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ENERGY MANAGEMENT IN CROP PRODUCTION USING A NOVEL FUZZY DATA ENVELOPMENT ANALYSIS MODEL

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Abstract. Data envelopment analysis is a relatively "data oriented" approach to measure the efficiency of a set of decision making units which transform multiple inputs into multiple outputs. However, some production processes may generate undesirable outputs like smoke pollution or waste. On the other hand, in many situations, such as a manufacturing system, a production process or a service system, inputs and outputs can be considered as a fuzzy variable. Thus, this paper has presented a new non-radial DEA model based on a modification of Enhanced Russell Model (ERM model) in the presence of an undesirable output in a fuzzy environment. Hereafter, a method for solving the proposed fuzzy DEA model based on the concept of alpha cut and possibility approach is presented. A useful stochastic closeness coefficient is also proposed to present a complete ranking. The proposed methodology is applied to evaluate the efficiencies of barley production farms in 22 provinces in Iran.

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1. Introduction

Energy management and reducing greenhouse gas emission is important in decreasing the environmental hazards of crop production. Usually greenhouse gas emission is not a desirable data and it is considered as an undesirable output in agriculture. When studying the efficiency of crop production, it is important to present a model that correctly measures the effects of this undesirable emission.

Koopmans [52] has mentioned that the production process may also generate undesirable outputs like smoke pollution or waste. An example can be a paper mill production unit where undesirable outputs namely pollutants such as biochemical oxygen demand, suspended solids, particulate and sulfur oxides are associated with the production. If the production is inefficient, the undesirable pollutants should be reduced to improve the inefficiency; therefore the undesirable and desirable outputs should be treated differently when evaluating the production performance of paper mills. The combination of life cycle analysis (LCA) with optimization techniques connects operational input efficiency to environmental impacts [65]. In a study by Mulwa *et al.* undesirable pollutant output was used in both hyperbolic and directional distance function DEA models to measure the total factor

Keywords. Data envelopment analysis (DEA), efficiency, decision making unit (DMU), fuzzy data, undesirable output, possibility approach.

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productivity in sugarcane farming in Kenya [73]. Arabi *et al.* used undesirable emission output to determine the productivity and eco-efficiency trends of power plants in Iran. A slack based measure using material balance principle was developed to enhance the Malmquist Luenburger index [6].

Agriculture contributes to the global CO_2 emission by 14% [65] and crop production requires a large quantity of fossil-based energy in the form of direct and indirect energy [49]. Optimizing finite agricultural energy resources can be done using mathematical programming techniques such as data envelopment analysis. Data envelopment analysis (DEA) is a nonparametric methodology to calculate the relative efficiency of decision making units (DMUs). The first DEA model, *i.e.* CCR model, was proposed by Charnes *et al.* [11] and is based on the work of Farrell [20]. This model is a radial model. Radial models have some disadvantages, such as, the failure to recognize weak efficient DMUs [37,38]. Another kind of DEA models are non-radial DEA models. One important non-radial DEA model is Enhanced Russell Model (ERM model) that was proposed by Pastor *et al.* [77]. A useful feature of this model is the ability to recognize weak efficient DMUs. On the other hand, this model has disadvantages like failure to rank efficient DMUs. Izadikhah *et al.* [39] proposed a modified version of ERM model that enables ERM model to rank efficient DMUs. Our proposed DEA methodology is an extension of their model that takes into consideration undesirable data. Also, data envelopment analysis has been applied to energy management in the production of various crops [50, 51, 54, 71, 72]. In a study, energy efficiency of grape production was calculated in a two-stage DEA and Tobit regression model. The average technical and pure technical efficiency of grape production in the studied area was 0.723 and 0.881, respectively. Authors also identified that the farmer's level of education influenced the efficiency of grape production [51].

This paper aims to evaluate the efficiency of crop production in barley farms in Iran farms in which the greenhouse gas emission is the undesirable output. However, in many situations, such as a manufacturing system, a production process or a service system, inputs and outputs are volatile and complex thus, it is difficult to measure them accurately. Instead, the data can be considered as a fuzzy variable. The concept of fuzzy theory was initialized by Zadeh [103]. After that many fuzzy approaches have been introduced in the DEA literature. Sengupta [89] applied principle of fuzzy set theory to introduce fuzziness in the objective function and the right-hand side vector of the conventional DEA model and developed the tolerance approach that was one of the first fuzzy DEA models. DEA and Fuzzy modeling were combined to study energy efficiency and sustainability of corn production in the south of Iran. The results recommended farmers to change their current trend of energy consumption to improve efficiency and sustainability [35]. Conventional DEA needs accurate measurement of inputs and outputs. However, the values of the input and output data in crop productions are sometimes imprecise or uncertain and since some data values in this paper were not exactly known, the data was stated as fuzzy data. Thus, firstly, the proposed model is extended to a fuzzy environment. Then, a method is presented for solving the proposed fuzzy DEA model based on the concept of alpha cut and possibility approach. Also for the purpose of final ranking, a stochastic closeness coefficient is offered. This coefficient is very useful and integrates all results of various values of α. Therefore, this paper has combined fuzzy inputs and fuzzy undesirable output and presented a novel fuzzy DEA model to estimate the efficiency of crop production in barley farms in Iran.

The main contributions of this paper are as follows: This paper extends a recent modified version of ERM model to deal with undesirable data. The proposed model modifies the problem of ERM model in ranking the efficient DMUs. In addition, this paper presents a new fuzzy DEA model based on the modified ERM in the presence of undesirable output. The model uses the concept of α-cut and possibility approach to defuzzification. Also for the purpose of final ranking, this paper proposes a stochastic closeness coefficient. This coefficient removes the difficulty of different ranking by various values of α. The proposed methodology is applied to evaluate the efficiency of barley production farms in 22 provinces in Iran.

This paper unfolds as follows: In Secton 2 literature review is presented. Section 3 briefly reviews some related important topics. Section 4 proposes our new DEA model in the presence of undesirable output. The proposed fuzzy DEA model is presented in Section 5. In Section 6, a case study is presented and the final conclusion appears in Section 7.

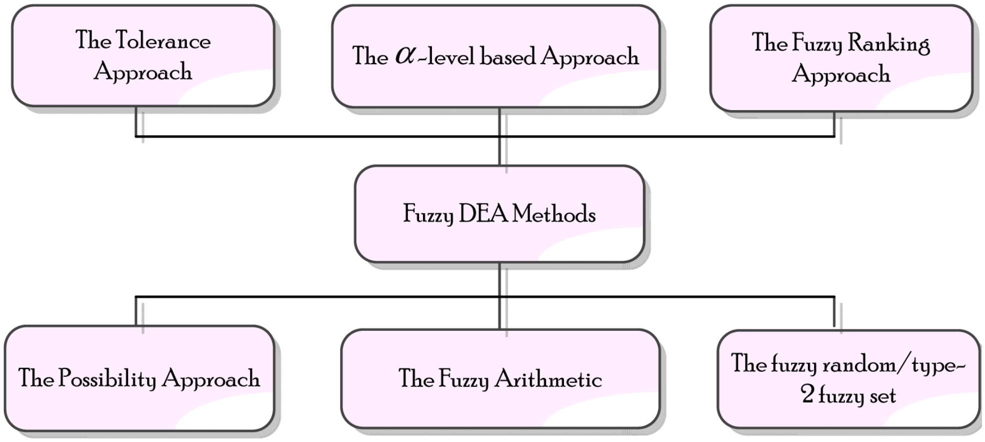


FIGURE 1. The fuzzy DEA methods.

2. Literature review

In this section we review some related articles prior to this study.

2.1. Fuzzy DEA methods

Sengupta [89, 90] was the first to introduce a fuzzy mathematical programming approach in which fuzziness was incorporated into the DEA model by defining tolerance levels on both the objective function and constraint violations.

The applications of fuzzy set theory in DEA are usually categorized into six groups [14]:

(1) The tolerance approach that is one of the first fuzzy DEA models that was developed by Sengupta [89] and further improved by Kahraman and Tolga [43].
(2) The a-level based approach that Girod [24] used for the first time to formulate the fuzzy BCC and free disposal hull (FDH) models which are radial measures of efficiency.
(3) The fuzzy ranking approach that was initially developed by Guo and Tanaka [26]. They proposed a fuzzy CCR model in which fuzzy constraints (including fuzzy equalities and fuzzy inequalities) were converted into crisp constraints by predefining a possibility level and using the comparison rule for fuzzy numbers.
(4) The possibility approach proposed by Guo *et al.* [28] who built fuzzy DEA models based on possibility and necessity measures.
(5) The fuzzy arithmetic approach that Wang *et al.* [99] pioneered. They proposed two fuzzy DEA models with fuzzy inputs and outputs by means of fuzzy arithmetic.
(6) The fuzzy random/type-2 fuzzy set developed by Qin *et al.* [84]. This DEA model with type-2 fuzzy inputs and outputs was introduced to deal with linguistic uncertainties as well as numerical uncertainties with respect to fuzzy membership functions. Figure 1 illustrates the various methods for solving fuzzy DEA model.

Table 1 summarizes the researches related to various methods for solving fuzzy DEA models. This table updates the survey of Emrouznejad *et al.* [14] on fuzzy DEA methods.

Table 1. Various methods for solving fuzzy DEA models.

	References	Description
The tolerance approach	Sengupta, (1992a, 1992b)	Uncertainty is incorporated into the DEA models by defining tolerance levels on constraint violations.
The α-level based approach	Kao and Liu, (2000); Agarwal, (2014); Lertworasirikul *et al.*, (2003); Emrouznejad *et al.*, (2011); Shokouhi *et al.*, (2010); Zhou *et al.*, (2012); Zerafat Angiz L. *et al.*, (2012) ; Zerafat Angiz L *et al.*, (2010); Fathi and Izadikhah, (2013a, (2013b); Tavana and Khalili-Damghani, (2014); Wanke *et al.*, (2016); Khalili-Damghani *et al.*, (2016); Hatami-Marbini *et al.*, (2016);	These methods use the concept of during solving the related fuzzy DEA models.
The fuzzy ranking approach	Saati and Memariani (2005); Soleimani-damaneh *et al.* (2006); Guo and Tanaka (2001a); Pei-Huang, (2006); Jahanshahloo *et al.*, (2009); Lee *et al.*, (2005); Molavi *et al.*, (2005); Dia(2004); León *et al.*, (2003); Lertworasirikul, (2002); Izadikhah *et al.*, (2017b); Hatami-Marbini *et al.*, (2017); Olfat *et al.*, (2016);	These methods consider a fuzzy variable that is associated with a possibility distribution like a random variable that is associated with a probability distribution.
The possibility approach	Payan and Shariff, (2013); Wang and Chin, (2011); Lin, (2010); Zhao and Yue, (2012); Nedeljkovic' and Drenovac, (2012); Khodabakhshi *et al.*, (2010); Wen and Li, (2009); Wen *et al.*, (2010); Hossainzadeh Lotfi *et al.*, (2011); Azadi *et al.*, (2015); Zerafat Angiz L *et al.*, (2015);	These methods consider a fuzzy variable that is associated with a possibility distribution like a random variable that is associated with a probability distribution.
The fuzzy arithmetic	Mirhedayatian, Jelodar, *et al.*, (2013); Mirhedayatian, Vahdat, *et al.*, (2013); Jafarian-Moghaddam and Ghoseiri, (2012); Wang *et al.*, (2009); Abdoli *et al.*, (2011); Han *et al.*, (2015); Puri and Yadav, (2015); Hongmei *et al.*, (2015); Shermeh *et al.*, (2016); Mashayekhi and Omrani, (2016)	In these methods decision makers are not allowed to convert a fuzzy fractional programming to a LP model using conventional methods.
The fuzzy random/type-2 fuzzy set	Zerafat Angiz L. *et al.* (2013); Tavana *et al.*, (2013); Bray *et al.*, (2015); Zhou *et al.*, (2016);	In these methods uncertainty is incorporated into the membership function of a fuzzy set.

2.2. DEA models in the presence of undesirable data

It often occurs that apart from consuming inputs and producing desirable outputs, the DMUs also generate undesirable outputs. That is rather common in many production settings in which pollution, noise, *etc.*, are unwillingly but inevitably generated. There are many DEA approaches that can handle this situation basically through the assumption of an appropriate technology. Figure 2 shows the various methods for considering undesirable data in data envelopment analysis models.

From Figure 2 it is clear that there are two main methods for considering undesirable data. They are: (i) The methods based on weak disposability and (ii) The methods based on data translation. The second method has

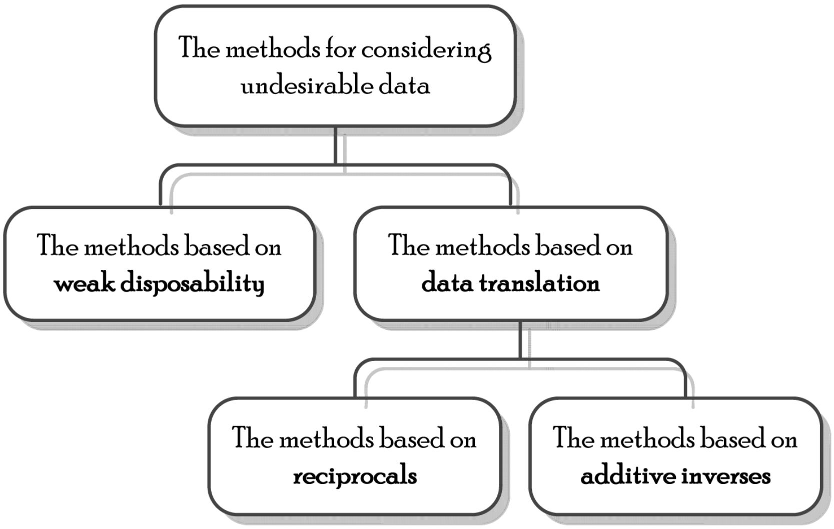


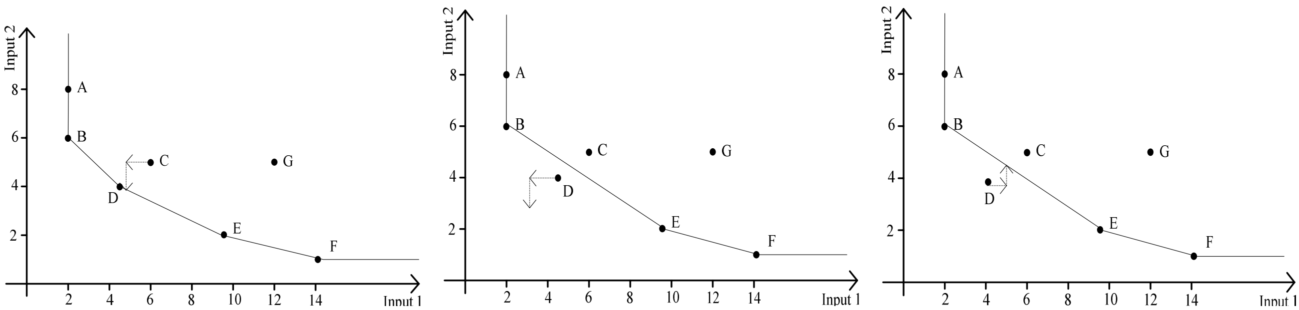
FIGURE 2. The various methods of considering undesirable data.

TABLE 2. The DEA methods that considered undesirable data.

Methods		References	Description
The methods based on weak disposability		Färe and Grosskopf, (2000), (2003), (2004) (2009); Färe *et al.*, (1993); Tyteca, (1997); Korhonen and Luptacik, (2004); Puri and Yadav, (2014b); Khalili-Damghani *et al.*, (2016)	Treat undesirable outputs in their original forms and assumes that these are weakly disposable
The methods based on data translation	The methods based on reciprocals	Golany and Roll, (1989)	Undesirable outputs are considered in the form of their reciprocals
	The methods based on additive inverses	Scheel, (2001); Seiford and Zhu, (2002); Hadi Vencheh *et al.*, (2005); Sahoo *et al.*, (2011); Sharp *et al.*, (2007); Kerstens and Van de Woestyne, (2011); Farzipoor Saen, (2010); Liu *et al.*, (2010); Aliakbarpoor and Izadikhah, (2012); Barros *et al.*, (2012); Li, Li, *et al.*, (2013); Li, Yang, *et al.*, (2013); Maghbouli *et al.*, (2014) Liu *et al.*, (2015); Song *et al.*, (2014); Aghayi, (2016); Puri and Yadav, (2016) , Ignatius *et al.*, (2016);	Undesirable outputs are considered in the form of their additive inverses. In these methods undesirable output (input) is considered as desirable input (output).

been used more than the first one. See Table 2 for a breif survey of methods that used undesirable data along with data envelopment analysis. Table 2 states the DEA works that used undesirable data. As it can be seen from Table 2, many previous works that used undesirable data are based on additive inverses.

Since undesirable data in assessing the firm's performance is important, many authors have presented fuzzy DEA model in the presence of undesirable data. Puri and Yadav [82] proposed a fuzzy MC-DEA model in which shared and undesirable fuzzy resources are incorporated. Also, Puri and Yadav [81] proposed a DEA model with undesirable outputs in fuzzy environment in view of the fact that input/output data are not always available in exact form in real life problems. In another work, Song *et al.* [95] presented a super–efficiency DEA model considering both desirable and undesirable outputs based on the classical slack–based measure (SBM) environmental efficiency evaluation model. Puri and Yadav [80] extended the cost efficiency and revenue efficiency models with undesirable output to fully fuzzy environments to account for real situations where



(a) Efficiency frontier before removing $\mathrm{DMU_D}$ (b) Efficiency frontier after removing $\mathrm{DMU_D}$ (c) A new movement direction

FIGURE 3. Illustration of DMUs in Example 1.

input–output data and their corresponding prices are not precisely known. Aghayi [3] introduced a method to evaluate the revenue efficiency of DMUs when there exists both desirable and undesirable data in fuzzy DEA.

Khalili–Damghani *et al.* [47] presented a fuzzy DEA framework for solving performance evaluation problems with coexisting desirable input and undesirable output data in the presence of simultaneous input–output projection. Ignatius *et al.* [36] proposed a DEA-based framework where the input and output data are characterized by symmetrical and asymmetrical fuzzy numbers and some of them are undesirable.

3. Preliminaries

In this section some required concepts are reviewed.

3.1. Enhanced Russell model

Assume that there are n DMUs where each $DMU_j (j = 1, \ldots, n)$, uses m inputs, $x_{ij} (i = 1, \ldots, m)$ to produce s outputs, $y_{rj} (r = 1, \ldots, s)$. Also, assume that data set are positive and deterministic. Non-radial ERM model is considered for measuring relative efficiency of DMU under evaluation, DMU_p, as follows [77]:

$$
\begin{aligned}
\rho_p^* = \min \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\varphi_r} \\
& s.t. \\
& \sum_{j=1}^{n}\lambda_j x_{ij} \leqslant \theta_i x_{ip}, i = 1, \ldots, m, \quad \sum_{j=1}^{n}\lambda_j y_{rj} \geqslant \varphi_r y_{rp}, r = 1, \ldots, s, \\
& \theta_i \leqslant 1, \varphi_r \geqslant 1, \forall i, r, \\
& \lambda_j \geqslant 0, j = 1, \ldots, n.
\end{aligned} \tag{3.1}
$$

Definition 3.1. (ERM-efficiency [77]). Optimal ρ_p^* of the model (3.1) is called ERM efficiency score of DMU_p. DMU_p is ERM efficient, if and only if $\rho_p^* = 1$. This condition is equivalent to $\theta_i^* = 1$ and $\varphi_r^* = 1$ for each i and r in any optimal solution.

3.2. A super-efficiency method based on modified ERM

It is showed that if the DMU under evaluation from reference set of the model (3.1) is removed, then correct super-efficiency score can not be obtained [38]. Consider example 1 and assume that the super-efficiency score of $\mathrm{DMU_D}$ is going to be measured. New efficiency frontier after removing $\mathrm{DMU_D}$ is shown in Figure 3b. The movement direction of $\mathrm{DMU_D}$ in the given model (3.1) is shown. It is clear that the ERM model does not have optimal solution and fails to obtain complete ranking. Therefore, if the super-efficiency score of a DMU that is located outside production possibility set (PPS) is to be measured the movement direction of DMU should be changed. As a result, an ERM model is needed that considers both movement directions, simultaneously. To solve this problem, Izadikhah *et al.* [39] proposed the following integer programming model that is a modified ERM model.

$$
\begin{aligned}
& R^* = \min \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\varphi_r} \\
& \quad s.t. \\
& \quad \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j x_{ij} \leqslant \theta_i x_{io}; \quad i=1,\ldots,m, \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j y_{rj} \geqslant \varphi_r y_{ro}; \quad r=1,\ldots,s, \\
& \quad \theta_i - 1 \leqslant M\delta; \quad i=1,\ldots,m, -\theta_i + 1 \leqslant M(1-\delta); \quad i=1,\ldots,m, \\
& \quad -\varphi_r + 1 \leqslant M\delta; \quad r=1,\ldots,s, \varphi_r - 1 \leqslant M(1-\delta); \quad r=1,\ldots,s, \\
& \quad \delta \in \{0,1\}, \\
& \quad \lambda_j \geqslant 0, \quad j=1,\ldots,n.
\end{aligned} \tag{3.2}
$$

In model (3.2), the binary variable δ guarantees that only one of the two groups of constraints is held:

$$
(I): \begin{cases} \theta_i \leqslant 1; & i=1,\ldots,m, \\ \varphi_r \geqslant 1; & r=1,\ldots,s, \end{cases} \quad \text{or} \quad (II): \begin{cases} \theta_i \geqslant 1; & i=1,\ldots,m, \\ \varphi_r \leqslant 1; & r=1,\ldots,s, \end{cases} \tag{3.3}
$$

If a DMU is located inside the PPS, constraints of group (I) will be active. If DMU is located outside the PPS, constraints of group (II) will be active. Figure 3c shows the new direction of movement for a DMU that is located outside PPS. It can be seen that in this method, DMU_D moves towards efficiency frontier and as a result, the proposed new model is able to rank all DMUs.

Example 3.2. This example shows that the ERM model cannot give a complete ranking among efficient DMUs. Consider seven DMUs that use two inputs to produce a single output which has unit value as depicted in Table 3.

As is shown in the last row of Table 3, it is clear that the ERM model fails to present a complete ranking among efficient DMUs. Figure 3 illustrates these DMUs.

Figure 2a shows that the DMUs {A, B, D, E, F} are efficient DMUs and their ERM efficiency scores are equal to unity showing that the ERM model is unable to discriminate them. Also, DMUs $\{C, G\}$ are inefficient and $\mathrm{DMU_A}$ is a weak efficient DMU. Figure 3a shows the movement direction of $\mathrm{DMU_C}$ onto the efficiency frontier according to model (3.1). Rankings obtained by model (3.2) are also summarized in Table 3. It is seen that the method Izadikhah *et al.* [39] proposed, presents a complete ranking among efficient DMUs and thus its discrimination power is greater than the ERM model. Also, comparing the rankings obtained by Andersen and Petersen [5] (AP method) and Izadikhah *et al.* [39] method, it can be seen that the first model fails to recognize inefficiency of $\mathrm{DMU_A}$ while the second model recognizes the inefficiency.

TABLE 3. Results of our proposed model compared with AP model.

DMUs	A	B	C	D	E	F	G
INPUT 1	2	2	6	4.5	9.5	14	12
INPUT 2	8	6	5	4	2	1	5
OUTPUT	1	1	1	1	1	1	1
ERM efficiency score	0.87	1	0.77	1	1	1	0.53
AP score	1	1.22	0.78	1.1	1.08	2	0.59
AP ranking	5	2	6	3	4	1	7
Efficiency score obtained from the model (3.2)	0.87	1.17	0.77	1.08	1.07	1.50	0.59
Ranking	5	2	6	3	4	1	7

3.3. Possibility approach

Zadeh [103] presented the concept of possibility approach in terms of fuzzy set theory. Here is a review of the definition of possibility space [7, 103].

Definition 3.3. (Possibility space) Let Θ be a nonempty set, and P the power set of Θ. Each element in P is called an event. To present an axiomatic definition of possibility, it is necessary to assign to each event A, a number $\pi(A)$which indicates the possibility that A will occur. Then the triplet $(\Theta, p, \pi)$is called a possibility space.

Definition 3.4. Let A be a fuzzy variable defined on a possibility space(Θ, p, π). The membership of this variable introduced by Zadeh is as follows:

$$\mu_A(s) = \pi(\theta_i \in \Theta_i | A(\theta_i) = s) = \sup \theta_i \in \Theta_i \{\pi(\theta_i) | A(\theta_i) = s\}, \forall s \in R$$

Definition 3.5. Let (Θ, p, π) be a possibility space such that $\Theta = \Theta_1 \times \ldots \times \Theta_n$, therefore, for any set A we have

$$\pi(A) = \sup_{\theta_i \in \Theta_i} \{\pi_i(A_i) | A = A_1 \times \ldots \times A_n, A_i \in p\}$$

Definition 3.6. Denote an $\alpha-$cut of fuzzy number A by A_α which is defined as follows:

$$A_\alpha = \{x \,| A(x) \geqslant \alpha\} \tag{3.4}$$

An $\alpha-$cut of A can be stated as $A_\alpha = [A_L^{-1}(\alpha), A_U^{-1}(\alpha)] = \left[[A]_\alpha^L, [A]_\alpha^U\right]$ for all $\alpha \in\ [0,1]$.

Considering the fuzzy theory, the following lemma can be very useful for interpreting the possibility function.

Lemma 3.7. *Let $A_1, \ldots, A_n$ be normal and convex fuzzy variables. For any given possibility level $\varepsilon_1, \varepsilon_2$ and $\varepsilon_3 \quad (0 \leqslant \varepsilon_i \leqslant 1)$ we have*

(i) $\pi(A_1 + \ldots + A_n \leqslant a) \geqslant \varepsilon_1$ *if and only if* $[A_1]_{\varepsilon_1}^L + \ldots + [A_n]_{\varepsilon_1}^L \leqslant a$
(ii) $\pi(A_1 + \ldots + A_n \leqslant a) \geqslant \varepsilon_2$ *if and only if* $[A_1]_{\varepsilon_2}^U + \ldots + [A_n]_{\varepsilon_2}^U \geqslant a$
(iii) $\pi(A_1 + \ldots + A_n \leqslant a) \geqslant \varepsilon_3$ *if and only if* $[A_1]_{\varepsilon_3}^L + \ldots + [A_n]_{\varepsilon_3}^L \leqslant a$ *and* $[A_1]_{\varepsilon_3}^U + \ldots + [A_n]_{\varepsilon_3}^U \geqslant a$

where $[A_j]_{\varepsilon_i}^L$ and $[A_j]_{\varepsilon_i}^U$ are the lower and upper bounds of the ε_i-level set of $A_j (j = 1, \ldots, n)$. The above lemma is very suitable for defuzzification of the fuzzy DEA model's constraints.

4. Modified ERM with undesirable data

Here, a model is developed to consider both desirable and undesirable outputs. Let us consider calculating the performance of a homogeneous set of n DMUs ($DMUj; j = 1, \dots, n$) in a production process, in which a vector of m inputs x_{ij} $(i = 1, \dots, m)$ is used to produce s outputs. The number of desirable outputs and their values are denoted by s_1 and y^g_{rj} $(r = 1, \dots s_1)$ respectively, and s_2 is the number of undesirable outputs and their values are denoted by y^b_{tj} $(t = 1, \dots, s_2)$ such that $s = s_1 + s_2$. Following Banker *et al.* [8], we can define a production possibility set, T, as follows:

$$T = \left\{(x_j, y^g_j, y^b_j) : x_j \quad \text{can produce} \quad y^g_j \quad \text{and} \quad y^b_j\right\}$$

It is preferred to produce desirable outputs as much as possible, not to produce undesirable outputs. Liu *et al.* [64] believed that it is very useful to regard the undesirable inputs and outputs as desirable outputs and inputs respectively. Considering this subject the following integer programming model that is a modified ERM model in the presence of undesirable data is proposed.

$$\begin{aligned}
&R^* = \min \frac{\frac{1}{m+s_2}\left(\sum_{i=1}^m \theta_i + \sum_{t=1}^{s_2} \gamma_t\right)}{\frac{1}{s_1}\sum_{r=1}^{s_1} \varphi_r}\\
&s.t.\\
&\sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j x_{ij} \leqslant \theta_i x_{ip}; \quad i = 1, \dots, m,\\
&\sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j y^b_{tj} \leqslant \gamma_t y^b_{tp}; \quad t = 1, \dots, s_2, \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j y^g_{rj} \geqslant \varphi_r y^g_{rp}; \quad r = 1, \dots, s_1,\\
&\theta_i - 1 \leqslant M\delta_1; \quad i = 1, \dots, m, -\theta_i + 1 \leqslant M(1-\delta_1); \quad i = 1, \dots, m,\\
&\gamma_t - 1 \leqslant M\delta_1; \quad t = 1, \dots, s_2, -\gamma_t + 1 \leqslant M(1-\delta_1); \quad t = 1, \dots, s_2,\\
&-\varphi_r + 1 \leqslant M\delta_2; \quad r = 1, \dots, s_1, \varphi_r - 1 \leqslant M(1-\delta_2); \quad r = 1, \dots, s_1,\\
&\delta_1 + \delta_1 = 1; \quad \delta_1, \delta_2 \in \{0, 1\},\\
&\theta_i, \lambda_j \geqslant 0; \quad \forall i, j.
\end{aligned} \tag{4.1}$$

In model (4.1), the binary variables δ_1 and δ_2 guarantee that only one of the two groups of constraints is held:

$$(I) : \begin{cases} \theta_i \leqslant 1; \quad i = 1, \dots, m; \quad \gamma_t \leqslant 1; t = 1, \dots, s_1,\\ \varphi_r \geqslant 1; \quad r = 1, \dots, s_2; \end{cases} \quad \text{or} \quad (II) : \begin{cases} \theta_i \geqslant 1; \ i = 1, \dots, m, \ \gamma_t \geqslant 1; \quad t = 1, \dots, s_1,\\ \varphi_r \leqslant 1; \ r = 1, \dots, s_2, \end{cases} \tag{4.2}$$

If a DMU is located inside the PPS, constraints of group (I) will be active. If DMU is located outside the PPS, constraints of group (II) will be active.

5. A new Fuzzy modified ERM with undesirable data

The classic DEA models can only be used for cases where the data are precisely measured while in real-world situations, the observed values of the input and output data are sometimes inexact, incomplete, vague or ambiguous. To reflect a kind of general sense or experience of experts, this type of inaccurate data can be represented as linguistic variables characterized by fuzzy numbers. The concept of fuzzy set theory was first developed by Zadeh [103] to deal with the issue of uncertainty in systems modeling. Fuzzy DEA is a powerful tool for evaluating the performance of DMUs in uncertain environments. In this section, we propose a new fuzzy DEA model in the presence of undesirable output for evaluating a set of DMUs with fuzzy inputs and outputs. Hence, we extend model (4.1) to a fuzzy model.

5.1. Justification of the model

The efficiency of a homogeneous set of n DMUs ($\mathrm{DMU_j}$; $j = 1, \ldots, n$) is to be assessed. Assume that $\mathrm{DMU_j}$ uses m fuzzy inputs $\tilde{x}_{ij}$ $(i = 1, \ldots, m)$ to produce s fuzzy outputs in which s_1 fuzzy outputs denoted by $\tilde{y}^g_{rj}$ $(r = 1, \ldots, s_1)$ are desirable (good) and s_2 fuzzy outputs denoted by $\tilde{y}^b_{tj}$ $(t = 1, \ldots, s_2)$ are undesirable (bad) such that $s = s_1 + s_2$. The proposed DEA model for calculating the efficiency of $\mathrm{DMU_p}$ is as follows:

$$
\begin{aligned}
& R^* = \min \frac{\frac{1}{m+s_2}\left(\sum_{i=1}^{m}\theta_i + \sum_{t=1}^{s_2}\gamma_t\right)}{\frac{1}{s_1}\sum_{r=1}^{s_1}\varphi_r} \\
& s.t. \\
& \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j \tilde{x}_{ij} \leqslant \theta_i \tilde{x}_{ip}; \quad i = 1, \ldots, m, \\
& \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j \tilde{y}^b_{tj} \leqslant \gamma_t \tilde{y}^b_{tp}; \quad t = 1, \ldots, s_2, \ \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j \tilde{y}^g_{rj} \geqslant \varphi_r \tilde{y}^g_{rp}; \quad r = 1, \ldots, s_1, \\
& \theta_i - 1 \leqslant M\delta_1; \quad i = 1, \ldots, m, -\theta_i + 1 \leqslant M(1-\delta_1); \quad i = 1, \ldots, m, \\
& \gamma_t - 1 \leqslant M\delta_1; \quad t = 1, \ldots, s_2, -\gamma_t + 1 \leqslant M(1-\delta_1); \quad t = 1, \ldots, s_2, \\
& -\varphi_r + 1 \leqslant M\delta_2; \quad r = 1, \ldots, s_1, \varphi_r - 1 \leqslant M(1-\delta_2); \quad r = 1, \ldots, s_1, \\
& \delta_1 + \delta_1 = 1; \quad \delta_1, \delta_2 \in \{0,1\}, \\
& \theta_i, \lambda_j \geqslant 0; \quad \forall i, j.
\end{aligned} \tag{5.1}
$$

This model is a fuzzy version of model (5.1) into which the fuzzy numbers are incorporated. This fuzzy integrated DEA model cannot be solved like a crisp model. It is needed to design a procedure to solve it.

5.2. Solving procedure

As it was mentioned before the proposed fuzzy DEA model cannot be solved like a crisp model. Thus, in order to solve it one can apply a possibility approach formulated in terms of fuzzy set theory proposed by Zadeh [103]. This procedure converts the fuzzy integrated DEA model to the standard linear programming (LP) by α-cut technique. In this case, each fuzzy coefficient can be viewed as a fuzzy variable and each constraint can be considered as a fuzzy event, see Azadi *et al.* [7]. Using possibility theory, possibilities of fuzzy events (*i.e.*, fuzzy constraints) can be determined. Regarding the proposed model and the concept of possibility space of the fuzzy event, some constraints are defined as crisp values and others are considered uncertain. Therefore, by introducing the predetermined acceptable levels of possibility ε_1, ε_2 and ε_3 for constraints, the proposed model is converted as follows:

$$\begin{aligned}
&\min \frac{\frac{1}{m+s_2}\left(\sum_{i=1}^{m}\theta_i+\sum_{t=1}^{s_2}\gamma_t\right)}{\frac{1}{s_1}\sum_{r=1}^{s_1}\varphi_r}\\
&s.t.\\
&\pi\left(\sum_{\substack{j=1\\ j\neq p}}^{n}\lambda_j\tilde{x}_{ij}-\theta_i\tilde{x}_{ip}\leqslant 0\right)\geqslant\varepsilon_1;\quad i=1,\ldots,m,\\
&\pi\left(\sum_{\substack{j=1\\ j\neq p}}^{n}\lambda_j\tilde{y}^b_{tj}-\gamma_t\tilde{y}^b_{tp}\leqslant 0\right)\geqslant\varepsilon_2;\quad t=1,\ldots,s_2,\pi\left(\sum_{\substack{j=1\\ j\neq p}}^{n}\lambda_j\tilde{y}^g_{rj}-\varphi_r\tilde{y}^g_{rp}\geqslant 0\right)\geqslant\varepsilon_3;\quad r=1,\ldots,s_1,\\
&\theta_i-1\leqslant M\delta_1;\quad i=1,\ldots,m,-\theta_i+1\leqslant M(1-\delta_1);\quad i=1,\ldots,m,\\
&\gamma_t-1\leqslant M\delta_1;\quad =1,\ldots,s_2,-\gamma_t+1\leqslant M(1-\delta_1);\quad t=1,\ldots,s_2,\\
&-\varphi_r+1\leqslant M\delta_2;\quad r=1,\ldots,s_1,\varphi_r-1\leqslant M(1-\delta_2);\quad r=1,\ldots,s_1,\\
&\delta_1+\delta_1=1;\quad \delta_1,\delta_2\in\{0,1\},\\
&\theta_i,\lambda_j\geqslant 0;\quad \forall i,j.
\end{aligned}\tag{5.2}$$

In model (5.2) parametersα_1, α_2 and α_3 are the predefined levels that the related constraints should take in order to attain the possibility level. According to Lemma 1, model (5.2) can be stated as follows:

$$\begin{aligned}
&\min \frac{\frac{1}{m+s_2}\left(\sum_{i=1}^{m}\theta_i+\sum_{t=1}^{s_2}\gamma_t\right)}{\frac{1}{s_1}\sum_{r=1}^{s_1}\varphi_r}\\
&s.t.\\
&\left(\sum_{\substack{j=1\\ j\neq p}}^{n}\lambda_j\tilde{x}_{ij}-\theta_i\tilde{x}_{ip}\right)^L_{\varepsilon_1}\leqslant 0;\quad i=1,\ldots,m,\\
&\left(\sum_{\substack{j=1\\ j\neq p}}^{n}\lambda_j\tilde{y}^b_{tj}-\gamma_t\tilde{y}^b_{tp}\right)^L_{\varepsilon_2}\leqslant 0;\quad t=1,\ldots,s_2,\left(\sum_{\substack{j=1\\ j\neq p}}^{n}\lambda_j\tilde{y}^g_{rj}-\varphi_r\tilde{y}^g_{rp}\right)^U_{\varepsilon_3}\geqslant 0;\quad r=1,\ldots,s_1,\\
&\theta_i-1\leqslant M\delta_1;\quad i=1,\ldots,m,-\theta_i+1\leqslant M(1-\delta_1);\quad i=1,\ldots,m,\\
&\gamma_t-1\leqslant M\delta_1;\quad t=1,\ldots,s_2,-\gamma_t+1\leqslant M(1-\delta_1);\quad t=1,\ldots,s_2,\\
&-\varphi_r+1\leqslant M\delta_2;\quad r=1,\ldots,s_1,\varphi_r-1\leqslant M(1-\delta_2);\quad r=1,\ldots,s_1,\\
&\delta_1+\delta_1=1;\quad \delta_1,\delta_2\in\{0,1\},\\
&\theta_i,\lambda_j\geqslant 0;\quad \forall i,j.
\end{aligned}\tag{5.3}$$

Consider in the proposed model, each fuzzy number is considered as a triangular fuzzy number. So, let $\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^U)$ is a triangular fuzzy number of the ith input of $\mathrm{DMU_j}\tilde{y}_{tj}^b = (y_{tj}^{bL}, y_{tj}^{bM}, y_{tj}^{bU})$ and $\tilde{y}_{rj}^g = (y_{rj}^{gL}, y_{rj}^{gM}, y_{rj}^{gU})$ are the triangular fuzzy numbers of the tth undesirable output and rth desirable output of $\mathrm{DMU_j}$. Also, without loss of generality, let us assume that$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \alpha$. By these transformations our model for evaluating $\mathrm{DMU_p}$ and measuring its super efficiency becomes as follows:

$$
\begin{aligned}
& R_p^\alpha = \min \frac{\frac{1}{m+s_2}\left(\sum_{i=1}^m \theta_i + \sum_{t=1}^{s_2} \gamma_t\right)}{\frac{1}{s_1}\sum_{r=1}^{s_1} \varphi_r} \\
& s.t. \\
& \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j(x_{ij}^L + \alpha(x_{ij}^M - x_{ij}^L)) - \theta_i(x_{ip}^L + \alpha(x_{ip}^M - x_{ip}^L)) \leqslant 0; \quad i = 1,\ldots,m, \\
& \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j(y_{tj}^{bL} + \alpha(y_{tj}^{bM} - y_{tj}^{bL})) - \gamma_t(y_{tp}^{bL} + \alpha(y_{tp}^{bM} - y_{tp}^{bL})) \leqslant 0; \quad t = 1,\ldots,s_2, \\
& \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j(y_{rj}^{gU} - \alpha(y_{rj}^{gU} - y_{rj}^{gM})) - \varphi_r(y_{rp}^{gU} - \alpha(y_{rp}^{gU} - y_{rp}^{gM})) \geqslant 0; \quad r = 1,\ldots,s_1, \\
& \theta_i - 1 \leqslant M\delta_1; \quad i = 1,\ldots,m, -\theta_i + 1 \leqslant M(1-\delta_1); \quad i = 1,\ldots,m, \\
& \gamma_t - 1 \leqslant M\delta_1; \quad t = 1,\ldots,s_2, -\gamma_t + 1 \leqslant M(1-\delta_1); \quad t = 1,\ldots,s_2, \\
& -\varphi_r + 1 \leqslant M\delta_2; \quad r = 1,\ldots,s_1, \varphi_r - 1 \leqslant M(1-\delta_2); \quad r = 1,\ldots,s_1, \\
& \delta_1 + \delta_1 = 1; \quad \delta_1, \delta_2 \in \{0,1\}, \\
& \theta_i, \lambda_j \geqslant 0; \quad \forall i, j.
\end{aligned} \tag{5.4}
$$

For each value of $\alpha \in [0,1]$ model (5.4) calculates a super efficiency score for DMUp. This value is called α-super efficiency. For the purpose of integrating the obtained scores and ranking DMUs, the following criterion ψ_p is proposed for each DMUp and is called stochastic closeness coefficient. This criterion is inspired by the closeness coefficient of TOPSIS method. Assume that *n+1* different value for $\alpha \in [0,1]$ as $\{\alpha_0, \alpha_1, \ldots, \alpha_n\}$are applied to obtain the α-super efficiencies. The selected values of α by $\varDelta$ *i.e.* $\varDelta = \{\alpha_0, \alpha_1, \ldots, \alpha_n\}$. This criterion are denoted as follows:

$$
\psi_p = \frac{\left(\frac{\sum_{\alpha\in\varDelta}^{R_p^\alpha}}{n+1} - \min_{\alpha,j}\left\{R_j^\alpha\right\}\right)}{\left(\frac{\sum_{\alpha\in\varDelta}^{R_p^\alpha}}{n+1} - \min_{\alpha,j}\left\{R_j^\alpha\right\}\right) + \left(\max_{\alpha,j}\left\{R_j^\alpha\right\} - \frac{\sum_{\alpha\in\varDelta}^{R_p^\alpha}}{n+1}\right)}
$$

In fact, $\min_{\alpha,j}\left\{R_j^\alpha\right\}$ is the worst result of α-super efficiencies among all DMUs and under all considered values of α, thus, it is a kind of negative ideal value. On the other hand, the best result of α-super efficiencies among all DMUs is $\max_{\alpha,j}\left\{R_j^\alpha\right\}$, hence, it is a kind of positive ideal value. The idea behind the criterion ψ_p is that if the average of obtained values for DMUp has the shortest distance from the positive ideal value and the farthest distance from the negative ideal value, then DMUp should have the best ranking situation. The stochastic

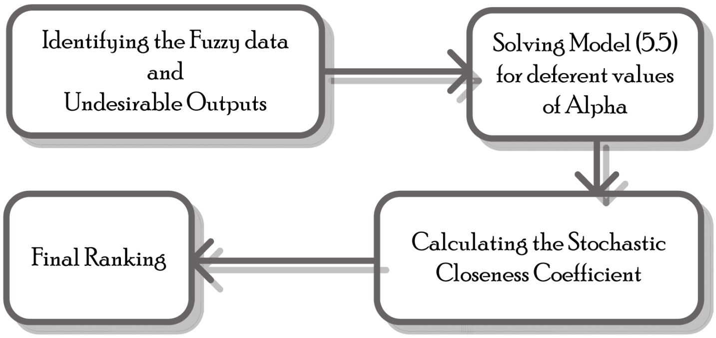


FIGURE 4. The flowchart of our proposed fuzzy DEA model.

closeness coefficient can simply be converted into the following relation:

$$\psi_p = \frac{\frac{\sum_{\alpha \in \Delta}^{R_p^\alpha}}{n+1} - \min_{\alpha,j}\left\{R_j^\alpha\right\}}{\max_{\alpha,j}\left\{R_j^\alpha\right\} - \min_{\alpha,j}\left\{R_j^\alpha\right\}} \tag{5.5}$$

Clearly, for each p we have $0 \leqslant \psi_p \leqslant 1$. And thus DMUs can be ranked according to decreasing order of the stochastic closeness coefficient. Figure 4 illustrates the flowchart of our proposed fuzzy DEA model.

6. An application in barley production farms

In this study, data were obtained from barley production farms in 22 provinces of Iran (Tab. 4). The dataset[3] dates back to 2012. Four energy inputs were considered in the study consisting of human labor, barley seed, machinery (including tractor and implements usage and diesel consumption) and fertilizers (including nitrogen, phosphate and potash fertilizers). To obtain the energy inputs, the input resources were transformed to their equivalent energy forms. There are different transforming coefficients proposed by experts to convert of *Labor*, *Machinery* and *Fertilizers* to their energy equivalents [46, 76]. Thus, experts' ideas were aggregated and a fuzzy number for each of these energy inputs was made. This study considered them as triangular fuzzy numbers except for *seed energy* which was stated as a crisp number.

The model outputs consisted of total production value of barley as desirable output and greenhouse gas emission as undesirable output. Emission of each DMU was calculated by multiplying each input with its corresponding emission factor. There is different emission coefficients proposed by experts to calculate emission of each input [46, 55]. Thus, experts' ideas were aggregated and a triangular fuzzy number for emission was made.

In order to solve model (5.4) with these data each crisp data is regarded as a triangular fuzzy number. The results of solving model (5.4) can be seen in Table 6. Model (5.4) was executed for some values of α *i.e.* $\alpha \in \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. In this section, the efficiency for each DMU was obtained by executing a GAMS program of model (5.4) at different levels of α for the year 2012. See the appendix for the GAMS code. For better comparison, these results are provided in Figure 5. This figure shows the super efficiency of barley production farms at the different levels of α in an integrated form. According to the fore mentioned table and figure, the barley production farms in provinces of Chaharmahal–Bakhtiari, Kermanshah, Esfahan, North Khorasan and Markazi have been recognized effective at different levels of α (They have maximum efficiency in different α).

Figure 5 shows the efficiency results at $\alpha \in \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. Table 6 and Figure 6 reveal that, five DMUs *i.e.* $\{4, 7, 9, 18, 21\}$ are overall efficient for every

[3]Annual agricultural statistics. Ministry of agriculture of Iran [in Persian], www.maj.ir 2012. [accessed 01.1016].

Table 4. Data Set.

DMU	Province	Inputs				Output	
		Labor	Machinery	Seed	Fertilizers	TPV	Emission
1	East Azarbayjan	(79.30, 90.63, 101.96)	(2502.15, 2812.46, 3122.78)	1675.7	(1172.85, 1329.17, 1373.64)	3792380	(197.53, 220.86, 238.57)
2	West Azarbayjan	(58.17, 66.48, 74.79)	(1839.90, 2076.29, 2312.69)	1839.5	(1165.25, 1316.35, 1359.35)	4714360	(163.85, 183.29, 197.29)
3	Ardebil	(176.44, 201.64, 226.85)	(3466.93, 3875.02, 4283.11)	1566.5	(1359.26, 1537.58, 1595.46)	5996460	(266.39, 295.76, 319.06)
4	Esfahan	(108.94, 124.50, 140.06)	(940.87, 1062.86, 1184.86)	1118.0	(2635.01, 2989.99, 3079.28)	5794870	(187.13, 211.39, 222.30)
5	Ilam	(58.31, 66.64, 74.97)	(1831.28, 2067.74, 2304.19)	1879.8	(1224.81, 1384.54, 1439.04)	1716830	(166.39, 186.40, 201.11)
6	Boushehr	(38.57, 43.42, 48.75)	(2527.65, 2770.57, 3013.48)	1296.1	(470.87, 533.32, 551.69)	1043690	(172.75, 187.91, 200.86)
7	Chaharmahal-Bakhtiari	(344.37, 393.57, 442.76)	(1796.16, 2020.36, 2244.56)	1857.7	(1486.14, 1686.90, 1736.99)	6123940	(176.70, 198.02, 211.78)
8	South Khorasan	(113.19, 129.36, 145.53)	(1188.95, 1347.05, 1505.15)	2332.2	(563.69, 639.83, 658.42)	828730	(92.81, 104.47, 113.25)
9	North Khorasan	(100.98, 115.40, 129.83)	(3092.92, 3444.46, 3796.01)	1756.3	(1045.21, 1186.45, 1227.98)	8390020	(231.60, 256.19, 275.76)
10	Khouzestan	(104.13, 119.01, 133.89)	(2085.56, 2333.07, 2580.58)	1457.3	(701.85, 797.04, 819.25)	2127200	(152.50, 169.52, 182.87)
11	Zanjan	(89.32, 102.08, 114.84)	(3097.13, 3451.20, 3805.28)	1296.1	(1808.98, 2050.45, 2121.28)	4539620	(270.27, 300.20, 321.54)
12	Semnan	(174.11, 198.98, 223.85)	(3461.66, 3853.72, 4245.78)	2476.5	(1759.62, 1991.23, 2053.28)	3492240	(289.53, 320.73, 343.41)
13	Qazvin	(181.52, 207.45, 233.38)	(2286.08, 2558.25, 2830.41)	1129.7	(1363.16, 1541.99, 1600.05)	6733160	(201.08, 223.81, 240.45)
14	Golestan	(63.57, 68.08, 74.59)	(4104.77, 4580.85, 5056.93)	2635.1	(3746.87, 4258.01, 4369.26)	10115770	(437.27, 486.70, 516.07)
15	Gilan	(119.50, 136.57, 153.64)	(1898.41, 2143.01, 2387.61)	1626.3	(2302.47, 2607.25, 2687.60)	5589960	(221.05, 248.78, 265.24)
16	Lorestan	(125.40, 143.32, 161.23)	(2014.17, 2274.22, 2534.26)	1952.6	(3231.17, 3658.97, 3786.10)	4499220	(295.12, 330.14, 350.04)
17	Mazandaran	(111.96, 127.95, 143.94)	(3212.67, 3574.93, 3937.19)	1974.7	(3139.41, 3558.32, 3674.70)	4802280	(362.29, 401.77, 426.03)
18	Markazi	(107.70, 123.09, 138.47)	(2969.41, 3326.04, 3682.67)	1303.9	(1602.53, 1800.59, 1898.83)	8709110	(257.67, 286.01, 309.25)
19	Hamedan	(41.71, 47.67, 53.63)	(3334.84, 3733.60, 4132.36)	1878.5	(2771.43, 3135.76, 3241.24)	6254150	(337.86, 376.35, 401.83)
20	Kordestan	(188.92, 215.91, 242.90)	(2435.55, 2717.55, 2999.54)	1727.7	(1409.85, 1582.64, 1639.71)	6385530	(214.15, 237.06, 254.13)
21	Kermanshah	(31.28, 35.75, 40.22)	(3332.58, 3738.01, 4143.44)	2042.3	(2398.34, 2718.58, 2806.89)	8337240	(324.23, 360.70, 385.51)
22	Kohkilouyeh–boyerahmad	(143.92, 164.48, 185.04)	(1510.89, 1720.18, 1929.46)	1617.2	(3208.54, 3631.02, 3755.41)	5860610	(254.48, 286.74, 303.99)

Table 5. The average amount of input and output data.

Data		Unit	Data average
Inputs	labo	MJ	(111.94, 127.93, 143.93)
	Machiner	MJ	(2496.84, 2794.61, 3092.38
	Seed	MJ	1747.2
	Fertilizer	MJ	(1843.97, 2088, 2157.97
Output	TP	Rial	526579
	Emission	Kg C_{O2}	(239.66, 266.95, 285.47)

Table 6. The efficiency of DMUs with respect to different value of α.

DMUs	$\alpha = 0.0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1.0$
1	0.5082	0.5079	0.5077	0.5075	0.5073	0.5071	0.5069	0.5067	0.5065	0.5063	0.5061
2	0.7509	0.7505	0.7501	0.7497	0.7493	0.7489	0.7486	0.7482	0.7479	0.7475	0.7472
3	0.6038	0.6037	0.6037	0.6036	0.6036	0.6035	0.6034	0.6034	0.6033	0.6033	0.6032
4	1.1941	1.1936	1.1931	1.1926	1.1922	1.1917	1.1913	1.1909	1.1905	1.1901	1.1897
5	0.2701	0.27	0.2698	0.2696	0.2695	0.2693	0.2692	0.2690	0.2689	0.2687	0.2686
6	0.2179	0.2181	0.2183	0.2186	0.2188	0.21	0.2192	0.2194	0.2196	0.2198	0.2200
7	1.00	1.0038	1.0035	1.0032	1.00	1.0028	1.0025	1.0023	1.0022	1.0020	1.0019
8	0.1698	0.1696	0.1694	0.1693	0.1691	0.1689	0.1687	0.1686	0.1684	0.1682	0.1681
9	1.1153	1.11	1.1148	1.1145	1.1143	1.1141	1.1138	1.1136	1.1134	1.1132	1.1130
10	0.33	0.3379	0.3378	0.3378	0.3377	0.3376	0.3375	0.3375	0.3374	0.3373	0.3373
11	0.5223	0.5222	0.5222	0.5222	0.5221	0.5221	0.5221	0.5220	0.5220	0.5220	0.5219
12	0.2977	0.2977	0.2978	0.2978	0.2978	0.2978	0.2978	0.2978	0.2978	0.2978	0.2978
13	0.85	0.8508	0.8506	0.8505	0.8503	0.8501	0.8500	0.8498	0.8497	0.8495	0.8494
14	0.8398	0.8407	0.8416	0.8425	0.8433	0.8442	0.8450	0.8459	0.8467	0.8475	0.8483
15	0.6584	0.6579	0.6574	0.6570	0.6565	0.6561	0.6557	0.6553	0.6549	0.6545	0.6541
16	0.4515	0.4513	0.4510	0.4508	0.4505	0.4503	0.4501	0.4498	0.4496	0.4494	0.4492
17	0.4156	0.4156	0.4156	0.4157	0.4157	0.4157	0.4157	0.4158	0.4158	0.4158	0.4158
18	1.0806	1.0805	1.0804	1.0804	1.0803	1.0803	1.0803	1.0802	1.0802	1.0801	1.0801
19	0.6994	0.6994	0.6994	0.6995	0.6995	0.6995	0.6996	0.6996	0.6996	0.6997	0.6997
20	0.7069	0.70	0.7070	0.7071	0.7072	0.7073	0.7074	0.7074	0.7075	0.7076	0.7076
21	1.2297	1.2277	1.2257	1.2235	1.2213	1.2191	1.2170	1.2150	1.2130	1.2111	1.2092
22	0.7035	0.70	0.7025	0.7019	0.7014	0.7009	0.7004	0.7000	0.6995	0.6990	0.6986

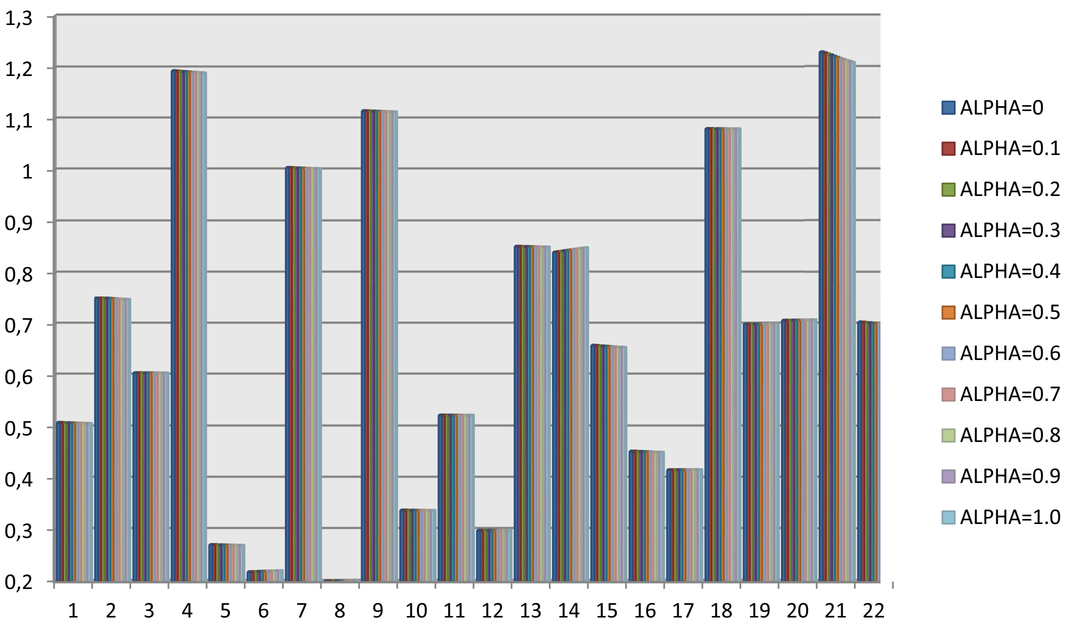


Figure 5. Column Chart of super efficiency results of DMUs at different $\propto \in (0, 1]$.

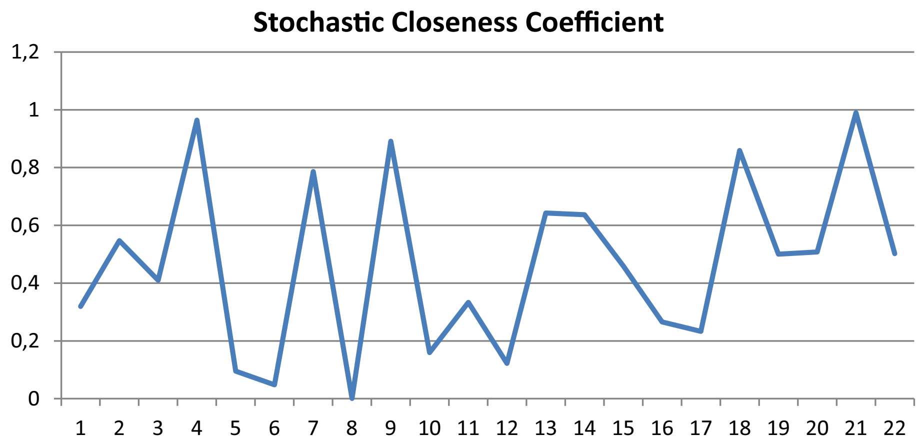


FIGURE 6. The results of stochastic Closeness Coefficients for different DMUs .

TABLE 7. Final Results and Ranking.

DMUs	Province	Stochastic Closeness Coefficient	Rank
1	East Azarbayjan	0.3193	15
2	West Azarbayjan	0.5471	8
3	Ardebil	0.4101	13
4	Esfahan	0.9642	2
5	Ilam	0.0954	20
6	Boushehr	0.0479	21
7	Chaharmahal-Bakhtiari	0.7863	5
8	South Khorasan	0.0008	22
9	North Khorasan	0.8911	3
10	Khouzestan	0.1597	18
11	Zanjan	0.3334	14
12	Semnan	0.1222	19
13	Qazvin	0.6425	6
14	Golestan	0.6368	7
15	Gilan	0.4597	12
16	Lorestan	0.2658	16
17	Mazandaran	0.2332	17
18	Markazi	0.8592	4
19	Hamedan	0.5006	11
20	Kordestan	0.5079	9
21	Kermanshah	0.990	1
22	Kohkilouyeh–boyerahmad	0.5019	10

$\alpha \in \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. Figure 6 reveals that all of these five DMUs are efficient for every α.

The values of the stochastic closeness coefficient and the final ranking of all DMUs are shown in the last two columns of Table 7. From this table, we can see the DMU #21 (Kermanshah Province) and DMU #8 (South Khorasan) have the best and the worst performances among all DMUs, respectively. A slightly deeper observation of studied farms' data shows that "Kermanshah Province" consumed the least value of "Labor" and produced a high value (third position) of "TPV". Furthermore, this farm performed well in other criteria. This is the reason behind recognizing "Kermanshah Province" as the best barley farm.

From the results of Table 7, we can see the DMUs 6 and 8, *i.e.* Boushehr and South Khorasan,are the two worst DMUs and have weak performances. One of the main reasons behind the results is that these DMUs produced the two worst amounts of the desirable output. This leads to obtain big values for φ and low values for θ, and therefore, these DMUs have small values of efficiency. The graphical representations of the stochastic closeness coefficient results using fuzzy input–output data are shown in Figure 6.

Figure 6 shows that DMU #21 (Kermanshah Province) and DMU #8 (South Khorasan) have the best and worst performances, respectively.

7. Conclusion

DEA is used to measure the relative efficiency of decision making units. However, some production processes may generate undesirable outputs like smoke pollution or waste. Hence, a new non-radial DEA model was presented based on a modification of ERM model in the presence of undesirable output. This study evaluated barley production farms in 22 provinces of Iran. Four energy inputs, namely human labor, barley seed, machinery and fertilizers energies were considered in the study. The output consisted of the total production value of barley as desirable output and greenhouse gas emission as undesirable output.

On the other hand, in many situations, such as in a manufacturing system, a production process or a service system, inputs and outputs can be given as fuzzy variables. Since the quantity of some of our data in this paper was not exactly known the data was stated as fuzzy data. Triangular fuzzy data was used to state the complexity of inputs and outputs. Fuzzy inputs and outputs included labor, machinery, Fertilizers and greenhouse gas emission.

A method for solving the proposed fuzzy DEA model was presented based on the concept of alpha cut and possibility approach. Also for the purpose of final ranking a stochastic closeness coefficient was offered. The performances of 22 barley production farms were measured by applying the proposed model and considering some different values of α. The obtained values were integrated by using the proposed stochastic closeness coefficient. Based on the results of the proposed stochastic closeness coefficient the best and the worst performances were determined. Table 6 and Figure 5 revealed that, barley farms in five provinces , *i.e.* {Esfaha, Chaharmahal-Bakhtiar, North Khorasan, Markaz, Kermanshah} were overall efficient for every α.

Furthermore, all provinces were ranked based on their performance in barley production. The barley production of Kermanshah province has the best performance. A more profound observation of farms' data showed that "Kermanshah Province" consumed the least value of "Labor" and produced a high value (third position) of "TPV". Besides this farm performed well in other criteria. This was the reason behind recognizing the "Kermanshah Province" as the best province in barley production. Results also showed that DMUs 6 and 8, *i.e.* Boushehr and South Khorasan,were the two worst DMUs and had weak performances because they produced the two worst amounts of the desirable output. This leads to obtaining large values for φ and small values for θ, and therefore, small efficiency values for these DMUs.

In this paper, a new DEA model was proposed to rank DMUs in the existence of undesirable output and fuzzy environment. It is recommended that the presented approach be used in other DEA models such as two stage DEA models and network DEA models. Also, one can incorporate stochastic data instead of fuzzy data into the suggested model. In this paper, the proposed model was applied for evaluating the performances of barley production farms. It seems that the suggested model can be used in other problems such as evaluating the sustainability of suppliers.

Appendix A

The GAMS code used for assessing the considered DMUs is as follows:

```
$title A Russell Model

$onsymxref
```

```
$onsymlist
$onuellist
$onuelxref
Sets
i "Inputs" /I1 * I5/
O /O1 * O1/
j "Units" /1 * 22/;
Alias(j,l); Alias(j,K);
Table XL(j,i)
$include "C:\Users\caspian \Fuzzy DEA \GAMS\XL.TXT";
Table XM(j,i)
$include "C:\Users\caspian \Fuzzy DEA \GAMS\XM.TXT";
Table XU(j,i)
$include "C:\Users\caspian \Fuzzy DEA \GAMS\XU.TXT";
Table YL(j,O)
$include "C:\Users\caspian \Fuzzy DEA \GAMS\YL.TXT";
Table YM(j,O)
$include "C:\Users\caspian \Fuzzy DEA \GAMS\YM.TXT";
Table YU(j,O)
$include "C:\Users\caspian \Fuzzy DEA \GAMS\YU.TXT";
Variables
Z, Teta(i), Phi(O), Lambda(j);
Positive Variable
Lambda;
BINARY VARIABLE
D;
Phi.lo(O)=0.02;
Parameters
ALPHA, JK(J), XXL(i), XXM(i), XXU(i), YYL(O), YYM(O), YYU(O), m, s;
m=Card(i);
s=Card(O);
FILE RESULT /C:\Users\caspian \Fuzzy DEA \GAMS\RESULT-FUZZYRussell-3.txt/ ;
Equations
```

```
Objective
Const1(i), Const2(O), Const3(O), Const4(i), Const5(O), Const6(i), Const7(i);
Objective... z=e=Sum(i,Teta(i)/m)/Sum(O,Phi(O)/s);
Const1(i)... Sum(j $JK(J),(XL(j,i)+ALPHA*(XM(J,I)-XL(J,I)))*Lambda(j)) -
Teta(i)*(XXL(I)+ALPHA*(XXM(I)-XXL(I))) =L= 0;
Const2(O)... Sum(j $JK(J),(YU(j,O)-ALPHA*(YU(J,O)-YM(J,O)))*Lambda(j)) =G= Phi(O)*(YYU(O)-
ALPHA*(YYU(O)-YYM(O)));
const3(O)...-Phi(O) + 1 =L= 10000 * D;
const4(i)... Teta(i) - 1 =l= 10000 * D;
const5(O)... Phi(O) - 1 =L= 10000 * (1-D);
const6(i)...-Teta(i) + 1 =l= 10000 * (1-D);
const7(i)... Teta(i)=g=0;
Model Russell_Model /All/;
Put Result;
Put #2 @12'z'/;
ALPHA=0;
WHILE(ALPHA<1.05,
PUT 'ALPHA= 'ALPHA; Put/;
Loop(l,
Loop(i,XXL(i)=xL(l,i));
Loop(i,XXM(i)=xM(l,i));
Loop(i,XXU(i)=xU(l,i));
Loop(O,YYL(O)=yL(l,O));
Loop(O,YYM(O)=yM(l,O));
Loop(O,YYU(O)=yU(l,O));
LOOP(K,JK(K)=1);
JK(L)=0;
Solve Russell_Model Using MINLP Minimizing z;
Put l.tl:6;
Put z.l:7:4;
Put/;
Display z.l;
);
ALPHA=ALPHA+0.1;
);
```

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