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# Addendum to «Prime graph components of finite almost simple groups».

MARIA SILVIA LUCIDO (*)

This is an addendum to the paper [2]. We refer to that paper for the definitions. We denote by $\Gamma(G)$ the prime graph of a finite group $G$.

The following groups have to be added to the list of the almost simple groups in which the prime graph is not connected in [2].

PROPOSITION 1. *Let $G$ be one of the following almost simple groups:*

$S < G \leqslant \mathrm{Aut}(S)$ *and $G$ does not split over $S$, with $S$ a simple non abelian group such that $\Gamma(S)$ is not connected;*

$PGL(2, q)$, *with $q = p^n$, $p$ an odd prime;*

$G = Sz(q)\langle\alpha\rangle$ *where $q = 2^f$, $f$ is an odd prime number and $\alpha$ is a field automorphism of order $f$;*

$G = Ree(q)\langle\alpha\rangle$ *where $q = 3^f$, $f$ is an odd prime number and $\alpha$ is a field automorphism of order $f$.*

$PSL(3, 4) = S < G < \mathrm{Aut}(S)$ *and* $|G/S| = 2$.

*Then $\Gamma(G)$ is not connected.*

PROOF. Let $G$ be an almost simple group $S < G \leqslant \mathrm{Aut}(S)$ such that $G$ does not split, we prove that $\Gamma(G) = \Gamma(S)$. As at the beginning of the proof of the case of Lie groups in [2], we can suppose that $|G/S| = p$. Let $x$ be an element of $G \setminus S$, then $x^p \in S$ but $x^p \neq 1$. Moreover we can suppose that $x$ is a $p$-element. Then $C_S(x) \leqslant C_S(x^p)$, therefore for any prime $r \in \pi(G)$ we have that $r \sim p$ in $\Gamma(G)$ if and only if $r \sim p$ in $\Gamma(S)$.

If $G = PGL(2, q)$ with $q = p^f$, $p$ an odd prime, then $\pi_1(G) = \pi(q^2 - 1)$ and $\pi_2(G) = \{p\}$.

Let $G = Sz(q)\langle\alpha\rangle$ where $q = 2^f$, $f$ is an odd prime number and $\alpha$ is a field automorphism of order $f$. If $S = Sz(q) = {}^2B_2(q)$, then the connected

(*) Indirizzo dell'A.: Dipartimento di Matematica e Informatica, Università di Udine, via delle Scienze 200, I-33100 Udine, Italy; e-mail: lucido@dimi.uniud.it

components of $\Gamma(S)$ not containing 2 are

$$\pi_2(S) = \pi(q-1), \quad \pi_3(S) = \pi(q-\sqrt{2q}+1), \quad \pi_4(S) = \pi(q+\sqrt{2q}+1).$$

If $\alpha$ is a field automorphism, then $\pi(C_S(\alpha)) = \pi(|^2B_2(2)|) = \{2, 5\}$ (see [1], Theorem 9.1). We observe that 5 divides $2^{2f}+1$. It can be proved that, if $f \equiv 1, 7$ (8), 5 divides $(q+\sqrt{2q}+1)$; or, if $f \equiv 3, 5$ (8), then 5 divides $(q-\sqrt{2q}+1)$. Therefore

$$\pi_1(G) = \{2, f\} \cup \pi_4(S) \quad \pi_2(G) = \pi_2(S) \quad \pi_3(G) = \pi_3(S) \quad \textit{if } f \equiv 1, 7 \ (8),$$

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Let $G = Ree(q)\langle\alpha\rangle$ where $q = 3^f$, $f$ is an odd prime number and $\gamma$ is a field automorphism of order $f$. If $S = Ree(q) = {}^2G_2(\overline{q}^2)$, then the connected components of $\Gamma(S)$ not containing 2 are

$$\pi_2(S) = \pi(q-\sqrt{3q}+1), \qquad \pi_3(S) = \pi(q+\sqrt{3q}+1).$$

If $\alpha$ is a field automorphism, then $\pi(C_S(\alpha)) = \pi(|^2G_2(3)|) = \{2, 3, 7\}$ (see [1], Theorem 9.1). We observe that 7 divides $3^{3f}+1$. It can be proved that, if $f \equiv 1, 11$ (12), 7 divides $(q+\sqrt{3q}+1)$; or, if $f \equiv 5, 7$ (12), then 7 divides $(q-\sqrt{3q}+1)$. Therefore

$$\pi_1(G) = \pi(q(q^2-1)f) \cup \pi_3(S) \qquad \pi_2(G) = \pi_2(S) \quad \textit{if } f \equiv 1, 11 \ (12),$$

$$\pi_1(G) = \pi(q(q^2-1)f) \cup \pi_2(S) \qquad \pi_2(G) = \pi_3(S) \quad \textit{if } f \equiv 5, 7 \ (12).$$

If $S = PSL(3, 4)$ and $G \leq \mathrm{Aut}(S)$ with $|G/S| = 2$, then we have the following three possibilities:

$G = S.2_1$, extension with a graph-field automorphism, then $\pi_1(G) = \{2, 3\}$, $\pi_2(G) = \{5\}$ and $\pi_3(G) = \{7\}$.

$G = S.2_2$, extension with a field automorphism, then $\pi_1(G) = \{2, 3, 7\}$, $\pi_2(G) = \{5\}$.

$G = S.2_3$, extension with a graph automorphism, then $\pi_1(G) = \{2, 3, 5\}$, $\pi_2(G) = \{7\}$.

## REFERENCES

[1] D. Gorenstein - R. Lyons, *The local structure of finite groups of characteristic 2 type*, Memoirs of the American Mathematical Society **276**, March 1983 (vol. 42).

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