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Errata aux exposés I et II

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SEMINAIRE MAUREY-SCHWARTZ 1975-1976

Exposés I et II

ERRATA

Exposé n° I

Sur la page de couverture, au lieu de 7 Octobre 1976, lire 7 Octobre 1975.

Exposé n° II

With apologies to D. R. Lewis and thanks to J. J. Uhl for pointing out to us that the lemma on page II.4 is not correct as stated, change the last line in the statement of the lemma to the following :
" Then there exists $\tau_o : M \to Y$, Bochner integrable, $\sup_m \|\tau_o(m)\| \leq \sup_m \|\tau(m)\|$ such that for all $y^* \in Y^*$, $y^* \circ \tau = y^* \circ \tau_o$ μ - a.e."

To prove the lemma in this form, let Σ be the σ-algebra of all μ-measurable subsets of M and define $\lambda : \Sigma \to Y^{**}$ by

$$<\lambda(E), y^*> = \int_E y^*(\tau(m))d\mu(m).$$

Since $\lambda(E)$ is sequentially continuous on the unit ball of Y^* in the $\sigma(Y^*, Y)$ topology, $\lambda(E)$ is in Y. (cf. corollaries 3 and 4, page 148 of Diestel's Geometry of Banach Spaces - Selected Topics, Berlin-Heidelberg-New-York, 1975). Thus λ is a Y valued measure, absolutely continuous with respect to μ. The proofs of claims 1 and 2 (which are correct except for the last two sentences of the verification of claim 2) show that λ is representable by a Bochner integrable function $\tau_o : K \to Y$. That is, τ_o is strongly measurable, essentially separably valued, and

$$\lambda(E) = \int_E \tau_o(m)d\mu(m).$$

In particular, for all $y^* \in Y^*$, all $E \in \Sigma$,

$$\langle \lambda(E), y^* \rangle = \int_E y^*(\tau_o(m))\,d\mu(m)$$

$$= \int_E y^*(\tau(m))\,d\mu(m).$$

Suppose $\|\tau(m)\| \leq 1$ for all $m \in M$. Let Y_o be a closed, separable, linear subspace of Y such that $\mu\{m : \tau_o(m) \in Y_o\} = 1$. By the Hahn-Banach theorem there exist

$$\{y_i^*\}_{i=1}^{\infty} \subseteq Y^* \ , \ \|y_i^*\| = 1 \ ,$$

$$\{y \in Y_o : \|y\| > 1\} \subseteq \bigcup_{i=1}^{\infty} \{y : |y : |y_i^*(y)| > 1\} \ .$$

Since $\{m : |y_i^*(\tau_o(m))| > 1\} \subseteq \{m : |y_i^*(\tau_o(m) - \tau(m))| > 0\}$ we have that

$$\{m : \tau_o(m) \notin Y_o \ \text{ or } \ \|\tau_o(m)\| > 1\}$$

has μ-measure zero. That is

$$\mu\{\tau_o(m)\| \leq 1\} = 1 \ .$$

In lemma on II.5, change "the following are equivalent" to "(i) implies (ii)".

In line on II.5 beginning "For convenience..." change "(i) to "(ii)".

In last line on II.5 change sentence "So T is μ-strongly measurable" to "So there exists $\sigma : K \to Y$ Bochner integrable, $\|\sigma(k)\| \leq \|T\|$ a.e., and $y^* \circ \sigma = T^* y^*$ μ - a.e. on K."

In the first line of II.6 change

$$\text{"}Vf = \int_{k \in K} f(k)T(k)\,d\mu(k)\text{"}$$

to

$$\text{"}Vf = \int_{k \in K} f(k)\sigma(k)\,d\mu(k)\text{"}.$$

In the third line of II.6 change

$$" \int f(k) y^*(T(k)) d\mu(k)"$$

to

$$"\int f(k) y^*(\sigma(k)) d\mu(k)"$$

In line 12 from bottom of II.6 change the sentence beginning "That is..." to "That is, there exists $\sigma : K \to L_1(S,\nu)$ that is Bochner integrable, $\|\sigma(k)\| \leq \|U\|$, for all $\psi \in L_\infty(S,\nu)$, $\psi \circ \sigma = U^* \psi$, and $UR^* f = \int f(k)\sigma(k) d\mu$ for all $f \in L_1(K,\mu)$.

In lines 10 and 9 from bottom of II.6 change "U" to "σ" .

In line 3 from bottom of II.6 change "U(k)" to σ(k)".