Séminaire sur les équations non linéaires
École Polytechnique

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Introduction

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INTRODUCTION

The main direction of all the papers in this volume consists in investigation of meromorphic solution for various completely integrable systems discrete as well as continuous in one and two space dimensions.

Here is a brief description of the papers in the volume. The general point of view as well as the definition of Hamiltonian structure is presented in exposé No 1. In this paper especially interesting are the examples of two dimensional completely integrable systems associated with linear differential operators of orders up to 5. The papers No 2 and 3 present detailed examination of the motion of poles of meromorphic solutions for Korteweg-de Vries (KdV) and Burgers-Hopf (BH) equations. In exposé No 4 meromorphic and especially new elliptic solutions are presented for two dimensional KdV and its higher analogues. In the papers No 5 and 6 we show the simplest examples of completely integrable Hamiltonians for arbitrary number of degrees of freedom that are equivalent to stationary KdV. These systems that for $n = 3$ have been considered in 1870 by Neuman appear in a number of physical problems (stationary Thirring model, Hartree-Fock, laser dynamics, etc). These papers in shorten form were published in C. R. Acad. Sci. (1978), Lett. Nuovo Cimento 22 (1978). However the text here contains also useful equation defining Jacobians of hyperelliptic curves. In the paper 7 we discuss meromorphic solutions of Benjamin-Ono equation and present interesting reduction to non-stationary Schrödinger equation. Exposé No 8 contains algebraic procedure of generating "soliton" solutions for an arbitrary two-dimensional Zaharov-Shabat systems. This algebraic procedure is used in No 9 in order to obtain multisoliton formulae using Wronskians. In the No 10 we show that the assumptions of meromorphicity of eigenfunctions for non-stationary Schrödinger implies that the poles of potential interact as particles with the force $G'x^{-3}$.

Riccati equation for non-stationary Schrödinger enables us to construct Bäcklung transformation for many-particle system with potential $G'x^{-2}$. This is done in No 11.

Considerable part of the results of this volume have been obtained by authors during their stay in France (September 1977 - February 1978). One of us D.V. Choodnovsky is extremely thankful to Ecole Polytechnique, D.G.R.S.T. and especially to the Director of Centre de Mathématiques, Professor L. Schwartz, for their kind hospitality, possibility to work at Ecole Polytechnique and to start this seminar. Four papers in this volume (No 3-6) were published as some of the exposé of our joint Herman-Choodnovsky-Choodnovsky seminar "L'analyse Diophantienne et Ses Applications". We hope to continue the publications of the proceedings of this seminar.

This seminar would be impossible withour friendly support of mathematicians and physicists from Ecole Polytechnique. We are extremely thankful for fruitful discussions and kind contacts to Professor J. Lascoux, we thank a lot to Professors A. Wightman and R. Sénéor.

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Some other papers have been prepared at the Department of Mathematics of Columbia Univeristy during authors seminar on Non-Linear Equations. We want to express our sincere gratitude and cordial thanks to Professor L. Bers for his permanent attention, active interest and support towards authors as well as the seminar. The works done at Columbia University were supported by Office of Naval Research Grant N 00014-78-C-0318 and National Science Foundation Grant MCS77-07660, to which tha authors are deeply thankful.

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