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EQUATIONS AUX DERIVEES PARTIELLES

CORNER MELLIN OPERATORS AND CONORMAL ASYMPTOTICS

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0. Introduction

A manifold with corners in our sense will be a space K which is outside of some one-dimensional sheleton $K_1 \subset K$ a C^∞ manifold, and K_1 contains a finite system of points $K_0 = \{v_1, \cdots v_n\}$, the corners of K, such that $K_1 \setminus K_0$ are the one-dimensional C^∞-edges, emanated from the corners. In addition there will be imposed locally close to any $v \in K_0$ the structure of a cone $(B \times [0,1))/(B \times \{0\})$ with a base B being a manifold with conical singularities. B will also be replaced by the stretched manifold $\mathbf{B}$ which has the structure $X \times [0,1)$ near any (stretched) conical point, with a C^∞ manifold X, closed, compact, which is the base of the cone to the corresponding conical point. Locally close to any $t \in K_1 \setminus K_0$ the space K has the structure of a wedge $\{(X \times [0,1))/(X \times \{0\}) \times (a,b)$, $t \in (a,b) \subset K_1 \setminus K_0$. For the analysis on K it will be convenient to talk about the corresponding stretched space $\mathbf{K}$, locally being near (stretched) corners like $\mathbf{B} \times [0,1)$; whereas $\mathbf{K}$ near the emanated edges looks like $X \times [0,1) \times (a,b)$. The points in the corresponding choice of coordinates are denoted by (x,r,t), and t will also be called the corner axis, r the cone axis variable. x sums over the base X of the "model cone" of the wedge.

The programme of the analysis of elliptic equations on $\mathbf{K}$ is to understand the nature of parametrices and the elliptic regularity in weighted Sobolev spaces with (and without) asymptotics. Analogous questions are natural on more general manifolds with singularities, i.e. with ligher edge and corner orders. In classical special cases the problems are well-understood. This concerns, in particular, conical singularities (cf. [K1], [S3,I], edges in form of boundaries of C^∞ manifolds (cf. [B1], [S1]. [S3], VII]), where the model cone is $\overline{\mathbf{R}}_+$ and plays the role of the inner normal to the boundary, or edges with non-trivial (stretched) model cones like $X \times \overline{\mathbf{R}}_+$ (cf.[S1], [S4]). The general edge theory is already rather complex, because of the additional data along the edges (similar to boundary and potential conditions as in [B1]) and of the variable asymptotics of solutions along the edges. Since $\mathbf{K} \setminus \{$ stretched corners$\}$ is a manifold with edges, all this does occur also for $\mathbf{K}$, but this theory has to be combined with ideas from the cone theory along the corner axis, now with operator-valued Mellin symbols operating globally along $\mathbf{B}$. Then we will also get a corner contribution to the asymptotics. The operators in consideration are degenerate in a typical way. For instance, the Laplace-Beltrami operator belonging to $dt^2 + t^2(dr^2 + r^2 g)$

with a Riemannian metric g on X will be of that sort. The ellipticity is to be described here in terms of three leading symbolic levels namely the interior, the edge and the corner ones, denoted by $\sigma^\mu_\psi, \sigma^\mu_\Lambda$ and σ^μ_M, respectively, μ being the order of the operator. Parametrices are obtained by inverting the symbols and there follow Fredholm operators between the corner Sobolev spaces.

1. Edge-corner degenerate differential operators

A differential operator on int **K** (near any stretched corner) will be called edge-corner degenerate if it is close to $t = 0$ of the form

$$A = \omega(r)t^{-\mu}r^{-\mu}\sum_{j+k\leq\mu} a_{jk}(r,t)(-r\frac{\partial}{\partial r})^j(-rt\frac{\partial}{\partial t})^k \tag{1.1}$$

$$+(1-\omega(r))t^{-\mu}\sum_{k=0}^{\mu} a_k(t)(-t\frac{\partial}{\partial t})^k$$

with certain $a_{jk}(r,t) \in C^\infty([0,1) \times [0,1), \mathrm{Diff}^{\mu-(j+k)}(X))$, $a_k(t) \in C^\infty([0,1), \mathrm{Diff}^{\mu-k}$ $(\mathrm{int}\mathbf{B}))$. Here Diff(...) is the space of differential operators on (...) with C^∞ coefficients in local coordinates. $\omega(r)$ is a cut-off function in r, i.e. in $C_0^\infty(\overline{\mathbf{R}}_+)$, $\omega(r) \equiv 1$ close to $r = 0$. In order to describe the operator-valued symbolic levels of (1.1) we need the weighted cone Sobolev spaces. $\mathcal{H}^{s,\beta}(\mathbf{B})$ for $s \in \mathbf{N}$, $\beta \in \mathbf{R}$ will denote the subspace of those $u \in H^s_{\mathrm{loc}}(\mathrm{int}\mathbf{B})$ for which (in the coordinates (x,r) near $r = 0$) $\omega(r)D^\alpha_x(r\partial/\partial r)^k r^{-\beta}u(x,r) \in r^{-n/2}L^2(X \times \mathbf{R}_+)$ for all $\alpha \in \mathbf{N}^n$, $k \in \mathbf{N}$, with $|\alpha| + k \leqq s(n = \dim X)$. For arbitrary $s \in \mathbf{R}$ we define the spaces $\mathcal{H}^{s,\beta}(\mathbf{B})$ by duality and interpolation. Moreover $\mathcal{K}^{s,\beta}(X^n)$ for $X^n = X \times \mathbf{R}_+ \ni (x,r)$ will denote the space of all $\omega u + (1-\omega)v$ with $\omega u(x,r)$ belonging to $\omega\mathcal{H}^{s,\beta}(\mathbf{B})$ and v to $H^s(X^n)$, where the latter space is the standard Sobolev space on X^n of smoothness s (in the special case of $X = S^n$, $X^n \cong \mathbf{R}^{n+1} \setminus \{0\}$ it corresponds to $H^s(\mathbf{R}^{n+1})$ far from the origin).

With (1.1) we can associate the ordinary homogeneous principal symbol $\sigma^\mu_\psi(A)$ of order μ which is a function on $T^*(\mathrm{int}\mathbf{K}) \setminus 0$. Moreover by inserting $z \in \mathbf{C}$ instead of $-t\partial/\partial t$ and freezing coefficients at $t = 0$ it follows the conormal symbol of A of order μ

$$\sigma^\mu_M(A)(z) = \omega(r)r^{-\mu}\sum_{j+k\leqq\mu} a_{jk}(r,0)(-r\frac{\partial}{\partial r})^j(rz)^k \tag{1.2}$$

$$+(1-\omega(r))\sum_{k=0}^{\mu} a_k(0)z^k .$$

This may be regarded as a z-dependent operator family

$$\sigma^\mu_M(A)(z) \ : \mathcal{H}^{s,\beta}(\mathbf{B}) \to \mathcal{H}^{s-\mu,\beta-\mu}(\mathbf{B}) \tag{1.3}$$

for arbitrary $s \in \mathbf{R}$, with any fixed $\beta \in \mathbf{R}$. Finally we can form the homogeneous principal edge symbol

$$\sigma_\Lambda^\mu(A)(t,\tau) = t^{-\mu} r^{-\mu} \sum_{j+k=\mu} a_{jk}(0,t)(-r\frac{\partial}{\partial r})^j(-irt\tau)^k \tag{1.4}$$

which is an operator family

$$\sigma_\Lambda^\mu(A)(t,\tau) \ : \mathcal{K}^{s,\beta}(X^n) \to \mathcal{K}^{s-\mu,\beta-\mu}(X^n) \tag{1.5}$$

$s, \beta \in \mathbf{R}$, parametrized by $(t,\tau) \in T^*(K_1 \setminus K_0) \setminus 0$. The homogeneity of (1.4) is to be understood in the sense

$$\sigma_\Lambda^\mu(A)(t,\lambda\tau) = \lambda^\mu x_\lambda \sigma_\Lambda^\mu(A)(t,\tau) x_\lambda^{-1} \tag{1.6}$$

for all $\lambda > 0$. Here $\{x_\lambda\}_{\lambda \in \mathbf{R}_+}$ is an operator family on $\mathcal{K}^{s,\beta}(X^n)$, defined by $(x_\lambda u)(x,r) = \lambda^{(n+1)/2} u(x,\lambda r)$.

Similarly to the edge theory in general the ellipticity with respect to the edge symbolic level does depend on the weight β. Moreover we will have to pose additional edge conditions, in general being of trace and potential type along the edge. That means that (1.5) has to be completed to a family of isomorphisms

$$e(t,\tau) \ := \begin{pmatrix} \sigma_\Lambda^\mu(A) & \sigma_\Lambda^\mu(K) \\ \sigma_\Lambda^\mu(T) & \sigma_\Lambda^\mu(Q) \end{pmatrix}(t,\tau) \ : \ \begin{matrix} \mathcal{K}^{s,\beta}(X^n) \\ \oplus \\ \mathbf{C}^N \end{matrix} \ \longrightarrow \ \begin{matrix} \mathcal{K}^{s-\mu,\beta-\mu}(X^n) \\ \oplus \\ \mathbf{C}^M \end{matrix} \tag{1.7}$$

for $\tau \neq 0$ and all t. Here it is to supposed $e(t,\lambda\tau) = \lambda^\mu (x_\lambda \oplus 1) e(t,\tau)(x_\lambda \oplus 1)^{-1}$ for all t, and $\tau \neq 0$, $\lambda > 0$. If (1.7) consists of isomorphisms then (1.5) is necessarily a family of Fredholm operators (with index $M - N$). This is the case if A is elliptic with respect to σ_ψ^μ (cf. the more precise description below) and of the "subordinated" leading conormal symbol with respect to every conical point of $\mathbf{B}$

$$(\sigma_M^\mu \sigma_\Lambda^\mu(A))(t,w) \ := t^{-\mu} \sum_{j=0}^{\mu} a_{j0}(0,t) w^j \ : H^s(X) \to H^{s-\mu}(X) \tag{1.8}$$

is a family of isomorphisms for all t, and $w \in \mathbf{C}$ with Re $w = (n+1)/2 - \beta$, for a certain $s \in \mathbf{R}$. The subscript M indicates the relation to the Mellin transform

$$Mu(w) = \int_0^\infty r^{w-1} u(r) dr \ , \tag{1.9}$$

and (1.8) comes from (1.1) where (apart from the weight factor $r^{-\mu}$) it was inserted $r = 0$ and $-r\partial/\partial r$ replaced by w, according to $wMu(w) = (M(-r\partial/\partial r)u)(w)$.

It is well-known that the interior ellipticity implies that (1.8) are isomorphisms for all $s \in \mathbf{R}$ and all $\beta \in \mathbf{R} \setminus \{\beta_j\}_{j \in \mathbf{Z}}$ with some sequence of exceptional weights, $|\beta_j| \to \infty$ as $|j| \to \infty$.

2. Parameter-dependent cone operators and corner Sobolev spaces

As noted in the beginning the finite-dimensional entries of (1.7) play the role of additional edge symbols of trace and potential type, similarly to Boutet de Monvel's theory in the case of boundary value problems, cf. [B1], [S1]. Their behaviour is very close to that of Green operators in the left upper corners to be introduced below. For rotational convenience we shall restrict our consideration from now on to left upper corners. The extension to the general case is straightforward.

The nature of the corner Sobolev spaces as well as of corner Mellin symbols will be determined to a large extent by the nature of certain families of cone pseudo-differential operators along **B**, with a parameter τ. Let us first remind of some notations on parameter-dependent pseudo-differential operators ($\psi D0's$) in the usual set-up. If $\Omega \subset \mathbf{R}^n$ is open, we have the standard space $L^\mu(\Omega)$ of $\psi D0's$ of order $\mu \in \mathbf{R}$ over Ω, defined in terms of symbols $S^\mu(\Omega_x \times \Omega_x, \times \mathbf{R}^n_\xi)$ (= Hörmander's symbol class for $\rho = 1$, $\delta = 0$). We shall content ourselves here with $L^\mu_{c\ell}(\Omega)$, the classical $\psi D0's$. Further $L^\mu_{c\ell}(\Omega\ ;\mathbf{R}^m)$ will denote the space of parameter-dependent $\psi D0's$, defined in terms of classical symbols $a(x,x',\xi,\tau) \in S^\mu_u(\Omega \times \Omega \times \mathbf{R}^n_\xi \times \mathbf{R}^m_\tau)$, $L^{-\infty}(\Omega\ ;\mathbf{R}^m)$ being identified with $\mathcal{S}(\mathbf{R}^m\ ,\ C^\infty(\Omega \times \Omega))$, the Schwartz space of $C^\infty(\Omega \times \Omega)$-valued functions on $\mathbf{R}^m$. Analogous notions make sense globally over X or $X^n = X \times \mathbf{R}_+$, i.e. we have the spaces $L^\mu_{c\ell}(X\ ;\mathbf{R}^m_\tau)$, $L^\mu_{c\ell}(X^n\ ;\mathbf{R}^m_\tau)$. We mainly need the cases $m = 0,1,2$. The spaces of parameter-dependent $\psi D0's$ are Fréchet in a natural way. If $U \subset \mathbf{C}$ is open and F any Fréchet space then $\mathcal{A}(U,F)$ will denote the space of F-valued holomorphic functions in U.

Definition 2.1.— *$M^\mu_0(X\ ;\mathbf{R}^m)$ for any $\mu \in \mathbf{R}$ will denote the space of all $h(w,\tau) \in \mathcal{A}(\mathbf{C}, L^\mu_{c\ell}(X\ ;\mathbf{R}^m_\tau))$, $w \in \mathbf{C}$, with $h(\delta + i\rho\ ,\tau) \in L^\mu_{c\ell}(X\ ;\mathbf{R}^{m+1}_{\rho,\tau})$ uniformly in every finite strip $c \leqq \delta \leqq c'$, $c,c' \in \mathbf{R}$.*

For $m = 0$ we get by definition $M^\mu_0(X)$. All our spaces in question have a natural locally convex topology. This will tacitly be used in the sequel. In particular the spaces of Definition 2.1 are Fréchet.

Now let us introduce the weighted Mellin $\psi D0's$ with parameters

$$op^\gamma_M(h)(\tau)u(r) = r^\gamma M^{-1}_{w\to r} h(w-\gamma,\tau) M_{r'\to w}((r')^{-\gamma}u(r')) \tag{2.1}$$

for $\gamma \in \mathbf{R}$, $h(w,\tau) \in M^\mu_0(X\ ;\mathbf{R}^m_\tau)$, cf.(1.9). The inverse Mellin transform is taken in the sense $(M^{-1}g)(r) = \int r^{-w} g(w)dw/2\pi i$, with integration along $\Gamma_{1/2}$. Here

$$\Gamma_\alpha = \{w \in \mathbf{C}\ : \mathrm{Re}\ w = \alpha\}\ . \tag{2.2}$$

If $\omega(r)$, $\omega_0(r)$ are arbitrary cut-off functions, then we get continuous operators $\omega r^{-\mu}\ \mathrm{op}^{\beta-n/2}_M(h)(\tau)\omega_0\ : \mathcal{K}^{s,\beta}(X^n) \to \mathcal{K}^{s-\mu,\beta-\mu}(X^n)$ for all $s \in \mathbf{R}$, dependent on τ as parameter ($n = \dim X$). The cone $\psi D0's$ in general also contain Mellin operators with meromorphic operator-valued symbols, associated with any sequence

$$R = \{(p_j, m_j, N_j)\}_{j\in\mathbf{Z}}\ , \tag{2.3}$$

$p_j \in \mathbf{C}$ being the poles of multiplicities $m_j + 1$, $|Re\ p_j| \to \infty$ as $|j| \to \infty$, and $N_j \subset L^{-\infty}(X)$ are finite-dimensional subspaces of finite-dimensional operators. Let $M_R^{-\infty}(X)$ denote the space of all $h(w) \in \mathcal{A}(\mathbf{C} \setminus \pi_{\mathbf{C}} R\ , L^{-\infty}(X))$, $\pi_{\mathbf{C}} R = U\{p_j\}$, such that $h(w)$ is meromorphic with poles at p_j of multiplicities $m_j + 1$ and Laurent coefficients at $(w - p_j)^{-(k+1)}$ in N_j, $0 \leqq k \leqq m_j$, and further $\chi(w)h(w)|_{\Gamma_\delta} \in L^{-\infty}(X\ ;\Gamma_\delta)$ uniformly in $c \leqq \delta \leqq c'$ for all $c, c' \in \mathbf{R}$, for any $\chi(w) \in C^\infty(\mathbf{C})$ with $\chi(w) = 0$ for dist $(w\ ;\pi_{\mathbf{C}} R) < \varepsilon$, $\chi(w) = 1$ for dist $(w\ ;\pi_{\mathbf{C}} R) > 2\varepsilon$ with some $\varepsilon > 0$. Here $L^{-\infty}(X\ ;\Gamma_\delta)$ is to be understood as $L^{-\infty}(X\ ;\mathbf{R})$ under the identification $\mathbf{R} \ni \rho \to \delta + i\rho \in \Gamma_\delta$. Occasionally we will write $sg(h) = \pi_{\mathbf{C}} R$.

The smoothing parameter-dependent Mellin operators of the cone theory are related to a given weight $\beta \in \mathbf{R}$ and some weight strip

$$\theta = (-k, 0] \quad \text{for some} \quad k \in \mathbf{N} \setminus \{0\} \tag{2.4}$$

They are of the form

$$m(\tau) = \omega(r[\tau])r^{-\nu} \sum_{j=0}^{k-1} r^j op_M^{\beta_j - n/2}(h_j)\varphi_j(\tau)\tilde{\omega}(r[\tau]) \tag{2.5}$$

with $\varphi_j(\tau)$ being a polynomial in $\tau \in \mathbf{R}$ of order $\leqq j$, $\tau \to [\tau]$ a strictly positive C^∞ function with $[\tau] = |\tau|$ for $|\tau| >$ const > 0, ω ,$\tilde{\omega}$ arbitrary cut-off functions, $h_j(w) \in M_{R_j}^{-\infty}(X)$ with certain R_j as mentioned, cf. (2.3), and weighs β_j satisfying $sg(h_j) \cap \Gamma_{(n+1)/2 - \beta_j} = \emptyset$, and $\beta_j + j \geqq, \beta - \beta_j \geqq 0$ for all $j = 0, \cdots, k-1$.

Parallel to the pattern of poles of meromorphic Mellin symbols there will be organized the asymptotic types of distributions over X^n close to $r = 0$. They are sequences $P = \{(p_j, m_j, L_j)\}_{j \in \mathbf{N}}$ with $p_j \in \mathbf{C}$, Re $p_j < (n+1)/2 - \beta$, Re $p_j \to -\infty$ as $j \to \infty$, and $L_j \subset C^\infty(X)$ are finite-dimensional subspaces. Then $\mathcal{K}_P^{s,\beta}(X^n)$ denotes the subspace of all $u(x,r) \in \mathcal{K}^{s,\beta}(X^n)$ with asymptotics $\sum_{j=0}^{\infty} \sum_{k=0}^{m_j} \zeta_{jk}(x) r^{-p_j} \log^k r$ for $r \to 0$, with certain $\zeta_{jk} \in L_j$, $0 \leqq k \leqq m_j$ (uniquely determined by u). The precise meaning of the asymptotics may be found, for instance, in [S1], [S3,I].

Proposition 2.2.— *(2.5) induces for every τ continuous operators*

$$m(\tau)\ : \mathcal{K}^{s,\beta}(X^n) \to \mathcal{K}^{\infty,\beta-\nu}(X^n)\ ,\ \mathcal{K}_P^{s,\beta}(X^n) \to \mathcal{K}_Q^{\infty,\beta-\nu}(X^n) \tag{2.6}$$

for every $s \in \mathbf{R}$ and every asymptotic type P with another asymptotic type Q, dependent on $m(\tau)$ and P. If $\tilde{m}(\tau)$ is of analogous structure as $m(\tau)$ with the same h_j , φ_j, but with another choice of β_j and cut-off's, then $g(\tau) := m(\tau) - \tilde{m}(\tau)$ induces continuous operators

$$g(\tau)\ : \mathcal{K}^{s,\beta}(X^n) \to \mathcal{K}_{Q_1}^{\infty,\beta-\nu}(X^n), g^*(\tau)\ : \mathcal{K}^{s,-\beta+\nu}(X^n) \to \mathcal{K}_{Q_2}^{\infty,-\beta}(X^n) \tag{2.7}$$

with certain asymptotic types Q_1, Q_2, dependent on g (not on s).

$g^*(\tau)$ in (2.7) refers to the scalar product of $\mathcal{K}^{0,0}(X^n)$. Let us set $\mathcal{S}_P^\gamma(X^n) = \omega\mathcal{K}_P^{\infty,\gamma}(X^n) + (1-\omega)\mathcal{S}(\bar{X}^n)$ with obvious meaning of the sum, $\mathcal{S}(\bar{X}^n) := C^\infty(X) \otimes_{\bar{n}} \mathcal{S}(\bar{\mathbf{R}}_+)$ ($\otimes_{\bar{n}}$ being the completed projective tensor product). Analogous notations such as $\mathcal{K}_P^{s,\gamma}(X^n)$, $\mathcal{S}_P^\gamma(X^n)$ make sense for finite asymptotic types P i.e. where $\pi_{\mathbf{C}}P$ is finite and contained in the weight strip $\theta = (-k, 0]$ on the left of $\Gamma_{(n+1)/2-\gamma}$, i.e. in $(n+1)/2-\gamma-k <$ Re $w < (n+1)/2-\gamma$. Remainders of finite asymptotic expansions then belong to

$$\mathcal{K}_\theta^{s,\gamma}(X^n) := \lim_{\substack{\varepsilon\to 0\\ \varepsilon>0}} \mathcal{K}^{s,\gamma+k-\varepsilon}(X^n).$$

The operator families (2.6), (2.7) are in fact operator-valued symbols with τ as covariable, according to the following general definition (here being given for "constant coefficients"). Let E be a Banach space, $\{x_\lambda\}_{\lambda\in\mathbf{R}_+} \subset C(\mathbf{R}_+, \mathcal{L}_\sigma(E))$ be a group of isomorphisms (σ indicating the strong operator topology in $\mathcal{L}(E)$). Example : $E = \mathcal{K}^{s,\beta}(X^n), (x_\lambda u)(x,r) = \lambda^{(n+1)/2}u(x,\lambda r)$. If $\tilde{E}$ is another Banach space with a corresponding group $\{\tilde{x}_\lambda\}_{\lambda\in\mathbf{R}_+}$, then $S^\nu(\mathbf{R}\,; E, \tilde{E})$ for $\nu \in \mathbf{R}$ denotes the space of all $a(\tau) \in C^\infty(\mathbf{R}, \mathcal{L}(E,\tilde{E}))$ with

$$\|\tilde{x}_{[\tau]}^{-1}\{D_\tau^j a(\tau)\}x_{[\tau]}\|_{\mathcal{L}(E,\tilde{E})} \leqq c[\tau]^{\nu-j} \quad \text{for all} \quad j \in \mathbf{N}$$

with constants $c = c(j) > 0$. The subspace of classical symbols $S_{c\ell}^\nu(\mathbf{R}\,; E, \tilde{E})$ is defined by $a(\tau) \sim \sum_{j=0}^\infty a_{\nu-j}(\tau)$ with $a_{\nu-j}(\lambda\tau) = \lambda^{\nu-j}\tilde{x}_\lambda a_{\nu-j}(\tau)x_\lambda^{-1}$ for all $\lambda \geqq 1$, $|\tau| \geqq$ const, for all j. We will now employ the straightforward extension of these notions to Fréchet spaces $\tilde{E}$, cf. [S1], [S3,VII].

Définition 2.3.— *Fix weight data $\eta = (\beta, \beta-\mu, \theta)$ with $\beta, \mu \in \mathbf{R}$ and (2.4), and let $\nu \in \mathbf{R}$. Then $C_G^\nu(X^n, \eta\,; \mathbf{R}_\tau)$ is the space of all $g(\tau) \in \bigcap_s S_{cl}^\nu(\mathbf{R}_\tau\,; \mathcal{K}^{s,\beta}(X^n), \mathcal{K}^{\infty,\beta-\mu}(X^n))$ which induce even elements*

$$g(\tau) \in S_{c\ell}^\nu(\mathbf{R}\,; \mathcal{K}^{s,\beta}(X^n), \mathcal{S}_{Q_1}^{\beta-\mu}(X^n)), g^*(\tau) \in S_{c\ell}^\nu(\mathbf{R}\,; \mathcal{K}^{s,-\beta+\mu}(X^n), \mathcal{S}_{Q_2}^{-\beta}(X^n)) \qquad (2.8)$$

for all $s \in \mathbf{R}$, with (finite) asymptotic types Q_1, Q_2 dependent on g, with $\pi_{\mathbf{C}}Q_1, \pi_{\mathbf{C}}Q_2$ in the θ-strip on the left of the corresponding weight lines $\Gamma_{(n+1)/2-\beta+\mu}$ and $\Gamma_{(n+1)/2+\beta}$, respectively. The $g(\tau)$ are called Green operator families (of the parameter-dependent cone theory).

Globally over $\mathbf{B}$ we will need the space $C_G^{-\infty}(\mathbf{B}, \eta\,; \mathbf{R})$ of all $g(\tau) \in \bigcap_s \mathcal{S}(\mathbf{R}_\tau, \mathcal{L}(\mathcal{H}^{s,\beta}(\mathbf{B}), \mathcal{H}_{Q_1}^{\infty,\beta-\mu}(\mathbf{B}))$ for which also $g^*(\tau) \in \bigcap_s \mathcal{S}(\mathbf{R}_\tau, \mathcal{L}(\mathcal{H}^{s,-\beta+\mu}(\mathbf{B}), \mathcal{H}_{Q_2}^{\infty,-\beta}(\mathbf{B}))$. Here $\mathcal{H}_Q^{s,\gamma}(\mathbf{B})$ is defined as the subspace of those $u \in \mathcal{H}^{s,\gamma}(\mathbf{B})$ such that $\omega u \in \mathcal{K}_Q^{s,\gamma}(X^n)$ for some cut-off function ω, supported by a tubular neighbourhood of $\partial\mathbf{B}(\cong X)$. The latter $*$ refers to the scalar product in $\mathcal{H}^{0,0}(\mathbf{B})$.

Next we shall formulate a result on a "Mellin operator convention" for parameter-dependent "degenerate" symbols. Let $\{U_j\}_{1\leqq j\leqq N}$ be an open finite covering of X by coordinate neighbourhoods, $\{\varphi_j\}_{1\leqq j\leqq N}$ be a subordinated partition of unity, and $\{\tilde{\varphi}_j\}_{1\leqq j\leqq N}$ be another system of functions in $C_0^\infty(U_j)$ with $\varphi_j\tilde{\varphi}_j = \varphi_j$ for all j.

Theorem 2.4.— *Let* $\{p_j(x,r,\xi,\rho,\tau)\}_{1 \leqq j \leqq N}$ *be symbols in* $S^{\nu}_{cl}(U_j \times \bar{\mathbf{R}}_+ \times \mathbf{R}^{n+2}_{\xi,\rho,\tau})$ *(x being local coordinates in* U_j*) and form* $q_j(x,r,\xi,\rho,\tau) := p_j(x,r,\xi,r\rho,r\tau)$. *Set*

$$f_1(\tau) = \sum_{j=1}^{N} \varphi_j op_{\psi,(x,r)}(q_j)(\tau)\tilde{\varphi}_j \tag{2.9}$$

($op_{\psi,(x,r)}(.)$ *indicates the pseudo-differential operator with respect to the* (x,r)*-variables, pull-backed to* U_j *; this depends on the parameter* τ*). Then there exists an* $m(r,w,\tau) \in C^{\infty}(\bar{\mathbf{R}}_+, M^{\nu}_0(X\,;\mathbf{R}_\tau))$ *such that for* $f_0(r,w,\tau) := m(r,w,r\tau)$ *we have* $op_M^{\beta-n/2}(f_0)(\tau) = f_1(\tau)$ *mod* $L^{-\infty}(X^n\,;\mathbf{R}_\tau)$.

Definition 2.5.— *Let* $\mu,\nu,\beta \in \mathbf{R}, \mu-\nu \in \mathbf{N}, \eta = (\beta,\beta-\mu,\theta)$. *Then* $C^{\nu}(\mathbf{B},\eta\,;\mathbf{R})$ *is the space of all operator families*

$$a(\tau) = \omega\{a_0(\tau) + a_1(\tau) + m(\tau) + g(\tau)\}\omega_0 + (1-\omega)f(\tau)(1-\omega_1) + g_0(\tau) \tag{2.10}$$

with cut-off functions $\omega(r), \omega_i(r), i = 0,1$, *satisfying* $\omega\omega_0 = \omega, \omega\omega_1 = \omega_1$, *and* $f(\tau) \in L^{\nu}_{c\ell}(\text{int } \mathbf{B}\,;\mathbf{R}_\tau)$,

$$a_0(\tau) = \omega(r[\tau])r^{-\nu}op_M^{\beta-n/2}(f_0)(\tau)\omega_0(r[\tau]), a_1(\tau) = (1-\omega(r[\tau]))r^{-\nu}f_1(\tau)(1-\omega_1(r[\tau])) \tag{2.11}$$

with f_0, f_1 *being as in Theorem 2.4, further* $m(\tau)$ *being of the form (2.5),*

$$g(\tau) \in C^{\nu}_G(X^n,\eta\,;\mathbf{R}), g_0(\tau) \in C^{-\infty}_G(\mathbf{B},\eta\,;\mathbf{R}) \,.$$

Note that the cut-off functions ω, ω_i in (2.10) may be chosen different from those in (2.11) without changing the class of operators.

We shall also write $C^{\nu}(\mathbf{B},\eta\,;\Gamma_\delta)$ when $\mathbf{R}_\tau$ is identified with $\Gamma_\delta = \delta + i\tau$.

Theorem 2.6.— *For every* $\beta,\mu \in \mathbf{R}$ *there exists an operator family* $b^{\mu}(\tau) \in C^{\mu}(\mathbf{B},\eta\,;\mathbf{R}_\tau)$ *with* $\eta = (\beta,\beta-\mu,\theta)$, *such that*

$$b^{\mu}(\tau) \,:\mathcal{H}^{s,\beta}(\mathbf{B}) \to \mathcal{H}^{s-\mu,\beta-\mu}(\mathbf{B}) \tag{2.12}$$

is an isomorphism for every fixed $\tau \in \mathbf{R}$, *and all* $s \in \mathbf{R}$.

The parameter-dependent cone operator families of the class $C^{\nu}(\mathbf{B},\eta\,;\mathbf{R})$ will be the starting point below for introducing the meromorphic corner Mellin symbols, where τ plays the role of Im z for the complex Mellin covariable z to the corner axis variable t. The special families (2.12) may be used for introducing the corner Sobolev spaces over $\mathbf{B}^n := \mathbf{B} \times \mathbf{R}_+$ (the infinite stretched corner with base $\mathbf{B}$).

Définition 2.7.— *Let* $s \in \mathbf{R}, \gamma = (\beta, \alpha) \in \mathbf{R}^2$. *Then* $\mathcal{H}^{s,\gamma}(\mathbf{B}^n)$ *is the completion of* $C_0^\infty(\text{int}\mathbf{B} \times \mathbf{R}_+)$ *with respect to the norm*

$$\left\{ \frac{1}{2\pi i} \int_{\Gamma_{(n+2)/2-\alpha}} \| b^s(\tau)(M_{t\to z}u)(z)\|^2_{\mathcal{H}^{0,\beta-s}(\mathbf{B})} dz \right\}^{1/2} \tag{2.13}$$

Remark 2.8. If $\tilde{b}^s(\tau)$ is another operator family with the mentioned properties then the associated norm is equivalent to (2.13).

Remark 2.9. The spaces $\mathcal{H}^{s,\gamma}(\mathbf{B}^n)$ are over $\mathbf{B} \times (t_1, t_2)$ for every $0 < t_1 < t_2 < \infty$ equivalent to the corresponding wedge Sobolev spaces from [S1], Chapter 3 (or [S4]). This enables us to introduce the global corner spaces $\mathcal{H}^{s,\gamma}(\mathbf{K})$ in terms of the corresponding properties under localizations close to $t = 0$ and onbide any neighbourhood of $t = 0$.

Remark 2.10. Analogous by to Definition 2.5 it makes sense also to introduce $C^\nu(\mathbf{B}, \eta\ ; \mathbf{R}^m)$ with the parameters $\tau = (\tau_1, \cdots, \tau_m) \in \mathbf{R}^m$. In particular, for $m = 0$ we get the usual class of cone *ψDO'S* $C^\nu(\mathbf{B}, \eta)$, cf. [S1], [S3,I]. As noted above all these spaces have a natural locally convex topology (here the inductive limit of Fréchet spaces).

3. Mellin operators and edge-corner asymptotics

Analogously to Definition 2.1 (for $m = 0$) we now introduce holomorphic operator-valued Mellin symbols where X is to be replaced by $\mathbf{B}$. Since $\mathbf{B}$ itself has singularities there will be inharited the weight data $\eta = (\beta, \beta - \mu, \theta)$, cf. Definition 2.5.

Definition 3.1.— $M_0^\nu(\mathbf{B}, \eta)$ *is the space of all*

$$h(Z) \in \bigcap_{s\in\mathbf{R}} \mathcal{A}(\mathbf{C}, \mathcal{L}(\mathcal{H}^{s,\beta}(\mathbf{B}), \mathcal{H}^{s-\nu,\beta-\mu}(\mathbf{B}))$$

with $h(Z)|_{\Gamma_\delta} \in C^\nu(\mathbf{B}, \eta\ ; \Gamma_\delta)$ *uniformly in every finite strip* $c \leqq \delta \leqq c'$, $c, c' \in \mathbf{R}$.

Similarly to the cone theory we will also have smoothing Mellin symbols with a meromorphic structure. They are associated with any system $R = \{(p_j, m_j, N_j)\}_{j\in\mathbf{Z}}$ with p_j, m_j being of the same meaning as in (2.3), but now $N_j \subset C_G(\mathbf{B}, \eta)$ are finite-dimensional spaces of finite-dimensional operators, where the asymptotics types are independent of j (here $C_G(\mathbf{B}, \eta)$ equals $C_G^{-\infty}(\mathbf{B}, \eta\ ; \mathbf{R}^m)$ for $m = 0$).

Now $M_R^{-\infty}(\mathbf{B}, \eta)$ is the space of all $h(Z) \in \mathcal{A}(\mathbf{C} \setminus \pi_{\mathbf{C}} R, \bigcap_{s\in\mathbf{R}} \mathcal{L}(\mathcal{H}^{s,\beta}(\mathbf{B}), \mathcal{H}^{\infty,\beta-\mu}(\mathbf{B})))$ which are meromorphic with poles at all p_j of multiplicities $m_j + 1$, with Laurent coefficients at $(z - p_j)^{(k+1)}$ belonging to N_j, for all $0 \leqq k \leqq m_j$ and $\chi(Z)h(Z)|_{\Gamma_\delta} \in C_G^{-\infty}(\mathbf{B}, \eta\ ; \Gamma_\delta)$ uniformly in $c \leqq \delta \leqq c'$ for all $c, c' \in \mathbf{R}$ and any $\pi_{\mathbf{C}} R$-excision function $\chi(Z)$.

The edge-corner asymptotics $P = (P_1, P_0)$ of distributions in $\mathcal{H}^{s,\gamma}(\mathbf{K}), s \in \mathbf{R}, \gamma = (\beta, \alpha) \in \mathbf{R}^2$, will consist of an edge and a corner part

$$P_1 = \{(\pi_\ell, \mu_\ell, \Lambda_\ell)\}_{\ell\in\mathbf{N}} \quad \text{and} \quad P_0 = \{(p_j, m_j, L_j)\}_{j\in\mathbf{N}},$$

respectively. Here $\pi_\ell \in \mathbf{C}, (n+1)/2 - \beta > \mathrm{Re}\ \pi_\ell, \mathrm{Re}\ \pi_\ell \to -\infty$ as $\ell \to \infty$, $\mu_\ell \in \mathbf{N}$, $\Lambda_\ell \subset C^\infty(X)$ is of finite dimension, and $p_j \in \mathbf{C}, (n+2)/2 - \alpha > \mathrm{Re}\ p_j, \mathrm{Re} p_j \to -\infty$ as $j \to \infty$, $m_j \in \mathbf{N}, L_j \subset \mathcal{H}^{\infty,\beta}_{P_1}(\mathbf{B})$ is of finite dimension. The "non-smooth singular functions" of the edge asymptotics along $K_1 \setminus K_0$, described by P_1, have the form

$$F^{-1}_{\tau\to t}\{[\tau]^{(n+1)/2}(r[\tau])^{-\pi_\ell}(F_{t\to\tau}\lambda_{\ell h})(x,\tau)\log^h(r[\tau])\omega(r[\tau])\} \tag{3.1}$$

with $\lambda_{\ell h}(x,t) \in H^s_{\mathrm{loc}}(\mathbf{R}_+, \Lambda_\ell), 0 \leqq h \leqq \mu_\ell$. Here $F_{t\to\tau}$ is the Fourier transform along the edge. Close to $t = 0$ we take instead of this the Mellin transform $M_{z\to t}$ in the weighted sense, with the weight $\alpha - (n+1)/2, \tau = \mathrm{Im}_z$, where the non-smooth singular functions are

$$M^{-1}_{z\to t}\{[\tau]^{(n+1)/2}(r[\tau])^{-\pi_\ell}(M_{z\to t}\lambda_{\ell h})(x,Z)\log^h(r[\tau])\omega(r[\tau])\}\ , \tag{3.2}$$

$z = (u+2)/2 - \alpha + i\tau$, $\lambda_{\ell h}(x,t) \in \mathcal{H}^{s,\alpha-(n+1)/2}(\mathbf{R}_+, \Lambda_\ell)$. This is compatible to (3.1) modulo smooth singular functions in intersections of coordinate neighbourhoods on $K_1 \setminus K_0$ (the latter ones are allowed in addition anyway ; P_1 is assumed to satisfy the shadow condition, cf. [S1], [S3]). (3.2) is the edge contribution to the corner asymptotics. The corner contribution P_0 to the asymptotics contains global data along $\mathbf{B}$. It is of the form

$$u(\tilde{x},t) \sim \sum_{j=0}^{\infty}\sum_{k=0}^{m_j} \zeta_{jk}(\tilde{x})t^{-p_j}\log^k t \quad \text{for} \quad t \to 0\ , \tag{3.3}$$

$\tilde{x} \in \mathrm{int}\mathbf{B}$, $\zeta_{jk} \in L_j$ for $0 \leqq k \leqq m_j$. Any pair $P = (P_1, P_0)$ will be called an edge-corner asymptotic type. We will denote by $\mathcal{H}^{s,\gamma}_P(\mathbf{K})$ the subspace of all $u \in \mathcal{H}^{s,\gamma}(\mathbf{K})$ with asymptotics of type P.

Now let us come to the edge-corner $\psi D0's$ themselves. The smoothing elements form a space $C_G(\mathbf{K},\eta)$, called edge-corner Green operators, associated with the weight data $\eta = (\beta, \beta-\mu, \theta\ ; \alpha, \alpha-\mu, \theta)$. It is defined as the set of all $G \in \bigcap_s \mathcal{L}(\mathcal{H}^{s,\gamma}(\mathbf{K}), \mathcal{H}^{\infty,\gamma-\mu}(\mathbf{K}))$ which induce continuous operators

$$G\ : \mathcal{H}^{s,\gamma}(\mathbf{K}) \to \mathcal{H}^{\infty,\gamma-\mu}_P(\mathbf{K})\ ,\ G^*, \mathcal{H}^{s,-\gamma+\mu}(\mathbf{K}) \to \mathcal{H}^{\infty,-\gamma}_Q(\mathbf{K})$$

for all $s \in \mathbf{R}$, for certain edge-corner asymptotics types P, Q, dependent of G (and associated with the corresponding weight). $*$ refers to the scalar product of $\mathcal{H}^{0,(0,0)}(\mathbf{K})$.

Definition 3.2.— *Let $\mu, \nu \in \mathbf{R}$, $\mu - \nu \in \mathbf{N}$, and η be as mentioned. Then $C^\nu(\mathbf{K},\eta)$, the space of all edge-corner $\psi DO's$ with constant discrete asymptotics, is the space of all $A = A_0 + A_1 + M + G$ with*

$$A_0 = \omega t^{-\nu} op_M^{\alpha-(n+1)/2}(h)\omega_0 \quad with \quad h(t,Z) \in C^\infty(\overline{\mathbf{R}}_+, M^\nu_0(\mathbf{B},\eta_1))$$

($\eta_1 = (\beta,\beta-\mu,\theta)$; $n+1 = \dim\mathbf{B}$), $A_1 = (1-\omega)P(1-\omega_1)$ with a $\psi D0$ P of the wedge class on $\mathbf{K} \setminus$ {stretched corners} (with constant discrete asymptotics, cf. [S1], [S4]), also

associated with the weight data η_1, and $\omega(t), \omega_0(t), \omega_1(t)$ being arbitrary cut-off functions, furthermore

$$M = \omega\{t^{-\nu} \sum_{j=0}^{k-1} t^j op_M^{\delta_j-(n+1)/2}(h_j)\}\omega, \quad \text{where} \quad \theta = (-k, 0],$$

with arbitrary $h_j(Z) \in M_{R_j}^{-\infty}(\mathbf{B}, \eta_1), \pi_{\mathbf{C}} R_j \cap \Gamma_{(n+2)/2-\delta_j} = \emptyset, \alpha - (\mu - \nu) - j \leqq \delta_j \leqq \alpha$ for all j and finally $G \in C_G(\mathbf{K}, \eta)$.

Remark 3.3. The differential operators (1.1) belong to $C^\mu(\mathbf{K}, \eta)$, where in this case M and G disappear.

Theorem 3.4.— *Every $A \in C^\nu(\mathbf{K}, \eta)$ with $\eta = (\beta, \beta - \mu, \theta\ ; \alpha, \alpha - \mu, \theta)$ induces continuous operators*

$$A\ : \mathcal{H}^{s,\gamma}(\mathbf{K}) \to \mathcal{H}^{s-\mu,\gamma-\mu}(\mathbf{K})\ ,\ A\ : \mathcal{H}_P^{s,\gamma}(\mathbf{K}) \to \mathcal{H}_Q^{s-\mu,\gamma-\mu}(\mathbf{K})$$

for $\gamma = (\alpha, \beta)$, every $s \in \mathbf{R}$, and every edge-corner asymptotic type P with another edge-corner asymptotic type Q, dependent on P and A (not on s).

Remark 3.5. The operator classes $C^\nu(\mathbf{K}, \eta)$ can be generalized to classes of matrices with additional trace and potential conditions, similarly to (1.7). Another generalization concerns the asymptotics in the continuous sense, formulated in terms of vector-valued analytic functionals (cf. [S1], [S3], [S4]).

Remark 3.6. The symbolic levels $\sigma_\psi^\nu, \sigma_\Lambda^\nu, \sigma_M^\nu$ as they have been described for $\nu = \mu$ in Section 1 make sense analogously also over $C^\nu(\mathbf{K}, \eta)$, and they are multiplicative under compositions of operators (which preserves the nature of our operators, where the order of the composition is the sum of orders of the factors)

Remark 3.7. If $\sigma_\psi^\mu, \sigma_\Lambda^\mu, \sigma_M^\mu$ vanish on $A \in C^\nu(\mathbf{K}, \eta)$, then $A\ : \mathcal{H}^{s,\gamma}(\mathbf{K}) \to \mathcal{H}^{s-\mu,\gamma-\mu}(\mathbf{K})$ is compact for all $s \in \mathbf{R}$.

4. Ellipticity and parametrices

Definition 4.1.— *$A \in C^\mu(\mathbf{K}, \eta)$ is called elliptic with respect to the weights $\gamma = (\beta, \alpha)$ if*

(i) $\sigma_\psi^\mu(A) \neq 0$ on $T^*(\text{int } \mathbf{K}) \setminus 0$, and if in addition it remains $\neq 0$ after putting $r\rho = \tilde{\rho}$ close to $r = 0, t\tau = \tilde{\tau}$ close to $t = 0$, or $r\rho = \tilde{\rho}$, $rt\tau = \tilde{\tau}_0$ close to $r = 0, t = 0$, now up to $r = 0, t = 0$, after removing the weight factors, and $(\mathcal{E}, \tilde{\rho}, \tilde{\tau}), \cdots$ being interpreted as new covariables,

(ii) $\sigma_\Lambda^\mu(A)(t, \tau)$ is bijective in the sense (1.5) for some s, for all $(t, \tau) \in T^*(K_1 \setminus K_0) \setminus 0$, and it remains bijective close to $t = 0$ up to $t = 0$ after putting $t\tau = \tilde{\tau}$ and removing the weight factor $t^{-\mu}$,

(iii) $\sigma_M^\mu(A)(z)$ is bijective in the sense (1.3) for some s, for all $z \in \Gamma_{(n+2)/2-\alpha}$.

Theorem 4.2.— *Let $A \in C^\mu(\mathbf{K},\eta)$ be elliptic (with respect to $\gamma = (\beta,\alpha)$, where $\eta = (\beta,\beta-\mu,\theta\ ;\alpha,\alpha-\mu,\theta)$. Then*

$$A\ :\mathcal{H}^{s,\gamma}(\mathbf{K}) \to \mathcal{H}^{s-\mu,\gamma-\mu}(\mathbf{K}) \tag{4.1}$$

is a Fredholm operator for all $s \in \mathbf{R}$.

For obtaining a parametrix of A within our class with constant discrete asymptotics we need an extra assumption, namely

$$\begin{cases} \text{the points } w \in \mathbf{C} \text{ of non}-\text{bijectivity of} \sigma_M^\mu \sigma_\Lambda^\mu(A)(t,w) \\ \text{in the sense of (1.8) are independent of } t\ . \end{cases} \tag{4.2}$$

Theorem 4.3.— *Under the condition (4.2) an elliptic operator $A \in C^\mu(\mathbf{K},\eta)$ has a parametrix $P \in C^{-\mu}(\mathbf{K},\zeta)$ with $\zeta = (\beta-\mu,\beta,\theta\ ;\alpha-\mu,\alpha,\theta)$, in the sense that $AP - 1 \in C_G(\mathbf{K},\eta_r), PA-1 \in C_G(\mathbf{K},\eta_\ell)$ with $\eta_r = (\beta-\mu,\beta-\mu,\theta\ ;\alpha-\mu,\alpha-\mu,\theta), \eta_\ell = (\beta,\beta,\theta\ ;\alpha,\alpha,\theta)$.*

Theorem 4.4.— *Let $A \in C^\mu(\mathbf{K},\eta)$ be elliptic. Then $Au = f \in \mathcal{H}^{s,\gamma-\mu}(\mathbf{K}), u \in \mathcal{H}^{-\infty,\gamma}(\mathbf{K})$ implies $u \in \mathcal{H}^{s+\mu,\gamma}(\mathbf{K})$ for all $s \in \mathbf{R}$. In addition, if (4.2) holds, then $Au = f \in \mathcal{H}_Q^{s,\gamma-\mu}(\mathbf{K})$ for some edge-corner asymptotic type and $u \in \mathcal{H}^{-\infty,\gamma}(\mathbf{K})$ implies $u \in \mathcal{H}_P^{s+\mu,\gamma}(\mathbf{K})$ for a resulting edge-corner asymptotic type P, independent of $s \in \mathbf{R}$.*

Remark 4.5. The concept of edge-corner $\psi DO's$ can be generalized to the continuous asymptotics, analogously to the corresponding cone and edge theory from [S1], [S4]. This larger class is closed under constructing parametrices of elliptic operators. In particular there always exists a parametrix in this sense for elliptic $A \in C^\mu(\mathbf{K},\eta)$ without the condition (4.2). Analogous results hold with additional edge trace and potential conditions.

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