RAIRO-Oper. Res. 53 (2019) 401–414
https://doi.org/10.1051/ro/2017030

RAIRO Operations Research
www.rairo-ro.org

IMPACT OF VEHICLE TRACKING ON A ROUTING PROBLEM WITH DYNAMIC TRAVEL TIMES

Jean Respen[1], Nicolas Zufferey[1,*] and Jean-Yves Potvin[2]

Abstract. This paper evaluates the benefits of data obtained *via* modern information technologies, such as global positioning systems, when solving a vehicle routing problem with dynamic customer requests and dynamic travel times. It is empirically demonstrated that substantial improvements are achieved over a previously reported model which does not assume the availability of such information. We also analyze how the system handles dynamic perturbations to the travel times that lead to earliness or lateness in the planned schedule.

Mathematics Subject Classification. 90B06.

Received June 1, 2016. Accepted April 21, 2017.

1. Introduction

Dynamic vehicle routing is attracting a growing attention in the research community. In these problems, some data are not know in advance, but are rather revealed in real-time while the routes are executed. Dynamically occurring customer requests have often been considered, but also dynamic customer demands and dynamic travel times. Since our work deals with dynamic customer requests and dynamic travel times, we provide a non exhaustive review of these variants in the following. Note that general considerations as well as surveys on different types of dynamic vehicle routing problems can be found in [6, 11, 15, 17, 18].

In [5], the authors propose a parallel tabu search heuristic for a vehicle routing problem with soft time windows in the presence of dynamic customer requests. In this work, a central dispatch office manages the planned routes. Furthermore, the vehicles are not aware of their planned routes and are informed of their next destination only when they have reached their current customer location. The optimization procedure runs in background and is interrupted when a vehicle reaches a customer or when a new customer request is received. At that point, the best known solution is returned and updated, based on the new information received, and a new optimization task is launched on the updated solution. An adaptive memory is also combined with the parallel

Keywords. Routing, transportation, dynamic travel times, diversion, time windows.

[1] Geneva School of Economics and Management, University of Geneva, 1211 Geneva 4, Switzerland.

[2] Département d'informatique et de recherche opérationnelle and CIRRELT, Université de Montréal, C.P. 6128, succ. Centre-ville, Montréal, Québec, Canada.

[*] Corresponding author: n.zufferey@unige.ch

tabu search to maintain a pool of interesting solution alternatives. It is shown that this algorithm improves over simple greedy heuristics when the optimization tasks can run long enough before they are interrupted. This work was later extended in [4] to address a courier service application where each new customer request is made of a pick-up and a delivery location, with a precedence constraint between the two locations.

The impact of diversion has also been studied in the literature. It consists in diverting a vehicle to a newly occurring customer request, close to the vehicle's current location, while en route to another destination. In [8], diversion is integrated within the tabu search heuristic reported in [5], and is shown to provide substantial improvements. Diversion is also considered in [7] where two different approaches are compared. The first approach, called sample-scenario planning, provides high-quality solutions, but at the expense of large computation times. At each step, a sample of likely-to-occur future customer requests is generated to obtain a number of scenarios. Robust planned routes are then computed based on these scenarios. The second method, called anticipatory-insertion heuristic, incorporates information about expected future customer requests when each new request is inserted into the current planned routes. In [2], it is argued that diversion can be a source of distraction for drivers and needs to be controlled. This is implemented through a penalty-based approach that accounts for the negative consequences of diversion.

As illustrated by the second method proposed in [7], the myopic behavior of methodologies developed for static problems can be alleviated by exploiting any probabilistic knowledge about the occurrence of future customer requests, either implicitly or explicitly. Different approaches are based on waiting and relocation strategies. In [1], for example, the vehicles can either wait at their current customer location or at any other site, to answer customer requests that are likely to occur in their vicinity. A similar idea is also found in [10]. Here, dummy customers in the planned routes stand for future, likely-to-occur, customer requests which are replaced by true requests when they occur. Another approach reported in the literature uses a short-term and a long-term objective, where the latter tends to introduce waiting times in the planned routes to facilitate the inclusion of future requests [14].

Dynamic travel times, where times can change due to road congestion, have also raised the attention of the research community. In [3], for example, a traffic management system forecasts the travel times, based on road conditions, and transmits this information to the dispatch office. The latter then takes appropriate actions in the context of a pickup and delivery problem, assuming that the communication between the dispatch office and the drivers is possible at all time. The authors also describe a general framework to account for dynamic travel times and report results based on traffic information from the city of Berlin, Germany.

The authors in [16] consider a vehicle routing problem with time windows and dynamic travel times. In this work, the travel times have three different components: static long-term forecasts (often referred to as time-dependent travel times in the literature), short-term forecasts, where the travel time on a link is modified with a random uniform value to account for any new information available when a vehicle is ready to depart from its current location, and dynamic perturbations caused by unforeseen events that may occur while traveling on a link (*e.g.*, an accident causing sudden congestion). A modification to a planned route is only possible when the vehicle is at a customer location. Hence, a planned route cannot be reconsidered while the vehicle is traveling on a link between two customer locations. An extension to this model is proposed in [12]. In this work, the position of each vehicle can be obtained when the lateness tolerance at a customer location is reached or when a new customer request occurs. Based on this information, the planned route of each vehicle is reconsidered, including the possibility of diversion (*i.e.*, redirecting a vehicle en route to its current destination). The results show that the setting of an appropriate lateness tolerance limit can provide substantial improvements. Here, we propose a further extension by assuming that the position of each vehicle is known at all time. This assumption allows the system to detect perturbations to the travel times and take appropriate actions much earlier.

This rest of the paper is organized as follows. A description of the problem is provided in Section 2. Next, Section 3 describes the two models in [12, 16] and explain the extension proposed here. Section 4 introduces travel time perturbations that lead to earliness in the planned schedule. The results obtained with the model in [12] and the new extension are then compared in Section 5. Finally, Section 6 concludes the paper and proposes future research avenues.

2. Problem description

The description of the problem is based on [12] where a fleet of vehicles performs routes, starting from and ending at a central depot, to collect goods at customer locations. Each customer must be visited exactly once by a vehicle within a (soft) time window. Some customer requests are said to be static, because they are known in advance and can be used to create initial planned routes. Other requests occur dynamically through the day and must be incorporated in real-time into the current solution. The ratio between the number of static requests and the total number of requests (static plus dynamic) is known as the degree of dynamism and is denoted d_{od} in the following [13]. Additional details on this topic can be found in [15].

Formally, let us consider a complete undirected graph $G = (V, E)$ with a set of vertices $V = \{0, 1, 2, \ldots, n\}$, where vertex 0 is the depot, and a set of arcs E. Each arc $(i, j) \in E$ is characterized by a travel time t_{ij}. Also, each vertex $i \in V \setminus \{0\}$ has a time window $[e_i, l_i]$. A vehicle can arrive before the lower bound e_i but must wait to start the service. Conversely, a vehicle can arrive after the upper bound l_i, but a (linear) penalty is incurred in the objective. We assume that K vehicles of virtually infinite capacity are available. Each vehicle performs a single route which must end before an upper bound l_0, otherwise another penalty is incurred in the objective.

The objective function f takes into account (1) the travel time, (2) the sum of lateness at customer locations and (3) the lateness at the depot. Denoting t_{i^k} the arrival time of vehicle k at customer $i \in V \setminus \{0\}$ (assuming that customer i is served by vehicle k) and by t_0^k the return time of vehicle k at the depot 0, the objective can be written as:

$$\begin{aligned} f(S) &= \sum_{k \in K} f(S^k) \\ &= \sum_{k \in K} \left(\alpha \sum_{p=1}^{m_k} t_{i^k_{p-1}, i^k_p} + \beta \sum_{p=1}^{m_k - 1} \max\{0, t_{i^k_{p-1}} - l_{i^k_{p-1}}\} + \gamma \max\{0, t_0^k - l_0\} \right) \end{aligned} \tag{2.1}$$

where $S = \bigcup_{k \in K} S^k$ represents a solution (a set of routes) and $S^k = \{i_0^k, i_1^k, \ldots, i_{m_k}^k\}$ is the route of vehicle $k \in K$, with $i_0^k = i_{m_k}^k = 0$. The weights α, β and γ are used to put more or less emphasis on travel time or lateness.

With regard to the static, time-dependent, component of the travel time, we split the operations day in three time periods for the morning, lunch time and afternoon. With each period is associated a coefficient that multiplies the average travel time (namely, 1.25 for the morning, 0.5 for the lunch time, and 1.25 for the afternoon). To guarantee that a vehicle leaving earlier from some customer location also arrives earlier at destination, which is known as the FIFO property, the travel times are adjusted when a boundary between two time periods is crossed, as in [9].

The travel times also suffer dynamic perturbations due, for example, to unexpected congestion. A dynamic perturbation is thus included based on a normal probability law with mean 0 and different standard deviations σ. Perturbations with negative values, leading to earliness, are reset to 0 in the first implementation, so that only lateness in the planned schedule can occur (as it is done in [12, 16]). In a second implementation, perturbations with negative values are also considered. Of course, the perturbation is unknown to the driver and the dispatch office.

3. Models

Three related models are presented in this section. The third model is an extension of the two previous ones.

3.1. Model 1

In Model 1 [16], a central dispatch office manages the planned route of each vehicle. It is assumed that communication between the drivers and the dispatch office takes place only at customer locations. When a driver has finished serving a customer, he communicates with the dispatch office to know his next destination. Hence, the drivers are not aware of their planned route, but only of the next customer to be served. The static requests are first used to construct initial routes through an insertion heuristic where, at each iteration, a customer is selected and inserted at the best possible place in the current routes (*i.e.*, with minimum increase in the objective value). At the end, a local search-based improvement procedure is triggered using CROSS exchanges [21], where sequences of customers are moved from one route to another. Finally, another local search-based improvement procedure is applied to each individual route, based on the relocation of each customer. Whenever a new dynamic request is received, the same insertion and reoptimization procedures are applied to update the planned routes.

Since travel times are dynamic, a lateness tolerance TL is defined, which is the maximum acceptable delay to a vehicle's planned arrival time at its current destination before some reassignment action is considered. For example, if we assume that s^k is the current destination of vehicle k and its planned arrival time is t_{s^k}, then $t_{s^k} + TL$ defines the tolerance time limit TTL^k of vehicle k. That is, if vehicle k has not reached customer s^k at time TTL^k, s^k is removed from its planned route and inserted in the planned route of some other vehicle l (note that vehicle k is not aware of this change and will continue toward s^k, as communication between the dispatch office and vehicles only take place at customer locations). If it happens that vehicle k still reaches s^k before vehicle l and while l is en route to s^k, then vehicle k serves s^k, but vehicle l will only know it when reaching s^k. A major drawback of this model thus relates to the limited communication scheme between the dispatch office and the vehicles.

3.2. Model 2

In [12], Model 1 was extended by adding diversion to allow any vehicle to be redirected to another customer, while en route to its current destination (if it provides some benefit with regard to the objective). When (a) a new customer request is received or (b) some vehicle k has reached its tolerance time limit, it is assumed that the dispatch office can obtain the current location of each vehicle to evaluate the benefits of a diversion. In case (a), a pure diversion of vehicle k to serve the newly occurring customer request is considered whereas, in case (b), the current destination of vehicle k is reassigned to another vehicle $l \neq k$. Vehicle k is then redirected to the customer that immediately follows (what was) its current destination in the planned route. Figure 1 illustrates these two cases. In Figure 1a, a new customer request s occurs while vehicle k is located at position x between vertices i^k_{p-1} and i^k_p. In this case, vehicle k will serve s before i^k_p if it is beneficial to do so. In Figure 1b, vehicle k has reached its tolerance time limit. Thus, its current destination $s = i^k_p$ is removed from its planned route and is reassigned to another vehicle l, while vehicle k is redirected to i^k_{p+1}. The results reported in [12] show that Model 2 significantly outperforms Model 1. Also, empirical results demonstrated that the best TL value is 0. That is, an appropriate action must be considered as soon as a perturbation to the planned schedule is detected.

3.3. Model 3

The new model proposed here extends Model 2 by assuming that the position of each vehicle is known at all time, and not only when the two types of situations described above for Model 2 occur (note that preliminary results regarding Model 3 have been presented in [19]). We have considered here an implementation of Model 3 where the dynamic perturbation component of the travel time is distributed uniformly along a link. Next, as soon as it is impossible for vehicle k to arrive at its current destination at time TTL^k, a reassignment action is considered. For example, let us assume that vehicle k departs from i to j at time t, with a travel time $t_{ij} = 5$ and a dynamic perturbation $\Delta t_{ij} = 3$. That is, the vehicle is planned to arrive at j at time $t + 5$, but will in fact arrive only at time $t + 8$. If $TL = 0$, then $TTL^k = t + 5$ and Model 2 will consider a reassignment at time $t + 5$ when it is observed that vehicle k has not yet reached j. On the other hand, by tracking the position of each vehicle, Model 3 can detect the problem much earlier.

From a more theoretical point of view, if we assume that a vehicle leaves i for j at time t under a uniformly distributed perturbation along arc (i, j) and that $\Delta t_{ij} > TL$, then the fraction $\bar{x}$ of the travel time performed

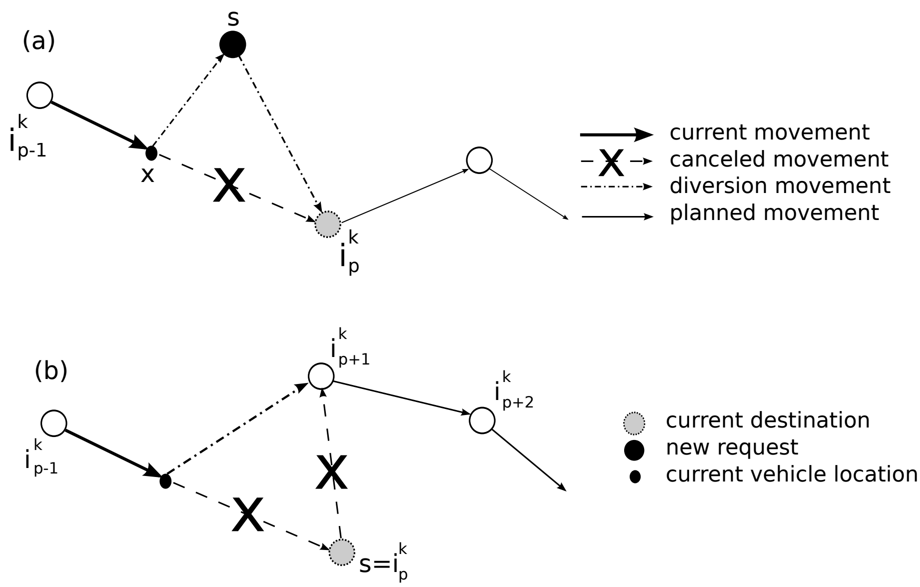

FIGURE 1. Diversion and reassignment actions.

under the perturbation before lateness can be detected is:

$$\bar{x} = \inf \left\{ x : x \cdot (t_{ij} + \Delta t_{ij}) + (1 - x) \cdot t_{ij} > t_{ij} + TL \right\}.$$

An equivalent formulation is

$$\bar{x} = \inf \left\{ x : x > \frac{TL}{\Delta t_{ij}} \right\}$$

given that

$$\begin{aligned} & x \cdot (t_{ij} + \Delta t_{ij}) + (1 - x) \cdot t_{ij} > t_{ij} + TL \\ \Leftrightarrow \quad & t_{ij} + x \cdot \Delta t_{ij} > t_{ij} + TL \\ \Leftrightarrow \quad & x > \frac{TL}{\Delta t_{ij}}. \end{aligned}$$

Thus, lateness will be detected at time $t + \bar{t}$, where $\bar{t} = \bar{x} \cdot (t_{ij} + \Delta t_{ij})$. When $TL = 0$, we have:

$$\bar{x} = \inf \{ x : x > 0 \}$$

and $\bar{x}$, as well as $\bar{t}$, tend toward 0. In practice, the vehicle needs to move for lateness to be detected by the dispatch office, and $\bar{t}$ would correspond to a small value greater than 0. Additional information about the way to simulate Model 3 is provided in Section 5.

4. Earliness

As stated earlier, only positive perturbations to the travel times that lead to lateness in a vehicle schedule were considered in [12], by resetting any negative value to 0. Negative perturbation values, leading to earliness in a vehicle schedule (*i.e.*, the vehicle will arrive earlier than expected at its current destination) have been tested here for both Models 2 and 3. If some vehicle l is late, then the earliness in the schedule of another vehicle k

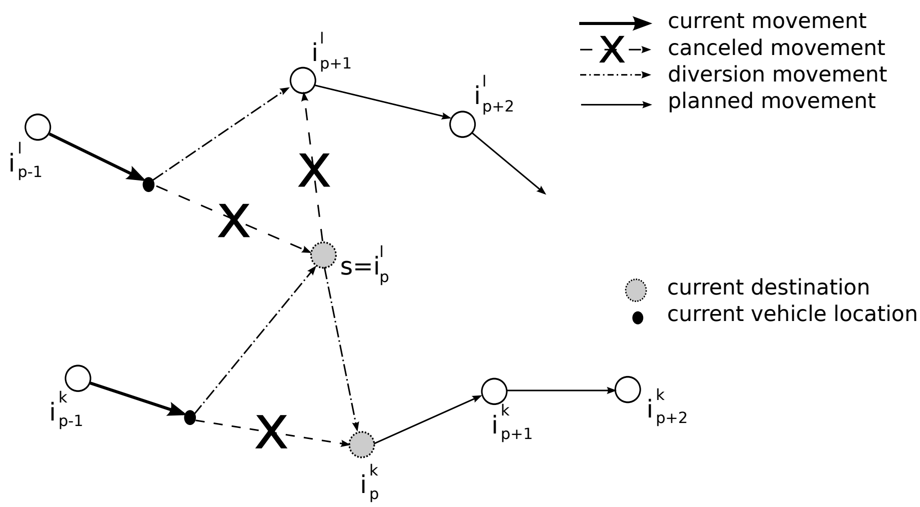

FIGURE 2. Integrating earliness into the model.

will automatically be exploited by the optimization procedure. That is, the current destination of vehicle l will likely be transferred to vehicle k. The benefits of Model 3 over Model 2 in this situation are the same as those mentioned for positive perturbation values: it will be possible to detect the earliness and lateness in the vehicle schedules before the vehicles reach their current destination and, consequently, react more promptly. Figure 2 illustrates this capability. In the figure, vehicle k is currently traveling between customers i^k_{p-1} and i^k_p and is ahead of its schedule. Similarly, another vehicle l is traveling between customers i^l_{p-1} and i^l_p and is late. Next, vehicle k can be redirected to i^l_p and vehicle l to i^l_{p+1} while both vehicles are en route.

5. Computational results

Tests were performed on a 3.4 GHz Intel Quad-core i7 with 8 GB of DDR3 RAM memory. The 100-customer Euclidean Solomon's benchmark instances [20] were used to compare Models 2 and 3. In these instances, the Euclidean distance is the same as the travel time (*i.e.*, one travel time unit in Solomon's instances corresponds to one Euclidean distance unit). Any dynamic customer request i was set to occur at time $e_i \cdot r$, where e_i is the lower bound of the time window at customer i and r is a random number between 0 and 1. Parameters α, β and γ were set to 1 in the objective. For these experiments, only the three classes of instances $R2$, $C2$ and $RC2$ with 11, 8 and 8 instances, respectively, were considered due to their large time horizon which allows for many customers per route. Note that customers are randomly generated in $R2$, clustered in $C2$ or both clustered and randomly generated in $RC2$. Note also that the computing times are not commented given that the optimization takes place within a fraction of a second.

The computational results were obtained through a discrete time simulation. In this simulation, and following the example discussed in Section 3.3, vehicle k has still to cover $\frac{7}{8}$ of the distance at time $t+1$, so that the planned arrival time at location j can be updated to $t+1+\frac{7}{8}\cdot 5$, which already exceeds $TTL^k = t+5$. Recall that in practice, the vehicle needs to move for lateness to be detected by the dispatch office, and, according to the theory presented in Section 3.3, $\bar{t}$ would correspond to a small value greater than 0. In our computational results, this is naturally achieved through our discrete time simulation (*i.e.*, lateness is detected at time $t+1$). In a sense, we have almost perfect knowledge about lateness when $TL = 0$, and the results reported here will quantify the benefits obtained in this case *versus* $TL > 0$. For sake of completeness, let us assume again that a vehicle is moving from i to j and that its position at the current time t is i'. If the vehicle is diverted to a new

TABLE 1. Results of Model 3 on class R2.

TL	$\sigma = 1$	$\sigma = 4$	$\sigma = 16$	$\sigma = 32$
0	1548.57 (52.76,8.86%)	2099.08 (54.65,15.77%)	4779.91 (67.24,45.86%)	7322.89 (77.35,65.35%)
$1 \cdot \sigma$	1623.17 (15.96,7.86%)	2288.34 (16.24,13.89%)	6641.03 (16.58,34.10%)	15201.17 (15.05,44.69%)
$2 \cdot \sigma$	1634.21 (2.05,9.73%)	2326.33 (2.04,12.50%)	6945.38 (1.75,18.75%)	15211.18 (1.33,27.40%)
$3 \cdot \sigma$	1640.09 (0.07,25.00%)	2335.04 (0.07,0.00%)	6925.79 (0.04,0.00%)	15260.43 (0.04,0.00%)
$1000 \cdot \sigma$	1641.17 (0.00,-)	2335.04 (0.00,-)	6925.79 (0.00,-)	15260.43 (0.00,-)
Improv.	5.64%	10.11%	31.00%	52.01%

TABLE 2. Results of Model 3 on class C2.

TL	$\sigma = 1$	$\sigma = 4$	$\sigma = 16$	$\sigma = 32$
0	2145.41 (51.65,6.53%)	2746.84 (51.68,7.21%)	6047.75 (53.70,12.90%)	10865.41 (58.20,26.29%)
$1 \cdot \sigma$	2419.65 (15.80,8.39%)	3054.61 (15.75,9.52%)	6722.66 (15.48,15.51%)	12808.44 (12.50,28.60%)
$2 \cdot \sigma$	2696.53 (2.03,11.11%)	3311.12 (2.03,9.88%)	7119.54 (1.75,18.57%)	13263.69 (0.90,33.33%)
$3 \cdot \sigma$	2725.66 (0.10,50.00%)	3349.47 (0.10,25.00%)	7197.18 (0.10,25.00%)	13326.74 (0.00,-)
$1000 \cdot \sigma$	2728.99 (0.00,-)	3359.17 (0.00,-)	7200.55 (0.00,-)	13326.74 (0.00,-)
Improv.	21.38%	18.23%	16.01%	18.47%

location j' at time t, then a perturbation needs to be generated for the travel from i' to j', because the vehicle does not follow anymore the arc (i, j).

Tables 1, 2, and 3 show the results on classes $R2$, $C2$, and $RC2$ respectively, for various lateness tolerances TL and σ values in Euclidean units, where σ is the variance of the dynamic perturbations to the travel times. The selected values for σ (namely 1, 4, 16, 32) account for small, medium, large and very large perturbations. We have considered that tolerances are functions of σ because we tend to be more tolerant to lateness in a very uncertain environment with large perturbations to the travel times (and conversely). Each entry in the tables correspond to the average objective value over ten runs, using ten different seeds, and over each instance of a given class. There is also a pair of numbers between parentheses: the first number is the average number of times a reassignment action was considered (per instance) and the second number is the percentage of reassignments that were undertaken because they proved to be beneficial. The last row with $TL = 1000 \cdot \sigma$ is an extreme case where no action is taken. That is, the planned routes are followed whatever the perturbation. The degree of dynamism d_{od} was set to 0.5 and only lateness with regard to the current schedule was allowed (*i.e.*, negative perturbations to the travel times were reset to 0). The last line in each table reports the improvement percentage in the objective function obtained with $TL = 0$ over $TL = 1000 \cdot \sigma$.

First, the results are consistent with regard to the variation of the tolerance level TL, as the number of times a reassignment action was considered (per instance) increases when TL decreases. Second, small TL values lead to better results, with the best value being $TL = 0$. This result was expected given that $TL = 0$ corresponds

TABLE 3. Results of Model 3 on class RC2.

TL	$\sigma = 1$	$\sigma = 4$	$\sigma = 16$	$\sigma = 32$
0	1515.47 (52.03,6.78%)	1878.14 (53.98,13.06%)	3975.85 (63.75,36.39%)	6205.70 (73.58,56.81%)
$1 \cdot \sigma$	1572.37 (15.85,4.73%)	2006.65 (16.03,10.30%)	5313.23 (15.38,27.80%)	11370.48 (13.75,36.91%)
$2 \cdot \sigma$	1590.77 (1.98,3.80%)	2037.86 (1.98,7.59%)	5490.76 (1.88,13.33%)	11641.22 (1.25,26.00%)
$3 \cdot \sigma$	1591.66 (0.10,0.00%)	2041.68 (0.10,0.00%)	5495.05 (0.10,25.00%)	11685.73 (0.08,66.67%)
$1000 \cdot \sigma$	1591.66 (0.00,-)	2041.68 (0.00,-)	5501.21 (0.00,-)	11734.49 (0.00,-)
Improv.	4.79%	8.01%	27.73%	47.12%

to (almost) perfect information about the occurrence of lateness. Third, the percentage PR of reassignments that provide an improvement generally increases with σ. This was also expected, given that the current plan is likely to be improved when large perturbations are encountered. Finally, PR tends to decrease with TL on the instances of classes $R2$ and $RC2$, because the later we are, the less likely we are to improve the situation with a reassignment action (due to the random component in the customer locations, the vehicles will tend to be randomly distributed in the network). This trend is not observed on class $C2$, because a late vehicle can possibly be helped by another vehicle located in the same cluster.

Table 4 shows the objective values as well as the percentage of improvement of Model 3 over Model 2 when $TL = 0$ for d_{od} values equal to 0.1, 0.3, 0.5, 0.7 and 0.9. Although we only show these results, additional experiments with other TL values led to the same observation, namely, that Model 3 is clearly superior to Model 2 due to its ability to detect perturbations to the current plan much earlier. The improvement is quite substantial in the case of $R2$ and $RC2$, and can even reach 30% for large σ values. The results are less impressive in the case of $C2$ (Model 3 is even worse than Model 2 for $d_{od} = 0.9$ and $\sigma = 16$). This observation can be explained by the geographical clustering of customers which seriously limits the benefit of redirecting a vehicle, for example to serve a distant customer in another cluster. Interestingly, it can also be observed that, for the non-clustered instances with smaller perturbations, the gap between the two models generally increases with the degree of dynamism d_{od} (number of dynamic requests). Table 5 summarizes the results shown in Table 4 by reporting the average improvement over all d_{od} values (*i.e.*, 0.1, 0.3, 0.5, 0.7 and 0.9) for each class of instances, as well as over all classes of instances (see the last line). This table shows that the benefit of Model 3 increases on the one hand when the clustering of the requests decreases (as it is the case, for example, in urban areas), and on the other hand when the traffic perturbations increase (which again appears more often in urban areas).

Tables 6, 7, and 8 are similar to Tables 1, 2, and 3 and report the results of Model 3 for various tolerance TL and σ values with $d_{od} = 0.5$ when negative perturbations to the travel times are allowed (leading to earliness in the schedule). Tables 9 and 10 are also similar to Tables 4 and 5. Table 9 reports the improvement of Model 3 over Model 2 with $TL = 0$ for d_{od} values equal to 0.1, 0.3, 0.5, 0.7 and 0.9 when negative perturbations are allowed. Table 10 summarizes the results shown in Table 9 by reporting the average improvement over all d_{od} values (*i.e.*, 0.1, 0.3, 0.5, 0.7 and 0.9) for each class of instances, as well as over all classes of instances (see the last line). Overall, the trends are the same as those observed previously but are somewhat accentuated, in particular for large σ values. In other words, the benefit of Model 3 increases when it accounts for travels that are performed faster than expected, which actually better captures real-case situations [22].

Recall that the objective function f takes into account three components: the travel time f_1, the sum f_2 of lateness at customer locations, and the lateness f_3 at the depot. Tables 11 and 12 present the average proportion of each objective function component f_i on all instances with $d_{od} = 0.5$ and $TL = 0$. Table 11 reports the results

TABLE 4. Improvement of Model 3 over Model 2 with $TL = 0$ for different d_{od} values.

d_{od}			$\sigma = 1$	$\sigma = 4$	$\sigma = 16$	$\sigma = 32$
0.1	**R2**	Model 2	1183.59	1653.41	4827.73	8606.15
		Model 3	1168.82	1598.71	4097.30	6545.20
		% Imprv.	1.26%	3.42%	17.83%	31.49%
	C2	Model 2	1195.68	1536.89	4073.00	8578.14
		Model 3	1171.77	1533.58	3920.30	8230.87
		% Imprv.	2.04%	0.22%	3.89%	4.22%
	RC2	Model 2	1267.60	1557.40	4002.04	7499.71
		Model 3	1259.96	1524.96	3512.06	5805.93
		% Imprv.	0.61%	2.13%	13.95%	29.17%
0.3	**R2**	Model 2	1381.69	1933.86	5334.02	9228.19
		Model 3	1346.87	1853.73	4520.57	7097.35
		% Imprv.	2.59%	4.32%	17.99%	30.02%
	C2	Model 2	1645.09	2014.47	5269.25	9705.95
		Model 3	1642.23	1970.29	5103.47	9466.89
		% Imprv.	0.17%	2.24%	3.25%	2.53%
	RC2	Model 2	1460.75	1857.16	4521.94	8105.48
		Model 3	1442.85	1775.30	3815.26	6338.51
		% Imprv.	1.24%	4.61%	18.52%	27.88%
0.5	**R2**	Model 2	1604.03	2245.95	5891.80	9636.36
		Model 3	1548.57	2099.08	4779.92	7322.89
		% Imprv.	3.58%	7.00%	23.26%	31.59%
	C2	Model 2	2193.26	2769.02	6221.36	11652.32
		Model 3	2145.41	2746.84	6047.76	10865.42
		% Imprv.	2.23%	0.81%	2.87%	7.24%
	RC2	Model 2	1551.26	1949.16	4621.90	8426.49
		Model 3	1515.47	1878.14	3975.86	6205.70
		% Imprv.	2.36%	3.78%	16.25%	35.79%
0.7	**R2**	Model 2	2014.79	2823.41	6553.96	10366.10
		Model 3	1932.65	2599.74	5292.18	8037.42
		% Imprv.	4.25%	8.60%	23.84%	28.97%
	C2	Model 2	2444.12	2806.64	6075.49	11937.20
		Model 3	2340.94	2790.25	5809.04	10929.91
		% Imprv.	4.41%	0.59%	4.59%	9.22%
	RC2	Model 2	1805.32	2343.49	5311.32	9153.11
		Model 3	1747.96	2185.97	4708.65	6948.25
		% Imprv.	3.28%	7.21%	12.80%	31.73%
0.9	**R2**	Model 2	2182.42	2994.81	6961.94	11386.24
		Model 3	2044.20	2769.36	5728.86	8902.86
		% Imprv.	6.76%	8.14%	21.52%	27.89%
	C2	Model 2	3498.62	4337.93	8702.07	15191.83
		Model 3	3414.02	4216.22	8941.81	14441.63
		% Imprv.	2.48%	2.89%	-2.68%	5.19%
	RC2	Model 2	2097.39	2782.93	6743.54	10730.58
		Model 3	2008.42	2554.65	5759.39	8285.90
		% Imprv.	4.43%	8.94%	17.09%	29.50%

TABLE 5. Average improvement of Model 3 over Model 2 with $TL = 0$ over all d_{od} values.

	$\sigma = 1$	$\sigma = 4$	$\sigma = 16$	$\sigma = 32$
$R2$	3.69%	6.30%	20.89%	29.99%
$C2$	2.27%	1.35%	2.38%	5.68%
$RC2$	2.38%	5.33%	15.72%	30.81%
Overall	2.78%	4.33%	13.00%	22.16%

TABLE 6. Results of Model 3 on class R2 using positive and negative perturbations.

TL	$\sigma = 1$	$\sigma = 4$	$\sigma = 16$	$\sigma = 32$
0	1408.23 (53.13,8.80%)	1576.70 (54.82,14.23%)	2951.06 (66.20,42.85%)	4156.69 (76.67,62.81%)
$1 \cdot \sigma$	1474.64 (16.20,7.58%)	1719.48 (16.35,14.17%)	4840.45 (16.71,34.33%)	12027.22 (15.36,45.03%)
$2 \cdot \sigma$	1488.51 (2.12,8.58%)	1753.46 (2.10,10.82%)	5176.51 (1.90,25.36%)	12765.64 (1.39,40.52%)
$3 \cdot \sigma$	1490.47 (0.13,14.29%)	1761.16 (0.12,15.38%)	5160.00 (0.09,10.00%)	12485.61 (0.07,12.50%)
$1000 \cdot \sigma$	1491.05 (0.00,-)	1762.37 (0.00,-)	5161.13 (0.00,-)	12489.44 (0.00,-)
Improv.	5.55%	11.78%	42.82%	66.72%

TABLE 7. Results of Model 3 on class C2 using positive and negative perturbations.

TL	$\sigma = 1$	$\sigma = 4$	$\sigma = 16$	$\sigma = 32$
0	2208.83 (52.21,6.22%)	2480.35 (52.43,7.20%)	4934.58 (54.14,12.65%)	8706.28 (57.91,24.15%)
$1 \cdot \sigma$	2433.25 (15.90,8.18%)	2697.11 (15.95,9.01%)	5550.78 (15.54,14.88%)	10736.38 (12.54,26.32%)
$2 \cdot \sigma$	2592.38 (2.09,10.18%)	2877.84 (2.08,12.65%)	5923.32 (1.73,21.01%)	11454.67 (0.89,29.58%)
$3 \cdot \sigma$	2639.34 (0.14,27.27%)	2932.43 (0.14,9.09%)	5999.76 (0.11,22.22%)	11449.34 (0.04,0.00%)
$1000 \cdot \sigma$	2638.97 (0.00,-)	2933.91 (0.00,-)	6018.05 (0.00,-)	11449.34 (0.00,-)
Improv.	16.30%	15.46%	18.00%	23.96%

obtained when only positive perturbations are allowed, whereas Table 12 reports similar results when both positive and negative perturbations are allowed. The proportions in each cell are given in the order f_1, f_2, f_3. For example, with only positive perturbations and $\sigma = 1$, Table 11 indicates that the contribution of f_1 (resp. f_2 and f_3) is 70.27% (resp. 27.21% and 2.52%) for the instances in class $R2$. Unsurprisingly, the following observations can be made: (1) f_1 has a major impact on f (particularly on the non-clustered instances), as its contribution usually ranges between 30% and 70%; (2) f_3 has the smallest contribution to f, as it is always below 12%; (3) the contribution of f_2 (and even f_3) increases with σ (indeed, if larger perturbations are applied to the travel times, more lateness is likely to occur at customer locations or at the depot); (4) the contribution

TABLE 8. Results of Model 3 on class RC2 with positive and negative perturbations.

TL	$\sigma = 1$	$\sigma = 4$	$\sigma = 16$	$\sigma = 32$
0	1444.06 (53.05,7.28%)	1570.02 (54.51,12.27%)	2602.30 (65.19,37.03%)	3583.52 (74.21,55.47%)
$1 \cdot \sigma$	1489.27 (15.96,6.50%)	1669.12 (16.15,12.00%)	3931.03 (15.61,29.78%)	8636.15 (13.38,39.81%)
$2 \cdot \sigma$	1503.83 (2.06,8.48%)	1712.14 (2.10,16.07%)	4122.88 (1.83,24.66%)	9111.62 (1.39,28.83%)
$3 \cdot \sigma$	1508.63 (0.14,0.00%)	1722.23 (0.14,0.00%)	4150.32 (0.14,18.18%)	9185.94 (0.05,25.00%)
$1000 \cdot \sigma$	1508.63 (0.00,-)	1722.23 (0.00,-)	4145.58 (0.00,-)	9184.10 (0.00,-)
Improv.	4.28%	8.84%	37.23%	61.00%

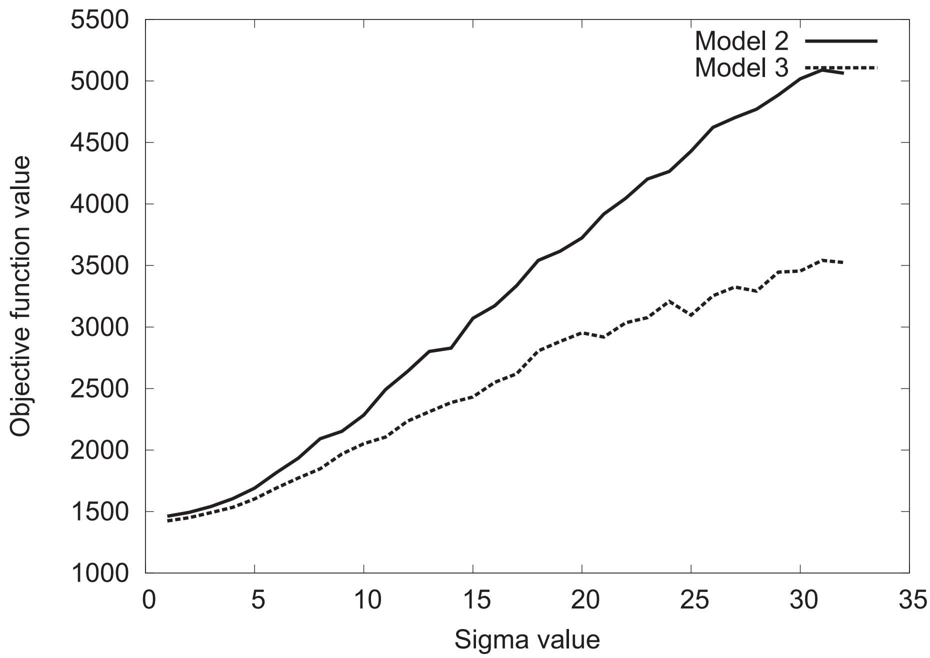

FIGURE 3. Average objective values of Models 2 and 3 on class $RC2$ with $TL = 0$, $d_{od} = 0.5$ and different σ values.

of f_2 and f_3 are reduced if negative perturbations are allowed (indeed, faster trips can compensate for slower trips, resulting in less lateness at customer locations).

Finally, Figure 3 illustrates the average objective values of Models 2 and 3 with $TL = 0$ and $d_{od} = 0.5$ for a large number of σ values taken between 1 and 32 using the instances of class $RC2$. In this figure, negative perturbations to the travel times are allowed. This figure clearly shows that the gap between the two models sharply increases with σ.

TABLE 9. Improvement of Model 3 over Model 2 with $TL = 0$ for different d_{od} values, using positive and negative perturbations.

d_{od}			$\sigma = 1$	$\sigma = 4$	$\sigma = 16$	$\sigma = 32$
	R2	Model 2	1093.38	1238.16	3091.47	5400.52
0.1		Model 3	1082.36	1208.02	2531.94	3723.15
		% Imprv.	1.02%	2.50%	22.10%	45.05%
	C2	Model 2	1180.54	1361.29	3160.62	6975.13
		Model 3	1167.54	1343.24	3095.66	6527.79
		% Imprv.	1.11%	1.34%	2.10%	6.85%
	RC2	Model 2	1205.74	1322.11	2749.25	4765.88
		Model 3	1199.42	1300.70	2298.89	3178.00
		% Imprv.	0.53%	1.65%	19.59%	49.96%
	R2	Model 2	1255.76	1445.00	3417.23	5638.62
0.3		Model 3	1229.63	1377.47	2679.75	3874.91
		% Imprv.	2.12%	4.90%	27.52%	45.52%
	C2	Model 2	1626.13	1767.72	3902.18	7793.65
		Model 3	1607.29	1735.47	3651.05	7159.94
		% Imprv.	1.17%	1.86%	6.88%	8.85%
	RC2	Model 2	1372.78	1541.12	3208.57	5321.58
		Model 3	1356.16	1481.81	2532.72	3579.10
		% Imprv.	1.23%	4.00%	26.68%	48.68%
	R2	Model 2	1454.72	1668.62	3741.13	5996.84
0.5		Model 3	1408.23	1576.70	2951.06	4156.69
		% Imprv.	3.30%	5.83%	26.77%	44.27%
	C2	Model 2	2269.21	2496.09	5110.58	9508.50
		Model 3	2208.83	2480.35	4934.58	8706.28
		% Imprv.	2.73%	0.63%	3.57%	9.21%
	RC2	Model 2	1478.08	1634.76	3338.04	5278.44
		Model 3	1444.06	1570.02	2602.30	3583.52
		% Imprv.	2.36%	4.12%	28.27%	47.30%
	R2	Model 2	1803.15	2053.27	4159.67	6545.16
0.7		Model 3	1728.48	1909.71	3264.42	4409.19
		% Imprv.	4.32%	7.52%	27.42%	48.44%
	C2	Model 2	2310.80	2422.14	4852.43	9672.17
		Model 3	2217.70	2336.18	4573.30	8745.50
		% Imprv.	4.20%	3.68%	6.10%	10.60%
	RC2	Model 2	1669.42	1853.11	3614.90	5818.96
		Model 3	1625.96	1747.68	2805.90	3807.39
		% Imprv.	2.67%	6.03%	28.83%	52.83%
	R2	Model 2	1965.55	2283.00	4631.17	7386.65
0.9		Model 3	1861.09	2099.15	3610.27	4990.11
		% Imprv.	5.61%	8.76%	28.28%	48.03%
	C2	Model 2	3323.86	3621.42	6403.54	12274.79
		Model 3	3261.67	3446.48	6610.18	11123.78
		% Imprv.	1.91%	5.08%	-3.13%	10.35%
	RC2	Model 2	2112.12	2381.04	5101.59	7644.64
		Model 3	2016.67	2195.41	3943.92	5532.53
		% Imprv.	4.73%	8.46%	29.35%	38.18%

TABLE 10. Average improvement of Model 3 over Model 2 with $TL = 0$ over all d_{od} values, using positive and negative perturbations.

	$\sigma = 1$	$\sigma = 4$	$\sigma = 16$	$\sigma = 32$
$R2$	3.28%	5.90%	26.42%	46.26%
$C2$	2.22%	2.52%	3.10%	9.17%
$RC2$	2.30%	4.85%	26.55%	47.39%
Overall	2.60%	4.42%	18.69%	34.28%

TABLE 11. Contribution (in %) of each objective function component (f_1, f_2, f_3) on the whole objective function value f with $TL = 0$ and $d_{od} = 0.5$ when only positive perturbations are allowed.

	$\sigma = 1$			$\sigma = 4$			$\sigma = 16$			$\sigma = 32$		
R2	70.27	27.21	2.52	58.83	36.92	4.25	35.83	57.71	6.46	26.82	66.06	7.12
C2	47.38	41.89	10.73	43.02	45.3	11.68	27.47	61.24	11.29	19.67	69.88	10.45
RC2	84.43	15.12	0.45	77.04	21.78	1.18	47.74	46.43	5.82	36.71	55.81	7.49

TABLE 12. Contribution (in %) of each objective function component (f_1, f_2, f_3) on the whole objective function value f with $TL = 0$ and $d_{od} = 0.5$ when both positive and negative perturbations are allowed.

Class	$\sigma = 1$			$\sigma = 4$			$\sigma = 16$			$\sigma = 32$		
R2	74.95	23.04	2.01	70.15	26.79	3.06	47.97	46.24	5.79	37.67	55.95	6.38
C2	48.42	40.85	10.73	45.76	42.93	11.3	30.66	58.5	10.84	21.85	67.84	10.31
RC2	86.6	13.06	0.34	83.54	15.83	0.63	63.15	33.53	3.32	50.53	44.17	5.31

6. CONCLUSION

This paper has investigated the benefits of exploiting data obtained *via* modern information technologies, such as global positioning systems, when solving a dynamic vehicle routing problem with time windows. The dynamic characteristics relate to the occurrence of new customer requests and perturbations to the travel times. We empirically quantified the benefits of (almost) perfect knowledge about lateness (*i.e.*, $TL = 0$ *versus* $TL > 0$). Our new model also proved to be significantly superior to another model where the location of each vehicle is only known at specific times during the operations day. In practice, some adjustments would be required to implement the new model because a driver is likely to be faced with many diversions during his working day. This situation could eventually be addressed through a limit on the number of diversions or through some penalty in the objective. In the future, we plan to investigate the impact of other probability distributions to model the travel time perturbations as well as other ways to distribute the perturbation along a link.

REFERENCES

[1] R. Bent and P. Van Hentenryck, Waiting and Relocation Strategies in Online Stochastic Vehicle Routing. In *IJCAI'07 Proc. of the 20th international joint conference on Artifical intelligence* (2007) 1816–1821.

[2] F. Ferrucci and S. Bock, A General Approach for Controlling Vehicle En-Route Diversions in Dynamic Vehicle Routing Problems. *Transp. Res. – Part B* **77** (2015) 76–87.

[3] B. Fleischmann, S. Gnutzmann and E. Sandvoß, Dynamic Vehicle Routing based on Online Traffic Information. *Trans. Sci.* **38** (2004) 420–433.

[4] M. Gendreau, F. Guertin, J.-Y. Potvin and R. Séguin, Neighborhood Search Heuristics for a Dynamic Vehicle Dispatching Problem with Pickups and Deliveries. *Transportation Research Part C: Emerging Technologies* **14** (2006) 157–174.

[5] M. Gendreau, F. Guertin, J.-Y. Potvin and É. Taillard, Parallel Tabu Search for Real-Time Vehicle Routing and Dispatching. *Trans. Sci.* **33** (1999) 381–390.

[6] M. Gendreau and J.-Y. Potvin, Dynamic Vehicle Routing and Dispatching. In Fleet Management and Logistics. Edited by T.G. Crainic and G. Laporte. Kluwer (1998) 115–126.

[7] G. Ghiani, E. Manni and B.W. Thomas, A Comparison of Anticipatory Algorithms for the Dynamic and Stochastic Traveling Salesman Problem. *Trans. Sci.* **46** (2012) 374–387.

[8] S. Ichoua, M. Gendreau and J.-Y. Potvin, Diversion Issues in Real-Time Vehicle Dispatching. *Transp. Sci.* **34** (2000) 426–438.

[9] S. Ichoua, M. Gendreau and J.-Y. Potvin, Vehicle Dispatching with Time-Dependent Travel Times. *Eur. J. Operat. Res.* **144** (2003) 379–396.

[10] S. Ichoua, M. Gendreau and J.-Y. Potvin, Exploiting Knowledge about Future Demands for Real-Time Vehicle Dispatching. *Trans. Sci.* **40** (2006) 211–225.

[11] A. Larsen, O.B.G. Madsen and M.M. Solomon, Recent Developments in Dynamic Vehicle Routing Systems. In The Vehicle Routing Problem: Latest Advances and New Challenges, edited by B. Golden, S. Raghavan and E. Wasil. In Vol. 42 of *Operations Research/Computer Science Interfaces.* Springer (2008) 199–218.

[12] S. Lorini, J.-Y. Potvin and N. Zufferey, Online Vehicle Routing and Scheduling with Dynamic Travel Times. *Comput. Oper. Res.* **38** (2011) 1086–1090.

[13] K. Lund, O.B.G. Madsen and J.M. Rygaard, *Vehicle Routing Problems with Varying Degrees of Dynamism.* Technical Report IMM-REP-1996-1. Technical University of Denmark (1996).

[14] S. Mitrović-Minić, R. Krishnamurti and G. Laporte, Double-Horizon Based Heuristics for the Dynamic Pickup and Delivery Problem with Time Windows. *Trans. Res. Part B: Methodological* **38** (2004) 669–685.

[15] V. Pillac, M. Gendreau, C. Guéret and A.L. Medaglia, A Review of Dynamic Vehicle Routing Problems. *Eur. J. Oper. Res.* **225** (2013) 1–11.

[16] J.-Y. Potvin, Y. Xu and I. Benyahia, Vehicle Routing and Scheduling with Dynamic Travel Times. *Comput. Oper. Res.* **33** (2006) 1129–1137.

[17] H.N. Psaraftis, Dynamic Vehicle Routing: Status and Prospects. *Ann. Oper. Res.* **61** (1995) 143 – 164.

[18] H.N. Psaraftis, M. Wen and C.A. Kontovas, Dynamic Vehicle Routing Problems: Three Decades and Counting. *Networks* **67** (2016) 3 – 31.

[19] J. Respen, N. Zufferey and J.-Y. Potvin, Online Vehicle Routing and Scheduling with Continuous Vehicle Tracking. In *Proc. of the ROADEF 2014, Bordeaux, France. February 26 to 28* (2014).

[20] M.M. Solomon, Algorithms for the Vehicle Routing and Scheduling Problem with Time Window Constraints. *Oper. Res.* **35** (1987) 254–265.

[21] É. Taillard, P. Badeau, M. Gendreau, F. Guertin and J.-Y. Potvin, A Tabu Search Heuristic for the Vehicle Routing Problem with Soft Time Windows. *Trans. Sci.* **31** (1997) 170–186.

[22] N. Zufferey, B.Y. Cho and R. Glardon, Dynamic Multi-Trip Vehicle Routing with Unusual Time-Windows for the Pick-Up of Blood Samples and Delivery of Medical Material. In *Proc. of the 5th International Conference on Operations Research and Enterprise Systems (ICORES 2016), Rome, Italy* (2016).