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POLYNOMIALS AND MULTILINEAR FORMS ON FULLY NUCLEAR SPACES

by Philipp J. BOLAND (*)

Abstract. - In this paper, we consider the spaces of continuous and hypo continuous n-linear forms and polynomials on fully nuclear spaces. We consider these spaces endowed with the topologies τ_0 (uniform convergence on compact sets) and τ_ω (Nachbin ported topology), and develop some duality relationships for these topologies.

1. Preliminaries.

In this paper, E will represent a fully nuclear space, that is E and E' are complete reflexive nuclear spaces over the field of complex numbers. Any Fréchet nuclear (FN) space or the strong dual of a Fréchet nuclear (DFN) space is fully nuclear. $\mathcal{D}$, $\mathcal{D}'$, $\prod_N \underset{\sim}{C} \times \sum_N \underset{\sim}{C}$ are other examples of fully nuclear spaces.

$\mathcal{L}(^m E)$ (respectively $\mathcal{L}_{HY}(^m E)$) is the space of continuous (hypo continuous, i. e. continuous on compact sets) m-linear forms on E . $\mathcal{L}_S(^m E)$ is the subspace of $\mathcal{L}(^m E)$ of symmetric m-linear forms. τ_0 is the topology of uniform convergence on compact sets. $P(^m E)$ (respectively $P_{HY}(^m E)$) is the space of continuous (hypo continuous m-homogeneous polynomials on E . τ_0 is also defined on $P(^m E)$ and $P_{HY}(^m E)$. τ_ω is the topology on $P(^m E)$ defined by all semi-norms p ported by compact subsets of E (p is ported by K if, for all open U , where $K \subset U \subset E$, there exists $C_U > 0$ such that $p(f) \leqslant C_U |f|_U$, for all $f \in P(^m E)$). τ_ω is a bornological topology on $P(^m E)$, and it may also be described as the topology defined by all semi-norms on $P(^m E)$ which are ported by zero [6].

If E is a K-space [10] (i. e. $f : E \longrightarrow X$ is continuous if f is continuous on the compact subsets of E), then $\mathcal{L}_{HY}(^m E) = \mathcal{L}(^m E)$ and $P_{HY}(^m E) = P(^m E)$. Every metrizable or DFN space is a K-space. $\prod_N \underset{\sim}{C} \times \sum_N \underset{\sim}{C}$ is not a K-space. The following example shows that $\mathcal{D}$ ($\mathcal{D} = \mathcal{D}(R)$ complexified) is not a K-space.

Example 1. - $P_{HY}(^2\mathcal{D}) \neq P(^2\mathcal{D})$.

Proof. - Define $p = \sum_{n=1}^\infty (\partial^n \delta_0)\delta_n$, where δ_a is the Dirac delta function at a . If K is compact in $\mathcal{D}$, there exists an m such that $K \subset \mathcal{D}_m$ (= complexification of $\{\varphi \in \mathcal{D} : \text{support } \varphi \subset [-m , m]\}$). Therefore $p_{|K} = \sum_{n=1}^m (\partial^n \delta_0)\delta_n$ which is continuous on K . Therefore $p \in P_{HY}(^2\mathcal{D})$. However, one may show that p is not bounded on any neighbourhood of zero, and hence $p \notin P(^2\mathcal{D})$ (see [5]).

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Note that, from the above $P(^2\mathbb{Q})$, τ_0 is not complete. We will now see that, if E is fully nuclear, then $(P_{HY}(^mE), \tau_0)$ is the completion of $(P(^m(E), \tau_0)$.

2. Density of continuous polynomials.

If U is an open absolutely convex set in E , then $E(U) = E/(p_u^{-1}(0))$, where p_u is the Minkowski functional of U . Similarly, if B is a closed absolutely convex bounded set in E , then $E(B) = \bigcup_n nB$ and E(B) is a normed space with unit ball B . We may identify $(E(U))'$ with $E'(U^0)$, and $E'(B^0)$ is a subspace of $(E(B))'_\beta$ (see [11]).

PROPOSITION 2. - Let E be fully nuclear. Then $(\mathcal{L}_{HY}(^mE), \tau_0) = \widehat{(\mathcal{L}(^mE), \tau_0)}$ and $(P_{HY}(^mE), \tau_0) = \widehat{(P(^mE), \tau_0)}$, for all m (where $\hat{F}$ is the completion of F).

Proof. - It suffices to show that $\mathcal{L}(^mE)$ is dense in $(\mathcal{L}_{HY}(^mE), \tau_0)$. Therefore, let $A \in \mathcal{L}_{HY}(^mE)$, $\epsilon > 0$, and K absolutely convex and compact be given. We want to find $A' \in \mathcal{L}(^mE)$ such that $|A - A'|_K < \epsilon$.

Without loss of generality, E(K) is Hilbert and therefore $E' \longrightarrow E'(K)$ has a dense image as E is reflexive. Now, there exists an absolutely convex compact set K_1 containing K such that $E(K) \longrightarrow E(K_1)$ is nuclear. Hence, there exist $(\varphi_n) \subseteq (E(K))'$ and $(y_n) \subseteq K_1$ such that, for all $x \in E(K)$,

$$x = \sum_n \varphi_n(x)\, y_n \quad \text{(convergence in } E(K_1)\text{)}$$

and

$$\sum_n |\varphi_n|_K\, p_{K_1}(y_n) = C < +\infty .$$

Now, A is continuous on K_1 (i. e. on $^mK_1$), and hence, for $x_1, \dots, x_m \in K$,

$$\begin{aligned} A(x_1, \dots, x_m) &= A(\textstyle\sum_n \varphi_n(x_1)\, y_n, \dots, \sum_n \varphi_n(x_m)\, y_n) \\ &= \textstyle\sum_{n_1,\dots,n_m} \varphi_{n_1}(x_1)\cdots\varphi_{n_m}(x_m)\, A(y_{n_1}, \dots, y_{n_m}) \\ &= \textstyle\sum_{n_1,\dots,n_m} \varphi_{n_1}\cdots\varphi_{n_m}(x_1, \dots, x_m)\, A(y_{n_1}, \dots, y_{n_m}) . \end{aligned}$$

As $\sum_n |\varphi_n|_K\, p_{K_1}(y_n) = C < +\infty$, it follows that there exists a finite set F of indices such that

$$|A - \textstyle\sum_F \varphi_{n_1}\cdots\varphi_{n_m}\, A(y_{n_1}, \dots, y_{n_m})|_K < \frac{\epsilon}{2} .$$

Since E' is dense in E'(K) , we can find continuous linear forms $\psi_{n_i} \in E'$ such that

$$|\varphi_{n_1}\cdots\varphi_{n_m}\, A(y_{n_1}, \dots, y_{n_m}) - \psi_{n_1}\cdots\psi_{n_m}|_K < \frac{\epsilon}{2|F|} ,$$

and therefore

$$|\textstyle\sum_F \varphi_{n_1}\cdots\varphi_{n_m}\, A(y_{n_1}, \dots, y_{n_m}) - \sum_F \psi_{n_1}\cdots\psi_{n_m}|_K < \frac{\epsilon}{2} .$$

Hence $|A - \sum_F \psi_{n_1}\cdots\psi_{n_m}|_K < \epsilon$, and therefore $\mathcal{L}_f(^mE)$ (the space of continuous m-linear forms of finite type on E) is dense in $(\mathcal{L}_{HY}(^mE), \tau_0)$.

Similarly, it follows that $P_f({}^mE)$ (and hence $P({}^mE)$) is dense in $(P_{HY}({}^mE) , \tau_0)$.

COROLLARY 3. - Let U be a balanced open set in the fully nuclear space E . Then, $H(U)$ is dense in $(H_{HY}(U) , \tau_0)$ (where $H(U)$ and $H_{HY}(U)$ are respectively the holomorphic and hypo analytic functions on U).

Proof. - This follows from proposition 2 and the Taylor series expansion of elements of $(H_{HY}(U) , \tau_0)$.

Remark 4. - If E is a fully nuclear space with an absolute basis, then it is considerably easier to prove proposition 2 (see [4]).

3. Duality for spaces of polynomials.

There is a natural algebraic isomorphism between $(P({}^mE) , \tau_0)'$ and $P({}^mE')$ when E is a fully nuclear space. The isomorphism is via the mapping

$$\beta : (P({}^mE) , \tau_0)' \longrightarrow P({}^mE') ,$$

where $\beta T(\varphi) = T(\varphi^m)$ (see [2]). Note that, because of proposition 2, $(P_{HY}({}^mE) , \tau_0)'$ and $P({}^mE')$ are also isomorphic.

We will now show that, when E is fully nuclear, then $(P({}^mE) , \tau_\omega)' \simeq P_{HY}({}^mE')$. Initially, however we need some definitions and a lemma.

Definition 5. - Let U be an absolutely convex neighbourhood of zero in E . Then

$$\mathcal{L}_N({}^mE(U)) = \{ A \ ; \ A \in \mathcal{L}({}^mE) , \ A = \sum_n \langle \ , a_n^1\rangle \cdots \langle \ , a_n^m\rangle ,$$
$$\text{where each } a_n^i \in (E(U))' \text{ and } \sum_n |a_n^1|_U \cdots |a_n^m|_U < +\infty \} .$$

We define the norm π_U on $\mathcal{L}_N({}^mE(U))$ by

$$\pi_U(A) = \inf\{\sum_n |a_n^1|_U \cdots |a_n^m|_U : A = \sum_n \langle \cdot , a_n^1\rangle \cdots \langle \ , a_n^m\rangle\} .$$

Similarly, we define

$$P_n({}^mE(U)) = \{p \in P({}^mE) , \ p = \sum_n \varphi_n^m ,$$
$$\text{where each } \varphi_n \in (E(U))' \text{ and } \sum_n |\varphi_n^m|_U < +\infty\} .$$

We define the norm π_U on $P_N({}^mE(U))$ by $\pi_U(p) = \inf\{\sum_n |\varphi_n^m|_U \ ; \ p = \sum_n \varphi_n^m\}$.

Note that, if $p \in P({}^mE)$ and A_p is the symmetric m-linear form $\in \mathcal{L}_S({}^mE)$ corresponding to p , then

$$p \in P_N({}^mE(U)) \Longleftrightarrow A_p \in \mathcal{L}_N({}^mE(U)) .$$

This follows since if $A = \sum_n \langle \ , a_n^1\rangle \cdots \langle \ , a_n^m\rangle \in \mathcal{L}_N({}^mE(U))$, where $\sum_n |a_n^1|_U \cdots |a_n^m|_U < +\infty$, then $(a_n^1 \cdots a_n^m)_S$ is such that $P_{(a_n^1 \cdots a_n^m)_S} \in P_f({}^mE(U))$ and

$$\pi_U(p_{(a_n^1 \cdots a_n^m)_S}) \leq \frac{m^m}{m!} \pi_U((a_n^1 \cdots a_n^m)_S) \leq \frac{m^m}{m!} \pi_U(a_n^1 \cdots a_n^m) = \frac{m^m}{m!} |a_n^1|_U \cdots |a_n^m|_U$$

where $(a_n^1 \dots a_n^m)_S$ is the symmetrization of $a_n^1 \dots a_n^m$ and $P_{(a_n^1 \dots a_n^m)_S}$ is the polynomial corresponding to $(a_n^1 \dots a_n^m)_S$). Moreover, we see that

$$\pi_U(p) \leq \frac{m^m}{m!} \pi_U(A_p) \leq \frac{m^m}{m!} \pi_U(p) \ , \text{ for } p \in P_N(^mE(U)) \ .$$

See [8].

Definition 6. – If B is an absolutely convex set in the fully nuclear space E, we define Σ_B on $\mathcal{L}_N(^mE(U))$ by

$\Sigma_B(A)$
$= \sup\{|\sum_n \langle z_1 , a_n^1\rangle \dots \langle z_m , a_n^m\rangle| \ ; \ A = \sum_n \langle \ , a_n^1\rangle \dots \langle \ , a_n^m\rangle$ and $z_i \in B^{00}\}$.

Note that, as E is reflexive, $\Sigma_B(A) = |A|_B$. Similarly, we mat define Σ_B on $P_N(^mE(U))$. Note that Σ_B may be infinite.

Now, we know that, if E is fully nuclear and U is an absolutely convex neighbourhood of zero in E, then there exists an absolutely convex neighbourhood V of zero such that $V \subset U$, $E(V) \longrightarrow E(U)$ is nuclear and $A \in \mathcal{L}_N(^mE(V))$, whenever $A \in \mathcal{L}(^mE(U))$. See [2].

LEMMA 7. – Let U be an absolutely convex neighbourhood of zero in the fully nuclear space E. Then, there exists $C > 0$ and absolutely convex neighbourhoods of zero W, V, where $W \subset V \subset U$ such that, if $A \in \mathcal{L}(^mE)$ and $|A|_U < +\infty$, then $A \in \mathcal{L}_N(^mE(W))$ and $\pi_\omega(A) \leq C^m \Sigma_V(A) = C^m|A|_V$, for all $m \geq 1$.

Proof. – We prove this only for the case $m = 2$. As E is fully nuclear, we may assume that $E(U)$ is pre-hilbert. Now, we can find absolutely convex pre-hilbertian neighbourhoods of zero W and V such that $W \subset V \subset U$ and

$$E(W) \xrightarrow{\text{dual nuclear}} E(V) \xrightarrow{\text{nuclear}} E(U) \ .$$

Hence, there exist $(V_n) \subseteq (E'(V^0))' = \widehat{E(V)}$ and $\langle\psi_n\rangle \subseteq E'(W^0)$ such that, for all $\varphi \in E'(V^0)$,

$$\varphi = \sum_n \langle\varphi , V_n\rangle\psi_n \quad (\text{convergence in } E'(W^0)\ )$$

and

$$\sum_n |V_n|_{V^0} \ |\psi_n|_W = C < +\infty \ .$$

Without loss of generality, we may assume $C \geq 1$.

Now, if $A \in \mathcal{P}(^2E)$ and $|A|_U < +\infty$, we know that $A \in \mathcal{L}_N(^2E(V))$ and hence $A \in \mathcal{L}_N(^2E(W))$. Therefore, there exists an $\alpha \in (\mathcal{L}_N(^2E(W)) \ , \pi_W)'$ such that $\pi_W(A) = \langle A , \alpha\rangle$ and $|\langle A' , \alpha\rangle| \leq \pi_W(A')$, for all $A' \in \mathcal{L}_N(^2E(W))$. In particular, if $\varphi_1 , \varphi_2 \in E'(V^0)$, then $\varphi_1 \varphi_2 \in \mathcal{L}(^2E(W))$ and

$$|\langle\varphi_1 \varphi_2 , \alpha\rangle| \leq |\varphi_1|_W \ |\varphi_2|_W \ .$$

Since $E(W) \longrightarrow E(V)$ is dual nuclear,

$$\langle\varphi_1 \varphi_2 , \alpha\rangle = \sum_n \langle\varphi_1 , V_n\rangle\langle\psi_n \varphi_2 , \alpha\rangle \ .$$

Now, as $A \in \mathfrak{L}_N({}^2E(V))$, A may be represented in the form

$$A = \sum_p \varphi_r \Phi_r \text{ , where } \varphi_r , \Phi_r \in E'(V^0) \text{ and } \sum_r |\varphi_r|_V |\Phi_r|_V < +\infty .$$

Therefore

$$\begin{aligned}\pi_\omega(A) &= \langle A , \alpha\rangle = \langle \textstyle\sum_r \varphi_r \Phi_r , \alpha\rangle = \sum_r \langle \varphi_r \Phi_r , \alpha\rangle \\ &= \textstyle\sum_r \sum_n \langle \varphi_r , V_n\rangle\langle \psi_n \Phi_r , \alpha\rangle = \sum_n \langle \psi_n(\sum_r \langle\varphi_r , V_n\rangle\Phi_r) , \alpha\rangle \\ &\leqslant \textstyle\sum_n |\psi_n|_W |\sum_r \langle\varphi_r , V_n\rangle\Phi_r|_W \leqslant \sum_n |\psi_n|_W |\sum_r \langle\varphi_r , V_n\rangle\Phi_r|_V \\ &\leqslant \textstyle\sum_n |\psi_n|_W |V_n|_{V^0} \Sigma_V(A) = C\Sigma_V(A) \leqslant C^2 \Sigma_V(A) .\end{aligned}$$

COROLLARY 8. - <u>Let</u> U <u>be an absolutely convex neighbourhood of zero in the fully nuclear space</u> E . <u>Then, there exist</u> $C' > 0$ <u>and absolutely convex neighbourhoods of zero</u> W <u>and</u> V , <u>where</u> $W \subset V \subset U$ <u>such that, if</u> $p \in P({}^mE)$ <u>and</u> $|p|_U < +\infty$, <u>then</u> $p \in P_N({}^mE(W))$ <u>and</u>

$$\pi_W(p) \leqslant (C')^m \epsilon_V(p) = (C')^m |p|_V , \text{ for all } m \geqslant 1 .$$

<u>Proof</u>. - Let p be given, and let A_p be the symmetric m-linear form corresponding to p . Let C , W , V be as in lemma 7. Then, $A_p \in \mathfrak{L}_N({}^mE(W))$ and

$$\pi_W(p) \leqslant \frac{m^m}{m!} \pi_W(A_p) \leqslant C^m \frac{m^m}{m!} \epsilon_V(A_p) \leqslant C^m \left(\frac{m^m}{m!}\right)^2 \epsilon_V(p) .$$

Hence, by choosing $C' = \sup_m (C^m(m^m/m!)^2)^{1/m}$,

$$\pi_W(p) \leqslant (C')^m \epsilon_V(p) .$$

PROPOSITION 9. - <u>Let</u> E <u>be a fully nuclear space. Then,</u> $(P({}^mE) , \tau_\omega)' \simeq P_{HY}({}^mE')$.

<u>Proof</u>. - We define $\beta : (P({}^mE) , \tau_\omega)' \longrightarrow P_{HY}({}^mE')$ by $\beta T(\varphi) = T(\varphi^m)$, for $T \in (P({}^mE) , \tau_\omega)'$ and $\varphi \in E'$. It is clear that β is well defined, and moreover as $P_f({}^mE)$ is dense in $(P({}^mE) , \tau_\omega)$ (see [4]), it follows that β is $|-|$.

We now show that β is surjective. Let $p' \in P_{HY}({}^mE')$. We define $T_{p'}$ on $P({}^mE)$ as follows. If $p \in P({}^mE)$, then $p \in P_N({}^mE(U'))$, for some U' . Hence, $p = \sum_n \varphi_n^m$, where $|\varphi_n^m|_{U'} < +\infty$. We define $T_{p'}(p) = \sum_n p'(\varphi_n)$. $T_{p'}$ is well defined on $P({}^mE)$ (see [8]), and we show that $T_{p'}$ is τ_ω-continuous. In fact, we show that $T_{p'}$ is ported by zero. Hence, let U be an absolutely convex neighbourhood of zero, and let C' , V and W be as in corollary 8. Then, if $p \in P({}^mE)$ and $|p|_U < +\infty$, we know p may be represented in the form $p = \sum_n \varphi_n^m$, where $\sum_n |\varphi_n^m|_W < +\infty$.

Hence,

$$|T_{p'}(p)| = |\textstyle\sum_n p'(\varphi_n)| \leqslant |p'|_{W^0} \sum_n |\varphi_n^m|_W .$$

Therefore

$$|T_{p'}(p)| \leqslant |p'|_{W^0} \pi_W(p) \leqslant |p'|_{W^0} (C')^m \epsilon_V(p) \leqslant |p'|_{W^0}(C')^m \epsilon_U(p) .$$

Hence $T_{p'}$ is ported by zero. Since $\beta T_{p'} = p'$, this complete the proof.

COROLLARY 10. - Let E be a fully nuclear space. Then

$$(P_{HY}(^mE)\ ,\ \tau_0)'_\beta = (P(^mE)\ ,\ \tau_\omega)\ .$$

Proof. - $(P_{HY}(^mE)\ ,\ \tau_0)$ is a complete nuclear space and hence, semi-reflexive (see [3]). By proposition 9, $(P(^mE')\ ,\ \tau_\omega)'$ is isomorphic to $P_{HY}(^mE)$. Let τ_β denote the topology on $P(^mE')$ induced by $(P_{HY}(^mE)\ ,\ \tau_0)'_\beta$. Then $(P(^mE')\ ,\ \tau_\omega)$ and $(P(^mE')\ ,\ \tau_\beta)$ have the same dual. τ_ω is bornological and hence Mackey. Since τ_0 is semi-reflexive on $P_{HY}(^mE)$, τ_β is barrelled and hence also Mackey. Therefore, $\tau_\beta = \tau_\omega$ on $P(^mE')$, completing the proof.

PROPOSITION 11. - Let E be fully nuclear. If τ_ω bounded subsets of $P(^mE)$ are locally uniformly bounded, then $(P(^mE)\ ,\ \tau_\omega)'_\beta = (P_{HY}(^mE')\ ,\ \tau_0)$.

Proof. - Let U an absolutely convex neighbourhood of zero in E and $\epsilon > 0$ be given. We will show that there exists a τ_ω bounded subset B of $P(^mE)$ such that $B^0 \subset \{p' ;\ p' \in P_{HY}(^mE)\ ,\ |p'|_{U^0} \leqslant \epsilon\}$. Let $B = \{p \in P(^mE)\ ,\ |p|_U \leqslant \frac{1}{\epsilon}\}$. Then, if $T \in B^0$, $|\beta T(\varphi)| = |T(\varphi^m)| \leqslant \epsilon$, for all $\varphi \in U^0$. Hence

$$B^0 \subset \{p' ;\ p' \in P_{HY}(^mE)\ ,\ |p'|_{U^0} \leqslant \epsilon\}\ .$$

Conversely, let B be a given bounded subset of $(P(^mE)\ ,\ \tau_\omega)$. By assumption, there exists an absolutely convex neighbourhood of zero U and $\alpha > 0$ such that $B \subset \{p ;\ p \in P(^mE)\ ,\ |p|_U < \alpha\}$. Let C', W, V be as in corollary 8, and let $\gamma = \{p' ;\ p' \in P_{HY}(^mE')\ ,\ |p'|_{W^0} \leqslant (1/((C')^m \alpha)\}$. Then

$$|T_{p'}(p)| \leqslant |p'|_{W^0}\ \pi_W(p) \leqslant \frac{1}{(C')^m \alpha}\ (C')^m\ |p|_U\ ,$$

for all $p \in P(^mE)$ such that $|p|_U < +\infty$. Hence $\gamma \subset B^0$ and this completes the proof.

4. Some examples.

Remark 12. - If E is a Fréchet nuclear space or a $\mathcal{D}$FN space, then τ_0 and τ_ω bounded subsets of $P(^mE)$ are locally uniformly bounded (see [1]). In this case, as τ_ω is bornological, we have that $\tau_{0,b} = \tau_\omega$, where $\tau_{0,b}$ is the bornological topology associated with τ_0. However, since a Fréchet nuclear space or a dual of Fréchet nuclear space is a K-space, it follows from corollary 10 and proposition 11 that $(P(^mE)\ ,\ \tau_0)$ is reflexive and hence $\tau_0 = \tau_\omega$ on $P(^mE)$, whenever E is a Fréchet nuclear or dual of Fréchet nuclear space.

Example 13. - The space $E = \prod_N \underset{\sim}{C} \times \sum_N \underset{\sim}{C}$ is a fully nuclear space which is neither a FN or $\mathcal{D}$FN space. Although τ_0 bounded subsets of $P(^mE)$ are not necessarily locally uniformly bounded.

Therefore

$$((P(^mE)\ ,\ \tau_0)'_\beta)'_\beta = (P_{HY}(^mE)\ ,\ \tau_0)$$

and $(P_{HY}(^mE)\ ,\ \tau_0)$ is reflexive. Note that $\tau_0 \neq \tau_\omega$ on $P(^mE)$, but

$(P(^m E), \tau_0)'_\beta = (P(^m E), \tau_\omega)$ and $(P(^m E), \tau_\omega)'_\beta = (P_{HY}(^m E), \tau_0)$.

Example 14. - Let $E = \mathbb{C}$. Then, E is a fully nuclear space and $P(^m \mathbb{C}) \neq P_{HY}(^m \mathbb{C})$ while $P(^m \mathbb{C}') = P_{HY}(^m \mathbb{C}')$, for $m > 1$. Also $\tau_0 = \tau_\omega$ on $P(^m \mathbb{C})$ while $\tau_{0,b} = \tau_\omega \neq \tau_0$ on $P(^m \mathbb{C}')$. τ_0 bounded subsets of $P(^m \mathbb{C}')$ are locally uniformly bounded. However, τ_0 bounded subsets of $P(^m \mathbb{C})$ are not locally uniformly bounded. Otherwise, by corollary 10 and proposition 11, τ_0 would be reflexive and this would imply $\tau_0 = \tau_{0,b}$ on $P(^m \mathbb{C}')$.

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