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AN INVENTORY MODEL OF A THREE PARAMETER WEIBULL DISTRIBUTED DETERIORATING ITEM WITH VARIABLE DEMAND DEPENDENT ON PRICE AND FREQUENCY OF ADVERTISEMENT UNDER TRADE CREDIT

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Abstract. This paper develops an inventory model for a deteriorating item with variable demand dependent on the selling price and frequency of advertisement of the item under the financial trade credit policy. Shortages are allowed and these are partially backlogged with a variable rate dependent on the duration of waiting time until to the arrival of next order. In this inventory model, the deterioration rate follows a three-parameter Weibull distribution. The corresponding inventory model is formulated and solved by using the well-known generalized reduced gradient method along with an algorithm. To validate the inventory model, two numerical examples are considered and solved. Finally, based on one numerical example, the impacts of different parameters are studied by a sensitivity analysis considering one parameter at a time and leaving the other parameters fixed.

Mathematics Subject Classification. 90B05.

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1. Introduction

Permissible delay in payment is a key factor in the competitive marketing strategy. In this direction, the suppliers offer different facilities to retailers in order to stimulate the business' sales. One such facility is to sale products on credit. In this case, the supplier offers to retailer a certain credit period. For this period, no interest is applied by the supplier. Nonetheless, after this period, a rate of interest is applied by the supplier according to the terms and conditions established in an agreement among supplier and retailer. This kind of inventory problem is also known as inventory problem with permissible delay in payments or trade credit

Keywords. Deterioration, Weibull distribution, advertise and price dependent demand, partially backlogged shortage, credit policy.

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TABLE 1. Summary of related literature of inventory models with permissible delay in payments.

Authors	Deterioration	Demand rate	Shortages	Level of permissible delay in payments	Inventory policies
Hwang and Shinn ([37])	Yes	Constant	No	Single	–
Chang, *et al.* ([16])	Yes	Constant	No	Single	–
Abad and Jaggi ([2])	No	Linearly time dependent	No	Single	–
Ouyang, *et al.* ([51])	Yes	Constant	No	Single	–
Huang ([33])	No	Linearly time dependent	No	Two level	—
Huang ([34])	No	Constant	No	Single	–
Huang ([35])	No	Constant	No	Two level	–
Sana and Chaudhuri ([59])	Yes	Selling price dependent	No	Single	–
Huang and Hsu ([36])	No	Constant	No	Two level partial trade credit	–
Ho *et al.* ([32])	No	Constant	No	Two level	–
Jaggi and Khanna ([39])	Yes	Inventory level dependent	Complete backlogging	Single	IFS
Jaggi and Kausar ([38])	No	Selling price dependent	Complete backlogging	Single partial trade credit	–
Jaggi and Mittal ([40])	Yes	Annual	Complete backlogging	Single	IFS
Guria, *et al.* ([30])	No	Selling price dependent	Complete backlogging	Single	IFS
Taleizadeh, *et al.* ([63])	No	Constant	Partial backlogging	Single	IFS
Wu, *et al.* ([67])	Yes	Constant	No	Two-level	–
Chen, *et al.* ([17])	Yes	Constant	No	Two-level	–
Bhunia, *et al.* ([11])	Yes	Linearly time dependent	Partial backlogging	Single	IFS &SFI
Bhunia, *et al.* ([12])	Yes	Stock dependent	Partial backlogging	Single	IFS
Bhunia and Shaikh ([14])	Yes	Selling price dependent	Partial backlogging	Single	IFS &SFI
Shah and Cárdenas–Barrón ([61])	Yes	Constant	No	Two-level	–
This paper	Yes	Frequency of advertisement and selling price dependent demand	Partial backlogging	Single	IFS

financing inventory problem. Perhaps, Haley and Higgins [31] were the first researchers to introduce the idea of this inventory problem. Then Goyal [29] proposed an EOQ inventory model allowing a permissible delay in payments. Later, Aggarwal and Jaggi [3] revisited and extended the Goyal [29]'s inventory model considering deteriorating items without shortages. Conversely, Jamal *et al.* [42] developed a more general EOQ inventory model that allows shortages with fully backlogging. After Jamal *et al.* [42], several works have been done by researchers and academicians. Some of these works are shown in Table 1.

Chung and Huang [18] and Liang and Zhou [46] developed inventory models for deteriorating items for two warehouse system considering permissible delay in payments. In both inventory models demand is considered as constant and shortages are not permitted. Bhunia and Shaikh [9] built a trade credit two warehouse inventory model with deteriorating items taking into account a uniform demand and the inventory follows shortage (IFS) policy with partially backlogged shortages. Basically, they have proposed an alternative approach in the formulation of trade credit inventory model. Most recently, Tiwari *et al.* [64] and Jaggi *et al.* [41] derived two-warehouse inventory models for non-instantaneous deteriorating items under trade credit policy.

In the literature of inventory, there exist some inventory models that have been built considering that the product's life time in storage is unlimited. In other words, when a product is in stock; this does not deteriorate and it can be used to meet demand. However, in many situations, this statement is not always true because some products deteriorate thru time. Therefore, the damage owed to deterioration effect must be considered in the development of the inventory models. In this context, Ghare and Schrader [22] studied an inventory model for exponentially decaying inventory. Then Emmons [21] developed an inventory model with variable deterioration that follows a two-parameter Weibull distribution. Several academicians and researchers have extended these inventory models, for example: Covert and Philip [19], Giri *et al.* [25], and Ghosh and Chaudhuri [23]; just to name a few works. Conversely, Chakrabarty *et al.* [15], Giri *et al.* [24], Sana *et al.* [58], Sana and Chaudhuri [57], among others have proposed inventory models with deteriorating items considering Weibull distributed deterioration. Misra [50] introduced an EOQ model with Weibull deterioration rate for a perishable product without shortages. Following this research direction, Deb and Chaudhuri [20], Mandal and Phaujdar [48], Goswami and Chaudhuri [27], Pal *et al.* [56], Goyal and Gunasekaran [28], Padmanabhan and Vrat [52], Giri *et al.* [26], Mandal and Maiti [49], Sarker *et al.* [60], Bhunia and Maiti [5], Bhunia and Maiti [6], Pal *et al.* [53], Pal *et al.* [54], Tripathy and Mishra [65], Kawale and Bansode [43], Bhunia *et al.* [8], Amutha and Chandrasekaran [4], Bhunia et al. [10] among others have considered a time proportional deterioration rate.

It is important to remark that the price and advertisement of an inventory item are key factors in order to change positively demand behaviour between clients. The advertisement of products *via* traditional and modern ways produces a motivational impact on the clients that impulse them to purchase more products. The product's price is also other decisive factor in buying a product. Therefore, it is natural to propose that product's demand can be a function of price and the advertisement costs. Few researchers have investigated the impacts of variations in price and the advertisement on product's demand rate. It is well-known that Kotler [44] was the first researcher that included marketing policies into inventory decisions. Afterwards, Ladany and Sternleib [45] analyzed the impacts on variations of price in the order quantity. But, they did not study the impacts of advertisement. Later, Subramanyam and Kumaraswamy [62], Urban [66], Goyal and Gunasekaran [28], Abad [1], Luo [47], Pal *et al.* [55], Bhunia and Shaikh [7], Bhunia *et al.* [13] developed inventory models incorporating the influence of variations on price and advertisement on product's demand.

This paper develops an inventory model for deteriorating items with variable demand dependent on the selling price and frequency of advertisement of the item under the financial trade credit policy. Shortages are allowed and there are partially backlogged with a variable rate dependent on the duration of waiting time until the arrival of next order. In this inventory model, the deterioration rate follows a three-parameter Weibull distribution. Additionally, the transportation cost of the items is fixed for a finite capacity of a transport mode such as a truck. The fixed cost is applied when a truck is used whether it is utilized fully or partially for a ceiling quantity or more. For quantities less than the ceiling quantity, a uniform cost per unit is charged. Taking into the consideration of these features, an inventory model is formulated and solved by generalized reduced gradient based method (GRG). Two numerical examples are solved in order to discuss the results and the significant features of the proposed inventory model. Finally, based on one numerical example, the impacts of different parameters are studied by a sensitivity analysis taking one parameter at a time and maintaining the other parameters unchanged.

2. Notation and assumptions

The following notation and assumptions are used to develop the proposed inventory model.

Notation.

Parameters	Description
k	Capacity of transport vehicle (units)
U	A quantity less than k
a, b, ν	Constants where $a, b, \nu > 0$
α, β, γ	Constants where $0 < \alpha \ll 1, \quad \beta > 0, \gamma \geq 0$
δ	Backlogging parameter
C_o	Ordering cost per order (\$/order)
C_h	Holding cost per unit per unit time (\$/unit/time unit)
C_s	Shortage cost per unit per unit time (\$/unit/time unit)
C_p	Purchasing cost per unit (\$/unit)
C_a	The advertising cost per advertisement (\$/advertisement)
C_t	Transportation cost of a full load transport vehicle (\$/ transport vehicle)
C_{tF}	Transportation cost per unit (\$/unit)
p	Selling price per unit (\$/unit)
m	Mark-up rate ($p = mC_p$) where m>1
$D(A, p)$	The demand rate dependent on selling price (p) and the frequency of advertisement (A). It is denoted by $D(A, p) = A^{\nu}(a - bp),\ a, b, \nu > 0$ with condition $0 < p < a/b$ (units/time unit)
M	Credit period offered by the supplier (time unit)
I_e	Rate of interest earned by retailer (percentage/time unit)
I_p	Rate of interest charged by the supplier to the retailer (percentage/time unit)
$I(t)$	Inventory level at time t (units)
$Z^{(.)}$	Profit (\$/time unit)

Dependent decision variables:	
S	Stock level at the beginning of stock-in period (units)
R	Shortage level (units)
$Q = S + R$	Order quantity (units)

Decision variables:	
t_1	Time at which the stock level attains to zero (time unit)
T	Cycle length (time unit)
n	Number of transport vehicles (an integer number)
A	Frequency of advertisement (number of advertisements/time unit $-$ an integer number)

Assumptions.

(i) The whole lot (Q) is supplied in one delivery.
(ii) The inventory planning horizon is infinite and the inventory system considers only one item and one stocking point.
(iii) Lead time is constant and known.
(iv) The ordering cost is constant and does not include transportation cost.
(v) The deteriorated units are neither refunded nor repaired.
(vi) The deterioration happens when the item is in-stock warehouse and its rate follows a three-parameter Weibull distribution.
(vii) α, β, γ are the parameters of the Weibull distribution whose probability density function is

$$f(t) = \alpha\beta\,(t-\gamma)^{\beta-1}\exp\left\{-\alpha\,(t-\gamma)^{\beta}\right\}$$

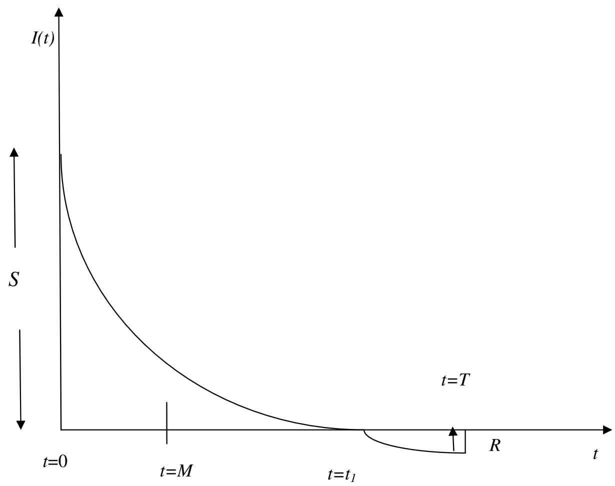

FIGURE 1. Inventory level in Scenario 1.

The instantaneous rate of deterioration of the on-hand inventory at any time t is $\theta(t)$ which follows the three–parameter Weibull distribution. So, $\theta(t) = \frac{f(t)}{1-F(t)} = \alpha\beta\,(t-\gamma)^{\beta-1}$ where $0 < \alpha \ll 1,\;\; \beta > 0, \gamma \geqslant 0$, and $F(t)$ is the cumulative distribution function of Weibull distribution.
For $\beta = 1,\; \theta(t)$ is constant and the Weibull distribution reduces to exponential distribution.
When$\beta > 1$ then $\theta(t)$ is an increasing function of t,
When$\beta < 1$then $\theta(t)$ is a decreasing function of t.
For $\gamma = 0$, the three-parameter Weibull distribution reduces to two-parameter Weibull distribution.

(viii) C_t is the transportation cost of a full load transport vehicle and C_{tF} is the transportation cost per unit.

(ix) U is the upper break point, where $U = \left\lfloor \frac{C_t}{C_{tF}} \right\rfloor$ $(< k)$; here $\left\lfloor \frac{C_t}{C_{tF}} \right\rfloor$ represents the greatest integer value which is less than or equal toC_t/C_{tF}.

(xi) Shortages are allowed and these are partially backlogged. The backlogging rate is dependent on the length of the waiting time until to the arrival of the next order. Considering this situation, the rate is defined as $[1+\delta\,(T-t)]^{-1}$ where $\delta > 0$.

3. Mathematical formulation of the inventory model

This section presents a description of the inventory model and its mathematical formulation. The inventory level thru time is represented in Figure 1. Here, it is considered that after satisfying the backorder quantity of the previous cycle, the on-hand inventory level is S at $t = 0$. Then immediately the inventory level, $I(t)$, starts to decrease due to both customers' demand and deterioration effect of the items. Notice that at time $t = t_1$the inventory level attains to zero. After the time $t = t_1$,the shortages happen and these are accumulated with a rate of $[1+\delta\,(T-t)]^{-1}$ where $\delta > 0$ until to the time $t = T$ when the next order enters to the inventory system. At time $t = T$ the maximum shortage level is R. Then the inventory cycle is repeated every Ttime units.

The $I\,(t)$ represents the instantaneous inventory level at any time $t \geqslant 0$. Then the inventory level $I\,(t)$ at any time t satisfies the following differential equations:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \theta(t)\,I(t) = -D\,(A,\;p)\,, \qquad 0 \leqslant t \leqslant t_1 \tag{3.1}$$

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = \frac{-D(A,p)}{1+\delta(T-t)}\,, \qquad t_1 < t \leqslant T \tag{3.2}$$

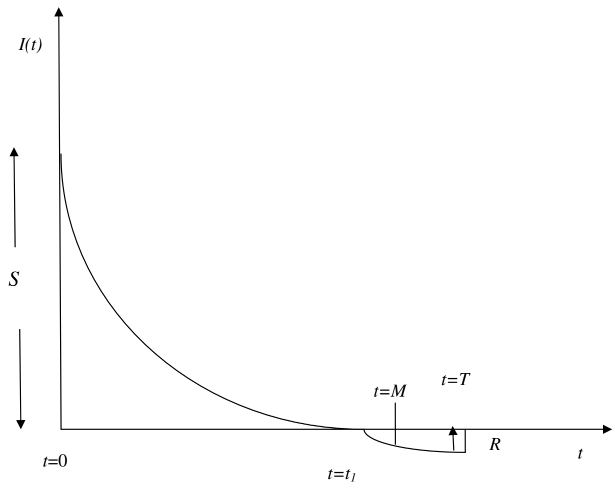

Figure 2. Inventory level in Scenario 2.

with the boundary conditions

$$I\,(t) = S \qquad \text{at} \qquad t = 0,\; I(t) = 0 \qquad \text{at} \qquad t = t_1, \tag{3.3}$$

$$I\,(t) = -R \qquad \text{at} \qquad t = T. \tag{3.4}$$

Furthermore, $I(t)$ is continuous at $t = t_1$.

Taking into account the boundary conditions (3.3) and (3.4), the solutions of the differential equations (3.1) and (3.2) are obtained and these are as follows:

$$I(t) = D(A,p)\mathrm{e}^{-\alpha(t-\gamma)^\beta} \int_t^{t_1} \mathrm{e}^{\alpha(t-\gamma)^\beta} \mathrm{d}t, \qquad 0 \leqslant t \leqslant t_1 \tag{3.5}$$

$$I(t) = \frac{D(A,p)}{\delta} \log|1 + \delta(T-t)| - R, \quad t_1 < t \leqslant T. \tag{3.6}$$

From the boundary condition (3.3), it is known that $I\,(t) = S$ at$t = 0$, therefore

$$S = D\,(A,\,p)\,\mathrm{e}^{-\alpha(-\gamma)^\beta} \int_0^{t_1} \mathrm{e}^{\alpha(t-\gamma)^\beta} \mathrm{d}t \tag{3.7}$$

From the continuity condition, consequently R is computed with

$$R = \frac{D(A,p)}{\delta} \log|1 + \delta(T - t_1)| \tag{3.8}$$

The total inventory holding cost (C_{hol}) for the cycle is calculated with

$$C_{\text{hol}} = C_h \int_0^{t_1} I\,(t)\,\mathrm{d}t \tag{3.9}$$

The total shortage cost (C_{sho}) over the cycle is determined with

$$C_{\text{sho}} = C_s \int_{t_1}^{T} [-I(t)]\mathrm{d}t \tag{3.10}$$

The total transportation cost (C_{tran}) is given by

$$C_{\text{tran}} = \begin{cases} nC_t + (S+R-nk)\,C_{tF} \text{ and } n^* = n \text{ when } nk < S+R \leqslant nk+U \\ (n+1)\,C_t \text{ and } n^* = n \text{ when } nk+U < S+R < (n+1)\,k \\ (n+1)\,C_t \text{ and } n^* = n+1 \text{ when } S+R = (n+1)\,k \end{cases} \tag{3.11}$$

where n^* is the number of full load vehicles.

The supplier offers a trade credit period to his/her retailer and it is M. Depending on the values of M and t_1 two scenarios happen: **Scenario 1.** $0 < M \leqslant t_1$ and **Scenario 2:** $t_1 < M$. These two scenarios are illustrated in Figures 1 and 2.

Now, both scenarios are discussed below.

Scenario 1. $0 < M \leqslant t_1$

In this scenario, the total amount payable to the supplier is $C_p\,(S+R)$ at time $t = M$.

Due to sales and interest earned, the total revenue is calculated with

$$U_1 = p\int_0^M D\mathrm{d}t + pI_e \int_0^M \int_0^t D\mathrm{d}u\mathrm{d}t + pR\,(1+I_eM) \tag{3.12}$$

Depending on the values of U_1 and $C_p\,(S+R)$, the following two cases may occur: **Case 1.1:** $U_1 \geqslant C_p\,(S+R)$ and **Case 1.2:** $U_1 < C_p\,(S+R)$.

Case 1.1. $U_1 \geqslant C_p\,(S+R)$

Here, the total accumulated amount at time $t = M$ is greater than or equal to the total purchasing cost.

In this case, the profit for the cycle is determined as follows:

$$Z^{(1.1)}\,(n, A, t_1, T) = \frac{X_1}{T} \tag{3.13}$$

Where $X_1 =$ < Excess amount after paying the amount to the supplier > + < interest earned for the rest amount during the interval $[M,T]$ > + < sales during the interval $[M,t_1]$ > + < interest earned during the interval $[M,t_1]$ > + < interest earned during the interval $[t_1,T]$ > $- TC$.

where the total cost TC of the system is computed with $TC =$ < ordering cost > + < advertisement cost > + < purchasing cost > + < holding cost > + < shortage cost > + < transportation cost >

Thus,

$$TC = C_o + C_aA + C_{\text{hol}} + C_{\text{sho}} + C_{\text{tran}} \tag{3.14}$$

and

$$X_1 = \{U_1 - C_p\,(S+R)\}\,\{1 + I_e\,(T-M)\} + Dp\,(t_1 - M)\left\{1 + \frac{1}{2}I_e\,(t_1 - M)\right\}\{1 + I_e\,(T - t_1)\} - TC \tag{3.15}$$

Hence, the corresponding optimization problem is expressed as follows:
Problem 1. Maximize $Z^{(1.1)}(n, A, t_1, T) = \frac{X_1}{T}$

$$\text{subject to} \qquad 0 < M \leqslant t_1 < T \tag{3.16}$$

Case 1.2. $U_1 < C_p(S + R)$

In this case, the total accumulated amount at time $t = M$ is less than the total purchase cost.

In this situation, two subcases may appear: **Subcase 1.2.1.** Partial payment is permitted at time $t = M$ and **Subcase 1.2.2.** Partial payment is not permitted at time $t = M$.

Subcase 1.2.1. Partial payment is permitted at time $t = M$.

In this subcase, assume that at time $t = B_1 (B_1 > M)$ the rest amount $C_p(S + R) - U_1$ is paid. Therefore, the retailer must pay the interest of the amount $C_p(S + R) - U_1$ during the interval $[M, B_1]$.

As a result, the total amount that retailer must pay at time $t = B_1$ is obtained with the following mathematical expression $\{C_p(S + R) - U_1\}\{1 + I_p(B_1 - M)\}$.

The total amount available to the retailer is determined as

$<$ sales amount during the interval $[M, B_1]$ $>+<$ the interest earned $>=$ $p\int_M^{B_1} D\mathrm{d}t + pI_e\int_M^{B_1}\int_M^{t} D\mathrm{d}u\mathrm{d}t$

At time $t = B_1$, the amount payable to the supplier is equal to the total amount available to the retailer, so

$$\{C_p(S + R) - U_1\}\{1 + I_p(B_1 - M)\} = p\int_M^{B_1} D\mathrm{d}t + pI_e\int_M^{B_1}\int_M^{t} D\mathrm{d}u\mathrm{d}t$$

The profit for the cycle is calculated with

$$Z^{(1.2.1)}(n, A, t_1, T) = \frac{X_2}{T} \tag{3.17}$$

where $X_2 =$ $<$ sales during the interval $[B_1, t_1]$ $>+<$ interest earned during the interval $[B_1, t_1]$ $>+<$ interest earned during the interval $[t_1, T]$ $>-$ TC

Thus,

$$X_2 = \left\{p\int_{B_1}^{t_1} D\mathrm{d}t + pI_e\int_{B_1}^{t_1}\int_{B_1}^{t} D\mathrm{d}u\mathrm{d}t\right\}\{1 + I_e(T - t_1)\} - TC \tag{3.18}$$

Here, the total cost TC of the inventory system is computed with equation (3.14).

Therefore, in this subcase, the corresponding constrained optimization problem is given by
Problem 2. Maximize $Z^{(1.2.1)}(n, A, t_1, T) = \frac{X_2}{T}$

$$\text{subject to} \qquad 0 < M \leqslant t_1 < T \tag{3.19}$$

Subcase 1.2.2. Partial payment is not permitted at time $t = M$.

In this subcase, the retailer must pay the credit amount to the supplier. Let this time point be B_2 where $B_2 > M$. In this situation, retailer has to pay the interest for the period $[M, B_2]$. Obviously, the amount payable to the supplier is equal to the total on hand amount available to the retailer at time $t = B_2$. Thus,

$$C_p(S + R)\{1 + I_p(B_2 - M)\} = p\int_0^{B_2} D\mathrm{d}t + pI_e\int_0^{B_2}\int_0^{t} D\mathrm{d}u\mathrm{d}t + pR(1 + I_eB_2)$$

Therefore, the profit for the cycle is obtained with

$$Z^{(1.2.2)}(n, A, t_1, T) = \frac{X_3}{T} \tag{3.20}$$

where $X_3 = <$ sales during the interval $[B_2, t_1] > + <$ interest earned during the interval $[B_2, t_1] > + <$ interest earned during the interval $[t_1, T] > -\ TC$

Mathematically speaking,

$$X_3 = \left\{ p \int_{B_2}^{t_1} D\mathrm{d}t + pI_e \int_{B_2}^{t_1} \int_{B_2}^{t} D\mathrm{d}u\mathrm{d}t \right\} \{1 + I_e (T - t_1)\} - TC \tag{3.21}$$

Here, again the total cost TC of the inventory system is calculated with equation (3.14).

As a result, in this subcase, the corresponding constrained optimization problem is written as follows:

Problem 3. Maximize $Z^{(1.2.2)} (n, A, t_1, T) = \frac{X_3}{T}$

$$\text{subject to} \qquad 0 < M \leqslant t_1 < T \tag{3.22}$$

Scenario 2. $t_1 < M \leqslant T$

The inventory level thru time for Scenario 2 is depicted in Figure 2.

Due to sales and interest earned, the total revenue is given by

$$U_2 = \left\{ p \int_{0}^{t_1} D\mathrm{d}t + pI_e \int_{0}^{t_1} \int_{0}^{t} D\mathrm{d}u\mathrm{d}t \right\} \{1 + I_e(M - t_1)\} + pR\,(1 + I_e M)$$

Here, it is assumed that $U_2 > C_p(S + R)$.

In this scenario, the profit for the cycle is calculated with

$$Z^{(2)} (n, A, t_1, T) = \frac{X_4}{T} \tag{3.23}$$

where $X_4 = <$ Excess amount $> + <$ interest earned for that excess amount during the time period $[M, T] >$ $-\ TC$

Again, the total cost TC of the inventory system is obtained using equation (3.14).

Consequently,

$$X_4 = \{U_2 - C_p\,(S + R)\}\,\{1 + I_e(T - M)\} - TC \tag{3.24}$$

So, the corresponding constrained optimization problem is formulated as follows:

Problem 4. Maximize $Z^{(2)} (n, A, t_1, T) = \frac{X_4}{T}$

$$\text{subject to} \qquad 0 < t_1 < M \leqslant T \tag{3.25}$$

Notice that the profit in the four optimization problems is a function of two continuous variables (t_1, T) and two integer variables (n, A). Clearly, the above functions are increasing functions with respect to mark-up rate (m). Therefore, the objective of the optimization problems is to determine the optimal values of A, n, t_1, and T by solving the mixed integer nonlinear optimization problems.

The earlier mentioned four optimization problems can be solved by using the well-known generalized reduced gradient (GRG) method and the following algorithm.

Algorithm.

Step 1. Input all the parameters.

Step 2. Determine the value of $U = \lfloor \frac{C_t}{C_{tF}} \rfloor$.

Step 3. Solve the optimization problems (3.16), (3.19), (3.22) and (3.25) by taking the transportation cost for the condition $nk < S + R \leqslant nk + U$

Step 4. If $S + R$ satisfies the condition $nk < S + R \leqslant nk + U$, then this is the optimal policy and goes to Step 7. Otherwise, go to Step 5.

Step 5. Solve the optimization problems (3.16), (3.19), (3.22) and (3.25) by taking the transportation cost for condition $nk + U < S + R \leqslant (n+1)k$.
Step 6. If $S+R$ satisfies the condition $nk+U < S+R \leqslant (n+1)k$, then this is the optimal policy go to Step 7. Otherwise, go to Step 3.
Step 7. Report the optimal solution and stop.

4. Numerical examples

To illustrate and validate the proposed inventory model, two numerical examples are solved.

Example 1. For this example the following data is considered:
$C_o = \$\,250$ per order, $C_h = \$1.5$ per unit per unit time, C_s =\$12 per unit per unit time, $C_p = \$20$ per unit, $C_a = \$\,50$ per advertisement, $C_t = \$\,100$ per transport vehicle, $C_{tF} = \$\,1.25$ per unit, $\alpha = 0.05$, $\beta = 2$, $\gamma = 2.5$, $a = 250$, $b = 0.5$, $\nu = 0.1, k = 100$ units, $M = 0.25$ year, $I_e = 9\,\%$ per year, $I_p = 13\,\%$ per year, $\delta = 1.5$.

Using the solution procedure (Algorithm) the optimal solution is obtained with the help of LINGO software for two different values of m (1.25 and 1.3). The optimum values of $n*$, A, t_1, T, S, R and Q along with maximum total profit are presented in Table 2.

Table 2. Optimal solution for Example 1.

m	n^*	A	S	R	Q	t_1	T	Z	B	Optimization problem
	10†	**15**	**901.8124**	**98.1876**	**1000**	**3.648212**	**4.05143**	**1620.335**	–	Problem 1
1.25	7	9	629.4909	70.50908	700	2.6233	2.909779	1248.128	1.820364	Problem 2
	6	6	578.8426	21.15736	600	2.50	2.578789	1160.843	1.796833	Problem 3
	–	–	–	–	–	–	–	–	–	Problem 4
	11†	**11**	**982.2297**	**117.7703**	**1100**	**3.869506**	**4.361151**	**1998.109**	–	Problem 1
1.30	8	13	712.0136	87.98641	800	2.891499	3.250581	1598.334	1.903951	Problem 2
	7	10	644.8375	55.16252	700	2.669146	2.882208	1510.233	1.80714	Problem 3
	–	–	–	–	–	–	–	–	–	Problem 4

† bold denotes the optimal solution, (–) indicates infeasible solution.

Example 2. Here, the $C_{tF} = 1.15$ and the rest of the parameters are same as Example 1.

5. Sensitivity analysis

For the numerical Example 1 solved in Section 4, a sensitivity analysis was performed to study the impact of changes of different parameters. This analysis was done numerically by varying the parameters from – 20% to + 20% considering one parameter at a time and maintaining the other parameters with their original values. The results of this sensitivity analysis are given in Table 4.

Form Table 4, the following observations are made:

(1) The profit (Z) is highly sensitive with respect to the demand parameter a, purchasing cost C_p and parameter γ. The profit (Z) is moderately sensitive regarding to $C_h, C_{\text{a and}}\ \nu$ whereas insensitive with respect to the parameter b, C_s and C_o.

(2) Cycle length (T) of the inventory system is highly sensitive with respect to the parameter γ while moderately sensitive with respect to demand parameters ν, a, C_a and C_p. It is less sensitive with regard to the parameters C_o, C_h, C_s and b.
(3) On the one hand, the shortage level (R) is highly sensitive with respect toγ, ν and a. On the other hand, the shortage level (R) is moderately sensitive with regard to the parameter b, C_o, C_p, C_s, C_a and C_h.
(4) The stock-level (S) is insensitive with respect to the parameter C_o. It is highly sensitive with respect to the parameters a, γ and ν. However it is moderately sensitive with respect to the parameters C_p, C_s, C_a, C_h and b.

TABLE 3. Optimal solution for Example 2.

m	n^*	A	S	R	Q	t_1	T	Z	B	Optimization problem
	10†	**15**	**909.9565**	**100.8748**	**1010.831**	**3.6816**	**4.0987**	**1620.431**	–	Problem 1
1.25	7	9	629.4909	70.50908	700	2.6233	2.909779	1248.128	1.820364	Problem 2
	6	6	578.8426	21.15736	600	2.50	2.578789	1160.843	1.796833	Problem 3
	–	–	–	–	–	–	–	–	–	Problem 4
	11†	**20**	**985.3021**	**118.8056**	**1104.108**	**3.881455**	**4.378738**	**1998.122**	–	Problem 1
1.30	8	13	712.0136	87.9864	800	2.8914	3.2505	1598.33	1.9039	Problem 2
	7	10	644.8375	55.16252	700	2.669146	2.882208	1510.233	1.80714	Problem 3
	–	–	–	–	–	–	–	–	–	Problem 4

† bold denotes the optimal solution, (–) indicates infeasible solution.

6. Practical application of proposed inventory model

With changing trends in this volatile era it is visualized that demand is not always increasing or decreasing. The demand keeps changes with selling price of product. Also, the frequency of advertisement has great positive impact on the demand of product. This is the reason of why every organization invest more on the advertisement in order to attract more and more customers. Furthermore, the effect of deterioration of physical goods cannot be ignored while deciding inventory policy, since it has direct impact on the order quantity. This paper provides sound base for the inventory models for deteriorating items where demand of product depends at the same time on both selling price and the frequency of advertisement. Moreover, in many developing countries, suppliers offer to retailers various credit terms to stimulate sales and hence reduce inventory. Specifically for fashionable products companies want to clear the stock before the season ends. As well due to a variety of competitive products available in the market, costumer's willingness to wait has declined, thus shortages are assumed to be partially backlogged. Consequently, there exists the need to consider all these factors together at the planning or developing of inventory models because these jointly affect the purchasing behavior of the retailers. In this sense, this paper takes into consideration all above mentioned concepts to develop a practical inventory model which surely provides a benefit to the organizations.

7. Conclusion

This paper develops an inventory model for deteriorating items with variable demand dependent on the selling price and the frequency of advertisement of the item. This inventory model considers transportation cost explicitly for replenishing the order quantity and permissible delay in payment. It is important to remark that this inventory model can be applied in many situations of real life.

TABLE 4. Sensitivity analysis with respect to different parameters.

Parameter	% Change of parameters	% Change in					
		A^*	S^*	R^*	t_1^*	T^*	Z^*
C_o	– 20	14	−0.09	−0.79	0.61	0.74	0.74
	– 10	15	0.01	−0.07	0.01	0.00	0.38
	10	15	−0.01	0.07	−0.01	0.00	−0.38
	20	17	−1.03	1.32	7.09	9.39	−1.47
C_h	– 20	18	9.25	16.85	7.27	8.46	6.09
	– 10	15	0.37	−3.36	0.37	−0.08	3.04
	10	14	−0.47	4.29	0.22	0.84	−3.01
	20	28	−0.84	7.71	−0.16	0.94	−6.00
C_s	– 20	17	7.03	5.27	5.72	9.99	1.02
	– 10	17	6.78	4.41	6.45	9.65	0.46
	10	14	0.4	−3.72	1.11	0.62	−0.34
	20	15	0.96	−8.82	0.97	−0.2	−2.47
C_p	– 20	11	−1.01	−0.08	−2.11	−2.15	−32.07
	– 10	13	0.01	0.01	0.91	0.95	−15.76
	10	19	1.35	1.15	4.39	5.35	15.94
	20	20	2.47	2.04	5.80	8.28	34.52
C_a	– 20	21	8.86	20.47	5.28	6.90	2.62
	– 10	19	8.67	17.15	2.85	5.40	1.21
	10	13	−0.23	2.11	1.20	1.55	−1.07
	20	12	−0.37	−2.76	−3.58	0.96	−2.04
a	– 20	12	−20.50	−15.45	−3.00	−3.92	−23.90
	– 10	13	−10.31	−7.20	−1.71	−2.20	−12.02
	10	19	18.81	30.89	2.01	3.54	12.21
	20	20	29.09	38.39	3.65	4.70	24.58
b	– 20	17	2.66	−2.33	−2.20	−2.24	1.21
	– 10	15	0.09	−0.81	−0.44	−0.56	0.59
	10	15	−0.09	0.81	0.45	0.57	−0.59
	20	14	−0.28	2.55	1.50	1.90	−1.20
ν	– 20	11	−9.58	−13.87	−2.19	−2.81	−5.76
	– 10	14	−0.54	4.99	2.85	3.66	−2.97
	10	18	9.09	18.36	4.09	5.35	3.40
	20	22	18.27	35.87	7.00	9.33	7.03
γ	– 20	8	−30.34	−22.73	−32.00	−31.43	−28.98
	– 10	10	−19.63	−15.73	−20.14	−20.55	−16.45
	10	24	27.63	51.80	27.25	30.78	15.76
	20	34	46.56	81.60	47.46	52.64	30.67

For future research, the proposed inventory model can be extended in several forms. For example, different types of variable demand dependent on displayed stock-level or time can be included. On the other hand, it can also be generalized by considering two level credit policy. Furthermore, the inventory model can be studied in fuzzy and interval environments. These are some interesting and challenging research directions that can be explored by researchers and academicians in a near future.

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