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DYNAMIC FUZZY DATA ENVELOPMENT ANALYSIS MODELS: CASE OF BUS TRANSPORT PERFORMANCE ASSESSMENT

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Abstract. In the transport field, two characteristics–inter-temporal dependency and fuzziness–need to be considered when assessing transport performance. First, input and output levels are inter-temporal dependent due to heavy capital investment and because quasi-fixed input can influence output levels over multiple periods. Second, conventional Data Envelopment Analysis (DEA) models are, in nature, formulated with quantitative variables. However, qualitative measurements that are characterized with "vagueness" or "fuzziness" are as important as quantitative variables for multi-period transport performance assessment. To rectify these problems, the present study extends previous research by proposing a Dynamic Fuzzy Data Envelopment Analysis (DFDEA) method for assessing the comparative efficiency where inter-temporal dependence exists in operating production processes with some "fuzzy" variables. An case study was conducted to evaluate the performance of city bus transport companies in Taipei, Taiwan. Results showed the superiority of the proposed DFDEA model by comparing the results with static models.

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1. Introduction

Data Envelopment Analysis (DEA) is a well–known non–parametric technique that employs the linear programming method to determine the relative efficiencies of a set of homogeneous and comparable decision making units (DMUs). DMUs are generally regarded as the evaluated units (*e.g.*, companies, organizations, departments) that perform the same or similar tasks in a production or service system within an industry or across different industries. The DEA technique has three distinct advantages: (1) it can measure the relative efficiency of DMUs that produce similar products with multiple inputs and outputs; (2) it does not impose functional relationships between input and output variables, or assumptions on the statistical distribution of error terms [1]; and (3) it can identify efficient performers and provide actionable measures for rectifying the inefficient counterparts [2,3]. Therefore, a variety of DEA models have been developed for performance evaluation, ranking, and benchmarking among DMUs in different fields [4].

Keywords. Fuzzy data envelopment analysis, dynamic data envelopment analysis, bus transport, dynamic efficiency, membership function.

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In the transport field, numerous studies have employed DEA models to evaluate the efficiency and/or effectiveness of airline services (*e.g.* [5–9]), air-express couriers (*e.g.* [10–18]), maritime services (*e.g.* [19, 20]), bus services (*e.g.* [21–34]), and rail transport services (*e.g.* [35–38]). Most of these studies have focused on measuring efficiency using only quantitative variables for a single year period. However comparative performance assessments for passenger or freight sector in different transport modes require both quantitative measures (*e.g.* bus number, operating revenue, service frequency) and qualitative measures (*e.g.* passenger satisfaction, driver quality, crew member attitude). Qualitative measures have largely been overlooked by previous studies that adopted Conventional Data Envelopment Analysis (CDEA) models. CDEA models are in nature, formulated with quantitative variables [34]. Selected studies in the transport field are reviewed in the Appendix.

In order to consider the fuzziness of variables, Fuzzy Data Envelopment Analysis (FDEA) models have been proposed over the past decade (*e.g.* [34, 39–41]). Hatami-Marbini *et al.*, [42] and Emrouznejad and Tavana [43] reviewed the FDEA method and presented a classification scheme with four primary categories, including the tolerance approach, the α-level approach, the fuzzy ranking approach, and the possibility approach. The α-level approach has been adopted by most of the previous studies that used two CDEA models which were used to evaluate the efficiency score of the lower-limit and the upper-limit separately with a specific α-level. For example, Kao and Liu's [47] modeling approach transformed a FDEA model to a family of CDEA models by applying the αlevel approach. However, the distorted fuzzy number for this type of FDEA can result in inconsistent efficiency frontier ranking or unreasonable efficiency score [34]. Further, results generated by the CDEA modeling approach would require another method to rank fuzzy sets and different fuzzy ranking methods may lead to different evaluation results [50]. To avoid the need to use another method to rank fuzzy sets that might result inconsistent results, Lan *et al.* [34] developed an integrated Fuzzy Data Envelopment Analysis Model that can generate a crisp efficiency score for each DMU by combining both lower- and upper-bound efficiency frontiers into a single model under a specific αlevel.

In this study, the α-level approach was adopted because it will allow operators and policymakers to choose the α-level to manage the level of fuzziness. For example, in public transport, customer satisfaction is one of the most important qualitative variables for transport operators to consider when evaluating performance. From the perspective of customers (*i.e.* passengers), a higher level of service (*e.g.* high service frequency) would be important for saving time on travel, which will lead to higher satisfaction. However, a higher level of service might lead to higher costs. From the perspective of operators or policymakers, customer satisfaction is not the only index taken into account when deciding on the level of service. That is, other indexes such as labor costs (*e.g.* hiring more drivers) or vehicle costs (*e.g.* purchasing more vehicles) need to be taken into account. The level of fuzziness of qualitative variables (*e.g.* customers satisfaction) would be an important control variable for operators and policymakers. Therefore, the α-level approach was used in this study.

The CDEA model is able to correctly measure efficiency if there are no quasi–fixed inputs and/or inter–temporal dependence between input and output variables in the production process. However, if the DMU cannot adjust the quasi–fixed input variables instantaneously to the optimal level in the CDEA model, the performance evaluation results may be biased [51]. Therefore, a dynamic framework for the CDEA model has been proposed to address this problem [51–54]–hereinafter termed as Dynamic Data Envelopment Analysis (DDEA). Emrouznejad and Thanassoulis [52] classified two types of multi–period production processes for DDEA: (1) multi–period without inter–temporal input–output dependence and (2) multi–period with inter–temporal input–output dependence.

The Malmquist index was developed by Färe *et al.* [55, 56], and is used to measure productivity change over time without inter-temporal input–output dependence [53]. It is also used to decompose the total productivity change between the two periods into technical change and efficiency change [52]. In the case of multi–period production processes with inter-temporal input–output dependence capital stock and lagged output should be considered in the production process. In order to capture the characteristics of inter–temporal input-output dependence, Emrouznejad and Thanassoulis [53] used an input-output "path" mapped out by operating units.

Based on previous research, this study used the input-output "path" to account for the instantaneous resource allocation within CDEA That is, the quasi-fixed input variables at the end of the period were treated as output variables in that period in the DDEA models The next question is then how the "fuzziness" of variables can be considered without generating inconsistent results. To answer this question, this study proposed a novel approach that can incorporate both lower- and upper–bound into a FDEA model under a DDEA framework. The proposed approach aimed to advance FDEA and DDEA models to evaluate the holistic performance of transport services. Specifically, the proposed approach used a Dynamic Fuzzy Data Envelopment Analysis (DFDEA) method, which integrated lower- and upper–bound efficiency into the objective functions and constraints to assess the comparative efficiency where inter–temporal dependence exists. This study used bus companies in Taipei, Taiwan as an example to demonstrate the applicability and superiority of the proposed DFDEA model. However, the proposed modeling approach can be applied to wider production systems with similar characteristics.

In this paper, FDEA and DDEA were extended to be a DFDEA model. A DDEA model for crisp data and a FDEA model will be reviewed in Section 2. The proposed DFDEA model will be derived in Section 3. The case study of Taipei city bus companies will be presented in Section 4 with policy implications. Finally, conclusions and directions for future studies will be discussed.

2. Preliminary models

2.1. DDEA model in crisp DEA

Consider n DMUs to be evaluated; each DMU utilizes m inputs to produce s outputs within the period t, $t=\tau,\dots,\tau+T$. m inputs can be divided into two sub-sets of periodspecific inputs and capital inputs, I_1 and I_2 respectively. The DDEA model for measuring the efficiency of DMU proposed by Emrouznejad and Thanassoulis [53] is as follows:

Model [DDEA]

$$\text{Min } \alpha=\frac{\sum_{t=\tau}^{\tau+T}\theta^t}{T}-\varepsilon\left(\sum_{t=\tau}^{\tau+T}\sum_{i\in I_1}S_i^{t-}+\sum_{t=\tau}^{\tau+T}\sum_{i\in I_2}\delta_i^{t-}+\sum_{t=\tau}^{\tau+T}\sum_{r=1}^{s}S_r^{t+}+\sum_{i\in I_2}\gamma_i^-+\sum_{i\in I_2}\gamma_i^+\right)$$

$$\text{s.t. D1}: \quad \sum_{j=1}^{n}\lambda_j x_{ij}^t=\theta^t x_{ik}^t-S_i^{t-}; i\in I_1, t=\tau,\dots,\tau+T$$

$$\text{D2}: \sum_{j=1}^{n}\lambda_j z_{ij}^t=\theta^t z_{ik}^t-\delta_i^{t-}; i\in I_2, t=\tau,\dots,\tau+T$$

$$\text{D3}: \sum_{j=1}^{n}\lambda_j y_{rj}^t=y_{ik}^t-S_r^{t+}; r=1,\dots s, t=\tau,\dots,\tau+T$$

$$\text{D4}: \sum_{j=1}^{n}\lambda_j Z_{ij}^{\tau+T}=Z_{ik}^{\tau+T}-\gamma_i^+; i\in I_2$$

$$\text{D5}: \sum_{j=1}^{n}\lambda_j Z_{ij}^{\tau-1}$$

$$\lambda_j\geqslant 0;\forall j, S_i^{t-}\geqslant 0;\delta_i^{t-}\geqslant 0(\forall t,\forall i\in I_1); S_r^{t+}\geqslant 0(\forall t,\forall i\in I_1);\gamma_i^+\geqslant 0,\gamma_i^-\geqslant 0(\forall i\in I_2),$$

where $I_1\subset\{1,\dots,m\}$ are inputs that do not have inter-temporal dependence with outputs, $I_2\subset\{1,\dots,m\}$ are inputs where the end amount will be converted, directly or indirectly, into more outputs in some future period. $Z_{ij}^{\tau-1}$ is the initial amount of type$i$ input for DMU$_j$. $Z_{ij}^{\tau+T}$ is the end amount of type$i$ input for DMU$_j$.

2.2. FDEA model

We also consider n DMUs to be evaluated; each DMU utilizes m inputs to produce s outputs. FDEA model for measuring the efficiency of DMU_k was proposed by Lan *et al.* [34]. The DDEA model is as follows:

Model [FDEA]

$$\text{Min}_{\theta,\lambda}\, \theta - \varepsilon\left(\sum_{i=1}^{m} s_{i_1}^- + \sum_{i=1}^{m} s_{i_2}^- + \sum_{r=1}^{s} s_{r_1}^+ + \sum_{r=1}^{s} s_{r_2}^+\right)$$

$$\text{s.t. F1}: \theta x_{ik\alpha}^L - \sum_{j=1}^{n} \lambda_j x_{ij\alpha}^L - s_{i_1}^- = 0$$

$$\text{F2}: \theta x_{ik\alpha}^U - \sum_{j=1}^{n} \lambda_j x_{ij\alpha}^U - s_{i_2}^- = 0$$

$$\text{F3}: \sum_{j=1}^{n} \lambda_j y_{rj\alpha}^L - y_{rk\alpha}^L - s_{r_1}^+ = 0$$

$$\text{F4}: \sum_{j=1}^{n} \lambda_j y_{rj\alpha}^U - y_{rk\alpha}^U - s_{r_2}^+ = 0$$

$$\text{F5}: \sum_{j=1}^{n} \lambda_j = 1$$

$$\text{F6}: \theta x_{ik} - \sum_{j=1}^{n} \lambda_j x_{ij} - s_i^- = 0$$

$$\text{F7}: \sum_{j=1}^{n} \lambda_j y_{rj} - y_{rk} - s_r^+ = 0$$

$$\lambda_j, s_{i_1}^-, s_{i_2}^-, s_i^-, s_{r_1}^+, s_{r_2}^+, s_r^+ \geqslant 0; r = 1,\ldots,s; i = 1,\ldots,m; j = 1,\ldots,n$$

$$\theta \text{ unrestricted in sign}$$

where θ represents the efficiency score of DMU k. If θ equals 1, then the DMU is regarded as relatively efficient; otherwise, it is relatively inefficient. λ_j is the influence from DMU j. $s_{i_1}^-, s_{i_2}^-$ are slack variables of the ith input and $s_{r_1}^+, s_{r_2}^+$ are slack variables of the rth output for lower-bound and upper-bound corresponding to a specific α-level, respectively Constraints F1 to F4 were applied for fuzzy data, and constraint F6 and F7 were applied for crisp data (refer to Lan *et al.* [34] for further details about FDEA models).

3. Model formulation

The DDEA model modifies the CDEA model by adding constraint D2, D4 and D5 which are formulated for crisp variables. These constraints are used to capture inter-temporal dependence. However, the variables z_{ij}^t, $Z_{ij}^{\tau-1}$ and $Z_{ij}^{\tau+T}$ could take crisp and/or fuzzy forms. Therefore, constraint D2, D4 and D5 could be extended to fuzzy forms by introducing a lower-bound and an upper-bound under a specific αlevel FDEA model and can be stated as follows:

$$\mathrm{D2^L}: \sum_{j=1}^{n} \lambda_j z_{ij\alpha}^{Lt} = \alpha^t z_{ik\alpha}^{Lt} - \delta_{i_1}^{t-}; \quad i_1 \in I_2, t = \tau, \ldots, \tau + T$$

$$\mathrm{D2^U}: \sum_{j=1}^{n} \lambda_j z_{ij\alpha}^{Ut} = \alpha^t z_{ik\alpha}^{Ut} - \delta_{i_2}^{t-}; \quad i_2 \in I_2, t = \tau, \ldots, \tau + T$$

$$\mathrm{D4^L}: \sum_{j=1}^{n} \lambda_j Z_{ij\alpha}^{L(\tau+T)} = Z_{ik\alpha}^{L(\tau+T)} - \gamma_{i_1}^{+}; \quad i_1 \in I_2$$

$$\mathrm{D4^U}: \sum_{j=1}^{n} \lambda_j Z_{ij\alpha}^{U(\tau+T)} = Z_{ik\alpha}^{U(\tau+T)} - \gamma_{i_2}^{+}; \quad i_2 \in I_2$$

$$\mathrm{D5^L}: \sum_{j=1}^{n} \lambda_j Z_{ij\alpha}^{L(\tau-1)} = Z_{ik\alpha}^{L(\tau-1)} - \gamma_{i_1}^{-}; \quad i \in I_2$$

$$\mathrm{D5^U}: \sum_{j=1}^{n} \lambda_j Z_{ij\alpha}^{U(\tau-1)} = Z_{ik\alpha}^{U(\tau-1)} - \gamma_{i_2}^{-}; \quad i \in I_2$$

Combining equation $\mathrm{D2^L}$, $\mathrm{D2^U}$, $\mathrm{D4^L}$, $\mathrm{D4^U}$, $\mathrm{D5^L}$, $\mathrm{D5^U}$ into the FDEA model with a dynamic framework of DDEA, the DFDEA model can be proposed as follows:

Model [DFDEA]

$$\begin{aligned}
\text{Min } \alpha = {} & \frac{\sum\limits_{t=\tau}^{\tau+T} \theta^t}{T} \\
& - \varepsilon \left(\begin{array}{l} \sum\limits_{t=\tau}^{\tau+T} \sum\limits_{i\in I_1} S_i^{t-} + \sum\limits_{t=\tau}^{\tau+T} \sum\limits_{i\in I_2} \delta_i^{t-} + \sum\limits_{t=\tau}^{\tau+T} \sum\limits_{r=1}^{s} S_r^{t+} + \sum\limits_{i\in I_2} \gamma_i^{-} + \sum\limits_{i\in I_2} \gamma_i^{+} + \sum\limits_{t=\tau}^{\tau+T} \sum\limits_{i\in I_1} s_{i_1}^{t-} + \sum\limits_{t=\tau}^{\tau+T} \sum\limits_{i\in I_1} s_{i_2}^{t-} \\ + \sum\limits_{t=\tau}^{\tau+T} \sum\limits_{i\in I_2} \delta_{i_1}^{t-} + \sum\limits_{t=\tau}^{\tau+T} \sum\limits_{i\in I_2} \delta_{i_2}^{t-} + \sum\limits_{t=\tau}^{\tau+T} \sum\limits_{r=1}^{s} s_{r_1}^{t+} + \sum\limits_{t=\tau}^{\tau+T} \sum\limits_{r=1}^{s} s_{r_2}^{t+} + \sum\limits_{i\in I_2} \gamma_{i_1}^{-} + \sum\limits_{i\in I_2} \gamma_{i_2}^{-} + \sum\limits_{i\in I_2} \gamma_{i_1}^{+} + \sum\limits_{i\in I_2} \gamma_{i_2}^{+} \end{array} \right),
\end{aligned}$$

$$\text{s.t. DF1}: \quad \sum_{j=1}^{n} \lambda_j x_{ij}^t = \theta^t x_{ik}^t - S_i^{t-}; i \in I_1, t = \tau, \ldots, \tau + T,$$

$$\text{DF2}: \sum_{j=1}^{n} \lambda_j z_{ij}^t = \theta^t z_{ik}^t - \delta_i^{t-}; i \in I_2, t = \tau, \ldots, \tau + T,$$

$$\text{DF3}: \sum_{j=1}^{n} \lambda_j y_{rj}^t = y_{ik}^t - S_r^{t+}; r = 1, \ldots s, t = \tau, \ldots, \tau + T,$$

$$\text{DF4}: \sum_{j=1}^{n} \lambda_j Z_{ij}^{\tau+T} = Z_{ik}^{\tau+T} - \gamma_i^{+}; i \in I_2,$$

$$\text{DF5}: \sum_{j=1}^{n} \lambda_j Z_{ij}^{\tau-1} = Z_{ik}^{\tau-1} - \gamma_i^{-}; i \in I_2,$$

$$\text{DF6}: \quad \sum_{j=1}^{n} \lambda_j x_{ij\alpha}^{Lt} = \theta^t x_{ik\alpha}^{Lt} - s_{i_1}^{t-}; i \in I_1, t = \tau, \ldots, \tau + T,$$

$$\text{DF7}: \quad \sum_{j=1}^{n} \lambda_j x_{ij\alpha}^{Ut} = \theta^t x_{ik\alpha}^{Ut} - s_{i_2}^{t-}; i \in I_1, t = \tau, \ldots, \tau + T,$$

$$\text{DF8}: \sum_{j=1}^{n} \lambda_j z_{ij\alpha}^{Lt} = \alpha^t z_{ik\alpha}^{Lt} - \delta_{i_1}^{t-}; i_1 \in I_2, t = \tau, \ldots, \tau + T,$$

$$\text{DF9}: \sum_{j=1}^{n} \lambda_j z_{ij\alpha}^{Ut} = \alpha^t z_{ik\alpha}^{Ut} - \delta_{i_2}^{t-}; i_2 \in I_2, t = \tau, \ldots, \tau + T$$

$$\text{DF10}: \sum_{j=1}^{n} \lambda_j y_{rj\alpha}^{Lt} = y_{ik\alpha}^{Lt} - s_{r_1}^{t+}; r = 1, \ldots s, t = \tau, \ldots, \tau + T,$$

$$\text{DF11}: \sum_{j=1}^{n} \lambda_j y_{rj\alpha}^{Ut} = y_{ik\alpha}^{Ut} - s_{r_2}^{t+}; r = 1, \ldots s, t = \tau, \ldots, \tau + T,$$

$$\text{DF12}: \sum_{j=1}^{n} \lambda_j Z_{ij\alpha}^{L(\tau+T)} = Z_{ik\alpha}^{L(\tau+T)} - \gamma_{i_1}^{+}; i_1 \in I_2,$$

$$\text{DF13}: \sum_{j=1}^{n} \lambda_j Z_{ij\alpha}^{U(\tau+T)} = Z_{ik\alpha}^{U(\tau+T)} - \gamma_{i_2}^{+}; i_2 \in I_2,$$

$$\text{DF14}: \sum_{j=1}^{n} \lambda_j Z_{ij\alpha}^{L(\tau-1)} = Z_{ik\alpha}^{L(\tau-1)} - \gamma_{i_1}^{-}; i \in I_2,$$

$$\text{DF15}: \sum_{j=1}^{n} \lambda_j Z_{ij\alpha}^{U(\tau-1)} = Z_{ik\alpha}^{U(\tau-1)} - \gamma_{i_2}^{-}; i \in I_2,$$

$$\lambda_j \geqslant 0;\ \forall j, S_i^{t-}, s_{i_1}^{t-}, s_{i_2}^{t-} \geqslant 0;\ \delta_i^{t-}, \delta_{i_1}^{t-}, \delta_{i_2}^{t-} \geqslant 0\,(\forall t, \forall i \in I_1);\ S_r^{t+}, s_{r_1}^{t+}, s_{r_2}^{t+} \geqslant 0\,(\forall t, \forall i \in I_1);$$
$$\gamma_i^{+}, \gamma_{i_1}^{+}, \gamma_{i_2}^{+}, \gamma_i^{-}, \gamma_{i_1}^{-}, \gamma_{i_2}^{-} \geqslant 0\,(\forall i \in I_2);$$

Following the above DFDEA procedures for Constant Returns To Scale (CRS), the DFDEA-VRS model for Variable Returns To Scale (VRS) can then be easily derived by simply adding a convexity constraint: $\sum_{j=1}^{n} \lambda_j = 1$.

4. Case study

To demonstrate the applicability and superiority of the proposed DFDEA modeling approach in terms of the benchmarking power over the CDEA modeling approaches, a case study on evaluating the dynamic efficiency of bus companies in Taipei, Taiwan is presented.

4.1. Data

The case study evaluates the efficiency and service effectiveness of 10 city bus companies in Taipei. Based on previous studies (*e.g.*, [9,10,32,34,38,57,58]), two crisp input variables were selected: number of buses (NB) and operating network (ON). Four output variables were selected: number of bus runs (NBR), operating revenue (OR), passenger-km (P–km), and passenger satisfaction (PS). Of the output variables, NBR, OR, an P–km were crisp variables and PS was a fuzzy qualitative variable.

Data were obtained from a questionnaire conducted by the Ministry of Transportation and Communications, R.O.C. in 2005 and in 2011 to evaluate the performance of city bus companies. In order to be consistent with

TABLE 1. Summary of descriptive statistics for 10 Taiwan's city bus companies 2005 and 2011.

Year	Item	Input		Output		
		NB	ON (km)	NBR	OR (NT$)	P–km
2005	Mean	245	633	1,357,783	513,424,653	381,508,167
	Std. Dev.	204	745	1,658,845	519,306,128	329,287,206
	Max.	704	2,570	4,883,988	1,536,823,756	913,099,891
	Min.	5	47	38,181	9,879,059	3,039,611
2011	Mean	277	646	991,798	645,298,532	608,782,370
	Std. Dev.	232	545	885,491	637,483,557	724,936,755
	Max.	796	1,771	2,732,334	1,952,532,018	2,203,607,386
	Min.	18	74	43,791	15,013,113	12,204,886

TABLE 2. Correlation coefficients among crisp input and output variables.

Year	2005					2011				
Variables	NB	ON (km)	NBR	OR (NT$)	P–km	NB	ON (km)	NBR	OR (NT$)	P–km
NB	1.00					1.00				
ON (km)	0.33	1.00				0.42	1.00			
NBR	0.94	0.36	1.00			0.95	0.32	1.00		
OR (NT$)	0.97	0.38	0.98	1.00		0.97	0.37	0.99	1.00	
P-km	0.93	0.28	0.86	0.93	1.00	0.97	0.38	0.96	0.99	1.00

fuzzy output data, this study only used crisp data in 2005 and 2011 to conduct a dynamic efficiency analysis. All the input and output data from the 10 bus companies were available from the 2005 and 2011 Annual Report published by the Ministry of Transportation and Communications, R.O.C. In order to capture the nature of the quasi-fixed input characteristic, NB was considered as a capital input variable. Table 1 presents the descriptive data from 2005 and 2011.

Table 2 shows the correlation coefficients among the crisp variables. All correlation coefficients between input and output variables in each year are significantly positive, confirming that the dataset satisfies the isotonicity property. The fuzzy variable of passenger satisfaction was rated as follows: "poor service," "fair service" and "good service," with half-overlapped triangular membership functions.

4.2. Efficiency scores

Table 3 presents the efficiency score of the bus companies with CRS using the DFDEA model and Table 4 presents the efficiency score of the bus companies with VRS using the DFDEA-VRS model Results in these two tables show a similar pattern. Table 3 shows that the proposed DFDEA model identified three bus companies (DMU 1, 6, and 10) to be dynamic efficient. However, the CDEA model in Table 3 identified three bus companies (DMU 1 6 and 10) to be efficient in 2005 and four (DMU 1, 3, 4 and 10) in 2011, whereas the CDEA model in Table 4 identified five bus companies (DMU 1, 6, 7, 9 and 10) to be efficient in 2005 and four (DMU 1, 6, 9 and 10) in 2011 This shows that CDEA models fail to capture the inter-temporal effects. For example, DMU 3 is not dynamic efficient but efficient if we only evaluate performance in 2011. These results indicate that the CDEA model overstated the resource allocation ability for DMU 3 because of the existence of the quasi-fixed input variable More specifically, DMU 4 cannot adjust its quasi-fixed input variable, *i.e.*, number of buses, to the optimal levels in 2011 The DFDEA model can identify the instantaneous effects by evaluating productive efficiency on the basis of an inter-temporal efficiency. It is interesting to note that the efficiency scores do not vary much with different αlevels, but one can still observe that the efficiency scores of inefficient bus companies decrease as the αlevel increases.

Table 3. Efficiency score of 10 city bus companies under various αlevels with CRS.

DMU	DFDEA			CDEA								
				Average efficiency in 2005 and 2011			2011			2005		
	0.0	0.5	1.0	0.0	0.5	1.0	0.0	0.5	1.0	0.0	0.5	1.0
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.531	0.531	0.529	0.487	0.483	0.480	0.544	0.537	0.532	0.429	0.428	0.428
3	0.911	0.911	0.911	0.941	0.941	0.941	1.000	1.000	1.000	0.881	0.881	0.881
4	0.667	0.667	0.663	0.785	0.784	0.782	1.000	1.000	1.000	0.570	0.567	0.563
5	0.742	0.742	0.742	0.778	0.778	0.778	0.695	0.695	0.695	0.861	0.861	0.861
6	1.000	1.000	1.000	0.876	0.875	0.874	0.751	0.749	0.748	1.000	1.000	1.000
7	0.661	0.659	0.657	0.672	0.670	0.669	0.561	0.559	0.557	0.782	0.781	0.781
8	0.802	0.801	0.799	0.823	0.817	0.812	0.744	0.732	0.722	0.902	0.901	0.901
9	0.886	0.886	0.886	0.903	0.903	0.903	0.919	0.919	0.919	0.886	0.886	0.886
10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 4. Efficiency score of 10 city bus companies under various αlevels with VRS.

DMU	DFDEA-VRS			CDEA								
				Average efficiency in 2005 and 2011			2011			2005		
	0.0	0.5	1.0	0.0	0.5	1.0	0.0	0.5	1.0	0.0	0.5	1.0
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.541	0.541	0.539	0.487	0.715	0.715	0.715	1.000	1.000	0.429	0.429	0.429
3	1.000	1.000	0.977	1.000	1.000	0.982	0.982	1.000	1.000	1.000	1.000	0.963
4	0.674	0.672	0.670	0.787	0.787	0.787	0.787	1.000	1.000	0.574	0.574	0.574
5	0.762	0.750	0.745	0.861	0.822	0.807	0.807	0.723	0.700	0.926	0.920	0.913
6	1.000	1.000	1.000	0.876	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
7	1.000	1.000	1.000	0.835	0.899	0.866	0.866	0.798	0.731	1.000	1.000	1.000
8	0.902	0.902	0.902	0.842	0.966	0.961	0.961	1.000	1.000	0.939	0.931	0.922
9	1.000	1.000	1.000	0.960	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

4.3. Scale analysis

Table 5 presents the scale efficiency score of these bus companies. The DFDEA model will generate only one scale value, whereas the CDEA models will generate two scale values corresponding to each evaluated period. The proposed DFDEA approach is superior for examining the scale property. Under the DFDEA approach, five bus companies were identified to be Decrease Returns to Scale (DRS), two bus companies were identified to be Increase Returns to Scale (IRS) and three bus companies were identified to be CRS (αlevel = 0.5). These results indicate whether a company needs to downsize, upsize, or remain the same. In contrast, with the separate CDEA models, scale efficiency score in different periods resulted in different numbers of bus companies as CRS, DRS, and IRS. For example, for DMU 2, the proposed DFDEA model suggests to increase its overall size (IRS). In contrast, the CDEA models suggest increasing the size (IRS) in 2005 but decreasing the size (DRS) in 2011. This information may puzzle decision makers.

TABLE 5. Scale efficiency score of 10 city bus companies at various αlevels.

DMU	DFDEA			CDEA					
				2011			2005		
	0.0	0.5	1.0	0.0	0.5	1.0	0.0	0.5	1.0
1	–	–	–	–	–	–	–	–	–
2	–	irs	irs	drs	drs	drs	–	irs	irs
3	drs	drs	drs	–	–	–	drs	drs	drs
4	irs	irs	irs	–	–	–	irs	irs	irs
5	drs	drs	drs	drs	drs	drs	drs	drs	drs
6	–	–	–	drs	drs	drs	–	–	–
7	drs	drs	drs	drs	drs	drs	drs	drs	drs
8	drs	drs	drs	drs	drs	drs	drs	drs	drs
9	drs	drs	drs	drs	drs	drs	drs	drs	drs
10	–	–	–	–	–	–	–	–	–

Note: "–" is crs.

4.4. Policy implications

The case study presented in this paper compared results from a DFDEA model and a DFDEA-VRS model This case study demonstrated that the proposed DFDEA model provides more consistent output when the target industry is characterized by inter-temporal dependencies and fuzziness. Two aspects of this modeling approach are worth highlighting: efficiency score and scale analysis. First, the efficiency score from the proposed DFDEA model not only can provide policymakers a coherent picture of the industry performance over the years but also can incorportate qualitative performance indicators (*e.g.*, customer satisfication). While the CDEA model cannot account for inter-temporal dependencies and fuzziness, the DFDEA model can. Furthermore, the DFDEA model can be used to evaluate the performance over multiple time periods and to consider heavy investment in the early stages (*e.g.* number of bus runs in the case study).

The scale analysis for DFDEA can also provide an integrated scale for each DMU over the analysis period. This integrated scale value provides policymakers or operators information on whether the company size should be changed. That is, to upsize if IRS, to downsize if DRS, and to remain the same if CRS. In other words, the DFDEA scale analysis results can provide a clear direction for policymakers and operators in the long term, and eliminate the potential confusion from CDEA models (refer to the DMU 2 analysis in Sect. 4.2). Therefore the proposed DFDEA models can better evaluate performance for the industry using a wider range of variables and take into account the nature of inter–temporal dependencies compared to CDEA models.

5. Conclusion

This paper developed a method for evaluating the performance of DMUs when the input and output variables are characterized by inter–temporal dependencies and fuzziness. Using the proposed DFDEA model, we can measure the dynamic efficiency for industries that have a capital fixity issue. Furthermore, considering the fuzziness of variables allows us to evaluate the holistic performance of transport service. The case study presented in this paper demonstrated the superiority of using the proposed DFDEA models to evaluate the comparative efficiency for DMUs under the circumstance that at least one of the variables is measured qualitatively (such as passenger satisfaction) or characterized by inter–temporal dependence (such as number of buses).

Several research directions for future studies can be identified. The proposed DFDEA model in this study was specified with the integration of FDEA models [34] and DDEA models [53]. Other specifications or even multi-objective specifications warrant further exploration. Further, this study only illustrated one case of bus companies in Taipei, Taiwan. Future research can validate the use of DFDEA models for different modes of transport and in different locations.

Appendix A. Summary of literature review for DEA model in transport field.

Ref. No	Author	Industry	Approach	Input variables	Output variables	Service variables	Model	DMU
1	Odeck and Alkadi (2001)	Transit	DEA	total number of seats	seat kilometers	–	BCC	Bus company
				fuel consumption	Vehicle kilometers			
				consumption equipment	Passenger kilometers			
					passengers			
5	Schefczyk (1993)	Airline	DEA	available ton kilometers	revenue passenger kilometers	–	CCR	Airlines
				facilities	cargo revenue			
				current assets	other revenue			
				other assets	–			
				labor				
				fuel				
				commissions to agents				
6	Charnes *et al.* (1996)	Airline	DEA REPF	seat-kilometer available	Passenger kilometer		BCC	Airlines
				cargo-ton kilometer available	–			
				fuel				
				labor				
7	Sengupta (1999)	Airline	DEA	available ton kilometers	revenue passenger kilometers	–	CCR	Airlines
				facilities	cargo revenue			
				current assets	other revenue			
				other assets	–			
				labor				
				fuel				
9	Chiou and Chen (2006)	Airline	DEA	fuel cost	number of flights	Passenger mile	CCR BCC	Airline
				personnel cost	seat-mile	embarkation passengers		
				aircraft cost-	–	–		
13	Peck *et al.* (1998)	Airport	DEA	labor expenses on airframes	flights arrivals delayed for mechanical reasons	–	BCC	Airlines
				labor expenses on aircraft engines				
				expenditures on airframe repairs	–			
				expenditures on engine repairs				
				material expenditures on airframes				
				material expenditures on engines				

Appendix A. (*continued*).

Ref. No	Author	Industry	Approach	Input variables	Output variables	Service variables	Model	DMU
15	Sarkis (2000)	Airport	DEA	airport operational costs	operational revenue		CCR BCC	Airport
				number of airport employees	passenger flow			
				gates	commercial			
				runways	general aviation movement			
					total cargo transportation			
17	Adler and Berechman (2001)	Airport	DEA	Questionnaire	Service satisfaction		BCC	City
				Haul charge	–			
				Connection times				
				Average delay time				
18	Barros and Dieke (2007)	Airport	DEA	labor costs	number of planes		CCR BCC	Airport
				capital invested	number of passenger			
				operational costs	cargo			
					aeronautical receipts			
					handling receipts			
					commercial receipts			
19	Tongzon (2001)	Port	DEA	number of berths, cranes and tugs	cargo throughput		CCR	City
				number of port authority employees	ship working rate			
				terminal area of the ports	–			
20	Cullinane *et al.* (2006)	Port	DEA SFA	terminal length	container throughput		CCR BCC	Country
				terminal area				
				quayside gantry				
				yard gantry				
				straddle carrier				
22	Fielding *et al.* (1985)	Transit	DEA	labor	vehicle hours	passengers	CCR	US City
				capital	vehicle miles	Passenger miles		
				fuel	capacity miles	operating revenue		
					service reliability	–		

Appendix A. (*continued*).

Ref. No	Author	Industry	Approach	Input variables	Output variables	Service variables	Model	DMU
25	Viton (1998)	Transit	DEA	fleet sizes	vehicle-miles	–	BCC	Transit industry
				number of gallons of fuel	Passenger trips			
				number of person-hours of transportation	vehicle hours			
				number of person-hours of maintenance				
				number of person-hours of administrative				
				capital				
				the cost of tires and other materials				
				the cost of services				
				the cost of utilities				
				the cost of insurance				
28	Karlaftis (2004)	Transit	DEA	Number of vehicles	Vehicle miles	Passenger miles	BCC	City
				gallons of fuel				
				Total employees				
32	Chiou *et al.* (2010)	Transit	IDEA	number of buses	number of bus runs	operating revenue	CCR BCC	Bus company
				operating network	Bus kilometer	number of passengers		
						Passenger kilometer		
						average number of on-board passengers per run		
33	Chiou *et al.* (2012)	Transit	RDEA	fuel cost	Operating revenue	–	CCR BCC	Bus company and Bus route
				labor	Passenger-km			
				bus number	–			
36	Coelli and Perelman (1999)	Railway	DEA SFA COLS	annual mean of monthly data on staff levels	passenger services		BCC	Company
				available freight wagons	freight services			
				coach transport capacities in tones	–			
				coach transport capacities in seats				
				total length of lines				

Appendix A. (*continued*).

Ref. No	Author	Industry	Approach	Input variables	Output variables	Service variables	Model	DMU
37	Lan and Lin (2005)	Railway	DEA	Lines	passenger train-kilometer	Passenger kilometers	BCC	Railway
				Passenger cars	freight-train-kilometer	Ton kilometers		
				Freight cars	–	–		
				Employees				
38	Lan and Lin (2003)	Railway	DEA	length of lines	Train kilometer	Passenger kilometer	CCR BCC EXO CAT	Railway
				number of locomotives and cars	–	Ton kilometer		
				number of employees				

Note:
1 SFA: Stochastic Froniter Analysis
2 CCR model [59] for CRS technology
3 BCC model [60] for VRS technology
4 EXO DEA: exogenously fixed inputs model
5 CAT DEA: To compare the performance measurements in a homogeneous environment can be formulated according to appropriate categorical variables.
6 COLS: A parametric frontier using corrected ordinary least squares
7 REPF: Robustly Efficient Parametric Frontier
8 IDEA: integrated data envelopment analysis
9 RDEA: Route-based data envelopment analysis

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