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# COORDINATED DECISION-MAKING ON MANUFACTURER'S EPQ-BASED AND BUYER'S PERIOD REVIEW INVENTORY POLICIES WITH STOCHASTIC PRICE-SENSITIVE DEMAND: A CREDIT OPTION APPROACH

Mona Jazinaninejad[1], Seyed Mohammad Seyedhosseini[1], Seyyed-Mahdi Hosseini-Motlagh[1,*] and Mohammadreza Nematollahi[1]

**Abstract.** Pricing and replenishment decisions are no longer internal issues within organizations and it is of high significance to coordinate such primary decisions from a supply chain (SC) perspective. In this study, for the first time, we develop a new method based on credit option to coordinate production, pricing, and periodic review policy decisions considering a manufacturer-buyer chain under price-sensitive stochastic demand. In the studied SC, the buyer applies a periodic review order-up-to level inventory policy and its decision variables are order-up-to level, selling price and review period. In this case, the manufacturer follows an economic production quantity (EPQ) policy and decides on production multiplier. Firstly, exact algorithms in addition to concavity analysis are addressed to find optimal decisions in centralized and decentralized structures. Afterwards, a credit option mechanism is proposed to obtain mutually acceptable decisions and create a win–win situation for the manufacturer and buyer. Numerical examples and sensitivity analysis along with a real case study are carried out to show the applicability and performance of the credit option contract. The results reveal the capability of the proposed credit option contract in coordinating pricing, production, and periodic review replenishment decisions; thereby encouraging both sides to accept the coordination mechanism. Moreover, the results indicate that the proposed credit option contract can even improve the SC profit more than centralized model under some circumstances.

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## 1. Introduction

In traditional business environments, each business entity mainly focuses on maximizing its own profitability which results in self-interested decisions. In supply chain (SC) context, most decisions made by a SC actor impact efficiency of other actors due to the extensive interactions between them. Profitability of an independent player in supply chain not only depends on its own actions but also is affected by decisions of other SC actors [20]. Joint decision-making in supply chain, which is called centralized model, creates more benefits for the total SC. However, in practice, obtaining such profits comprises some complexities [19]. Most of supply

*Keywords.* Pricing, supply chain coordination, periodic review replenishment, stochastic price-sensitive demand, credit option.

[1] School of Industrial Engineering, Iran University of Science and Technology, University Road, Hengam Street, Resalat Square, Narmak, Tehran, Iran.
[*] Corresponding author: motlagh@iust.ac.ir

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chains are mainly consist of independent members which pursue their own interests and consequently they may refuse to make decisions which reduce their profitability. Therefore, an incentive scheme should be designed to convince SC actors to accept the joint decision-making structure and reconsider their individually optimized decisions. Several types of coordination contracts such as buy back, two part tariff, quantity discount, revenue sharing, and quantity flexibility have been proposed to increase the profitability of decentralized SC systems. These coordination schemes adopt practical solutions for persuading all SC members to make globally optimum decisions from the entire SC point of view.

In SC coordination literature, there has been great attention given to the coordination of replenishment decisions, especially under economic order quantity (EOQ) and continuous inventory systems. However, literature on coordination of periodic review inventory systems is scant [36]. In many real world cases, it is observed that retailers employ a periodic review policy for replenishing goods. In such a case, the inventory level is reviewed in constant intervals and enough inventory is ordered to raise the inventory up to a fixed level [44]. Johari *et al.* [27] studied the supplier–retailer SC coordination under periodic review inventory system. However, they did not consider pricing decisions. There are various decisions in the supply chain that should be coordinated. For instance, one of the most significant decisions is pricing that play a substantial role in the profitability of SC especially when customer demand is price dependent [41]. In the current study, for the first time, we aim to coordinate production multiplier, periodic review replenishment, and pricing decisions in a manufacturer-buyer chain applying a credit option contract.

In today's competitive market, demand of many products is sensitive to their price. The price sensitivity of demand, which is also named as "price elasticity", measures the demand sensitivity to one percentage change in price [23]. Generally, product price plays a complex and principal role in consumers' evaluation of a product. Price elasticity can be positive or negative. For the fashion products, the price elasticity is positive and for the typical products it is negative [31]. Optimal pricing strategy leads to a brand's success through improving the brand image to the consumers [30], which in turn benefits all supply chain members. Thus, to reach more market share, SC managers should properly make pricing decisions. In the SC context, coordinating pricing decision can significantly improve the entire SC benefit [45]. The upstream SC members profit and market share of whole SC are affected by changing the retailer's selling price [28]. In the traditional business environment, a retailer often individually decides on the selling price without considering the other SC members. However, the optimal pricing strategy determined by the retailer may not be the best decision from the entire SC perspective. Therefore, an incentive scheme should be designed to satisfy the retailer to make pricing decision from entire SC perspective.

In the current study, a credit option contract as a motivation scheme is proposed to coordinate replenishment, production, and pricing decisions under the periodic review policy in a manufacturer-buyer SC. In such a case, the buyer faces a stochastic price sensitive demand and replenishes his/her inventory based on the periodic review system. The buyer decides on the selling price, review period, and order-up-to-level simultaneously. The manufacturer follows an economic production quantity (EPQ) replenishment system and decides on the production multiplier. The pricing and replenishment decisions made by the buyer not only influences his/her profitability, but also impacts on the manufacturer and the entire SC profitability. In the decentralized structure, we model the investigated SC where each member aims to earn the maximum amount of his/her profit regardless of another SC member. Afterwards, the centralized structure is proposed as a benchmark for obtaining the optimal pricing and periodic review replenishment decisions from the entire SC point of view. Solution procedures are addressed to find the optimal SC decision variables under both centralized and decentralized structures. To coordinate the channel, a credit option contract is proposed to satisfy the buyer to accept the coordination scheme. In the proposed credit option contract, the manufacturer provides a credit period for the buyer to encourage him/her to optimize his/her pricing and replenishment decision variables from the whole SC perspective. In order to approach to real world, it is presumed that the manufacturer and the buyer act in various business environments and hence their annual rates of return on investments are various. The acceptable interval of credit period under which the manufacturer and the buyer have enough incentive to take part in the coordination scheme is studied. The applicability of the investigated models are evaluated using a real industrial case study.

The remarkable contribution of this study is synchronous coordination of the manufacturer's production multiplier under EPQ policy and the buyer's pricing and replenishment decisions under periodic review policy that has not yet investigated in the literature. To the best of authors' knowledge, this is the first research that addresses the effect of credit option contract on production, pricing, and replenishment variables under both EPQ and periodic review inventory systems. In addition, the partial credit option contract as a coordination mechanism is developed in this paper for the first time. The structure of this study is as follows. Section 2 reviews the relevant literature. In Section 3, problem definition and used notations are presented. The decentralized, centralized and coordinated decisions-making structures are investigated in Section 4, respectively. Numerical results and sensitivity analysis are presented in Section 5. A real industrial case is investigated in Section 6. Two model extensions are proposed in Section 7. The conclusion and future study recommendations are addressed in Section 8.

## 2. Literature review

Supply chain coordination has been one of the most popular issues in recent years. In the literature, various coordination contracts such as quantity discount (Chaharsooghi *et al.* [9]; Ferhan Çebi [7]; Yang *et al.* [46]), revenue sharing (Arani Vafa *et al.* [4]; Cai *et al.* [6]; Lu *et al.* [32]), return policies (Ai *et al.* [1]; Zhang *et al.* [48]), sales rebate (Saha [39]), wholesale price (Chen *et al.* [12]), cost sharing (Johari and Hosseini-Motlagh, [29]) collaborative decision-making (Nematollahi *et al.* [36]) are applied to coordinate different decisions. For more information, interested readers can refer to Giri *et al.* [17], who make a comparison among different coordination contracts.

In real business environments, it is usual that the seller allows the buyer to pay his/her purchasing cost within a permissible period after receiving orders [43]. Credit option (delay in payment) is one of the most important contracts which has been extensively used for coordinating supply chains. In this contract, the upstream member allows the downstream member to postpone paying his/her purchase cost as long as the downstream member commits to optimize his/her decision variables in such a way that optimize the entire SC performance.

Goyal [18] firstly proposed an EOQ model by applying the credit option. Afterwards, Jaber and Osman [24] expanded profit sharing and the credit options scenario in a two-echelon SC model. They assumed that the permissible credit period is a decision variable and thus replenishment decisions between two levels are coordinated. Chaharsooghi and Heydari [8] later investigated the coordination of order quantity and reorder point in a two echelon SC under backorder inventory model using credit option. Further, Heydari [19] coordinated a single-upstream single-downstream supply chain using delay in payment contract. In this study, demand and also lead times were assumed uncertain. Gao *et al.* [16] later studied a supplier–retailer chain with stochastic demand. They applyed a credit option contract to coordinate the SC. Subsequently, Moussawi-Haidar *et al.* [34] formulated a coordination scheme with discounted interest rate and permissible delay period in a three level SC. To minimize the entire SC cost, they optimized the inventory and financial decisions. In another study, Aljazzar *et al.* [2] investigated the coordination of a three level SC (supplier–manufacturer–retailer) under credit option contract. Their study examined nine various scenarios of credit period offered to the SC members. Recently, Aljazzar *et al.* [3] studied combining price discounts and permissible delay in payments scheme to coordinate supplier–manufacturer–retailer chain. In their study, the discounts and the credit period offered along the SC were supposed to be decision variables. Further, Hojati *et al.* [22] have investigated the delay in payment mechanism to coordinate the periodic review policy decisions.

Some studies in the literature of SC coordination have investigated the coordination of inventory and pricing decisions. Boyacı and Gallego [5] analyzed the coordination of pricing policies along with lot sizing in a SC under deterministic price-dependent demand. Sajadieh and Akbari-Jokar [40] later proposed a mechanism to integrate inventory-production model within two-level supply chain. Their results showed that SC coordination could make a reduction in the selling price. In another research, Chen *et al.* [10] modeled coordination of a two level SC for fashionable product within profit sharing along with three-parameter risk contract. The retail price, order quantity, and amount of capacity reservation were considered to be decision variables. Subsequently, Chen and Bell [11] studied a decentralized manufacturer–retailer chain where the retailer decided on the selling price and order quantity. In their model, customers demand was stochastic and depended on the retail price. Afterwards, Du *et al.* [14] studied a coordination model in a manufacturer–retailer chain by delay in payment discount and wholesale price. The decision variables of this study were production batch size, retail price and order quantity.

Previous researches on coordination of inventory and pricing decisions are compared in Table 1. According to Table 1, some studies in the field of pricing strategy do not consider the uncertainty in demand and they

TABLE 1. Proposed research *vs.* the literature on the joint pricing and inventory decisions.

| Reference | Supply chain structure | Inventory system | Decisions | Demand type | Coordination mechanism |
|---|---|---|---|---|---|
| Boyacı and Gallego [22] | Single wholesaler–Multiple retailers | Single wholesaler: Lot for lot<br>Multiple retailers: Economic order quantity (EOQ) | – Pricing<br>– Replenishment | – Deterministic<br>– Price dependent | Wholesale price discount |
| Sajadieh and Akbari-Jokar [40] | Vendor–Buyer | Vendor: Lot for lot<br>Buyer: EOQ | – Pricing<br>– Replenishment | – Deterministic<br>– Price dependent | Wholesale price discount |
| Chen *et al.* [10] | Supplier–Retailer | Supplier: Lot for lot<br>Retailer: Newsvendor | – Pricing<br>– Order quantity<br>– Amount of capacity reservation in the first stage | – Stochastic<br>– Price dependent | Profit sharing |
| Chen and Bell [11] | Manufacturer–Retailer | Manufacturer: Lot for lot<br>Retailer: News vendor | – Pricing<br>– Order quantity | – Stochastic<br>– Price dependent | Buyback policy |
| Giri and Maiti [17] | Manufacturer–Retailer | Manufacturer: –<br>Retailer: – | – Pricing<br>– Replenishment<br>– Credit period | – Deterministic<br>– Price and credit period dependent | Vertical Integration |
| Johari *et al.* [28] | Supplier–Retailer | Supplier: Lot for lot<br>Retailer: Periodic review policy | – Pricing<br>– Replenishment | – Stochastic<br>– Price dependent | Crashing lead time |
| Du *et al.* [14] | Supplier–Buyer | Supplier: EPQ<br>Buyer: EOQ | – Pricing<br>– Production batch size<br>– Replenishment | – Deterministic<br>– Price dependent | Wholesale price discount and delay in payment |
| Johari *et al.* [26] | Supplier–Retailer | Supplier: Lot for lot<br>Retailer: Periodic review policy | – Pricing<br>– Replenishment | – Stochastic<br>– Price dependent | Quantity discount |
| Heydari and Norouzinasab [21] | Supplier–Retailer | Supplier: Lot for lot<br>Retailer: EOQ | – Pricing<br>– Order quantity | – Stochastic<br>– Price dependent | Wholesale price discount |
| Ebrahimi *et al.* [15] | Supplier–Retailer | Supplier: Lot for lot<br>Retailer: Periodic review policy | –Promotional effort<br>– Replenishment | – Stochastic<br>– Promotional effort dependent | Delay in payment |
| Current study | Manufacturer–Buyer | Manufacturer: EPQ<br>Buyer: Periodic review order-up-to level inventory | – Pricing<br>– Order-up-to-level<br>– Review period<br>– Production multiplier | – Stochastic<br>– Price dependent | Credit option contract |

presumed that the demand is deterministic and price sensitive (see [14, 17, 22, 40]). While in the real word the SC actors face with the unpredictable demand because of the uncertainty. To address the more realistic demand some studies in the literature considered the stochastic demand under various inventory policies (see Tab. 1, [10, 11, 21]). As shown in Table 1, a few papers (see [11, 28]) consider a stochastic price-dependent demand under periodic review policy. The current study differs from these studies since we coordinate production, periodic review inventory, and pricing decisions. Heydari and Norouzinasab [21] investigated a discount model to coordinate ordering and pricing decisions in a two-echelon supply chain under EOQ inventory policy. They assumed that demand was price dependent and stochastic while lead times were fixed. The periodic review policy is one of the applicable replenishment policies which is extensively applied in the supermarkets, grocery stores, pharmacies and so on [35]. Johari *et al.* [28] studied coordination of pricing and periodic review replenishment decisions in a supplier–retailer chain. They proposed a crashing lead time policy to coordinate the both SC members' decisions through using the fast transportation mode. Further, Johari *et al.* [26] proposed a quantity discount scheme to coordinate a supplier–retailer chain. They assumed that the retailer applied a periodic review policy and customer demand was stochastic and depends on retail price. Although two above studies [26, 28] investigated the pricing and periodic review replenishment policy, they do not investigate the credit option contract as a motivation mechanism. Recently, Ebrahimi *et al.* [15] investigated coordination of SC including one supplier and one retailer under periodic review system by a delay in payment contract. They assumed that demand was stochastic and it is sensitive to the retailer's promotional effort.

Due to the mentioned literature of SC coordination under periodic review system, there is no study that considered the manufacturer production system and they investigated only supplier–retailer chain (see [15, 26, 28]). To the best of authors' knowledge, simultaneous coordination of production multiplier, periodic review replenishment, and pricing decisions is not yet investigated (see Tab. 1). Moreover, according to the literature highlighted in Table 1, a credit option contract for coordinating production, periodic inventory and pricing decisions has not yet been developed. Moreover, we simultaneously investigate the EPQ and periodic review inventory systems that it is not yet addressed in the literature (see Tab. 1). In this paper, however, we aim to analytically analyze the role of a credit option contract in coordination of production multiplier, pricing, and periodic review decisions in a manufacturer-buyer SC.

In this research, for the first time, a credit option scheme as a coordination method is proposed to coordinate the production multiplier, periodic review replenishment, and pricing decision variables in a manufacturer-buyer chain under stochastic price-sensitive demand. Moreover, in this study, it is considered that the manufacturer follows an EPQ inventory policy and the buyer applies the periodic review policy that has not yet investigated in the literature. We model the addressed SC under coordinated, centralized, and decentralized structures. Solution procedures and concavity analyses are proposed to prove the optimality of the decentralized and centralized solutions. In order to approach a realistic model, it is supposed that the SC members face different interest rates. Then, maximum and minimum permissible credit periods that are acceptable to both SC members are obtained. Therefore, the proposed credit option scheme results in win–win situation for all SC members.

## 3. Definition of the problem

This study considers a two-level SC including one manufacturer and one buyer in which the manufacturer produces one type of product (see Fig. 1). The demand of buyer is stochastic and depends on the selling price which follows a normal distribution $\left(\mu, \sigma^2\right)$. The annual cunsomer demand is linearly depends on the selling price given by $D\left(P_l\right) = \beta_1 - \beta_2 P_l$, where $\beta_1$ is market size and $\beta_2$ is the price-sensitivity coefficient of demand. The selling price, which is set by the buyer, impacts on the market share of the SC and consequently influences the profitability of the manufacturer. The buyer uses a periodic review policy for replenishing products. Under such a system, every $T$ units of time, inventory level is reviewed and enough items are ordered to raise the inventory up to the order-up-to level $R$ at each review epoch. The order-up-to-level $R$ which is sum of the expected demand during protection interval $(T + L)$ and safety stock $SS$ can be calculated as $R = D\left(P_l\right)(T + L) + k\sigma\sqrt{T + L}$ [37]. Lead time for the buyer is constant and deterministic and does not overpass a review period $T$ $(L < T)$.

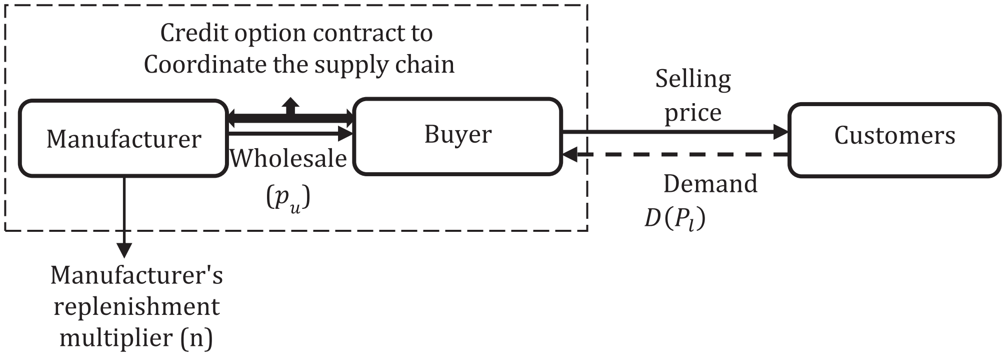



Figure 1. Manufacturer-buyer SC under consideration.

The buyer faces shortage with partial backordering and its decision variables are the length of review period, selling price and order-up-to-level to the customers. However, the manufacturer applies an EPQ system and only decides on production multiplier $n$. The production multiplier is a positive integer indicating the amount of manufacturer production as a multiple of buyer's order quantity. On the other hand, when the buyer orders $DT$ unit of product, the manufacturer produces $nDT$ unit by a finite production rate $P$. Thus, determining the best amount for $n$ is an important decision for the manufacturer. Under the decentralized structure, each SC member independently optimizes its decisions to increase its own profitability. However, these self-interested decisions will decrease the performance of the whole SC system. In this study, we investigate a credit option contract as a coordination scheme to not only raise the profit of the entire SC but also increase the profit of each SC member in comparison to the decentralized one.

### 3.1. Notations

In order to model the considered SC, the following notations are used in this paper.

| | |
|---|---|
| $\beta_1$ | Market size |
| $\beta_2$ | Price-sensitivity coefficient of demand |
| $\sigma^2$ | Variance of the demand |
| $A_l$ | Ordering cost of buyer per order |
| $h_l$ | Buyer's unit inventory holding cost per unit time |
| $h_u$ | Manufacturer's unit inventory holding cost per unit time |
| $D(P_l)$ | Annual customer's demand |
| $L$ | Length of the Lead time |
| $T$ | Length of the review period (decision variable) |
| $R$ | Order-up-to-level of the buyer (decision variable) |
| $\pi$ | Unit shortage cost |
| $\mathrm{P_l}$ | Selling price of the buyer per unit (decision variable) |
| $p_u$ | Selling price of the manufacturer per unit |
| $S_u$ | Setup cost of the manufacturer per unit |
| $P$ | Annual production rate of the manufacturer |
| $p_m$ | Production cost of the manufacturer per unit |
| $i_l$ | Buyer's annual rate of return on investment |
| $i_u$ | Manufacturer's annual rate of return on investment |
| $n$ | Manufacturer's production multiplier (decision variable) |
| $Y$ | Protection interval $(T+L)$ demand that has a normal distribution function with infinite mean $D(T+L)$ and standard deviation $\sigma\sqrt{T+L}$. |
| $\alpha$ | Fraction of the shortage that will be lost, $0<\alpha<1$. |

## 4. Model formulation

In the following, three various decision-making strategies are studied as follows. In Section 4.1, the decentralized model as a traditional decision making structure is explored. The results of the decentralized model determine the minimum satisfactory profit for the manufacturer and the buyer to accept the coordination scheme. The centralized model as a benchmark is studied in Section 4.2. In the last sub-section of Section 4, the proposed credit option scheme as a coordination contract is addressed.

### 4.1. Decentralized structure

Under the decentralized structure, the manufacturer and the buyer individually optimize their own profits. In the following, the expected profit functions of the buyer and the manufacturer are maximized in the decentralized model. The buyer follows a periodic review policy and its decision variables are selling price, order-up-to level and review period. The buyer faces holding, ordering, and shortage costs. As in Montgomery *et al.* [33], the expected holding and shortage costs per year for the buyer can be calculated as $h_l\left[R - DL - \frac{DT}{2} + \alpha E(X-R)^+\right]$ and $\frac{\pi+\alpha(P_l-p_u)}{T}E(Y-R)^+$, respectively, where $E(Y-R)^+$ is the expected shortage quantity during the protection interval. The expected annual profit of the buyer is given by $\mathit{\Pi}_l\left(T,R,P_l\right)$ = earned revenue – ordering cost – expected holding cost per year – expected shortage cost. Thus, we have:

$$\mathit{\Pi}_l\left(T,R,P_l\right) = \left(P_l - p_u\right)\left(\beta_1 - \beta_2 P_l\right) - \frac{A_l}{T} - h_l\left[R - \left(\beta_1 - \beta_2 P_l\right)L - \frac{\left(\beta_1 - \beta_2 P_l\right)T}{2} + \alpha E\left(Y-R\right)^+\right] - \frac{\pi + \alpha\left(P_l - p_u\right)}{T}E\left(Y-R\right)^+. \tag{4.1}$$

The protection interval demand $Y$ follows a normal distribution with mean $D(T+L)$ and standard deviation $\sqrt{T+L}$ . Therefore, the expected shortage quantity can be formulated as:

$$E\left(Y-R\right)^+ = \int_R^\infty \left(Y-R\right)f_Y\left(Y\right)\mathrm{d}_y = \int_k^\infty \sigma\sqrt{T+L}\left(Z-k\right)f_z\left(z\right)\mathrm{d}_z = \sigma\sqrt{T+L}G\left(k\right) > 0 \tag{4.2}$$

where $f_z\left(z\right)$ defines the standard normal distribution function and $G\left(k\right) = \varphi\left(k\right) - k\left[1 - \emptyset\left(k\right)\right]$, where $\emptyset\left(k\right)$ and $\varphi\left(k\right)$ define the cumulative distribution and standard normal functions, respectively. For simplicity, in the following, the order-up-to-level $R$ is replaced by safety factor $k$. Then, the function of the buyer's profit (4.1) can be converted to:

$$\mathit{\Pi}_l\left(T,k,P_l\right) = \left(P_l - p_u\right)\left(\beta_1 - \beta_2 P_l\right) - \frac{A_l}{T} - h_l\left[\frac{\left(\beta_1 - \beta_2 P_l\right)T}{2} + k\sigma\sqrt{T+L} + \alpha\sigma\sqrt{T+L}G\left(k\right)\right] - \frac{\pi + \alpha\left(P_l - p_u\right)}{T}\sigma\sqrt{T+L}G\left(k\right). \tag{4.3}$$

In order to optimize the profit functions of the buyer, the manufacturer, and the entire SC, in the following, a set of propositions are provided. If we can prove the concavity of the expected annual total profit functions of the SC members and entire SC, the global optimal amount of the decision variables are achieved from the first derivative of the profit functions.

**Proposition 4.1.** *The profit function of the buyer is concave w.r.t.* $k$ *and* $P_l$ *for a given* $T$ *when*

$$2\beta_2\left[h_l\alpha\sigma\sqrt{T+L}G\left(k\right) + \frac{\pi + \alpha\left(P_l - p_u\right)}{T}\sigma\sqrt{T+L}\emptyset\left(k\right)\right] > \left(\frac{\alpha\sigma\sqrt{T+L}\left(\emptyset\left(k\right) - 1\right)}{T}\right)^2. \tag{4.4}$$

*Proof.* See Appendix A. □

As the buyer's profit function is concave w.r.t. $k$ and $\mathrm{P_l}$, the global optimal amount of the buyer's safety factor and selling price are as follows

$$1-\emptyset\,(k)=\frac{Th_l}{Th_l\alpha+\pi+\alpha\,(P_l-p_u)} \tag{4.5}$$

$$P_l=\frac{\beta_1+\beta_2\mathrm{P_u}+\frac{h_l\beta_2 T}{2}-\frac{\alpha\sigma\sqrt{T+L}G(k)}{T}}{2\beta_2}. \tag{4.6}$$

Since the values of $k$ and $P_l$ are circulatory depending on each other, then a repetitive procedure is proposed to compute the optimum amounts of SC members' decision variables as follows:

**Step 1:** Set $T=\varepsilon$ (the lowest viable value for $T$).

**Step 2:** Let $P_l=p_u$ (the lowest viable value for $P_l$).

**Step 3:** Compute $k$ using (4.5).

**Step 4:** Compute $P_l$ using (4.6) based on the obtained $k$.

**Step 5:** Repeat third and fourth steps to converge.

**Step 6:** If $T>\frac{\pi+\alpha(P_l-p_u)}{h_l(1-\alpha)}$, then value of $T$, with the maximum buyer's profit is optimal and terminate the repetitive procedure; otherwise, set $T=T+\varepsilon$, $P_l=p_u$ and go to step 2.

**Step 7:** Obtain values of $k$ and, $P_l$ using equations (4.5) and (4.6), respectively.

In the decentralized structure, the manufacturer follows EPQ system and decides on the production multiplier $n$. The setup cost of per production run is $\frac{S}{nT}$. When the first $DT$ units of products are produced, the manufacturer will deliver them to the buyer. Afterwards, the manufacturer will deliver on every, $T$ units of time until inventory is exhausted. Based on Joglekar [25], the average inventory per unit of time is formulated as:

$$\left\{nDT\left[\frac{DT}{P}+(n-1)T\right]-\frac{(nDT)^2}{D}-\frac{D^2T^2}{D}\left[1+2+\cdots+(n-1)\right]\right\}\frac{D}{nDT}=$$
$$\frac{DT}{2}\left[\frac{D}{P}\,(2-n)+(n-1)\right]. \tag{4.7}$$

Thus, the manufacturer inventory cost per unit of time is $h_u\frac{DT}{2}\left[\frac{D}{P}\,(2-n)+(n-1)\right]$. The manufacturer annual profit function is given by $\mathit{\Pi}_u\,(n)=$ revenue – setup cost- inventory cost – production cost. Hence we have:

$$\mathit{\Pi}_u\,(n)=(p_u-p_m)\,(\beta_1-\beta_2P_l)-\frac{S_u}{nT}-h_u\frac{(\beta_1-\beta_2P_l)\,T}{2}\left[\frac{(\beta_1-\beta_2P_l)}{P}\,(2-n)+(n-1)\right]. \tag{4.8}$$

In the following, Proposition 4.2 is addressed to calculate the global optimal amount of the manufacturer's production multiplier ($n$). First the concavity of the manufacturer's profit function is proved. Afterwards, the optimal amount of $n$ is obtained.

**Proposition 4.2.** *The profit function of the manufacturer is concave w.r.t. n.*

*Proof.* See Appendix B. □

By setting the first derivative of the manufacturer's profit function w.r.t. $n$ equal to zero, the optimum value of $n$ can be formulated as:

$$n=\sqrt{\frac{S_u}{\frac{T^2h_u(\beta_1-\beta_2P_l)}{2}\left[1-\frac{(\beta_1-\beta_2P_l)}{P}\right]}}. \tag{4.9}$$

In order to achieve optimal solution, $n$ should be a discrete variable [38], then the integer before and after $n$ that results greatest profit for the manufacturer will be optimal.

## 4.2. Centralized structure

Under joint decision making model, it is presumed that there is a central decision maker that optimizes the pricing and replenishment decisions from the whole SC perspective [47]. Under such a hypothetical case, both the manufacturer and buyer jointly determine their decisions. Thus, in the centralized decision making, the individual benefit of members is not important and therefore some SC members may incur loses. However, the centralized structure can be considered as a criterion for measuring the SC performance and determining the optimal decisions from the entire viewpoint. In this section, the replenishment and pricing strategies are determined from the whole SC perspective. Expected annual profit function of the supply chain is $\Pi_{\mathrm{SC}}(T,k,P_l,n)$ and it is the sum of the profit functions of the buyer and manufacturer:

$$\begin{aligned}\Pi_{\mathrm{SC}}(T,k,P_l,n) &= \Pi_l(T,k,P_l)+\Pi_u(n)\\ &=(P_l-P_m)(\beta_1-\beta_2P_l)-\frac{A_l}{T}\\ &\quad-h_l\left[\frac{(\beta_1-\beta_2P_l)T}{2}+k\sigma\sqrt{T+L}+\alpha\sigma\sqrt{T+L}G(k)\right]\\ &\quad-\frac{\pi+\alpha(P_l-p_u)}{T}\sigma\sqrt{T+L}G(k)-\frac{S_u}{nT}\\ &\quad-h_u\frac{(\beta_1-\beta_2P_l)T}{2}\left[\frac{(\beta_1-\beta_2P_l)}{P}(2-n)+(n-1)\right].\end{aligned} \tag{4.10}$$

To calculate the optimal amounts of the selling price, safety factor, and production multiplier under the whole SC viewpoint, the following proposition is provided. Proposition 4.3 investigates the concavity of the expected annual profit function of the supply chain.

**Proposition 4.3.** *The expected annual profit function of the supply chain is concave w.r.t. $n$, $k$ and $P_l$ for a given $T$.*

*Proof.* See Appendix C. □

Setting the first order derivatives of the supply chain expected annual profit function equal to zero, the optimal values of $P_l$, $k$, and $n$ in centralized decision making can be calculated as:

$$P_l=\frac{\left(\beta_1+p_m+\frac{\beta_2h_lT}{2}-\frac{\alpha\sigma\sqrt{T+L}G(k)}{T}+\frac{\beta_2h_uT(n-1)}{2}+\frac{h_u\beta_1\beta_2T(2-n)}{P}\right)}{\left(2\beta_2+\frac{Th_u\beta_2^2(2-n)}{P}\right)} \tag{4.11}$$

$$1-\emptyset(k)=\frac{Th_l}{Th_l\alpha+\pi+\alpha(P_l-p_u)} \tag{4.12}$$

$$n=\sqrt{\frac{S_u}{\frac{T^2h_u(\beta_1-\beta_2P_l)}{2}\left[1-\frac{(\beta_1-\beta_2P_l)}{P}\right]}}. \tag{4.13}$$

Since the values of $k$, $P_l$ and $n$ are circulatory depending on each other, then a repetitive procedure is proposed to compute the optimum amounts of variables as follows:

**Step 1:** Set $T=\varepsilon$ (the lowest viable value for T)
**Step 2:** Let $P_l=P_u$ (the lowest viable value for $P_l$)
**Step 3:** Compute $k$ using (4.12)
**Step 4:** Compute $n$ using (4.13)
**Step 5:** Compute $P_l$ using (4.11) based on obtained $k$ and $n$
**Step 6:** Repeat (3)–(5) steps to converge

**Step 7:** Compute supply chain profit function at the previous integer and next integer of $n$. Then the value that results greater profit function is optimum value of $n$.

**Step 8:** If $T > \frac{\pi+\alpha(P_l-p_u)}{h_l(1-\alpha)}$, then a value of $T$ with the maximum profit of the supply chain is optimal and finish the repetitive procedure; otherwise, set $T = T + \varepsilon$ and $P_l = P_u$, and then go to step (2).

**Step 9:** Obtained values of $k$, $P_l$ and $n$ are optimum.

In the rest of the paper, superscript "dec", "cen" and "coo" explain the optimal values of the decentralized, centralized, and coordinated structures, respectively. Although the centralized structure raises the entire SC profit in comparison with the decentralized one ($\mathit{\Pi}_{\text{SC}}\left(T^{\text{cen}}, k^{\text{cen}}, P_l^{\text{cen}}, n^{\text{cen}}\right) > \mathit{\Pi}_{\text{SC}}\left(T^{\text{dec}}, k^{\text{dec}}, P_l^{\text{dec}}, n^{\text{dec}}\right)$), using the centralized solution decreases the buyer's profit ($\mathit{\Pi}_l\left(T^{\text{cen}}, k^{\text{cen}}, P_l^{\text{cen}}\right) < \mathit{\Pi}_l\left(T^{\text{dec}}, k^{\text{dec}}, P_l^{\text{dec}}\right)$). In such a case, the manufacturer should propose an appropriate incentive scheme to compensate the buyers loses. Thus, in the next sub-section, a credit option contract as a coordination mechanism is investigated to convince the buyer to accept the globally optimal strategy from the entire SC point of view.

### 4.3. Coordinated scheme: Credit option contract

There are different incentive schemes for inducing the buyer to take part in centralized structure. Basically, a coordination scheme should be desirable for all SC actors. In this paper, we use a credit option contract as a coordination mechanism. In such a case, the manufacturer offers a permissible credit period to the buyer as long as the buyer make optimal decisions from entire SC viewpoint. The buyer's interest rate of return is $i_l$ and he/she invest in a market by $i_l$. Then, the buyer's expected profit function under coordinated model is as follows:

$$\begin{aligned}\mathit{\Pi}_l\left(T^{\text{cen}}, k^{\text{cen}}, P_l^{\text{cen}}, t\right) = {} & \left(P_l^{\text{cen}} - p_u\right)\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right) + i_l p_u\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right)t - \frac{A_l}{T^{\text{cen}}} \\ & - h_l\left[\frac{\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right)T^{\text{cen}}}{2} + k^{\text{cen}}\sigma\sqrt{T^{\text{cen}} + L} + \alpha\sigma\sqrt{T^{\text{cen}} + L}G\left(k^{\text{cen}}\right)\right] \\ & - \frac{\pi + \alpha\left(P_l^{\text{cen}} - p_u\right)}{T^{\text{cen}}}\sigma\sqrt{T^{\text{cen}} + L}G\left(k^{\text{cen}}\right)\end{aligned} \tag{4.14}$$

where the buyer's total annual revenue is showed by the first term. The second term demonstrates annual profit of investing unpaid money during the permissible period. The third and fourth terms demonstrate the ordering cost and inventory cost of the buyer. The fifth term shows the annual expected shortage cost. In such a case, $t$ indicates the agreed credit option period. If profitability of the buyer under the coordinated structure be less than the decentralized one, he/she will accept the coordinated scheme. Thus, it is important to set the delay period long enough to be appealing for the buyer to take part. The manufacturer suffers extra costs due to the delay in payments offered to the buyer. Therefore, the manufacturer cannot invest its money during credit period. By offering the credit period, the manufacturer faces the cost $i_u p_u\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right)t$ per replenishment period, where $i_u$ is the rate of return for the manufacturer in the market. Hence, the profit function of the manufacturer under coordination model is as follows:

$$\begin{aligned}\mathit{\Pi}_u\left(T^{\text{cen}}, n^{\text{cen}}, t\right) = {} & \left(p_u - p_m\right)\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right) - i_u p_u\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right)t - \frac{S_u}{n^{\text{cen}}T^{\text{cen}}} \\ & - h_u\frac{\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right)T^{\text{cen}}}{2}\left[\frac{\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right)}{P}\left(2 - n^{\text{cen}}\right) + \left(n^{\text{cen}} - 1\right)\right].\end{aligned} \tag{4.15}$$

The manufacturer's cost will increase as he/she offers longer delay period. In such a case, transferring from the decentralized to the centralized structure incurs losses for the buyer. Hence, without sufficient incentives, both SC participants will not accept the coordinated scheme. In the following, the structure of the proposed credit option contract is described. Under this contract, the buyer accept to change its decisions from $\left(T^{\text{dec}}, k^{\text{dec}}, P_l^{\text{dec}}\right)$ to $\left(T^{\text{cen}}, k^{\text{cen}}, P_l^{\text{cen}}\right)$ and the manufacturer from $\left(n^{\text{dec}}\right)$ to $\left(n^{\text{cen}}\right)$. In this regard, the buyer should use coordinator coefficients $Q_T = T^{\text{cen}}/T^{\text{dec}}$, $Q_k = k^{\text{cen}}/k^{\text{dec}}$, $Q_{P_l} = P_l^{\text{cen}}/P_l^{\text{dec}}$ on the review period, safety factor, and selling price decisions, respectively and the manufacturer should apply coordinator coefficient $Q_n = n^{\text{cen}}/n^{\text{dec}}$ on the replenishment multiplier decision. Through proposing coordinator coefficients under the decentralized structure, supply chain coordination will be achieved. Meanwhile, a credit option contract is developed to guarantee increment of both members' profit after using the above mentioned coefficients. The length of the credit period should be adjust so that it be acceptable to both members.

As the buyer incurs loses through the centralized decision making, he/she takes part in the proposed credit option contract if and only if his/her profit raises by participating in the scheme. In mathematical terms, the buyer's profit after participation $\Pi_l\left(T^{\text{cen}}, k^{\text{cen}}, P_l^{\text{cen}}, t\right)$ must be greater than the decentralized one $\Pi_l\left(T^{\text{dec}}, k^{\text{dec}}, P_l^{\text{dec}}\right)$. The condition for the participation of the buyer is shown in equation (4.16). The buyer wants to increase his/her profit as much as possible. While the manufacturer wants to decrease his/her loses that causes from participating in the coordination scheme as much as possible. Thus, if the credit period is less than a determined threshold (minimum credit period), then the buyer does not participate in the scheme and makes his/her locally optimum decision. Therefore, in the following, Proposition 4.4 is proposed to calculate the minimum credit period from the buyer's viewpoint.

$$\Pi_l\left(T^{\text{cen}}, k^{\text{cen}}, P_l^{\text{cen}}, t\right) \geq \Pi_l\left(T^{\text{dec}}, k^{\text{dec}}, P_l^{\text{dec}}\right) \tag{4.16}$$

**Proposition 4.4.** *The minimum credit period that convince the buyer to take part in the credit option scheme is:*

$$t_{\min} = (A - B)/i_l p_u\left(\beta_1 - \beta_2 P_l^{\text{dec}} Q_{P_l}\right) \tag{4.17}$$

*where,*

$$\begin{aligned} A = {} & \left(P_l^{\text{dec}} - p_u\right)\left(\beta_1 - \beta_2 P_l^{\text{dec}}\right) - \frac{A_l}{T^{\text{dec}}} - \frac{\pi + \alpha\left(P_l^{\text{dec}} - p_u\right)}{T^{\text{dec}}}\sigma\sqrt{T^{\text{dec}} + L}G\left(k^{\text{dec}}\right) \\ & - h_l\left[\frac{\left(\beta_1 - \beta_2 P_l^{\text{dec}}\right)T^{\text{dec}}}{2} + k^{\text{dec}}\sigma\sqrt{T^{\text{dec}} + L} + \alpha\sigma\sqrt{T^{\text{dec}} + L}G\left(k^{\text{dec}}\right)\right] \end{aligned} \tag{4.18}$$

$$\begin{aligned} B = {} & \left(Q_{P_l} P_l^{\text{dec}} - p_u\right)\left(\beta_1 - \beta_2 Q_{P_l} P_l^{\text{dec}}\right) - \frac{A_l}{Q_T T^{\text{dec}}} \\ & - h_l\left[\frac{\left(\beta_1 - \beta_2 Q_{P_l} P_l^{\text{dec}}\right) Q_T T^{\text{dec}}}{2} + Q_k k^{\text{dec}}\sigma\sqrt{Q_T T^{\text{dec}} + L} + \alpha\sigma\sqrt{Q_T T^{\text{dec}} + L}G\left(Q_k k^{\text{dec}}\right)\right] \\ & - \frac{\pi + \alpha\left(Q_{P_l} P_l^{\text{dec}} - p_u\right)}{Q_T T^{\text{dec}}}\sigma\sqrt{Q_T T^{\text{dec}} + L}G\left(Q_k k^{\text{dec}}\right). \end{aligned} \tag{4.19}$$

*Proof.* By replacing equation (4.3) into equation (4.16), the minimum permissible delay period that convinces the buyer to participate in the scheme is obtained. □

By proposing a credit option contract, the manufacturer incurs loses due to lose the opportunity of investment during the delay period. The manufacturer participates in the coordination contract if and only if its profit is more than before participation. The manufacturer's condition for participating is determined in equation (4.20). Thus the manufacturer does not participate in the credit option scheme if the credit period is more than a determined threshold (maximum credit period). On the other hand, the manufacturer wants to set a lower credit period as much as possible and then the maximum credit period is determined from his/her viewpoint. In the following, Proposition 4.5 is addressed to calculate the maximum allowable credit period.

$$\Pi_u\left(T^{\text{cen}}, n^{\text{cen}}, t\right) \geq \Pi_u\left(T^{\text{dec}}, n^{\text{dec}}\right) \tag{4.20}$$

**Proposition 4.5.** *The maximum allowable delay period from manufacturer perspective to participate in the coordination scheme is:*

$$t_{\max} = (C - D)/i_u p_u \left(\beta_1 - \beta_2 Q_{P_l} P_l^{\text{dec}}\right) \tag{4.21}$$

$$C = (p_u - p_m)\left(\beta_1 - \beta_2 Q_{P_l} P_l^{\text{dec}}\right) - \frac{S_u}{Q_n n^{\text{dec}} Q_T T^{\text{dec}}} - h_u \frac{\left(\beta_1 - \beta_2 Q_{P_l} P_l^{\text{dec}}\right) Q_T T^{\text{dec}}}{2}\left[\frac{\left(\beta_1 - \beta_2 Q_{P_l} P_l^{\text{dec}}\right)}{P}\left(2 - Q_n n^{\text{dec}}\right) + \left(Q_n n^{\text{dec}} - 1\right)\right] \tag{4.22}$$

$$D = (p_u - p_m)\left(\beta_1 - \beta_2 P_l^{\text{dec}}\right) - \frac{S_u}{n^{\text{dec}} T^{\text{dec}}} - h_u \frac{\left(\beta_1 - \beta_2 P_l^{\text{dec}}\right) T^{\text{dec}}}{2}\left[\frac{\left(\beta_1 - \beta_2 P_l^{\text{dec}}\right)}{P}\left(2 - n^{\text{dec}}\right) + \left(n^{\text{dec}} - 1\right)\right]. \tag{4.23}$$

*Proof.* By replacing equation (4.8) into equation (4.20), the maximum delay period that convince the manufacturer to participate in the scheme is calculated. □

SC coordination can be achieved by choosing a credit period in the interval $(t_{\min}, t_{\max})$. Based on the permissible delay period, $t_{\text{sharing}}$ that is between the lower and upper bounds of the credit period, the achieved profits will be shared between the manufacturer and the buyer according to their bargaining powers. When $t_{\text{sharing}}$ approaches to $t_{\min}$, the manufacturer achieves more profit and when $t_{\text{sharing}}$ gets $t_{\min}$, then all the extra profits will be obtained by the manufacturer. Whereas $t_{\text{sharing}}$ becomes closer to $t_{\max}$, the buyer archives more profit and when $t_{\text{sharing}}$ sets $t_{\max}$, all received profit of the SC coordination will be obtained by the buyer. In numerical examples, to divide the coordination profit between the SC members, it is presumed that the credit period is implemented by setting $t_{\text{sharing}}$ as middle of the permissible interval of the credit period. In such a case, credit period is defined as $t_{\text{sharing}} = (t_{\min} + t_{\max})/2$.

## 5. Numerical examples and sensitivity analysis

In the following, using a set of numerical examples, the applicability of the proposed coordination model is studied. Table 2 demonstrates parameters of the four addressed test problems under different circumstances. Results of running three decision making structures (*i.e.* decentralized, centralized, and coordinated) for test problems are demonstrated in Table 3.

According to the results of Table 3, comparing the centralized and the decentralized structures indicates that the joint decision making increases the profit of entire SC compared to the decentralized structure. As shown in Table 3, the selling price of the buyer in the decentralized structure is more than the centralized one as the buyer tries to increase his/her profit by setting a higher selling price. Under the centralized structure, there is one decision-maker that determines the SC variables from the entire SC perspective. The result of centralized structure in Table 3 indicates that the decision-maker decreases the selling price of the buyer in the centralized structure in comparison with the decentralized one. From the viewpoint of whole SC, a decrease in selling price leads to more demand and it is profitable for the whole SC. Moreover, the results indicate that the centralized structure on the pricing and periodic review inventory decisions significantly increase the entire SC profit compared to the decentralized one. However, as illustrated in Table 3, transition from the decentralized structure to the centralized one decrease the buyer's profit. Therefore, the buyer refuses to accept the centralized structure.

As shown in Table 3, the proposed coordination mechanism based on the credit option compensates the buyer's losses. The proposed credit option contract improves both SC actors' profit as well as entire SC profit in the investigated SC. The rates of return of the mufacturer and the buyer are critical parameters in achieving the channel coordination. As shown in Table 3, when $i_u = i_l$, the whole SC profit under the coordinated structure

TABLE 2. Four investigated test problems.

| Parameters | $A_l$ | $h_l$ | $i_l$ | $\beta_1$ | $\beta_2$ | $h_u$ | $S_u$ | $i_u$ | $p_u$ | $P$ | $p_m$ | $L$ | $\pi$ | $\alpha$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Test 1 | 80 | 40 | 20% | 3000 | 10 | 35 | 95 | 15% | 200 | 18000 | 150 | 1/365 | 1.5 | 0.4 | 200 |
| Test 2 | 110 | 45 | 8% | 10000 | 28 | 55 | 150 | 11% | 280 | 25000 | 250 | 2/365 | 1.4 | 0.5 | 400 |
| Test 3 | 70 | 35 | 15% | 6000 | 15 | 40 | 100 | 15% | 220 | 21000 | 180 | 3/365 | 1.6 | 0.35 | 300 |
| Test 4 | 100 | 50 | 16% | 16000 | 33 | 70 | 120 | 7% | 300 | 25000 | 270 | 5/365 | 1.4 | 0.45 | 1000 |

TABLE 3. Comparison of the decentralized, centralized and coordinated structures.

| | Test 1 | Test 2 | Test 3 | Test 4 |
|---|---|---|---|---|
| Decentralized model | | | | |
| $T$ | 22.77 | 22.50 | 25.66 | 18.47 |
| $k$ | 1.21 | 1.14 | 1.45 | 1.57 |
| $P_l$ | 249.74 | 318.32 | 310.16 | 392.21 |
| $n$ | 2 | 1 | 1 | 1 |
| $R$ | 94.94 | 191.47 | 228.46 | 596.32 |
| $\Pi_l$ | 19613.99 | 30893.24 | 113892.62 | 253639.47 |
| $\Pi_u$ | 23819.39 | 30097.20 | 52356.70 | 88671.17 |
| $\Pi_{\text{SC}}$ | 43433.38 | 60990.44 | 166249.33 | 342310.65 |
| Centralized model | | | | |
| $T$ | 15.84 | 22.58 | 20.94 | 12.23 |
| $k$ | 1.06 | 0.85 | 1.44 | 1.69 |
| $P_l$ | 224.79 | 302.95 | 290.09 | 377.14 |
| $n$ | 2 | 1 | 1 | 1 |
| $R$ | 80.47 | 191.36 | 218.82 | 535.24 |
| $\Pi_l$ | 13440.49 | 24332.45 | 107603.33 | 245389.02 |
| $\Pi_u$ | 35935.85 | 42939.50 | 64051.43 | 102452.17 |
| $\Pi_{\text{SC}}$ | 49376.34 | 67271.96 | 171654.76 | 347841.20 |
| Coordinated model | | | | |
| $T$ | 15.84 | 22.58 | 20.94 | 12.23 |
| $k$ | 1.06 | 0.85 | 1.44 | 1.69 |
| $P_l$ | 224.79 | 302.95 | 290.09 | 377.14 |
| $n$ | 2 | 1 | 1 | 1 |
| $\Pi_l$ | 24604.26 | 32190.65 | 116271.25 | 263764.80 |
| $\Pi_u$ | 27563.02 | 32134.48 | 55383.51 | 94412.77 |
| $\Pi_{\text{SC}}$ | 52167.28 | 64325.14 | 171654.76 | 358177.57 |
| $\Delta P$ | 8733.89 | 3334.69 | 5405.43 | 15866.92 |
| $t_{\min}$ | 74.89 | 68.47 | 37.84 | 11.23 |
| $t_{\max}$ | 196.02 | 100.29 | 78.46 | 67.39 |
| $t_{\text{sharing}}$ | 135.45 | 84.38 | 58.15 | 39.31 |

is equal to the centralized one (see Test 3). When $i_u > i_l$, then the proposed delay in payment results in more cost for the manufacturer compared to the additional profit gained by the buyer (see Tab. 3, Test 2). If $i_u$ is much larger than $i_l$, hence it is expected that the credit option scheme could not achieve SC coordination. When $i_u < i_l$, the savings obtained from the proposed delayed in payments contract even exceeds the centralized structure (see Test 1 and Test 4). Hence, the proposed scheme leads to more entire SC profitability in comparison with the centralized one when $i_u < i_l$. Increasing the whole SC profitability even more than the centralized structure can be considered as one of the unique features of the coordination scheme.

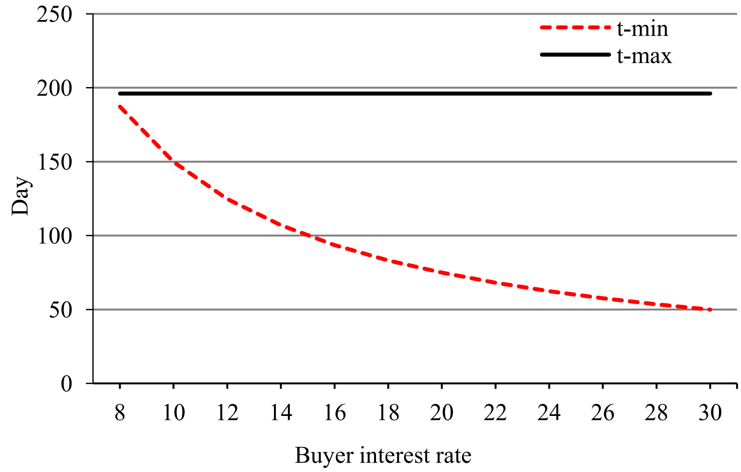



FIGURE 2. Changing on coordinating parameter $t$ by increasing $i_l$ per fixed $i_u = 15\%$.

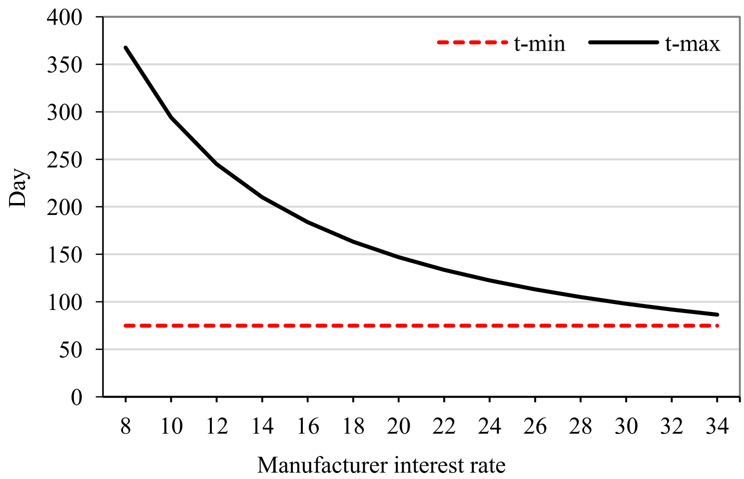



FIGURE 3. Changing on coordinating parameter $t$ by increasing $i_u$ per fixed $i_l = 20\%$.

In the following, a set of sensitivity analyses is provided to evaluate the effects of key parameters ($i_u$, $i_l$, $\beta_2$, $h_u$, $p_u$, and $\sigma$). Because of the high importance of the rates of return on investment in the investigated models, a set of sensitivity analysis on these parameters is carried out. Using Test 1, the required data for Figures 2 and 3 is provided. Figure 2 plots the trend of variable $t$ over increasing $i_l$. According to Figure 2, the interval $[t_{\min}, t_{\max}]$ is not empty under various amounts of the buyer interest rate and is empty only under very small values of the buyer rates of return ($i_l$ is less than 8% with respect to $i_u = 15\%$). The results indicate that achieving channel coordination will be more convenient as the buyer's interest rate increases. As shown in Figure 2, by increasing the buyer's interest rate, loses of the buyer will be compensated using a shorter credit period (See $i_l = 20\%$, $t = 74$ day and $i_l = 30\%$, $t = 50$ day).

Figure 3 shows the changes of the credit period by changing the manufacturer rate of return $i_u$. As the manufacturer rate of return $i_u$ increases, $t_{\max}$ approaches $t_{\min}$. As shown in Figure 3, as the manufacturer rates of return grows, the offered credit time by the manufacturer is diminished. This means that when the manufacturer rates of return is high, he/she cannot offer a long credit period to the buyer as it incurs a great loss to the manufacturer. According to Figures 2 and 3, it can be concluded that under small value of the interest rates, the delay period needs

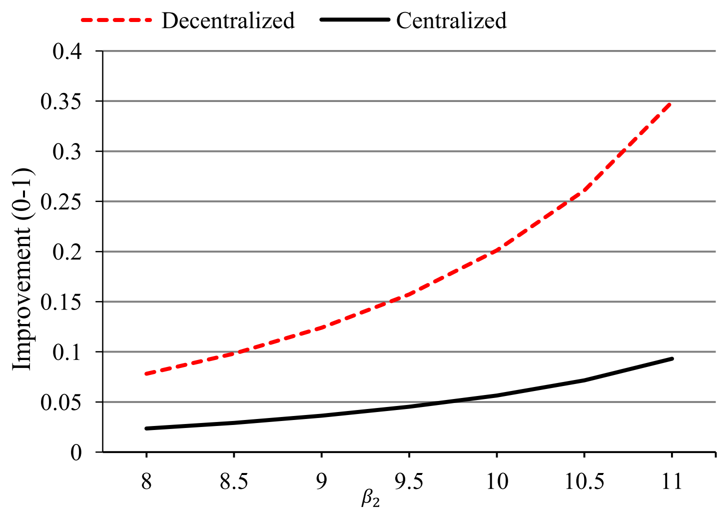



FIGURE 4. Comparison of improvement percentage of SC profit in the decentralized and coordinated structures based on $\beta_2$.

to be enhanced to offset the impact of low interest rates. In contrast, a short period of credit option is sufficient for achieving channel coordination under high value of the interest rates.

Furthermore, a sensitivity analysis is investigated to investigate the impact of $\beta_2$ on the improvement percentage of the entire SC profit by applying the proposed credit option model in comparison with the decentralized and centralized models. The data for this sensitivity analysis is taken from Test 1. Price-sensitivity coefficient of demand, $\beta_2$ demonstrates the effect of selling price on customers' demand. By increasing the price-sensitivity coefficient of demand, the SC profit decreases as the consumers are more sensitive to selling price. However, as shown in Figure 4, the proposed credit option contract can significantly improve the whole SC profit in compared to decentralized and centralized structures even under high values of $\beta_2$ (see $\beta_2 = 11$ and 11.5).

To analyze the proposed scheme from inventory policy point of view, the effects of manufacturer's unit inventory holding cost ($h_u$) on replenishment decision making is conducted in Figures 5 and 6. Figure 5(a)–(c) depicts the changing on review period $T$, safety factor $k$, and manufacturer's annual inventory cost by increasing $h_u$ using Test 4. As shown in Figure 5(a) and (b), increase in manufacturer's unit inventory holding cost has no effect on buyer's replenishment decision making ($T$, $k$) under the decentralized structure. Under this situation the buyer decides on review period and safety factor from its viewpoint. Therefore, the manufacturer's annual inventory cost significantly increases under decentralized model as $h_u$ increases. Implementing the joint decision making variables by credit option scheme, the buyer review the inventory level in smaller intervals and higher safety factor as $h_u$ increases. In such a case, the trend of increasing the manufacturer's annual inventory cost in comparison with the decentralized one decline. Therefore, the proposed credit option contract can share the risk of manufacturer's high inventory cost between the buyer and the manufacturer by changing the buyer's replenishment policy.

Using Test 3, Figure 6(a)–(c) depicts the effects of the manufacturer's unit inventory holding cost ($h_u$) on review period $T$, safety factor $k$, and production multiplier $n$, respectively. As the manufacturer's unit inventory holding cost increases, production multiplier that is the manufacture's decision variable decreases under both the decentralized and centralized structures. As illustrated in Figure 6(c), production multiplier under the decentralized model is always less than or equal to the centralized one. According to Figure 6(a) and (b), increase the manufacturer's unit inventory holding cost has no effect on the buyer's replenishment variables ($T$ and $k$) under the decentralized structure. While under the joint decision making structure, the review period, safety stock, and production multiplier are changed according to profitability of the entire SC. For instance, as $h_u$ grows from 15 to 25, $n$ is constant (see Fig. 6(c)) and then we expect that the manufacture's annual inventory

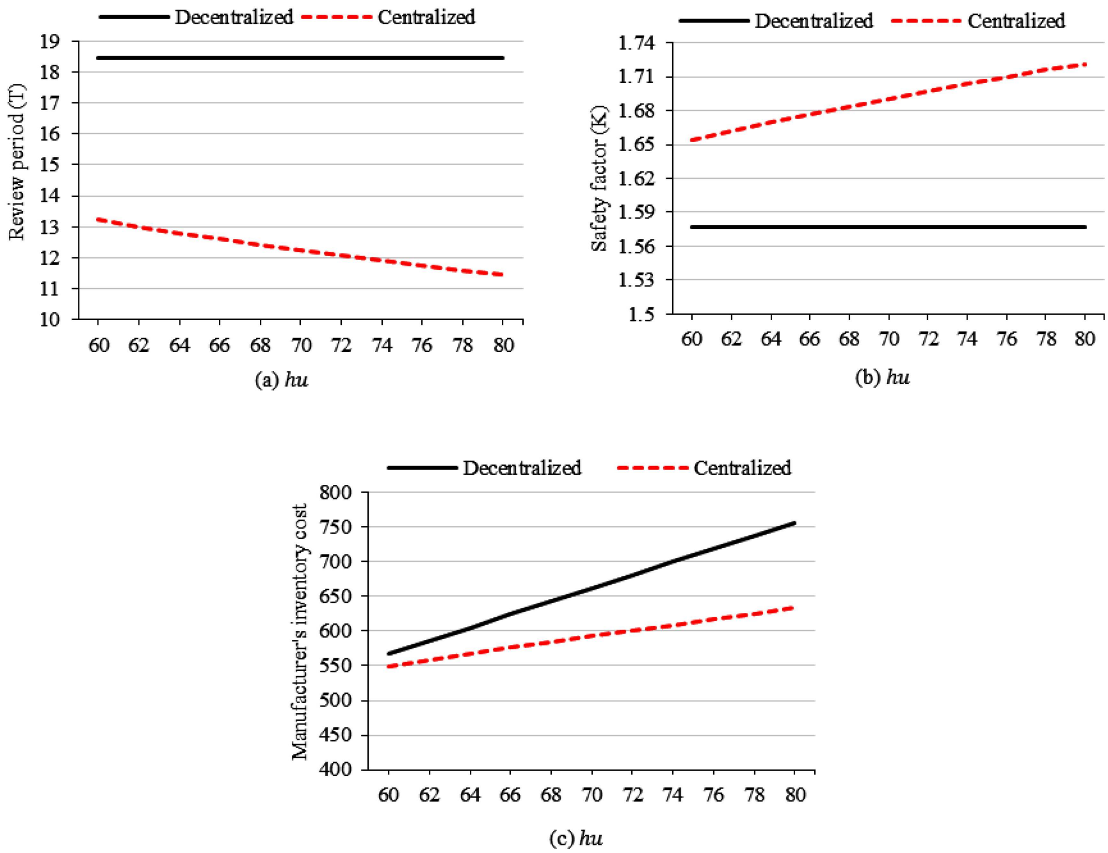



FIGURE 5. Effect of increase in manufacturer's unit inventory holding cost ($h_u$) on: (a) review period, (b) safety factor, and (c) manufacturer's inventory cost under decentralized and centralized structures.

cost increases. While in such a case as shown in Figure 6(a) and (b), the buyer reviewed the inventory level under the shorter $T$ and the higher safety factor. Under this situation, implementing the credit option contract based on the centralized decision variables can share the risk of manufacturer's high inventory cost between the manufacturer and the buyer.

Using Test 1, the effect of increasing the manufacturer's selling price (wholesale price) on buyer's selling price is shown in Figure 7. Under the decentralized decision making, as the wholesale price increases, the buyer adopts a higher selling price. As the consumers are sensitive to the selling price, increase in selling price decreases the consumers demand. Therefore, the diminished demand causes more cost to SC members and the entire SC. As demonstrated in Figure 7, under the centralized structure, as the wholesale price increases, the selling has not a significant change and it is decreased a little. This situation is beneficial for the consumers in comparison to the decentralized decision making. Therefore, applying credit option contract based on centralized decision variables can diminish the negative impacts of decentralized decision making on SC actors and also consumers.

In the investigated SC, the consumers demand is stochastic and the standard deviation ($\sigma$) of demand shows the uncertainty level that SC members encounter with. As high level of demand uncertainty could negatively impact on the efficiency of SC, in the following, a sensitivity analysis with respect to demand uncertainty is conducted. Figures 8 and 9(a) and (b) explore the impacts of uncertainty on the investigated SC in Test 3. Figure 8 compares the coordinated SC profit versus the decentralized SC profit under different values of demand uncertainty. According to Figure 8, under all amounts of $\sigma$, the SC profitability of the coordinated structure

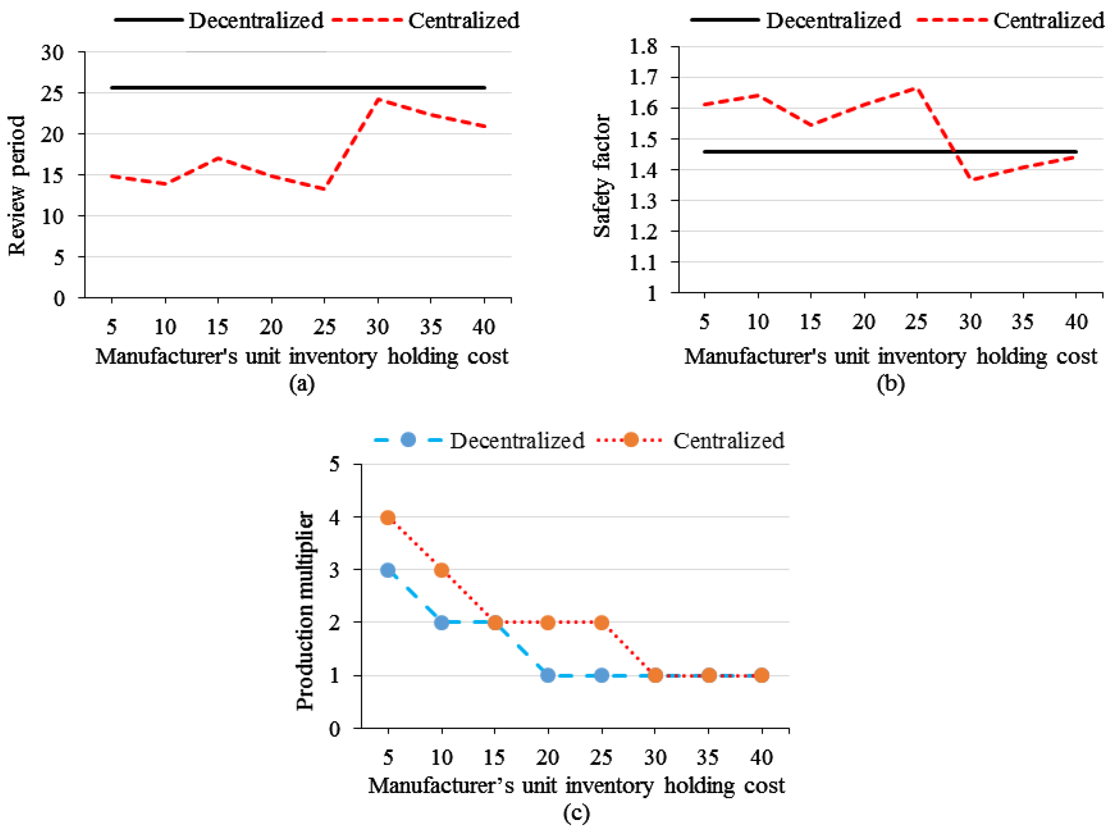



FIGURE 6. Effect of increase in manufacturer's unit inventory holding cost ($h_u$) on: (a) review period, (b) safety factor, and (c) production multiplier under decentralized and centralized structures.

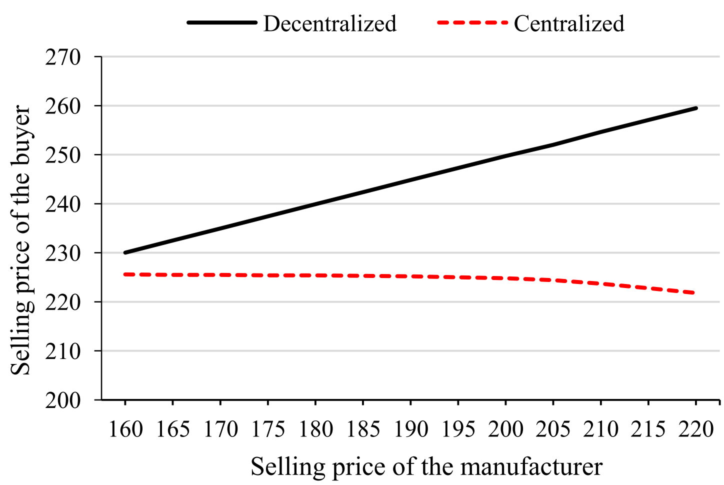



FIGURE 7. Changing on selling price of the buyer by increasing selling price of the manufacturer.

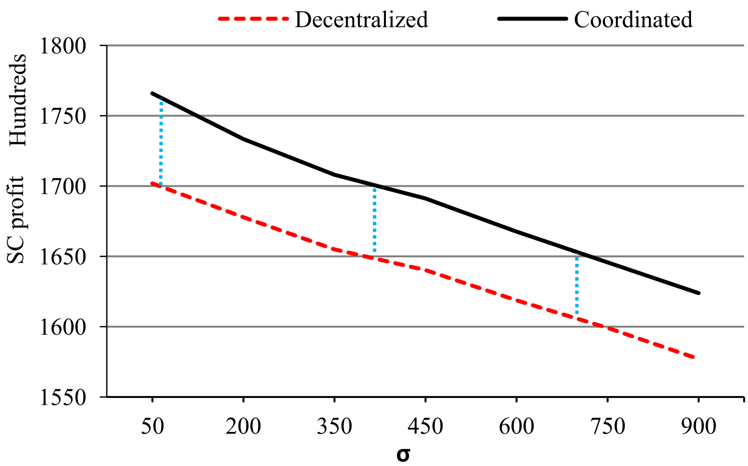



FIGURE 8. Comparison of entire SC profit based on uncertainty ($\sigma$) in the decentralized and coordinated models.

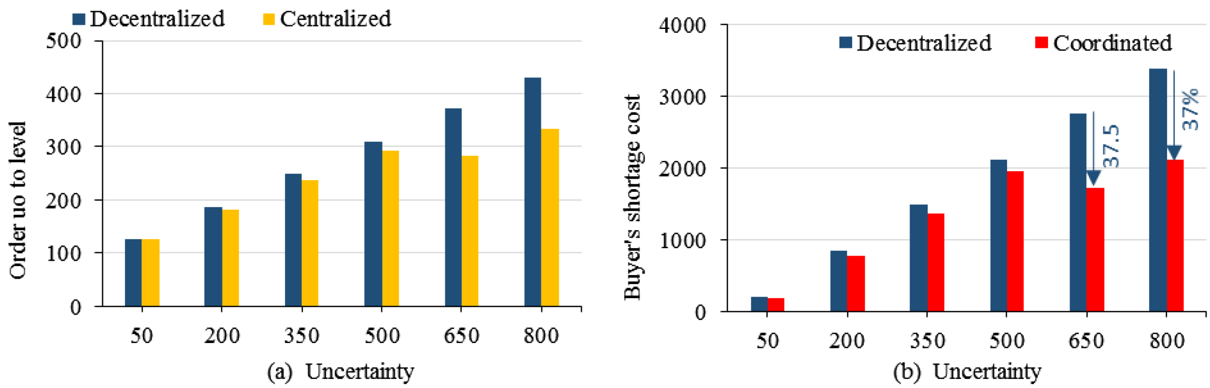
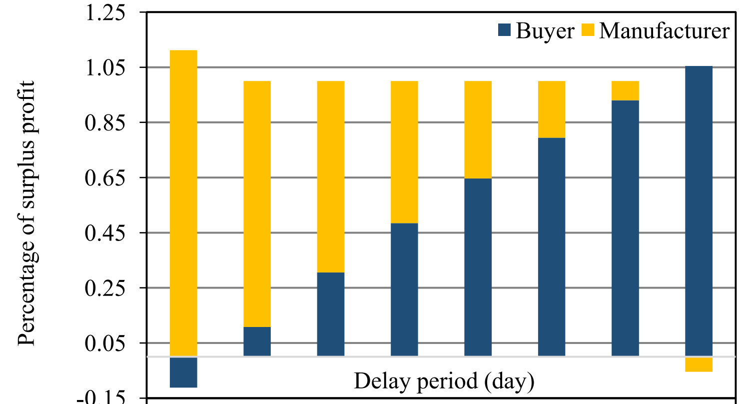



FIGURE 9. Comparison of (a) order up to level ($R$) and (b) buyer's shortage cost in the decentralized and centralized models under uncertainty ($\sigma$) growing.

is more than the decentralized model. In addition, by growing the demand uncertainty, the SC profitability of the both models decreases significantly. Nevertheless, the proposed coordination scheme could compensate the negative effects of high demand uncertainty.

In a periodic review policy, order up to level ($R$) is one of the critical variables that significantly impacts on the amounts of shortage and holding cost. Optimizing the order up to level prevents the risk of both low and high inventory level. If $R$ be greater than the optimal value, the holding cost increases and if $R$ be less than the optimal value, the shortage cost increases and consequently loyalty of the consumers diminishes. Figure 9(a) illustrates the amount of order up to level ($R$) in the decentralized and centralized structures under different amounts of demand uncertainty. As shown in Figure 9(a), in low demand uncertainty, the amounts of $R$ under both models are near together (see Fig. 9(a), $\sigma = 50$). By growing the demand uncertainty, order up to level increases but its increase in the decentralized model is significant. The high amount of $R$ diminishes the risk of demand uncertainty but it also increases the holding cost. As shown in Figure 9(a), the centralized solution is able to respond the consumer demand by setting a lower amount of order up to level. This means that in the case of high demand uncertainty, the buyer's holding cost under centralized model decreases in comparison with the




| | 65 | 85 | 105 | 125 | 145 | 165 | 185 | 205 |
|---|---|---|---|---|---|---|---|---|
| Manufacturer | 1.11 | 0.89 | 0.69 | 0.52 | 0.35 | 0.21 | 0.07 | -0.05 |
| Buyer | -0.11 | 0.11 | 0.31 | 0.48 | 0.65 | 0.79 | 0.93 | 1.05 |

FIGURE 10. The percentage share of surplus profit between SC members based on permissible delay periods.

decentralized one. Figure 9(b) illustrates the comparison of the shortage cost of the buyer in the decentralized and centralized models by increasing $\sigma$. As demonstrated in Figure 9(b), growing the demand uncertainty leads to more shortage cost in both models. However, by growing the demand uncertainty, the deviation between the decentralized and centralized shortage cost increases. As demonstrated in Figure 9(b), under high demand uncertainty (see $\sigma = 650$ and 800), the centralized model significantly decreases the shortage cost (about 37% decreases). This means that under high demand uncertainty, applying the joint decision making is more useful.

In a coordination plan, it is of high importance to fairly divide the surplus profit among SC members. Figure 10 indicates the percentage share of the surplus profit allocated to the manufacturer and buyer under different credit periods. As shown in Table 3, the minimum and maximum credit period for Test 1 are $t_{\min} = 74.89$ day and $t_{\max} = 196.02$ day, respectively. For any credit period less than 74.89 day and more than 196.02 day, SC coordination cannot be achieved (For example 65 day and 205 day in Figure 10). In the case of 65 day, the buyer will not accept the proposed incentive scheme as only the manufacturer achieves the surplus profit. Although the channel is coordinated at 85 day delay period, the manufacturer obtains 89% of the surplus profit and consequently this case might not be fair to the buyer. On the other hand, in the case of 185 day, the buyer achieves 93% of the surplus profit and it might not be fair to the manufacturer. In this study, it is assumed that the SC members agree on the average of minimum and maximum acceptable credit periods. However, as can be observed in Figure 10, the proposed credit option contract is flexible enough and helps the SC members to share the surplus profit between themselves in a fair manner.

## 6. A real case study

In this section, we operationally apply the proposed models in a dairy manufacturing company named "Kalleh". Kalleh produces a new dairy product which is called "Parmesan cheese". Kalleh applies an EPQ system and its annual production rate is $P = 20000$/Parmesan. Setup cost per production is $S_u = \$35$ and the unit production cost per Parmesan cheese is $P_m = \$40$/Parmesan. The manufacturer wholesales the products through a supermarket at price $P_u = \$50$/Parmesan. The supermarket uses a periodic review inventory policy for replenishing Parmesan cheese. Under such a system, inventory level is reviewed every $T$ units of time and enough Parmesan cheeses are ordered to raise the inventory up to the order-up-to level $R$ at each review period. Kalleh only decides on its production multiplier $n$ that is a positive integer representing the amount of Kalleh production cycle as a multiple of supermarket's order quantity. Supermarket's ordering cost

Table 4. Results of running the case study in decentralized, centralized and coordinated structures.

| Results | $T$ | $K$ | $P_l$ | $n$ | $\Pi_l$ | $\Pi_u$ | $\Pi_{SC}$ | $\Delta\Pi$ | $t_{\min}$ | $t_{\max}$ | $t_{\text{sharing}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Decentralized | 30.41 | 1.16 | 340.10 | 1 | 95400.45 | 52109.04 | 147509.50 | – | – | – | – |
| Centralized | 22.24 | 1.19 | 324.82 | 1 | 87862.72 | 65250.46 | 153113.19 | 5603.69 | – | – | – |
| Coordinated | 22.24 | 1.19 | 324.82 | 1 | 99025.39 | 56320.32 | 155345.72 | 7836.22 | 23.73 | 63.60 | 43.01 |

per order is $A_l = \$100$/order and lead time for him is $L = 3$ days. Unit inventory holding cost per unit time of the supermarket and Kalleh are $h_l = \$40$/Parmesan and $h_u = \$35$/Parmesan, respectively. The supermarket faces partial shortage where the fraction of the shortage that will be lost is $\alpha = 0.4$ and unit shortage cost is $\pi = \$2$/Parmesan.

The results of studying the consumers purchase behavior showed that the stochastic consumers' demand follows a normal distribution and it depends on the supermarket's selling price $P_l$. The selling price, which is set by the supermarket, impacts on the market share of the cheese and consequently influences profitability of the Kalleh. Supermarket decides on the selling price offered to the consumers, length of review period, and order-up-to-level. In Parmesan case, primary demand for the supermarket follows a normal distribution with mean 12000 and standard deviation 500 and supermarket's price-sensitivity coefficient of demand is $\beta_2 = 30$, which is estimated based on interviews with experts. Hence, the price dependent demand at supermarket follow a normal distribution as $(12000 - 30P_l, 500)$.

Under the traditional business environment, Kalleh and supermarket independently optimize their decisions. However, these self-interested decisions decrease the performance of the whole Parmesan SC. The results of solving the Parmesan cheese SC problem under three decision-making models are illustrated in Table 4. Although the centralized model can enhance the whole Parmesan SC profitability ($\Delta\Pi = 5603.69\$$), it decreases the profitability of the supermarket ($87862.72\$ < 95400.45\$$). The supermarket accepts the optimal decisions of centralized model if his profit under the coordination model increases compared to his decentralized profit (95400.45$). Thus, Kalleh should propose an incentive mechanism for inducing the supermarket to accept the centralized decisions. In the studied case study, Kalleh offers a credit option to the supermarket to encourage him to participate in the coordination scheme. Kalleh and supermarket can invest in different markets and their annual rate of return on investment are $i_l = 15\%$ and $i_u = 12\%$, respectively. The proposed credit option contract should be acceptable to both supermarket and Kalleh.

From the supermarket point of view, the minimum acceptable delay in payment to participate in the coordination plan is $t_{\min} = 23.73$ day. Furthermore, the maximum delay in payment that is acceptable for Kalleh is $t_{\max} = 63.60$ day. If the delay period approaches to $t_{\max}$, supermarket can achieve more surplus profit than Kalleh and when delay period approaches to $t_{\min}$, Kalleh can achieve more than supermarket. Thus, a fair profit sharing scheme can achieve using average of $t_{\min}$ and $t_{\max}$ . By applying the coordination scheme, the whole Parmesan SC profit deviation between the coordinated and decentralized model is $\Delta\Pi = 7836.22\$$. Moreover, profits of both members increase under the coordinated model compared to the decentralized setting.

As shown in Table 4, under the coordinated model, the selling price of Parmesan $P_l = 324.82$ is less than the selling price of the decentralized model $P_l = 340.10$ and it benefits the consumers. By decreasing the selling price, the market demand of Parmesan cheese increases from $D = 1797$/Parmesan under the decentralized model to $D = 2257$/Parmesan under the coordinated model. The results show that increase in Parmesan demand leads to more profit for both Parmesan SC members and consumers are also satisfied.

## 7. Model extensions

In the following, two model extensions are addressed to improve the investigated models. In Section 7.1, profit sharing strategy is studied to achieve a win–win situation in the credit option contract. In Section 7.2, partial credit option scheme is developed to examine the effect of the manufacturer's lack of cash on the permissible delay period.

### 7.1. Model extension 1: Profit sharing strategy

There are some strategies to divide the surplus profit resulting from the implementation of coordination scheme. SC members want to apply the fairly strategy that leads to a win–win situation. The win–win situation is set to be based on the bargaining power of the SC members. Therefore, in the investigated SC, setting the best value of $t_{\text{sharing}}$ that fairly divides the surplus profit is of high importance. In Section 4.3, we analyzed the middle strategy to calculate the value of $t_{\text{sharing}}$. Although the middle amount of credit option is acceptable for the buyer and the manufacturer, the SC members' bargaining powers are not considered and this situation is more beneficial for one of the SC members. Therefore, in this model extension, we analyze the profit sharing strategy that shares the surplus profit based on the bargaining powers. In this strategy, it is assumed that the bargaining powers of the buyer ($\varphi$) and the manufacturer $(1-\varphi)$ are based on their share of entire SC profit under the decentralized model ($\varphi = \Pi_l^{\text{dec}}/\Pi_{\text{SC}}^{\text{dec}}$). The buyer obtains $\varphi$ percent of the surplus profit $\Delta = \Pi_{\text{SC}}^{\text{coo}} - \Pi_{\text{SC}}^{\text{dec}}$ and the manufacturer receives $(1-\varphi)$ percent of it. Therefore, by substituting the equations (4.3) and (4.8) of the decentralized model and equations (4.14) and (4.15) of the coordinated one the amount of surplus profit $\Delta$ is calculated as follows:

$$\Delta = \Pi_{\text{SC}}^{\text{coo}} - \Pi_{\text{SC}}^{\text{dec}} = \Pi_{\text{SC}}^{\text{cen}} - \Pi_{\text{SC}}^{\text{dec}} + (i_l - i_u)p_u\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right)t. \tag{7.1}$$

The formulation of the buyer's profit under the coordinated model is sum of the buyer's decentralized profit and his/her share of surplus profit that is calculated in equation (7.2).

$$\Pi_l^{\text{coo}} = \Pi_l^{\text{dec}} + \varphi\Delta = \Pi_l^{\text{dec}} + \varphi(\Pi_{\text{SC}}^{\text{cen}} - \Pi_{\text{SC}}^{\text{dec}} + (i_l - i_u)p_u\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right)t) \tag{7.2}$$

By setting the buyer's profit under coordinated model (Eq. (4.14)) equal to that of profit sharing model (Eq. (7.2)), the credit option under the profit sharing strategy is obtained as follows.

$$t_{\text{sharing}} = \frac{\Pi_l^{\text{dec}} - \Pi_l^{\text{cen}} + \varphi(\Pi_{\text{SC}}^{\text{cen}} - \Pi_{\text{SC}}^{\text{dec}})}{p_u\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right)\left(i_l - \varphi(i_l - i_u)\right)} \tag{7.3}$$

Results of running the profit sharing strategy in comparison with the middle strategy are shown in Table 5. According to Table 5, Test 1, the credit option under the profit sharing strategy is less than that of the middle strategy. It means that the middle strategy is in the interest of the buyer and the manufacturer interested in the profit sharing strategy. As shown in Table 5, Test 3, the credit option under the middle strategy is less than the profit sharing strategy $(58.15 < 68.05)$. In such a case, however, the buyer interested in the profit sharing strategy, this strategy cause some loses for the manufacturer. According to Table 5, for all Test problems, the win–win situation is achieved under the profit sharing strategy based on SC members' bargaining powers.

### 7.2. Model extension 2: Partial credit option

In the real world, some suppliers or manufacturers aim to keep their business continuity by offering a partial credit option to the buyer [42]. In such a case, the buyer pays a partial amount, $\tau$ percent, as the products are received and he/she pays the rest of the payment, $(1-\tau)$ percent, at the credit period proposed by the manufacturer [13]. In some studies in the field of inventory management, the partial trade credit is addressed. While in the field of SC coordination, only the full trade credit is studied. In the following, for the first time, we analyze the partial credit option as the coordination scheme. Therefore, in the SC members' coordinated profit function (Eqs. (4.14) and (4.15)), term that indicates the buyer's profits and manufacturer's loses in the credit period are changed to $i_l p_u (1-\tau)\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right)t$ and $i_u p_u (1-\tau)\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right)t$, respectively. Similar to Proposition 4.4, the minimum credit period that convince the buyer to take part in the partial credit option scheme is as follows:

$$t_{\min} = \left(\Pi_l^{\text{dec}} - \Pi_l^{\text{cen}}\right)/i_l p_u (1-\tau)\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right). \tag{7.4}$$

TABLE 5. Results of running the profit sharing strategy in in comparison with the middle strategy.

| | Test 1 | Test 2 | Test 3 | Test 4 |
|---|---|---|---|---|
| Credit period | | | | |
| $t_{\min}$ | 74.89 | 68.47 | 37.84 | 11.23 |
| $t_{\max}$ | 196.02 | 100.29 | 78.46 | 67.39 |
| Middle strategy | | | | |
| $t_{\text{sharing}}$ | 135.45 | 84.38 | 58.15 | 39.31 |
| $\Pi_l$ | 24604.26 | 32190.65 | 116271.25 | 263764.80 |
| $\Pi_u$ | 27563.02 | 32134.48 | 55383.51 | 94412.77 |
| $\Pi_{\text{SC}}$ | 52167.28 | 64325.14 | 171654.76 | 358177.57 |
| $\Delta P$ | 8733.89 | 3334.69 | 5405.43 | 15866.92 |
| Profit sharing strategy | | | | |
| $\varphi$ | 0.45 | 0.50 | 0.68 | 0.74 |
| $t_{\text{sharing}}$ | 121.15 | 87.92 | 68.05 | 45.30 |
| $\Pi_l$ | 23424.97 | 32519.82 | 114134.71 | 266562.21 |
| $\Pi_u$ | 28447.49 | 31681.87 | 54137.71 | 93188.90 |
| $\Pi_{\text{SC}}$ | 51872.46 | 64201.70 | 168272.43 | 359751.11 |
| $\Delta P$ | 8439.08 | 3211.25 | 2023.10 | 17440.46 |

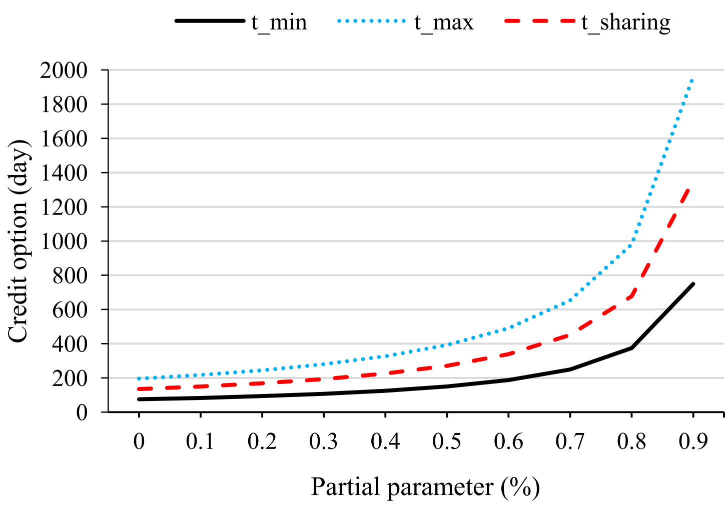



FIGURE 11. Changing on the minimum, maximum, and sharing amount of credit period by increasing the partial parameter $\tau$.

Therefore, according to Proposition 4.5, the maximum credit period that guarantee the manufacturer's participation in the partial credit option contract is as follows:

$$t_{\max} = \left(\Pi_u^{\text{cen}} - \Pi_u^{\text{dec}}\right)/i_u p_u\left(1-\tau\right)\left(\beta_1 - \beta_2 P_l^{\text{cen}}\right). \tag{7.5}$$

Using the middle strategy, the amount of credit option $t_{\text{sharing}}$ obtains from the proposed partial credit option scheme perspective. Figure 11 demonstrate the results of running the model for Test 1. The amount of credit period under $\tau = 0$ is equal to that of full credit option. As $\tau$ increases, the permissible credit period (deviation between $t_{\min}$ and $t_{\max}$) grows and thus $t_{\text{sharing}}$ significantly increases in comparison to full credit option one ($\tau = 0$). By increase in $\tau$ the manufacturer's lack of cash is more critical and he/she obtains more cash on the placing an order and thus sets a greater credit period to the buyer. The proposed partial credit option scheme offers a wider period of feasible credit period ($t_{\min}$, $t_{\max}$) to the buyer compared to the full credit option one. Therefore, the proposed strategy is more beneficial from both SC members perspective.

## 8. Conclusion

In this paper, a credit option contract as an incentive mechanism was proposed to simultaneously coordinate production, pricing, and periodic review replenishment decisions in a two-echelon SC. The investigated SC included one manufacturer and one buyer. The manufacturer used an EPQ system and the buyer used the periodic review order-up-to level policy for replenishing items. The demand was assumed to be stochastic and price sensitive. The investigated SC was modeled under three models, *i.e.* decentralized, centralized, and coordinated models. Under decentralized model, each SC actor individually made its decisions regardless of the other SC actor. While, in the centralized structure, the SC decisions were optimized from the whole SC perspective. Although the centralized decision-making could increase the profitability of the entire SC, the buyer could incur losses and consequently he/she would refuse to apply the centralized solution. Hence, an incentive mechanism based on credit option was proposed to convince the buyer to take part in the coordination scheme and optimize its decisions from the whole SC perspective. Also coordination mechanism investigated based on the different interest rates of both SC members. The minimum and maximum allowable credit period under which two SC participants had sufficient motivation to accept coordination plan were obtained. To calculate optimal decision variables under the decentralized and centralized models, solution procedures were proposed along with concavity analysis. A set of numerical examples and a real case study were carried out to demonstrate the applicability and performance of the developed scheme. Furthermore, when the interest rates of buyer was more than that the manufacturer, the profitability of whole SC in the coordinated model exceeded even more than the centralized decision making structure (see Tab. 3, Test 3). Moreover, the results demonstrated that the proposed credit option contract could enhance the SC profitability under high demand uncertainty towards the decentralized model and compensate the negative effects of high demand uncertainty (see Fig. 8). The proposed coordination mechanism diminishes negative effects of growing price-sensitivity coefficient on the whole SC profitability compared to the decentralized structure (see Fig. 4). The credit option contract can share the risk of manufacturer's high inventory cost between the SC members by changing the buyer's replenishment policy (see Figs. 5 and 6). In the case of high demand uncertainty, the centralized decision making is able to respond the customer stochastic demand by setting a lower amount of order up to level in comparison with the decentralized one (see Fig. 9(a)). In such a case, the centralized structure significantly decreases the shortage cost (see Fig. 9(b)). It can be inferred that under high demand uncertainty, implementing the proposed credit option scheme along with the centralized decision variables is useful. One of the future research directions is to extend the work by considering other demand functions. In addition to price, other parameters such as lead time length and product quality could affect the demand. Moreover, applying other coordination contracts such as quantity discount for coordinating the investigated SC is a promising area for future research.

## Appendix A. Proof of Proposition 4.1

To prove concavity of the buyer profit function w.r.t. $k$, and $\mathrm{P_l}$ for a given $T$, the Hessian matrix of the buyer's profit function should be calculated. If the principal minors are alternatively negative and positive, *i.e.*, the $k$th order leading principal minor $\mathrm{H}_k$ follows the sign of $(-1)^k$ then the profit function $\Pi_l$ is concave. The associated Hessian matrix of $\Pi_l$ is:

where

$$H\left(\Pi_l\left(k, P_l\right)\right) = \begin{bmatrix} \frac{\partial^2 \Pi_l}{\partial P_l^2} & \frac{\partial^2 \Pi_l}{\partial P_l \partial k} \\ \frac{\partial^2 \Pi_l}{\partial k \partial P_l} & \frac{\partial^2 \Pi_l}{\partial k^2} \end{bmatrix}$$

$$H_{11} = \frac{\partial^2 \Pi_l}{\partial P_l^2} = -2\beta_2 \tag{A.1}$$

$$\frac{\partial^2 \Pi_l}{\partial k \partial P_l} = \frac{\partial^2 \Pi_l}{\partial P_l \partial k} = -\frac{\alpha\sigma\sqrt{T+L}\left(\emptyset\left(k\right)-1\right)}{T} \tag{A.2}$$

$$\frac{\partial^2 \Pi_l}{\partial k^2} = -h_l \alpha\sigma\sqrt{T+L}\varphi\left(k\right) - \frac{\pi+\alpha\left(P_l-p_u\right)}{T}\sigma\sqrt{T+L}\varphi\left(k\right) \tag{A.3}$$

$$H_{22} = \left(-2\beta_2\right)\left(-h_l\alpha\sigma\sqrt{T+L}\varphi\left(k\right) - \frac{\pi+\alpha\left(P_l-p_u\right)}{T}\sigma\sqrt{T+L}\varphi\left(k\right)\right) - \left(-\frac{\alpha\sigma\sqrt{T+L}\left(\emptyset\left(k\right)-1\right)}{T}\right)^2 \tag{A.4}$$

$$2\beta_2\left(h_l\alpha\sigma\sqrt{T+L}\varphi\left(k\right) + \frac{\pi+\alpha\left(P_l-p_u\right)}{T}\sigma\sqrt{T+L}\varphi\left(k\right)\right) \geq \left(\frac{\alpha\sigma\sqrt{T+L}\left(\emptyset\left(k\right)-1\right)}{T}\right)^2 \tag{A.5}$$

The first principal minor ($H_{11}$) has a negative value. The second principal minor is positive under the condition (A.5). The condition (A.5) holds under a wide range of reasonable parameters and examined test problem. Therefore, the buyer profit function is concave w.r.t. $k$, and $P_l$ for a given $T$.

## Appendix B. Proof of Proposition 4.2

In order to prove the concavity of manufacturer profit function w.r.t. $n$, taking the first and second derivatives of $\Pi_u\left(n\right)$ w.r.t. $n$ gives:

$$\frac{\partial \Pi_u\left(n\right)}{\partial n} = \frac{S_u}{n^2 T} - h_u \frac{\left(\beta_1-\beta_2 P_l\right)T}{2}\left[1 - \frac{\left(\beta_1-\beta_2 P_l\right)}{P}\right] \tag{B.1}$$

$$\frac{\partial^2 \Pi_u\left(n\right)}{\partial n^2} = -\frac{2S_u}{n^3 T} < 0. \tag{B.2}$$

The second order derivative is negative, thus $\Pi_u\left(n\right)$ is concave w.r.t. $n$.

## Appendix C. Proof of Proposition 4.3

To show concavity of SC profit w.r.t. variables $k$, $P_l$, and $n$ for a given $T$, the Hessian matrix for the SC profit function w.r.t. variables $k$, $P_l$, and $n$ variables should be calculated as follows. If the Hessian matrix is negative definite, the proposition will be proved. To show concavity, it is temporarily assumed that the variable $n$ is a continuous variable.

$$H\left(\Pi_{\text{SC}}\right) = \begin{bmatrix} \frac{\partial^2 \Pi_{\text{SC}}}{\partial n^2} & \frac{\partial^2 \Pi_{\text{SC}}}{\partial n \partial k} & \frac{\partial^2 \Pi_{\text{SC}}}{\partial n \partial P_l} \\ \frac{\partial^2 \Pi_{\text{SC}}}{\partial k \partial n} & \frac{\partial^2 \Pi_{\text{SC}}}{\partial k^2} & \frac{\partial^2 \Pi_{\text{SC}}}{\partial k \partial P_l} \\ \frac{\partial^2 \Pi_{\text{SC}}}{\partial P_l \partial n} & \frac{\partial^2 \Pi_{\text{SC}}}{\partial P_l \partial n} & \frac{\partial^2 \Pi_{\text{SC}}}{\partial P_l^2} \end{bmatrix}$$

where,

$$H_{11} = \frac{\partial^2 \Pi_{\text{SC}}}{\partial n^2} = \frac{-2S_u}{n^3 T} < 0. \tag{C.1}$$

And the second principle minor is as follows:

$$H_{22} = \frac{\partial^2 \Pi_{\text{SC}}}{\partial n^2} \times \frac{\partial^2 \Pi_{\text{SC}}}{\partial k^2} - \left(\frac{\partial^2 \Pi_{\text{SC}}}{\partial k \partial n}\right)^2 = \left(\frac{-2S_u}{n^3 T}\right) \times \left(-h_l\alpha\sigma\sqrt{T+L}\varphi\left(k\right) - \frac{\pi+\alpha\left(P_l-p_u\right)}{T}\sigma\sqrt{T+L}\varphi\left(k\right)\right) - 0 > 0 \tag{C.2}$$

And the third principle minor is negative when:

$$
\begin{aligned}
H_{33} =& \frac{\partial^2 \Pi_{\mathrm{SC}}}{\partial n^2} \times \left[\frac{\partial^2 \Pi_{\mathrm{SC}}}{\partial k^2} \times \frac{\partial^2 \Pi_{\mathrm{SC}}}{\partial P_l^2} - \left(\frac{\partial^2 \Pi_{\mathrm{SC}}}{\partial k \partial P_l}\right)^2\right] - \frac{\partial^2 \Pi_{\mathrm{SC}}}{\partial n \partial k} \times \left[\frac{\partial^2 \Pi_{\mathrm{SC}}}{\partial k \partial n} \times \frac{\partial^2 \Pi_{\mathrm{SC}}}{\partial P_l^2} - \frac{\partial^2 \Pi_{\mathrm{SC}}}{\partial k \partial P_l} \times \frac{\partial^2 \Pi_{\mathrm{SC}}}{\partial P_l \partial n}\right] \\
&+ \frac{\partial^2 \Pi_{\mathrm{SC}}}{\partial n \partial P_l} \times \left[\frac{\partial^2 \Pi_{\mathrm{SC}}}{\partial k \partial n} \times \frac{\partial^2 \Pi_{\mathrm{SC}}}{\partial P_l \partial n} - \frac{\partial^2 \Pi_{\mathrm{SC}}}{\partial k^2} \times \frac{\partial^2 \Pi_{\mathrm{SC}}}{\partial n \partial P_l}\right] \\
=& \frac{-2S_u}{n^3 T} \times \left[\left(-h_l \alpha \sigma \sqrt{T+L}\varphi(k) - \frac{\pi + \alpha(P_l - p_u)}{T} \sigma \sqrt{T+L}\varphi(k)\right)\right. \\
&\times \left. \left(-2\beta_2 - \frac{h_u T \beta_2^2 (3-n)}{2P}\right) - \left(\frac{-\alpha\sigma\sqrt{T+L}(\Phi(k)-1)}{T}\right)^2\right] - 0 + \left(\frac{-h_u T \beta_2 (2\beta_2 - 2\beta_2 P_l - P)}{2P}\right) \\
&\times \left[0 - \left(-h_l \alpha \sigma \sqrt{T+L}\varphi(k) - \frac{\pi + \alpha(P_l - p_u)}{T} \sigma \sqrt{T+L}\varphi(k)\right) \times \left(\frac{-h_u T \beta_2 (2\beta_2 - 2\beta_2 P_l - P)}{2P}\right)\right]
\end{aligned}
\tag{C.3}
$$

The condition is tested numerically and observed that it would be satisfied for reasonable parameter values. Then, by satisfying condition (C.3), Hessian matrix of the SC expected annual profit function is negative definite.

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