

Lie Ideals and Jordan Triple Derivations in Rings

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ABSTRACT - In this paper we prove that on a 2-torsion free semiprime ring R every Jordan triple (resp. generalized Jordan triple) derivation on Lie ideal L is a derivation on L (resp. generalized derivation on L).

1. Introduction.

This paper is motivated by the work of Jing and Lu [13]. Throughout the present paper, R will denote an associative ring with center $Z(R)$. A ring R is said to be 2-torsion free, if whenever $2x = 0$ with $x \in R$, implies $x = 0$. Recall that R is prime if for any $a, b \in R$, $aRb = \{0\}$ implies that $a = 0$ or $b = 0$, and R is semiprime if for all $a \in R$, $aRa = \{0\}$ implies $a = 0$. An additive subgroup L of R is said to be a Lie ideal of R if $[L, R] \subseteq L$. A Lie ideal L is said to be square-closed if $a^2 \in L$ for all $a \in L$. An additive mapping $\delta : R \rightarrow R$ is called a derivation (resp. a Jordan derivation) if $(xy)^\delta = x^\delta y + xy^\delta$ (resp. $(x^2)^\delta = x^\delta x + xx^\delta$) holds for all $x, y \in R$. A famous result due to Herstein [11, Theorem 3.3] states that a Jordan derivation of a prime ring of characteristic not equal to 2 must be a derivation. A brief proof of Herstein's result can also be found in [8]. This result was extended to 2-torsion free semiprime rings by Cusack [10] and subsequently, by Bresar [7].

Following [6], an additive mapping $\delta : R \rightarrow R$ is called a Jordan triple derivation if $(xyx)^\delta = x^\delta yx + xy^\delta x + xyx^\delta$ holds for all $x, y \in R$. One can

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easily prove that any Jordan derivation on an arbitrary 2-torsion free ring is a Jordan triple derivation (see for example [8] where further references can be found). Bresar has proved the following result.

THEOREM 1.1 (6, Theorem 4.3). *Let R be a 2-torsion free semiprime ring and let $\delta : R \rightarrow R$ be a Jordan triple derivation. In this case δ is a derivation.*

An additive mapping $F : R \rightarrow R$ is said to be a generalized derivation (resp. a generalized Jordan derivation) on R if there exists a derivation $\delta : R \rightarrow R$ such that $(xy)^F = x^F y + xy^\delta$ (resp. $(x^2)^F = x^F x + xx^\delta$) holds for all $x, y \in R$. An additive mapping $F : R \rightarrow R$ is said to be a generalized Jordan triple derivation on R if there exists a Jordan triple derivation $\delta : R \rightarrow R$ such that $(xyx)^F = x^F yx + xy^\delta x + xyx^\delta$ holds for all $x, y \in R$. Motivated by the above result Jing and Lu have proved the following generalization of Theorem 1.1.

THEOREM 1.2 (13, Theorem 3.5). *Let R be a 2-torsion free prime ring, then every generalized Jordan triple derivation on R is a generalized derivation.*

Very recently, Vukman [18] extended the result mentioned above for 2-torsion free semiprime rings. In the present paper, our objective is to generalize Theorem 1.1 and 1.2 on Lie ideal of R .

2. Jordan Triple Derivations.

It is obvious to see that every derivation is a Jordan triple derivation, but the converse need not to be true in general. Motivated by the Theorem 1.1, in the present section, it is shown that on a 2-torsion free semiprime ring R every Jordan triple derivation on Lie ideal L is a derivation on L . More precisely, we prove the following:

THEOREM 2.1. *Let R be a 2-torsion free semiprime ring and $L \not\subseteq Z(R)$ be a square-closed Lie ideal of R . If an additive mapping $\delta : R \rightarrow R$ satisfies*

$$(aba)^\delta = a^\delta ba + ab^\delta a + aba^\delta \text{ for all } a, b \in L$$

and $L^\delta \subseteq L$, then δ is a derivation on L .

To facilitate our discussion, we shall begin with the following lemmas:

LEMMA 2.1 (4, Lemma 4). *Let $L \not\subseteq Z(R)$ be a Lie ideal of a 2-torsion free prime ring R and $a, b \in R$. If $aLb = \{0\}$, then $a = 0$ or $b = 0$.*

LEMMA 2.2 (17, Lemma 2.4). *Let R be a 2-torsion free semiprime ring, L be a Lie ideal of R such that $L \not\subseteq Z(R)$ and let $a \in L$. If $aLa = \{0\}$, then $a^2 = 0$ and there exists a nonzero ideal $K = R[L, L]R$ of R generated by $[L, L]$ such that $[K, R] \subseteq L$ and $Ka = aK = \{0\}$.*

COROLLARY 2.1. *Let R be a 2-torsion free semiprime ring, L be a Lie ideal of R such that $L \not\subseteq Z(R)$ and let $a, b \in L$.*

- (1) *If $aLa = \{0\}$, then $a = 0$.*
- (2) *If $aL = \{0\}$ (or $La = \{0\}$), then $a = 0$.*
- (3) *If L is square-closed, and $aLb = \{0\}$, then $ab = 0$ and $ba = 0$.*

PROOF. (1) Since $Ka = R[L, L]Ra = \{0\}$ and $a^2 = 0$ by Lemma 2.2, we have $0 = [[a, x], a]ya = axaya$ for all $x, y \in R$, and so, we have $axayaxa = 0$. Since R is semiprime, we get $axa = 0$ for all $x \in R$. Moreover, by semiprimeness of R , we get $a = 0$.

(2) It is clear by (1).

(3) If $aLb = \{0\}$, then we have $baLba = \{0\}$ obviously, and so we have $ba = 0$ by (1). Moreover, since $abLab \subseteq aLb = \{0\}$, we get $ab = 0$.

LEMMA 2.3 (17, Lemma 2.5). *Let R be a 2-torsion free ring, L be a Lie ideal of R and let $a, b \in L$. If $aub + bua = 0$ for all $u \in L$, then $aubLaub = \{0\}$.*

LEMMA 2.4 (17, Lemma 2.7). *Let G_1, G_2, \dots, G_n be additive groups, R a 2-torsion free semiprime ring and $L \not\subseteq Z(R)$ be a Lie ideal of R . Suppose that mappings $S : G_1 \times G_2 \times \dots \times G_n \rightarrow R$ and $T : G_1 \times G_2 \times \dots \times G_n \rightarrow R$ are additive in each argument. If $S(a_1, a_2, \dots, a_n)xT(a_1, a_2, \dots, a_n) = 0$ for all $x \in L$, $a_i \in G_i$ $i = 1, 2, \dots, n$, then $S(a_1, a_2, \dots, a_n)xT(b_1, b_2, \dots, b_n) = 0$ for all $x \in L$, $a_i, b_i \in G_i$ $i = 1, 2, \dots, n$.*

LEMMA 2.5. *Let δ be a Jordan triple derivation and L be a Lie ideal of R . For arbitrary $a, b, c \in L$, we have*

$$(abc + cba)^\delta = a^\delta bc + ab^\delta c + abc^\delta + c^\delta ba + cb^\delta a + cba^\delta.$$

PROOF. We have

$$(2.1) \quad (aba)^\delta = a^\delta ba + ab^\delta a + aba^\delta \text{ for all } a, b \in L.$$

We compute $W = \{(a + c)b(a + c)\}^\delta$ in two different ways. On one hand, we find that $W = (a + c)^\delta b(a + c) + (a + c)b^\delta(a + c) + (a + c)b(a + c)^\delta$, and on the other hand $W = (aba)^\delta + (abc + cba)^\delta + (cbc)^\delta$. Comparing two expressions, we obtain the required result.

REMARK 2.1. It is easy to see that every Jordan derivation of a 2-torsion free ring satisfies (2.1) (see [1] for reference).

For the purpose of this section, we shall write; $\Delta(abc) = (abc)^\delta - a^\delta bc - ab^\delta c - abc^\delta$, and $\Delta(abc) = abc - cba$. We list a few elementary properties of δ and Δ :

- (i) $\Delta(abc) + \Delta(cba) = 0$
- (ii) $\Delta((a + b)cd) = \Delta(acd) + \Delta(bcd)$ and $\Delta((a + b)cd) = \Delta(acd) + \Delta(bcd)$
- (iii) $\Delta(a(b + c)d) = \Delta(abd) + \Delta(acd)$ and $\Delta(a(b + c)d) = \Delta(abd) + \Delta(acd)$
- (iv) $\Delta(ab(c + d)) = \Delta(abc) + \Delta(abd)$ and $\Delta(ab(c + d)) = \Delta(abc) + \Delta(abd)$

PROPOSITION 2.2. *Let R be a 2-torsion free semiprime ring and $L \not\subseteq Z(R)$ be a square closed Lie ideal of R . If $\Delta(abc) = 0$ holds for all $a, b, c \in L$, then δ is a derivation of L .*

PROOF. Let assume $\Delta(abc) = 0$ for all $a, b, c \in L$, that is,

$$(abc)^\delta = a^\delta bc + ab^\delta c + abc^\delta.$$

Let $M = abxab$. We have

$$(2.2) \quad \begin{aligned} M^\delta &= \{a(bxa)b\}^\delta = (a)^\delta(bxa)b + ab^\delta xab + abx^\delta ab \\ &+ (abx)a^\delta b + (abxa)b^\delta \text{ for all } x, a, b \in L. \end{aligned}$$

On the other hand,

$$(2.3) \quad M^\delta = \{(ab)x(ab)\}^\delta = (ab)^\delta(xab) + (ab)x^\delta(ab) + (abx)(ab)^\delta.$$

Comparing (2.3) with (2.2) we get

$$\{(ab)^\delta - a^\delta b - ab^\delta\}(xab) + (abx)\{(ab)^\delta - a^\delta b - ab^\delta\} = 0$$

that is, $a^b xab + abxa^b = 0$, where a^b stands for $(ab)^\delta - a^\delta b - ab^\delta$. Thus, by Lemma 2.3 we find that $(a^b xab)y(a^b xab) = 0$ for all $a, b, x, y \in L$. Now, by

Corollary 2.1, we obtain $a^b xab = 0$ for all $a, b, x \in L$. Now by Lemma 2.4, we obtain $a^b xcd = 0$ for all $a, b, c, d, x \in L$. Hence, by using Corollary 2.1, we get $a^b = 0$ for all $a, b \in L$ that is, δ is a derivation on L .

LEMMA 2.6. *Let L be a Lie ideal of R . For any $a, b, c, x \in L$, we have*

$$\Delta(abc)x\Delta(abc) + \Delta(abc)x\Delta(abc) = 0.$$

PROOF. For any $a, b, c, x \in L$, suppose that $N = abcxba + cbaabc$. Now we find

$$\begin{aligned} N^\delta &= \{a(bcxcb)a + c(baxab)c\}^\delta = \{a(bcxcb)a\}^\delta + \{c(baxab)c\}^\delta \\ &= a^\delta bcxcb + ab^\delta cxcba + abc^\delta xcba \\ &\quad + abcx^\delta cba + abcx^\delta cba + abcx^\delta cba \\ &\quad + abcx^\delta cba + c^\delta baxabc + cb^\delta axabc \\ &\quad + cba^\delta xabc + cba^\delta xabc + cba^\delta xabc \\ &\quad + cba^\delta xabc + cba^\delta xabc + cba^\delta xabc. \end{aligned}$$

On the other hand, we have

$$\begin{aligned} N^\delta &= \{(abc)x(cba) + (cba)x(abc)\}^\delta \\ &= (abc)^\delta x(cba) + (abc)x^\delta(cba) + (abcx)(cba)^\delta \\ &\quad + (cba)^\delta(xabc) + (cba)x^\delta(abc) + (cbax)(abc)^\delta. \end{aligned}$$

By comparing last two expressions, we get

$$-\Delta(cba)(xcba) + \Delta(cba)(xabc) + (abcx)\Delta(cba) - (cbax)\Delta(cba) = 0.$$

This implies that $\Delta(abc)x\Delta(abc) + \Delta(abc)x\Delta(abc) = 0$ for all $a, b, c \in L$.

LEMMA 2.7. *Let R be a semiprime ring and $L \not\subseteq Z(R)$ be a square closed Lie ideal of R . Then $\Delta(abc)x\Delta(rst) = 0$ holds for all $a, b, c, r, s, t, x \in L$.*

PROOF. By Lemma 2.6 we have $\Delta(abc)x\Delta(abc) + \Delta(abc)x\Delta(abc) = 0$ for all $a, b, c, x \in L$. Thus, we get $\Delta(abc)x\Delta(abc)L\Delta(abc)x\Delta(abc) = \{0\}$ by Lemma 2.3, and hence we obtain $\Delta(abc)x\Delta(abc) = 0$, for all $a, b, c, x \in L$ by Corollary 2.1. Now, we find that $\Delta(abc)x\Delta(rst) = 0$, for all $a, b, c, r, s, t, x \in L$ by Lemma 2.4.

LEMMA 2.8. *Let R be a 2-torsion free semiprime ring and L be a square closed Lie ideal of R . If $\Delta(abc) = 0$ for all $a, b, c \in L$, then $L \subseteq Z(R)$.*

PROOF. Assume that $L \not\subseteq Z(R)$. We have $\Delta(abc) = 0$ for all $a, b, c \in L$, that is, $abc = cba$. Replacing b by $2tb$, we get $2atbc = 2ctba$ for all

$a, b, c, t \in L$. Again replacing t by $2tw$ and using the fact that R is 2-torsion free, we get $atwbc = ctwba$ and hence $a(twb)c = a(bwt)c = a(bc)tw = awtbc$. Thus, we find that $a[t, w]bc = 0$ for all $a, b, c, t, w \in L$. By Corollary 2.1, we get $[t, w] = 0$ for all $t, w \in L$, that is, L is a commutative Lie ideal of R . And so, we have $[a, [a, t]] = 0$ for all $t \in R$ and hence by Sublemma on page 5 of [11], $a \in Z(R)$. Hence $L \subseteq Z(R)$, a contradiction. This completes the proof of the Lemma.

PROOF OF THEOREM 2.1. Suppose $\delta : R \rightarrow L$ is a Jordan triple derivation on L . Our goal will be to show that δ is a derivation of associative triple systems. We know that $\Delta(abc) = -\Delta(cba)$. Hence

$$\begin{aligned} 2\Delta(abc)x\Delta(abc) &= \Delta(abc)x\{\Delta(abc) - \Delta(cba)\} \\ &= \Delta(abc)x\{\Delta(abc)^\delta + \Delta(c^\delta ba) \\ &\quad + \Delta(cb^\delta a) + \Delta(cba^\delta)\}. \end{aligned}$$

By Lemma 2.7, the above relation reduces to

$$(2.4) \quad 2\Delta(abc)x\Delta(abc) = \Delta(abc)x\Delta(abc)^\delta$$

Similarly we obtain

$$(2.5) \quad 2\Delta(abc)x\Delta(abc) = \Delta(abc)^\delta x\Delta(abc)$$

Now we have

$$\begin{aligned} 0 &= \{\Delta(abc)x\Delta(abc) + \Delta(abc)x\Delta(abc)\}^\delta \\ &= \Delta(abc)^\delta x\Delta(abc) + \Delta(abc)x^\delta \Delta(abc) \\ &\quad + \Delta(abc)x\Delta(abc)^\delta + \Delta(abc)^\delta x\Delta(abc) \\ &\quad + \Delta(abc)x^\delta \Delta(abc) + \Delta(abc)x\Delta(abc)^\delta \end{aligned}$$

and according to Lemma 2.6 and (2.4), and (2.5), we get

$$0 = 4\Delta(abc)x\Delta(abc) + \Delta(abc)^\delta x\Delta(abc) + \Delta(abc)x\Delta(abc)^\delta.$$

We multiply the relation above from left by $\Delta(abc)x\Delta(abc)y$ for all $a, b, c, x, y \in L$ and by Lemma 2.7 we obtain $4\Delta(abc)x\Delta(abc)y\Delta(abc)x\Delta(abc) = 0$ for all $a, b, c, x, y \in L$. Since R is a 2-torsion free, it follows that $\Delta(abc)x\Delta(abc)y\Delta(abc)x\Delta(abc) = 0$, that is, $\Delta(abc)x\Delta(abc)L\Delta(abc)x\Delta(abc) = \{0\}$. By Corollary 2.1, we find that $\Delta(abc)x\Delta(abc) = 0$ and again using Corollary 2.1, we find that $\Delta(abc) = 0$ for all $a, b, c \in L$ and hence by Proposition 2.1, we get the required result.

THEOREM 2.2. *Let R be a 2-torsion free prime ring and $L \not\subseteq Z(R)$ be a nonzero square-closed Lie ideal of R . If an additive mapping $\delta : R \rightarrow L$ satisfies that*

$$(aba)^\delta = a^\delta ba + ab^\delta a + aba^\delta \text{ for all } a, b \in L,$$

then δ is a derivation on L .

PROOF. By Lemma 2.7, we have $\Delta(abc)x\Delta(rst) = 0$ for all $a, b, c, r, s, t, x \in L$, that is, $\Delta(abc)L\Delta(rst) = \{0\}$. Now, either $\Delta(abc) = 0$ or $\Delta(rst) = 0$ by Lemma 2.1. If $\Delta(rst) = 0$ for all $r, s, t \in L$, then we get $L \subseteq Z(R)$ by Lemma 2.8, a contradiction. On the other hand if $\Delta(abc) = 0$ for all $a, b, c \in L$, then we get the required result by Proposition 2.1.

3. Generalized Jordan Triple Derivations.

An additive mapping $\mu : R \rightarrow R$ is called a Jordan triple left centralizer on L if $(aba)^\mu = a^\mu ba$ for all $a, b \in L$, and called a Jordan left centralizer on L if $(a^2)^\mu = a^\mu a$ for all $a \in L$.

THEOREM 3.1. *Let R be a 2-torsion free semiprime ring and $L \not\subseteq Z(R)$ be a square-closed Lie ideal. If $\mu : R \rightarrow R$ is a Jordan triple left centralizer on L , then μ is a Jordan left centralizer on L .*

PROOF. By the hypothesis, we have

$$(3.1) \quad (aba)^\mu = a^\mu ba \text{ for all } a, b \in L.$$

Replacing a by $a + c$ in (3.1), we find that

$$\{(a + c)b(a + c)\}^\mu = a^\mu ba + c^\mu ba + a^\mu bc + c^\mu bc \text{ for all } a, b, c \in L.$$

On the other hand, we obtain

$$\{(a + c)b(a + c)\}^\mu = (abc + cba)^\mu + a^\mu bc + c^\mu ba \text{ for all } a, b, c \in L.$$

Combining last two expressions, we get

$$(3.2) \quad (abc + cba)^\mu = a^\mu bc + c^\mu ba \text{ for all } a, b, c \in L.$$

Now replacing c by a^2 in (3.2), we get

$$(3.3) \quad (aba^2 + a^2ba)^\mu = a^\mu ba^2 + (a^2)^\mu ba.$$

Again replacing b by $ab + ba$ in (3.1), we get

$$(3.4) \quad (aba^2 + a^2ba)^\mu = a^\mu aba + a^\mu ba^2.$$

Combining (3.3) and (3.4), we obtain

$$\{(a^2)^\mu - a^\mu a\}ba = 0 \text{ for all } a, b \in L.$$

By setting $\Omega^a = (a^2)^\mu - a^\mu a$, we have

$$(3.5) \quad \Omega^a ba = 0 \text{ for all } a, b \in L.$$

Thus, by Corollary 2.1, we obtain

$$(3.6) \quad \Omega^a a = 0 = a\Omega^a \text{ for all } a \in L.$$

Linearizing (3.6), we get

$$(3.7) \quad \Omega^{a+b}a + \Omega^{a+b}b = 0 \text{ for all } a, b \in L.$$

Now, we compute $\Omega^{a+b} = \{(ab + ba)^\mu - a^\mu b - b^\mu a\} + (a^2)^\mu - a^\mu a + (b^2)^\mu - b^\mu b$ and hence we have

$$(3.8) \quad \Omega^{a+b} = \Phi(a, b) + a^\mu + b^\mu \text{ for all } a, b \in L,$$

where $\Phi(a, b) = (ab + ba)^\mu - a^\mu b - b^\mu a$. Thus, in view of (3.8), the expression (3.7) implies that

$$(3.9) \quad \Omega^a b + \Phi(a, b)a + \Omega^b a + \Phi(a, b)b = 0.$$

Again, replacing a by $-a$ in (3.9), we get

$$(3.10) \quad \Omega^a b + \Phi(a, b)a - \Omega^b a - \Phi(a, b)b = 0.$$

Adding (3.9) with (3.10), we find that

$$(3.11) \quad \Omega^a b + \Phi(a, b)a = 0.$$

On right multiplication of (3.11) by Ω^a , we obtain

$$0 = \Omega^a b\Omega^a + \Phi(a, b)a\Omega^a = \Omega^a b\Omega^a.$$

Thus, we obtain $\Omega^a = 0$ for all $a \in L$ by Corollary 2.1. This gives that μ is a Jordan left centralizer.

THEOREM 3.2. *Let R be a 2-torsion free semiprime ring and $L \not\subseteq Z(R)$ be a square-closed Lie ideal. If $F : R \rightarrow R$ is a generalized Jordan triple derivation on L with a Jordan triple derivation δ such that $L^\delta \subseteq L$, then F is a generalized Jordan derivation on L .*

PROOF. Since F is a generalized Jordan triple derivation on L . Therefore we have

$$(3.12) \quad (aba)^F = a^F ba + ab^\delta a + aba^\delta \text{ for all } a, b \in L.$$

In (3.12), we take δ as a Jordan triple derivation on L . Since R is a 2-torsion free semiprime ring, so in view of Theorem 2.1 δ is a derivation on L . Now we write $\Gamma = F - \delta$. Then we have

$$\begin{aligned} (aba)^\Gamma &= (aba)^{F-\delta} \\ &= (aba)^F - (aba)^\delta \\ &= (a^F - a^\delta)ba = a^{F-\delta}ba = a^\Gamma ba \text{ for all } a, b \in L. \end{aligned}$$

And so, we have $(aba)^\Gamma = a^\Gamma ba$ for all $a, b \in L$. In other words, Γ is a Jordan triple left centralizer on L . Since R is a 2-torsion free semiprime ring, one can conclude that Γ is a Jordan left centralizer by Theorem 3.1. Hence F is of the form $F = \Gamma + \delta$, where δ is a derivation and Γ is a Jordan left centralizer on L . Hence F is a generalized Jordan derivation on L .

THEOREM 3.3. *Let R be a 2-torsion free prime ring and $L \not\subseteq Z(R)$ be a square-closed Lie ideal. If $F : R \rightarrow R$ is a generalized Jordan triple derivation on L , then F is a generalized derivation on L .*

PROOF. By Theorem 3.2 and Theorem of [2] we get the required result.

In conclusion, it is tempting to conjecture as follows:

CONJECTURE 3.1. *Let R be a 2-torsion free semiprime ring and $L \not\subseteq Z(R)$ be a Lie ideal of R . If $F : R \rightarrow R$ is a generalized Jordan triple derivation on L , then F is a generalized derivation on L .*

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