology

Based on equality and inequality of powers within players of the supply chain, there are two cases, where Case 1 is unequal powers of the supply chain players and Case 2 is equal powers of the supply chain players.

Case 1: Unequal powers for the supply chain players

Within unequal powers of supply chain players, there are two sub-cases, where Subcase 1.1 is as the vendor as the leader and the multi-buyer as the follower and Subcase 1.2 is as the multi-buyer as the leader and the vendor as the follower.

Subcase 1.1: Vendor as leader and multi-buyer as the follower

In this subcase, the vendor is the leader, thus the vendor is watching the optimum values of all decision variables of the multi-buyer. Observing these optimum values and using these data, the vendor optimizes his total cost to reach the optimum level. Thus, the optimization will start from back substitution process as follows:

The partial derivatives of $TB_mC(n_\xi, q_\xi, \alpha, k_\xi, L_\xi)$ with respect to q_ξ, k_ξ, and L_ξ where α is not continuous variable, are as

$$\frac{\partial TB_mC(n_\xi, q_\xi, \alpha, k_\xi, L_\xi)}{\partial q_\xi} = -\frac{W_1}{W_2}\frac{1}{q_\xi^2} + \frac{r_{b\xi}C_{b\xi}}{4}W_2 \tag{5.1}$$

$$\frac{\partial TB_mC(n_\xi, q_\xi, \alpha, k_\xi, L_\xi)}{\partial k_\xi} = r_{b\xi}C_{b\xi}\sigma_\xi\sqrt{L_\xi} + \frac{2\Pi_\xi D_\xi \sigma_\xi\sqrt{L_\xi}}{(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))}\left(\Phi_\xi(k_{\text{jai}}) - 1\right) \tag{5.2}$$

$$\frac{\partial TB_mC(n_\xi, q_\xi, \alpha, k_\xi, L_\xi)}{\partial L_\xi} = \frac{\Pi_{\text{jai}} D_\xi \sigma_\xi \psi_\xi(k_{\text{jai}})}{(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))}\frac{1}{\sqrt{L_\xi}} + r_{b\xi}C_{b\xi}k_\xi\sigma_\xi\frac{1}{2\sqrt{L_\xi}} + \frac{2D_\xi}{(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))}\frac{\partial C_\xi(L_{\text{jai}})}{\partial L_\xi}. \tag{5.3}$$

The values of these parameters q_ξ, n_ξ, and k_ξ can be obtained by setting these derivatives equal to zero as follows:

$$q_\xi = \frac{2}{W_2}\sqrt{\frac{W_1}{r_{b\xi}C_{b\xi}}} \tag{5.4}$$

$$\Phi_\xi(k_{\text{jai}}) = 1 - \frac{r_{b\xi}C_{b\xi}(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))}{2\Pi_{\text{jai}}D_\xi}. \tag{5.5}$$

The second order partial derivative of L_ξ is

$$\frac{\partial^2 TB_mC(n_\xi, q_\xi, \alpha, k_\xi, L_\xi)}{\partial L_\xi^2} = -\frac{D_\xi}{2}\Pi_\xi\sigma_\xi\psi_\xi(k_{\text{jai}})L_\xi^{-3/2} + \frac{2D_\xi}{(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))}\frac{\partial^2 C_\xi(L_\xi)}{\partial L_\xi^2} - \frac{1}{4}r_{b\xi}C_{b\xi}k_\xi\sigma_\xi L_\xi^{-3/2} < 0. \tag{5.6}$$

The second term derivative is less than zero, indicating that L_ξ is concave function. Hence, the minimum value can be found at the end point of the interval $[L_\xi, L_{\xi-1}]$.

For sufficient condition, Hessian matrix can be used. All principal minors have to be positive. Substituting all these optimum values into the vendor's equation and the equation becomes a function of a single variable.

The equation is then,

$$\begin{aligned}\mathrm{TVC}(K_\zeta) = \sum_{\zeta=1}^{y}\sum_{\xi=1}^{m}\Bigg[&\left(\frac{K_\zeta}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}+\frac{S_0e^{-rK_\zeta}}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\right)\\ &+\frac{r_vC_v}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\left(\left(\sum_{\xi=1}^{m}Q_\xi\right)\left(\sum_{\xi=1}^{m}\frac{q_\xi}{P}+\sum_{\xi=1}^{m}\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2D_\xi}\right)\right.\\ &\left.-\frac{1}{2P}\left(\sum_{\xi=1}^{m}Q_\xi\right)^2-\frac{1}{2D_\xi}\left(\sum_{\xi=1}^{m}q_\xi\right)^2\left[1+\frac{\alpha^2n_\xi(n_\xi-1)(2n_\xi-1)}{6}\right]\right)\\ &+I_s\frac{Q_\xi}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}+P_r\frac{Q_\xi}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\\ &+\left(\frac{n_\xi F}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}+\frac{V_\xi Q_\xi}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\right)\\ &+\left(\frac{n_\xi S_b}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}+\frac{V_{b\xi}Q_\xi}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\right)\Bigg] \quad (5.7)\end{aligned}$$

where, q_ξ, n_ξ, and α are already in the optimized points. Now, the value of decision variable K_ζ can be calculated as follows:

$$\frac{\partial\mathrm{TVC}(K_\zeta)}{\partial K_\zeta}=\frac{1}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\left(1+S_0e^{-rK_\zeta}(-r)\right). \quad (5.8)$$

Now, the optimum value is

$$K_\zeta=\frac{\ln(rS_0)}{r}\cdot \quad (5.9)$$

For a sufficient condition,

$$\frac{\partial^2\mathrm{TVC}(\cdot)}{\partial K_\zeta^2}=\frac{1}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}S_0e^{-rK_\zeta}(r)^2>0. \quad (5.10)$$

[See Appendix L for all values]

Subcase 1.2: Multi-buyer as the leader and vendor as the follower

For this subcase, the multi-buyer is the leader and the vendor is the follower. Thus, as before, the optimization will starts from the vendor and the optimum values will be used in the multi-buyer equation to calculate the multi-buyer's minimum cost.

Therefore, the partial derivatives of $\mathrm{TVC}(n_\xi,q_\xi,\alpha,K_\zeta)$ are taken with respect to K_ζ, and q_ξ as

$$\frac{\partial\mathrm{TVC}(n_\xi,q_\xi,\alpha,K_\zeta)}{\partial K_\zeta}=\frac{1}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\left(1+S_0e^{-rK_\zeta}(-r)\right) \quad (5.11)$$

$$\frac{\partial\mathrm{TVC}(n_\xi,q_\xi,\alpha,K_\zeta)}{\partial q_\xi}=\left[-\frac{1}{(q_\xi)^2}U_1+U_2\right]. \quad (5.12)$$

Then, the values of these parameters K_ζ, q_ξ can be obtained by setting the above derivatives equal to zero as

$$K_\zeta = \frac{\ln(rS_0)}{r} \tag{5.13}$$

$$q_\xi = \sqrt{\frac{U_1}{U_2}}. \tag{5.14}$$

After substituting all these optimum values in the buyer's cost equation, the equation becomes a function of two variables k_ξ, and L_ξ.

Thus, these decision variables are calculated by taking derivatives of $TB_mC(k_\xi, L_\xi)$ with respect to k_ξ, and L_ξ.

$$\frac{\partial TB_mC(k_\xi, L_\xi)}{\partial k_\xi} = r_{b\xi}C_{b\xi}\sigma_\xi\sqrt{L_\xi} + \frac{2\Pi_\xi D_\xi\sigma_\xi\sqrt{L_\xi}}{(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))}(\Phi_\xi(k_{\text{jai}}) - 1) \tag{5.15}$$

$$\frac{\partial TB_mC(k_\xi, L_\xi)}{\partial L_\xi} = \frac{\Pi_{\text{jai}}D_\xi\sigma_\xi\psi_\xi(k_{\text{jai}})}{(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))}\frac{1}{\sqrt{L_\xi}} + r_{b\xi}C_{b\xi}k_\xi\sigma_\xi\frac{1}{2\sqrt{L_\xi}} + \frac{2D_\xi}{(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))}\frac{\partial C_\xi(L_{\text{jai}})}{\partial L_\xi}. \tag{5.16}$$

Similarly, as on the last case, the second term partial derivative of L_ξ is

$$\frac{\partial^2 TB_mC(n_\xi, q_\xi, \alpha, k_\xi, L_\xi)}{\partial L_\xi^2} = -\frac{D_\xi}{2}\Pi_\xi\sigma_\xi\psi_\xi(k_{\text{jai}})L_\xi^{-3/2} + \frac{2D_\xi}{(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))}\frac{\partial^2 C_\xi(L_{\text{jai}})}{\partial L_\xi^2} - \frac{1}{4}r_{b\xi}C_{b\xi}k_\xi\sigma_\xi L_\xi^{-3/2} < 0.$$

As before, L_ξ is a concave function. Hence, the minimum value can be found at the end point of the interval $[L_\xi, L_{\xi-1}]$.

The value of k_ξ is

$$\Phi_\xi(k_\xi) = 1 - \frac{r_{b\xi}C_{b\xi}(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))}{2\Pi_{\text{jai}}D_\xi}. \tag{5.17}$$

For the global minimum, to calculate the Hessian matrix, the principal minors have to be calculated. (See Appendix M for all values)

Case 2: Equal powers for the supply chain players

For equal powers of the vendor and multi-buyer, the total cost function is optimized simultaneously. Thus, the total cost function is

$$TC_{vb}(\cdot) = \sum_{\zeta=1}^{y}\sum_{\xi=1}^{m}\Bigg[\left(\frac{K_\zeta}{\left(\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)} + \frac{S_0 e^{-rK_\zeta}}{\left(\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\right) + \frac{r_v C_v}{\left(\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\left(\left(\sum_{\xi=1}^{m}Q_\xi\right)\left(\sum_{\xi=1}^{m}\frac{q_\xi}{P} + \sum_{\xi=1}^{m}\frac{\alpha q_\xi n_\xi(n_\xi - 1)}{2D_\xi}\right) - \frac{1}{2P}\left(\sum_{\xi=1}^{m}Q_\xi\right)^2 - \frac{1}{2D_\xi}\left(\sum_{\xi=1}^{m}q_\xi\right)^2\left[1 + \frac{\alpha^2 n_\xi(n_\xi - 1)(2n_\xi - 1)}{6}\right]\right)$$

$$+ I_s \frac{Q_\xi}{\left(\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2(P - D_\xi)}\right)} + P_r \frac{Q_\xi}{\left(\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2(P - D_\xi)}\right)}$$

$$+ \left(\frac{n_\xi F}{\left(\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2(P - D_\xi)}\right)} + \frac{V_\xi Q_\xi}{\left(\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2(P - D_\xi)}\right)} \right)$$

$$+ \left(\frac{n_\xi S_b}{\left(\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2(P - D_\xi)}\right)} + \frac{V_{b\xi} Q_\xi}{\left(\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2(P - D_\xi)}\right)} \right) \Bigg]$$

$$+ \sum_{\xi=1}^{m} \left[\frac{2A_\xi D_\xi}{(2q_\xi + \alpha q_\xi n_\xi (n_\xi - 1))} + r_{b\xi} C_{b\xi} \left[\frac{(2q_\xi + \alpha q_\xi n_\xi (n_\xi - 1))}{4} \right.\right.$$

$$\left.\left. + k_\xi \sigma_\xi \sqrt{L_\xi} \right] + \frac{2\Pi_{\text{jai}} D_\xi \sigma_\xi \sqrt{L_\xi} \psi_\xi(k)}{(2q_\xi + \alpha q_\xi n_\xi (n_\xi - 1))} + \frac{2D_\xi C_\xi(L_{\text{jai}})}{(2q_\xi + \alpha q_\xi n_\xi (n_\xi - 1))} \right] \cdot \tag{5.18}$$

Therefore, the partial derivatives of $TC_{vb}(n_\xi, q_\xi, \alpha, k_\xi, L_\xi, K_\zeta)$ are taken with respect to q_ξ, k_ξ, L_ξ, and K_ζ as

$$\frac{\partial TC_{vb}(n_\xi, q_\xi, \alpha, k_\xi, L_\xi, K_\zeta)}{\partial q_\xi} = \sum_{\xi=1}^{m} \left[-\frac{U_3}{q_\xi^2} + U_4 \right] \tag{5.19}$$

$$\frac{\partial TC_{vb}(n_\xi, q_\xi, \alpha, K_\zeta, L_\xi, K_\zeta)}{\partial k_\xi} = r_{b\xi} C_{b\xi} \sigma_\xi \sqrt{L_\xi} + \frac{2\Pi_\xi D_\xi \sigma_\xi \sqrt{L_\xi}}{(2q_\xi + \alpha q_\xi n_\xi (n_\xi - 1))} \left(\Phi_\xi(k_\xi) - 1 \right) \tag{5.20}$$

$$\frac{TC_{vb}(n_\xi, q_\xi, \alpha, k_\xi, L_\xi, K_\zeta)}{\partial L_\xi} = \frac{\Pi_{\text{jai}} D_\xi \sigma_\xi \psi_\xi(k_{\text{jai}})}{(2q_\xi + \alpha q_\xi n_\xi (n_\xi - 1))} \frac{1}{\sqrt{L_\xi}} + r_{b\xi} C_{b\xi} k_{\text{jai}} \sigma_\xi \frac{1}{2\sqrt{L_\xi}}$$
$$+ \frac{2D_\xi}{(2q_\xi + \alpha q_\xi n_\xi (n_\xi - 1))} \frac{\partial C_\xi(L_{\text{jai}})}{\partial L_\xi} \tag{5.21}$$

$$\frac{\partial TC_{vb}(n_\xi, q_\xi, \alpha, k_\xi, L_\xi, K_\zeta)}{\partial K_\zeta} = \frac{1}{\left(\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi (n_\xi - 1)}{2(P - D_\xi)}\right)} \left(1 + S_0 e^{-rK_\zeta} (-r) \right). \tag{5.22}$$

The values of these parameters q_ξ, k_ξ, and K_ζ can be obtained by equating these derivatives to zero as

$$q_\xi = \sqrt{\frac{U_3}{U_4}} \tag{5.23}$$

$$\Phi_\xi(k_\xi) = 1 - \frac{r_{b\xi} C_{b\xi} (2q_\xi + \alpha q_\xi n_\xi (n_\xi - 1))}{2\Pi_{\text{jai}} D_\xi} \tag{5.24}$$

$$K_\zeta = \frac{\ln r S_0}{r} \cdot \tag{5.25}$$

The second order partial derivative with respect to L_ξ is

$$\frac{\partial^2 TB_m C(n_\xi, q_\xi, \alpha, K_\zeta, L_\xi)}{\partial L_\xi^2} = -\frac{D_\xi}{2} \Pi_\xi \sigma_\xi \psi_\xi(k_{\text{jai}}) L_\xi^{-3/2}$$
$$+ \frac{2D_\xi}{(2q_\xi + \alpha q_\xi n_\xi (n_\xi - 1))} \frac{\partial^2 C_\xi(L_{\text{jai}})}{\partial L_\xi^2}$$
$$- \frac{1}{4} r_{b\xi} C_{b\xi} k_{\text{jai}} \sigma_\xi L_\xi^{-3/2} < 0. \tag{5.26}$$

TABLE 2. Parametric values of multi-buyer and vendor.

Parameter(s) $((\xi = 1, \ldots, 4))$	Values	Parameter(s)	Values
D_ξ (units/year)	$1600, 1610, 1615, 1620$	C_v (\$/unit)	10
A_ξ (\$/order)	$200, 210, 205, 220$	S_b (\$/shipment/year)	0.21
$r_{b\xi}$ (\$/unit/unit time)	$0.2, 0.2, 0.2, 0.2$	F (\$/shipment/year)	0.1
$C_{b\xi}$ (\$/unit)	$1, 5, 2, 1$	P_r (\$/unit/year)	0.01
Π_{jai} (\$/unit)	$50, 50, 50, 50$	r_v (\$/unit/unit/time)	0.01
σ_ξ	$7, 7, 7, 7$	I_s (\$/unit/year)	0.01
V_ξ (\$/unit)	$0.01, 0.02, 0.01, 0.03$	r (\$/year)	0.27
$V_{b\xi}$ (\$/unit)	$0.01, 0.02, 0.02, 0.01$	S_0 (\$/setup)	1600
P (\$/unit/year)	2500		

TABLE 3. Lead time data.

Lead time component j	Normal duration b_j (days)	Minimum duration a_j (days)	Unit crashing cost c_j (\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

As the second term of the derivative is less than zero, it follows that L_ξ is a concave function. Hence, the minimum value can be found at the end point of the interval $[L_\xi, L_{\xi-1}]$.
For a sufficient condition, the global minimum can be obtained by using the Hessian matrix. [Assuming $TC_{vb}(n_\xi, q_\xi, \alpha, k_\xi, L_\xi, K_\zeta)$=$TC_{vb}(\cdot)$], (See Appendix T for all values).

6. Numerical example

Example 6.1. The input values are given in Tables 2 and 3. The optimum solution is given by Table 4.

Table 4 provides the comparison of results under three different cases. In Subcase 1.1, the follower is multi-buyer and numerical output is \$2400.54 which is calculated first. As vendor is leader thus, another result is \$1808.88 for vendor side and two parties total cost \$4209.42. On the other Subcase 1.2, similarly, follower's result (vendor) is \$1732.09 and output of multi-buyer (leader) is \$2974.04. Therefore, total evaluating cost is \$4706.13. Again, it is obtained the numerical value from total cost function (\$4146.17) after adding separate two players' cost equations in Case 2.

7. Sensitivity analysis

The sensitivity analysis of the key parameters of the model is given with respect to the above example in Table 5.

- Little bit changes are shown in the parameters like buyers' holding cost $r_{b\xi}$, variable transportation cost V_ξ, premium cost P_r, S_b, vendor's inspection cost I_s, and unit backordering cost of multi-buyer Π_2, respectively,

Table 4. The optimum results for all cases.

Decision variables ($\xi, \zeta = 1, \ldots, 4$)	Subcase 1.1	Subcase 1.2	Case 2
n_ξ	$3, 2, 3, 2$	$2, 3, 2, 2$	$2, 2, 2, 2$
q_ξ (\$/unit)	$50, 59.99, 69.99, 80$	$77.02, 56.05, 45.02, 63.91$	$540, 306.18, 438.31, 517.30$
k_ξ	$3.36, 3.08, 3.24, 3.34$	$3.49, 2.67, 3.45, 3.55$	$3.39, 3.05, 3.23, 3.38$
K_ζ (\$/year)	$22.48, 22.48, 22.48, 22.48$	$23.75, 22.62, 24.84, 22.40$	$22.43, 21.54, 22.46, 21.08$
α	12	14	3
L_2 (\$/week)	6	6	6
$C_2(L)$ (\$/unit)	5.6	5.6	5.6
Total cost (\$/year)	$TB_mC = 2400.54$, TVC = 1808.88 $TC_{\text{SCM}} = 4209.42$	$TB_mC = 2974.04$, TVC = 1732.09 $TC_{\text{SCM}} = 4706.13$	$TC_{vb} = 4146.17$

Table 5. Sensitivity analysis for Subcase 1.1.

Parameter	Change (in %)	$\xi = 1$ (in %)	$\xi = 2$ (in %)	$\xi = 3$ (in %)	$\xi = 4$ (in %)	Parameter	Change (in %)	TC_{SCM} (in %)
	−50%	−5.09	−12.33	−7.45	−5.35	S_0	−50%	−0.83
	−25%	−2.34	−5.69	−3.43	−2.45		−25%	−0.35
$r_{b\xi}$	+25%	+2.07	+5.08	+3.05	+2.18		+25%	+0.61
	+50%	+3.96	+9.72	+5.83	+4.16		+50%	+1.10
	−50%	−5.08	−12.33	−3.45	−5.08	C_v	−50%	−24.73
	−25%	−2.33	−5.69	−3.43	−2.33		−25%	−12.37
$C_{b\xi}$	+25%	+2.08	+5.08	+3.05	+2.08		+25%	+12.37
	+50%	+3.92	+9.72	+5.83	+3.92		+50%	+24.63
	−50%	−0.88	−1.94	−1.86	−0.89	r_v	−50%	−24.63
	−25%	−0.37	−0.90	−0.86	−0.38		−25%	−2.36
$V_{b\xi}$	+25%	+0.64	+1.17	+1.13	+0.64		+25%	+2.36
	+50%	+1.14	+2.20	+2.12	+1.16		+50%	+24.63
	−50%	−0.88	−1.34	−0.86	−2.94	P_r	−50%	−6.99
	−25%	−0.37	−0.90	−0.36	−1.40		−25%	−3.43
V_ξ	+25%	+0.64	+1.17	+0.63	+1.67		+25%	+3.69
	+50%	+1.14	+2.20	+1.13	+3.20		+50%	+7.26
	Change (in %)	TC_{SCM} (in %)		Change (in %)	TC_{SCM} (in %)		Change (in %)	TC_{SCM} (in %)
S_b	−50%	−0.02	I_s	−50%	−6.99	Π_2	−50%	−0.40
	−25%	−0.05		−25%	−3.43		−25%	−0.17
	+25%	+0.21		+50%	+3.69		+25%	+0.14
	+50%	+0.29		+50%	+7.26		+25%	+0.26

TABLE 6. Sensitivity analysis for Subcase 1.2.

Parameter	Change (in %)	$\xi = 1$ (in %)	$\xi = 2$ (in %)	$\xi = 3$ (in %)	$\xi = 4$ (in %)	Parameter	Change (in %)	TC_{SCM} (in %)
	−50%	−1.36	−1.36	−1.36	−1.36	S_0	−50%	−1.21
	−25%	−0.68	−0.68	−0.68	−0.68		−25%	−0.60
$r_{b\xi}$	+25%	+0.68	+0.68	+0.68	+0.68		+25%	+0.60
	+50%	+1.36	+1.36	+1.36	+1.36		+50%	+1.21
	−50%	−2.15	−21.06	−2.68	−1.82	C_v	−50%	−12.17
	−25%	−1.07	−10.52	−1.34	−0.91		−25%	−5.56
$C_{b\xi}$	+25%	+1.07	+10.51	+1.33	+0.91		+25%	+4.90
	+50%	+2.14	+21.00	+2.66	+1.81		+50%	+11.19
	−50%	−1.08	−2.08	−2.13	−1.06	r_v	−50%	−12.16
	−25%	−0.54	−1.04	−1.07	−0.53		−25%	−5.56
$V_{b\xi}$	+25%	+0.54	+1.04	+1.07	+0.53		+25%	+4.90
	+50%	+1.08	+2.08	+2.13	+1.06		+50%	+12.17
	−50%	−1.08	−2.09	−1.07	−3.18	P_r	−50%	−12.83
	−25%	−0.54	−1.04	−0.53	−1.59		−25%	−2.90
V_ξ	+25%	+0.54	+1.04	+0.53	+1.59		+25%	+0.04
	+50%	+1.08	+2.09	+1.07	+3.18		+50%	+1.48
	Change (in %)	TC_{SCM} (in %)		Change (in %)	TC_{SCM} (in %)		Change (in %)	TC_{SCM} (in %)
S_b	−50%	−0.18	I_s	−50%	−7.44	Π_2	−50%	−0.38
	−25%	−0.09		−25%	−3.72		−25%	−0.17
	+25%	+0.09		+50%	+3.72		+25%	+0.14
	+50%	+0.18		+50%	+7.44		+25%	+0.28

then the changes make total supply chain cost TC_{SCM} in the feasible region by alternating percentages from ±25% to ±50%.

- Changing values of the parameters like unit purchasing cost parameter $C_{b\xi}$, variable carbon emission cost value $V_{b\xi}$, and initial setup cost S_0 are increased gradually in positive direction with respect to changing percentages at ±25%, ±50%. Thus, it implies total cost TC_{SCM} function is also in feasible range.
- The percentage of unit production cost and holding cost for vendor r_v, are symmetric in nature with respect to increasing percentages that implies the total cost function is the at equilibrium position.

The sensitivity analysis of the key parameter of the model is shown in the following under the Table 6.

- It is found that almost all parameteric values of initial setup cost S_0, Buyers' holding cost parameter $r_{b\xi}$, S_b, vendor's inspection cost I_s, multi-buyers' variable carbon emission cost $V_{b\xi}$, variable transportation cost V_ξ alter equal values with opposite directions from negative to positive direction of ±25%, and ±50% percentages. Therefore, total cost TC_{SCM} has at symmetric region at these parameters.
- On other side, the unit purchasing cost parameter $C_{b\xi}$, unit production cost C_v, and holding cost for vendor r_v, change with small gap of sensitivity values from −50% to +50% for all values of $\xi = 1, 2, \ldots 4$. It concludes the consistant solution of the total cost TC_{SCM}.
- The changes of premium cost P_r, and unit backordering cost of Π_2 have large gap in between outputs along with increasing percentages from negative to positive which provides the total cost TC_{SCM} value is also followed a feasible region.

The following sensitivity analysis of the parametric values is given with respect to the Table 7.

- The impact cost of holding cost of buyer $r_{b\xi}$, and unit purchasing cost $C_{b\xi}$ on TC_{vb} are huge at an extreme points *i.e.*, are also affecting the total cost by ±1% at extreme points almost ±10%.

TABLE 7. Sensitivity analysis for Case 2.

Parameter	Change (in %)	$\xi=1$ (in %)	$\xi=2$ (in %)	$\xi=3$ (in %)	$\xi=4$ (in %)	Parameter	Change (in %)	TC_{vb} (in %)
	−50%	−1.63	−1.63	−1.63	−1.63	S_0	−50%	−0.78
	−25%	−0.79	−0.79	−0.79	−0.79		−25%	−0.38
$r_{b\xi}$	+25%	+0.75	+0.75	+0.75	+0.75		+25%	+0.38
	+50%	+1.46	+1.46	+1.46	+1.46		+50%	+0.76
	−50%	−1.62	−5.69	−2.91	−1.62	C_v	−50%	−8.87
	−25%	−0.78	−2.68	−1.39	−0.78		−25%	−3.98
$C_{b\xi}$	+25%	+0.74	+2.68	+1.30	+0.74		+25%	+3.98
	+50%	+1.45	+5.59	+2.51	+1.45		+50%	+8.87
	−50%	−2.70	−1.35	−1.35	−2.70	r_v	−50%	−7.86
	−25%	−1.35	−0.67	−0.67	−1.35		−25%	−2.97
$V_{b\xi}$	+25%	+1.35	+0.67	+0.67	+1.35		+25%	+2.97
	+50%	+2.70	+1.35	+1.35	+2.70		+50%	+7.86
	−50%	−2.70	−1.35	−2.70	−2.03	P_r	−50%	−2.79
	−25%	−1.35	−0.67	−1.35	−1.02		−50%	−1.35
V_ξ	+25%	+1.35	+0.67	+1.35	+1.02		−50%	+1.35
	+50%	+2.70	+1.35	+2.70	+2.03		−50%	+2.70
	Change (in %)	TC_{vb} (in %)		Change (in %)	TC_{vb} (in %)		Change (in %)	TC_{vb} (in %)
S_b	−50%	−0.09	I_s	−50%	−3.25	Π_2	−50%	−0.05
	−25%	−0.04		−25%	−1.63		−25%	−0.08
	+25%	+0.04		+50%	+1.63		+25%	+0.07
	+50%	+0.09		+50%	+3.25		+25%	+0.13

- If it is analyzed the lower impact on the objective, then it is observed that Π_2 and initial setup cost S_0 are affecting with the minor change of just almost ±0.3%. It indicates that it has the very minor impact on the total cost of the whole supply chain.
- When changing production C_v and holding cost r_v of vendor, $V_{b\xi}$, V_ξ, P_r, S_b, and I_s increases in equal and opposite direction almost of ±50%, it is found that the total cost of supply chain is showing an equilibrium position with a change of almost ±5% in the optimal value.

Here, some of the optimum values should be integer values, but results are found as floating values. Thus, it is taken all possible combinations of n_1, n_2, n_3, n_4, α, and q_1, q_2, q_3, q_4 for rounding off. Among all possible combinations of those values, the smallest corresponding cost is considered. Then, the integer values of decision variables are then substituted in the other's cost equation to calculate the values.

8. Managerial insights

Managerial decisions conclude a proper way how the industry managers would gain more profit. Generally, business starts with atleast two parties together at which every party has similar power. Managers have to decide what are the good policies for their industries. Even though supply chain is there for maintaining the joint profit, but the players are with the thinking of their own profits. Thus for some business sectors vendors are leader and for some sectors buyers are leaders. Based on the situation or status of the industry, using our strategy, the managers can reduce the total cost. Through our modelling, we prove that total cost will be minimized when buyers are follower and vendor is leader subject to the inequality of power within them; otherwise centralized supply chain always gives less cost.

Vendor (leader)	Multi-buyer (follower)	Total cost
\$1808.88	\$2400.54	\$4209.42

Vendor (follower)	Multi-buyer (leader)	Total cost
\$2974.04	\$1732.09	\$4706.13

9. Conclusions

This model studied a two-echelon supply chain model with a single-vendor and multi-buyer. The controllable lead time was used to reduce the lead time. Unequal shipment sizes, fixed and variable transportation costs as well as carbon emission costs were used to ensure the supply chain is always a two-echelon chain. The premium cost was utilized to attract more customers for gaining more. This model was solved analytically and for inequality of powers of supply chain players, Stackelberg game policy were considered. The model obtained the minimum cost at the optimal solutions. Numerical studies proved that the outcomes has a huge impact on reality. It was found that the total cost is lower in the case of joint total cost with power equality. The model has a limitation in that a constant demand for both the vendor and buyer was assumed. Thus, this model can be extended to stochastic demand for multi-vendor and multi-buyer with multi-product under variable backorder in a supply chain with queueing structures.

Appendix

Appendix A

$$\begin{aligned} W_1 &= 2A_\xi D_\xi + 2\Pi_\xi D_\xi \sigma_\xi \sqrt{L_\xi}\psi_\xi(k_{\mathrm{jai}}) + 2D_\xi C_\xi(L_{\mathrm{jai}}) \\ W_2 &= (2+\alpha n_\xi(n_\xi-1)). \end{aligned}$$

Appendix U

$$\begin{aligned} U_1 &= \sum_{\zeta=1}^{y}\sum_{\xi=1}^{m}\left(\frac{K_\zeta + S_0 e^{-rK_\zeta} + n_\xi F + n_\xi S_b}{\left(\frac{1}{P}+\frac{\alpha n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\right) \\ U_2 &= \sum_{\xi=1}^{m}\frac{r_v C_v}{\left(\frac{1}{P}+\frac{\alpha n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\left(\left(\frac{2+\alpha n_\xi(n_\xi-1)}{2}\right)\left(\frac{1}{P}+\frac{\alpha n_\xi(n_\xi-1)}{2D_\xi}\right)\right. \\ &\quad \left. -\frac{1}{2P}\left(\frac{2+\alpha n_\xi(n_\xi-1)}{2}\right)^2 - \frac{1}{2D_\xi}\left[1+\frac{\alpha^2 n_\xi(n_\xi-1)(2n_\xi-1)}{6}\right]\right) \\ U_3 &= \sum_{\zeta=1}^{y}\sum_{\xi=1}^{m}\left(\frac{K_\zeta + S_0 e^{-rK_\zeta} + n_\xi F + n_\xi S_b}{\left(\frac{1}{P}+\frac{\alpha n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\right. \\ &\quad \left. + \frac{2A_\xi D_\xi + 2\Pi_{\mathrm{jai}} D_\xi \sigma_\xi \sqrt{L_\xi}\psi_\xi + 2D_\xi C_\xi(L_{\mathrm{jai}})}{(2+\alpha n_\xi(n_\xi-1))}\right) \\ U_4 &= \sum_{\xi=1}^{m} r_{b\xi} C_{b\xi}\frac{(2+\alpha n_\xi(n_\xi-1))}{4} \end{aligned}$$

$$+\frac{r_vC_v}{\left(\frac{1}{P}+\frac{\alpha n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\left(\left(\frac{2+\alpha n_\xi(n_\xi-1)}{2}\right)\left(\frac{1}{P}+\frac{\alpha n_\xi(n_\xi-1)}{2D_\xi}\right)\right.$$
$$\left.-\frac{1}{2P}\left(\frac{2+\alpha n_\xi(n_\xi-1)}{2}\right)^2-\frac{1}{2D_\xi}\left[1+\frac{\alpha^2 n_\xi(n_\xi-1)(2n_\xi-1)}{6}\right]\right).$$

Appendix L

The first order principal minor of $|H|$ is

$$|H_{11}|_{(q_\xi,k_\xi)}=\left|\frac{\partial^2 TB_mC(\cdot)}{\partial(q_\xi)^2}\right|=\frac{2W_1}{W_2}\frac{1}{q_\xi^3}>0.$$

The second order principal minor of $|H|$ is

$$\begin{aligned}|H_{22}|_{(q_\xi,k_\xi)}&=\frac{\partial^2 TB_mC(\cdot)}{\partial q_\xi^2}\frac{\partial^2 TB_mC(\cdot)}{\partial k_\xi^2}-\left(\frac{\partial^2 TB_mC(\cdot)}{\partial q_\xi\partial k_\xi}\right)^2\\&=\left[\frac{2W_1}{W_2}\frac{1}{q_\xi^3}\right]\left[\frac{2\Pi_\xi D_\xi\sigma_\xi\sqrt{L_\xi}}{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))}\phi_\xi(k_{\text{jai}})\right]\\&\quad-\left(\frac{2\Pi_\xi D_\xi\sigma_\xi\sqrt{L_\xi}}{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))^2}(\Phi_\xi(k_{\text{jai}})-1)(2+\alpha n_\xi(n_\xi-1))\right)^2\\&=\left[\frac{2\Pi_\xi D_\xi\sigma_\xi\sqrt{L_\xi}}{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))}\frac{1}{q_\xi^3}\right]\left(\frac{2W_1}{W_2}\phi_\xi(k_{\text{jai}})\right.\\&\quad\left.-\frac{(\Phi_\xi(k_{\text{jai}})-1)^2 2\Pi_\xi D_\xi\sigma_\xi\sqrt{L_\xi}}{(2+\alpha n_\xi(n_\xi-1))}\right)>0\end{aligned}$$

where

$$\left(\frac{2W_1}{W_2}\phi_\xi(k_{\text{jai}})-\frac{(\Phi_\xi(k_{\text{jai}})-1)^2 2\Pi_\xi D_\xi\sigma_\xi\sqrt{L_\xi}}{(2+\alpha n_\xi(n_\xi-1))}\right)$$
$$=\frac{2\phi_\xi(k_{\text{jai}})(2A_\xi D_\xi+2\Pi_\xi D_\xi\sigma_\xi\sqrt{L_\xi}\psi_\xi(k_{\text{jai}})+2D_\xi C_\xi(L_{\text{jai}}))-((\Phi_\xi(k_{\text{jai}})-1)^2 2\Pi_\xi D_\xi\sigma_\xi\sqrt{L_\xi})}{(2+\alpha n_\xi(n_\xi-1))}.$$

Here, $\phi_\xi(k_{\text{jai}})$, $\psi(k_{\text{jai}})>0$ and
$2\phi_\xi(k_{\text{jai}})(2A_\xi D_\xi+2\Pi_\xi D_\xi\sigma_\xi\sqrt{L_\xi}\psi_\xi(k_{\text{jai}})+2D_\xi C_\xi(L_{\text{jai}}))-((\Phi_\xi(k_{\text{jai}})-1)^2 2\Pi_\xi D_\xi\sigma_\xi\sqrt{L_\xi})>0$ for all $k_\xi>0$

$$\begin{aligned}\frac{\partial^2 TB_mC(\cdot)}{\partial(q_\xi)^2}&=\frac{2W_1}{W_2}\frac{1}{q_\xi^3}\\\frac{\partial^2 TB_mC(\cdot)}{\partial(k_\xi)^2}&=\frac{2\Pi_\xi D_\xi\sigma_\xi\sqrt{L_\xi}}{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))}\frac{\partial\Phi_\xi(k_{\text{jai}})}{\partial k_\xi}\\\frac{\partial^2 TB_mC(\cdot)}{\partial q_\xi\partial k_\xi}&=-\frac{2\Pi_\xi D_\xi\sigma_\xi\sqrt{L_\xi}}{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))^2}(\Phi_\xi(k_{\text{jai}})-1)(2+\alpha n_\xi(n_\xi-1))\\\frac{\partial^2 TB_mC(\cdot)}{\partial(K_\zeta)^2}&=\sum_{\xi=1}^{m}\frac{1}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}S_0e^{-rK_\zeta}(r)^2.\end{aligned}$$

Appendix M

The first order principal minor of $|H|$ is

$$\begin{aligned}|H_{11}|_{(q_\xi,K_\zeta)} &= \left|\frac{\partial^2\mathrm{TVC}(\cdot)}{\partial(q_\xi)^2}\right| \\ &= \frac{2U_1}{q_\xi^3} > 0.\end{aligned}$$

The second order principal minor of $|H|$ is

$$\begin{aligned}|H_{22}|_{(q_\xi,K_\zeta)} &= \frac{\partial^2\mathrm{TVC}(\cdot)}{\partial q_\xi^2}\frac{\partial^2\mathrm{TVC}(\cdot)}{\partial K_\zeta^2} - \left(\frac{\partial^2\mathrm{TVC}(\cdot)}{\partial q_\xi \partial K_\zeta}\right)^2 \\ &= \left[\frac{2U_1}{q_\xi^3}\right]\left[\sum_{\xi=1}^{m}\frac{1}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}S_0e^{-rK_\zeta}(r)^2\right] \\ &\quad - \left(\sum_{\xi=1}^{m}\frac{1}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}(1+S_0e^{-rK_\zeta}(-r))\frac{1}{(q_\xi)^2}\right)^2 \\ &= \sum_{\xi=1}^{m}\frac{1}{\left(\frac{1}{P}+\frac{\alpha n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}\frac{1}{(q_\xi)^4}\left[\frac{2q_\xi^2U_1S_0e^{-rK_\zeta}(r)^2}{q_\xi^2}\right. \\ &\quad \left. - \frac{1}{(q_\xi)^4}\left[\frac{(1+S_0e^{-rK_\zeta}(-r))^2}{q_\xi^2}\right]\right] > 0\end{aligned}$$

$$\frac{\partial^2\mathrm{TVC}(\cdot)}{\partial(q_\xi)^2} = \frac{2U_1}{q_\xi^3}$$

$$\frac{\partial^2\mathrm{TVC}(\cdot)}{\partial(K_\zeta)^2} = \sum_{\xi=1}^{m}\frac{1}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}S_0e^{-rK_\zeta}(r)^2$$

$$\frac{\partial^2\mathrm{TVC}(\cdot)}{\partial q_\xi \partial K_\zeta} = -\sum_{\xi=1}^{m}\frac{1}{\left(\frac{q_\xi}{P}+\frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}(1+S_0e^{-rK_\zeta}(-r))\frac{1}{(q_\xi)^2}.$$

Again, for ξth buyers decision variable

$$\frac{\partial^2 TB_mC(\cdot)}{\partial(k_\xi)^2} = \frac{2\Pi_\xi D_\xi\sigma_\xi\sqrt{L_\xi}}{(2q_\xi+\alpha q_\xi n_\xi(n_\xi-1))}\frac{\partial\Phi_\xi(k_{\mathrm{jai}})}{\partial k_\xi} > 0.$$

Appendix T

The first order principal minor of $|H|$ is

$$\begin{aligned}|H_{11}|_{(q_\xi,k_\xi,K_\zeta)} &= \left|\frac{\partial^2 TC_{vb}(\cdot)}{\partial(q_\xi)^2}\right| \\ &= \frac{2U_3}{q_\xi^3} > 0.\end{aligned}$$

The second order principal minor of $|H|$ is

$$|H_{22}|_{(q_\xi,k_\xi,K_\zeta)} = \frac{\partial^2 TC_{vb}(\cdot)}{\partial q_\xi^2}\frac{\partial^2 TC_{vb}(\cdot)}{\partial k_\xi^2} - \left(\frac{\partial^2 TC_{vb}(\cdot)}{\partial q_\xi\partial k_\xi}\right)^2$$

$$= \left[\frac{2U_3}{q_\xi^3}\right]\left[\frac{2\Pi_\xi D_\xi \sigma_\xi \sqrt{L_\xi}}{(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))}\frac{\partial \Phi_\xi(k_{\text{jai}})}{\partial k_\xi}\right]$$
$$- \left(\frac{2\Pi_\xi D_\xi \sigma_\xi \sqrt{L_\xi}}{(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))^2}(\Phi_\xi(k_{\text{jai}}) - 1)(2 + \alpha n_\xi(n_\xi - 1))\right)^2$$
$$= \left[\frac{2\Pi_\xi D_\xi \sigma_\xi \sqrt{L_\xi}}{(2 + \alpha n_\xi(n_\xi - 1))}\frac{1}{q_\xi^4}\right]\left(2U_3\frac{\partial \Phi_\xi(k_{\text{jai}})}{\partial k_\xi}\right.$$
$$\left. - \frac{(2(\Phi_\xi(k_{\text{jai}}) - 1)\Pi_\xi D_\xi \sigma_\xi \sqrt{L_\xi})^2}{(2 + \alpha n_\xi(n_\xi - 1))}\right).$$

The above expression is simplified which is greater than zero.

The third order principal minor of $|H|$ is

$$|H_{33}|_{(q_\xi,k_\xi,K_\zeta)} = \begin{vmatrix} \frac{\partial^2 TC_{vb}(\cdot)}{\partial(q_\xi)^2} & \frac{\partial^2 TC_{vb}(\cdot)}{\partial q_\xi \partial k_\xi} & \frac{\partial^2 TC_{vb}(\cdot)}{\partial q_\xi \partial K_\zeta} \\ \frac{\partial^2 TC_{vb}(\cdot)}{\partial k_\xi \partial q_\xi} & \frac{\partial^2 TC_{vb}(\cdot)}{\partial(k_\xi)^2} & \frac{\partial^2 TC_{vb}(\cdot)}{\partial k_\xi \partial K_\zeta} \\ \frac{\partial^2 TC_{vb}(\cdot)}{\partial K_\zeta \partial q_\xi} & \frac{\partial^2 TC_{vb}(\cdot)}{\partial K_\zeta \partial k_\xi} & \frac{\partial^2 TC_{vb}(\cdot)}{\partial K_\zeta^2} \end{vmatrix}$$
$$= -\frac{\partial^2 TC_{vb}(\cdot)}{\partial K_\zeta \partial q_\xi}\frac{\partial^2 TC_{vb}(\cdot)}{\partial q_\xi \partial K_\zeta}\frac{\partial^2 TC_{vb}(\cdot)}{\partial(k_\xi)^2} + \frac{\partial^2 TC_{vb}(\cdot)}{\partial(K_\zeta)^2}|H_{22}|$$
$$= X_1 + \frac{\partial^2 TC_{vb}(\cdot)}{\partial(K_\zeta)^2}|H_{22}| > 0$$

$$\text{where} \qquad X_1 = \sum_{\xi=1}^{m}\left(\frac{1}{\left(\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}(1 - S_0 e^{-rK_\zeta}(r))\frac{1}{(q_\xi)^2}\right)$$
$$\left(\frac{1}{\left(\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}(S_0 e^{-rK_\zeta}\mathrm{r} - 1)\frac{1}{(q_\xi)^2}\right)$$
$$\frac{2\Pi_\xi D_\xi \sigma_\xi \sqrt{L_\xi}}{(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))}\frac{\partial \Phi_\xi(k_{\text{jai}})}{\partial k_\xi}.$$

Also, the second part has $\frac{\partial^2 TC_{vb}(\cdot)}{\partial(K_\zeta)^2}$ is positive and $|H_{22}|$ is previously is shown to be positive.

$$\frac{\partial^2 TC_{vb}(\cdot)}{\partial(q_\xi)^2} = \frac{2U_3}{q_\xi^3}$$
$$\frac{\partial^2 TC_{vb}(\cdot)}{\partial(k_\xi)^2} = \frac{2\Pi_\xi D_\xi \sigma_\xi \sqrt{L_\xi}}{(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))}\frac{\partial \Phi_\xi(k_{\text{jai}})}{\partial k_\xi}$$
$$\frac{\partial^2 TC_{vb}(\cdot)}{\partial(K_\zeta)^2} = \sum_{\xi=1}^{m}\frac{1}{\left(\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}S_0 e^{-rK_\zeta}(r)^2$$
$$\frac{\partial^2 TC_{vb}(\cdot)}{\partial q_\xi \partial k_\xi} = \frac{\partial^2 TC_{vb}(\cdot)}{\partial k_\xi \partial q_\xi} = -\frac{2\Pi_\xi D_\xi \sigma_\xi \sqrt{L_\xi}}{(2q_\xi + \alpha q_\xi n_\xi(n_\xi - 1))^2}(\Phi_\xi(k_{\text{jai}}) - 1)(2 + \alpha n_\xi(n_\xi - 1))$$
$$\frac{\partial^2 TC_{vb}(\cdot)}{\partial k_\xi \partial K_\zeta} = 0$$
$$\frac{\partial^2 TC_{vb}(\cdot)}{\partial q_\xi \partial K_\zeta} = -\sum_{\xi=1}^{m}\frac{1}{\left(\frac{q_\xi}{P} + \frac{\alpha q_\xi n_\xi(n_\xi-1)}{2(P-D_\xi)}\right)}(1 + S_0 e^{-rK_\zeta}(-r))\frac{1}{(q_\xi)^2}.$$

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