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Large deviations for multiple Wiener–Itô integral processes

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Abstract

For $m \geq 1$ let $I_m(h)$ denote the multiple Wiener–Itô integral of order m of a square integrable symmetric kernel h. In this paper we consider different conditions on a time-dependent family of kernels $\{h_t, 0 \leq t \leq 1\}$ which guarantee that the process $I_m(h_t)$ has continuous sample paths and that the probability measures induced by $\epsilon^{m/2} I_m(h_t)$ satisfy a large deviations principle in $C([0,1])$.

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1 Introduction

The aim of this paper is to study the sample path continuity of stochastic processes X_t which can be represented by multiple Wiener-Itô integrals of time dependent kernels, and the large deviations from 0 of $\{\epsilon^{m/2}X_\cdot, \epsilon > 0\}$ in $C([0,1])$.

Let $(T, \mathcal{A}, \mu)$ be a σ–finite separable atomless measure space and denote by H the Hilbert space $L^2(T, \mathcal{A}, \mu)$ with inner product $< \cdot, \cdot >_H$ and norm $\| \cdot \|_H$. Let $W = W(f), f \in H$, be a zero-mean Gaussian field defined on a complete probability space $(\Omega, \mathcal{F}, P)$ such that $E\, W(f_1)W(f_2) =< f_1, f_2 >_H$ for all $f_1, f_2 \in H$. Fix $m \geq 1$ and let $L^2_s(T^m)$ be the space of all real valued square integrable symmetric functions on T^m (that is, $f(\theta_{\pi(1)}, \ldots, \theta_{\pi(m)}) = f(\theta_1, \ldots, \theta_m)$ for any permutation π of $\{1, \ldots, m\}$). Given a family of kernels $h_t \in L^2_s(T^m), 0 \leq t \leq 1$ define the process $X_t = I_m(h_t)$, the m-th multiple Wiener-Itô integral of h_t with respect to W (cf. [6]). For the sake of uniformity when $m = 0$ we interpret $L^2_s(T^m)$ to be $\mathbb{R}$ and $I_m(h) = h,\ h \in \mathbb{R}$.

If $\varphi_i \in H, 1 \leq i \leq l$, the tensor product $\varphi_1 \otimes \ldots \otimes \varphi_l$ (or $\varphi^{\otimes l}$ if $\varphi_1 = \ldots = \varphi_l = \varphi$) denotes the element $f(\theta_1, \ldots, \theta_l)$ of $L^2(T^l)$ defined by $\varphi_1(\theta_1) \ldots \varphi_l(\theta_l)$. Whenever the dimension j is selfunderstood we shall also use the shorthand notation $\underline{\theta} = (\theta_1, \ldots, \theta_j)$ as, for example, in $\int_{T^j} \psi(\underline{\theta}) d\mu(\underline{\theta})$. If μ is the Lebesgue measure we write $d\underline{\theta}$ instead of $d\mu(\underline{\theta})$.

In this work we first consider three different sufficient conditions on the family $\{h_t, 0 \leq t \leq 1\}$ which will guarantee the sample path continuity of X_t. As explained in Section 3, these conditions differ in nature and there is no implication among them. Next we establish, assuming nothing more than the sample path continuity of X_t, that the family $\{\epsilon^{m/2}X_\cdot, \epsilon > 0\}$ satisfies a large deviations principle (LDP) in $C([0,1])$with rate function

$$\Lambda(f) = \frac{1}{2} \inf\{\| \varphi \|^2_H \mid \beta_{T,m}(\varphi; h_t) = f(t) \ \ \forall t\} \tag{1.1}$$

where for any $l \in \mathbb{N}$ and $g \in L^2_s(T^l)$

$$\beta_{T,l}(\psi; g) = \int_{T^l} g(\underline{\theta}) \psi^{\otimes l}(\underline{\theta}) d\mu(\underline{\theta}), \qquad \psi \in H. \tag{1.2}$$

By this we mean, as usual, that for any Borel set G in $C([0,1])$

$$\begin{aligned} -\Lambda(\overset{\circ}{G}) &\leq \liminf_{\epsilon \to 0} \epsilon \log P(\epsilon^{m/2} I_m(h_\cdot) \in G) \\ &\leq \limsup_{\epsilon \to 0} \epsilon \log P(\epsilon^{m/2} I_m(h_\cdot) \in G) \\ &\leq -\Lambda(\bar{G}) \end{aligned} \tag{1.3}$$

where for any set $F \in C([0,1])$, $\Lambda(F) = \inf_{f\in F} \Lambda(f)$.

The above LDP is implied by recent results of M. Ledoux in [8] who, employing different methods, considers large deviations related to families of Banach space valued homogeneous random variables on an abstract Wiener space. The methods used in the present work rely heavily on an extended contraction principle and as a result deal with the continuity structure of the multiple integral operation. We will expand on this point below and in Section 4.

To heuristically motivate the normalizing factor $\epsilon^{m/2}$ and rate function Λ suggested in (1.1) assume for the moment

(A) $(T, \mathcal{A}, \mu) = (I = [0,1], \mathcal{B}, leb)$ and $(W_\theta)_{\theta\in I}$ is a standard Brownian motion with $W(f) = \int_I f(\theta) dW_\theta$, $f \in H$. (Here and throughout $\mathcal{B}$ is the Borel σ–algebra and *leb* is the Lebesgue measure.)

Formally writing

$$\epsilon^{m/2} I_m(h.) = \beta_{I,m}(\epsilon^{1/2}\dot{W}; h.) = \epsilon^{m/2} \int_{I^m} h.(\underline{\theta})(\dot{W}^{\otimes m})_{\underline{\theta}}\, d\underline{\theta}\,, \qquad (1.4)$$

(although, almost surely, this makes no sense) a LDP with rate function Λ given in (1.1) can be expected if one applies the contraction principle (cf. [16]) to the family $\epsilon^{1/2}W$ which is known to satisfy a LDP in $C_0([0,1])$ with rate function $\Lambda_W(\varphi) = \frac{1}{2} \| \dot{\varphi} \|_H^2$ whenever meaningful, ∞ otherwise.

This argument will be made precise later. We do, however, wish to emphasize already at this point that even if sense could be made out of (1.4) (for example "integrating by parts") this equation isn't quite correct since terms of lower degree of homogeneity are involved in what is usually known as the Hu–Meyer formula ([5]). However it will turn out that these correction terms are insignificant in the exponential scale we are dealing with.

The contraction principle mentioned above and its various extensions assume some sort of continuity (or "approximate" continuity) of the mapping $W \in C_0([0,1]) \longrightarrow I_m(h.) \in C([0,1])$. Simple continuity of this map occurs in the first of the above mentioned three situations we shall deal with. Roughly speaking, they are:

1. (the "regular" case) Assuming **(A)**, h_t is generated for every t by a multimeasure ν_t and the mapping $t \longrightarrow \nu_t$ is continuous for an appropriate weak topology on multimeasures.

2. (the "slow growth" case) The mapping $t \longrightarrow h_t \in L_s^2(T^m)$ satisfies a certain growth condition on its modulus of continuity (which is implied, for example, by Hölder continuity of any order).

3. (the "adapted case") (A) is assumed and $h_t = h\mathbf{1}_{[0,t]^m}$ for some $h \in L_s^2(T^m)$.

Multimeasures were considered in [12] in connection with the continuity of scalar multiple Wiener–Itô integrals. In our setup, the regular case provides a LDP by direct application of the contraction principle, but is also instrumental in dealing with the other two cases via an "extended" contraction principle in which the mapping $W \in C_0([0,1]) \longrightarrow I_m(h.) \in C([0,1])$ is allowed to be "approximately" continuous.

These general themes will be laid out in Section 2 which is devoted to preliminaries. In it we summarize the main properties of multimeasures which will be needed later, as well as some of the more important general large deviations techniques. In Section 3 we enumerate and discuss the three sets of sufficient conditions on $\{h_t, 0 \leq t \leq 1\}$ mentioned above from which path continuity of $I_m(h_t)$ may be deduced. This is done respectively in subsections 3.1, 3.2 and 3.3. This section also includes an important tail estimate for $\| I_m(h.) \|$ due to C. Borell.

Section 4 deals with the large deviation principle. In broad lines, the method employed is to approximate a given time dependent kernel by a sequence of "regular" kernels – as denoted in subsection 3.1 – for which a simple contraction principle holds, while the Borell tail estimate takes care of the approximation error as well as of the lower order terms present in the Hu–Meyer expansion mentioned earlier.

Finally, in Section 5 some examples and applications will be presented; in particular, in the scalar case a more explicit form of the large deviations rate function may be obtained. In another direction, some particular stochastic processes studied by T. Mori and H. Oodaira in [10] and [11] can be incorporated into our general framework.

2 Preliminaries

In this section we shall state some preliminary results. The first lemma will enable us in many instances to assume (A) without loss of generality. For this purpose we shall temporarily include the particular field W in the notation of the multiple Wiener–Itô integral, i.e. $I_m(h;W)$. Let $J : H = L^2(T) \longrightarrow \tilde{H} \equiv L^2(I)$ be a fixed isometry and define

$$J^{\otimes m} : H^{\otimes m} \subset L^2(T^m) \longrightarrow L^2(I^m)$$

by

$$J^{\otimes m}(\sum_{k=1}^{K} a_k \varphi_{k,1} \otimes \ldots \otimes \varphi_{k,m}) = \sum_{k=1}^{K} a_k J\varphi_{k,1} \otimes \ldots \otimes J\varphi_{k,m}$$

which can be extended by continuity as a surjective isometry (also denoted $J^{\otimes m}$) on all of $L^2(T^m)$ which satisfies $J^{\otimes m}(L_s^2(T^m)) = L_s^2(I^m)$. For the given Gaussian field $\{W(f),\ f \in H\}$ define the corresponding Gaussian field $\{\tilde{W}(\tilde{f}),\ \tilde{f} \in \tilde{H}\}$ by $\tilde{W}(\tilde{f}) = W(J^{-1}\tilde{f})$. Clearly $E\tilde{W}(\tilde{f}_1)\tilde{W}(\tilde{f}_2) = <\tilde{f}_1, \tilde{f}_2>_{\tilde{H}}$ and

$$I_m(J^{\otimes m}h;\tilde{W}) = I_m(h;W) \qquad a.s. \qquad \forall h \in H. \tag{2.1}$$

Lemma 2.1 *Let $(T, \mathcal{A}, \mu)$ and W be given as in the Introduction, $J, J^{\otimes m}$, and $\tilde{W}$ as above. Let $\{h_t, 0 \le t \le 1\}$ be a measurable $L_s^2(T^m)$-valued family of kernels. Then, almost surely,*

$$\tilde{X}_t \equiv I_m(J^{\otimes m}h_t;\tilde{W}) \;=\; X_t \equiv I_m(h_t;W) \qquad a.e.(t).$$

Proof: Define

$$\begin{aligned} N &= \{(t,\omega) \in [0,1] \times \Omega \mid X_t(\omega) \neq \tilde{X}_t(\omega)\} \\ N_t &= \{\omega \in \Omega \mid (t,\omega) \in N\} \\ N_\omega &= \{t \in [0,1] \mid (t,\omega) \in N\}. \end{aligned}$$

Then by (2.1)

$$\begin{aligned} 0 &= \int_0^1 P(N_t)dt \\ &= \int_0^1 \left(\int_\Omega 1_{N_t}(\omega)dP(\omega)\right) dt \\ &= \int_\Omega \left(\int_0^1 1_{N_\omega}(t)dt\right) dP(\omega) \\ &= \int_\Omega leb(N_\omega)dP(\omega) \end{aligned}$$

from which we conclude that $leb(N_\omega) = 0$ a.s. □

2.1 General facts about large deviations

Proposition 2.2 (Schilder's theorem [15]) *Let $\{W_t, t \in I\}$ be a standard Brownian motion. Then $\{\epsilon^{1/2}W_., \epsilon > 0\}$ satisfies a LDP in $C_0([0,1])$ with rate function*

$$\Lambda_0(f) = \begin{cases} \frac{1}{2}\int_0^1 \dot{f}(\theta)^2 d\theta & \text{if } f \in H_0^{1,2} \\ \infty & \text{otherwise} \end{cases} \tag{2.2}$$

where $H_0^{1,2}$ is the space of functions in $C_0([0,1])$whose weak first derivatives are in H.

Proposition 2.3 (Extended contraction principle, [3]) *Let S and S' be two metric spaces and $\{U_\epsilon, \epsilon > 0\}$ (resp. $\{V_\epsilon, \epsilon > 0\}$) be a family of S-valued (resp. S'-valued) random variables.*
Assume $\{U_\epsilon, \epsilon > 0\}$ satisfies a LDP in S with rate function $\Lambda(s)$, $s \in S$, such that for all $r \in [0, \infty)$ the set

$$L_r = \{s \in S \mid \Lambda(s) \leq r\} \tag{2.3}$$

is compact in S.
Moreover, assume that the continuous $F_n : S \longrightarrow S'$, $n \in \mathbb{N}$, and the measurable $F : L_\infty \equiv \bigcup_{r \geq 0} L_r \longrightarrow S'$ satisfy,

(i) $F_n \mid_{L_r} \longrightarrow F \mid_{L_r}$ *uniformly for all* $r \in [0, \infty)$,
(ii) $\lim_{n\to\infty} \limsup_{\epsilon\to 0} \epsilon \log P(\mid F_n(U_\epsilon) - V_\epsilon \mid \geq \delta) = -\infty \quad \forall \delta > 0.$

Then $\{V_\epsilon, \epsilon > 0\}$ satisfies a LDP in S' with rate function

$$\Lambda'(s') = \inf\{\Lambda(s) \mid F(s) = s'\}. \tag{2.4}$$

Remark 2.4 *(a) The classical contraction principle corresponds to the case $F_n = F\ \forall n$, $V_\epsilon = F(U_\epsilon)$. Another particular instance occurs when $S' = S$ and $F_n(s) = s$ for all $n \in \mathbb{N}$ and $s \in S$ in which case one obtains the well known fact that exponentially close families of random variables (i.e. (ii) is satisfied) share the same large deviations principle.*

We also remark that Proposition 2.3 was formulated in slightly more restrictive terms in [3]; however the same proof works just as well for the present formulation.

(b) *Large deviations estimates often involve quantities of the type*

$$\mathcal{L}(a) = \limsup_{\epsilon \longrightarrow \infty} \epsilon \log a(\epsilon)$$

(as for example in condition (ii) of Proposition 2.3), where $a(\epsilon)$ is some nonnegative function. For future reference it will be useful to record the straightforward fact that whenever $a = \sum_{k=1}^{l} a^{(k)}$ then

$$\mathcal{L}(a) \leq \max_{1 \leq k \leq l} \mathcal{L}(a^{(k)}). \tag{2.5}$$

2.2 Continuous extensions of multiple Wiener-Itô integrals

The multiple Wierner-Itô integrals we are concerned with are measurable functionals of W, the Brownian motion. It is suggested by (but certainly not only by) Proposition 2.3 that it is important to understand when (i.e. for which kernels h) this functional has a continuous version. For the scalar case, that is when h doesn't depend on t, this question was fully answered in [12] and we shall now summarize the main relevant results obtained there.

For the remainder of this subsection, thus, we assume **(A)** and moreover that there exists an $h \in L^2(I^m)$ such that for each $t \in [0,1]$, $h_t(\cdot) = h(\cdot)$ almost everywhere in I^m.

Proposition 2.5 *There exists a continuous $F : C_0(I) \longrightarrow \mathbb{R}$ such that $I_m(h) = F(W)$ a.s. if and only if there exists a multimeasure ν_h on $\mathcal{B}^m$ that generates h in the sense that*

$$h(\underline{\theta}) = \nu_h((\theta_1, 1], \ldots, (\theta_m, 1]) \qquad a.e.(I^m). \tag{2.6}$$

A multimeasure of order l on I is a function $\nu(B_1, \ldots, B_l), B_i \in \mathcal{B}$, which is separately a measure in each component; not all multimeasures can be extended to measures on $(I^l, \mathcal{B}^l)$. A more complete account of this concept as well as of the corresponding integration theory may be found in [12] and references therein. We shall just point out the following facts for future use:

(i) For any multimeasure ν it turns out that

$$\| \nu \|_{MM} \equiv \sup \sum_{i_1, \ldots, i_l = 1}^{K} \epsilon_{i_1} \ldots \epsilon_{i_l} \nu(A_{i_1}, \ldots, A_{i_l}) < \infty$$

where the supremum runs over all measurable partitions $\{A_1, \ldots, A_K\}$ of I and all choices of signs $\{\epsilon_i\} \in \{-1, 1\}^K$. Moreover, equipped with the norm $\| \ \|_{MM}$ the space of all such multimeasures, which we shall denote here $MM(I;l)$ is a Banach space which may be identified with the topological dual of $C_0([0,1])^{\otimes_\pi l}$, the l-th projective tensor power of $C_0([0,1])$.

(ii) For $f_1, \ldots, f_l \in L^\infty([0,1])$ and $\nu \in MM(I;l)$

$$\left| \int_{I^l} \left(\otimes_{i=1}^l f_i \right) d\nu \right| \leq \| \nu \|_{MM} \prod_{i=1}^{l} \| f_i \|_\infty . \tag{2.7}$$

(iii) Any $\nu \in MM(I;l)$ induces for each $1 \leq k \leq [l/2]$ a multimeasure $\nu^{(k)} \in MM(I, l-2k)$ defined by

$$\nu^{(k)}(B_1, \ldots, B_{l-2k}) = \tag{2.8}$$
$$\int_{I^l} (\theta_1 \wedge \theta_2) \ldots (\theta_{2k-1} \wedge \theta_{2k}) 1_{B_1}(\theta_{2k+1}) \ldots 1_{B_{l-2k}}(\theta_l) d\nu(\underline{\theta})$$

for $B_1, \ldots, B_{l-2k} \in \mathcal{B}$. Furthermore, for each appropriate k there exists a positive γ_k such that

$$\| \nu^{(k)} \|_{MM} \leq \gamma_k \| \nu \|_{MM} \qquad \forall \nu \in MM(I;l). \tag{2.9}$$

(iv) In Proposition 2.5 the function F is given by

$$F(\varphi) = \sum_{k=0}^{[m/2]} \alpha_{m,k} \int_{I^{m-2k}} \varphi^{\otimes(m-2k)}(\underline{\theta}) d\nu_h^{(k)}(\underline{\theta}) \tag{2.10}$$

with

$$\alpha_{m,k} = \left(-\frac{1}{2} \right)^k \frac{m!}{k!(m-2k)!}. \tag{2.11}$$

Here and throughout $\nu_h^{(0)} = \nu_h$.

(v) Whenever a kernel h is generated by a multimeasure ν as in (2.6) it turns out that it possesses a k–th trace $h^{(k)} \in L_s^2(I^{m-2k})$ (see [12] for the definition of this trace), which is given by

$$h^{(k)}(\theta_1, \ldots, \theta_{m-2k}) = \int_{I^m} \prod_{i=1}^{k} (\tau_{2i-1} \wedge \tau_{2i}) \prod_{j=1}^{m-2k} 1_{(\theta_j, 1]}(\tau_{2k+j}) d\nu(\underline{\tau}) \tag{2.12}$$

and moreover $\nu_h^{(k)} = \nu_{h^{(k)}}$, $0 \leq k \leq [m/2]$. Furthermore the integral involved in the k–th summand of (2.10) is the $(m-2k)$th multiple Stratonovich integral of $h^{(k)}$, denoted $\overset{\circ}{I}_{m-2k}\,(h^{(k)})$, so that (2.10) is nothing else but the Hu-Meyer formula (cf. [5], [7])

$$I_m(h) = \sum_{k=1}^{[m/2]} \alpha_{m,k}\, \overset{\circ}{I}_{m-2k}\,(h^{(k)}). \tag{2.13}$$

As mentioned earlier, there are examples of multimeasures which are not identifiable with *measures* (the question here is whether the set function $\tilde{\nu}(\prod_{k=1}^{l} B_k) = \nu(B_1, \ldots, B_l)$ can be extended to a measure on $(I^l, \mathcal{B}^l)$); however, such examples do not come by naturally (see for example [1]). The multimeasures exhibited in the next example *can* thus be extended to measures, and they will later serve as approximation to arbitrary kernels.

Example 2.6 *Let $h \in L_s^2(I^m)$ have continuous derivatives of all orders and assume its support is contained in the interior of I^m. Then there exists a measure ν_h which generates h (i.e. satisties (2.6)). Its density is given by $(-1)^m \frac{\partial^m h}{\partial\theta_1 \ldots \partial\theta_m}$ and thus*

$$\| \nu \|_{MM} = \| \frac{\partial^m h}{\partial\theta_1 \ldots \partial\theta_m} \|_{L^1(I^m)} \,. \tag{2.14}$$

The traces $h^{(k)}$ can be computed explicitly from (2.12); one obtains

$$h^{(k)}(\underline{\theta}) = \int_{I^k} h(\underline{\theta}, \tau_1, \tau_1, \ldots, \tau_k, \tau_k) d\underline{\tau}$$

(here the dimensions of $\underline{\theta}$ and $\underline{\tau}$ are respectively (m-2k) and k; also $h^{(0)} = h$) and (2.10) becomes

$$I_m(h) = (-1)^m \sum_{k=0}^{[m/2]} \alpha_{m,k} \int_{I^{m-2k}} \frac{\partial^{m-2k} h^{(k)}}{\partial\theta_1 \ldots \partial\theta_{m-2k}} (\underline{\theta}) (W^{\otimes(m-2k)})_{\underline{\theta}}\; d\underline{\theta}.$$

Corollary 2.7 *The class of kernels h which satisfy the conditions of Proposition 2.5 is dense in $L_s^2(I^m)$.*

3 Path continuity of the integral process

In this section we shall consider the issue of sample path continuity of the process

$$X_t = I_m(h_t). \tag{3.1}$$

Three sets of sufficient conditions on the time dependent kernels

$$\{h_t(\underline{\theta}),\ \underline{\theta} \in T^m,\ t \in [0,1]\}$$

will be provided which ensure the path continuity of X_t neither one of which imply any of the other two. The first one imposes a certain regularity structure in the $\underline{\theta}$ variable ((2.6) holds) and only a mild continuity assumption on the t variable. This case is particularly important in the next section because it provides precisely those kernels for which not only $X_\cdot \in C([0,1])$ but also $W \in C_0([0,1]) \longrightarrow X \in C([0,1])$ is continuous. The second set of conditions makes no assumption on the $\underline{\theta}$ variable (other than square integrability) but imposes restrictions on the modulus of continuity in the t variable. Lastly, in the third it is the very particular joint behavior of h in the $\underline{\theta}$ and t variables which causes X to be a continuous martingale.

The basic space to which all our time dependent kernels will belong is $C([0,1]; L^2_s(T^m))$, which we shall denote $C_{2,m}(T)$ (or C_2 when m and T are selfunderstood), namely

$$C_2 = \{h_t(\underline{\theta}) \mid t \in [0,1] \longrightarrow h_t(\cdot) \in L^2_s(T^m) \text{ is continuous}\} \tag{3.2}$$

and equip C_2 with the norm $\| h \|_{\infty,2} \equiv \sup_{0\leq t\leq 1} \| h_t \|_{L^2(T^m)}$. The reason for restricting ourselves to this space arises from the following

Lemma 3.1 *If the process X_t given by (3.1) has continuous sample paths almost surely, then $h_t(\underline{\theta}) \in C_2$.*

This lemma will follow as a corollary from a remarkable Fernique–type result obtained by C. Borell ([2]), which will also serve us later for some large deviations estimates, and which we now state.

Lemma 3.2 *Assume the process X_t in (3.1) has continuous sample paths almost surely. Then $X \in L^2(\Omega; C([0,1]))$ and moreover,*

$$\limsup_{x\to\infty} x^{-2/m} \log P(\| X \|_\infty \geq x) \leq -\frac{1}{2} \| h \|_{\infty,2}^{-m/2} . \tag{3.3}$$

Proof of lemma 3.1: Let $t_n \to t$ in $[0,1]$. Then $Y_n \equiv (X_{t_n} - X_t)^2 \to 0$ almost surely. Moreover $Y_n \leq 4 \parallel X \parallel_\infty^2$ and by Lemma 3.2 $\parallel X \parallel_\infty^2$ is integrable. Thus $Y_n \to 0$ in L^1 which is equivalent to $\parallel (h_{t_n} - h_t)(\cdot) \parallel_{L^2(T^m)} \to 0$. □

3.1 The "regular" case

Here we shall assume **(A)**. As in subsection 2.2, the first step will be to identify time dependent kernels h for which the integral process is given by a continuous $C([0,1])$–valued functional of the underlying Brownian motion.

Definition 3.3 *We shall say that a time dependent kernel $h \in \mathcal{C}_2$ is* regular *if for every $0 \leq t \leq 1$ there exists a multimeasure $\nu_t \in MM(I;m)$ which generates $h_t(\cdot)$, i.e.*

$$h_t(\underline{\theta}) = \nu_t((\theta_1, 1], \ldots, (\theta_m, 1]) \tag{3.4}$$

and $t \longrightarrow \nu_t$ is continuous in the weak star topology of MM.

Proposition 3.4 *Assume $h \in \mathcal{C}_2$ is regular. Then there exists a continuous $F : C_0([0,1]) \longrightarrow C([0,1])$ such that $F(W)$ is a modification of X.*

Remark 3.5 *Whenever h is regular it will be assumed without further mention that the process X has continuous sample paths.*

Proof: First note that by the uniform boundedness principle and (2.9)

$$S \equiv \sup_{\substack{t\in[0,1] \\ 0\leq k\leq[m/2]}} \parallel \nu_t^{(k)} \parallel_{MM} < \infty. \tag{3.5}$$

Define $F : C_0([0,1]) \longrightarrow C([0,1])$ by

$$F(\varphi)_\cdot = \sum_{k=0}^{[m/2]} \alpha_{m,k} \int_{I^{m-2k}} \varphi^{\otimes(m-2k)}(\underline{\theta}) d\nu_\cdot^{(k)}(\underline{\theta}) \qquad 0 \leq t \leq 1.$$

By (2.10) and Proposition 2.5 we see that $F(W)$ is indeed a modification of X. Now, by (2.8), $F(\varphi)$ may also be written

$$\begin{aligned} F(\varphi)_t \;&= \int_{I^m} \Big[\textstyle\sum_{k=0}^{[m/2]} \alpha_{m,k} \quad \varphi(\theta_1)\ldots\varphi(\theta_{m-2k}) \\ &\qquad\qquad \theta_{m-2k+1} \wedge \theta_{m-2k+2} \cdots \theta_{m-1} \wedge \theta_m\Big] \, d\nu_t(\underline{\theta}) \end{aligned}$$

and since the integrand can easily be seen to belong to $C_0([0,1])^{\otimes_\pi m}$, $F(\varphi)$ is a continuous function for every φ. Moreover, for every $\varphi, \psi \in C_0([0,1])$, we may write

$$F(\psi)_t - F(\varphi)_t = \sum_{k=0}^{[m/2]} \alpha_{m,k} \int_{I^{m-2k}} \sum_{i=1}^{m-2k} \psi^{\otimes(i-1)}(\psi - \varphi)\varphi^{\otimes(m-2k-i)} d\nu_t^{(k)}$$

so when $\| \psi - \varphi \|_\infty \leq 1$, and by (2.7) and (3.5),

$$\| F(\psi) - F(\varphi) \|_\infty \leq \left[\sum_{k=0}^{[m/2]} (2m-k) \mid \alpha_{m,k} \mid (\| \varphi \|_\infty + 1)^{m-2k-1} \right] \| \psi - \varphi \|_\infty$$

which proves that F is continuous. □

We are at present unable to prove that a time dependent kernel h for which $I_m(h_t)$ is a continuous $C([0,1])$–valued function of the Brownian motion is necessarily regular. Fix $t \in [0,1]$. Since the point mass measure at t is a continuous functional on $C([0,1])$, the existence of a multimeasure ν_t which generates $h_t(\cdot)$ follows from the results stated is subsection 2.2. However, it is not clear to us whether ν_t is necessarily weak star continuous in t.

Example 3.6 *For each $t \in [0,1]$ let $h_t(\cdot) \in C^\infty(I^m)$ have closed support in the interior of I^m, and moreover assume that*

$$t \in [0,1] \longrightarrow \frac{\partial^m h_t}{\partial\theta_1 \ldots \partial\theta_m} \in L^1(I^m)$$

is continuous. Then following Example 2.6, h is regular. (Actually, the associated multimeasure is norm continuous in t).

We conclude this subsection with a simple observation. When $h \in C_2$ is regular, for fixed $t \in [0,1]$ and $1 \leq k \leq [m/2]$, the k–th trace $h_t^{(k)}$ of h_t exists (see item (v) following Proposition 2.5), and the Hu–Meyer formula (2.13) holds. We claim that for each $0 \leq k \leq [m/2]$, $h_\cdot^{(k)}$ is also a regular kernel of the appropriate order and that the tail behavior of the Stratonovich multiple integral $\overset{\circ}{I}_{m-2k}\ (h_\cdot^{(k)})$ is similar to that of the corresponding Itô multiple integral. The following weakened statement will suffice for our purposes.

Lemma 3.7 *Assume h is a regular kernel in C_2. Then for each $0 \leq k \leq [m/2]$*

(a) $h_{\cdot}^{(k)}$ *is a regular kernel in* $C_{2,m-2k}(I)$

(b) $\overset{\circ}{I}_{m-2k}\,(h_{\cdot}^{(k)})$ *has continuous sample paths a.s.*

(c)

$$\limsup_{x\to\infty} x^{-2/(m-2k)} \log P\left(\|\overset{\circ}{I}_{m-2k}\,(h_{\cdot}^{(k)})\|_\infty > x\right) < 0. \tag{3.6}$$

Proof: Statement (a) immediately implies (b) and (c) via the "inversion formula" (cf. [5]; here $l = m - 2k$ and $g = h^{(k)}$):

$$\overset{\circ}{I}_l\,(g) = \sum_{i=0}^{[l/2]} (-1)^k \alpha_{l,i} I_{l-2i}(g^{(i)}). \tag{3.7}$$

and applying (2.5) to the sum in (3.7). As for (a), $h_t^{(k)}$ is indeed generated by the multimeasure $\nu_t^{(k)}$, for each $0 \le t \le 1$ (see item (v) following Proposition 2.5). To verify that $\nu_t^{(k)}$ is weak star continuous in t, let $g \in C_0([0,1])^{\otimes_\pi (m-2k)}$. Then by (2.8)

$$\begin{aligned} \gamma_t &\equiv \int_{I^{m-2k}} g(\tau_1,\ldots,\tau_{m-2k})\nu_t^{(k)}(d\tau_1,\ldots,d\tau_{m-2k}) \\ &= \int_{I^m} g(\tau_1,\ldots,\tau_{m-2k})(\theta_1\wedge\theta_2)\ldots(\theta_{2k-1}\wedge\theta_{2k}) \\ &\qquad\qquad \nu_t(d\tau_1,\ldots,d\tau_{m-2k},d\theta_1,\ldots,d\theta_{2k}). \end{aligned}$$

Since $f(\theta,\theta') \equiv \theta\wedge\theta' \in C_0([0,1])^{\otimes_\pi 2}$ it follows that $f^{\otimes k}\otimes g \in C_0([0,1])^{\otimes_\pi m}$ and thus γ_t is continuous in t since $t \to \nu_t$ was assumed to be weak star continuous. □

3.2 The "slow growth" case

Here (A) is not assumed. In this subsection we shall obtain continuity of X's sample paths by imposing growth conditions on $t \in [0,1] \longrightarrow h_t \in L_s^2(T^m)$, the tool we use being the metric entropy concept introduced by Dudley. Namely, let

$$\rho(u) = \sup_{|t-s|\le u} \| h_t - h_s \|_{L^2(T^m)}$$

and

$$\rho^{-1}(y) = \inf\{u : \rho(u) \ge y\}.$$

It then follows from [4] that X_t will be almost surely continuous as long as

$$\sum_{n=0}^{\infty} 2^{-n} \left(-\log \rho^{-1}(2^{-n})\right)^{m/2} < \infty. \tag{3.8}$$

Condition (3.8) is satisfied, for example, if $\rho(u) = O\left((-\log u)^{-\alpha}\right)$ as $u \to 0$, for any $\alpha > m/2$. It should be pointed out that sample path continuity was proved by T. Mori and H. Oodaira in [10] using Kolmogorov's criterion in the case where $\rho(u) = O\left(u^{\beta}\right)$ as $u \to 0$, for any $\beta > 0$. Mori and Oodaira also proved (3.3) in this particular case. This Hölder assumption satisfies (3.8) and is clearly stronger than the logarithmic growth mentioned above.

3.3 The "adapted case"

We now again assume (A). The kernels we shall deal with here are of the form

$$h_t(\underline{\theta}) = \mathrm{h}(\underline{\theta}) 1_{[0,t]^m}(\underline{\theta})$$

for some $\mathrm{h} \in L^2_s(I^m)$ and we shall call such kernels *adapted.* Clearly adapted kernels are in $\mathcal{C}_2$. It is well known that in this situation $X_t = I_m(h_t)$ may be represented as the Itô integral

$$X_t = m! \int_0^t \int_0^{\theta_m} \cdots \int_0^{\theta_2} \mathrm{h}(\underline{\theta}) dW_{\theta_1} \ldots dW_{\theta_{m-1}} dW_{\theta_m} \tag{3.9}$$

which implies that X has a modification which is a continuous martingale with increasing process

$$< X >_t = (m!)^2 \int_0^t \left\{ \int_0^{\theta_m} \cdots \int_0^{\theta_2} \mathrm{h}(\underline{\theta}) dW_{\theta_1} \ldots dW_{\theta_{m-1}} \right\}^2 d\theta_m. \tag{3.10}$$

It is worth mentioning that in this case the tail estimate (3.3) may be obtained directly by using an exponential inequality for martingales (see, for example, [14, (IV.37.8)] .

4 The large deviations

We recall the definition (3.1) of the integral process X_t. In this section we shall show that whenever X has continuous sample paths a.s., the family

$\{\epsilon^{m/2}X, \epsilon > 0\}$ satisfies a LDP in $C([0,1])$ as suggested in the Introduction, with the rate function (1.1). To prove this fact we apply the extended contraction principle (Proposition 2.3) to Schilder's theorem (Propsition 2.2), approximating the given kernel by regular kernels and using Borell's tail estimate (3.3) to verify condition (ii) in Proposition 2.3.

The above result is actually implied by recent work of M. Ledoux ([8]; it is assumed there *a priori* that $E(\sup_{0\leq t\leq 1} \mid X_t \mid)^2 < \infty$ but this always holds in our case due to Lemma 3.2). While both approaches rely on the asymptotically homogeneous property of the multiple integral functional, the explicit manifestation of this fact provided here by the Hu–Meyer formula ([5]) as well as the more systematic nature of contraction principles should, hopefully, provide some added insight.

In order to apply Proposition 2.3 we first need the following approximation lemma.

Lemma 4.1 *The regular time dependent kernels are dense in $\mathcal{C}_2$.*

Proof: Let $h \in \mathcal{C}_2$ be arbitrary. We shall approximate h by regular kernels of the type exhibited in Example 3.6. First observe that due to the continuity in the t variable, the families $\{h_t^2,\ t \in [0,1]\}$ and $\{\hat{h}_t^2,\ t \in [0,1]\}$ are compact in $L^1(I^m)$ and in $L^1(\mathbb{R}^m)$ respectively. Here $\hat{\cdot}$ denotes the Fourier transform.

Next, we claim that it may be assumed that for some $\delta > 0$ and for all $t \in [0,1]$, h_t is supported in $D_\delta \equiv [\delta, 1-\delta]^m$. Otherwise h may be approximated in $\mathcal{C}_2$ by such kernels. Indeed, for any $n \geq 4$ let $q_n \in C^\infty(I^m)$ be such that $0 \leq q_n \leq 1$, $q_n \mid_{D_{2/n}} \equiv 1$, $q_n \mid_{D^C_{1/n}} \equiv 0$ and define $g_n \in \mathcal{C}_2$ by

$$g_{n_t}(\underline{\theta}) = h_t(\underline{\theta})q_n(\underline{\theta})$$

which is clearly supported in $D_{1/n}$. Moreover

$$\| h - g_n \|^2_{\infty,2} \leq \sup_{t\in[0,1]} \int_{D^C_{2/n}} h_t^2(\underline{\theta})d\underline{\theta} \stackrel{n\to\infty}{\longrightarrow} 0$$

because of the compactness argument mentioned above.

Now, choose any nonnegative $j \in C^\infty(\mathbb{R}^m)$ with compact support which satisfies $\int_{\mathbb{R}^m} j(\underline{x})d\underline{x} = 1$, set $j_n(\underline{x}) = n^m j(n\underline{x})$, $n \in \mathbb{N}$ and define

$$h^{(n)} = h * j_n \qquad n \in \mathbb{N}$$

(where the convolution is in the $\underline{\theta}$ variables). For large enough n and for all $t \in [0,1]$, $h_t^{(n)}$ is C^∞ and supported in the interior of I^m. Moreover, using a standard convolution inequality,

$$\begin{aligned} & \| \frac{\partial^m h_t^{(n)}}{\partial\theta_1 \dots \theta_m} - \frac{\partial^m h_s^{(n)}}{\partial\theta_1 \dots \theta_m} \|_{L^2(I^m)} \\ = & \| \frac{\partial^m j_n}{\partial\theta_1 \dots \theta_m} * (h_t - h_s) \|_{L^2(I^m)} \\ \leq & \| \frac{\partial^m j_n}{\partial\theta_1 \dots \theta_m} \|_{L^1(\mathbb{R}^m)} \| h_t - h_s \|_{L^2(I^m)} \end{aligned} \tag{4.1}$$

so that for large enough n, $h^{(n)}$ is indeed regular as in Example 3.6.

Finally, for any $t \in [0,1]$ and $R > 0$,

$$\begin{aligned} \| h_t - h_t^{(n)} \|^2_{L^2(I^m)} & \leq \| \hat{h}_t(1 - \hat{j}_n) \|^2_{L^2(\mathbb{R}^m)} \\ & \leq \int_{|\underline{\xi}| \leq R} \hat{h}_t^2(\underline{\xi})(1 - \hat{j}_n)^2(\underline{\xi}) d\underline{\xi} + \int_{|\underline{\xi}| > R} \hat{h}_t^2(\underline{\xi}) d\underline{\xi}. \end{aligned}$$

Once again our earlier compactness argument enables us to choose R large enough to make the second integral arbitrarily small uniformly for all $t \in [0,1]$, and we may then choose n large enough to make the first integral small, since $\hat{j}_n \longrightarrow 1$ uniformly on compacts. This shows that $\| h - h^{(n)} \|_{\infty,2} \longrightarrow 0$. □

Theorem 4.2 *Let $h \in C_2$ and assume $I_m(h.)$ has continuous sample paths almost surely. Then the family $\{V_\epsilon = \epsilon^{m/2} I_m(h.), \epsilon > 0\}$ satisfies a LDP in $S' = C([0,1])$ with rate function given by (1.1).*

Remark 4.3 *The estimate (3.3) which, as will be seen, is originally needed for the proof of this theorem, can then be viewed as a consequence of it. Indeed the large deviations principle (1.3) applied to the set $G = (B_{C([0,1])}(1))^C$, the complement of the unit ball in $C([0,1])$, yields (setting $x = \epsilon^{-m/2}$ and using the notation (1.2))*

$$\lim_{x \to \infty} x^{-2/m} \log P(\| I_m(h.) \|_\infty > x) = -\frac{1}{2} \sup_{\|\psi\|_H \leq 1} \{\| \beta_{T,m}^{-2/m}(\psi; h.) \|_\infty\}.$$

(Actually, this stronger version of (3.3) was the one stated in [2]).

Proof: By Lemma 2.1 we may assume **(A)** without loss of generality. In order to apply Proposition 2.3 set $S = C_0([0,1])$ and $U_\epsilon = \epsilon^{1/2}W$, $\epsilon > 0$. From Proposition 2.2,

$$L_r = \{\varphi \in S \mid \frac{1}{2} \| \dot{\varphi} \|_H^2 \leq r\} \qquad r \geq 0.$$

By Lemma 4.1 choose a sequence $(h_n) \subset \mathcal{C}_2$ of *regular* kernels such that $h_n \to h$ in $\mathcal{C}_2$ and for each $n \in \mathbb{N}$ and $t \in [0,1]$ denote by $(\nu_n)_t$ the multimeasure which generates $(h_n)_t$ in the sense of (3.4).

Next, define $F_n : S \longrightarrow S'$ by

$$F_n(\varphi). = \int_{I^m} \varphi^{\otimes m}(\underline{\theta}) d(\nu_n).(\underline{\theta}).$$

Restricted to $L_\infty = \{\varphi \in S \mid \dot{\varphi} \in H\}$ we can integrate by parts to obtain

$$F_n(\varphi). = \beta_{I,m}(\dot{\varphi}; h_{n.}) = \int_{I^m} \dot{\varphi}^{\otimes m}(\underline{\theta})(h_n).(\underline{\theta}) d\underline{\theta} \, .$$

Defining F on L_∞ by $F(\varphi)_t = \beta_{I,m}(\dot{\varphi}; h_t)$, $t \in [0,1]$, it may readily be verified that for each $(s,t) \in [0,1]^2$, $| F(\varphi)_t - F(\varphi)_s | \leq \| h_t - h_s \|_{L^2(I^m)} \| \dot{\varphi} \|_H^m$ so that indeed $F : L_\infty \longrightarrow S' = C([0,1])$. Moreover by the same Cauchy–Shwartz argument

$$\| F(\varphi) - F_n(\varphi) \|_\infty = \| \beta_{I,m}(\dot{\varphi}; h - h_n) \|_\infty \leq \| h - h_n \|_{\infty,2} \| \dot{\varphi} \|_H^m \tag{4.2}$$

so that condition (i) in Proposition 2.3 is satisfied.

Concerning (ii), we recall the notation $\mathcal{L}(a) = \limsup_{\epsilon \to 0} \epsilon \log a(\epsilon)$ and shall apply (2.5) below. For arbitrary $\delta > 0$ and small enough $\epsilon > 0$, and by Lemma 3.2

$$\begin{aligned}
&\mathcal{L}\left(P(\| F_n(U_\epsilon) - V_\epsilon \|_\infty \geq \delta)\right) = \mathcal{L}\left(P(\epsilon^{m/2} \| \overset{\circ}{I}_m (h_n) - I_m(h) \|_\infty \geq \delta)\right) \\
&\leq \mathcal{L}\Bigg(P(\| I_m(h) - I_m(h_n) \|_\infty \geq \frac{\delta}{2\epsilon^{m/2}}) \\
&\qquad + P(\sum_{k=1}^{[m/2]} | \alpha_{m,k} | \| \overset{\circ}{I}_{m-2k} (h_n^{(k)}) \|_\infty \geq \frac{\delta}{2\epsilon^{m/2}})\Bigg) \\
&\leq -\frac{1}{2}(\delta/2)^{2/m} \| h - h_n \|_{\infty,2}^{-m/2} \\
&\qquad \vee \max_{1 \leq k \leq [m/2]} \mathcal{L}\left(P(\| \overset{\circ}{I}_{m-2k} (h_n^{(k)}) \|_\infty \geq \frac{\delta}{m | \alpha_{m,k} | \epsilon^{m/2}}\right) .
\end{aligned} \tag{4.3}$$

However, each one of the $[m/2]$ terms involved in the last maximum is $-\infty$. Indeed, for a given $1 \leq k \leq [m/2]$ we may split the corresponding term into

$$\left(\lim_{\epsilon \to 0} \epsilon^{-\frac{2k}{m-2k}}\right) \mathcal{L}\left(P(\|\overset{\circ}{I}_{m-2k}(h_n^{(k)})\|_\infty \geq \frac{\delta}{m \mid \alpha_{m,k} \mid \epsilon^{\frac{m-2k}{2}}})\right)$$

which tends to $-\infty$ since by (3.6) the second factor is strictly negative. Thus from (4.3) we obtain

$$\lim_{n \to \infty} \limsup_{\epsilon \to 0} \epsilon\, P(\epsilon^{m/2} \|\overset{\circ}{I}_m(h_n) - I_m(h)\|_\infty \geq \delta) = -\infty.$$

Our result now follows directly from Proposition 2.3 by recalling that the family $\{U_\epsilon, \epsilon > 0\}$ satisfies a LDP with rate function given by (2.2). □

5 Examples

Example 5.1 Assume $\mathbf{h} \in L_s^2(T^m)$ and $h_t \equiv \mathbf{h}$,$0 \leq t \leq 1$. Obviously $h. \in \mathcal{C}_2$ and $I_m(h.)$ has continuous sample paths. We view the process $I_m(h.)$ as the scalar random variable $X = I_m(\mathbf{h})$. From Theorem 4.2 we conclude that $\{\epsilon^{m/2} X,\ \epsilon > 0\}$ satisfies a LDP in $\mathbb{R}$ with rate function

$$\Lambda(x) = \frac{1}{2}\{\|\phi\|_H \mid \beta_{T,m}(\phi, \mathbf{h}) = x\} \quad , \quad x \in \mathbb{R}. \tag{5.1}$$

(recall the definition of the function β given in (1.2)) which is easily seen to satisfy

$$\Lambda(\alpha x) = \alpha^{2/m} \Lambda(x), \qquad \alpha > 0. \tag{5.2}$$

Being lower semicontinuous, Λ must necessarily be of the form

$$\Lambda(x) = \begin{cases} \Lambda_+ x^{2/m} & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ \Lambda_-(-x)^{2/m} & \text{if } x < 0 \end{cases} \tag{5.3}$$

Moreover, straightforward manipulations yield

$$\begin{aligned} \Lambda_+ &= \Lambda_+(h) &= \frac{1}{2}(\sup\{\beta_{I,m}(\varphi; h) \mid \|\varphi\|_H = 1\})^{-2/m} \\ \Lambda_- &= \Lambda_-(h) &= \frac{1}{2}(-\inf\{\beta_{I,m}(\varphi; h) \mid \|\varphi\|_H = 1\})^{-2/m}. \end{aligned}$$

If $\mathbf{h} \equiv 0$ then $\Lambda_+ = \Lambda_- = +\infty$ so for the following observations we shall assume $\mathbf{h} \not\equiv 0$.

(i) For odd m, X is a symmetric random variable and $\Lambda_+ = \Lambda_- \in (\| \mathbf{h} \|^{-2/m}, \infty)$. For even m, X can be highly skewed (for example, for any $\varphi \in H$, $I_2(\varphi^{\otimes 2}) = I_1(\varphi)^2 - \| \varphi \|_H^2$ is bounded below) and indeed Λ_+ and Λ_- might differ. Moreover one (but at most one) of them might be $+\infty$.

(ii) In this scalar case a tail estimate of the form of (3.3) is given in [9], namely, there exist $\kappa_m > 0$ and $0 < \gamma_m < 1/2$ such that

$$P(| I_m(\mathrm{h}) |> x) \leq \kappa_m \exp\left\{ -\gamma_m \left(\frac{x}{\| h \|} \right)^{2/m} \right\}$$

(Although the result was stated somewhat differently in [9, Section 8], the formulation above appears implicitly in the proof there. A similar bound may be found in [13]). The LDP then yields a finer one-sided tail estimate: for any $\epsilon > 0$ there exists an $x_0 > 0$ such that

$$e^{-(\Lambda_+ + \epsilon)x^{2/m}} \leq P(I_m(h) > x) \leq e^{-(\Lambda_+ - \epsilon)x^{2/m}} \qquad \forall x > x_0$$

with a similar estimate holding for the negative tails (Λ_- replacing Λ_+).

(iii) For $m > 2$, $\Lambda(x)$ is not a convex function.

Example 5.2 As special cases of Theorem 4.2 we are able to obtain LDPs for the stochastic processes considered in [10] and [11] with two particular types of kernels. Recall from subsection 3.2 that Hölder continuity of the time dependent kernel as a function from $[0,1]$ to $L_s^2(T^m)$ guarantees almost sure continuous sample paths of the integral process.

(1) In [10] T. Mori and H. Oodaira considered the large deviations of self–similar processes represented by multiple Wiener–Itô integrals as follows:

$$X_t = \int_{\mathbb{R}^m} Q_t(u_1, \ldots, u_m) dW_{u_1} \ldots dW_{u_m}$$

where for some $\alpha \in (0,1)$ and any $t \geq 0$ and $\underline{u} \in \mathbb{R}^m$ the symmetric kernel Q satisfies

$$Q_{ct}(cu_1, \ldots, cu_m) = c^{\alpha - m/2} Q_t(u_1, \ldots, u_m) \qquad c > 0$$

and

$$Q_{t+k}(u_1, \ldots, u_m) - Q_t(u_1, \ldots, u_m) = Q_k(u_1 - t, \ldots, u_m - t) \qquad k \geq 0.$$

Theorem 4.2 gives a LDP for $\{\epsilon^{m/2} I_m(Q_t),\ \epsilon > 0\}$ since

$$\begin{aligned} \| Q_{t+k} - Q_t \|^2_{L^2(\mathbb{R}^m)} &= k^{2\alpha-m} \int_{\mathbb{R}^m} \left(Q_1(\frac{u_1 - t}{k}, \ldots, \frac{u_m - t}{k}) \right)^2 d\underline{u} \\ &= k^{2\alpha} \| Q_1 \|^2_{L^2(\mathbb{R}^m)} . \end{aligned}$$

(2) Mori and Oodaira [11] have also studied the large deviations of a class of stochastic processes represented by multiple Wiener–Itô integrals with respect to a two–parameter Wiener process $W(\cdot,\cdot)$. Let $T = [0,1]^2$ and for $\mathbf{h} \in L^2_s([0,1]^m)$ define

$$h_t((u_1, v_1), \ldots, (u_m, v_m)) = \mathbf{h}(u_1, \ldots, u_m) \mathbf{1}_{[0,t]}(v_1), \ldots, \mathbf{1}_{[0,t]}(v_m)$$

and

$$X^\epsilon_t = \epsilon^{m/2} \int_{([0,1]^2)^m} h_t((u_1, v_1), \ldots, (u_m, v_m)) dW_{u_1, v_1} \ldots dW_{u_m, v_m} .$$

A LDP for $\{X^\epsilon,\ \epsilon > 0\}$ follows from Theorem 4.2 in this case since

$$\begin{aligned} \| h_t - h_s \|_{L^2(I^{2m})} &= \| \mathbf{h} \|_{L^2(I^m)} \left(\int_{[0,1]^m} (\mathbf{1}_{[0,t]}(\underline{v}) - \mathbf{1}_{[0,s]}(\underline{v}))^2 d\underline{v} \right)^{1/2} \\ &= \| \mathbf{h} \|_{L^2(I^m)} | t^m - s^m |^{1/2} \leq m^{1/2} \| \mathbf{h} \|_{L^2(I^m)} | t - s |^{1/2} . \end{aligned}$$

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