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CHANNEL COORDINATION THROUGH QUALITY IMPROVEMENT WITH BRAND HALO EFFECT

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Abstract. Product quality depends on the quality investment of the manufacturer and quality decisions of the supplier. Therefore, many firms and researchers pay considerable attention to supply quality management. Considering a supply chain that includes two competing suppliers and one manufacturer, this paper investigates the influences of competition and the "brand halo" effect on the quality strategies of channel members, and explores the potential coordinating power of the bilateral participation contract. Utilizing differential game theory, this paper compares and analyzes the quality strategies of all channel members under three different scenarios: (i) decentralized scenario within a subsidy program, (ii) integrated scenario, and (iii) bilateral participation contract. Our results confirm the following results. (1) The manufacturer may not grant a subsidy to the supplier if two final products are highly competitive. (2) Supply chain members are more likely to join the bilateral participation contract if the "brand halo" effect is large. (3) The bilateral participation contract can achieve perfect coordination if the competition is weak or if a transfer payment policy exists.

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1. Introduction

Product quality affects consumer buying behaviors. In certain industries, quality competition has gradually replaced price competition [15,36]. Firms have been paying considerable attention to quality investment. However, product quality depends on the quality investment of the manufacturer and quality decisions of the supplier [13,38]. In the electronics sector in the Republic of Ireland, supply chain relationship quality positively affected the design quality of the final product [14]. Therefore, firms need to collaborate with their upstream or downstream firms to improve final product quality. In practice, many famous firms are involved in quality improvement collaboration. For example, Intel selected certain suppliers to participate in its quality control projects [39]. In addition, GE granted an annual budget of 200–400 million for its Six Sigma program, a vital part of which aims at improving supplier product quality [41].

Meanwhile, scholars have also been paying substantial attention to supply quality management [1,5,7,11,17,18,26,37,43,46,48–50]. However, the aforementioned studies mainly consider a "single supplier and single manufacturer" network. Few papers consider competition in supply quality management [12,31,47]. In reality, more than one supplier offers components and parts to the manufacturer, and the manufacturer always produces

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final products with different sub-brands. For example, Dell Inc. produces and sells the laptops with four different sub-brands (Inspiron, XPS Laptops, Alienware, and Chromebook) in his online shop. Therefore, it is necessary to study the supply quality management under the competition environment.

On the other hand, consumers' purchasing behaviors not only depend on the product quality but also on the brand of product. Consumers are willing to pay a price premium for their preferred brand [20]. However, the brand image of the final product is also affected by the brand image of supplier due to the "brand halo" effect. "Brand halo" effect is usually exploited for brand extension, namely, consumer bias toward certain products due to favorable experience with other products made by the same firm. Therefore, the reputation of the supplier also has a positive effect on the final product's brand image. As the product brand image has a significant influence on consumers' purchasing behaviors, the "brand halo" effect should be considered in supply quality management, which enables decision makers to make more rational and effective decisions. Therefore, we consider the "brand halo" effect in this study and utilize a differential game to investigate the optimal quality decisions in a supply chain with two competing suppliers and one manufacturer. In addition, an uncoordinated supply chain always performs worse in quality improvement than a coordinated one [22]. Finally, this study also proposes a bilateral participation contract to coordinate this supply chain system. This study seeks to resolve the following problems: (1) What is the influences of the "brand halo" effect on channel members' optimal decisions? (2) What are the impacts of competition on the manufacturer's subsidy policy? (3) What is the potential coordinating power of the bilateral participation contract?

The paper is structured as follows. We present the existing literature on supply quality management in Section 2. Problem definition is given in Section 3. Section 4 presents the basic models for the scenarios. Section 4 deals with our calculation and analysis regarding the solutions in three different scenarios (Decentralized scenario within a subsidy program, integrated scenario and the bilateral participation contract). Section 6 gives numerical analysis. Section 7 concludes this paper. All proofs of the results are in the Appendix A.

2. Literature review

This study is related to supply quality management and brand halo effect in supply chain management.

2.1. Supply quality management

In operations management, a set of studies discussed quality improvement from a supply chain perspective [7, 8, 18, 19, 24, 29, 32, 33, 42, 43, 48, 50]. Tapiero [43] offered a strategic collaborative approach to quality control in a "supplier–buyer" supply chain by utilizing Neyman–Pearson theory. Zhu *et al.* [50] studied the roles of producers and buyers in quality improvement, and their results corroborated that buyer participation has a remarkable effect on the profits of all channel members. Hsieh and Liu [18] studied the quality decisions of all channel members in four different games and explored the influences of inspection-related information on decisions and profits of all channel members. Hu *et al.* [19] investigated a fuzzy random newsboy problem with imperfect quality, and their results verified that the repurchase behavior of the manufacturer could improve the total supply chain profit. De Giovanni [8] utilized the differential game to study the optimal quality improvement and advertising decisions in a supply chain with a manufacturer and a retailer. Xie *et al.* [48] investigated the effects of different supply chain strategies and risk-averse behaviors of channel members on quality improvement with uncertain demands. Dai *et al.* [7] investigated product quality and warranty protection period decisions in a supply chain and confirmed that the warranty cost share rate has different effects on optimal solutions under channel members' warranties. Leng *et al.* [24] studied a quality-assurance problem in a "manufacturer-retailer" supply chain and investigated the role of a retailer in assuring product quality, their results indicated that retailer's quality gatekeeping was contributed to reduce the defective rate. Maiti and Giri [29] studied the optimal pricing and quality decisions in a closed-loop supply chain under four different decision structures. Soni *et al.* [42] studied the influences of lost sales reduction and quality improvement in an imperfect production process. The aforementioned papers on supply quality management were developed in a "single upstream firm and single downstream firm" framework, whereas the current study focuses on a supply chain with two competing suppliers and a manufacturer and studies the optimal quality decisions of all channel members. Moreover, the current study also proposes a contract to coordinate this supply chain.

Hitherto, various studies focused on the coordination of supply chain with quality improvement [1, 5, 10, 11, 17, 26, 27, 32–35, 40, 46, 49]. Lim [26] probed the contract design problem of quality control with incomplete information and explored the optimal contract that maximizes the producer profit. Singer *et al.* [40] considered a "supplier–retailer" supply chain and studied the quality strategies of channel members and provided transfer contracts for the supplier–retailer alliance. Balachandran and Radhakrishnan [1] examined a supply chain in which the quality choice of the supplier is not observable to the manufacturer. Chao *et al.* [5] investigated the effects of two quality cost-sharing contracts between a manufacturer and a supplier on product quality. Xiao *et al.* [46] provided a revenue-sharing contract to coordinate a supply chain system with the quality assurance policy. El Ouardighi [11] utilized a two-stage game to study the potential coordinating power of the revenue-sharing contract in supply quality management. He *et al.* [17] investigated the influences of reference effects on quality improvement strategies and offered a contract to coordinate this supply chain. Lu *et al.* [27] studied the optimal quality and price decisions with reference price and proposed a contract to coordinate the supply chain. Though the above literature proposed contract to coordinate the supply chain, the above studies were based on a no-competitive context, and did not explore the "brand halo" effect on supply quality management.

The following studies considered competition in the supply quality management [9, 12, 21, 47]. Xie *et al.* [47] considered two competing supply chains and explored the mechanism on the selection of supply chain structure and quality improvement decisions. Dong *et al.* [9] explored the quality management in two supply chains (dyadic supply chain and multi-level supply chain) with outsourced manufacturing, and the result showed that agency cost is critical in driving the firm's optimal choice of quality management approach. The above two studies concentrated on the competition between two supply chains, however the "brand halo" effect was not incorporated in their model and the supply chain coordination was also not explored. El Ouardighi and Kim [12] utilized a differential game to examine a "one supplier and two price-competing manufacturers" supply chain and analyzed how each channel member should assign resources for quality improvement over time. Our study also uses a differential game to investigate supply quality management, while there are many differences between the above researches and the current study. "Brand halo" effect in the supply quality management is considered in this study, and a bilateral participation contract is proposed to coordinate the supply chain under supplier competition. Johari and Hosseini-Motlagh [21] proposed a promotion cost sharing contract to coordinate a supply chain with one manufacturer and two competing retailers. Different from the above literature, this study investigates the optimal quality decisions in a supply chain with two competing suppliers.

2.2. The "brand halo" effect in supply chain management

Another research topic related to the present study is the "brand halo" effect in supply chain management. Bendixen *et al.* [3] demonstrated that brand equity was contributed to extend the "brand halo" to other product lines. Betts and Taran [4] investigated the impacts of "brand halo" effect on the durable goods prices. Tafani *et al.* [44] empirically examined the influences of the "brand halo" effect on vertical product line extension strategies, Madden *et al.* [28] found that "brand halo" effect was more pervasive for product quality than for corporation social responsibility (CSR) associations. Gou *et al.* [16] considered the "brand halo" effect between two horizontal firms and studied cooperative advertising strategies for horizontal inter-firm cooperation between the two firms, and their results showed that the manager would ignore the "brand halo" effect if he believed future product sales are risky. Above literature review showed that researchers studied the "brand halo" effect together with CSR, price and cooperative advertising from the angle of the supply chain, however few studies considered "brand halo" effect in the supply quality management.

2.3. Research gap and contribution

The main contributions of this study are as follows. First, almost studies related to the supply quality management focused on the "single upstream firm and single downstream firm", few studies considered competition in the supply quality management [9, 12, 21, 47], whereas our study focuses on the competition between two suppliers and investigates the supply chain members' optimal quality decisions. Second, different from supply chain coordination literature in Table 1, our study explores the potential coordinating power of the bilateral participation contract in

TABLE 1. Current study vs. the literature on supply quality management.

Article	Supply chain structure	Model properties	Coordination mechanism
Tapiero [43]	A supplier and a buyer	Static	N/A
Zhu *et al.* [50]	A supplier and a buyer	Static	N/A
Hsieh and Liu [18]	A supplier and a manufacturer	Static	N/A
Hu *et al.* [19]	A manufacturer and a retailer	Static	N/A
De Giovanni [8]	A manufacturer and a retailer	Dynamic	N/A
Xie *et al.* [48]	A supplier and a manufacturer	Static	N/A
Dai *et al.* [7]	A supplier and a manufacturer	Static	N/A
Leng *et al.* [24]	A manufacturer and a retailer	Static	N/A
Maiti and Giri [29]	A manufacturer and a retailer	Static	N/A
Lim [26]	A supplier and a producer	Static	Price rebate and warranty contracts
Singer *et al.* [40]	A supplier and a retailer	Static	Transfer contract
Balachandran and Radhakrishnan [1]	A supplier and a manufacturer	Static	Warranty contract
Chao *et al.* [5]	A supplier and a manufacturer	Static	Recall Cost Sharing Contracts
Xiao *et al.* [46]	A manufacturer and a retailer	Static	Revenue-sharing contract
El Ouardighi [11]	A supplier and a manufacturer	Dynamic	Wholesale price and revenue-sharing contracts
He *et al.* [17]	A supplier and a manufacturer	Dynamic	Total cost-sharing contract
Lu *et al.* [27]	A manufacturer and a retailer	Static	Revenue sharing contract
Xie *et al.* [47]	Two competing supply chains	Static	N/A
Dong *et al.* [9]	Two supply chains	Static	N/A
El Ouardighi and Kim [12]	A supplier and two competing manufacturers	Dynamic	Wholesale price and revenue-sharing contracts
Johari and Hosseini-Motlagh [21]	Two competing retailers and a manufacturer	Static	Promotion cost sharing contract
Our Study	Two competing suppliers and a manufacturer	Dynamic	Bilateral participation contract

a supply chain with two competing suppliers and one manufacturer. Third, our study utilizes a differential game to investigate the influences of the "brand halo" effect on channel members' quality decisions and profits.

3. Problem definition

In this study, we consider a supply chain consisting of two competing suppliers and one manufacturer, where all channel members invest in product quality. In this supply chain system, two competing suppliers provide components to the manufacturer simultaneously, then the manufacturer produces and sells two final products to the consumers with different sub-brands. And each final product requires two different components (*i.e.*, one is provided by the manufacturer, and the other is offered by supplier i) and affected by both components' quality levels. Due to the "brand halo" effect, the sales of the final product i also depend on the supplier's goodwill stock. And the cost of quality improvement is increasing and convex in quality improvement efforts.

Then, we utilize a differential game to study the equilibrium solutions of all supply chain members in three different scenarios. (i) Decentralized scenario within a subsidy program. In this scenario, each channel member optimizes his own current value of profit within a subsidy program. And the decision sequences of all supply chain members are presented as follows. The manufacturer firstly grants subsidies to the suppliers to motivate their quality improvement spending. Then, all supply chain members decide quality improvement efforts simultaneously. (ii) Integrated scenario. In this scenario, all supply chain members are integrated as a single firm and maximize the current value of the whole supply chain system. (ii) Bilateral participation contract. In this scenario, we study the potential coordinating power of the bilateral participation contract. Finally, we use a numerical analysis to investigate the effects of competition and "brand halo" effect on channel members' decisions and profits.

3.1. Notations

t	Time t, $t \geq 0$
$G_{si}(t)$	Goodwill stock of supplier i at time $t, i=1,2$
$G_{mi}(t)$	Goodwill stock of final product i at time $t, i=1,2$
$x_m(t)$	Component m's quality level offered by the manufacturer over time t
$x_{si}(t)$	Component si's quality level provided by the supplier over time t
λ_{si}	The effect of component si's quality level on the final product
λ_m	The effect of component m's quality level on the final product
$S_i(t)$	Sales of final product i along time t
$\theta \in [0,1]$	Competition intensity denoting the competitive effect of product's quality
$\chi \in [0,1]$	Competition intensity denoting the competitive effect of product's goodwill
$\phi_i \in [0,1]$	Manufacturer's subsidy rate for the supplier i's quality improvement cost
$\rho_{si} \geq 0$	Marginal profit of the supplier i
$\rho_{mi} \geq 0$	Marginal profit of the manufacturer that is obtained by selling final product i
$\delta > 0$	Diminishing rate of goodwill
$r > 0$	Discount rate
π_{si}, π_m	Profit functions of supplier i and manufacturer, respectively
J_{si}, J_m	Current values of profit functions for supplier i and manufacturer, respectively

3.2. Assumptions

(1) All channel members invest in product quality.
(2) Two competing suppliers provide components to the manufacturer simultaneously.
(3) The manufacturer produces and sells two final products to the consumers with different sub-brands.
(4) Each final product requires two different components (*i.e.*, one is provided by the manufacturer, and the other is offered by supplier i).
(5) The sales of final product depend on its quality and goodwill stock.

4. Basic model

Based on the model of Nair and Narasimhan [30], we can the change of goodwill stock for supplier i is as follows:

$$\mathrm{d}G_{si}(t)/\mathrm{d}t = x_{si}(t) - \delta G_{si}(t),$$
$$G_{si}(0) = G_{si}^0 \geq 0, i = 1, 2, \tag{4.1}$$

where, G_{si}^0 represents the initial goodwill stock of supplier i, and δ is a positive constant, which is the diminishing rate of goodwill.

The change of the goodwill stock of final product i is the following:

$$\mathrm{d}G_{mi}(t)/\mathrm{d}t = \lambda_{si}x_{si}(t) + \lambda_m x_m(t) - \delta G_{mi}(t),$$
$$G_{mi}(0) = G_{mi}^0 \geq 0, i = 1, 2, \tag{4.2}$$

where, λ_{si} and λ_m are positive constants. Item $\lambda_{si}x_{si}(t) + \lambda_m x_m(t)$ implies the quality level of final product i. Constant G_{mi}^0 indicates the initial goodwill stock of product i.

The sales of final product $S_i(t)$ depend on the final product quality and goodwill stock. Given that this study aims to characterize the decisions of supply chain members for quality improvement efforts, the final product price is assumed fixed. This assumption is found in related literature [17, 47]. In addition, the sales of final product are also influenced by the "brand halo" effect, which leads to the positive effect of supplier goodwill on the final product sales [3, 16]. Therefore, the sales of product i are given by the following:

$$S_i(t) = \alpha_i + G_{mi} + \beta G_{si} + \lambda_{si}x_{si} + \lambda_m x_m$$
$$- \chi G_{m(3-i)} - \theta(\lambda_{s(3-i)}x_{s(3-i)} + \lambda_m x_m), i = 1, 2, \tag{4.3}$$

where, α_i represents the basic market size of product i, which is irrespective of product quality and goodwill levels. In equation (4.3), G_{mi} measures the effect of the goodwill of product i on its sales, and item $\lambda_{si}x_{si} + \lambda_m x_m$ indicates the influence of the quality of product i on its sales. Let χ (or θ) denotes the competitive effect of competitive product's goodwill (or quality), and $0 < \chi, \theta < 1$ represents that product's own quality and goodwill has a larger effect on its sales than that of the competitive product's goodwill and quality, this assumption is found in related literature [2, 30]. Item βG_{si} implies the "brand halo" effect of supplier i on the sales of product i.

The cost of quality improvement is increasing and convex in quality improvement efforts, and the cost is quadratic of the quality improvement efforts of the firm [17]. Accordingly, we have the cost functions of quality improvement efforts as follows:

$$C(x_{si}) = x_{si}^2, C(x_m) = x_m^2, i = 1, 2. \tag{4.4}$$

Without accounting for quality improvement cost, let ρ_{si} denote the marginal profit of supplier i, and let ρ_{mi} denote the marginal profit of the manufacturer that was obtained by selling final product i. Therefore, the profit functions of all channel members are the following:

$$\pi_{si}(t) = \rho_{si}S_i(t) - x_{si}^2(t), i = 1, 2, \tag{4.5}$$

$$\pi_m(t) = \rho_{m1}S_1(t) + \rho_{m2}S_2(t) - x_m^2(t). \tag{4.6}$$

5. Equilibria

In this section, we will analyze the equilibrium quality decisions of all channel members in the following three different scenarios. We use superscripts "s, I, and c" to denote the aforementioned scenarios, respectively.

5.1. Decentralized scenario within a subsidy program

We consider a traditional subsidy program in the supply chain system, that is, the manufacturer grants a subsidy to the supplier to motivate his quality improvement spending. This subsidy program is commonly implemented in many industries. Let $\phi_i (0 \leq \phi_i \leq 1, i=1,2)$ denote the subsidy rate. The profit functions of the channel members are the following:

$$\pi_{si}^s(t) = \rho_{si} S_i(t) - \phi_i x_{si}^2(t), i=1,2, \tag{5.1}$$

$$\pi_m^s(t) = \rho_{m1} S_1(t) + \rho_{m2} S_2(t) - x_m^2(t) - \phi_1 x_{s1}^2(t) - \phi_2 x_{s2}^2(t). \tag{5.2}$$

Assuming an infinite time horizon for the problem and with a discount rate $r > 0$, supplier i wants to maximize the current value, that is,

$$\max J_{si}^s = \int_0^{+\infty} \mathrm{e}^{-rt} \pi_{si}^s(t) \mathrm{d}t, i=1,2, \tag{5.3}$$

and for the manufacturer, we have as follow:

$$\max J_m^s = \int_0^{+\infty} \mathrm{e}^{-rt} \pi_m^s(t) \mathrm{d}t. \tag{5.4}$$

Considering equations (4.1) and (4.2), we can get the current value Hamiltonian for supplier $i (i=1,2)$ as following:

$$\begin{aligned} H_{si}^s = \pi_{si}^s &+ \mu_{i1}(x_{s1} - \delta G_{s1}) + \mu_{i2}(x_{s2} - \delta G_{s2}) + \mu_{i3}(\lambda_{s1} \\ &x_{s1} + \lambda_m x_m - \delta G_{m1}) + \mu_{i4}(\lambda_{s2} x_{s2} + \lambda_m x_m - \delta G_{m2}), \end{aligned} \tag{5.5}$$

and the manufacturer's current value Hamiltonian is as follow:

$$\begin{aligned} H_m^s = \pi_m^s &+ \mu_{31}(x_{s1} - \delta G_{s1}) + \mu_{32}(x_{s2} - \delta G_{s2}) + \mu_{33}(\lambda_{s1} \\ &x_{s1} + \lambda_m x_m - \delta G_{m1}) + \mu_{34}(\lambda_{s2} x_{s2} + \lambda_m x_m - \delta G_{m2}), \end{aligned} \tag{5.6}$$

where μ_{ij} represents co-state variables in the problem of channel members corresponding to goodwill levels.

Proposition 5.1. *Under the decentralized scenario, if subsidy rate ϕ_i is fixed, then the quality levels of all components along time t are all constants, that is,*

$$x_{si}^s(t) = \frac{1}{2(1-\phi_i)}(\lambda_{si}\rho_{si} + \frac{\rho_{si}(\lambda_{si} + \beta)}{r+\delta}), i=1,2, \tag{5.7}$$

$$x_m^s(t) = \frac{\lambda_m(1-\theta)(\rho_{m1}+\rho_{m2})}{2} + \frac{\lambda_m(1-\chi)(\rho_{m1}+\rho_{m2})}{2(r+\delta)}. \tag{5.8}$$

The following insights can be obtained based on Proposition 5.1. (a) When all else is equal, the marginal profit of the supplier positively influences its own quality improvement spending and does not affect that of the other channel members. (b) Similarly, the marginal profit of the manufacturer positively influences its own quality level only. (c) The subsidy rate positively affects the quality spending of the supplier. Accordingly, if the supplier has a high subsidy rate, then he will increase the equilibrium quality level. Meanwhile, the subsidy rate has no influence on the quality improvement spending of the manufacturer. The manufacturer will not reduce the investment in product quality improvement even if he provides high subsidy rates to suppliers. (d) When all else is equal, competition intensity χ (or θ) negatively affects the quality improvement spending of the manufacturer. However, if the competition intensity is weak, then the manufacturer will increase quality improvement efforts. Competition intensity does not affect the quality improvement of the supplier if all else is equal. (e) Parameter λ_{si} positively affects the quality spending of the supplier. A large value of this parameter implies that supplier i produces key component to the manufacturer and that the supplier will considerably spend on component si. (f) Discount rate r negatively affects the quality spending of all channel members. A high discount rate means that the channel member is myopic and will reduce his quality improvement spending. (g) Diminishing rate δ also negatively affects the quality improvement efforts of channel members. If the consumer easily forgets product goodwill, then the supply chain members will reduce quality spending. (h) When all else is equal, the quality efforts of the manufacturer do not depend on "brand halo" effect β. While this effect positively influences the quality spending of the supplier, who will increase his quality efforts if the "brand halo" effect is significant.

Subsequently, we can calculate all accumulated goodwill over time t, which is given by Proposition 5.2.

Proposition 5.2. *Under the decentralized scenario and if all the quality efforts of supply chain members are constants, i.e.,* $x_{si}(t)=x_{si}^{s}$ *and* $x_m(t)=x_m^s$*, then the goodwill stock for supplier* i *along time* t *is as follows:*

$$G_{si}(t)=G_{si}^{ss}+D_i\mathrm{e}^{-\delta t}, i=1,2. \tag{5.9}$$

The stock of goodwill for final product i *along time* t *is the following:*

$$G_{mi}(t)=G_{mi}^{ss}+E_i\mathrm{e}^{-\delta t}, i=1,2, \tag{5.10}$$

where $G_{si}^{ss}=x_{si}^{s}/\delta$, $G_{mi}^{ss}=(\lambda_{si}x_{si}^{s}+\lambda_m x_m^s)/\delta$, $D_i=G_{si}^{0}-x_{si}^{s}/\delta$, *and* $E_i=G_{mi}^{0}-G_{mi}^{ss}$. G_{si}^{ss} *is the steady goodwill of supplier* i *when* $t\to\infty$, *and* G_{mi}^{ss} *is the steady goodwill of product* i *when* $t\to\infty$.

We can get the following insights based on equation (5.9). (i) The steady state goodwill of the supplier is determined by his equilibrium quality efforts. An increase in the equilibrium quality efforts of the supplier will increase his steady state goodwill. (ii) The diminishing rate negatively affects the steady state goodwill of the supplier. Equation (5.10) verifies that the steady state goodwill of product i is influenced by its quality level and the diminishing rate of goodwill. The first factor positively affects the steady state goodwill of the final product. A high-quality level will bring considerable goodwill to the final product. While the last factor negatively affects the steady state goodwill, namely, if the consumer easily forgets the final product, the steady state goodwill of the final product will drop.

Substituting equations (5.7)–(5.10) with equation (5.4), we can get the present value of manufacturer profit J_m. Given the differentiation between J_m and ϕ_i, the optimal subsidy rates can be calculated, which is given by Proposition 5.3.

Proposition 5.3. *Under the decentralized scenario, optimal subsidy rate* ϕ_i^s *of the manufacturer is as follow:*

$$\phi_i^s=\begin{cases}\frac{A_i-B_i}{A_i+B_i} & \text{if } A_i-B_i>0\\ 0 & \text{else}\end{cases} \tag{5.11}$$

where $A_i=2\lambda_{si}(\rho_{mi}-\theta\rho_{m,3-i})(r+\delta)+2\lambda_{si}(\rho_{mi}-\chi\rho_{m,3-i})+2\rho_{mi}\beta$ *and* $B_i=\rho_{si}\lambda_{si}(r+\delta)+\rho_{si}(\beta+\lambda_{si}), i=1,2.$

Proposition 5.3 shows that subsidy rate ϕ_1^s is determined by the following factors. (a) Competition intensity θ(or χ). Differentiating ϕ_1^s from θ and χ), we have $\partial\phi_1^s/\partial\theta<0$ and $\partial\phi_1^s/\partial\chi<0$. Competition intensity negatively affects the optimal subsidy rate. If fierce competition exists between two final products, then the manufacturer reduces the subsidy rate to avoid internal conflicts. (b) Supplier $i's$ marginal profit ρ_{si}. Differentiating ϕ_1^s from ρ_{s1} and ρ_{s2}, we have $\partial\phi_1^s/\partial\rho_{s1}<0$ and $\partial\phi_1^s/\partial\rho_{s2}=0$. If the profitability of the supplier is low, then the manufacturer grants a high subsidy rate to motivate the supplier to enhance the product quality level. If the supplier's marginal is high, then he will increase the quality improvement efforts. Therefore, the manufacturer does not need to grant a high subsidy rate to motivate the supplier. Additionally, the marginal profit of supplier 2 has no influence on optimal subsidy rate ϕ_1^s. (c) Manufacturer's marginal profit ρ_{mi}. Differentiating ϕ_1^s from ρ_{m1} and ρ_{m2}, we have $\partial\phi_1^s/\partial\rho_{m1} > 0$ and $\partial\phi_1^s/\partial\rho_{m2} < 0$. If the manufacturer obtains a high marginal profit by selling product 1, then he gains considerable incentives to improve the subsidy rate given to supplier 1. However, if the manufacturer obtains a high marginal profit by selling product 2, then he reduces subsidy rate ϕ_1^s. (d) "Brand halo" effect β. A large value of parameter means a significant "brand halo" effect. Differentiating ϕ_1^s from β, we have $\partial\phi_1^s/\partial\beta > 0$. The manufacturer grants a high subsidy rate to the supplier if a significant "brand halo" effect exists. (e) Parameter λ_{s1}. Differentiating ϕ_1^s from λ_{s1}, we have $\partial\phi_1^s/\partial\lambda_{s1} < 0$. If supplier 1 is a key component producer, then the manufacturer tends to offers a low subsidy rate to supplier 1. Generally, the investment of a key supplier on quality improvement is maintained at a high level. Therefore, the manufacturer does not need to grant a high subsidy rate to motivate the supplier to increase his quality efforts. For optimal subsidy rate ϕ_2^s, we can get similar results.

Substituting equation (5.11) with equation (5.7), the equilibrium quality level of supplier i is as follows:

$$x_{si}^s = \frac{1}{2(1-\phi_i^s)}\left(\lambda_{si}\rho_{si} + \frac{\rho_{si}(\lambda_{si}+\beta)}{r+\delta}\right). \tag{5.12}$$

Furthermore, substituting equations (5.8)–(5.12) with equations (5.3) and (5.4), respectively, we obtain the current values of all supply chain members as J_{si}^s, J_m^s, and the current value of the whole supply chain system as J^s.

5.2. Integrated scenario

All supply chain members are integrated as a single firm; therefore, the integrated system objective function is as follows:

$$\max J^I = \int_0^{+\infty} \mathrm{e}^{-rt}(\pi_m(t) + \pi_{s1}(t) + \pi_{s2}(t))\mathrm{d}t. \tag{5.13}$$

Solving this decision problem, we can obtain Proposition 5.4.

Proposition 5.4. *When all supply chain members are integrated, the equilibrium quality levels of all components along time t are constants, that is,*

$$\begin{aligned} x_{si}^I(t) = {} & \frac{\lambda_{si}(\rho_{si}+\rho_{mi}) - \theta\lambda_{si}(\rho_{s,3-i}+\rho_{m,3-i})}{2} \\ & + \frac{(\beta+\lambda_{si})(\rho_{si}+\rho_{mi}) - \chi\lambda_{si}(\rho_{s,3-i}+\rho_{m,3-i})}{2(r+\delta)}, \end{aligned} \tag{5.14}$$

$$\begin{aligned} x_m^I(t) = {} & \frac{\lambda_m(1-\theta)(\rho_{s1}+\rho_{s2}+\rho_{m1}+\rho_{m2})}{2} \\ & + \frac{\lambda_m(1-\chi)(\rho_{s1}+\rho_{s2}+\rho_{m1}+\rho_{m2})}{2(r+\delta)}. \end{aligned} \tag{5.15}$$

An examination of Proposition 5.4 leads to the following management insights. (a) The quality levels of all components are influenced by the marginal profit of the whole supply chain. (b) Competition intensity negatively affects the quality levels of all components. (c) Comparing Proposition 5.1 with Proposition 5.4, given that $x_m^I > x_m^s$, we find the integrated system will considerably spend on component m compared with that in the decentralized scenario. (d) Comparing equation (5.14) with equation (5.7), we can obtain $\triangle x_{si} = x_{si}^I - x_{si}^s = (\lambda_{si}((\rho_{si} - 2\theta\rho_{s,3-i})(r+\delta) + \rho_{si} - 2\chi\rho_{s,3-i}) + \beta\rho_{si})/4(r+\delta)$. The afore-mentioned expression affirms that the integrated system would then increase the quality level of component si if conditions $\theta < (\rho_{s1}/2\rho_{s2})$ and $\chi < (\rho_{s1}/2\rho_{s2})$ are satisfied, which also means that the integrated system may then increase the quality levels of component si if the competition intensity is weak. Otherwise, the integrated system may decrease the quality level of component to avoid internal conflicts. Similarly, we can obtain the current value of the integrated system as J^I. Together with decentralized scenario, we can find the profit of whole supply chain is improved, and the supply chain surplus is given by $J^{Is} = J^I - J^s$.

5.3. Bilateral participation contract

We now introduce a bilateral participation contract to coordinate the supply chain system. Under this contract, the manufacturer shares part of the quality expenditure of the supplier i with subsidy rates $\phi_i (0 \le \phi_i \le 1)$, and the supplier i also offers sharing rates $\psi_i (0 \le \psi_i \le 1)$ to the product quality cost of the manufacturer. And this contract works as follows. Firstly, two suppliers and manufacturer jointly determine the subsidy rate ϕ_i and sharing rate ψ_i. Secondly, all channel members are responsible for determining their quality improvement efforts. Therefore, the objective function of supplier i is changed to the following:

$$\max J_{si}^c = \int_0^{+\infty} \mathrm{e}^{-rt} (\rho_{si} S_i(t) - \psi_i x_m^2 - (1-\phi_i) x_{si}^2) \mathrm{d}t, i = 1, 2, \tag{5.16}$$

and for the manufacturer, we have the following:

$$\max J_m^c = \int_0^{+\infty} \mathrm{e}^{-rt} (\rho_{m1} S_1(t) + \rho_{m2} S_2(t) - (1-\psi_1-\psi_2) x_m^2 - \phi_1 x_{s1}^2 - \phi_2 x_{s2}^2) \mathrm{d}t. \tag{5.17}$$

Solving this decision problem, we can obtain the equilibrium solutions as follows:

Proposition 5.5. *When the suppliers and the manufacturer sign a bilateral participation contract, the equilibrium quality levels of all components along time t are constants, that is,*

$$x_{si}^c(t) = \frac{1}{2(1-\phi_i)} \left(\lambda_{si} \rho_{si} + \frac{\rho_{si}(\lambda_{si} + \beta)}{r+\delta} \right), i = 1, 2, \tag{5.18}$$

$$x_m^c(t) = \frac{\lambda_m (1-\theta)(\rho_{m1} + \rho_{m2})}{2(1-\psi_1-\psi_2)} + \frac{\lambda_m (1-\chi)(\rho_{m1}+\rho_{m2})}{2(r+\delta)(1-\psi_1-\psi_2)}. \tag{5.19}$$

Proposition 5.5 shows that the quality level of component si in the bilateral participation contract has a structure similar to that in the decentralized scenario. When the supplier grants a sharing rate to the manufacturer, the quality level of component m is higher than that in the decentralized scenario. If the equilibrium quality levels under the proposed contract are equal to that in the integrated scenario ($x_{si}^c = x_{si}^I$ and $x_m^c = x_m^I$), then the profits of whole supply chain under the proposed contract are equal to that in the integrated scenario. Under this situation, this supply chain can be coordinated. We then solve these equations ($x_{si}^c = x_{si}^I$ and $x_m^c = x_m^I$) and obtain Proposition 5.6.

Proposition 5.6. *If subsidy rate ϕ_i and sharing rate ψ_i take the following values:*

$$\psi_1^c + \psi_2^c = \frac{\rho_{s1} + \rho_{s2}}{\rho_{s1} + \rho_{s2} + \rho_{m1} + \rho_{m2}}, \tag{5.20}$$

$$\psi_1^c + \psi_2^c = \frac{C}{C+B}, \tag{5.21}$$

where $C = \lambda_{si}(\rho_{mi} - \theta\rho_{m,3-i})(r+\delta) - \rho_{s,3-i}\theta\lambda_{si}(r+\delta) + \rho_{mi}(\beta+\lambda_{si}) - \chi\lambda_{si}(\rho_{s,3-i} + \rho_{m,3-i})$, and $B = \rho_{s1}\lambda_{s1}(r+\delta) + \rho_{si}(\beta+\lambda_{si})$. then the quality levels of all supply chain members under a bilateral participation contract are the same as that under the integrated system. Therefore, the bilateral participation contract can coordinate this supply chain system.

Equation (5.20) shows the following implications: (i) sharing rate ψ_i is not affected by competition and the "brand halo" effect, and its value only depends on the marginal profit of the channel members; (ii) when the supplier has a positive marginal profit, then the supplier grants a sharing rate to the manufacturer; and (iii) the total sharing rate $(\psi_1+\psi_2)$ is equal to the rate between the supplier's marginal profit and the whole supply chain' marginal profit. If the supplier has a high marginal profit, then he offers a high sharing rate to the manufacturer. From equation (5.21), we can get similar results with Proposition 5.3, and the only difference is that subsidy rate ϕ_{s1}^c is influenced by the marginal profit of supplier 2 under the bilateral participation contract. In addition, the manufacturer grants a high subsidy rate to supplier 1 if the marginal profit of supplier 2 is low. The manufacturer does not grant a subsidy to the supplier if two final products are very competitive, and the bilateral participation contract may not coordinate this system.

Similarly, we can get the current values of all supply chain members as J_{si}^c, J_m^c, and the current value of the whole supply chain system as J^c. According to the numerical analysis, the proposed contract may improve all channel members' profits in most situation, however channel member's profit may be suffered in this proposed contract in certain conditions. Under this situation, we may need introduce a transfer payment policy to ensure all channel members' profits are improved, *i.e.*, a fixed amount $(T_i, i=1,2)$ transferred from manufacturer to supplier i (or vice versa, if T_i is negative) to ensure a fair distribution of supply chain surplus J^{Is}. Then, the current value of each channel member is

$$J_m^* = J_m^c - T_1 - T_2, \tag{5.22}$$

$$J_{s1}^* = J_{s1}^c + T_1, \tag{5.23}$$

$$J_{s2}^* = J_{s2}^c + T_2. \tag{5.24}$$

In our study, we assume that all channel members equally allocate the supply chain surplus J^{Is}, this assumption is found in the literature [6,23,31]. Therefore, if the manufacturer provides a transfer payment to the supplier i, then let $J_{s1}^* = J_{s1}^s + J^{Is}/3$ and $J_{s2}^* = J_{s2}^s + J^{Is}/3$, we can obtain the transfer payment T_i as follows:

$$T_1 = J_{s1}^s + J^{Is}/3 - J_{s1}^c, \tag{5.25}$$

$$T_2 = J_{s2}^s + J^{Is}/3 - J_{s2}^c. \tag{5.26}$$

To achieve the perfect supply chain coordination, a bilateral participation contract can be setup with parameters of ϕ_i, ψ_i and T_i given by equations (5.20), (5.21), (5.25) and (5.26) respectively.

6. Numerical analysis

Numerical analysis is performed to illustrate the following: (i) the influence of competition intensity on the profit for each channel member and the whole supply chain and (ii) the effects of the bilateral participation contract on the profit for supply chain members and the whole supply chain. In numerical analysis, the following parameters are fixed with the following values: $\alpha_1=200, \alpha_2=300, \lambda_{s1}=3, \lambda_{s2}=3, \lambda_m=4, \beta=0.3, r=0.3, \delta=0.2, \chi=0.3, \rho_{s1}=5, \rho_{s2}=4, \rho_{m1}=6, \rho_{m2}=5, G_{s1}^0=100, G_{s2}^0=110, G_{m1}^0=200$, and $G_{m2}^0=220$. Let $\psi_i^c=\rho_{si}/(\rho_{s1}+\rho_{s2}+\rho_{m1}+\rho_{m2})$ denote the sharing rate of supplier i, which means that the sharing rate of supplier i is affected by his marginal profit. If the supplier has a high marginal profit, then the supplier grants a high sharing rate. This assumption is found in previous research (*e.g.*, [45]). The values of the other parameters are then adjusted.

Let competition intensity θ denote the value of x-axis, and change this value from 0 to 1. Figure 1 presents the relationships between the quality level of component si and competition intensity θ in different scenarios.

Figure 1 illustrates the following marginal insights. (i) As competition intensity θ increases, the equilibrium quality levels of components $s1$ and $s2$ drop in decentralized scenario and proposed contract. (ii) Commonly, the quality level of component si in the traditional subsidy program is lower than that under the bilateral participation contract. (iv) If the competition is weak, then the quality level of component si in the traditional

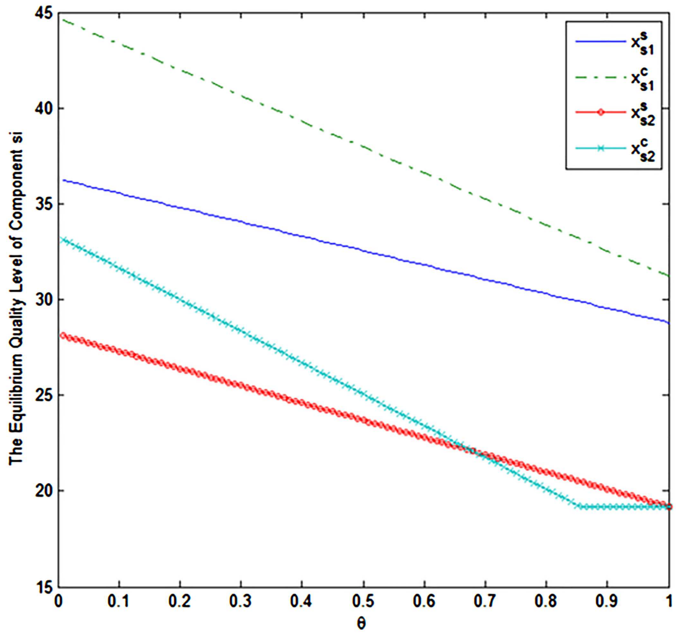

FIGURE 1. The relationships between the component si's quality level and the competition intensity θ.

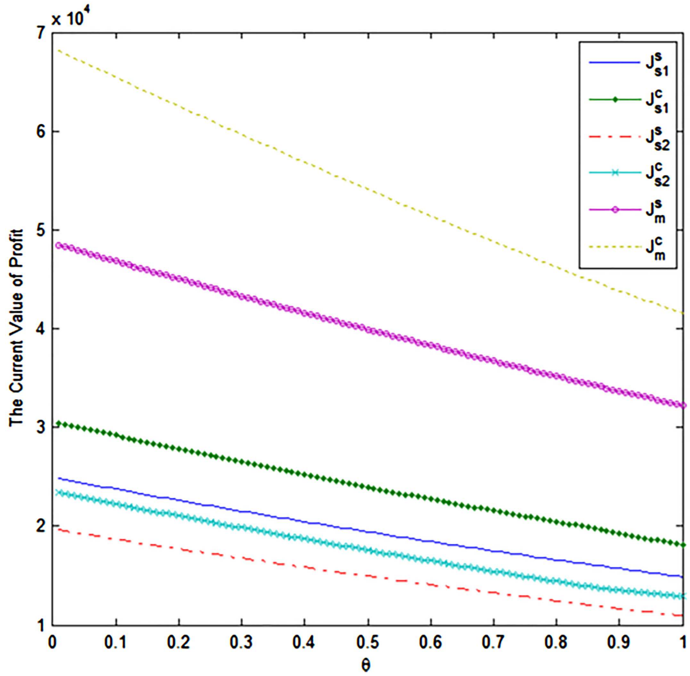

FIGURE 2. The relationships between the current value of profit and the competition intensity θ.

subsidy program is always lower than that under the proposed contract; otherwise, the quality level in the traditional subsidy program is higher than that under the proposed contract.

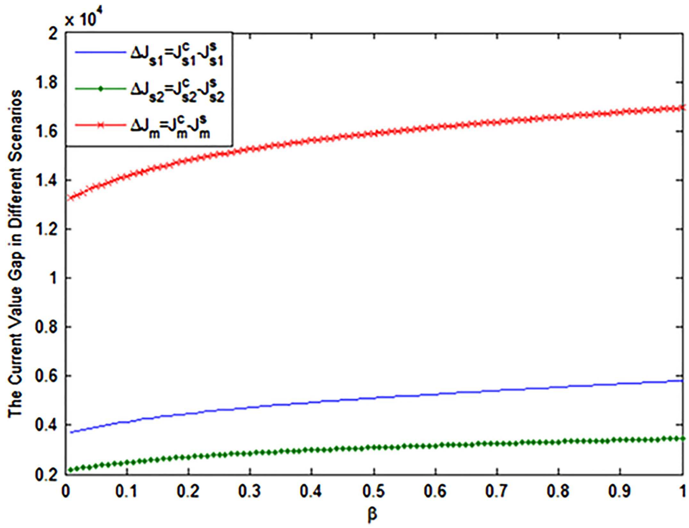

FIGURE 3. The relationships between the current value of profit and the "brand halo" effect β.

Figure 2 depicts the relationships between the current value of the supply chain member and the competition intensity θ in different scenarios.

Figure 2 indicates the following insights. (i) An increase of the competition intensity leads to the profit drop for all channel members in different scenarios. Therefore, the manufacturer should let the two final products focus on different sub-markets (*i.e.*, low- and high-end markets) to avoid a strong competition between the two sub-brands. For example, Lenovo produces different sub-brand computers (*e.g.*, THINKPAD, IDEAPAD, YOGA, and LENOVO), and different sub-brand computers may attract different consumer groups to purchase the products. (ii) The current value of the supply chain member under the bilateral participation contract is larger than that in the decentralized scenario, which proves the effectiveness of the bilateral participation contract. Therefore, all channel members can obtain an extra profit if they sign this supply chain contract. (iii) The profit improvement of channel members increases if the competition intensity decreases; thus, the improvement is not evident when both sub-brands are highly competitive. Therefore, the bilateral participation contract is considerably effective if the competition is weak.

In the Figure 3, we assume that $\theta = 0.4$. Then let "brand halo" effect β denote the value of x-axis, and change this value from 0 to 1, let the current value gap of each channel member denote the value of y-axis, where the item $\triangle J_{s1} = J_{s1}^c - J_{s1}^s$ implies the current value gap of supplier 1 between decentralized scenario and bilateral participation contract. Therefore, Figure 3 presents the relationships current value gap of each channel member and "brand halo" effect β. Figure 3 brings the following managerial implications: (i) Compared with that in the decentralized scenario, the bilateral participation contract improves the performance of each channel member. (ii) An increase of the "brand halo" effect β leads to the current value gap increases for all channel members, therefore channel members have a more incentive to join the bilateral participation contract if the "brand halo" effect is large. (iii) Figure 3 also proves that the bilateral participation contract can coordinate this system perfectly if the competition intensity is not high.

7. CONCLUSION

This study investigates the influences of competition and the "brand halo" effect on supply chain member quality strategies and explores the potential coordinating power of the bilateral participation contract in a supply chain with two suppliers and one manufacturer. Utilizing differential game theory, this study further analyzes the equilibrium solutions of all firms in the following scenarios, namely, (1) decentralized scenario within a subsidy program, (2) integrated scenario, and (3) bilateral participation contract.

Our results confirm the following managerial insights. (i) The "brand halo" effect positively influences manufacturer subsidy rates and channel members' quality efforts. Supply chain members are more likely to join the bilateral participation contract if the "brand halo" effect is large. (iii) Competition intensity plays an important role in the subsidy policy of the manufacturer. If two final products are competitive, then the manufacturer may not grant a subsidy to the supplier. (iv) The bilateral participation contract can achieve perfect coordination if the competition is weak or if a transfer payment policy exists.

A number of caveats should be noted regarding this study. First, this study does not explore the pricing decisions of channel members. Investigating the pricing strategies would be desirable in future studies. Second, we only explore the potential coordinating power of the bilateral participation contract and introducing other contracts would be highly interesting. Third, we focus on a "two suppliers and one manufacturer" supply chain network. The supply chain network can be extended to a "multiple suppliers and multiple manufacturers" network later.

Appendix A.

Proof of Proposition 5.1. In the decentralized scenario within a subsidy program, the current value Hamiltonian of supplier 1 is as follows:

$$H_{s1}^{s} = \pi_{s1}^{s} + \mu_{11}(x_{s1} - \delta G_{s1}) + \mu_{12}(x_{s2} - \delta G_{s2}) + \mu_{13}(\lambda_{s1} x_{s1} + \lambda_m x_m - \delta G_{m1}) + \mu_{14}(\lambda_{s2} x_{s2} + \lambda_m x_m - \delta G_{m2}), \tag{A.1}$$

Necessary conditions for equilibrium are the following:

$$\frac{\partial H_{s1}^{s}}{\partial x_{s1}} = 0, \tag{A.2}$$

$$\mathrm{d}\mu_{11}/\mathrm{d}t = r\mu_{11} - \frac{\partial H_{s1}^{s}}{\partial G_{s1}} = 0, \tag{A.3}$$

$$\mathrm{d}\mu_{12}/\mathrm{d}t = r\mu_{12} - \frac{\partial H_{s1}^{s}}{\partial G_{s2}} = 0, \tag{A.4}$$

$$\mathrm{d}\mu_{13}/\mathrm{d}t = r\mu_{13} - \frac{\partial H_{s1}^{s}}{\partial G_{m1}} = 0, \tag{A.5}$$

$$\mathrm{d}\mu_{14}/\mathrm{d}t = r\mu_{14} - \frac{\partial H_{s1}^{s}}{\partial G_{m2}} = 0. \tag{A.6}$$

Equation (A.2) implies the following:

$$x_{s1}(t) = \frac{1}{2(1-\phi_1)}(\lambda_{s1}\rho_{s1} + \mu_{11} + \lambda_{s1}\mu_{13}). \tag{A.7}$$

Solving equations (A.3)–(A.6), we can obtain the following:

$$\mathrm{d}\mu_{11}/\mathrm{d}t = (r+\delta)\mu_{11} - \beta\rho_{s1}, \tag{A.8}$$

$$\mathrm{d}\mu_{12}/\mathrm{d}t = (r+\delta)\mu_{12}, \tag{A.9}$$

$$\mathrm{d}\mu_{13}/\mathrm{d}t = (r+\delta)\mu_{13} - \rho_{s1}, \tag{A.10}$$

$$\mathrm{d}\mu_{14}/\mathrm{d}t = (r+\delta)\mu_{14} + \chi\rho_{s1}. \tag{A.11}$$

Differentiating equation (A.7) w.r.t. from time and substituting for the time derivative of μ_{11} and μ_{13} in (A.8) and (A.10), respectively, we can obtain the following:

$$\mathrm{d}x_{s1}(t)/\mathrm{d}t = (r+\delta)x_{s1} - \frac{1}{2(1-\phi_1)}(\lambda_{s1}\rho_{s1}(r+\delta) + \rho_{s1}(\lambda_{s1}+\beta)). \tag{A.12}$$

Subsequently, we can derive the time path of $x_{s1}(t)$ as follows:

$$x_{s1}(t) = C_1\mathrm{e}^{(r+\delta)t} + \frac{1}{2(1-\phi_1)}\left(\lambda_{s1}\rho_{s1} + \frac{\rho_{s1}(\lambda_{s1}+\beta)}{r+\delta}\right). \tag{A.13}$$

Given that the value of quality level given in equation (A.13) should satisfy the free-boundary condition, then:

$$\lim_{t\to\infty} x_{s1}(t) < \infty. \tag{A.14}$$

Therefore, condition (A.14) implies that $C_1 = 0$. Thereafter, we have the equilibrium quality level of supplier 1, that is,

$$x_{s1}^{s}(t) = \frac{1}{2(1-\phi_1)}\left(\lambda_{s1}\rho_{s1} + \frac{\rho_{s1}(\lambda_{s1}+\beta)}{r+\delta}\right). \tag{A.15}$$

Similarly, given supplier 2 and the profit maximization problems of the manufacturer, the equilibrium quality level of supplier 2 is given by the following:

$$x_{s2}^{s}(t) = \frac{1}{2(1-\phi_2)}\left(\lambda_{s2}\rho_{s2} + \frac{\rho_{s2}(\lambda_{s2}+\beta)}{r+\delta}\right). \tag{A.16}$$

and the quality level of the manufacturer is given by the following:

$$x_{m}^{s}(t) = \frac{\lambda_m(1-\theta)(\rho_{m1}+\rho_{m2})}{2} + \frac{\lambda_m(1-\chi)(\rho_{m1}+\rho_{m2})}{2(r+\delta)}. \tag{A.17}$$

Proof of Proposition 5.2. We can get the general solution of equation (4.1) as follows:

$$G_{si}(t) = G_{si}^{ss} + D_i\mathrm{e}^{-\delta t}, i = 1, 2, \tag{A.18}$$

where $G_{si}^{ss} = x_{si}^{s}/\delta$.

Considering $t = 0$ in equation (A.18) and utilizing the initial conditions of equation (4.1), we have $D_i = G_{si}^{0} - x_{si}^{s}/\delta, i = 1, 2$.

Substituting equation (A.18) with equation (4.2), we can get the general solution of equation (4.2) as follows:

$$G_{mi}(t) = G_{mi}^{ss} + E_i\mathrm{e}^{-\delta t}, i = 1, 2, \tag{A.19}$$

where $G_{mi}^{ss} = (\lambda_{si}x_{si}^{s} + \lambda_m x_m^{s})/\delta$,and $E_i = G_{mi}^{0} - G_{mi}^{ss}$.

Substituting equations (A.15)–(A.18), (5.11), and (5.12) with equations (5.3) and (5.4), we obtain the current values of all supply chain members as follows: J_{si}^{s}, J_{m}^{s}.

Proof of Proposition 5.3. In the integrated scenario, we have the current value Hamiltonian for the integrated system:

$$\begin{aligned} H^I = \pi^I &+ \eta_1(x_{s1} - \delta G_{s1}) + \eta_2(x_{s2} - \delta G_{s2}) + \eta_3(\lambda_{s1}x_{s1} \\ &+ \lambda_m x_m - \delta G_{m1}) + \eta_4(\lambda_{s2}x_{s2} + \lambda_m x_m - \delta G_{m2}), \end{aligned} \tag{A.20}$$

According to the Pontryagin's maximum principle, necessary conditions for equilibrium are as follows:

$$\frac{\partial H^I}{\partial x_{s1}} = 0, \tag{A.21}$$

$$\frac{\partial H^I}{\partial x_m} = 0, \tag{A.22}$$

$$\mathrm{d}\eta_1/\mathrm{d}t = r\eta_1 - \frac{\partial H^I}{\partial G_{s1}} = 0, \tag{A.23}$$

$$\mathrm{d}\eta_2/\mathrm{d}t = r\eta_2 - \frac{\partial H^I}{\partial G_{s2}} = 0, \tag{A.24}$$

$$\mathrm{d}\eta_3/\mathrm{d}t = r\eta_3 - \frac{\partial H^I}{\partial G_{m1}} = 0, \tag{A.25}$$

$$\mathrm{d}\eta_4/\mathrm{d}t = r\eta_4 - \frac{\partial H^I}{\partial G_{m2}} = 0. \tag{A.26}$$

Equation (A.21) implies the following:

$$x_{s1}(t) = \frac{1}{2}\big(\lambda_{s1}(\rho_{s1} + \rho_{m1}) - \theta\lambda_{s1}(\rho_{s2} + \rho_{m2})\eta_1 + \lambda_{s1}\eta_3\big). \tag{A.27}$$

Solving equations (A.23) and (A.25), we can obtain the following:

$$\mathrm{d}\eta_1/\mathrm{d}t = (r + \delta)\eta_1 - \beta(\rho_{s1} + \rho_{m1}), \tag{A.28}$$

$$\mathrm{d}\eta_3/\mathrm{d}t = (r + \delta)\eta_3 - (\rho_{s1} + \rho_{m1}) + \chi(\rho_{s2} + \rho_{m2}). \tag{A.29}$$

Differentiating equation (A.27) w.r.t. from time and substituting for the time derivative of η_1 and η_3 in (A.28) and (A.29), respectively, we can obtain the following:

$$\begin{aligned}\mathrm{d}x_{s1}(t)/\mathrm{d}t = (r + \delta)x_{s1} - \frac{1}{2}(r + \delta)(\lambda_{s1}(\rho_{s1} + \rho_{m1})\\ - \theta\lambda_{s1}(\rho_{s2} + \rho_{m2})) - \frac{1}{2}(\lambda_{s1} + \beta)(\rho_{s1} + \rho_{m1})\\ - \frac{1}{2}\chi\lambda_{s1}(\rho_{s2} + \rho_{m2}).\end{aligned} \tag{A.30}$$

Subsequently, we can derive the time path of $x_{s1}(t)$ as follows:

$$\begin{aligned}x_{s1}(t) = M_1 \mathrm{e}^{(r+\delta)t} + \frac{\lambda_{s1}(\rho_{s1} + \rho_{m1}) - \theta\lambda_{s1}(\rho_{s2} + \rho_{m2})}{2}\\ + \frac{(\beta + \lambda_{s1})(\rho_{s1} + \rho_{m1}) - \chi\lambda_{s1}(\rho_{s2} + \rho_{m2})}{2(r + \delta)}.\end{aligned} \tag{A.31}$$

Given that the value of quality level given in equation (A.31) should satisfy the free-boundary condition, then:

$$\lim_{t\to\infty} x_{s1}(t) < \infty. \tag{A.32}$$

Therefore, condition (A.32) implies that $M_1 = 0$. Thereafter, we have the equilibrium quality level of component $s1$ as follows:

$$\begin{aligned}x^I_{s1}(t) = \frac{\lambda_{s1}(\rho_{s1} + \rho_{m1}) - \theta\lambda_{s1}(\rho_{s2} + \rho_{m2})}{2}\\ + \frac{(\beta + \lambda_{s1})(\rho_{s1} + \rho_{m1}) - \chi\lambda_{s1}(\rho_{s2} + \rho_{m2})}{2(r + \delta)}.\end{aligned} \tag{A.33}$$

Similarly, the equilibrium quality levels of components $s2$ and m are the following

$$x_{s2}^I(t) = \frac{\lambda_{s2}(\rho_{s2}+\rho_{m2}) - \theta\lambda_{s2}(\rho_{s1}+\rho_{m1})}{2} + \frac{(\beta+\lambda_{s2})(\rho_{s2}+\rho_{m2}) - \chi\lambda_{s2}(\rho_{s1}+\rho_{m1})}{2(r+\delta)}. \tag{A.34}$$

$$x_m^I(t) = \frac{\lambda_m(1-\theta)(\rho_{s1}+\rho_{s2}+\rho_{m1}+\rho_{m2})}{2} + \frac{\lambda_m(1-\chi)(\rho_{s1}+\rho_{s2}+\rho_{m1}+\rho_{m2})}{2(r+\delta)}. \tag{A.35}$$

(A.36)

Proof of Proposition 5.4. Under the bilateral participation contract, the current value Hamiltonian for the manufacturer is

$$H_m^c = \pi_m^c + \nu_{11}(x_{s1} - \delta G_{s1}) + \nu_{12}(x_{s2} - \delta G_{s2}) + \nu_{13}(\lambda_{s1}x_{s1} + \lambda_m x_m - \delta G_{m1}) + \nu_{14}(\lambda_{s2}x_{s2} + \lambda_m x_m - \delta G_{m2}). \tag{A.37}$$

Necessary conditions for equilibrium are the following:

$$\frac{\partial H_m^c}{\partial x_m} = 0, \tag{A.38}$$

$$\mathrm{d}\nu_{11}/\mathrm{d}t = r\nu_{11} - \frac{\partial H_m^c}{\partial G_{s1}} = 0, \tag{A.39}$$

$$\mathrm{d}\nu_{12}/\mathrm{d}t = r\nu_{12} - \frac{\partial H_m^c}{\partial G_{s2}} = 0, \tag{A.40}$$

$$\mathrm{d}\nu_{13}/\mathrm{d}t = r\nu_{13} - \frac{\partial H_m^c}{\partial G_{m1}} = 0, \tag{A.41}$$

$$\mathrm{d}\nu_{14}/\mathrm{d}t = r\nu_{14} - \frac{\partial H_m^c}{\partial G_{m2}} = 0. \tag{A.42}$$

Similarly, we can have the quality level of component m:

$$x_m^c(t) = \frac{\lambda_m(1-\theta)(\rho_{m1}+\rho_{m2})}{2(1-\psi_1-\psi_2)} + \frac{\lambda_m(1-\chi)(\rho_{m1}+\rho_{m2})}{2(r+\delta)(1-\psi_1-\psi_2)}. \tag{A.43}$$

and the quality level of component si is:

$$x_{si}^c(t) = \frac{1}{2(1-\phi_i)}\left(\lambda_{si}\rho_{si} + \frac{\rho_{si}(\lambda_{si}+\beta)}{r+\delta}\right), i = 1, 2, \tag{A.44}$$

For equation (4.1), we can get the general solution of equation (4.1) as follows:

$$G_{si}(t) = G_{si}^{ss} + F_i \mathrm{e}^{-\delta t}, i = 1, 2. \tag{A.45}$$

where $G_{si}^{ss} = x_{si}^c/\delta$.

Considering $t = 0$ in equation (A.45) and utilizing the initial conditions of equation (4.1), we have $F_i = G_{si}^0 - x_{si}^c/\delta, i = 1, 2$.

Similarly, we can obtain the general solution of Equation (4.2) as the following:

$$G_{mi}(t) = G_{mi}^{ss} + H_i e^{-\delta t}, i = 1, 2, \tag{A.46}$$

where $G_{mi}^{ss} = (\lambda_{si} x_{si}^c + \lambda_m x_m^c)/\delta$, and $H_i = G_{mi}^0 - G_{mi}^{ss}$.

Substituting equations (A.43)–(A.46), (5.20), and (5.21) with equations (5.16) and (5.17), we get the current values of all supply chain members as follows: J_{si}^c, J_m^c.

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